ENGINEERING MATHEMATICS

ALL BRANCHES

Addition theorem, Conditional Probability

DPP-02 Solution

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Q1. A box contains 5 black balls and 3 red balls. A total of three balls are picked from the box one after another, without replacing them back. The probability of getting two black balls and one red ball is

A. 3/8

B. 2/15

C. 15/28

D. 1/2

Box → 5B, 3R (8 balls)

$$\underset{\text{I}}{P(RBB)} + \underset{\text{II}}{P(BRB)} + \underset{\text{III}}{P(BBR)}$$

$$\frac{3}{8}\times\frac{5}{7}\times\frac{4}{6} + \frac{5}{8}\times\frac{3}{7}\times\frac{4}{6} + \frac{5}{8}\times\frac{4}{7}\times\frac{3}{6}$$

$$3\times\left(\frac{3\times4\times5}{8\times7\times6}\right) = \frac{15}{28}$$

Q2. Seven car accidents occurred in a week, what is the probability that they all occurred on the same day?

A. $1/7^7$

B. $1/7^6$

C. $1/2^7$

$$\text{Total no. of ways} = \underset{\substack{\downarrow\\ 7\,\text{car}}}{\_\_}\ \underset{\substack{\downarrow\\ 7\,\text{car}}}{\_\_}\ \_\_\ \_\_\ \_\_\ \_\_\ \_\_$$

$$n(S) = 7^7$$

$$n(E) = M + T + W + T + F + S + S$$

$$= 7$$

$$P(E) = \frac{7}{7^7} = \frac{1}{7^6}$$

Q3. From a pack of regular playing cards, two cards are drawn at random. What is the probability that both cards will be kings, if first card is NOT replaced?

A. 1/26

B. 1/52

C. 1/169

✓ D. 1/221

4 Kings

52 cards

$$P(E) = \frac{{}^{4}C_2}{{}^{52}C_2} = \frac{\frac{4 \times 3}{1 \times 2}}{\frac{52 \times 51}{1 \times 2}} = \frac{1}{22}$$

Q4. A box contains 10 screws, 3 of which are defective. Two screws are drawn at random with replacement. The probability that none of the two screws will be defective

A. 100%

B. 50%

C. 49%

D. None of these

10 Screws

3 Def

7 Non-defective

$$S_1 \times S_1$$

$$\frac{7}{10} \times \frac{7}{10} = 49\%$$

Q5. A box contains 5 black and 5 red balls. Two balls are randomly are picked one after another from the box, without replacement. The probability for both balls being red is

A. 1/90

B. ½

C. 19/90

D. 2/9

$$P(R_1 \cap R_2) = P(R_1) \cdot P(R_2/R_1)$$

$$= \frac{5}{10} \times \frac{4}{9} = \frac{2}{9}$$

Q6. A fair unbiased coin was tossed in succession 4 times and resulted in following outcomes i) Head ii)I Head iii) Head iv) Head. The probability of obtaining a tail when the coins is tossed again is

A. 0

B. ½

C. 4/5

D. 1/3

$$H\,H\,H\,H \begin{cases} \to H \\ \to T \end{cases}$$

$$p(T) = 1/2$$

$$1.\,1.\,1.\,1. \times \frac{1}{2} = \frac{1}{2}$$

$$\frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2} = \frac{1}{32}$$

Q7. Consider two events E_1 and E_2 such that $\underbrace{P(E_1) = 1/2}$, $\underbrace{P(E_2) = 1/3}$ and $\underbrace{P(E_1 \cap E_2) = 1/5}$. Which of the following statements is true?

✗ A. $P(E_1 \cup E_2) = 2/3$

✗ B. E_1 and E_2 are independent

✓ C. E_1 and E_2 are not independent

✗ D. $P(E_1/E_2) = 4/5$

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$= \frac{1}{2} + \frac{1}{3} - \frac{1}{5}$$

$$P(E_1 \cup E_2) = \frac{19}{30}$$

$$P(E_1 \cap E_2) = P(E_1) \cdot P(E_2)$$

$$= \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{6} \neq \frac{1}{5} \quad \therefore \text{not independent}$$

$$P(E_1/E_2) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{1/5}{1/3} = 3/5$$

Q8. E_1 and E_2 are events in a probability space satisfying the following constraints $P(E_1) = P(E_2)$; $P(E_1 \cup E_2) = 1$; E_1 and E_2 are independent then $P(E_1) =$

A. 0

B. 1/4

C. 1/2

D. 1

$$P(E_1 \cup E_2) = P(E_1) + P(E_2) - P(E_1 \cap E_2)$$

$$P(E_1) + P(E_2) - P(E_1) \cdot P(E_2)$$

$$1 = x + x - x \cdot x$$

$$1 = 2x - x^2$$

$$x^2 - 2x + 1 = 0$$

$$(x-1)^2 = 0$$

$$x = 1, 1$$

$$P(E_1) = P(E_2) = 1$$

Q9. If P and Q are two random events, then the following is TRUE

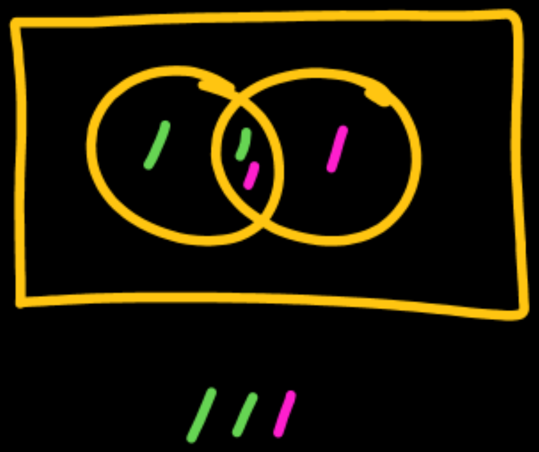
A. Independence of P and Q implies that probability $(P \cap Q) = 0$

B. Probability $(P \cup Q) \geq$ Probability (P) + Probability (Q)

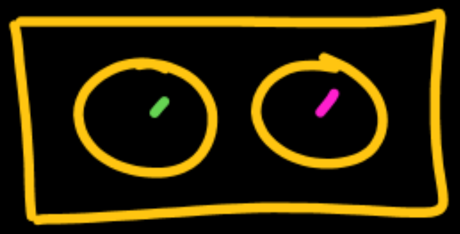
C. If P and Q are mutually exclusive, then they must be independent

D. Probability $(P \cap Q) \leq$ Probability (P)

$P \cap Q = 0$ (Mutually exclusive)



$P(P \cup Q) \leq P(P) + P(Q)$



$P(P \cap Q) \leq P(P)$

$\leq P(Q)$

Q10. A box contains 20 defective items and 80 non-defective items. If two items are selected at random without replacement, what will be the probability that both items are defective?

A. 1/5

B. 1/25

C. 20/99

D. 19/495

20 D 80 N.D

$P(D_1) \cdot P(D_2/D_1)$

$P(D_1 \cap D_2) = \frac{20}{100} \times \frac{19}{99} = \frac{19}{495}$

Thank you

GW

Soldiers !