# ENGINEERING MATHEMATICS

## ALL BRANCHES

Sandwich Theorem & Double Limits

Calculus

DPP-03 Solution

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**Question 1**

Evaluate: $\lim_{x \to 0}\left(\frac{1}{x^2}-\frac{1}{\sin^2 x}\right)$

**A** 1/3

**B** –1/2

**C** –1/3

**D** –1/4

$\lim_{x\to 0} \frac{\sin^2 x - x^2}{x^2 \sin^2 x} \quad \left(\frac{0}{0}\right)$

$\lim_{x\to 0} \frac{2\sin x \cos x - 2x}{x^2(2\sin x\cos x) + 2x\sin^2 x}$

$\lim_{x\to 0} \frac{\sin 2x - 2x}{x^2 \sin 2x + 2x\sin^2 x} \quad \left(\frac{0}{0}\right)$

$\frac{2\cos 2x - 2}{2x^2\cos 2x + 2x\sin 2x + 2x\sin 2x + 2\sin^2 x}$

$\lim_{x\to 0} \frac{\cos 2x - 1}{x^2\cos 2x + 2x\sin 2x + \sin^2 x} \quad \left(\frac{0}{0}\right)$ form

$\lim_{x\to 0} \frac{-2\sin 2x}{-2x^2\sin 2x + 2x\cos 2x + 4x\cos 2x + 2\sin 2x + \sin 2x}$

$$\lim_{x\to 0} \frac{-2\sin 2x}{-2x^2\sin^2 2x + 6x\cos 2x + 3\sin 2x} \quad \left(\frac{0}{0}\text{ form}\right)$$

$$\lim_{x\to 0} \frac{-4\cos 2x}{-4x^2\sin 4x - 4x\sin^2 2x - 12x\sin 2x + 6\cos 2x + 6\cos 2x}$$

$$\frac{-4\times 1}{0-0-0+6+6} = -\frac{4}{12} = -\frac{1}{3}$$

**Question 2**

Find: $\lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{\frac{1}{x^2}}$

A $e^{1/3}$

B $e^{1/2}$

C $e^{2/3}$

D None

$$y = \lim_{x \to 0} \left(\frac{\tan x}{x}\right)^{1/x^2}$$

$$\log y = \lim_{x \to 0} \frac{\log(\tan x / x)}{x^2} \qquad \left(\frac{0}{0}\right)$$

$$\lim_{x \to 0} \frac{\frac{1}{\frac{\tan x}{x}}\left[\frac{x\sec^2 x - \tan x}{x^2}\right]}{2x} \qquad \left[\because \lim_{x \to 0} \frac{\tan x}{x} = 1\right]$$

$$\lim_{x \to 0} \frac{x\sec^2 x - \tan x}{2x^3} \qquad \left(\frac{0}{0}\right) \text{ form}$$

$$\frac{2x\sec^2 x \tan x + \cancel{\sec^2 x} - \cancel{\sec^2 x}}{6x^2}$$

$$\lim_{x\to 0} \frac{\sec^2 x \tan x}{3x} \qquad \left(\frac{0}{0} \text{ form}\right)$$

$$\log y = \lim_{x\to 0} \frac{\sec^2 x . \sec^2 x + (2\sec^2 x \tan x)\tan x}{3}$$

$$\log y = \frac{1+0}{3} = \frac{1}{3}$$

$$\boxed{y = e^{1/3}}$$

## Question 3

The value of $\lim_{x \to 8} \dfrac{x^{\frac{1}{3}} - 2}{(x-8)}$

A 1/16

B 1/12

C 1/8

D 1/4

$\lim_{x \to 8} \dfrac{x^{1/3} - 2}{x - 8}$ $\left(\frac{0}{0} \text{ form}\right)$

$\lim_{x \to 8} \frac{1}{3} x^{1/3 - 1}$

$\frac{1}{3} \cdot (8)^{-2/3}$

$= \frac{1}{3} \frac{1}{(2^3)^{2/3}} = \frac{1}{3 \times 4} = \frac{1}{12}$

## Question 4

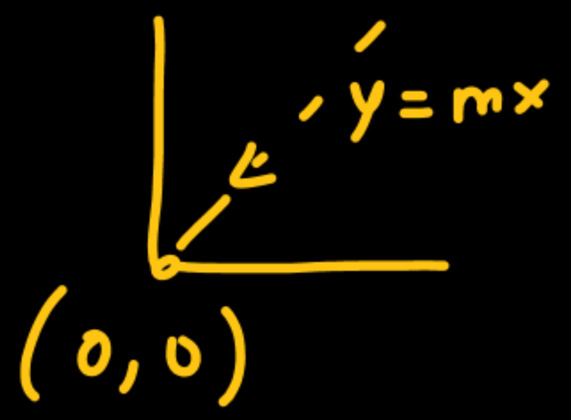
What is the value of $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \dfrac{xy}{x^2 + y^2}$

$x \to 0, \; y \to mx$

$$\lim_{\substack{x \to 0 \\ y \to mx}} \frac{x(mx)}{x^2 + (mx)^2} = \frac{m x^2}{x^2(1+m^2)}$$

$$= \frac{m}{1+m^2}$$

(Since limit depends on $m$ $\therefore$ it does not exist).




**A** 1

**B** –1

**C** 0

**D** Limit does not exist

Question 5

Evaluate: $\lim_{x \to 0} \dfrac{x - \sin x}{x^3}$

A 1/2

B 2

C 1/6

D 0

$\lim_{x \to 0} \dfrac{x - \sin x}{x^3} \quad \left(\dfrac{0}{0}\right)$

$\lim_{x \to 0} \dfrac{1 - \cos x}{3x^2} \quad \left(\dfrac{0}{0}\right)$

$\dfrac{\sin x}{6x} \quad \left(\dfrac{0}{0}\right)$

$\lim_{x \to 0} \dfrac{\cos x}{6}$

$= \dfrac{1}{6}$

**Question 6**

Evaluate the limit of the following function as $(x, y) \to (0, 0)$

$$f(x,y)=\frac{x^3y^3}{x^2+y^2}$$

A Limit exists

B Limit does not exist

C Limit is dependent of path chosen

D None

Along $y = mx$, $(x,y) \to (0,0)$

$$\lim_{\substack{x\to 0\\ y\to mx}} \frac{x^3y^3}{x^2+y^2}$$

$$= \frac{x^3(mx)^3}{x^2+(mx)^2}$$

$$\lim_{\substack{x\to 0\\ y\to mx}} \frac{m^3x^6}{x^2(1+m^2)} = x^4\left(\frac{m^3}{1+m^2}\right)$$

$$= 0$$

This limit is independent of m.

$\therefore$ limit exist.

Question 7

Evaluate the limit for the functions $f(x, y) = \frac{x^3 + y^3}{x - y}$ $(x, y) \neq (0, 0)$ at origin.

A Limit exists

B Limit does not exist

C Limit is independent of path chosen

D None

Along $y = mx$ $(x, y) \to (0, 0)$

$$\lim_{\substack{x \to 0 \\ y \to mx}} \frac{x^3 + y^3}{x - y}$$

$$\frac{x^3 + (mx)^3}{x - mx}$$

$$\frac{x^3(1 + m^3)}{x(1 - m)}$$

$$\lim_{x \to 0} x^2 \left(\frac{1 + m^3}{1 - m}\right)$$

$$= 0$$

Along $y = x - mx^3$

$$\frac{x^3+y^3}{x-y}$$

$$\frac{x^3+(x-mx^3)^3}{x-(x-mx^3)}$$

$$\frac{x^3+x^3-m^3x^9-3mx^5+3m^2x^7}{mx^3}$$

$$\lim_{x\to 0} \frac{2}{m} - m^2x^6 - 3x^2 + 3mx^4$$

$$\lim_{x\to 0} \frac{2}{m}$$ (dependent on m)

## Question 8

Evaluate the limit of $f(x,y) = \dfrac{x^2 - y^2}{x^2 + y^2}$, as (x, y) → (0, 0).

A Limit is independent of path chosen

B Limit exists

C Limit does not exist

D None

$$\lim_{\substack{x \to 0 \\ y \to mx}} \frac{x^2 - y^2}{x^2 + y^2}$$

$$\frac{x^2 - (mx)^2}{x^2 + (mx)^2}$$

$$\frac{x^2(1-m^2)}{x^2(1+m^2)}$$

$$= \frac{1-m^2}{1+m^2}$$

$\therefore$ limit DNE $\because$ depends on m.

# Thank you

# GW Soldiers !