CS & IT ENGINEERING

Discrete Maths
Graph Theory

Lecture No. 09

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TOPICS TO BE COVERED

01 Annalysis In Connectivity

02 Various definition in Connectivity

03 Edge Connectivity

04 Vertex Connectivity

05 Largest inequality theorem

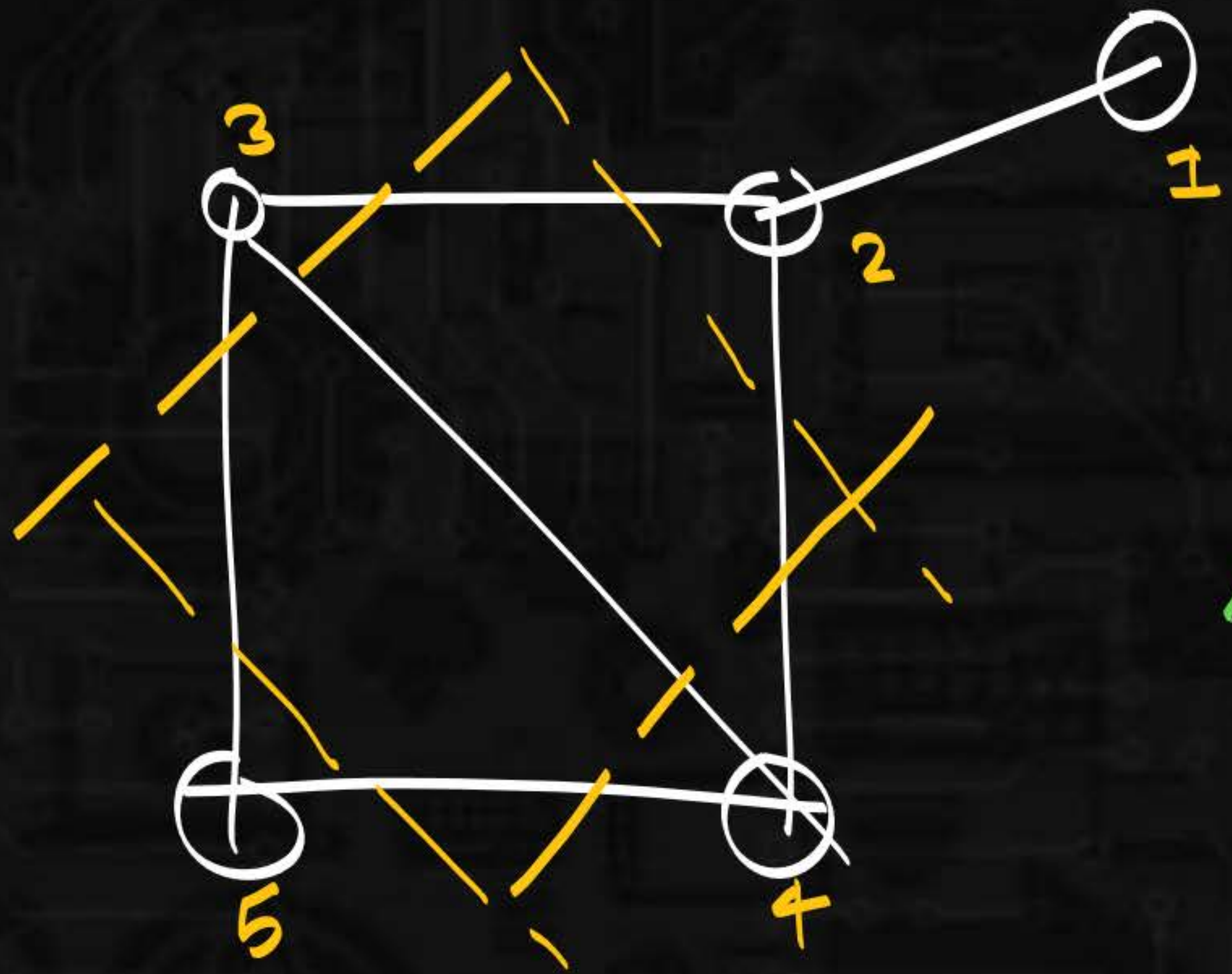
Connectivity in Graphs

cut edge
→ 1 edge.

*Cut set
→ set of edges.
1 → cut edge.
2
3
4

edge connectivity:
→ min + set of edges.

Connectivity in Graphs



edge con

$\boxed{\lambda(G) = 1}$

Cutset

size 1 → {12} cut edge.

Scan

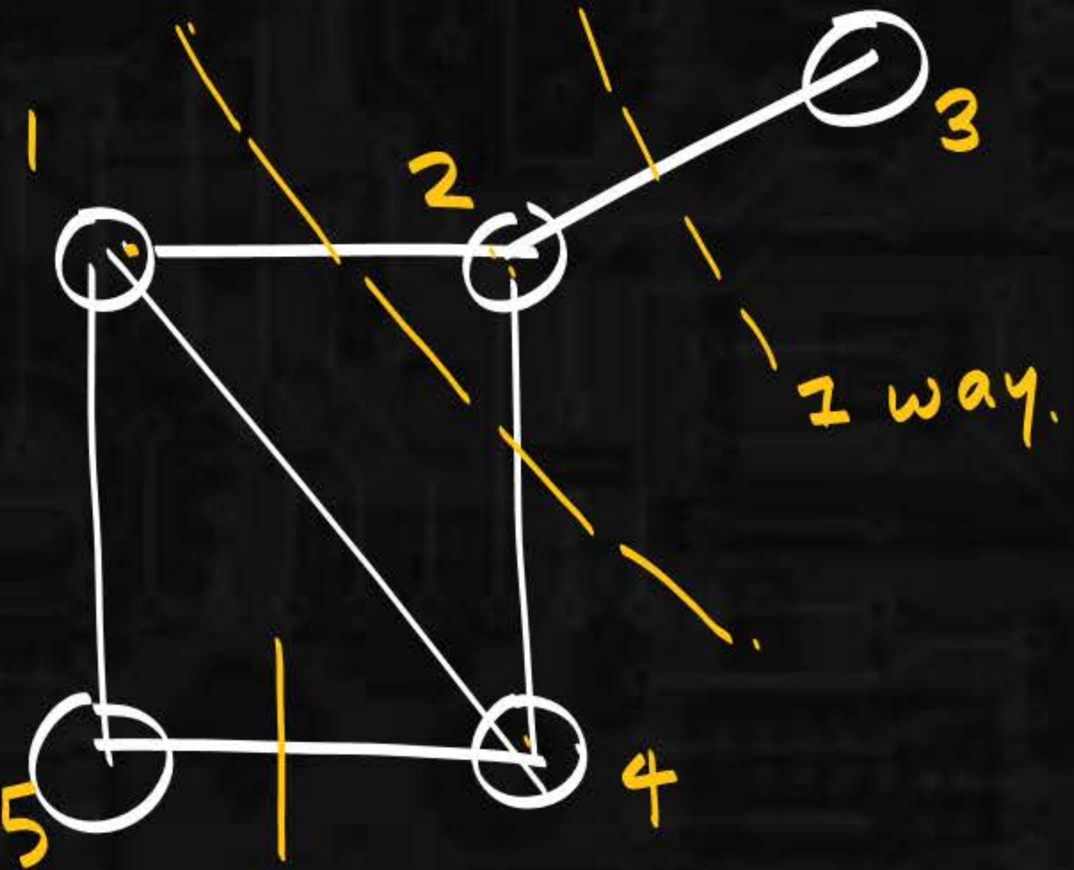
min

size 2 → {32, 24}

→ {35, 54}

size 3 → {

Connectivity in Graphs



{12, 24} cutset

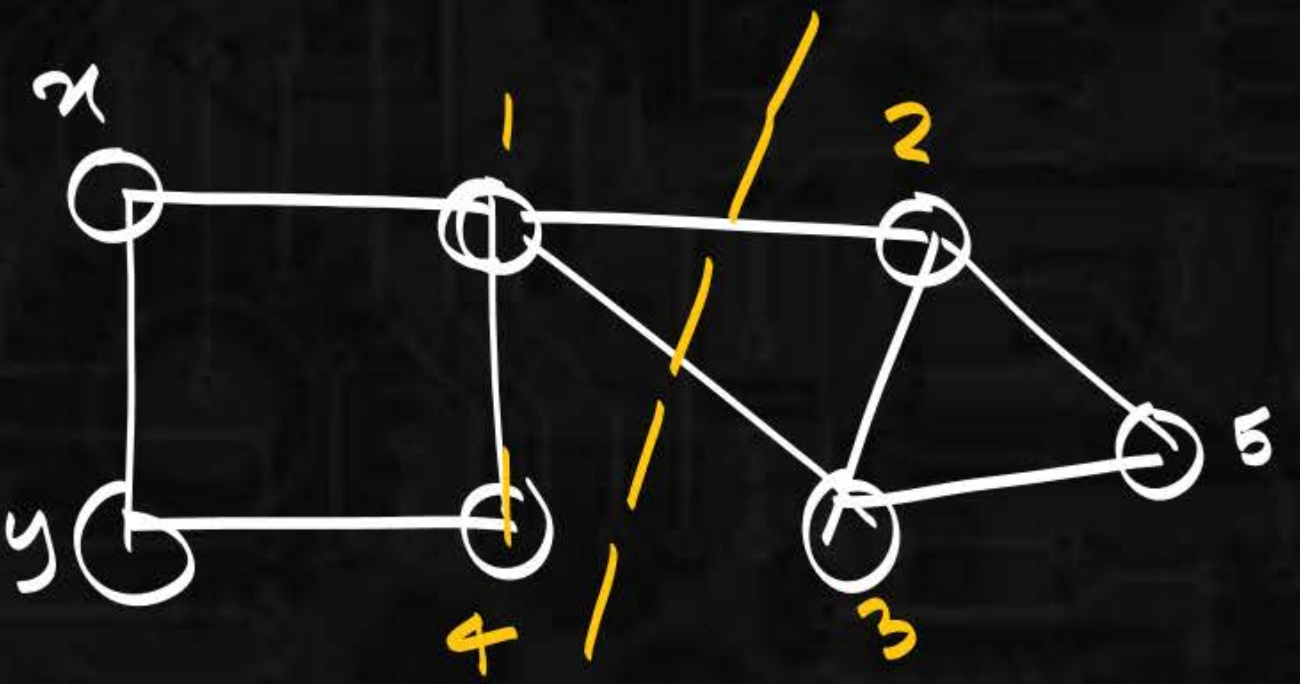
{$\boxed{12, 24}$, 45} →

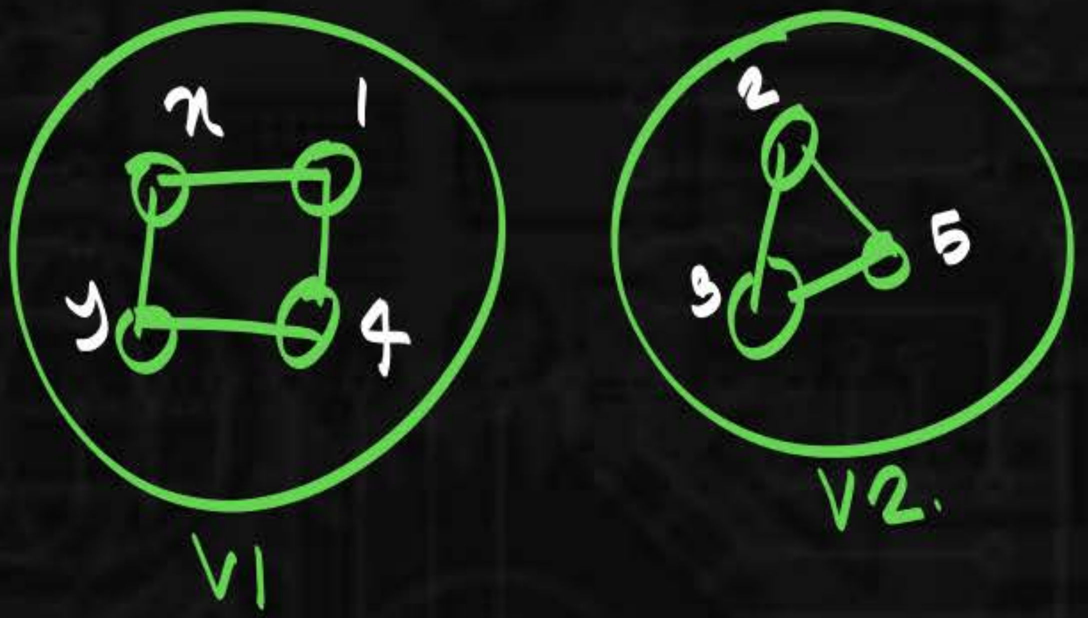
↓ Cutset.

not cutset

proper subset.

Connectivity in Graphs





{12, 13} → cutset ✓

{$\boxed{12, 13}$, 14} not cutset ?

Cutset

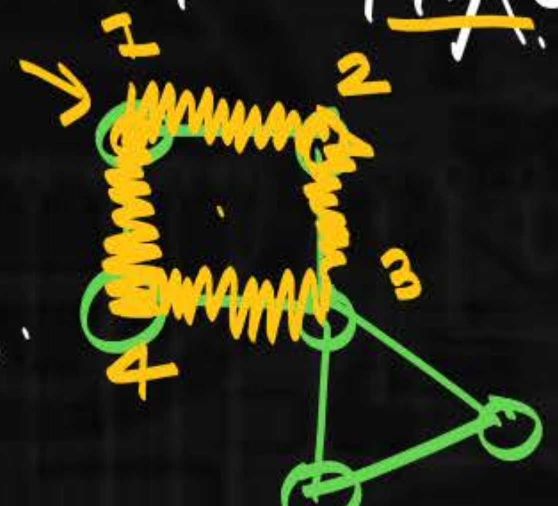
Connectivity in Graphs

Euler Graph :

Trail : alternating sequences of vertices & edges R.V | RXE

1 - 2 - 3 - 4 - 1

Closed Trail : Trail + starting = ending vertex.



Euler circuit : Closed Trail + cover all edges exactly once.

Connectivity in Graphs

Euler ckt:

{ Closed Trail.
all edges.

st

1 → 2 → 3 → 4 → 5 → 3 → 6 → 1

end

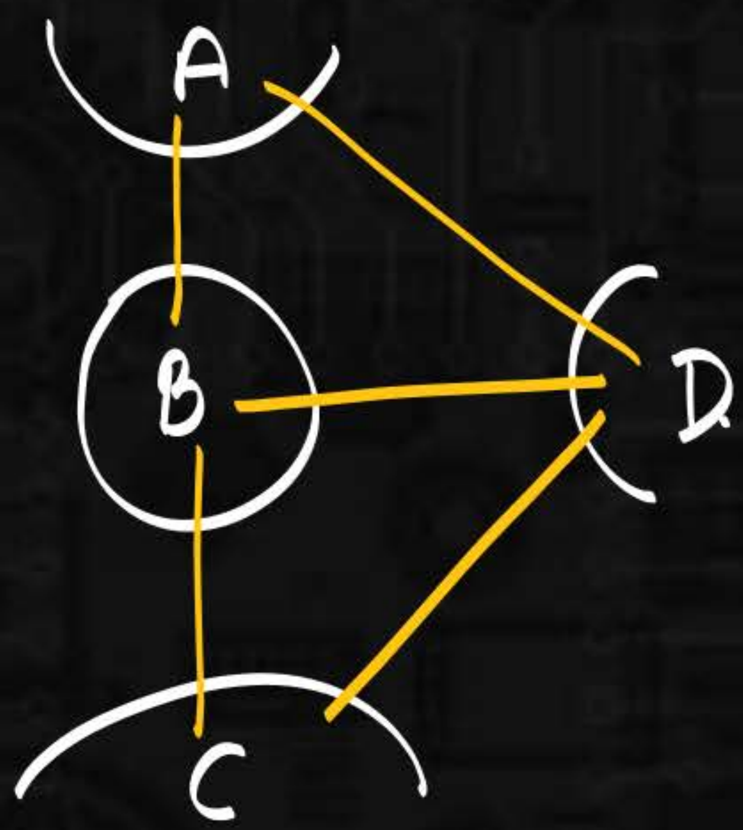
Repst^n → Euler ckt

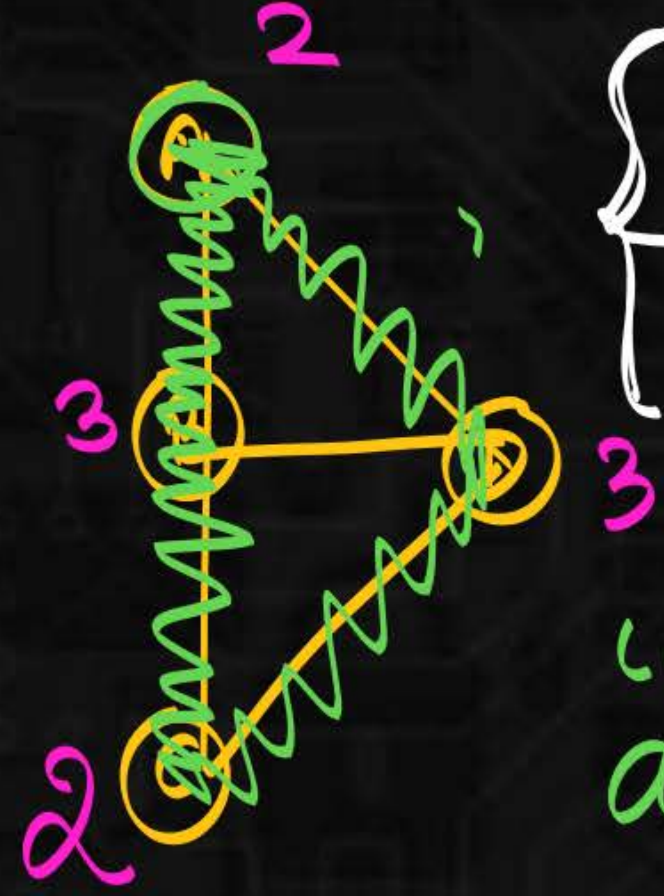
$G = (V, E)$

Euler Graph.

Connectivity in Graphs

1736:





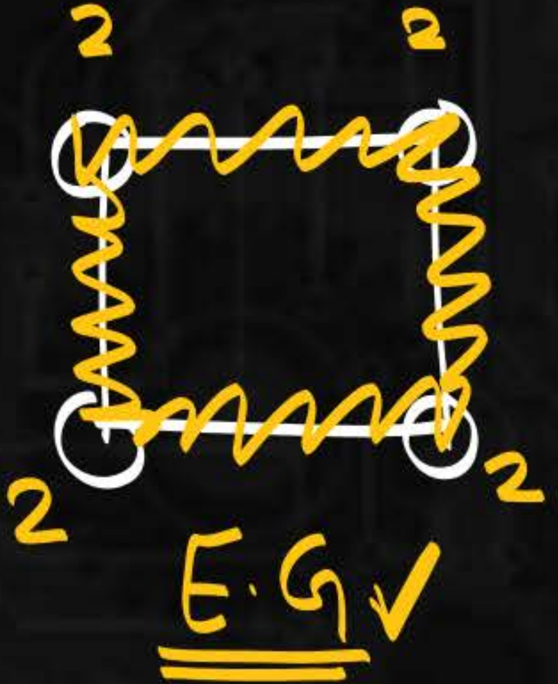
→ A
→ bridge.

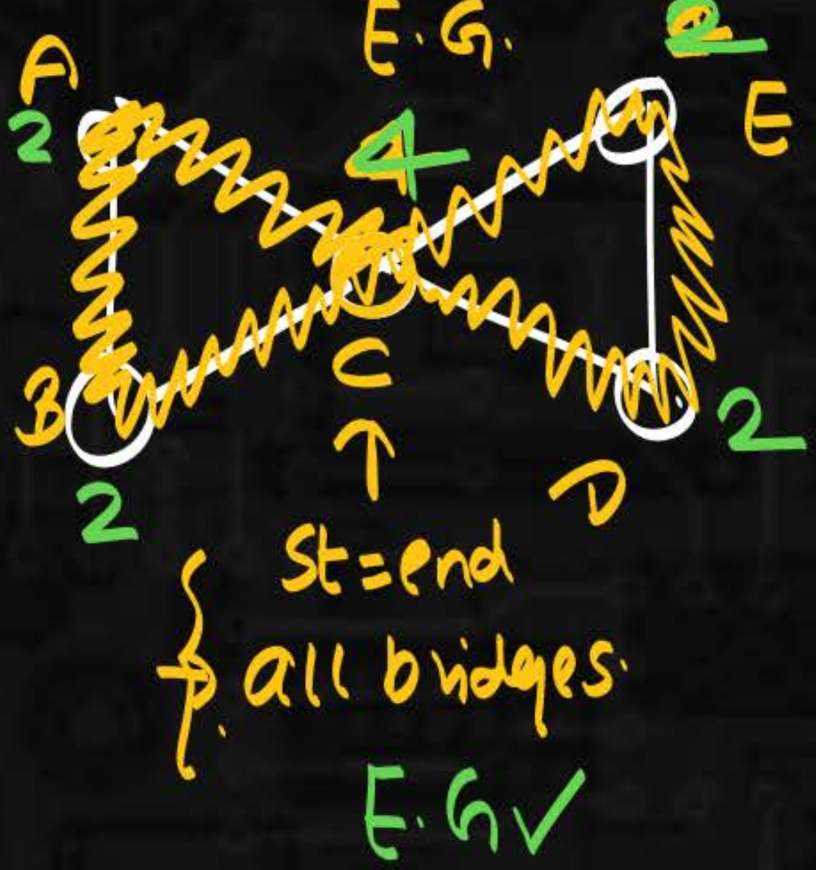
closed ✓
all edges X.

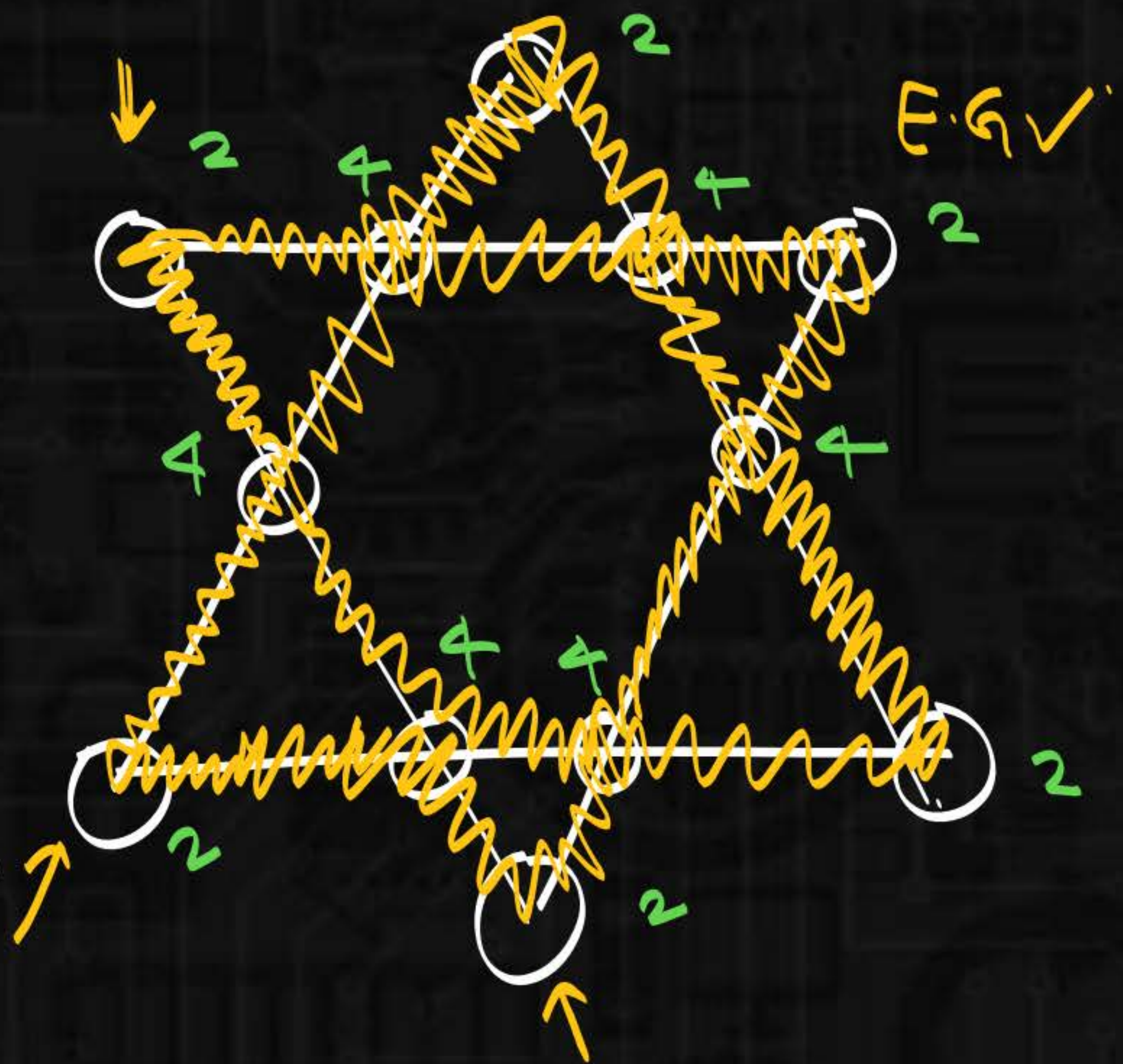
Euler Graph

→ Closed Trail.
→ all edges.

Connectivity in Graphs





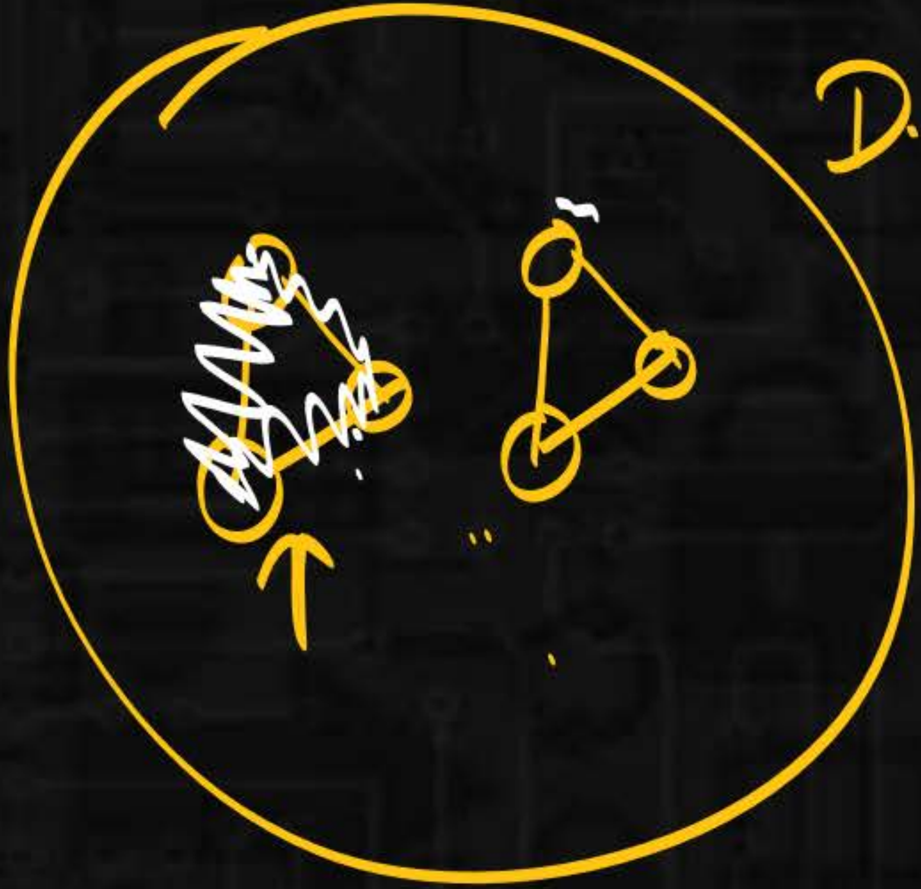


Connectivity in Graphs

nontrivial connected Graph is Euler Graph iff degrees of all vertices are even.

K1

D. C.



Degree = 0
even degree.

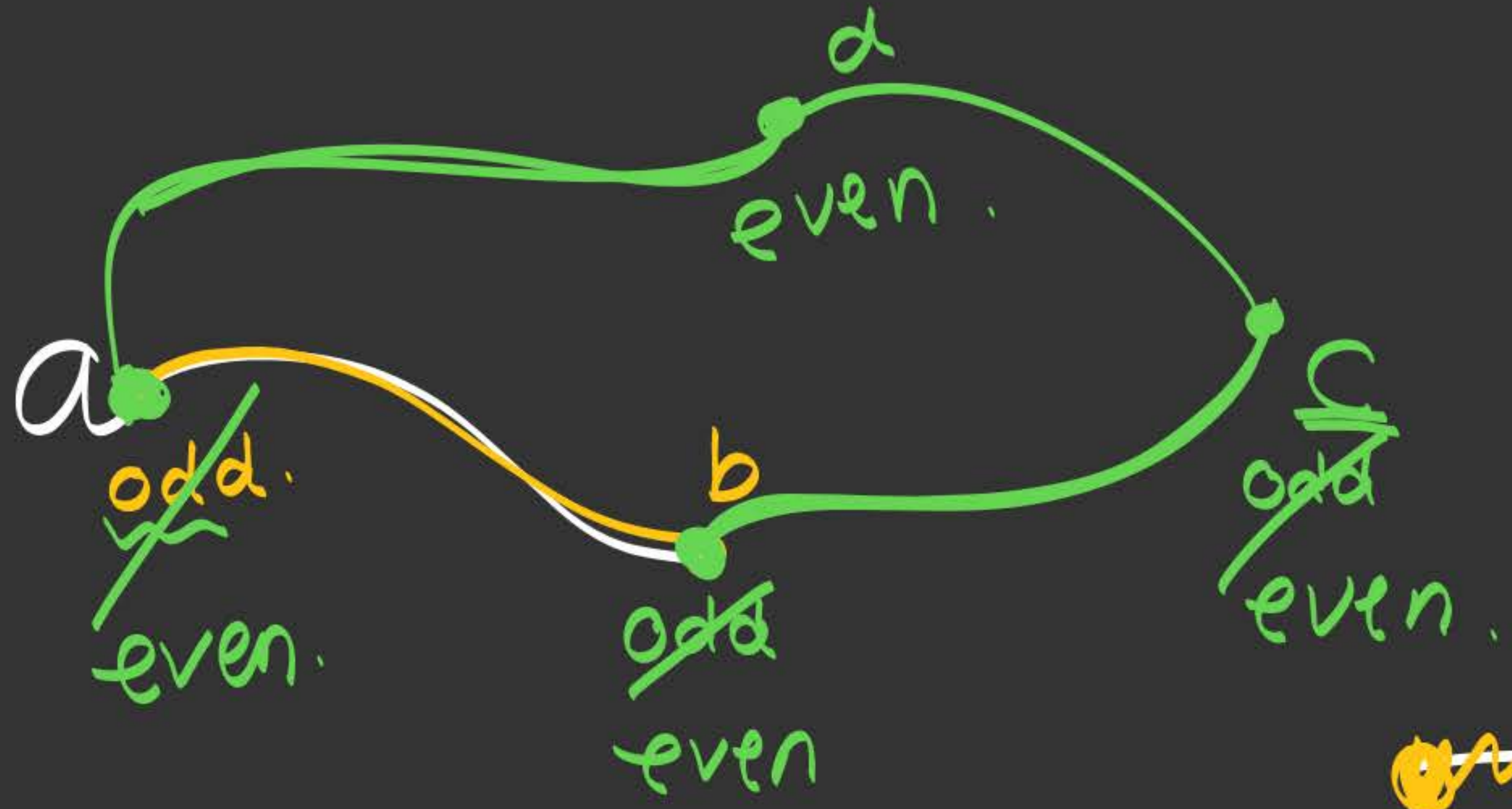
Trivial → Default.

Trivial Graph.

$$G = (\underbrace{V}_{\text{set}}, \underbrace{E}_{\text{set}})$$

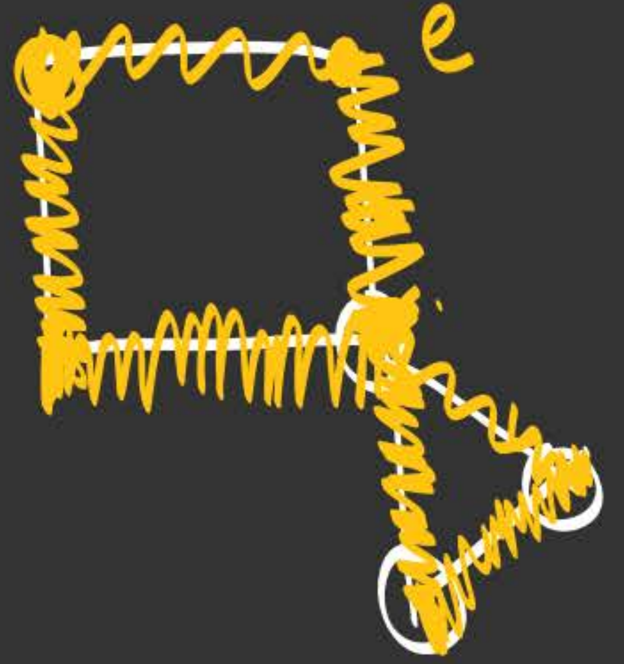
$$\begin{cases} V \neq \phi \\ E = \phi \end{cases} \rightarrow$$

Trivial Graph.



closed ✓
all edges ✓

Stop → a.



Connectivity in Graphs

Graph is Euler Graph iff degrees of all vertices are even.

+

<u>Connected.</u>

+

<u>non trivial.</u>

Doubt1:

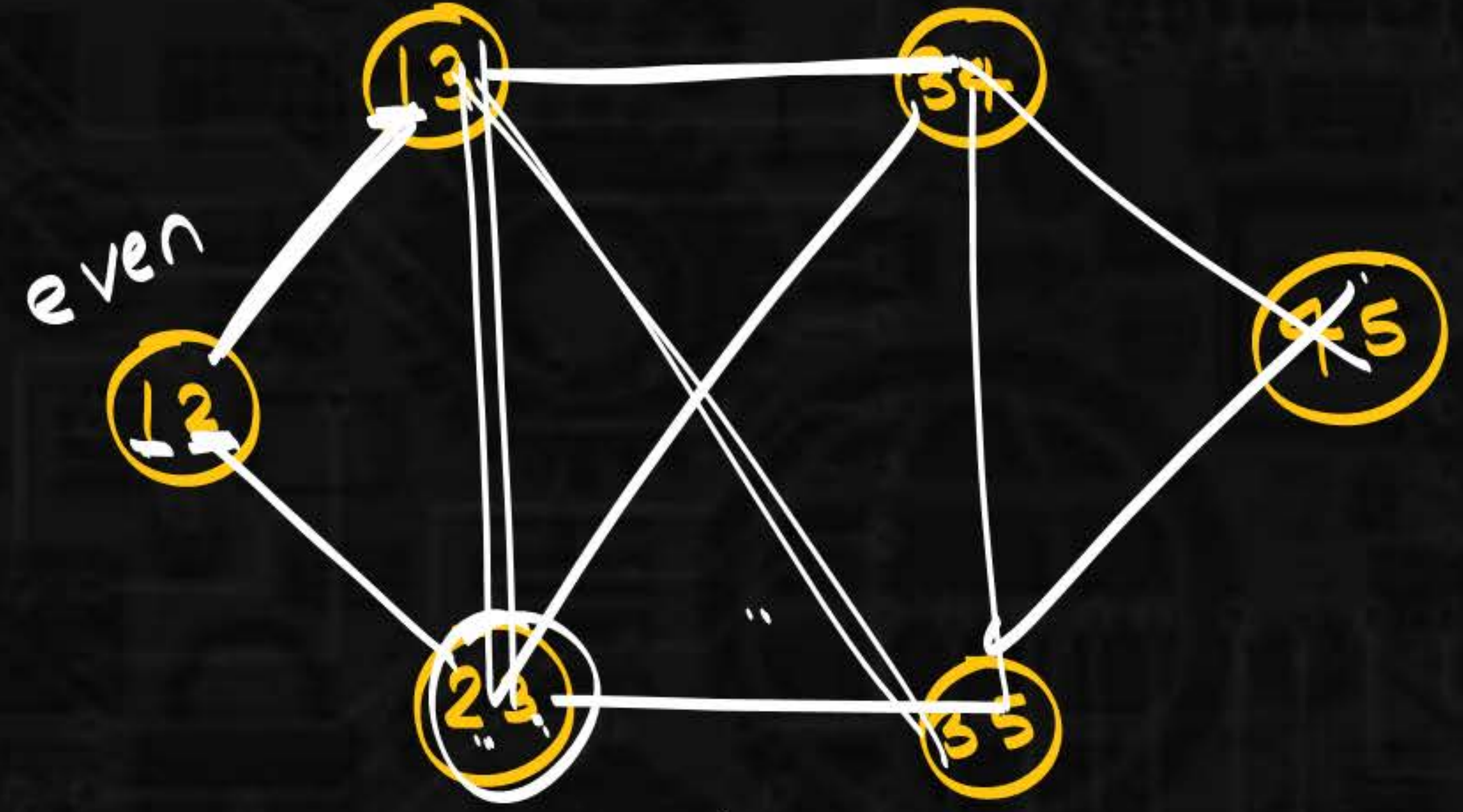
<u>Doubt2</u>:

Connectivity in Graphs

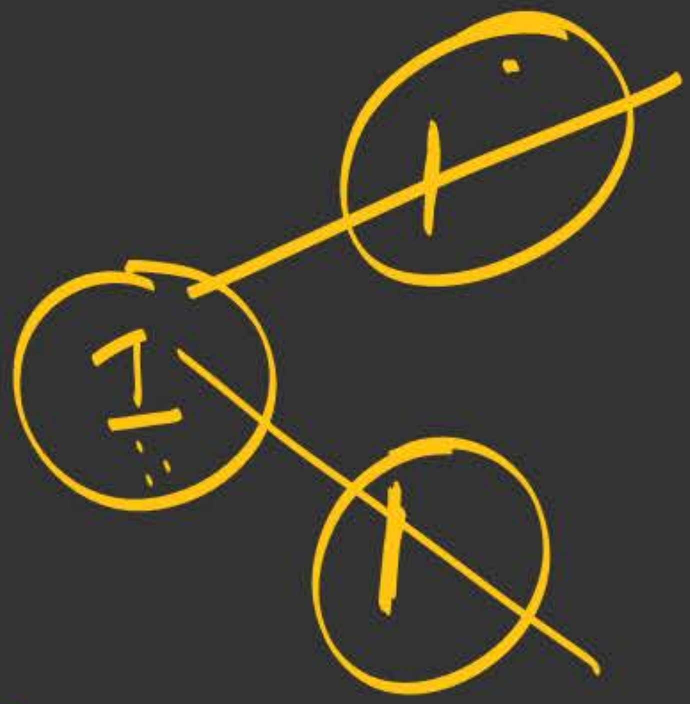
7. Degree → 2

(1,)

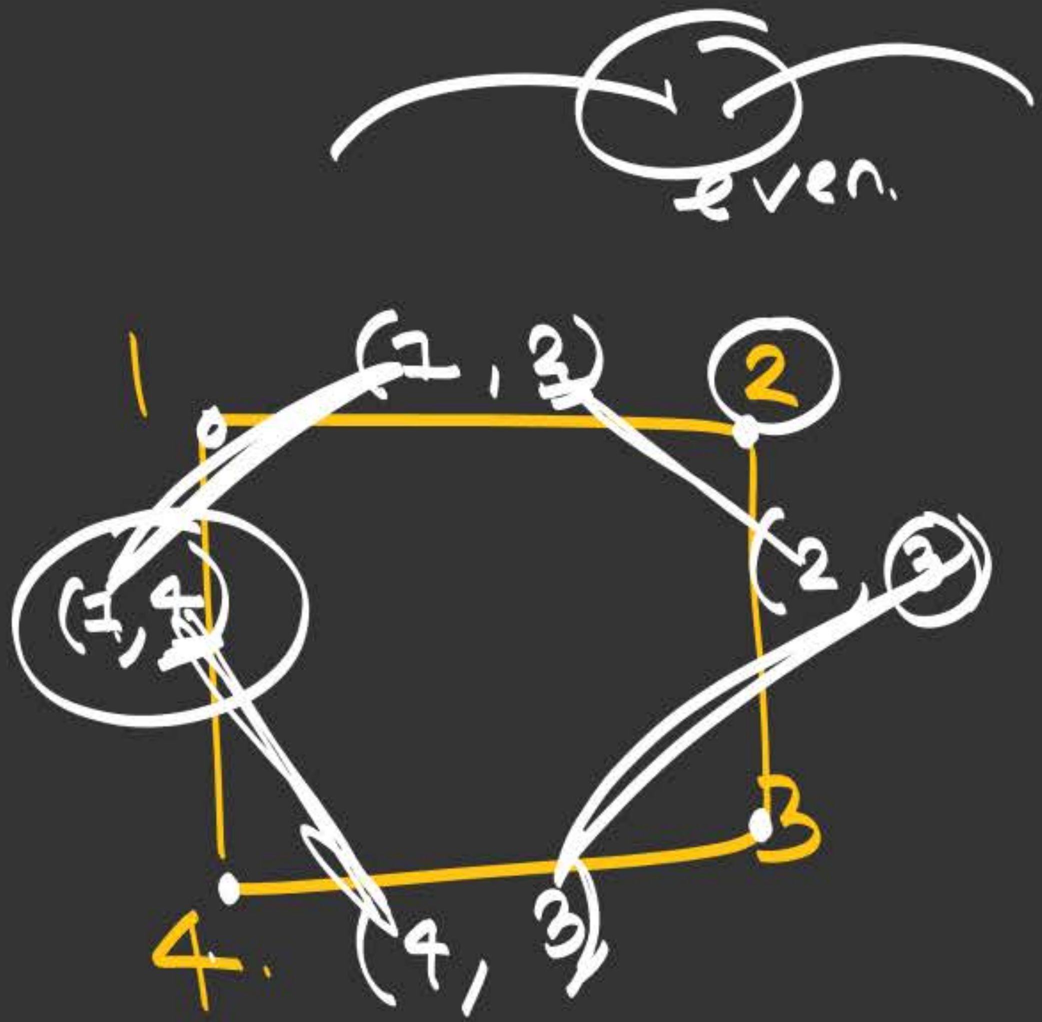
(1,)

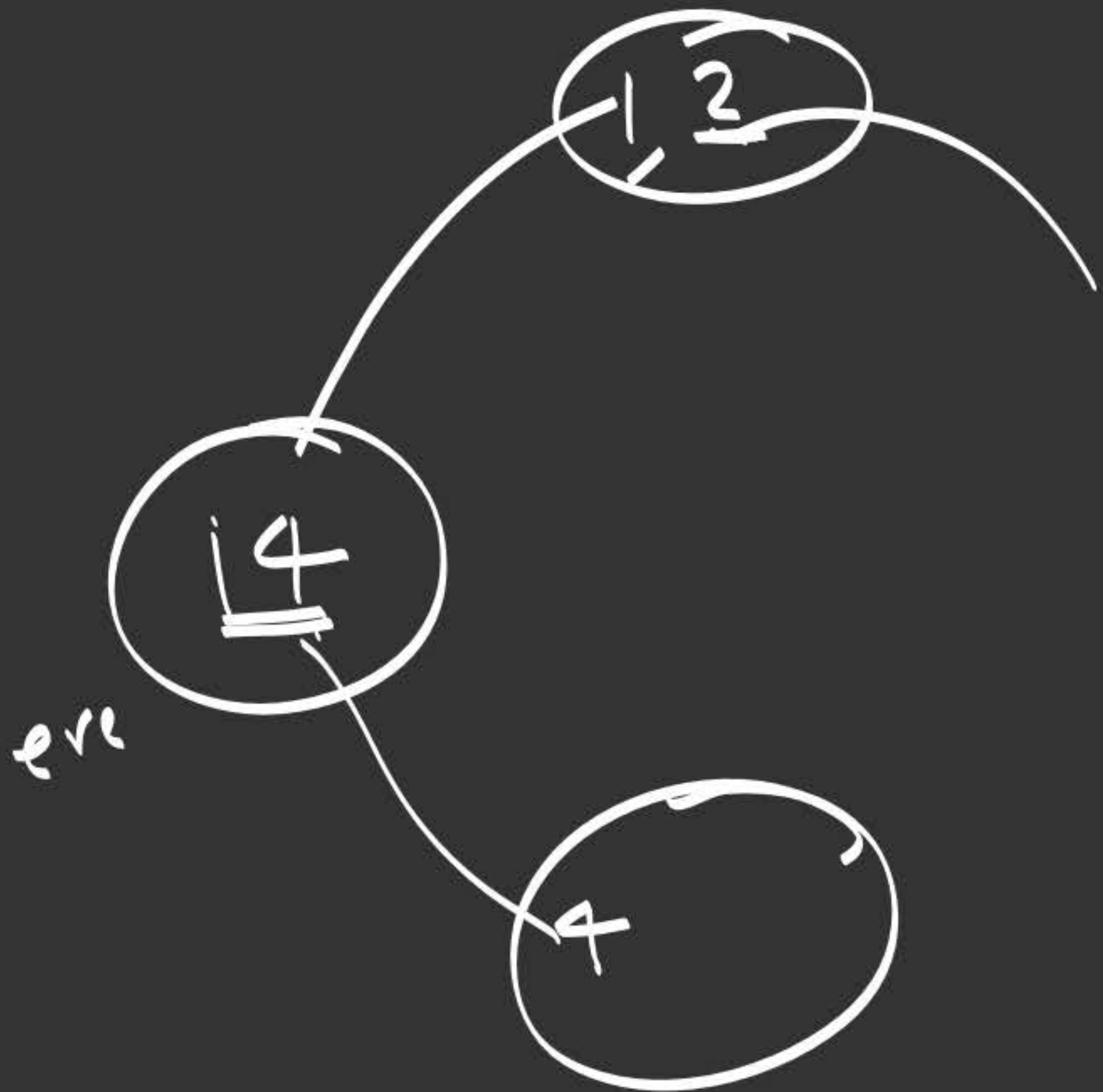


Line Graph of Euler Graph will always be Euler Graph.



degree → 2
2 baehe



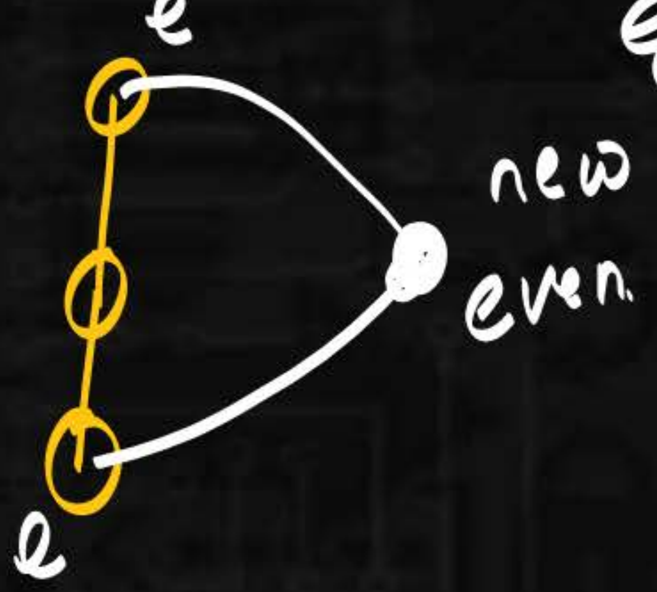


Connectivity in Graphs

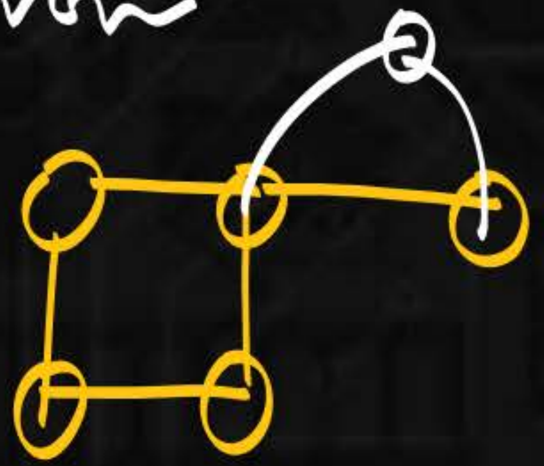
Thm 2:

Consider a connected graph having some odd degree vertices add new vertex make it connect to all odd degree vertices, then the resultant graph would be.

(GATE)



Eulerian

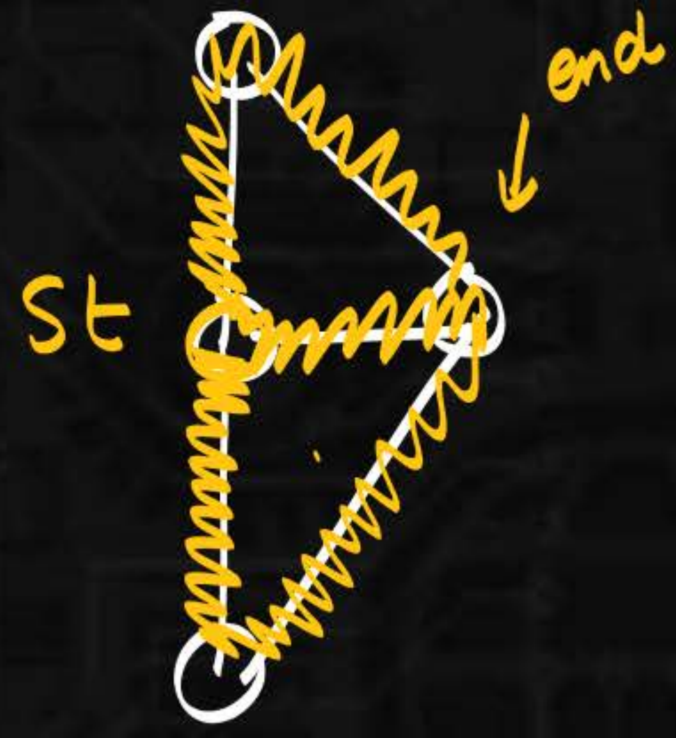


Euler Graph

Connectivity in Graphs

Trail :

Open Trail : Trail + st ≠ ending vertex



closed

$\boxed{\text{all edges}}$ ✓ → open.

Euler line : Open Trail + covering all edges exactly once.

Unicursal line

Euler path.

Connectivity in Graphs

EG X.

all edges ✓
+
open.

Connectivity in Graphs

covering all edges.

Closed.

Euler ckt

Open.

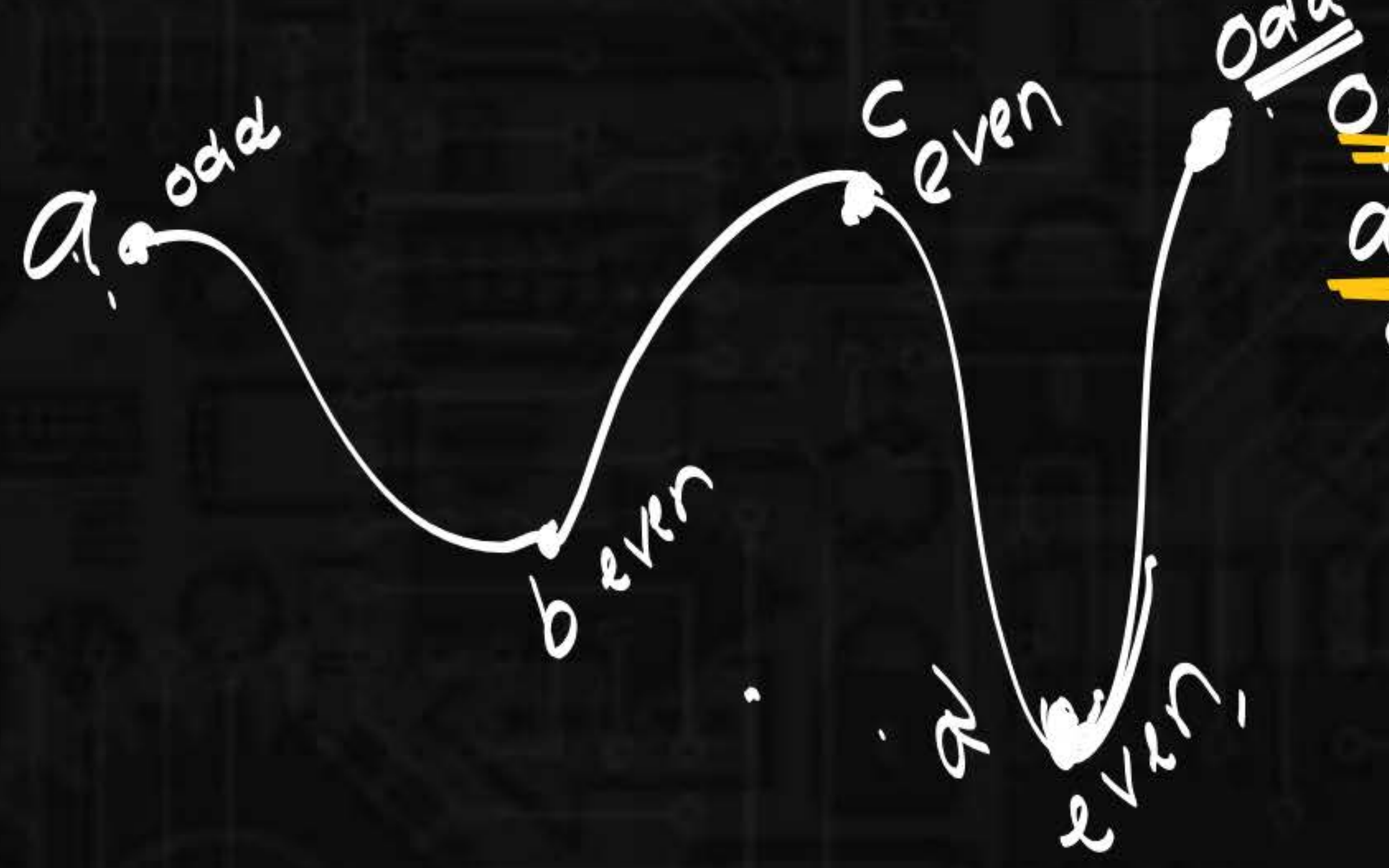
Euler line / Euler path:

Connectivity in Graphs

note: start → odd degree
end → odd degree

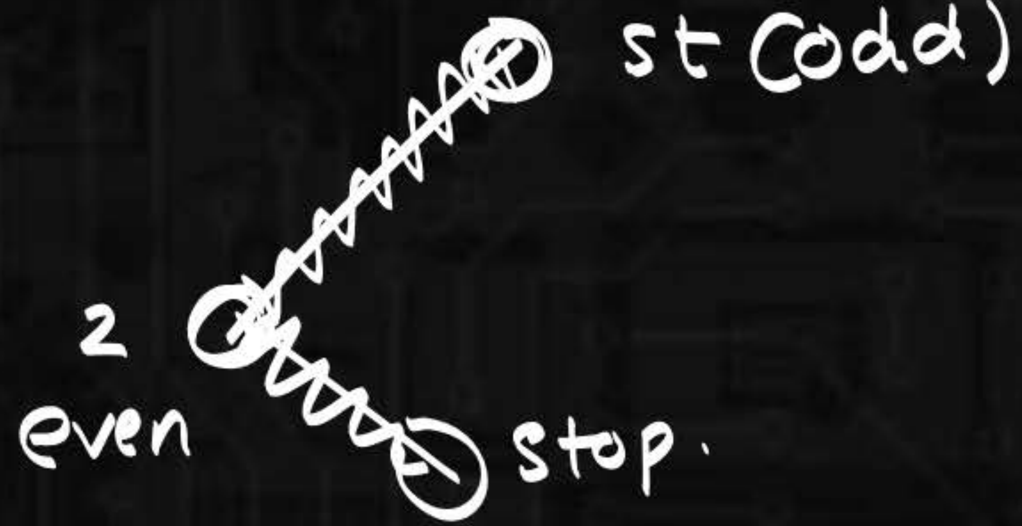
Thm:

Graph contains Euler line iff it contains exactly 2 odd degree vertex.



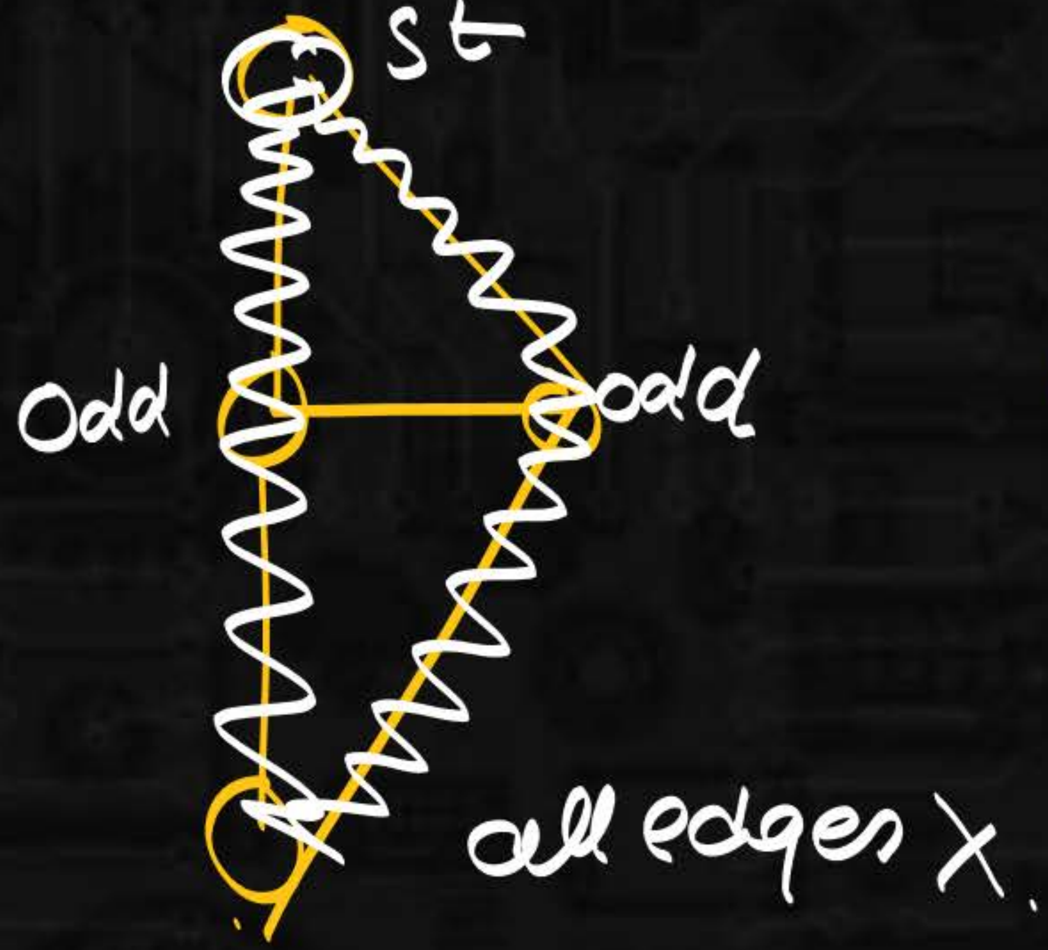
open
all edges
stop → not on a

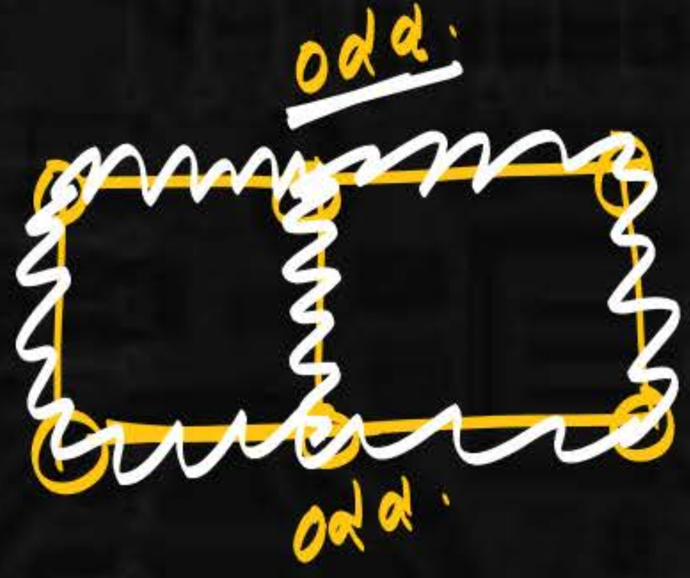
Connectivity in Graphs



St → a.
Stop → b
all edges.

Connectivity in Graphs





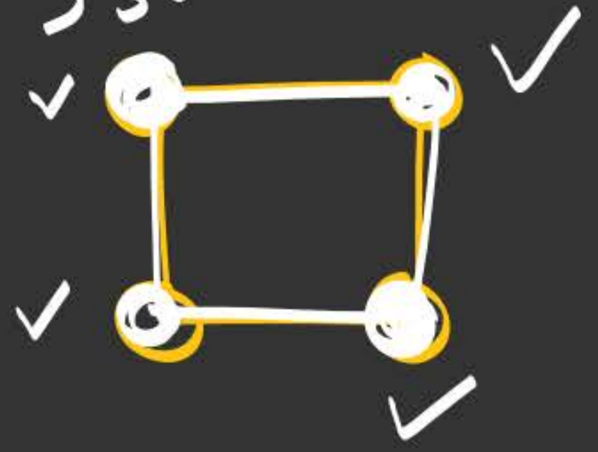
Connectivity in Graphs

Path :

Closed path : Path + st = ending vertex.

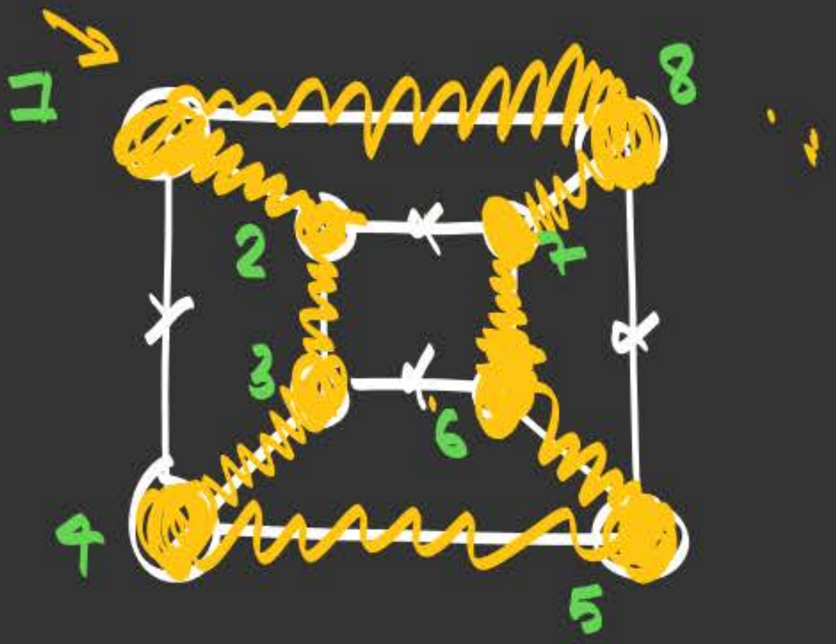
Hamiltonian ckt: Closed path + cover all vertices exactly once.

> st=ending.



closed + all vertices.

H.C



closed + all vertices.

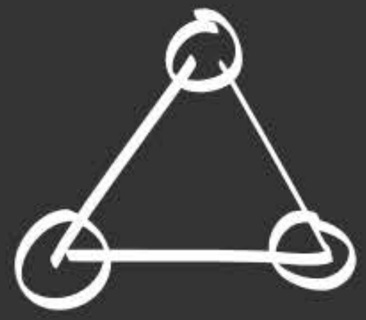
H.C.

Graph contains Hamiltonian ckt then Graph is called Hamiltonian Graph.

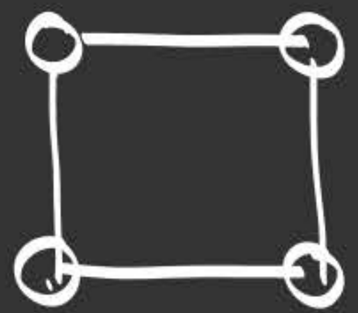
H.C

1 − 2 − 3 − 4 − 5 − 6 − 7 − 8 − 1

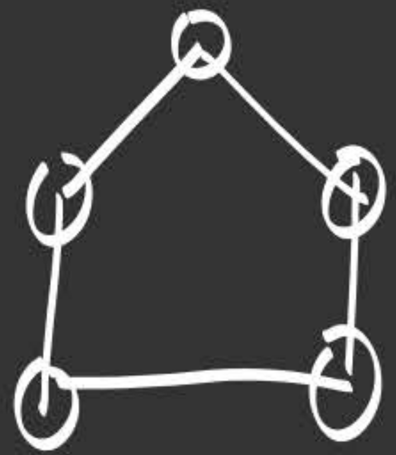
$G = (\underline{V}, \underline{E})$



H.C ✓

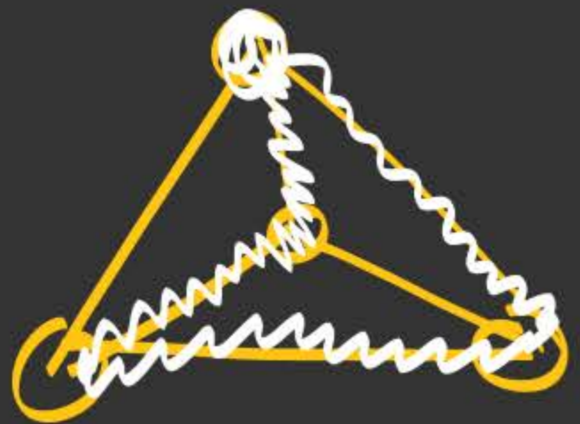


H.C ✓

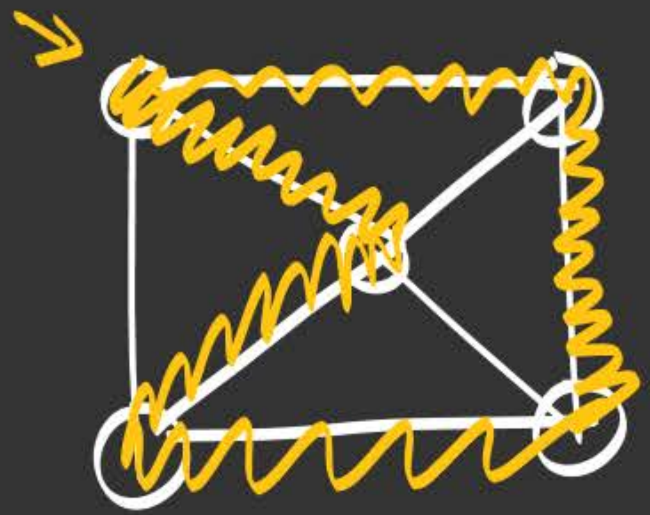


H.c ✓

all cn are H.G.



W4 → H.G.



w5 → H.G ✓

all wn → H.G.

k1

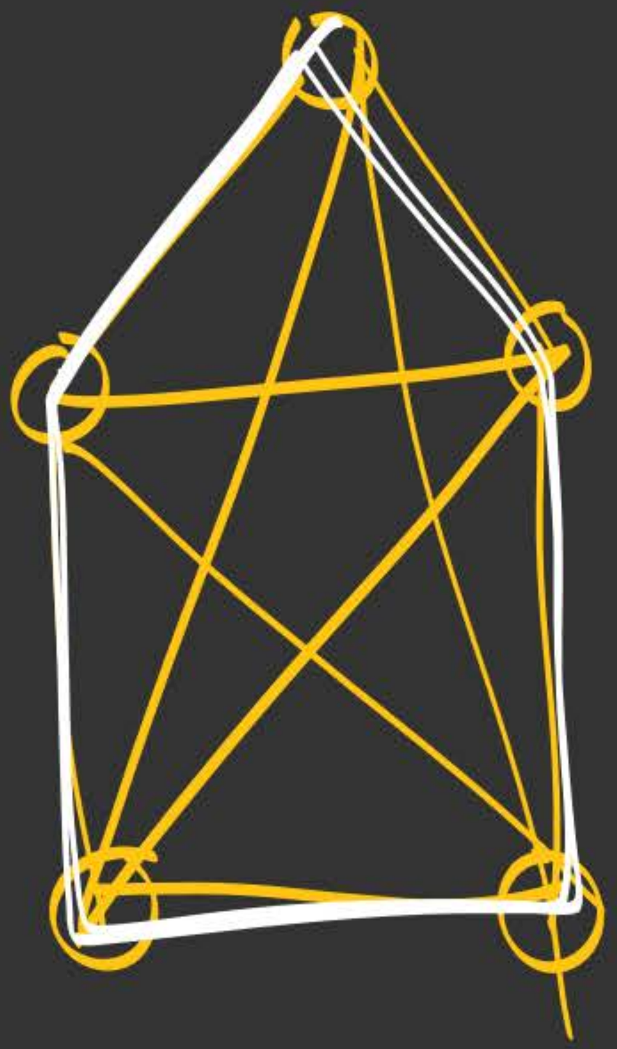
X.

X

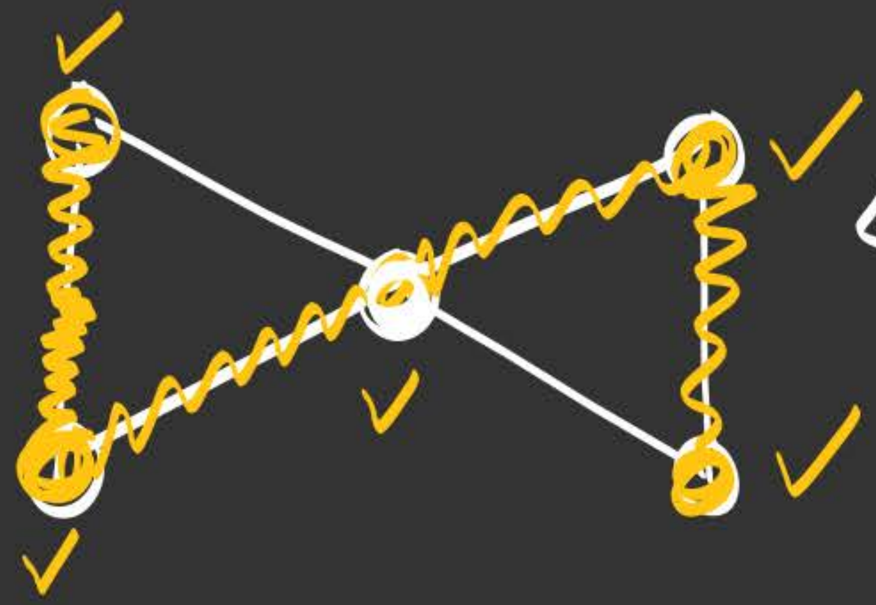
k2.

k3 ✓

H.G ✓



all $K_n \to$ H.G $(n \geq 3)$



Hamiltonian Path. :

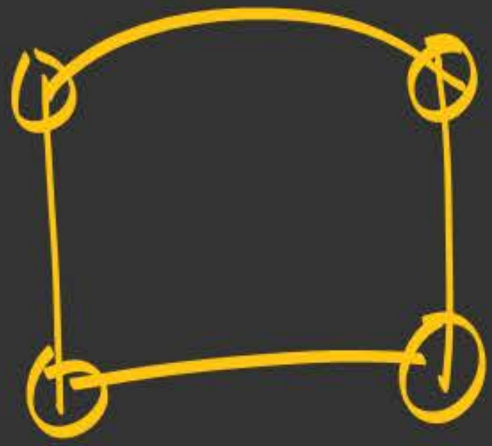
open Path + covers all vertices exactly once.

not Hamiltonian Graph.

all vertices ✓
closed. ✗

all vertices ✓
closed.

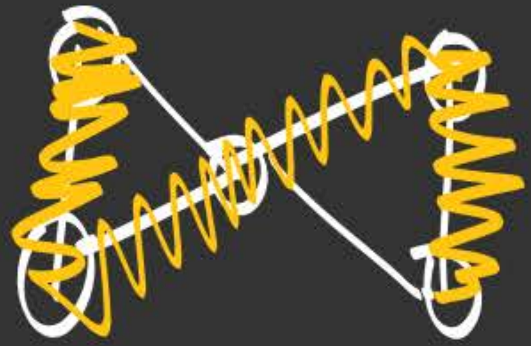
Every Hamiltonian ckt contains H.Path. ✓



all vertices open

→ Every Hamiltonian path contains Hamiltonian ckt.

(False)



Covering

- all edges.
  - closed
    - Euler ckt.
      - Euler Graph.
  - open.
    - Euler line.
    - Euler Path.
- all vertices.
  - Closed.
    - Hamiltonian ckt
  - open.
    - Hamiltonian path.


THANK YOU GW
SOLDIERS !