CS & IT ENGINEERING

Discrete Maths
Set Theory

Lecture No 01

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TOPICS TO BE COVERED

01 Basics operations in sets

02 set theory laws

03 Different operations thm

04 Infinite union

05 Union on intervals

Set: collection of an objects.

$A = \{1, 2, 3\}$

- Repeₜn are not allowed
- → order is not imp.
- $\{1, 2, 3\} = \{3, 2, 1\}$

$1 \in A$

$4 \notin A$.

A : set.

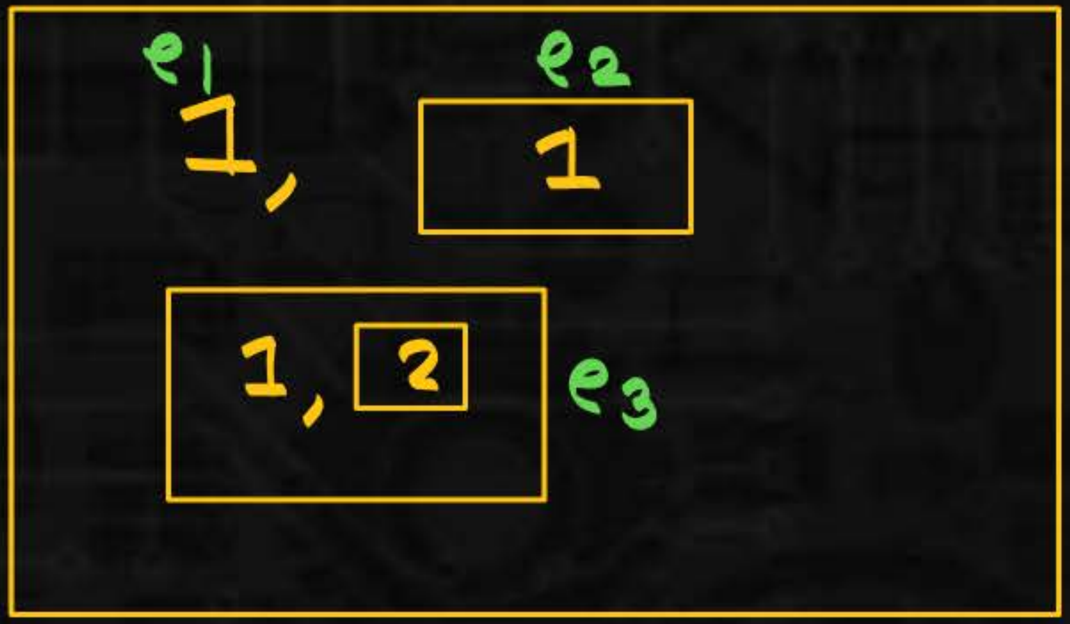
a : object

member

element

element $\in$ Set

$$A = \{ \overset{e_1}{1}, \overset{e_2}{\{1\}}, \overbrace{\{1, \{2\}\}}^{e_3} \}$$



$1 \in A$ (T)   e1 : 1.

$\{1\} \in A$ (T) $\{1, \{2\}\} \in A$ (T)

$|A| = 3$

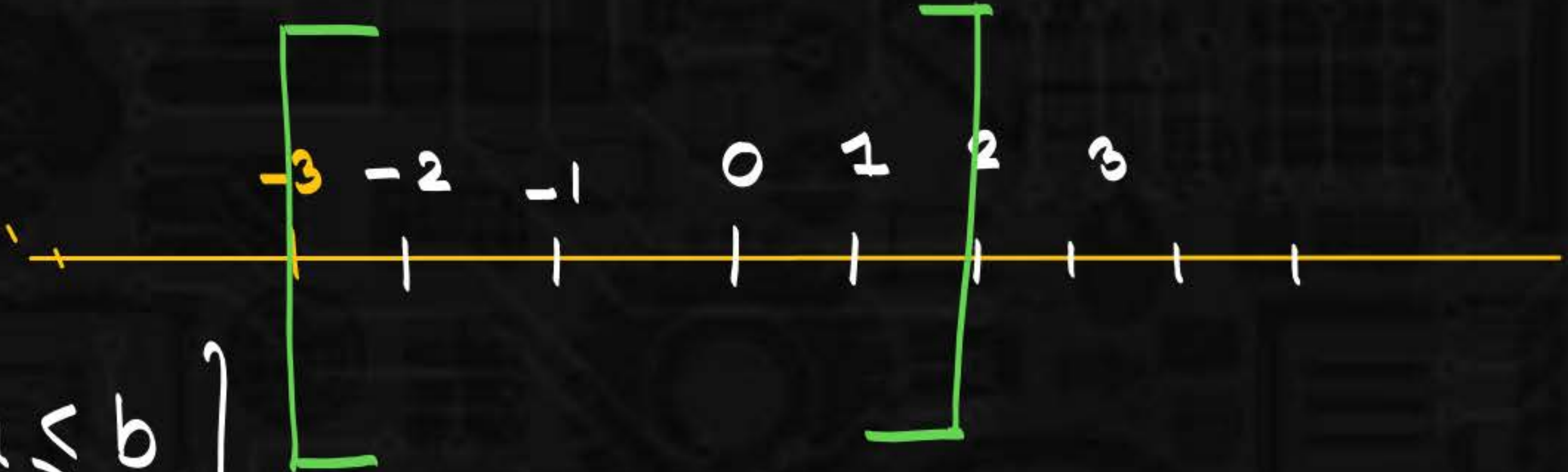
Rep :

Roster : $A = \{1, 2, 3\}$ $\qquad$ $A : \{1, 2, 3 \ldots\ldots 20\}$

Set builder : $\{x \mid \text{condtn}\}$ $\qquad$ $\{x \mid 1 \leq x \leq 20\}$

$\{x \mid 1 \leq x \leq 3\}$

Closed Interval:



$$[a, b] = \{x \mid a \leq x \leq b\}$$

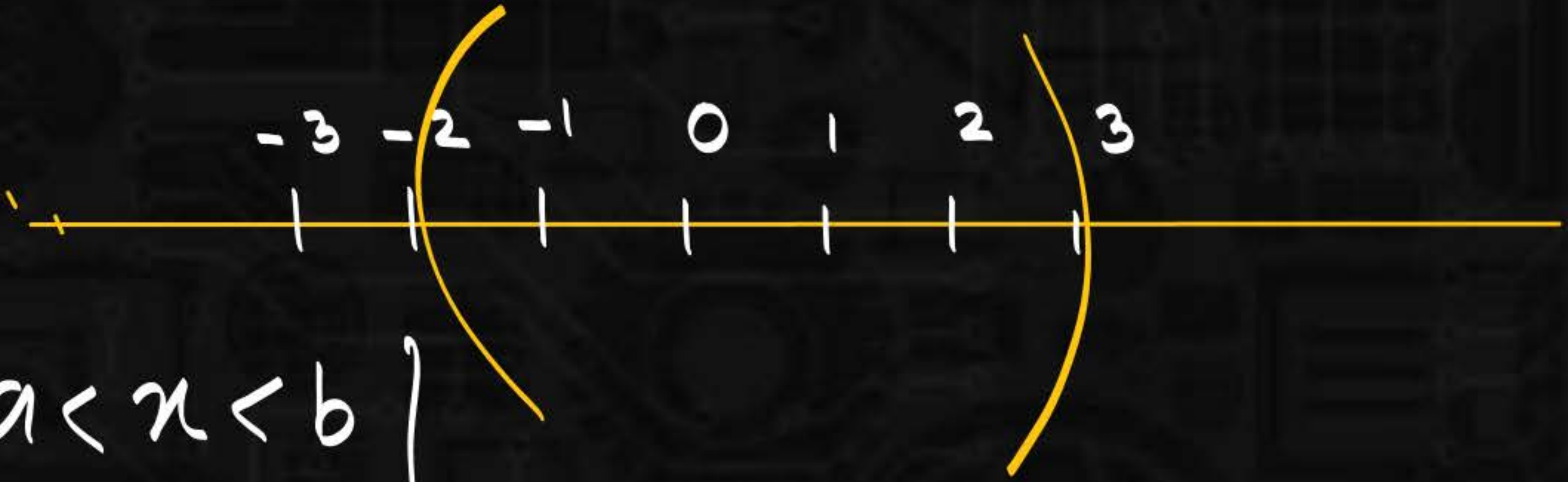
$$[-3, 2] = \{-3, -2, -1, 0, 1, 2\}$$

$$A = \{1, 2, 3\}$$

$$A = \{x \mid 1 \leq x \leq 3\}$$

$$[1, 3] = \{1, 2, 3\}$$

Open Interval:



$$(-2, 3) = \{x \mid a < x < b\}$$

$$= \{-1, 0, 1, 2\}$$

$[-2, 3)$

$= \{-2, -1, 0, 1, 2\}$

Subset : ($\subseteq$)

$A \subseteq B$

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$

if $x \in A$ then $x \in B$

$A \subseteq B$ $\forall x \left(\boxed{x \in A} \rightarrow x \in B \right)$

F $\longrightarrow$

T.

any set :

$\begin{cases} \phi \subseteq A \\ A \subseteq A \checkmark \end{cases}$

$\forall x \left(x \in \phi \rightarrow x \in A \right)$

Empty set/null set:

$A = \{\ \}$ $\quad |A| = 0$

$A_1 = \{\{\ \}\}$ $\quad |A| = 1.$

$A = \{\ \}$ $\quad |A| = 0$

$A_1 = \{\{\ \}\}$

not empty set

$A = \{\ \}$

proper subset ($\subset$)

$\forall x(x \in A \rightarrow x \in B \wedge \exists x(x \in B \wedge x \notin A))$

$A = \{1, 2, 3\}$ $B = \{1, 2, 3, 4\}$

Subset:

$A \subseteq B$

$|A| \leq |B|$

proper subset:

$A \subset B$

$|A| < |B|$

$$A = \{ \overset{e_1}{x}, \overset{e_2}{\{x\}}, \overset{e_3}{\{\{x\}\}} \}$$

element $\in$ set

$x \in A$ (T)

set $\subseteq$ set

$\{x\} \in A$ (T) $\quad \{x\} \subseteq A$ (T)

$\{$element$\} \subseteq$ set.

$A = \{1, \{1\}, \{2\}\}$ $e_1 : 1$ $e_2 : \{1\}$ $e_3 : \{2\}$

a) $1 \in A$ (T)

b) $\{1\} \in A$ (T)

c) $\{1\} \subseteq A$ (T) e_1

d) $\{\underbrace{\{1\}}_{e_2}\} \subseteq A$ (T)

e) $\{2\} \in A$ (T)

f) $\{2\} \subseteq A$ (false)

g) $\{\underbrace{\{2\}}_{e_3}\} \subseteq A$. (T)

h) $\{\underbrace{\{2\}}_{e_3}\} \subset A$ (T)

Except f all are True.

a) $\phi \in \phi$ (false)

b) $\phi \subset \phi$ (false)

c) $\phi \subseteq \phi$ (True)

d) $\phi \in \{\phi\}$ (T)

e) $\phi \subset \{\phi\}$ (T)

f) $\phi \subseteq \{\phi\}$ (T)

$\phi \subset$ anything except ϕ.

$\phi \subseteq$ anything.

$A = \{1, 2, 3\}$ $\quad$ $\underbrace{\text{powerset}}$: $P(A) / 2^A$

$\hookrightarrow$ Collection of subset

$\emptyset$ $\quad$ $\{1, 2\}$

$\{1\}$ $\quad$ $\{2, 3\}$

$\{2\}$ $\quad$ $\{1, 3\}$

$\{3\}$ $\quad$ $\{1, 2, 3\}$

$$P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1 2 3\}\}$$

Total no. of elements in $P(A) = 2^A$

$$\left\{\begin{array}{l} \emptyset \in P(A) \\ \emptyset \subseteq P(A) \end{array}\right.$$

$\emptyset \subseteq$ anyset.

Operations in Set

Set difference $(-)$

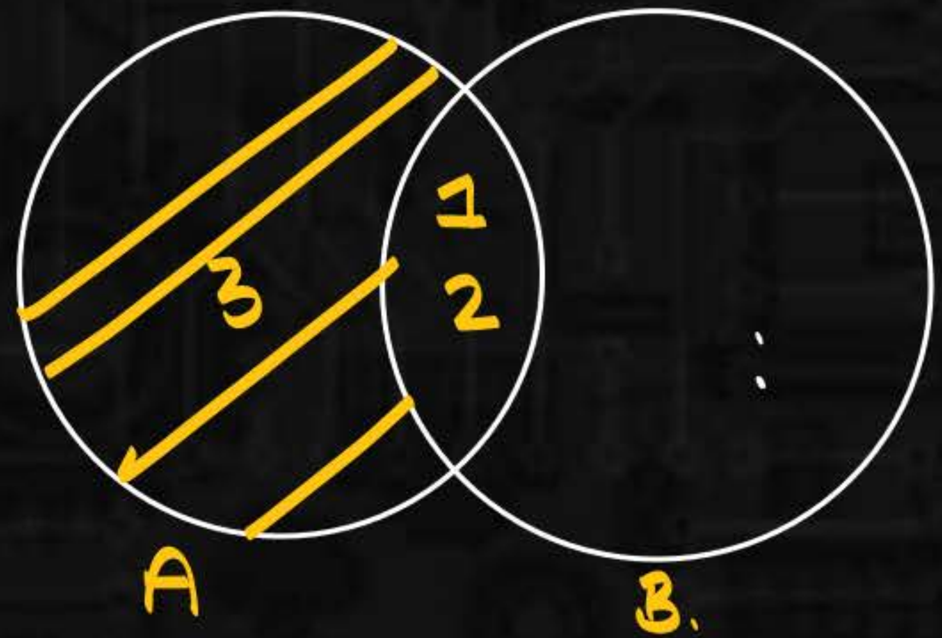
Symmetric difference $(\Delta / \oplus)$

$A = \{1, 2, 3\}$ $B = \{1, 2\}$

$A - B = \begin{matrix} \cancel{1} \\ \cancel{2} \\ 3 \end{matrix} - \begin{matrix} \cancel{1} \\ \cancel{2} \end{matrix} = \{3\}$

$B - A = \begin{matrix} \cancel{1} \\ \cancel{2} \end{matrix} - \begin{matrix} \cancel{1} \\ \cancel{2} \\ 3 \end{matrix} = \phi$

$A = \{1, 2, 3\} \quad B = \{1, 2\}$



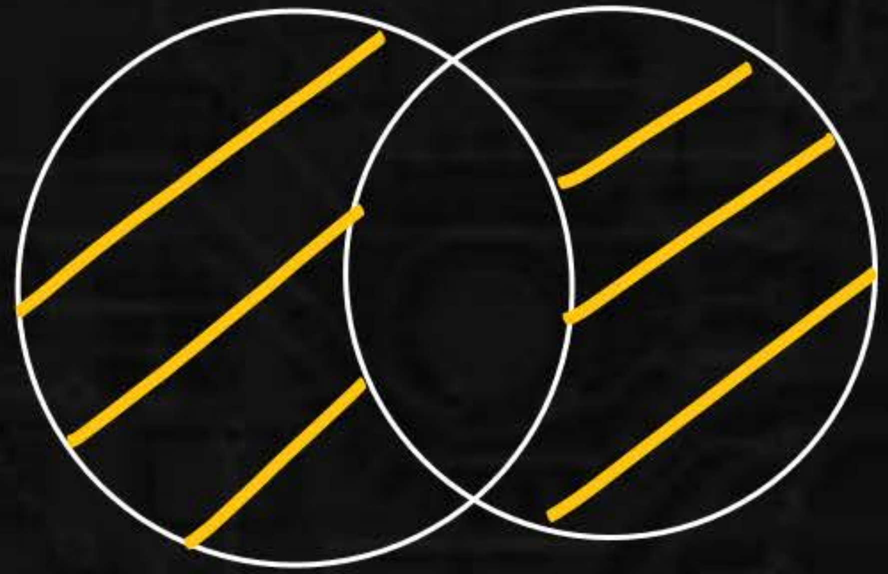
$A - B = A - (A \cap B)$

$= \begin{matrix}1\\2\\3\end{matrix} - \begin{matrix}1\\2\end{matrix} = \{3\}$

$B - A = \begin{matrix}\cancel{1}\\\cancel{2}\end{matrix} - \begin{matrix}\cancel{1}\\\cancel{2}\\3\end{matrix} = \phi$

$\Delta / \oplus$

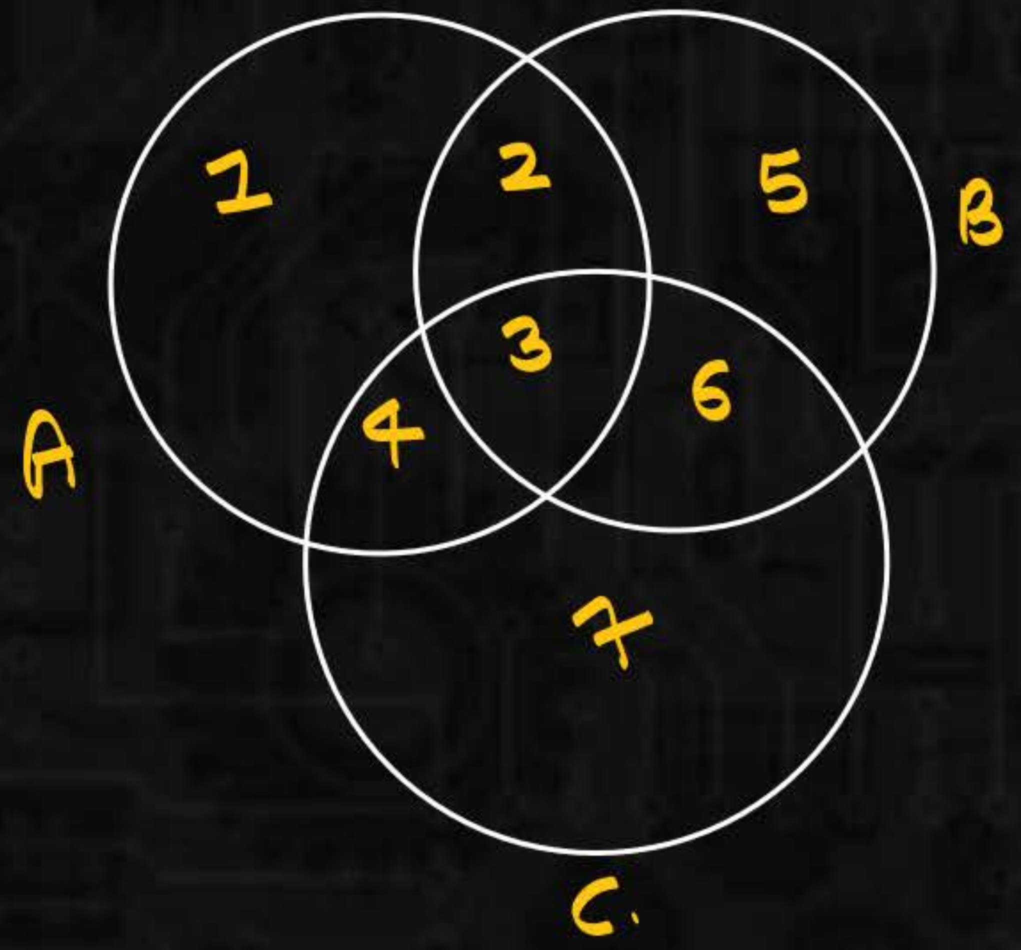
$$A \Delta B = (A - B) \cup (B - A)$$



$$= (A \cup B) - (A \cap B)$$

$$A \Delta B = B \Delta A$$

$$A \Delta (B \Delta C) = (A \Delta B) \Delta C.$$

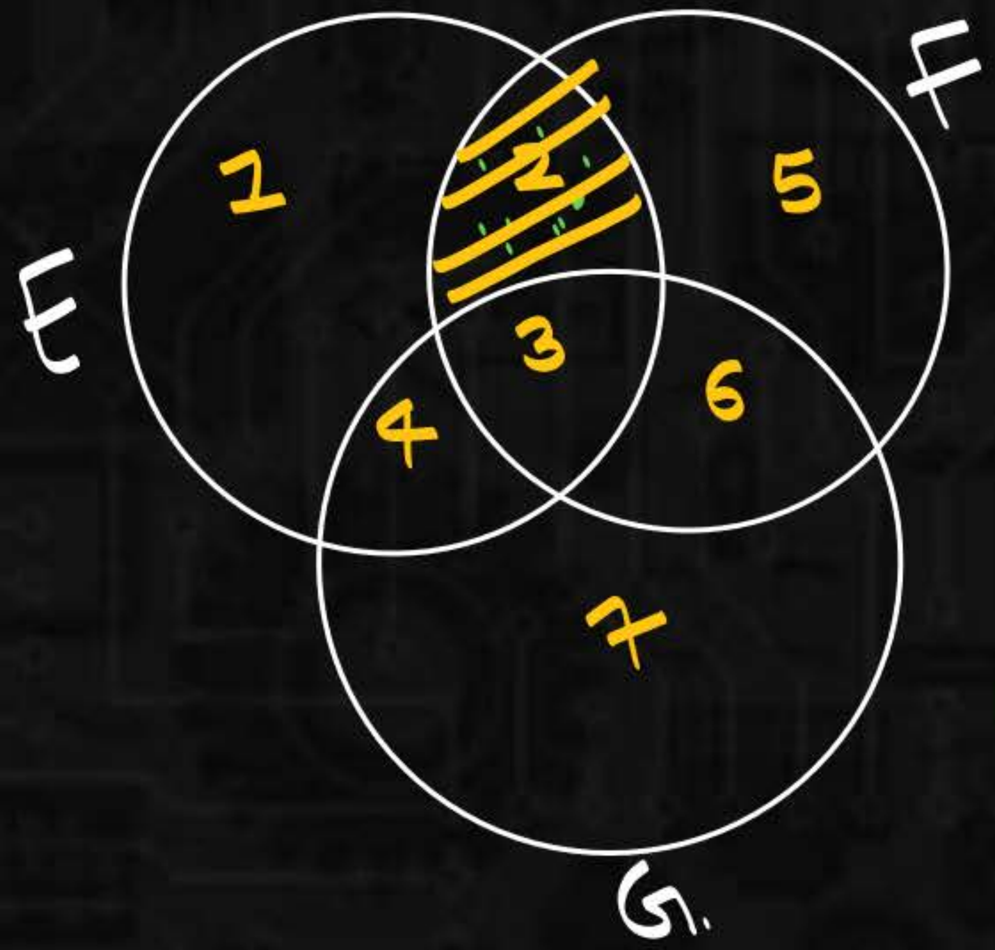


$(A - B) - C$

$$\underbrace{\begin{matrix} 1 \\ \cancel{2} \\ \cancel{3} \\ 4 \end{matrix} - \begin{matrix} \cancel{2} \\ \cancel{3} \\ 5 \\ 6 \end{matrix}}$$

$$\underbrace{\begin{matrix} 1 \\ \cancel{4} \end{matrix} - \begin{matrix} \cancel{4} \\ 3 \\ 6 \\ 7 \end{matrix}}$$

$1.$



$$x = (E\cap F) - (F\cap G)$$

$$y = (E - (E\cap G)) - (E - F)$$

$$x = y$$

$$(E\cap F) - (F\cap G)$$

$$\underbrace{\{1,2,3,4\} \cap \{2,5,3,6\}}_{2,\ \cancel{3}} - \underbrace{\{2,3,5,6\} \cap \{3,6,4,7\}}_{\cancel{3},\ 6} = \{2\}$$

$$E - (E\cap G) - (E - F)$$

$$\underbrace{\underbrace{\{1,2,\cancel{3},\cancel{4}\} - \{\cancel{3},\cancel{4}\}}_{\{\cancel{1},\ 2\}} - \underbrace{\{1,\cancel{2},\cancel{3},4\} - \{\cancel{2},\cancel{3},5,6\}}_{\{\cancel{1},\ 4\}}}_{\{2\}}$$

D.R.

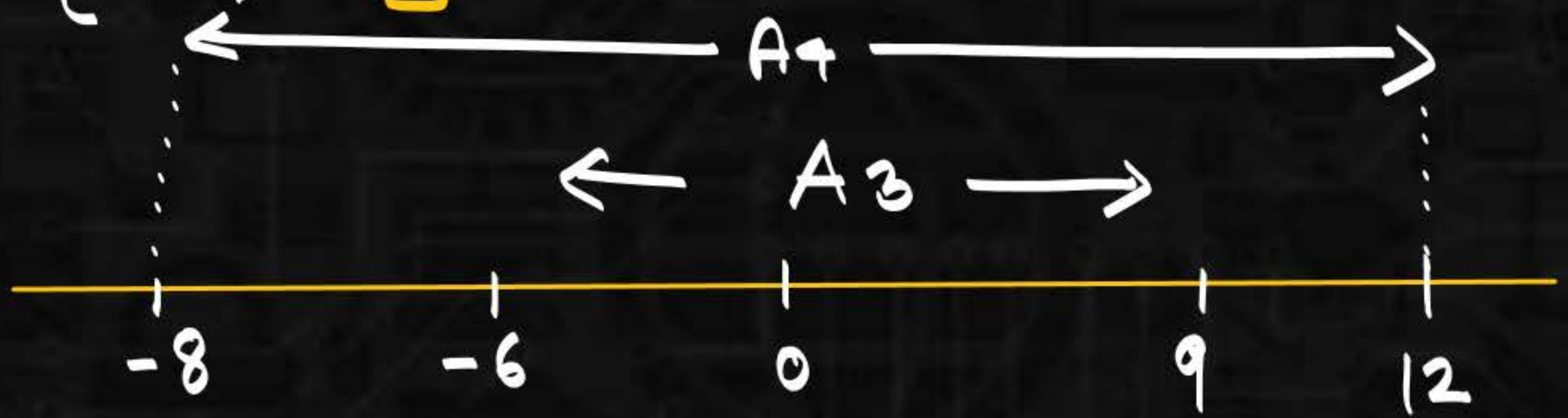
$$A_n = [-2n, 3n]$$

$$\bigcup_{i=1}^{7} A_i = A_7.$$

$$\bigcap_{i=1}^{7} A_i = A_1.$$

$n = 3$

$$A_3 = [-6, 9] \qquad A_4 = [-8, 12]$$



$$A_3 - A_4 = \phi$$

$$A_4 - A_3 = [-8, -6) \cup (9, 12]$$

$$\bigcup_{i=1}^{7} A_i = A_1 \cup A_2 \cup A_3 \ldots \quad A_7 = [-14, 21]$$

$$\bigcap_{i=1}^{7} A_i = A_1 \cap A_2 \ldots A_7 = A_1 = [-2, 3]$$

6. Prove each of the following results without using Venn diagrams or membership tables. (Assume a universe $\mathcal{U}$.)

a) If $A \subseteq B$ and $C \subseteq D$, then $A \cap C \subseteq B \cap D$ and $A \cup C \subseteq B \cup D$.

b) $A \subseteq B$ if and only if $A \cap \overline{B} = \emptyset$.

c) $A \subseteq B$ if and only if $\overline{A} \cup B = \mathcal{U}$.

7. Prove or disprove each of the following:

a) For sets $A, B, C \subseteq \mathcal{U}$, $A \cap C = B \cap C \Rightarrow A = B$.

b) For sets $A, B, C \subseteq \mathcal{U}$, $A \cup C = B \cup C \Rightarrow A = B$.

c) For sets $A, B, C \subseteq \mathcal{U}$, $[(A \cap C = B \cap C) \wedge (A \cup C = B \cup C)] \Rightarrow A = B$.

d) For sets, $A, B, C \subseteq \mathcal{U}$, $A \triangle C = B \triangle C \Rightarrow A = B$.

8. Using Venn diagrams, investigate the truth or falsity of each of the following, for sets $A, B, C \subseteq \mathcal{U}$.

a) $A \triangle (B \cap C) = (A \triangle B) \cap (A \triangle C)$

b) $A - (B \cup C) = (A - B) \cap (A - C)$

c) $A \triangle (B \triangle C) = (A \triangle B) \triangle C$

9. If $A = \{a, b, d\}$, $B = \{d, x, y\}$, and $C = \{x, z\}$, how many proper subsets are there for the set $(A \cap B) \cup C$? How many for the set $A \cap (B \cup C)$?

6. Determine whether each of the following statements is true or false. For each false statement, give a counterexample.

a) If A and B are infinite sets, then $A \cap B$ is infinite.

b) If B is infinite and $A \subseteq B$, then A is infinite.

c) If $A \subseteq B$ with B finite, then A is finite.

d) If $A \subseteq B$ with A finite, then B is finite.

7. A set A has 128 subsets of even cardinality. (a) How many subsets of A have odd cardinality? (b) What is $|A|$?

8. Let $A = \{1, 2, 3, \ldots, 15\}$.

a) How many subsets of A contain all of the odd integers in A?

b) How many subsets of A contain exactly three odd integers?

c) How many eight-element subsets of A contain exactly three odd integers?

d) Write a computer program (or develop an algorithm) to generate a random eight-element subset of A and have it print out how many of the eight elements are odd.

9. Let $A, B, C \subseteq \mathcal{U}$. Prove that

$$(A \cap B) \cup C = A \cap (B \cup C) \text{ if and only if } C \subseteq A.$$

12. Let $A = \{1, 2, 3, 4, 5, 7, 8, 10, 11, 14, 17, 18\}$.

a) How many subsets of A contain six elements?

b) How many six-element subsets of A contain four even integers and two odd integers?

c) How many subsets of A contain only odd integers?


THANK YOU GW
SOLDIERS !