ENGINEERING MATHEMATICS

ALL BRANCHES

Limit & its Application

Calculus

DPP-02 Solution

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Question 1

The value of $\lim\limits_{x\to 0}\dfrac{x\left(e^{x}-1\right)+2(\cos x-1)}{x(1-\cos x)}=$ _____.

$\left(\frac{0}{0}\text{ form}\right)$

$$\lim_{x\to 0}\frac{x e^{x}+(e^{x}-1)-2\sin x}{x(\sin x)+(1-\cos x)} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$\lim_{x\to 0}\frac{x e^{x}+e^{x}+e^{x}-2\cos x}{x\cos x+\sin x+\sin x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$\lim_{x\to 0}\frac{x e^{x}+e^{x}+e^{x}+e^{x}+2\sin x}{-x\sin x+\cos x+\cos x+\cos x}$$

$$\frac{0+1+1+1+0}{0+1+1+1}=\frac{3}{3}=1$$

Question 2

The value of $\left[\lim_{x\to\infty}\left(\frac{1}{\sin x}-\frac{1}{\tan x}\right)\right]$ is

A 0

B 1

C 2

D ∞

$$= \lim_{x\to\infty}\left(\frac{1}{\sin x}-\frac{\cos x}{\sin x}\right)$$

$$\lim_{x\to\infty}\left(\frac{1-\cos x}{\sin x}\right) = \frac{2\sin^2 x/2}{2\sin\frac{x}{2}\cos\frac{x}{2}}$$

$$\lim_{x\to\infty}\ \frac{\sin x/2}{\cos x/2}$$

$$\lim_{x\to\infty}\ \tan x/2$$

∴ This limit does not exist

∵ $\sin 2x = 2\sin x\cos x$

∵ $1-\cos 2x = 2\sin^2 x$

$1+\cos 2x = 2\cos^2 x$

Question 3

Value of the function $\lim_{x \to a} (x-a)^{(x-a)}$ is given by

A 1

B 0

C ∞

D a

$$y = \lim_{x \to a} (x-a)^{x-a}$$

Taking log both sides;

$$\log y = \lim_{x \to a} (x-a)\log(x-a)$$

$$\log y = \lim_{x \to a} \left[\frac{\log(x-a)}{\frac{1}{x-a}}\right] \quad \left(\frac{\infty}{\infty} \text{ form}\right)$$

$$\log y = \lim_{x \to a} \frac{\frac{1}{x-a}}{-\frac{1}{(x-a)^2}} = \lim_{x \to a} -(x-a)$$

$$\log y = 0$$

$$\boxed{y = e^{0} = 1}$$

Question 4

The value of the function $f(x) = \lim_{x \to 0} \frac{x^3+x^2}{2x^3-7x^2}$ is

A Zero

B $-\frac{1}{7}$

C $\frac{1}{7}$

D Infinite

$$\lim_{x \to 0} \frac{x^3+x^2}{2x^3-7x^2} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \frac{3x^2+2x}{6x^2-14x} \quad \left(\frac{0}{0} \text{ form}\right)$$

$$\lim_{x \to 0} \frac{6x+2}{12x-14} = -\frac{1}{7}$$

Question 5

$\lim_{x\to\infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2}$ equal

A ∞

B 0

C 2

D Does not exist

$$\lim_{x\to\infty} \frac{x^3 - \cos x}{x^2 + (\sin x)^2}$$

$$\lim_{x\to\infty} \frac{1 - \frac{\cos x}{x^3}}{\frac{1}{x} + \frac{(\sin x)^2}{x^3}}$$ ($\cos x \to 0$, $\frac{1}{x} \to 0$, $\frac{(\sin x)^2}{x^3} \to 0$)

(Divide Nr and Dr by x^3).

$$\lim_{x\to\infty} \frac{1 - 0}{0 + 0} = \infty$$

Question 6

$\lim_{x\to 0}\left(\frac{1-\cos x}{x^2}\right)$ is

A 1/4

B 1/2

C 1

D 2

$$\left[\lim_{x\to 0}\frac{1-\cos x}{x^2}=\frac{1}{2}\right]$$

$$\lim_{x\to 0}\frac{\sin x}{2x} \qquad \left(\frac{0}{0}\text{ form}\right)$$

$$\lim_{x\to 0}\frac{\cos x}{2}$$

$$=\boxed{\frac{1}{2}}$$

Question 7

The value of $\lim_{x \to 8} \frac{x^{1/3}-2}{(x-8)}$

A 1/16

B 1/12

C 1/8

D 1/4

$\lim_{x \to 8} \frac{x^{1/3} - 2}{x - 8}$ $\left(\frac{0}{0} \text{ form}\right)$

$\lim_{x \to 8} \frac{1}{3} x^{1/3-1}$

$\frac{1}{3}(8)^{-2/3} = \frac{1}{3(2^3)^{2/3}} = \frac{1}{3 \times 4}$

$= \frac{1}{12}$

Question 8

$$\lim_{x\to 0}\frac{e^x-\left(1+x+\frac{x^2}{2}\right)}{x^3}=$$

A 0

B 1/6

C 1/3

D 1

$$\lim_{x\to 0}\frac{e^x-\left(1+x+\frac{x^2}{2}\right)}{x^3}\quad\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{e^x-(1+x)}{3x^2}\quad\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{e^x-1}{6x}\quad\left(\frac{0}{0}\right)$$

$$\lim_{x\to 0}\frac{e^x}{6}=\boxed{\frac{1}{6}}$$

Alternatively,

By Series expansion,

$$\lim_{x\to 0}\frac{e^x-\left(1+x+\frac{x^2}{2}\right)}{x^3}$$

$$\frac{\left(1+x+\frac{x^2}{2!}+\frac{x^3}{3!}+\cdots\right)-\left(1+x+\frac{x^2}{2}\right)}{x^3}$$

$$\lim_{x\to 0}\left(\frac{1}{3!}+\frac{x}{4!}+\frac{x^2}{5!}+\cdots\right)$$

$$\frac{1}{3!}=\frac{1}{6}$$

Thank you

GW

Soldiers !