ENGINEERING MATHEMATICS

ALL BRANCHES

Calculus
Partial Differentiation & Euler's Theorem

DPP-08 Solution

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Question 1

If $u = e^{xyz}$, then $\frac{\partial^3 u}{\partial x \partial y \partial z}$ is equal to

A $e^{xyz}[1 + xyz + 3x^2y^2z^2]$

B $e^{xyz}[1 + xyz + x^3y^3z^3]$

C $e^{xyz}[1 + 3xyz + x^2y^2z^2]$

D $e^{xyz}[1 + 3xyz + x^3y^3z^3]$

$$u = e^{xyz}$$

$$\frac{\partial u}{\partial x} = e^{xyz} \cdot yz$$

$$\frac{\partial^2 u}{\partial y\, \partial x} = e^{xyz} \cdot z + xyz^2 e^{xyz} = e^{xyz}(z + xyz^2)$$

$$\frac{\partial^2 u}{\partial z \partial y \partial x} = e^{xyz}(1 + 2xyz) + e^{xyz} xy(z + xyz^2)$$

$$= e^{xyz}[1 + 2xyz + xyz + x^2y^2z^2]$$

$$\boxed{\frac{\partial^3 u}{\partial x \partial y \partial z} = e^{xyz}[1 + 3xyz + x^2y^2z^2]}$$

Question 2

If $z = f(x + ay) + \phi(x - ay)$, then

A $\dfrac{\partial^2 z}{\partial x^2} = a^2 \dfrac{\partial^2 z}{\partial y^2}$

B $\dfrac{\partial^2 z}{\partial y^2} = a^2 \dfrac{\partial^2 z}{\partial x^2}$

C $\dfrac{\partial^2 z}{\partial y^2} = -\dfrac{1}{a^2} \dfrac{\partial^2 z}{\partial x^2}$

D $\dfrac{\partial^2 z}{\partial x^2} = -a^2 \dfrac{\partial^2 z}{\partial y^2}$

$$\frac{\partial z}{\partial x} = 1.f'(x+ay) + 1.\phi'(x-ay)$$

$$\frac{\partial^2 z}{\partial x^2} = 1.f''(x+ay) + 1.\phi''(x-ay)$$

$$\frac{\partial z}{\partial y} = a f'(x+ay) - a\phi'(x-ay)$$

$$\frac{\partial^2 z}{\partial y^2} = a^2 f''(x+ay) + a^2\phi''(x-ay)$$

$$\frac{\partial^2 z}{\partial y^2} = a^2 \frac{\partial^2 z}{\partial x^2}$$

Question 3

If $u = tan^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ equals

A $2\cos 2u$

B $\frac{1}{4}\sin 2u$

C $\frac{1}{4}\tan u$

D $2\tan 2u$

$$f(u) = \tan u = \left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right) = \frac{x(1+y/x)}{x^{1/2}(1+(y/x)^{1/2})} = x^{1/2} f\left(\frac{y}{x}\right)$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{n f(u)}{f'(u)}$$

$$= \frac{1}{2}\cdot\frac{\tan u}{\sec^2 u} = \frac{1}{2}\frac{\frac{\sin u}{\cos u}}{\frac{1}{\cos^2 u}} = \frac{1}{2}\sin u\cos u$$

$$= \frac{1}{4}2\sin u\cos u$$

$$= \frac{1}{4}\sin 2u$$

Question 4

If $u = tan^{-1}\frac{x^3+y^3+x^2y-xy^2}{x^2-xy+y^2}$, then the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ is

A $\frac{1}{2}\sin 2u$

B $\sin 2u$

C $\sin u$

D 0

$$f(u) = \tan u = \left(\frac{x^3+y^3+x^2y-xy^2}{x^2-xy+y^2}\right) \rightarrow \text{Homogenous fn. of degree 1.}$$

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{n\, f(u)}{f'(u)}$$

$$= 1\, \frac{\tan u}{\sec^2 u} = \frac{\frac{\sin u}{\cos u}}{\frac{1}{\cos^2 u}}$$

$$= \sin u \cos u$$

$$= \frac{\sin 2u}{2}$$

Question 5

If $u = \underbrace{\phi\left(\frac{y}{x}\right)}_{v} + \underbrace{x\psi\left(\frac{y}{x}\right)}_{w}$, then the value of $x^2\frac{\partial^2 u}{dx^2} + 2xy\frac{\partial^2 u}{dxdy} + y^2\frac{\partial^2 u}{\partial y^2}$, is

$v \rightarrow$ Degree $0 \rightarrow n$

$w \rightarrow$ Degree $1 \rightarrow n$

A. 0

B. u

C. $2u$

D. $-u$

$$\rightarrow \quad x^2\frac{\partial^2 v}{\partial x^2} + 2xy\frac{\partial^2 v}{\partial x \partial y} + y^2\frac{\partial^2 v}{\partial y^2} = n(n-1)v = 0$$

$$\rightarrow \quad x^2\frac{\partial^2 w}{\partial x^2} + 2xy\frac{\partial^2 w}{\partial x \partial y} + y^2\frac{\partial^2 w}{\partial y^2} = n(n-1)w = 0$$

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2} = 0 + 0 = 0$$

Question 6

If $z = e^x \sin y$, $x = \log_e t$ and $y = t^2$, then $\frac{dz}{dt}$ is given by the expression

A $\frac{e^x}{t}(\sin y - 2t^2 \cos y)$

B $\frac{e^x}{t}(\sin y + 2t^2 \cos y)$

C $\frac{e^x}{t}(\cos y + 2t^2 \sin y)$

D $\frac{e^x}{t}(\cos y - 2t^2 \sin y)$

$$z = e^x \sin y \qquad x = \log_e t \qquad y = t^2$$

$$z \rightarrow x, y \longrightarrow t$$

$$\frac{dz}{dt} = \frac{\partial z}{\partial x} \cdot \frac{dx}{dt} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dt}$$

$$\frac{\partial z}{\partial x} = e^x \sin y$$

$$\frac{\partial z}{\partial y} = e^x \cos y$$

$$\frac{dx}{dt} = \frac{1}{t}$$

$$\frac{dy}{dt} = 2t$$

$$\frac{dz}{dt} = e^x \sin y \cdot \frac{1}{t} + e^x \cos y \, (2t)$$

$$\frac{dx}{dt} = \frac{e^x}{t}\left[\sin y + 2t^2 \cos y\right]$$

Question 7

If $z = z(u, v), u = x^2 - 2xy - y^2, v = a$, then

A $(x+y)\frac{\partial z}{\partial x} = (x-y)\frac{\partial z}{\partial y}$

B $(x-y)\frac{\partial z}{\partial x} = (x+y)\frac{\partial z}{\partial y}$

C $(x+y)\frac{\partial z}{\partial x} = (y-x)\frac{\partial z}{\partial y}$ ✓

D $(y-x)\frac{\partial z}{\partial x} = (x+y)\frac{\partial z}{\partial y}$

$z \rightarrow u, v \longrightarrow x, y$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial x} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial x}$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u} \cdot \frac{\partial u}{\partial y} + \frac{\partial z}{\partial v} \cdot \frac{\partial v}{\partial y}$$

$\frac{\partial u}{\partial x} = 2x - 2y$ $\qquad \frac{\partial v}{\partial x} = 0$

$\frac{\partial u}{\partial y} = -2x - 2y$ $\qquad \frac{\partial v}{\partial y} = 0$

$$\frac{\partial z}{\partial x} = \frac{\partial z}{\partial u}\cdot(2x-2y) + \frac{\partial z}{\partial v}\cdot 0$$

$$\frac{\partial z}{\partial y} = \frac{\partial z}{\partial u}\cdot(-2x-2y) + \frac{\partial z}{\partial v}\cdot 0$$

$$-2(x+y)\cdot\frac{\partial z}{\partial x} = \frac{\partial z}{\partial y}\ 2(x-y)$$

$$\boxed{x+y\ \frac{\partial z}{\partial x} = (y-x)\frac{\partial z}{\partial y}}$$

Question 8

If $f(x, y) = 0, \phi(y, z) = 0$, then

A $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} = \frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y} \cdot \frac{dz}{dx}$

B $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \frac{\partial f}{\partial x} = \frac{\partial f}{\partial x} \cdot \frac{dz}{dx}$

C $\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \cdot \frac{dz}{dx} = \frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y}$

D None of these

$$\frac{dy}{dx} = \frac{-\frac{\partial f}{\partial x}}{\frac{\partial f}{\partial y}}, \quad \frac{dz}{dy} = \frac{-\frac{\partial \phi}{\partial y}}{\frac{\partial \phi}{\partial z}}$$

$$\frac{\cancel{dy}}{dx} \cdot \frac{dz}{\cancel{dy}} = \frac{\cancel{\partial f/\partial x}}{\cancel{\partial f/\partial y}} \cdot \frac{\cancel{\partial \phi/\partial y}}{\cancel{\partial \phi/\partial z}}$$

$$\frac{\partial f}{\partial y} \cdot \frac{\partial \phi}{\partial z} \cdot \frac{dz}{dx} = \frac{\partial f}{\partial x} \cdot \frac{\partial \phi}{\partial y}$$

Question 9

$$3x^2dx + 3y^2dy + 3ax\,dy + 3ay\,dx = 0$$

$$\frac{dy}{dx} = \frac{-3(x^2+ay)}{3(y^2+ax)}$$

If $z = \sqrt{x^2+y^2}$ and $x^3 + y^3 + 3axy = 5a^2$, then at $x = a, y = a, \frac{dz}{dx}$ is equal to

$$z \to x, y \to x$$

$$\frac{dz}{dx} = \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} \cdot \frac{dy}{dx}$$

$$\frac{dz}{dx} = \frac{\cancel{2}x}{\cancel{2}\sqrt{x^2+y^2}} + \frac{\cancel{2}y}{\cancel{2}\sqrt{x^2+y^2}}\left(-\frac{x^2+ay}{y^2+ax}\right)$$

$$\left.\frac{dz}{dx}\right|_{(a,a)} = \frac{a}{\sqrt{a^2+a^2}} + \frac{a}{\sqrt{a^2+a^2}}\left(-\cancel{\frac{a^2+a^2}{a^2+a^2}}\right)$$

$$= 0$$

A) $2a$

B) 0

C) $2a^2$

D) a^3

Question 10

If $x = r\cos\theta, y = r\sin\theta$ where r and θ are the functions of t, then $\frac{dx}{dt}$ is equal to

$x, y \to r, \theta \to t$

$$\frac{dx}{dt} = \frac{\partial x}{\partial r}\cdot\frac{dr}{dt} + \frac{\partial x}{\partial \theta}\cdot\frac{d\theta}{dt}$$

$$\frac{dx}{dt} = \cos\theta\frac{dr}{dt} - r\sin\theta\frac{d\theta}{dt}$$

A $r\cos\theta\frac{dr}{dt} - r\sin\theta\frac{d\theta}{dt}$

B $\cos\theta\frac{dr}{dt} - r\sin\theta\frac{d\theta}{dt}$

C $r\cos\theta\frac{dr}{dt} + \sin\theta\frac{d\theta}{dt}$

D $r\cos\theta\frac{dr}{dt} - \sin\theta\frac{d\theta}{dt}$

Thank you

GW Soldiers !