ENGINEERING MATHEMATICS

ALL BRANCHES

Calculus

Infinite Series

DPP-07 Solution

By- CHETAN SIR

Question 1

The summation of series, $S = 2 + \frac{5}{2} + \frac{8}{2^2} + \frac{11}{2^3} + \ldots + \infty$ $-1)$

A 4.50

B 6.0

C 6.75

D 10.0

$$\frac{S}{2} = \frac{2}{2} + \frac{5}{2^2} + \frac{8}{2^3} + \cdots \quad -2)$$

$$\left(S - \frac{S}{2}\right) = 2 + \frac{5-2}{2} + \frac{8-5}{2^2} + \frac{11-8}{2^3} + \cdots$$

$$\frac{S}{2} = 2 + \frac{3}{2} + \frac{3}{2^2} + \frac{3}{2^3} + \cdots$$

$$\frac{S}{2} = 2 + \frac{3}{2}\left[1 + \frac{1}{2} + \frac{1}{2^2} + \cdots\right]$$

$$\frac{S}{2} = 2 + \frac{3}{2}\left[\frac{1}{1-\frac{1}{2}}\right]$$

$$S_\infty = \frac{a}{1-r}$$

$$\frac{S}{2} = 2 + \frac{3}{2}\left[\frac{1}{1/2}\right]$$

$$S = 2 \times 5 = 10$$

Question 2

For $|x| \ll 1$, $\cot h(x)$ can be approximated as

A x

B x^2

C $\dfrac{1}{x}$

D $\dfrac{1}{x^2}$

$$-1 < x < 1$$

$$\coth x = \frac{\cosh x}{\sinh x} = \frac{e^x + e^{-x}}{e^x - e^{-x}}$$

$$= \frac{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) + \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)}{\left(1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots\right) - \left(1 - x + \frac{x^2}{2!} - \frac{x^3}{3!} + \cdots\right)}$$

$$= \frac{\cancel{2}\left(1 + \frac{x^2}{2!} + \frac{x^4}{4!} + \cdots\right)}{\cancel{2}\left(x + x^3/3! + x^5/5! + \cdots\right)}$$

Neglecting higher powers of x,

we get $\coth x = \frac{1}{x}$

Question 3

For the function e^{-x}, the linear approximation around $x = 2$ is

A. $(3 - x)\, e^{-2}$

B. $1 - x$

C. $[3 + 2\sqrt{2} - (1 + \sqrt{2})x]\, e^{-2}$

D. e^{-2}

$$e^{-x} = e^{-2-x+2} = e^{-2} \cdot e^{-(x-2)}$$

$$e^{-2}\left[1 - (x-2) + \frac{(x-2)^2}{2!} - \frac{(x-2)^3}{3!} + \cdots\right.$$

$$e^{-2}[1 - x + 2]$$ [∵ of linear approx. higher powers neglected]

$$e^{-2}[3 - x]$$

Question 4

The limit of the following series as 'x' approaches $\frac{\pi}{2}$ is,

$$f(x)=x-\frac{x^3}{3!}+\frac{x^5}{5!}-\frac{x^7}{7!}+\ldots\ldots$$

A $\frac{2\pi}{3}$

B $\frac{\pi}{2}$

C $\frac{\pi}{3}$

D 1 ✓

$\therefore f(x) = \sin x$

$f(x) = \lim_{x\to\pi/2} \sin x.$

$= \sin \pi/2 = 1$

Question 5

$\sin x$ when expanded in powers of $(x - \frac{\pi}{2})$ is

A $1 + \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^3}{3!} + \frac{(x-\frac{\pi}{2})^4}{4!} + \ldots$

B $1 - \frac{(x-\frac{\pi}{2})^2}{2!} + \frac{(x-\frac{\pi}{2})^4}{4!} - \ldots$

C $(x-\frac{\pi}{2})^2 + \frac{(x-\frac{\pi}{2})^3}{3!} + \frac{(x-\frac{\pi}{2})^5}{5!} + \ldots$

D None of these

$$f(x) = f(a) + (x-a)f'(a) + \frac{(x-a)^2}{2!}f''(a) + \frac{(x-a)^3}{3!}f'''(a) + \frac{(x-a)^4}{4!}f''''(a) + \cdots$$

$$= 1 + \left(x-\frac{\pi}{2}\right)\cdot 0 + \frac{(x-\pi/2)^2}{2!}\cdot(-1) + \frac{(x-\pi/2)^3}{3!}\cdot 0 + \frac{(x-\pi/2)^4}{4!}\cdot 1$$

At $x = \pi/2$

$f(x) = \sin x. = \sin \pi/2 = 1$

$f'(x) = \cos x = \cos \pi/2 = 0$

$f''(x) = -\sin x = -\sin \pi/2 = -1$

$f'''(x) = -\cos x = -\cos \pi/2 = 0$

$f''''(x) = +\sin x = \sin \pi/2 = 1$

$$\sin x = 1 - \frac{(x-\pi/2)^2}{2!} + \frac{(x-\pi/2)^4}{4!} - $$

Question 6

The sum of the infinite series, $1+\frac{1}{2}+\frac{1}{3}+\frac{1}{4}+\ldots$ is,

A π

B infinity

C 4

D $\frac{\pi^2}{2}$

$\log(1-x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} - \cdots \infty$

$\log(1-x) = -\left(x + \frac{x^2}{2} + \frac{x^3}{3} + \frac{x^4}{4} + \cdots \infty\right)$

At $x=1$

$\log(1-1) = -\left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \infty\right)$

$\therefore \left(1 + \frac{1}{2} + \frac{1}{3} + \frac{1}{4} + \cdots \infty\right) = -\log 0 = \infty$

Question 7

For $x=\frac{\pi}{6}$, the sum of the series $\sum_{1}^{\infty}(\cos x)^{2n}=\cos^2 x+\cos^4 x+\ldots$ is

A π

B 3

C ∞

D 1

Common ratio $=\cos^2 x$

$$S_\infty=\frac{a}{1-r}=\frac{\cos^2 x}{1-\cos^2 x}$$

$$=\frac{\cos^2 x}{\sin^2 x}=\cot^2 x$$

$\cot x=\frac{1}{\tan x}$

At $x=\frac{\pi}{6}$; $\cot^2\left(\frac{\pi}{6}\right)=\left(\sqrt{3}\right)^2=3$

Question 8

The infinite series $3 + 1 + \frac{1}{3} + \frac{1}{9} + \cdots$

$|r| > 1 \Rightarrow$ Divergent series.

A converges

B diverges

C oscillates

D unstable

$\therefore \quad |r| < 1$

$\Rightarrow$ Convergent series

$$S_\infty = \frac{a}{1-r} = \frac{3}{1-1/3}$$

$$= \frac{3}{2/3} = \frac{9}{2} = 4.5 \text{ (Finite)}$$

$\therefore$ Convergent.

Thank you

GW Soldiers !