ENGINEERING MATHEMATICS

ALL BRANCHES

Linear Dependancy of Vectors
Linear Algebra
DPP-06 Solution

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Question 1

The number of linearly independent solutions of the system of

equations $\begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = 0$ is equal to

- A 1
- B 2
- C 3
- D 0

$x_1 + 2x_3 = 0$

$x_1 - x_2 = 0$

$2x_1 - 2x_2 = 0$

Let $x_1 = K$

$\therefore x_2 = K$ and $x_3 = -K/2$

Homogenous system

$\rightarrow \rho(A) = n$

Trivial soln.

$\rightarrow \rho(A) < n$

Non-trivial soln.

(Infinite soln.)

$$\left.\begin{matrix}\text{No. of L.I. solutions}\\ \text{No. of free variables}\end{matrix}\right\} = n - r$$

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 1 & -1 & 0 \\ 2 & -2 & 0 \end{bmatrix}$$

Check $|A|_{3\times 3} = 1(0) + 0 + 2(-2+2) = 0 \quad \therefore \rho(A) < 3$

Check $|A|_{2\times 2} = \begin{vmatrix} 0 & 2 \\ -1 & 0 \end{vmatrix} = 2 \neq 0 \quad \therefore \rho(A) = 2$

$\therefore$ No. of LI solutions $= n - r = 3 - 2 = 1$

Question 2

Consider the following two statements:

(i) The maximum number of linearly independent column vectors of a matrix A is called the rank of A.

$\rho(A) =$ No. of L I rows
No. of L I columns

(ii) If A is an n × n square matrix, it will be nonsingular if rank A = n.

With reference to the above statements, which of the following applies?

A. Both the statements are false

B. Both the statements are true

C. I is true but II is false

D. I is false but II is true

Question 3

If the rank of a (5×6) matrix Q is 4, then which one of the following statements is correct?

$Q^{-1} = \frac{\text{adj } Q}{|Q|}$ if $|Q| = 0$ then QQ^T are not invertible

A. Q will have four linearly independent rows and four linearly independent columns.

B. Q will have four linearly independent rows and five linearly independent columns.

$\rho(A) = \begin{matrix}\text{No. of LI rows} \\ \text{No. of LI colums}\end{matrix} = 4$

C. QQ^T will be invertible

D. Q^TQ will be invertible

Question 4

If $A \in R_{n \times n}$, det $A \neq 0$, then

$\Rightarrow \rho(A) = n =$ No. of LI rows/columns

A. A is nonsingular and the rows and columns of A are linearly independent.

B. A is non singular and the rows A are linearly independent.

C. A is non singular and the A has one zero rows.

D. A is singular.

Question 5

Which one of the following describes the relationship among the three vectors, $\hat{i}+\hat{j}+\hat{k}$, $2\hat{i}+3\hat{j}+\hat{k}$ and $5\hat{i}+6\hat{j}+4\hat{k}$?

A The vectors are mutually perpendicular.

B The vectors are linearly dependent.

C The vectors are linearly independent.

D The vectors are unit vectors.

$$\vec{X}_1 = \hat{i} + \hat{j} + \hat{k} = [1 \quad 1 \quad 1]^T$$
$$\vec{X}_2 = 2\hat{i} + 3\hat{j} + \hat{k} = [2 \quad 3 \quad 1]^T$$
$$\vec{X}_3 = 5\hat{i} + 6\hat{j} + 4\hat{k} = [5 \quad 6 \quad 4]^T$$

$$A = [X_1 \quad X_2 \quad X_3] = \begin{bmatrix} 1 & 2 & 5 \\ 1 & 3 & 6 \\ 1 & 1 & 4 \end{bmatrix}$$

Check $|A|_{3\times 3} = 1(12-6) - 1(8-5) + 1(12-15)$
$= 6 - 3 - 3 = 0 \quad \therefore \rho(A) < 3$

$|A|_{2\times 2} = \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1 \neq 0 \quad \therefore \rho(A) = 2$

$\therefore \rho(A) <$ no. of vectors $\Rightarrow$ So set of vectors are L.D.

Question 6

Find the value of λ for which the following vectors are linearly dependent.

$$X_1 = \begin{bmatrix} 1 \\ -2 \\ \lambda \end{bmatrix} \quad X_2 = \begin{bmatrix} 2 \\ -1 \\ 5 \end{bmatrix} \quad X_3 = \begin{bmatrix} 3 \\ -5 \\ 7\lambda \end{bmatrix}$$

A 3/14

B 5/14 ✓

C 1/14

D None of these

For set of vectors to be linearly dependent;
$\rho(A) <$ No. of vectors

$$A = [X_1 \ \ X_2 \ \ X_3] = \begin{bmatrix} 1 & 2 & 3 \\ -2 & -1 & -5 \\ \lambda & 5 & 7\lambda \end{bmatrix}$$

Since $\rho(A) < 3 \quad \therefore |A|_{3\times3} = 0$

$$|A| = 1(-7\lambda+25)-2(-14\lambda+5\lambda)+3(-10+\lambda) = 0$$

$$-7\lambda+25+18\lambda-30+3\lambda=0$$

$$14\lambda = 5$$

$$\boxed{\lambda = 5/14}$$

Question 7

For the following vectors

$$X_1 = \begin{bmatrix} 3 \\ 1 \\ -4 \end{bmatrix} \quad X_2 = \begin{bmatrix} 2 \\ 2 \\ -3 \end{bmatrix}$$

Statement I: Rank of matrix is 2

Statement II: Vectors are linearly independent.

Which of the following statements true?

A. I

B. II

C. I & II

D. None

$$A = [X_1 \quad X_2] = \begin{bmatrix} 3 & 2 \\ 1 & 2 \\ -4 & -3 \end{bmatrix}$$

Largest minor order $\rightarrow 2 \times 2$

Check $|A|_{2\times 2} = \begin{vmatrix} 3 & 2 \\ 1 & 2 \end{vmatrix} = 4 \neq 0$

$\therefore \rho(A) = 2$

$\therefore \rho(A) =$ No. of vectors $= 2$

$\therefore$ The vectors are L.I.

Question 8

Which of the following set of vectors is linearly dependent?

A $(1, 0, 1)\ (-1, 1, 0)\ (5, -1, 2)$

B $(1, 2, 0)\ (1, 1, 1)\ (2, 0, 1)$

C $(1, 1, -1)\ (2, -3, 5)\ (-2, 1, 4)$

D $(2, 3, -1)\ (-4, 2, -6)\ (5, -4, 9)$

Option a) $\begin{bmatrix} 1 & 0 & 1 \\ -1 & 1 & 0 \\ 5 & -1 & 2 \end{bmatrix}$ $\quad \Delta = 1(2-0)+0+(1-5) \neq 0$

b) $\begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 1 \\ 2 & 0 & 1 \end{bmatrix}$ $\quad \Delta \neq 0$

c) $\begin{bmatrix} 1 & 1 & -1 \\ 2 & -3 & 5 \\ -2 & 1 & 4 \end{bmatrix}$ $\quad \Delta \neq 0$

d) $\begin{bmatrix} 2 & 3 & -1 \\ -4 & 2 & -6 \\ 5 & -4 & 9 \end{bmatrix}$ $\quad \Delta = 2(18-24)-3(-36+30)-1(16-10)$

$= -12+18-6 = 0$

$\Delta = 0 \quad \therefore \rho(A) < 3$

$\therefore$ Vectors are L.D.

$\rho(A) =$ no. of vectors

$\therefore$ Vectors of L I.

$\rho(A) <$ no. of vectors

$\therefore$ Vectors are L D.

$\therefore \rho(A) < 3$

$\Rightarrow |A|_{3\times3} = 0$

Thank you

GW Soldiers !