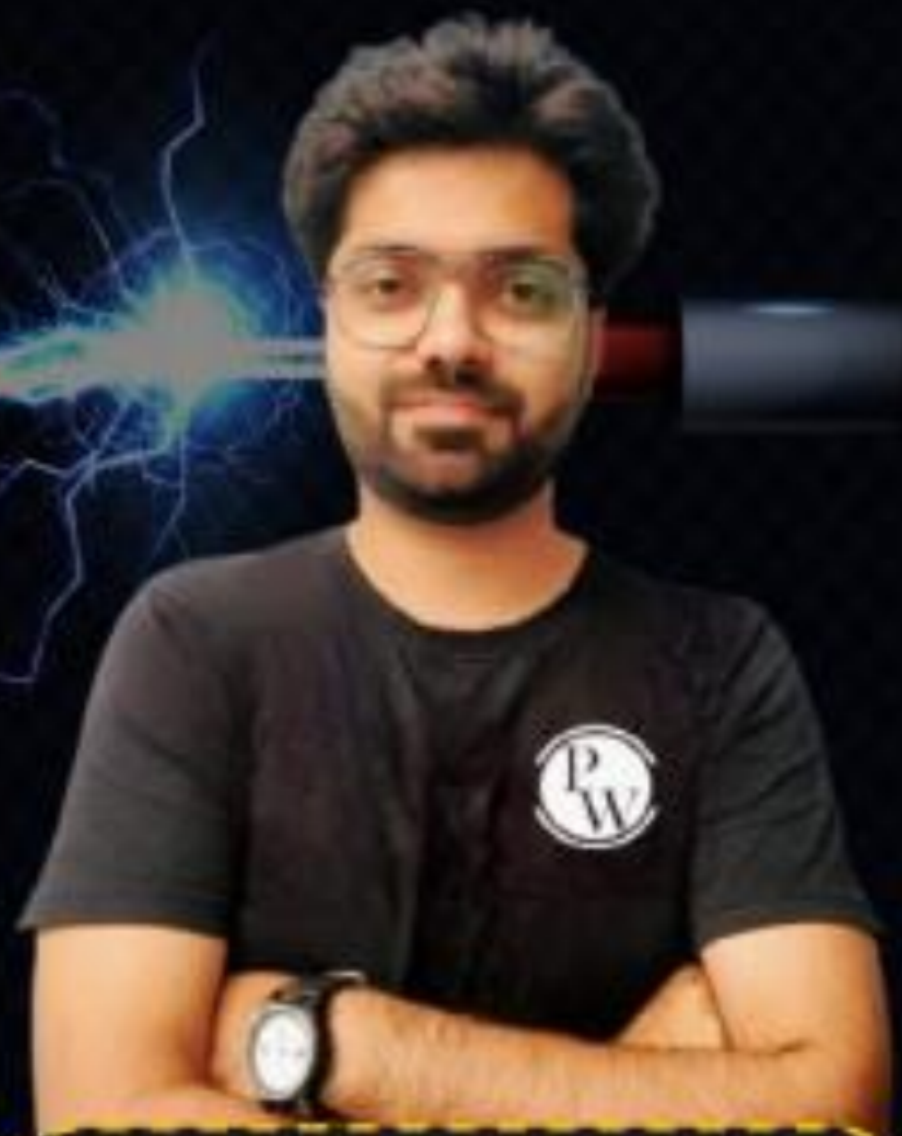
ENGINEERING MATHEMATICS

ALL BRANCHES

Probability

Binomial Theorem, Total probability theorem, Baye's Theorem

DPP-03 Solution



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Question 1

In a given day in the rainy season, it may rain 70% of the time. If it rains, chance that a village fair make a loss on that day is 80%. However, if it does not rain, chance that the fair will make a loss on that day is only 10%. If the fair has not made a loss on a given day in the rainy season, what is the probability that it has not rained on that day?

A $\frac{3}{10}$

B $\frac{9}{11}$

C $\frac{14}{17}$

D $\frac{27}{41}$

$P(E_1) = 0.7$

$P(E_2) = 0.3$

$A \rightarrow$ Fare has not made a loss

P (Fare has not made a loss when it rains) = 100 − 80%

$P\left(A/E_1\right) = 20\% \quad (100-80)\%$

$P\left(A/E_2\right) = 90\% \quad (100-10)\%$

By T.P.T. ; $\Rightarrow P(A) = P(E_1) \cdot P\left(A/E_1\right) + P(E_2) \cdot P\left(A/E_2\right)$

$= 0.7 \times 0.2 + 0.3 \times 0.9$

By Baye's theorem $P\left(E_2/A\right) = \dfrac{P(E_2 \cap A)}{P(A)} = \dfrac{0.3 \times 0.9}{0.7 \times 0.2 + 0.3 \times 0.9} = \dfrac{27}{41}$

Question 2

A can solve 90% of the problems given in a book and B can solve 70%. What is the probability that at least one of them will solve a problem, selected at random from the book?

A 0.16

B 0.63

C 0.97

D 0.20

$p(A) = 90\%$

$p(B) = 70\%$

$p(\text{at least one of them}) = 1 - p(\text{None of them})$

$1 - p(\bar{A} \cap \bar{B})$

$1 - p(\bar{A}) \cdot p(\bar{B})$

$1 - 10\% \cdot 30\%$

$1 - \frac{1}{10} \cdot \frac{3}{10} = \frac{97}{100} = 0.97$

Question 3

A speaks truth in 75% an B in 80% of the cases. In what percentage of cases are they likely to contradict each other narrating the same incident?

A 5%

B 45%

C 35%

D 15%

$p(A) = 75\%$

$p(B) = 80\%$

$$p(\text{Contradict}) = p(A \cap \bar{B}) + p(\bar{A} \cap B)$$

$$= p(A) \cdot p(\bar{B}) + p(\bar{A}) \cdot p(B)$$

$$= 0.75 \times 0.2 + 0.25 \times 0.8$$

$$= 0.35$$

35%

Question 4

A lot contains 20 articles. The probability that the lot contains exactly 2 defective articles in 0.4 and the probability that the lot contains exactly 3 defective articles is 0.6. Articles are drawn at random one by one without replacement and tested till all the defective articles are found. What is the probability that the testing procedure ends at the twelfth testing? ________

$E_1 \Rightarrow 20 \rightarrow 2D \quad p(2D) = 0.4$

$E_2 \Rightarrow 20 \rightarrow 3D \quad p(3D) = 0.6$

$A \rightarrow$ testing process end in 12^{th} testing.

$$P(A) = P(E_1) \cdot P(A/E_1) + P(E_2) \cdot P(A/E_1)$$

$$= 0.4 \cdot \left(\frac{{}^2C_1 \; {}^{18}C_{10}}{{}^{20}C_{11}}\right) \times \frac{1}{9} + 0.6 \cdot \left(\frac{{}^3C_2 \cdot {}^{17}C_9}{{}^{20}C_{11}}\right) \times \frac{1}{9}$$

$$= 0.0506$$

$2D \rightarrow \underbrace{10N.D, 1D}_{11} \mid \underbrace{1D \quad 8ND}_{9}$

$3D \rightarrow 9ND, 2D \mid 1D \quad 8ND$

Question 5

The chances of defective screws in three boxes A, B, C are $\frac{1}{5}, \frac{1}{6}, \frac{1}{7}$ respectively. A box is selected at random and screw drawn from it at random is found to be defective. Then the probability that it came from box A is ________.

T.P.T.

$$P(D) = P(A).P(D/A) + P(B).P(D/B) + P(C).P(D/C)$$

$$= \frac{1}{3} \times \frac{1}{5} + \frac{1}{3} \times \frac{1}{6} + \frac{1}{3} \times \frac{1}{7} = \frac{107}{630}$$

B.T.

$$P(A/D) = \frac{P(A \cap D)}{P(D)} = \frac{1/15}{107/630} = 42/107$$

Question 6

Four fair six-sided dice are rolled. The probability that the sum of the results being 22 is $\frac{x}{1296}$. The value of X is 10.

$$\text{Sum} = 22 \begin{cases} \rightarrow (6,6,6,4) = \frac{4!}{3!} = 4 \\ \rightarrow (6,6,5,5) = \frac{4!}{2!\,2!} = 6 \end{cases}$$

$$S = 6^4$$
$$= 1296$$

$$P(\text{Sum}=22) = \frac{4+6}{1296} = \frac{10}{1296} = \frac{x}{1296}$$

Question 7

What is the chance that a leap year, selected at random, will contain 53 Saturdays.

A $\frac{2}{7}$

B $\frac{3}{7}$

C $\frac{1}{7}$

D $\frac{5}{7}$

$366 \text{ days} = 52 \text{ weeks} + 2 \text{ days}$

S M
M T
T W
W T
T F
F S
S S

7

$\frac{2}{7}$

Question 8

In a housing society, half of the families have a single child per family, while the remaining half have two children per family. The probability that a child picked at random, has a sibling is $\underline{0.67}$.

$\frac{1}{2}$ $\qquad$ $\frac{1}{2}$

T.P.T. $\qquad (1) \to x \qquad (2) \to 2x$

$$P(C) = P(E_1) \cdot P(C/E_1) + P(E_2) \cdot P(C/E_2) = \frac{1}{2}x + \frac{1}{2}(2x)$$

B.T.

$$P\left(E_2/C\right) = \frac{P(E_2 \cap C)}{P(C)} = \frac{\frac{1}{2}\cdot(2x)}{\frac{1}{2}x + \frac{1}{2}(2x)} = \frac{2}{3} = 0.667$$

Question 9

Consider a dice with the property that the probability of a face with n dots showing up is proportional to n. The probability of the face with three dots showing up is $\underline{1/7}$.

1
2
3
4
5
6

$$P(n \text{ dots}) \propto n$$

$$P(n \text{ dots}) = Kn.$$

$$P(1)+P(2)+P(3)+P(4)+P(5)+P(6)=1$$

$$K+2K+3K+4K+5K+6K=1$$

$$\boxed{K=1/21}$$

$$P(3 \text{ dots}) = 3K = 3\times\frac{1}{21} = \frac{1}{7}$$

Question 10

A man can kill a bird once in a three shots. On this assumption he fires three shots. What is the chance that a bird is killed? ________

$p = \frac{1}{3}$ $q = \frac{2}{3}$

P(At least one shot kills the bird) $= 1 - P$(Not even a single shot kills bird)

$1/3$

$2/3$

$3/3$

$1 - q \cdot q \cdot q$

$1 - \frac{2}{3} \cdot \frac{2}{3} \cdot \frac{2}{3} = \frac{19}{27}$

Thank you

GW Soldiers !