ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-01

Probability

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Topics to Be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM

} Basic probability (Part - I)

- STATISTICS – I (PROBABILITY DISTRIBUTIONS) } Part - II
- STATISTICS – II (CORRELATION AND REGRESSION) } Part - III

Basic Concept

1. Random Experiment: An experiment whose outcome can't be predicted.

   Ex:- Biased dice outcome always 1. (Not R.E.)

   Unbiased dice outcome = $\{1, 2, 3, 4, 5, 6\}$ R.E.

2. Sample Space: The set of all possible outcomes of an experiment.

   Dice $\rightarrow$ $S = \{1, 2, 3, 4, 5, 6\}$

Basic Concept

3. Sample Point/ Event Point: The elements of sample space are sample points or event points.

Mixed events
$$\begin{cases} E_1 = \text{Even number} = \{2,4,6\} & \Rightarrow p(E_1) = 3/6 = 1/2 \\ E_2 = \text{Odd number} = \{1,3,5\} & \Rightarrow p(E_2) = 3/6 = 1/2 \\ E_3 = \text{Prime number} = \{2,3,5\} & \end{cases}$$

4. Event: Every subset of sample space is event.

Total no. of events that can be possible with n sample points $= \underline{2^n}$

PROBABILITY BASICS

Types of Events

a. Simple Event/ Elementary event:

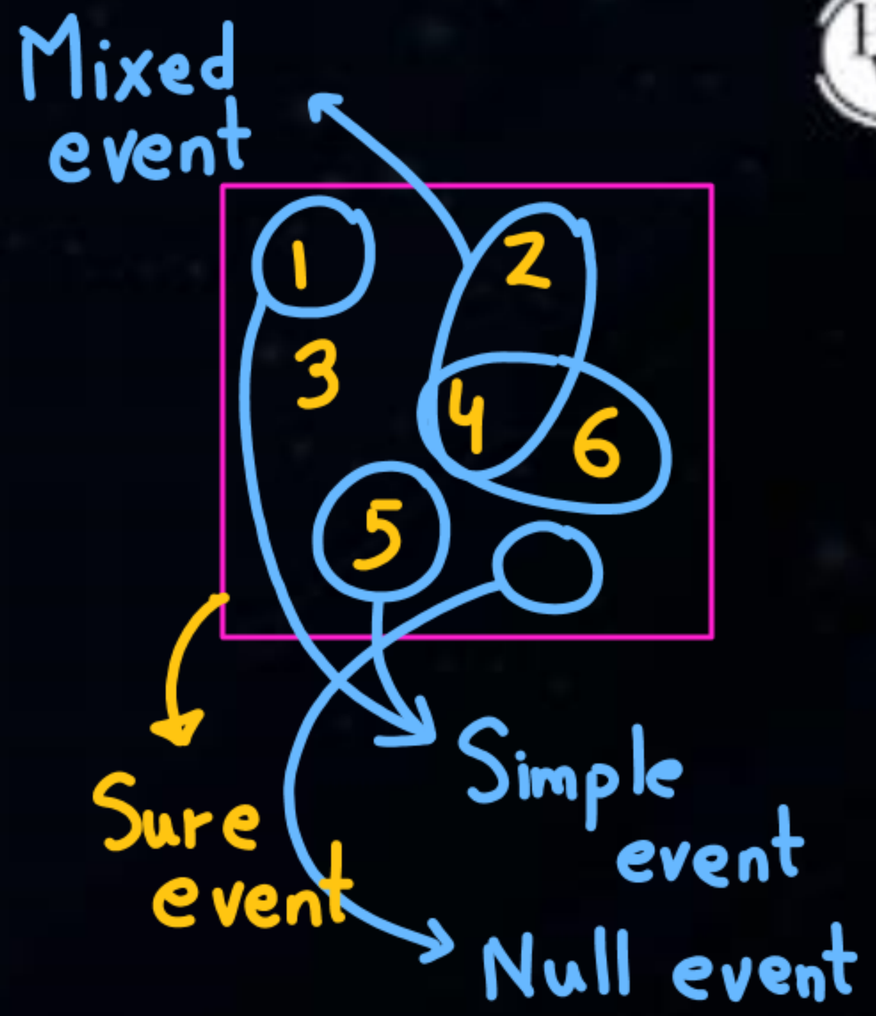
1 sample point

b. Mixed event/Compound event:

> 1sample point

c. Sure event: $P(S) = 1$

$E = \{1,2,3,4,5,6\}$ $P(E) = 1$



Types of Events

d. Impossible event/Null event:

$$P(\phi) = 0$$

Ex:- E = Getting 7 on a dice ; $P(E) = 0$

e. Equally likely events: Those events whose chances of happening are equal. $P(H) = P(T) = 1/2$

E_1 & E_2 are equally likely. $= 1/2$ E_1 & E_3 are not equally likely events.

E_3 = Perfect square $= \{1, 4\} = \frac{2}{6}$ E_2 & E_3

Types of Events

Mutually inclusive events :- $A \cap B \neq \phi$

f. Mutually exclusive events: Those events are said to be mutually exclusive when they cannot occur together.

$$A \cap B = \phi$$

For mutually exclusive events $A_1, A_2, A_3 \ldots\ldots A_n$.

$$P(A_1 \cup A_2 \cup A_3 \cup \ldots A_n) = P(A_1) + P(A_2) + P(A_3) + \ldots P(A_n)$$

g. Exhaustive Events:

Events that together cover all the possible outcomes of an experiment.

$E_1, E_2, E_3 \ldots E_n$ are exhaustive events.

$$E_1 \cup E_2 \cup E_3 \cup \ldots E_n = \cup\{E_i\}_{i=1}^{i=n} = S$$

Types of Events

h. Mutually exclusive & Exhaustive events

$E_1, E_2, E_3, \ldots \; E_n.$

$\rightarrow E_i \cap E_j = \phi$ (Pairwise disjoint)

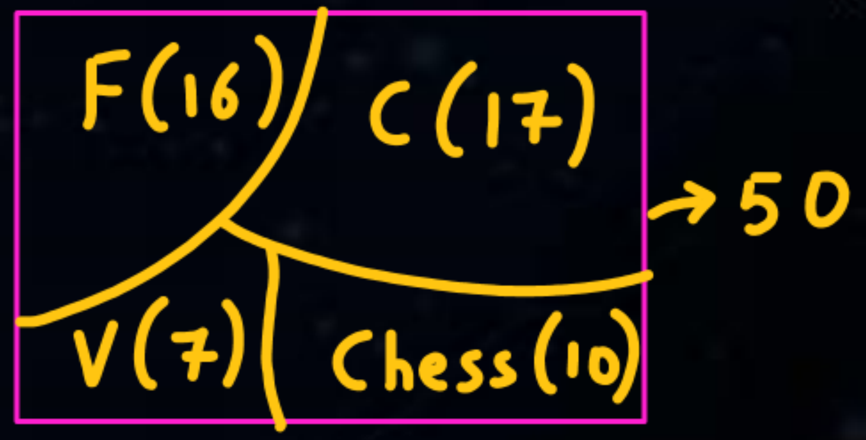
$\rightarrow E_1 \cup E_2 \cup E_3 \ldots \cup E_n = S$

$n(E_1) + n(E_2) + n(E_3) - \ldots \; n(E_n) = S$

$$\boxed{p(E_1) + p(E_2) + p(E_3) + \ldots p(E_n) = 1}$$

Ex:-

In a class, 50 students
- → 16 Football
- → 17 Cricket
- → 7 Volleyball
- → 10 Chess



Single student does not play more than one game.

↳ Mutually exclusive.

Soln:-

$$n(F) + n(C) + n(V) + n(Chess) = 50$$

Mutually exclusive & exhaustive events.

$$P(F) + P(C) + P(V) + P(Chess) = 1$$

$$\frac{16}{50} \quad \frac{17}{50} \quad \frac{7}{50} \quad \frac{10}{50}$$

$$P(F) = \frac{n(F)}{n(S)}$$

[PROBABILITY BASICS]

Compound Events → When 2 or more events are in relation with each other when it occurs in conjunction with each other's joint occurence.

Types of Compound events

1. Independent event: If 2 events happen such that occurence of one event does not depend on other.

2. Dependent event: . . . . . depends on other.

5B
3W

P(Drawing white ball & black ball with replacement)

$$P(W \cap B) = P(W) \cdot P(B)$$

$$\underset{(W)}{\frac{3}{8}} \times \underset{(B)}{\frac{5}{8}}$$

Independent events.

P(Drawing white ball & black ball w/o replacement)

$$P(W \cap B) = P(W) \cdot P(B/W)$$

$$\underset{(W)}{\frac{3}{8}} \times \underset{(B)}{\frac{5}{7}}$$

Dependent events.

Conditional Probability: If A and B are two events in a sample space such that B occurs only if A has already occured.

P(B occurs when A has already occured)

$$\boxed{P(B/A) = \frac{P(A \cap B)}{P(A)}}$$

Independent events :-

$$P(A \cap B) = P(A).P(B)$$

$$P(A \cap B \cap C) = P(A).P(B).P(C)$$

$$P(A \cap B \cap C \cap D) = P(A).P(B).P(C).P(D)$$

[PROBABILITY BASICS]

Conditional Probability:

$$P(B/A) = \frac{P(A \cap B)}{P(A)}$$

Dependent events :-

$$P(A \cap B) = P(A) . P(B/A)$$

$$P(A \cap B \cap C) = P(A) . P(B/A) . P(C/B \cap A)$$

$$P(A \cap B \cap C \cap D) = P(A) . P(B/A) . P(C/B \cap A) . P(D/C \cap B \cap A)$$

Ex :-

5B
4W
3R

$\rightarrow P(B \cap W \cap R) = P(B) . P(W/B) . P(R/W \cap B)$

$$= \frac{5}{12} \times \frac{4}{11} \times \frac{3}{10}$$

w/o replacement

Ex:- $P(ESE) = \frac{1}{12}$, $P(GATE) = \frac{1}{6}$

Probability of cracking GATE when ESE has occurred $= \frac{1}{10}$

,Find the probability of cracking both exams?

$$P(ESE \cap GATE) = P(ESE) \cdot P(GATE/ESE)$$

$$\frac{1}{12} \times \frac{1}{10} = \frac{1}{120}$$

GATE & ESE are dependent.

$$P(ESE \cap GATE) = P(ESE) \cdot P(GATE)$$

$$\frac{1}{12} \times \frac{1}{6} = \frac{1}{72}$$

GATE & ESE are independent.

Algebraic Operations

	Operations		
1. Complementary event A	$A^c, A', \bar{A}$		
2. Event A or B (Either A or B or both)	$A \cup B$		या (+)
3. Event A and B	$A \cap B$		और (x)
4. Event neither A nor B	$\overline{(A \cup B)} = \bar{A} \cap \bar{B}$		

PROBABILITY BASICS

$S \rightarrow$ Sample Space

$E \rightarrow$ Event

$$p(E) = \frac{\text{No. of favourable outcomes}}{\text{Total no. of outcomes}} \qquad \boxed{\frac{n(E)}{n(S)} = p(E)}$$

- $0 \leq p(E) \leq 1$
- $p(S) = 1$
- $p(\phi) = 0$
- $p(E) + p(E') = 1$

PROBABILITY BASICS

Coins:

1 Coin → {H, T} (2)

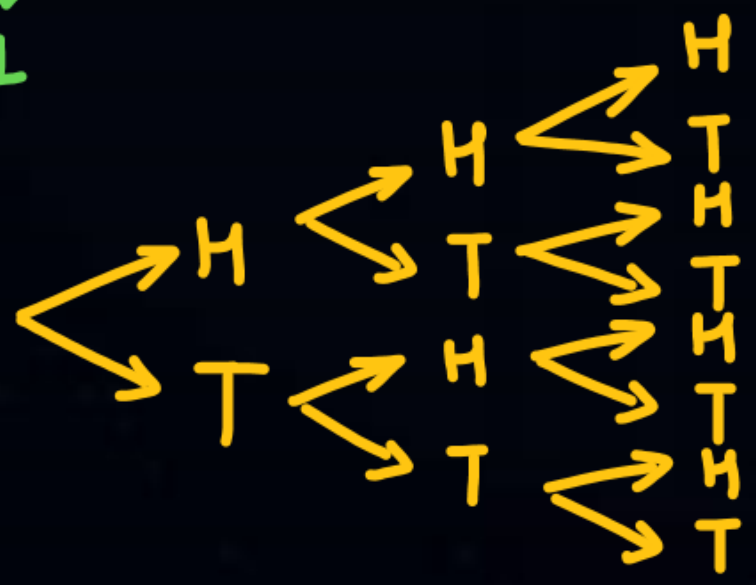
0H	1H
↓	↓
1	1

2 Coins

$\left\{\begin{matrix} H, H \\ H, T \\ T, H \\ T, T \end{matrix}\right\}$ (4)

0H	1H	2H
↓	↓	↓
1	2	1

3 coins



$\left\{\begin{matrix} H & H & H \\ H & H & T \\ H & T & H \\ H & T & T \\ T & H & H \\ T & H & T \\ T & T & H \\ T & T & T \end{matrix}\right\}$ (8)

0H	1H	2H	3H
↓	↓	↓	↓
1	3	3	1

0H	1H	2H	3H	4H
↓	↓	↓	↓	↓
1	4	6	4	1

$P(2H/4\text{coins}) = \frac{6}{16}$

Ex:- 2 Coins tossed, then find Probability

i) Exactly one head $= (HT, TH) = 2/4 = 1/2$

ii) At least one head $= (1H, 2H) = (HT, TH, HH) = 3/4$
(min.)

iii) At most one head $= (0H, 1H) = (TT, HT, TH) = 3/4$
(max.)

Ex:- 3 Coins tossed, then find Probability

i) Exactly one head $= (HTT, THT, TTH) = 3/8$

ii) At least one head $= (1H, 2H, 3H) = 1 - (0H) = 1 - P(TTT) = 1 - \frac{1}{8} = \frac{7}{8}$

$1 - p(\text{no head})$

iii) At most one head

$= (0H, 1H) = p(0H) + p(1H) = \frac{1}{8} + \frac{3}{8} = \frac{4}{8} = \frac{1}{2}$

PROBABILITY BASICS

DICE

1 Dice → {1, 2, 3, 4, 5, 6}

2 (1) 3 (2) 4 (3) 5 (4) 6 (5) 7 (6)

2 Dice
$$\left\{\begin{array}{cccccc}
(1,1) & (1,2) & (1,3) & (1,4) & (1,5) & (1,6) \\
(2,1) & (2,2) & (2,3) & (2,4) & (2,5) & (2,6) \\
(3,1) & (3,2) & (3,3) & (3,4) & (3,5) & (3,6) \\
(4,1) & (4,2) & (4,3) & (4,4) & (4,5) & (4,6) \\
(5,1) & (5,2) & (5,3) & (5,4) & (5,5) & (5,6) \\
(6,1) & (6,2) & (6,3) & (6,4) & (6,5) & (6,6)
\end{array}\right\}$$

8 (5)
9 (4)
10 (3)
11 (2)
12 (1)

36 outcomes

PROBABILITY BASICS

Ex:- 2 dices are thrown, Find probability of

i) $P(\text{Sum} = 7) = 6/36 = 1/6$

ii) P(sum at least 7) $= \underset{(6)}{\text{Sum } 7} + \underset{(5)}{\text{Sum } 8} + \underset{(4)}{\text{Sum } 9} + \underset{(3)}{\text{Sum } 10} + \underset{(2)}{\text{Sum } 11} + \underset{(1)}{\text{Sum } 12}$

$= 21/36$

iii) P (sum is double digit)

$\text{Sum } 10 + \text{Sum } 11 + \text{Sum } 12 = \frac{3+2+1}{36} = 6/36 = 1/6$

iv) P(sum is either 8 or 9)

$= \text{Sum } 8 + \text{Sum } 9$

$\frac{5}{36} + \frac{4}{36} = \frac{9}{36} = \frac{1}{4}$

Ex:- Find the probability that the difference of number shown is 1

Thank you

GW

Soldiers !