ENGINEERING MATHEMATICS

ALL BRANCHES

Differential Equation
Solution of higher order
linear DE

DPP-03 Solution

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Question 1

The solution of the differential equation $(x - y^2)\,dx + 2xy\,dy = 0$ is

Non-exact

$$\frac{\partial M}{\partial y} = -2y \qquad \frac{\partial N}{\partial x} = 2y$$

A $ye^{2/x} = A$

B $xe^{y^2/x} = A$

C $xe^{x/y^2} = A$

D $ye^{x/y^2} = A$

$$\frac{1}{N}\left(\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}\right) = f(x) \text{ alone}$$

$$\frac{1}{2xy}(-2y - 2y) = -\frac{4y}{2xy} = -\frac{2}{x} \quad f(x) \text{ alone.}$$

$$\text{I.F.} = e^{\int -\frac{2}{x}dx} = x^{-2}$$

$$\int \left(\frac{1}{x} - \frac{y^2}{x^2}\right)dx + \int 2\frac{y}{x}\,dy \Rightarrow \log x + \frac{y^2}{x} = \log A$$

$$A = x e^{y^2/x}$$

Question 2

The solution of the differential equation $2x\frac{dy}{dx} = 2 - y$ is

A $y = 2 - \sqrt{\frac{c}{x}}$

B $y = 2 + \sqrt{\frac{c}{x}}$

C $y = 2 - c\sqrt{x}$

D $y = 2 + c\sqrt{x}$

$$\int \frac{2\,dy}{2-y} = \int \frac{dx}{x}$$

$$-2\log(2-y) = \log x - \log c$$

$$(2-y)^{-2} = x/c$$

$$\frac{c}{x} = (2-y)^2$$

$$2 - y = \sqrt{\frac{c}{x}}$$

$$y = 2 - \sqrt{\frac{c}{x}}$$

Question 3

The general solution of the differential equation $\frac{dy}{dx} = \frac{y}{x} + \tan\frac{y}{x}$

A $\cos\frac{y}{x} = c$

B $\sin\frac{y}{x} = c$

C $\sin\frac{y}{x} = cx$

D $\cos\frac{y}{x} = cx$

$$\frac{dy}{dx} - \frac{y}{x} = \tan\frac{y}{x}$$

$$y/x = v \qquad \frac{dy}{dx} = v.1 + x\frac{dv}{dx}$$

$$v + x\frac{dv}{dx} = v + \tan v$$

$$\int \frac{dv}{\tan v} = \int \frac{dx}{x}$$

$$\log \sin v = \log x + \log c$$

$$\sin y/x = cx$$

Question 4

The DE $xdy - ydx + 2x^3\,dx = 0$ has the solution

A $y + x^3 = c_1x$

B $-y + x^3 = c_2x$

C $y^3 - x^3 = c_4x$

D $y - x^3 = c_3x$

$$x\,dy - y\,dx + 2x^3\,dx = 0$$

$$\frac{dy}{dx} - \frac{y}{x} + 2x^2 = 0$$

$$\frac{dy}{dx} - \frac{y}{x} = -2x^2$$

$P = -\frac{1}{x}$; $Q = -2x^2$

$$I.F. = x^{-1}$$

$$y.(I.F.) = \int Q\,(I.F.)$$

$$y\,(x^{-1}) = -\int 2x^2 \cdot x^{-1}$$

$$\frac{y}{x} = -\frac{x^2}{2} + c$$

$$y = -x^3 + cx$$

$$y + x^3 = cx$$

Question 5

Solution of the DE
$(2D+1)^2 y = 4e^{-\frac{x}{2}}$ is

A. $y = (c_1 + c_2 x)e^{-\frac{x}{2}}$

B. $y = \left(c_1 + c_2 x + \frac{1}{2}x^2\right)e^{-\frac{x}{2}}$

C. $y = \left(c_1 + c_2 x + \frac{1}{4}x^2\right)e^{-\frac{x}{2}}$

D. None of these

A.E. $(2D+1)^2 = 0$

$D = -1/2, -1/2$

C.F. $= (c_1 + c_2 x)e^{-\frac{1}{2}x}$

$P.I. = \frac{1}{(2D+1)^2} \cdot 4e^{-x/2}$

$= x \cdot \frac{1}{4(2D+1)} \; 4e^{-x/2}$

$P.I. = x^2 \cdot \frac{1}{2} \cdot e^{-x/2}$

$y = CF + PI. = (c_1 + c_2 x)e^{-x/2} + \frac{1}{2}x^2 e^{-x/2}$

Question 6

The solution of the differential equation $(D^2 + 1)^2 y = 0, D = \frac{d}{dx}$, is

A $(A_1 + A_2x)\cos x + (A_3 + A_4x)\sin x$

B $e^x(A\cos x + B\sin x)$

C $(A_1 + A_2)\cos x + (A_3 + A_4)\sin x$

D $A\cos x + B\sin x$

$(D^2+1)^2 = 0$

$[(D+i)(D-i)]^2 = 0$

$D = 0 \pm i\ ; 0 \pm i$ $\quad \alpha \pm i\beta$

$y = e^{0x}[(C_1+C_2x)\cos x + (C_3+C_4x)\sin x]$

Question 7

The solution of the differential equation $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{3x}$ is given by

A $y = c_1e^x + c_2e^{2x} + \frac{1}{2}e^{3x}$

B $y = c_1e^{-x} + c_2e^{-2x} + \frac{1}{2}e^{3x}$

C $y = c_1e^{-x} + c_2e^{2x} + \frac{1}{2}e^{3x}$

D $y = c_1e^x + \frac{1}{2}e^{-3x}$

$D^2 - 3D + 2 = 0$

$(D-2)(D-1) = 0$

$D = 1, 2$

$C.F. = C_1e^x + C_2e^{2x}$

$P.I. = \frac{1}{D^2-3D+2} \cdot e^{3x} = \frac{1}{3^2-3\times3+2} \cdot e^{3x} = \frac{e^{3x}}{2}$

$y = CF + PI$

Question 8

The particular integral of the differential equation

$(D^3 - D)y = e^x + e^{-x}, D = \frac{d}{dx}$ is

A $\frac{1}{2}(e^x + e^{-x})$

B $\frac{1}{2}x(e^x + e^{-x})$

C $\frac{1}{2}x^2(e^x + e^{-x})$

D $\frac{1}{2}x^2(e^x - e^{-x})$

$$P.I. = \frac{1}{D^3 - D}\cdot(e^x + e^{-x})$$

$$x\cdot\frac{1}{(3D-1)}(e^x + e^{-x})$$

$$x\left[\frac{e^x}{2} - \frac{e^{-x}}{4}\right]$$

$$\frac{x}{2}\left[e^x - \frac{e^{-x}}{2}\right]$$

Question 9

The particular integral for the differential equation

$\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1 + x^2$ is given by

A $\frac{1}{9}x^3 + \frac{1}{4}x^2 = \frac{25}{12}x$

B $-\frac{x^3}{18} + \frac{x^2}{36} - \frac{25}{108}x$

C $x^3 - \frac{1}{2}x^2 - \frac{25}{9}x$

D $\frac{1}{3}x^2 + \frac{1}{12}x^2 - \frac{25}{36}x$

$$P.I. = \frac{1}{D^3 - D^2 - 6D}(1+x^2)$$

$$= \frac{1}{-6D\left(1 + \frac{D - D^2}{6}\right)}(1+x^2)$$

$$= -\frac{1}{6D}\left[1 + \frac{D-D^2}{6}\right]^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left[1 - \left(\frac{D-D^2}{6}\right) + \left(\frac{D-D^2}{6}\right)^2 \cdots\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1 + x^2 - \frac{x}{3} + \frac{1}{3} + \frac{1}{18} + \cdots\right]$$

$D(1+x^2) = 2x$

$D^2(1+x^2) = 2$

$D^3(1+x^2) = 0$

$$-\frac{1}{6}\left[x+\frac{x^3}{3}-\frac{x^2}{6}+\frac{7}{18}x\right]$$

$$-\frac{x^3}{18}+\frac{x^2}{36}-\frac{25}{108}x$$

Question 10

The solution of the differential equation $xdy - ydx = \sqrt{x^2 + y^2}dx$ is given by

A $y = \frac{c_1}{x} + \sqrt{x^2 - y^2}$

B $y = c_2x^2 - \sqrt{x^2 + y^2}$

C $y = \frac{c_3}{x^2} + \frac{1}{\sqrt{x^2+y^2}}$

D $y = \frac{c_4}{x} - \frac{1}{\sqrt{x^2-y^2}}$

$$\frac{dy}{dx} - \frac{y}{x} = \sqrt{1 + (y/x)^2}$$

$$y = vx$$

$$\frac{dy}{dx} = v \cdot 1 + x\frac{dv}{dx}$$

$$y + x\frac{dv}{dx} - y = \sqrt{1 + v^2}$$

$$\int \frac{dv}{\sqrt{1+v^2}} = \int \frac{dx}{x}$$

$$\log\left(v + \sqrt{1+v^2}\right) = \log x + \log c$$

$$v + \sqrt{1+v^2} = cx$$

$$\frac{y}{x} + \sqrt{1 + \frac{y^2}{x^2}} = cx \qquad y = cx^2 - \sqrt{x^2+y^2}$$

Thank you

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Soldiers !