ENGINEERING MATHEMATICS

ALL BRANCHES

Differential Equations

Partial Differential Equations, 1D & 2D heat equation & Laplace equation, Cauchy's & Legendre's homogenous LDE, Variation of Parameters

DPP-04 Solution

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Question 1

The general solution of $\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 10\cos x$ is

A $y = c_1 e^{-x} + c_2 e^{2x} - 3\cos x - \sin x$

B $y = c_1 e^{x} + c_2 e^{2x} - 3\cos x$

C $y = c_1^{-x} + c_2 e^{2x} - 3x + \sin x$

D $y = c_1 e^{x} + c_2 e^{-2x} - 3\cos x - \sin x$

$$m^2 - m - 2 = 0$$
$$(m-2)(m+1) = 0$$
$$m = -1, 2$$

$$P.I. = \frac{1}{D^2 - D - 2} \cdot 10\cos x$$

$D^2 \to -a^2$

$$= \frac{1}{-1^2 - D - 2} 10\cos x$$

$$= -10 \frac{1}{D+3} \frac{(D-3)}{(D-3)} \cos x$$

$$-10\,\frac{(D-3)}{D^2-9}\cos x$$

$$\cancel{-10}\,\frac{(D-3)}{\cancel{-1^2-9}}\cos x = -\sin x - 3\cos x$$

$$y = C.F. + P.I. = C_1 e^{-x} + C_2 e^{2x} - \sin x - 3\cos x$$

Question 2

The solution of the equation $x^2\frac{d^2y}{dx^2}-3x\frac{dy}{dx}+4y=0$ is

$D \to d/dx$

$D' \to d/dz$

$z = \log x$

$(x^2D^2-3xD+4)y=0$

$(D'(D'-1)-3D'+4)y=0$

$D'^2-D'-3D'+4=0$

$m^2-4m+4=0$

$(m-2)^2=0 \qquad m=2,2$

$y=(c_1+c_2z)e^{2z}$

$(c_1+c_2\log x)e^{2\log_e x}$

$y=(c_1+c_2\log x)x^2$

A. $y=(c_1+c_2x)e^{2x}$

B. $y=(c_1+c_2x)e^{x}$

C. $y=(c_1+c_2x)\log x$

D. $y=(c_1+c_2\log x)x^2$ ✓

Question 3

The solution of the equation $xp + 2y = pxy, \left(p = \frac{dy}{dx}\right)$ is

A $xy^2 = Ae^y$

B $xy^2 = Ae^x$

C $x^2y = Ae^y$

D $xy = Ae^y$

$$xp(1-y) + 2y = 0$$

$$x\frac{dy}{dx}(1-y) + 2y = 0$$

$$\int\frac{(1-y)}{y}dy + 2\int\frac{dx}{x} = 0$$

$$\log y - y + 2\log x = \log A$$

$$\log\frac{yx^2}{A} = y$$

$$x^2y = Ae^y$$

Question 4

If $y = x$ is a solution of $x^2y'' + xy' - y = 0$, then the second linearly independent solution of the above equation is

A $\frac{1}{x}$

B $\frac{1}{x^2}$

C x^2

D x^n

$(x^2D^2 + xD - 1)y = 0$

$(D'(D'-1) + D' - 1)y = 0$

$D'^2 - 1 = 0$ (A.E.)

$D' = \pm 1$

$y = c_1 e^z + c_2 e^{-z}$

$= c_1 e^{\log x} + c_2 e^{-\log x}$

$y = c_1 x + \frac{c_2}{x}$

$x, \frac{1}{x}$

$c_1 x + c_2 \frac{1}{x}$

Question 5

The family of conic represented by the solution of the DE $\underline{(4x+3y+1)}dx + \underline{(3x+2y+1)}dy = 0$ is

A Circles

B Parabolas

C Hyperbolas

D Ellipses

$\frac{\partial M}{\partial y} = 3 \qquad \frac{\partial N}{\partial x} = 3 \qquad$ Exact

$\int 4x+3y+1 \, dx + \int (2y+1) dy$

$4\frac{x^2}{2} + 3yx + x + 2\frac{y^2}{2} + y = c$

$2x^2 + 3xy + y^2 + x + y = c$

$ax^2 + 2hxy + by^2 + \dots = c$

$a = 2, b = 1, h = 3/2$

$h^2 - ab = (3/2)^2 - 2.1 = 1/4 > 0$ Hyperbola

Question 6

$D = 2 - 6t$

Consider the following second-order differential equation: $D^2 = -6$

$$y'' - 4y' + 3y = 2t - 3t^2$$

The particular solution of the differential equation is

A. $-2 - 2t - t^2$

B. $-2t - t^2$

C. $2t - 3t^2$

D. $-2 - 2t - 3t^2$

$$P.I. = \frac{1}{D^2 - 4D + 3} \cdot (2t - 3t^2) = \frac{1}{3\left(1 + \frac{D^2}{3} - \frac{4D}{3}\right)} \cdot (2t - 3t^2)$$

$$\frac{1}{3}\left(1 + \overline{\frac{D^2}{3} - \frac{4D}{3}}\right)^{-1}(2t - 3t^2)$$

$$\frac{1}{3}\left[1 - \left(\frac{D^2}{3} - \frac{4D}{3}\right) + \left(\frac{D^2}{3} - \frac{4D}{3}\right)^2 - \cdots\right](2t - 3t^2)$$

$$\frac{1}{3}\left[1 - D^2/3 + 4D/3 + \frac{16}{9}D^2\right](2t - 3t^2)$$

$$\frac{1}{3}\left[2t - 3t^2 + \frac{13}{9}(-6) + \frac{4}{3}(2 - 6t)\right] = -2 - 2t - t^2$$

Question 7

The differential equation $\frac{d^2y}{dx^2} + 16y = 0$ for $y(x)$ with the two boundary conditions $\frac{dy}{dx}|x = 0 \Rightarrow 1$ and $\frac{dy}{dx}_{x=\frac{\pi}{2}} \Rightarrow -1$ has

A. No solution

B. Exactly two solutions

C. Exactly one solution

D. Infinitely many solutions

$m^2 + 16 = 0$

$(m + 4i)(m - 4i) = 0$

$m = 0 \pm 4i$

$y = e^{0x}[C_1 \cos 4x + C_2 \sin 4x]$

$y' = -4C_1 \sin 4x + 4C_2 \cos 4x$

(1) $1 = 4C_2 \Rightarrow \boxed{C_2 = 1/4}$

(2) $-1 = -4C_1 \sin 2\pi + 4C_2 \cos 2\pi$

$\Rightarrow \boxed{C_2 = -1/4}$

We are not sure about C_2 Hence no soln.

Question 8

Consider the differential equation $3y''(x) + 27y(x) = 0$ with initial conditions $y(0) = 0$ and $y'(0) = 2000$. The value of y at $x = 1$ is ____.

$$3m^2 + 27 = 0$$
$$m^2 + 9 = 0$$
$$m = 0 \pm 3i$$

$$y = e^{0x}\left[C_1 \cos 3x + C_2 \sin 3x\right]$$

(1) $0 = 1\left[C_1(1) + C_2(0)\right]$

$$\boxed{C_1 = 0}$$

$$y' = -3C_1 \sin 3x + 3C_2 \cos 3x$$

$$2000 = 3C_2 \qquad \boxed{C_2 = 2000/3}$$

$$y = \frac{2000}{3} \sin 3x$$

$$y(1) = \frac{2000}{3} \sin(3)$$
$$= 94.08$$

Question 9

The general solution of the differential equation

$$\frac{d^4y}{dx^4} - 2\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = 0$$

A $y = (c_1 - c_2x)e^x + c_3\cos x + c_4\sin x$

B $y = (c_1 + c_2x)e^x - c_3\cos x + c_4\sin x$

C $y = (c_1 + c_2x)e^x + c_3\cos x + c_4\sin x$

D $y = (c_1 + c_2x)e^x + c_3\cos x - c_4\sin x$

$$m^4 - 2m^3 + 2m^2 - 2m + 1 = 0$$

$$(m-1)(m^3 - m^2 + m - 1) = 0$$

$$(m-1)(m-1)(m^2+1) = 0$$

$$m = 1, 1, 0 \pm i$$

$$y = (c_1 + c_2 x)e^{x} + e^{0x}[c_3 \cos x + c_4 \sin x]$$

Question 10

The solution of the differential equation

$\frac{d^2y}{dx^2} - \frac{dy}{dx} - 2y = 3e^{2x}$, where, $y(0) = 0$ and $y'(0) = -2$ is

A $y = e^{-x} - e^{2x} + xe^{2x}$

B $y = e^{x} - e^{-2x} - xe^{2x}$

C $y = e^{-x} + e^{2x} + xe^{2x}$

D $y = e^{x} - e^{-2x} + xe^{2x}$

$m^2 - m - 2 = 0$

$m = -1, 2$

$D \to a$

$y = C_1 e^{-x} + C_2 e^{2x}$

$P.I. = \frac{1}{D^2 - D - 2} \cdot 3e^{2x} = \frac{1}{\cancel{2^2 - 2 - 2}} \cdot 3e^{2x}$ $\to 0$

$= \frac{x}{2D-1} \cdot 3e^{2x} = \frac{x}{\cancel{3}} \cdot \cancel{3}e^{2x}$

$y = c_1 e^{-x} + c_2 e^{3x} + x e^{2x}$

(1) $C_1 + C_2 = 0$

(2) $y' = -C_1 e^{-x} + 3C_2 e^{3x} + x e^{x} + e^{x}\cdot 1$

$-2 = -C_1 + 3C_2 + 1$

$C_1 = 1 \quad C_2 = -1$

$y = e^{-x} - 1e^{3x} + xe^{2x}$

Question 11

Consider the differential equation $\frac{dy}{dx}=(1+y^2)x$

The general solution with constant c is

A $y=\tan\frac{x^2}{2}+\tan c$

B $y=\tan^2\left(\frac{x}{2}+c\right)$

C $y=\tan^2\left(\frac{x}{2}\right)+c$

D $y=\tan\left(\frac{x^2}{2}+c\right)$

$$\int\frac{dy}{1+y^2}=\int x\,dx$$

$$\tan^{-1}y=\frac{x^2}{2}+c$$

$$y=\tan\left(\frac{x^2}{2}+c\right)$$

Question 12

The solution of $x\frac{dy}{dx}+y=x^4$ with the condition $y(1)=\frac{6}{5}$ is

$$\frac{dy}{dx}+\frac{1}{x}.y=x^3 \qquad \frac{dy}{dx}+Py=Q.$$

$$\text{I.F.}=e^{\int \frac{1}{x}dx}=x$$

$$y.x=\int x^3.x$$

$$y.x=\frac{x^5}{5}+c$$

$$y=\frac{x^4}{5}+\frac{c}{x}$$

$$\frac{6}{5}=\frac{1}{5}+\frac{c}{1}$$

$$\boxed{c=1}$$

$$y=\frac{x^4}{5}+\frac{1}{x}$$

A. $y=\frac{x^4}{5}+\frac{1}{x}$

B. $y=\frac{4x^4}{5}+\frac{4}{5x}$

C. $y=\frac{x^4}{5}+1$

D. $y=\frac{x^5}{5}+1$

Question 13

The integrating factor of the equation $(x^2y^3 + xy)\frac{dy}{dx} = 1$ is

A ey^2

B $e^{\frac{1}{2}y^2}$

C $e^{\frac{1}{2}x^2}$

D $e^{-\frac{1}{2}y^2}$

$$\frac{dx}{dy} = x^2y^3 + xy$$

$$\frac{dx}{dy} - yx = y^3x^2 \quad \text{(Bernoulli D.E.)}$$

$$\frac{1}{x^2}\cdot\frac{dx}{dy} - y\frac{1}{x} = y^3 \qquad \text{Divide } x^2$$

$$-\frac{dz}{dy} - y\cdot z = y^3 \qquad \frac{1}{x} = z$$

$$\frac{dz}{dy} + y z = -y^3 \qquad -\frac{1}{x^2}\frac{dx}{dy} = \frac{dz}{dy}$$

$P = y$, $Q = -y^3$

$$\text{I.F.} = e^{\int y\,dy} = e^{y^2/2}$$

Question 14

$z \to f(x,y) \qquad p \to \frac{\partial z}{\partial x} \quad q \to \frac{\partial z}{\partial y}$

The general integral of the partial differential equation $y^2p - xyq = x(z - 2y)$ is

$\frac{2}{3} = \frac{4}{6} = \frac{6}{9}$

A. $\phi(x^2 + y^2, y^2 - yz) = 0$

B. $\phi(x^2 - y^2, y^2 + yz) = 0$

C. $\phi(xy, yz) = 0$

D. $\phi(x + y, \ln x - z) = 0$

$Pp + Qq = R$

$P = y^2 ; Q = -xy ; R = x(z-2y)$

By Lagrange's Auxiliary eqn.

$$\frac{dx}{P} = \frac{dy}{Q} = \frac{dz}{R}$$

$$\frac{dx}{y^2} = \frac{dy}{-xy} = \frac{dz}{x(z-2y)} = \frac{x\,dx + y\,dy}{0}$$

$$\frac{dx}{y^2} = \frac{x\,dx + y\,dy}{0} \quad \Big| \quad \frac{dy}{-\cancel{x}y} = \frac{dz}{\cancel{x}(z-2y)}$$

$$x\,dx + y\,dy = 0$$

$$x^2 + y^2 = c_1$$

$$z\,dy - 2y\,dy = -y\,dz$$

$$\int z\,dy + y\,dz = \int 2y\,dy$$

$$\int d(yz) = y^2 + c$$

$$yz - y^2 = c_2$$

$$\phi(c_1, c_2) = 0$$

$$\phi(x^2+y^2,\ y^2-yz) = 0$$

Question 15

Match each differential equation in Group I to its family of solution curves from Group II

Group I	Group II
A. $\frac{dy}{dx}=\frac{y}{x}$	1. Circles
B. $\frac{dy}{dx}=-\frac{y}{x}$	2. Straight lines
C. $\frac{dy}{dx}=\frac{x}{y}$	3. Hyperbolas
D. $\frac{dy}{dx}=-\frac{x}{y}$	

A) A – 2, B – 3, C – 3, D – 1

B) A – 1, B – 3, C – 2, D – 1

C) A – 2, B – 1, C – 3, D – 3

D) A – 3, B – 2, C – 1, D – 2

$$\frac{dy}{dx} = \frac{y}{x} \Rightarrow \frac{dy}{y} = \frac{dx}{x}$$

$$\log y = \log x + \log c$$

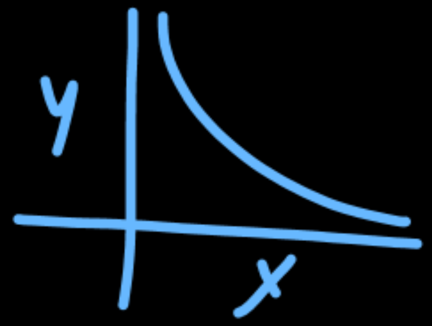
$$\boxed{y = cx}$$

$$\frac{dy}{dx} = -\frac{y}{x} \Rightarrow \frac{dy}{y} = -\frac{dx}{x}$$

$$\log y = -\log x + \log c$$

$$y = \frac{c}{x}$$

$$\boxed{xy = c}$$



Question 16

$T \to x, t$

The one dimensional heat conduction partial differential equation $\frac{\partial T}{\partial t} = \frac{\partial^2 T}{\partial x^2}$ is

$$A \frac{\partial^2 T}{\partial x^2} + B \frac{\partial^2 T}{\partial x \partial t} + C \frac{\partial^2 T}{\partial t^2} + f(x, t, T, T_t, T_x, c) = 0$$

On comparison $A = 1;\ B = 0,\ C = 0$

$$B^2 - 4AC = 0^2 - 4(1) \times 0 = 0$$

Parabolic.

A. Parabolic

B. Hyperbolic

C. elliptic

D. mixed

Question 17

The partial differential equation $\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial y^2} + \frac{\partial \phi}{\partial x} + \frac{\partial \phi}{\partial y} = 0$ has

Order = 2
Degree = 1

A. degree 1 and order 2

B. degree 1 and order 1

C. degree 1 and order 1

D. degree 2 and order 2

Question 18

The type of partial differential equation $\frac{\partial^2 P}{\partial x^2}+\frac{\partial^2 P}{\partial y^2}+3\frac{\partial^2 P}{\partial x dy}+2\frac{\partial P}{\partial x}-\frac{\partial P}{\partial y}=0$ is

$A P_{xx} + C P_{yy} + B P_{xy} + \cdots = 0$

$A = 1 \quad C = 1 \quad B = 3$

$B^2 - 4AC = 3^2 - 4(1)(1) = 5 > 0$

Hyperbolic P.D.E.

A elliptic

B parabolic

C hyperbolic

D none of these

Question 19

The solution is the following partial differential equation $\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$ is

A $\sin(3x - y)$

B $3x^2 + y^2$

C $\sin(3x - 3y)$

D $(3y^2 - x^2)$

Option a) $u_x = 3\cos(3x - y)$

$u_{xx} = -9\sin(3x - y)$ — (1)

$u_y = -1\cos(3x - y)$

$u_{yy} = -\sin(3x - y)$ — (2)

$$\frac{u_{xx}}{u_{yy}} = 9$$

$$\frac{\partial^2 u}{\partial x^2} = 9\frac{\partial^2 u}{\partial y^2}$$

Question 20

$p = \frac{\partial z}{\partial x} = a \quad | \quad q = \frac{\partial z}{\partial y} = b$

The complete integral of $(z - px - qy)^3 = pq + 2(p^2 + q)^2$ is

$y = px + \phi(p)$

$y = cx + \phi(c)$

General Clairaut's eqn.

$z = px + qy + \phi(p, q)$

& it's general soln.

$z = ax + by + \phi(a, b)$

$z = px + qy + \sqrt[3]{pq + 2(p^2 + q)^2}$

$z = ax + by + \sqrt[3]{ab + 2(a^2 + b)^2}$

A. $z = ax + by + \sqrt[3]{pq + 2(p^2 + q)^2}$

B. $z = ax + by + \sqrt[3]{ab + 2(a^2 + b)^2}$

C. $z = ax + by + \sqrt[3]{ab} + \sqrt[3]{2(a^2 + b)^2}$

D. $z = ax + by + c$

Thank you

GW

Soldiers !