ENGINEERING MATHEMATICS

ALL BRANCHES

Vector Calculus

Basics of Vector & DEL Operator

DPP-01 Solution

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Question 1

The angle θ_{AB} between the vectors $\vec{A} = 3a_x + 4a_y + a_z$ and $\vec{B} = 2a_y - 5a_z$ is nearly

A. $83.7°$

B. $73.7°$

C. $63.7°$

D. $53.7°$

$\vec{A} \cdot \vec{B} = |\vec{A}||\vec{B}| \cos\theta_{AB}$

$\vec{A} \cdot \vec{B} = 3 \times 0 + 4 \times 2 + 1 \times -5 = 3$

$|\vec{A}| = \sqrt{3^2 + 4^2 + 1^2} = \sqrt{26}$

$|\vec{B}| = \sqrt{2^2 + (-5)^2} = \sqrt{29}$

$\theta = \cos^{-1}\left(\frac{\vec{A} \cdot \vec{B}}{|\vec{A}||\vec{B}|}\right) = \cos^{-1}\left(\frac{3}{\sqrt{26}\sqrt{29}}\right) = 83.7°$

Question 2

The angle between two vectors $x_1 = \underbrace{[2\ 6\ 14]'}$ and $x_2 = \underbrace{[-12\ \ 8\ 16]'}$ in radian is <u>0.7</u>2

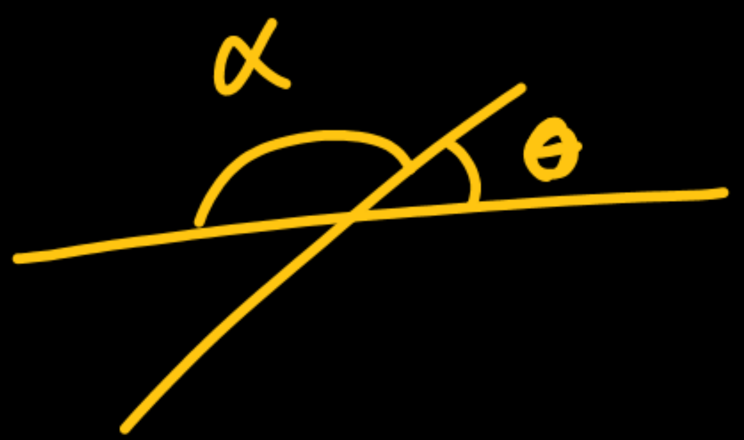
$$\theta = \cos^{-1}\left(\frac{x_1 \cdot x_2}{|\vec{x}_1|\,|\vec{x}_2|}\right) = \cos^{-1}\left(\frac{-24+48+224}{\sqrt{2^2+6^2+14^2}\,\sqrt{(-12)^2+8^2+16^2}}\right)$$

$$= 0.72 \text{ radians}$$

Question 3

The smaller angle (in degrees) between the planes $x + y + z = 1$ and $2x - y + 2z = 0$ is $\underline{54.73^\circ}$

$$\vec{N_1} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{N_2} = 2\hat{i} - \hat{j} + 2\hat{k}$$



$$\theta = \cos^{-1}\left(\frac{\vec{N_1} \cdot \vec{N_2}}{|\vec{N_1}| \cdot |\vec{N_2}|}\right) = \cos^{-1}\left(\frac{2 - 1 + 2}{\sqrt{1^2+1^2+1^2}\sqrt{2^2+(-1)^2+2^2}}\right)$$

$$\theta = 54.73^\circ$$

Question 4

Consider the time-varying vector $I = \hat{x}15\,cos(\omega t) + \hat{y}5\,sin(\omega t)$ in Cartesian coordinates, where. ω > 0 is a constant. When the vector magnitude |I| is at its minimum value, the angle θ that I makes with the *x* axis (in degrees, such that 0 < θ < 180) is $\underline{+90^\circ}$.

Min Amp. $= |5\hat{y}|$
$= 5$

$\omega = \frac{2\pi}{T}$

$\omega t = \frac{2\pi}{T} t$

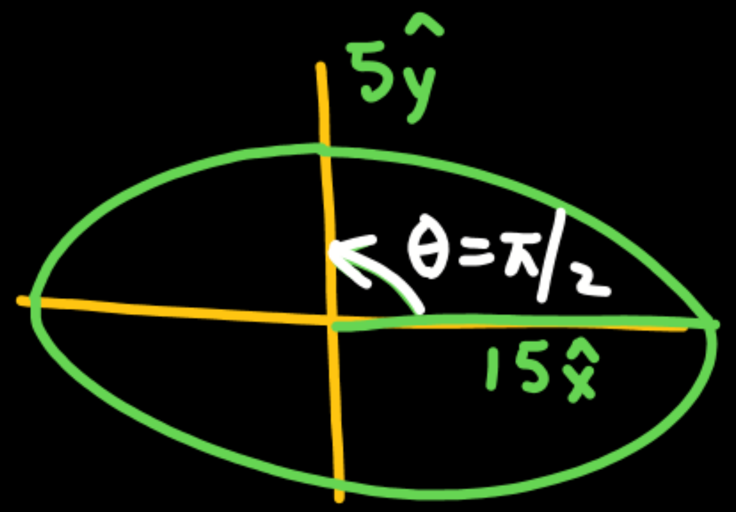
$t = 0$; $I = 15\hat{x}$

$t = T/4$; $\omega = \pi/2$; $I = 5\hat{y}$

$t = T/2$; $\omega = \pi$; $I = -15\hat{x}$

$t = 3T/4$; $\omega = 3\pi/2$; $I = -5\hat{y}$

$t = T$; $\omega = 2\pi$; $I = 15\hat{x}$



Question 5

A particle move along a curve whose parametric equations are:

$x = t^3 + 2t$, $y = -3e^{-2t}$ and $z = 2 \sin (5t)$, where x, y and z show variations of the distance covered by the particle (in cm) with time t (in s). The magnitude of the acceleration of the particle (in cm/s^2) at $t = 0$ is 12.

$$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k} = (t^3+2t)\hat{i} + (-3e^{-2t})\hat{j} + (2\sin 5t)\hat{k}$$

$$\frac{d\vec{r}}{dt} = (3t^2+2)\hat{i} + 6e^{-2t}\hat{j} + 10\cos 5t\,\hat{k}$$

At $t=0$

$$\vec{a} = \frac{d^2\vec{r}}{dt^2} = (6t)\hat{i} - 12e^{-2t}\hat{j} - 50\sin 5t\,\hat{k} = |-12\hat{j}|$$

Question 6

If P, Q and R are three points having coordinates (3, –2, –1), (1, 3, 4), (2, 1, –2) in XYZ space (O being the origin of the coordinate system) then distance of point P from plane OQR is)

P Q R

A	3
B	7
C	5
D	9

O Q R [0,0,0] [1,3,4] [2,1,-2]

$ax + by + cz = 0$ [Plane from origin]

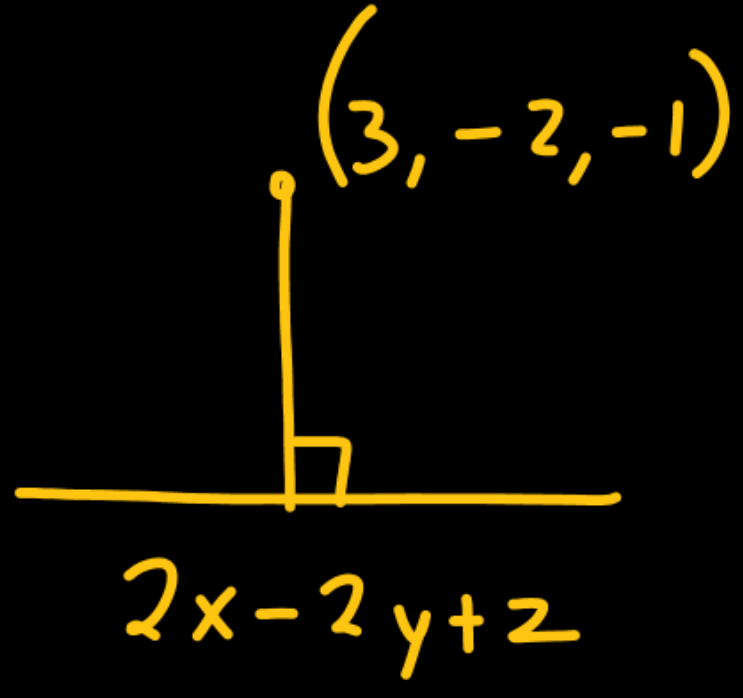
$x + \left(\frac{b}{a}\right)y + \left(\frac{c}{a}\right)z = 0$

$x + \alpha y + \beta z = 0$

$\left.\begin{aligned} 1 + 3\alpha + 4\beta &= 0 \quad -1) \\ 2 + 1\alpha - 2\beta &= 0 \quad -2) \end{aligned}\right\}$ Solving $\alpha = -1$, $\beta = 1/2$

$$x - y + \frac{1}{2}z = 0$$

$$2x - 2y + z = 0 \quad (\text{Plane})$$



$$\therefore \text{Distance} = \left| \frac{2(3) - 2(-2) + (-1)}{\sqrt{2^2 + (-2)^2 + 1^2}} \right|$$

$$= \boxed{3}$$

Question 7

A particle, starting from origin at $t = 0$s, is traveling along axis with velocity

$$V = \frac{\pi}{2}\cos\left(\frac{\pi}{2}t\right) \mathrm{m/s}$$

$$\int \frac{dS}{dt} = \int \frac{\pi}{2} \cos\left(\frac{\pi}{2} t\right)$$

At $t = 3$s, the difference between the distance covered by the particle and the magnitude of displacement from the origin is___.

Displacement; $S = \sin\left(\frac{\pi}{2} t\right)$

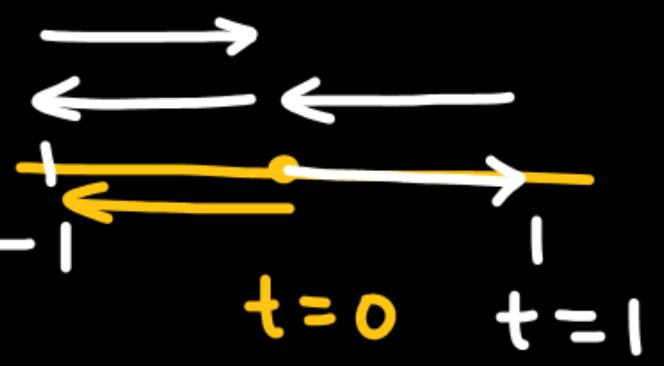
At $t=0 \quad s=0$
$t=1 \quad s=1$
$t=2 \quad s=0$
$t=3 \quad s=-1$

After 3 sec

Distance $= 3$

Displacement $= -1$

Diff. $= 3 - |-1| = 2$ units



Question 8

If $\vec{a}$ and $\vec{b}$ are two arbitrary vectors with magnitudes a and b, respectively, $|\bar{a} \times \bar{b}|^2$ will be equal to

A $a^2b^2 - (\vec{a} \cdot \vec{b})^2$

B $ab - \vec{a} \cdot \vec{b}$

C $a^2b^2 + (\vec{a} \cdot \vec{b})^2$

D $ab + \vec{a} \cdot \vec{b}$

$|\vec{a} \times \vec{b}|^2 = |ab\sin\theta\,\hat{n}|^2$

$|a|^2|b|^2\sin^2\theta\,|\hat{n}|^2$

$|a|^2|b|^2(1-\cos^2\theta)$

$|a|^2|b|^2 - |a|^2|b|^2\cos^2\theta$

$\boxed{|a \times b|^2 = a^2b^2 - (a.b)^2}$

Thank you

GW

Soldiers !