ENGINEERING MATHEMATICS

ALL BRANCHES

Eigen Values & Vectors

Linear Algebra

DPP-08 Solution

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Question 1

One of the eigen vectors of the matrix $A=\begin{bmatrix} 2 & 2 \\ 1 & 3 \end{bmatrix}$ *is*

A $\begin{bmatrix} 2 \\ -1 \end{bmatrix}$

B $\begin{bmatrix} 2 \\ 1 \end{bmatrix}$

C $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

D $\begin{bmatrix} 1 \\ -1 \end{bmatrix}$

Question 2

The matrix $\begin{bmatrix} 1 & 2 & 4 \\ 3 & 0 & 6 \\ 1 & 1 & p \end{bmatrix}$ has one eigen value equal to 3. The sum of the other two eigen value

A p

B $p-1$

C $p-2$

D $p-3$

let eigen values be $\lambda_1, \lambda_2, 3$

Theorem:- Sum of all eigen values = Trace of matrix

$$\lambda_1 + \lambda_2 + 3 = 1 + 0 + p$$

$$\boxed{\lambda_1 + \lambda_2 = p - 2}$$

Question 3

The eigen vector of the matrix $\begin{bmatrix} 1 & 2 \\ 0 & 2 \end{bmatrix}$ are written in the form $\begin{bmatrix} 1 \\ a \end{bmatrix}$ *and* $\begin{bmatrix} 1 \\ b \end{bmatrix}$. What is a+b?

$a + b = 0 + \frac{1}{2}$

$= \frac{1}{2}$

A 0

B $\frac{1}{2}$

C 1

D 2

Characterstic eqn. $|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & 2 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)(2-\lambda) = 0$$

$$\lambda = 1, 2$$

We Know, $(A - \lambda I)X = 0$

So for $\lambda = 1$ $\begin{bmatrix} 1-1 & 2 \\ 0 & 2-1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow x_2 = 0$ let $x_1 = K$

$\lambda = 1 \Rightarrow$ E. vector $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ 0 \end{bmatrix} = K\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\boxed{\therefore a = 0}$

So for $\lambda = 2$ $\begin{bmatrix} 1-2 & 2 \\ 0 & 2-2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix} \Rightarrow -x_1 + 2x_2 = 0$

let $x_1 = K$ $x_2 = K/2$

$\lambda = 2 \Rightarrow$ E. vector $= \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} K \\ K/2 \end{bmatrix} = K\begin{bmatrix} 1 \\ 1/2 \end{bmatrix}$ $\boxed{\therefore b = 1/2}$

Question 4

If a square matrix A is real and symmetric, then the eigen values

A. are always real

B. are always real and positive

C. are always real and nonnegative

D. occur in complex conjugate pairs

Let symmetric matrix $A = \begin{bmatrix} a & c \\ c & a \end{bmatrix}$

$$\therefore |A - \lambda I| = 0$$

$$\begin{vmatrix} a-\lambda & c \\ c & a-\lambda \end{vmatrix} = 0$$

$$(a-\lambda)^2 - c^2 = 0$$

$$(a-\lambda)^2 = c^2$$

$$a - \lambda = \pm c$$

$$\lambda_1 = a - c$$

$$\lambda_2 = a + c$$

Hence real eigen values

Question 5

The number of linearly independent eigen vectors of $\begin{bmatrix} 2 & 1 \\ 0 & 2 \end{bmatrix}$ is

A 0

B 1

C 2

D infinite

Characterstic eqn. $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & 1 \\ 0 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)^2 = 0$$

$$\lambda = 2, 2$$

Since we have only eigenvolue $\therefore$ only one L. I. eigen vector will exist.

Question 6

The eigen value of a skew-symmetric matrix are

A. always zero

B. always pure imaginary

C. either zero or pure imaginary in conjugate pairs

D. always real

Question 7

The Eigen values of following matrix are $\begin{bmatrix} -1 & 3 & 5 \\ -3 & -1 & 6 \\ 0 & 0 & 3 \end{bmatrix}$

Sum of eigen values = Trace

$= -1 - 1 + 3 = 1$

A. $3, 3 + 5j, 6 - j$ $\neq 1$

B. $-6 + 5j, 3 + j, 3 - j$ $\neq 1$

C. $3 + j, 3 - j, 5 + j$ $\neq 1$

D. $3, -1 + 3j, -1 - 3j$ $= 1$

Question 8

All the four entries of the 2×2 matrix $P = \begin{bmatrix} p_{11} & p_{12} \\ p_{21} & p_{22} \end{bmatrix}$ are nonzero,

and one of its eigenvalue is zero. Which of the following statements is true?

A $p_{11}p_{12} - p_{12}p_{21} = 1$

B $p_{11}p_{22} - p_{12}p_{21} = -1$

C $p_{11}p_{22} - p_{12}p_{21} = 0$

D $p_{11}p_{22} + p_{12}p_{21} = 0$

Product of eigen values = Determinant

let eigen values
λ_1, λ_2

$$\lambda_1 . \lambda_2 = P_{11} P_{22} - P_{12} P_{21}$$

$$\boxed{0 = P_{11} P_{22} - P_{12} P_{21}}$$

$\because \lambda_1 = 0$

Thank you

GW Soldiers !