CS & IT ENGINEERING

Discrete Maths

Graph Theory

Lecture No. - 12

By- SATISH YADAV SIR

TOPICS TO BE COVERED

01 Matching set

02 Maximal matching set

03 Matching no.

04 Covering set

05 Covering number

Dominating set:

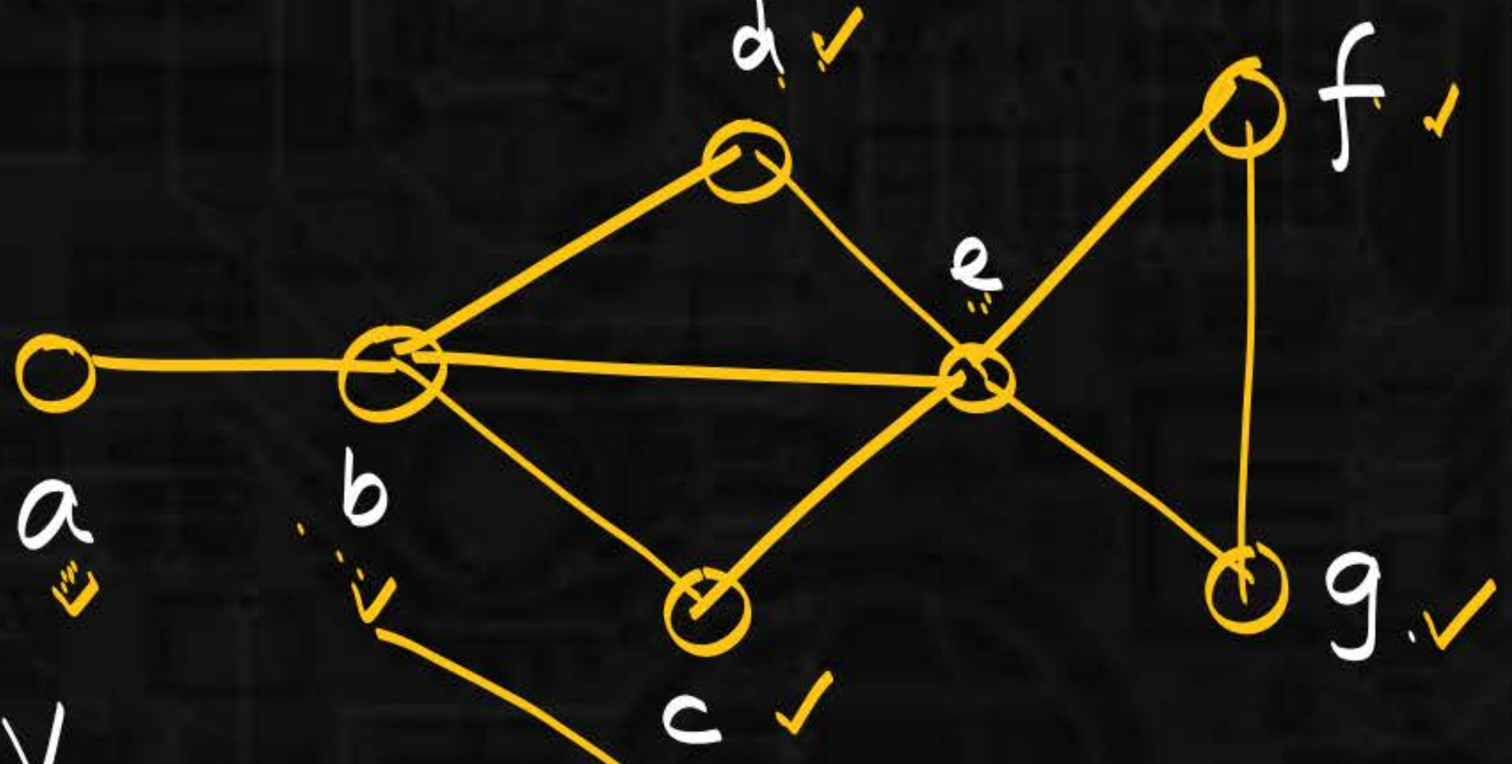
$G = (V, E)$

Dominating set $D \subseteq V$

if we take any vertex of V

either that vertex belongs to D

or it adjacent belongs to D.



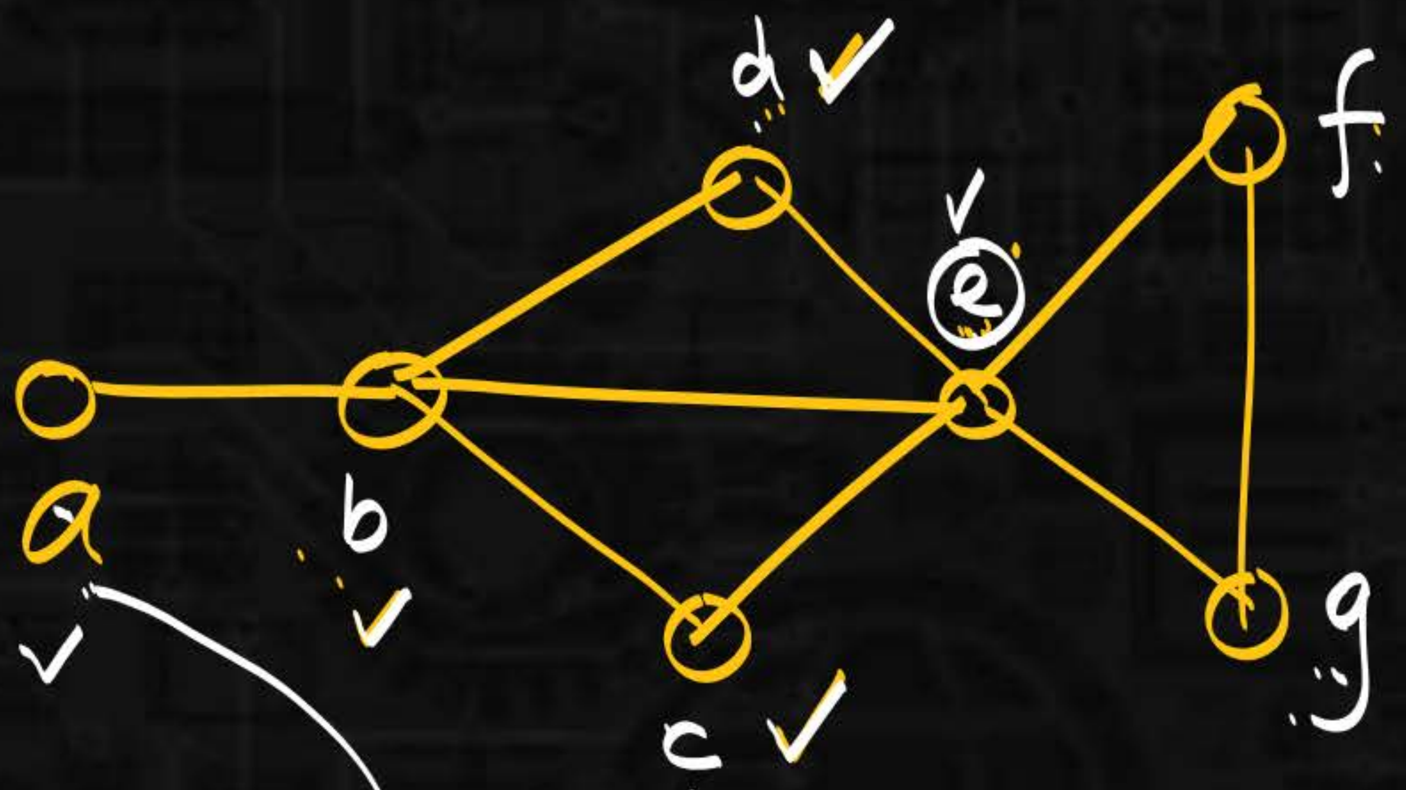
$D = \{b, e, f\}$

$D = \{b, e, f\}$: DS

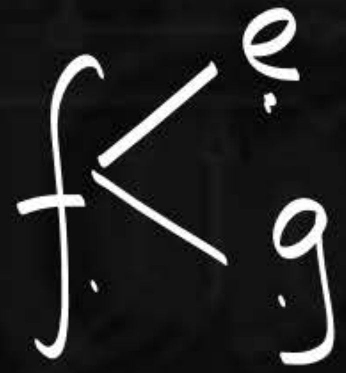
$D_2 = \{a, e\}$ DS

$D_3 = \{a, e, \underline{f}\}$ DS

$\{a, c, d\}$ not DS



$\{a, c, d\}$ f is not happy.

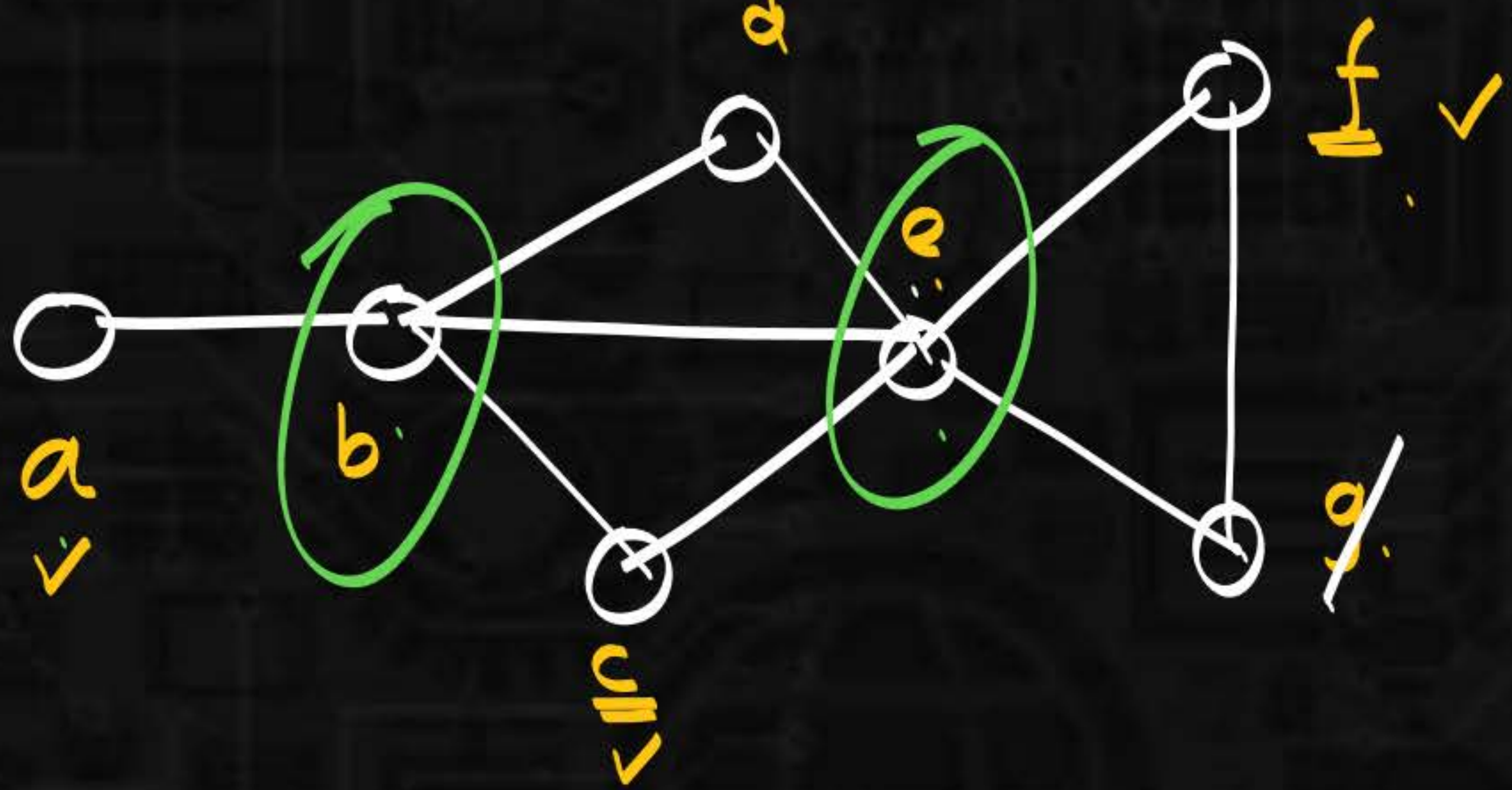


$\{\cancel{a}\ b\ c\ d\ e\ f\ g\} \rightarrow$ DS ✓

$\{b\ \cancel{c}\ d\ e\ f\ g\}$ - DS.

$\{b\ d\ e\ \cancel{f}\ g\}$ - DS

$\{b\ d\ e\ \cancel{g}\} \rightarrow \{b\ \cancel{d}\ e\} \rightarrow \{be\} \rightarrow$ minimal Dominating set



→ all vertices of graph will be Dominating set.

minimal/dominating set (MDS)

Dominating set such that we can not remove new vertex from this.

minimal ≠ minimum

minimal is not related to size but property

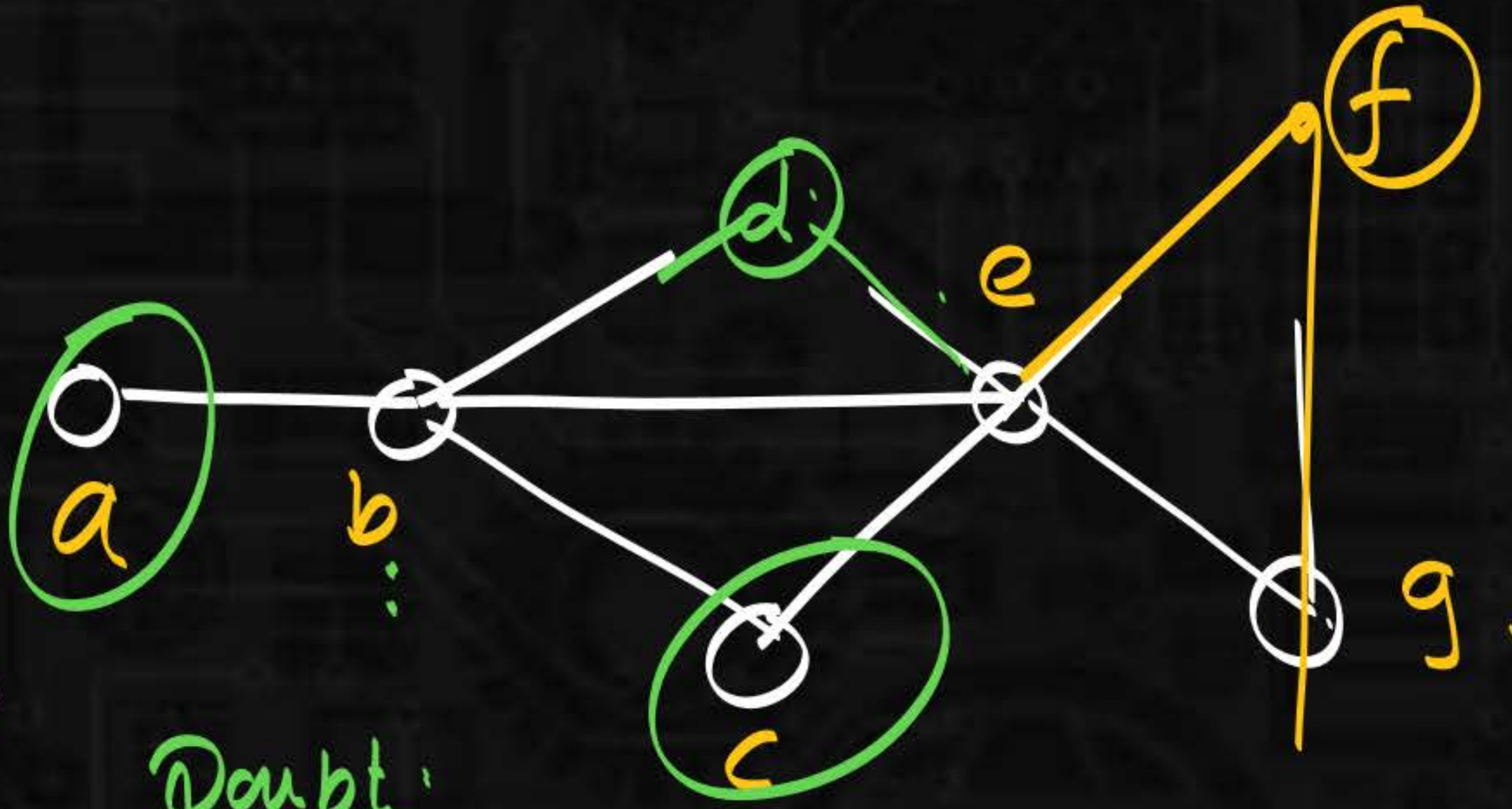
maximal → can not add

minimal → can not remove.

MDS :

{be}✓ {bf}✓

{ae}✓ {bg}✓

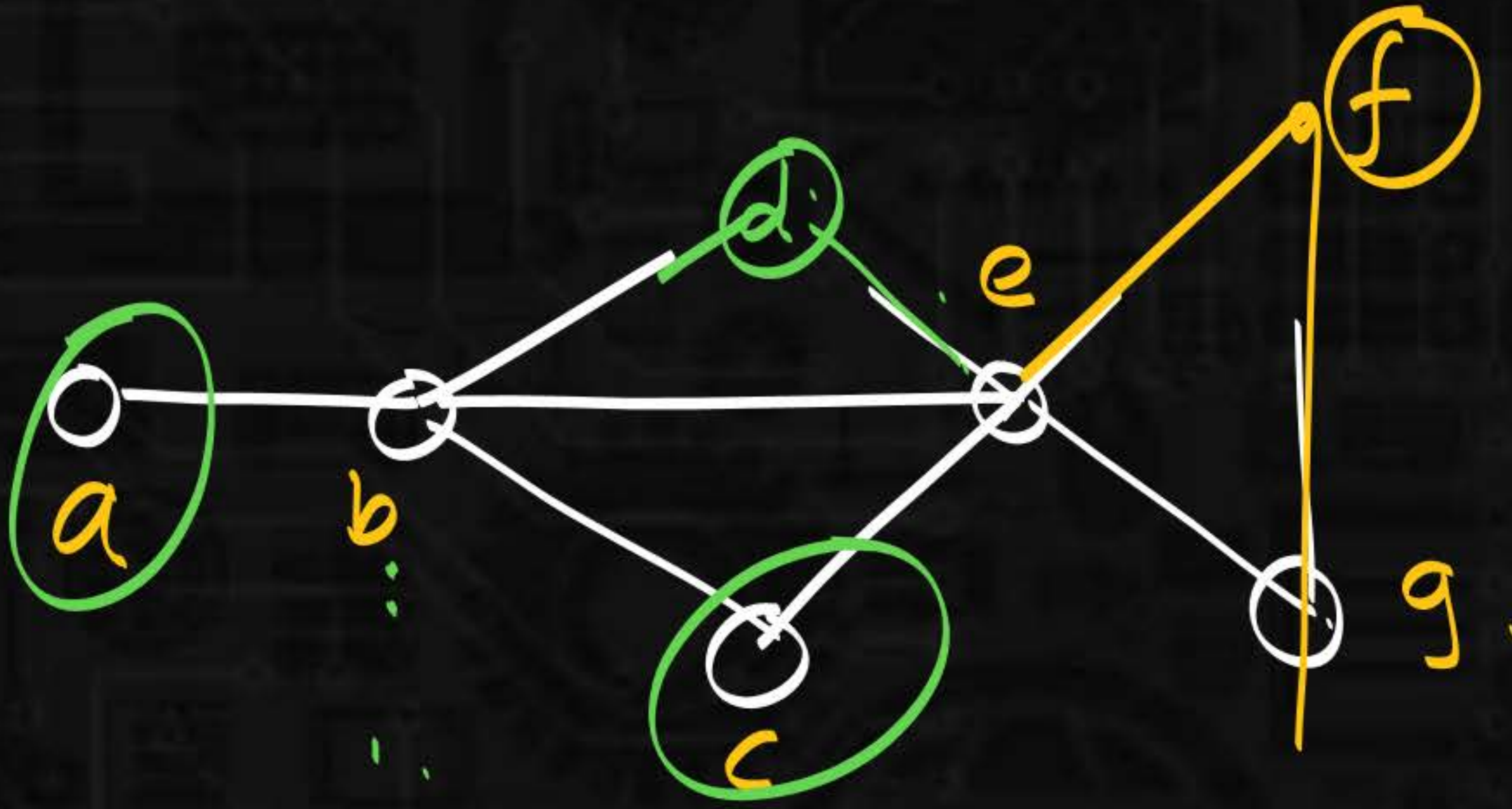


Doubt :

{acdf} MDS

MDS: $\begin{cases} \{a, c, d, f\} \\ \{a, e\} \\ \{b, f\} \\ \{b, g\} \\ \boxed{\{b, e\}} \end{cases}$

→ size: 2.



Domination no: $(\alpha(G))$

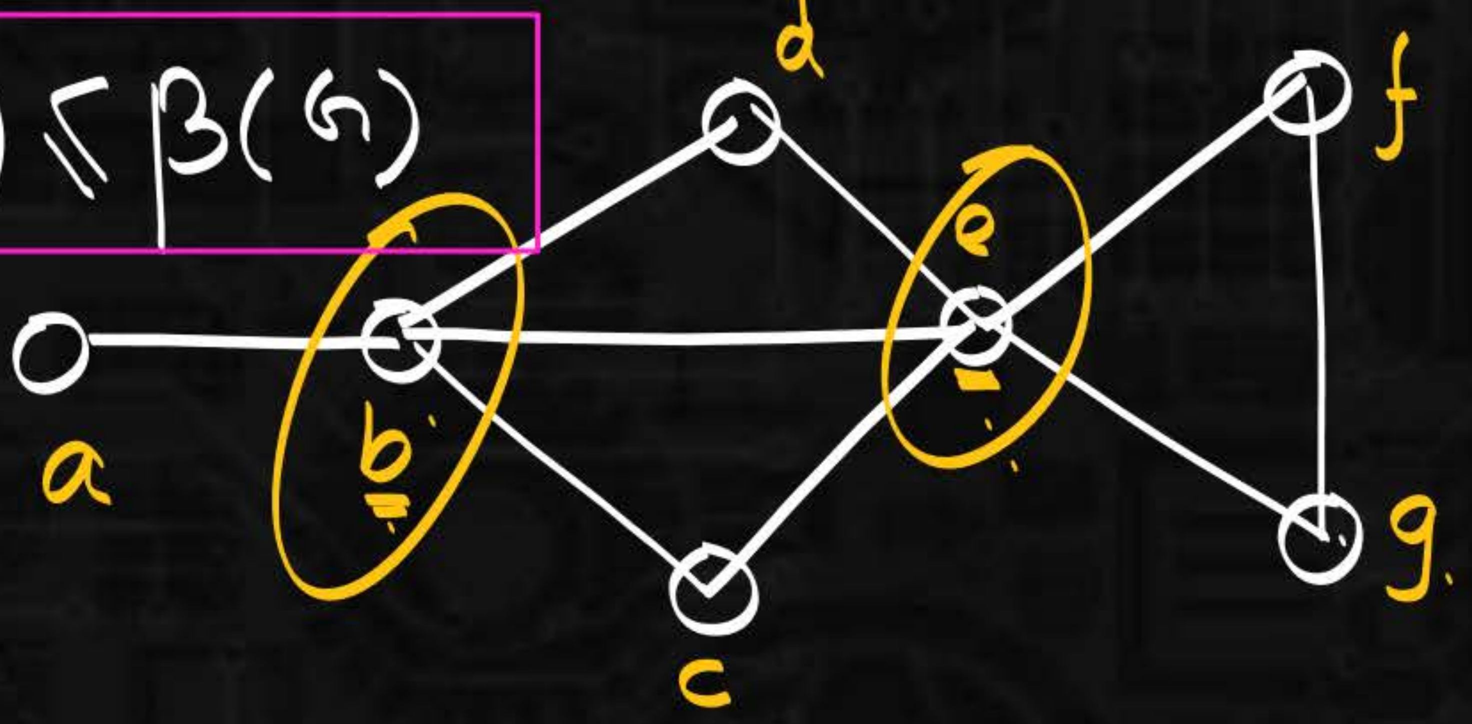
no. of vertices present in smallest MDS.

$\alpha(G) = 2$.

non adj. $\beta(G)=4$

MIS	MDS
{a, c, d, f}	{a, c, d, f}
{b, f}	{b, f}
{b, g}	{b, g}
{a e}	{a e}
	{b, e}

$$\boxed{\alpha(G) \leq \beta(G)}$$



1. → Every MIS will always be MDS.
2. → Every MDS will be MIS (false)
   eg: {b, e}

Domination ($\alpha(G)$) $\leq$ Independence no ($\beta(G)$)

6ft

5ft

3ft

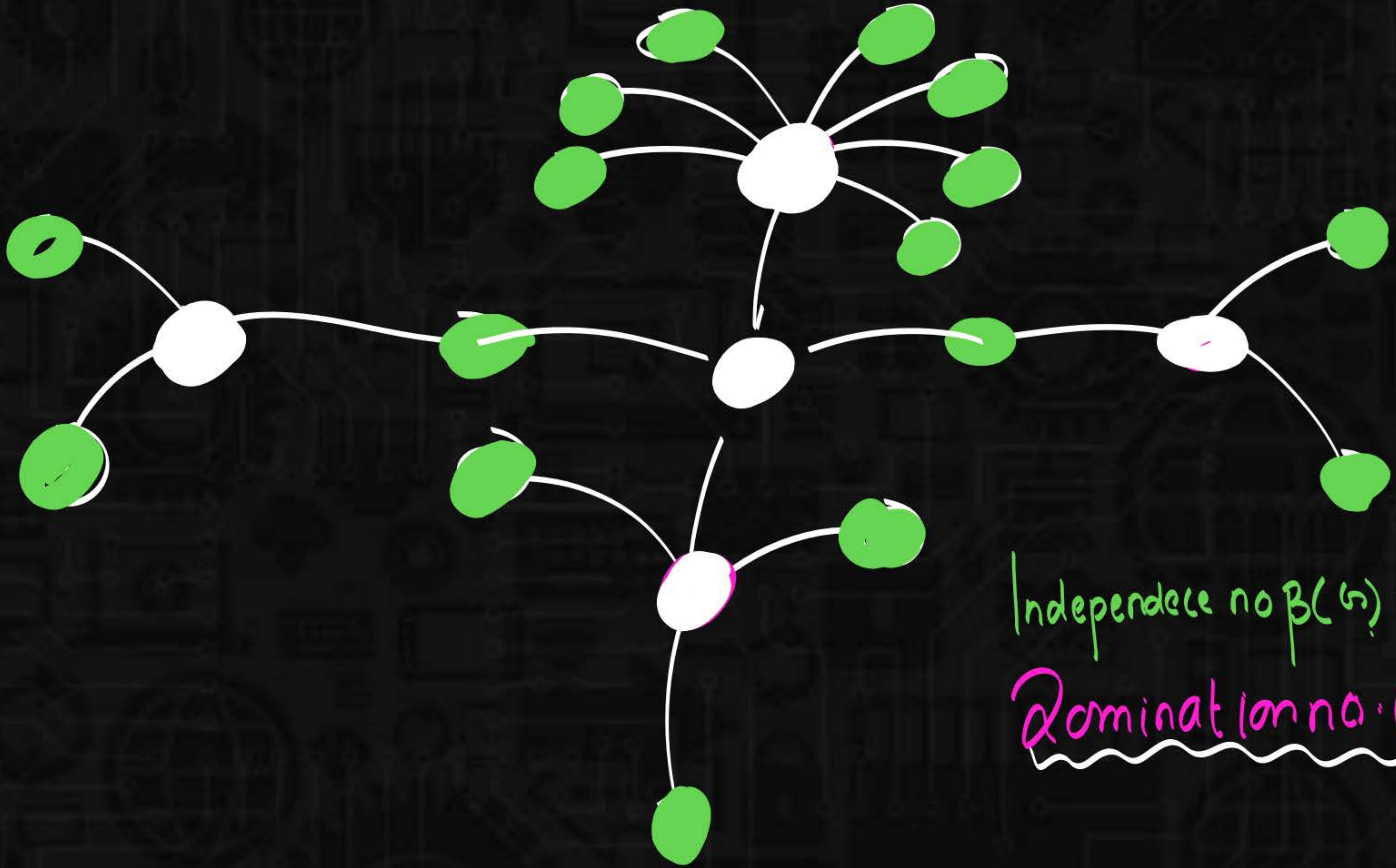
6ft

5ft

3ft

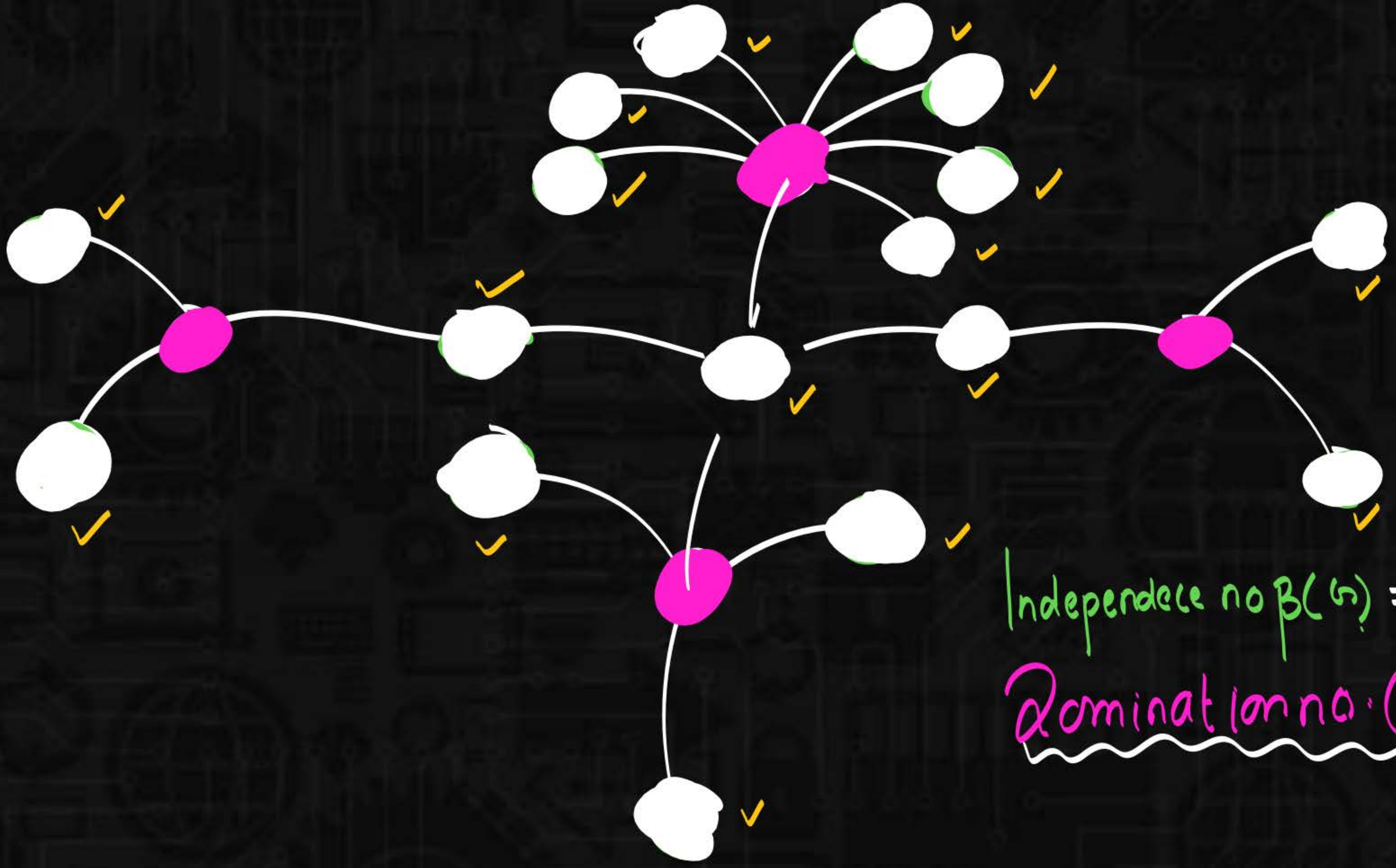
2f

3f



Independence no $\beta(G) = 16$

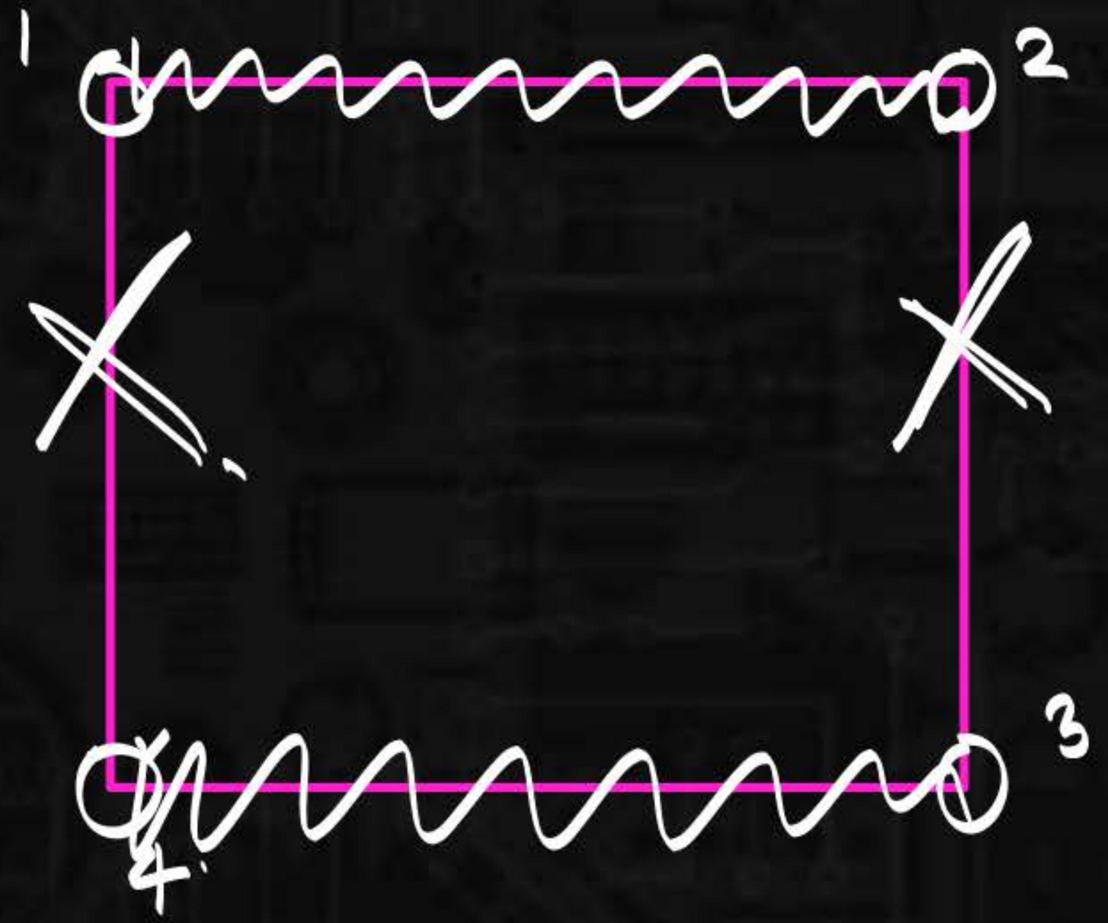
Domination no. $(\alpha(G) = 4$



Independece no $\beta(G) = 16$

Domination no. $(\alpha(G) = 4$.

Independent edge set : set of non adjacent edges.
(matching set)



{12} → matching set.

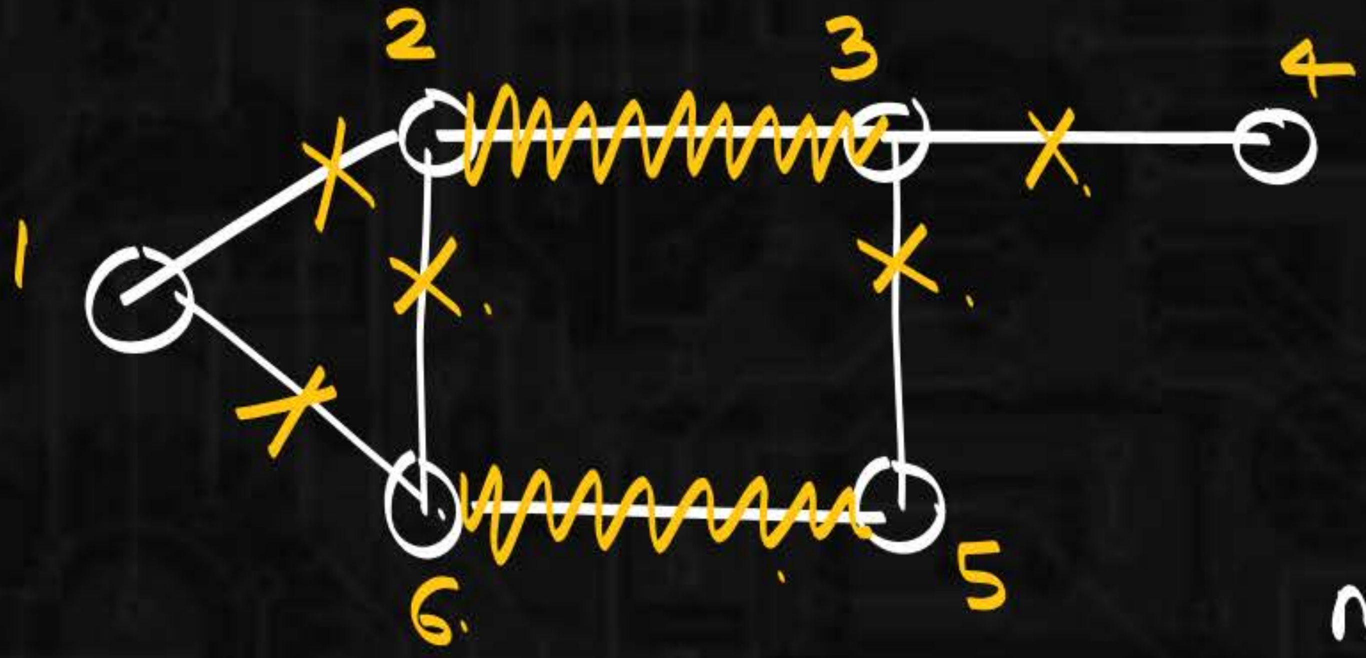
{12, 43} → matching set.

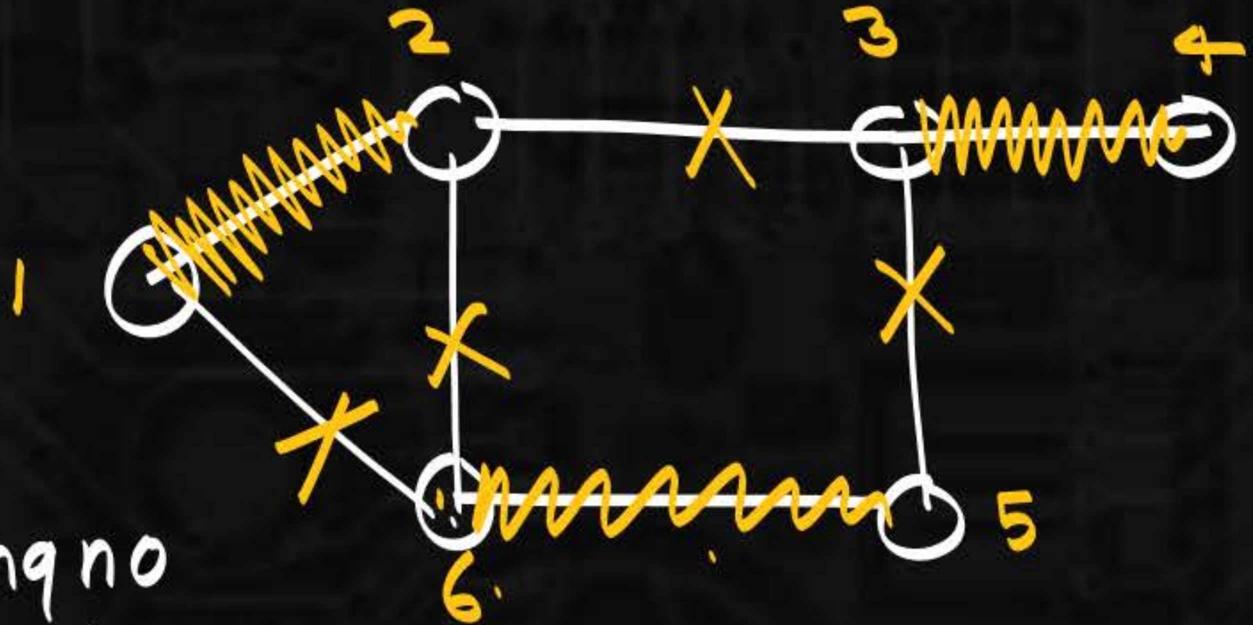
↳ maximal matching set

matching set: set of non adjacent edges.

maximal matching set : matching set such that we can not add new edge into this.

maximal → it is not related to size but property.





{23} - ms.

{23, 65} maximal matching set

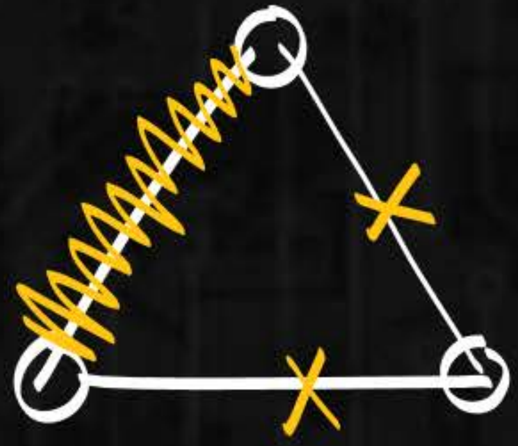
matching no ($m(G)$)

no. of edges present in largest maximal matching set.

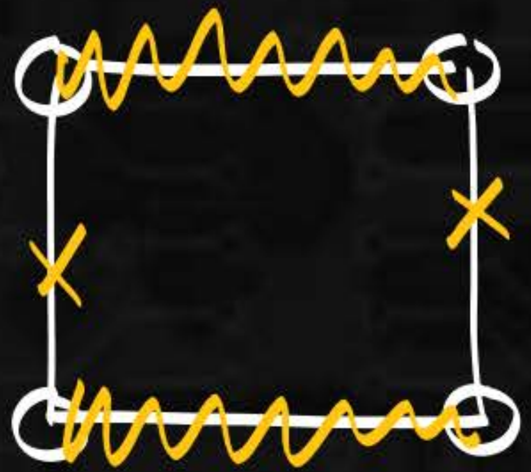
{34} - ms.

{34, 65} - ms.

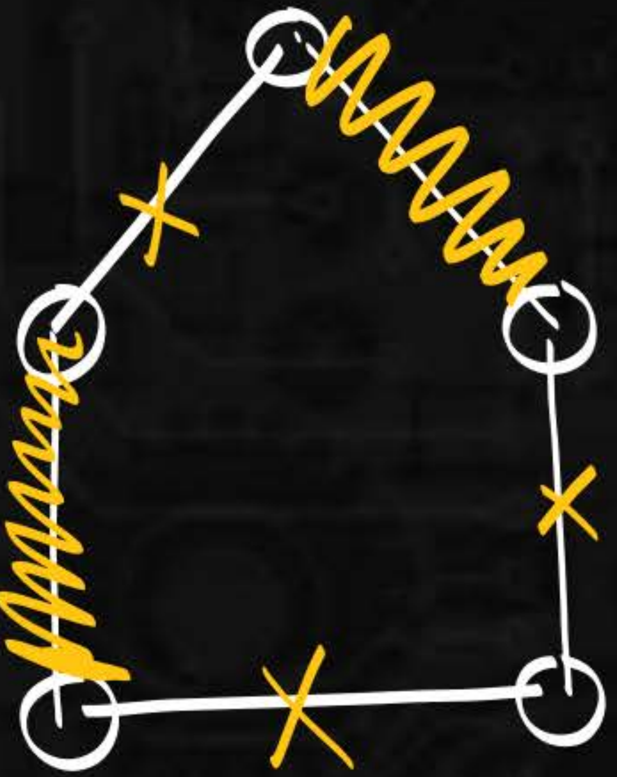
{34, 65, 12} - mms.



C_3



C_4



C_5

$m(C_3) = 1$

$m(C_4) = 2$

$m(C_5) = 2$

$$m(C_n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$C_3 \to 1$

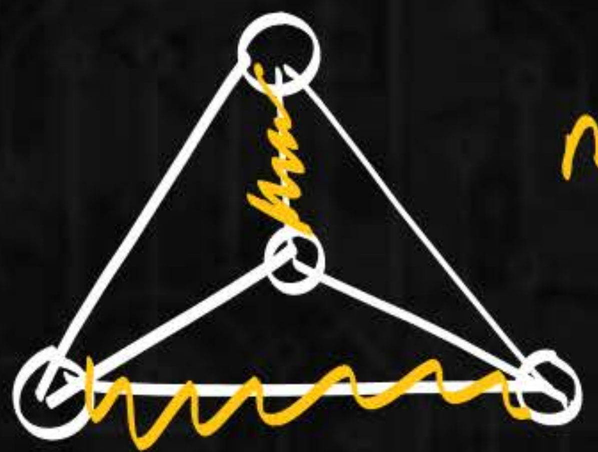
$C_4 \to 2$

$C_5 \to 2$

$5/2 = \lfloor 2.5 \rfloor$

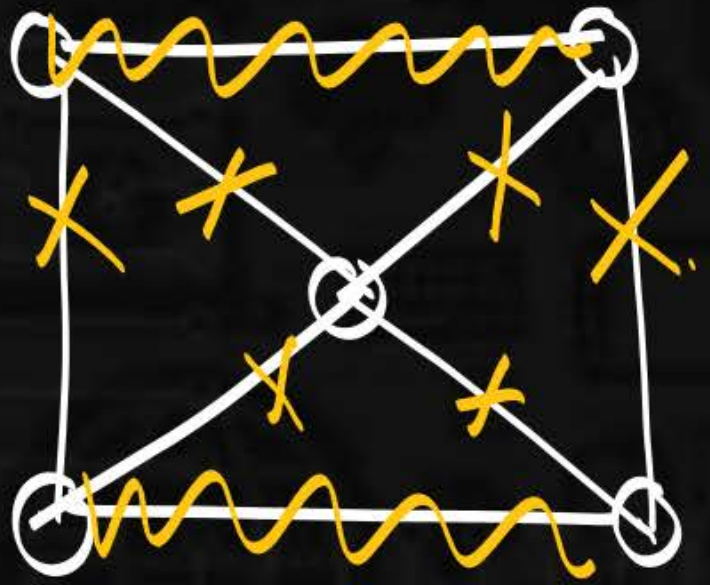
Wheel Graph.

W4



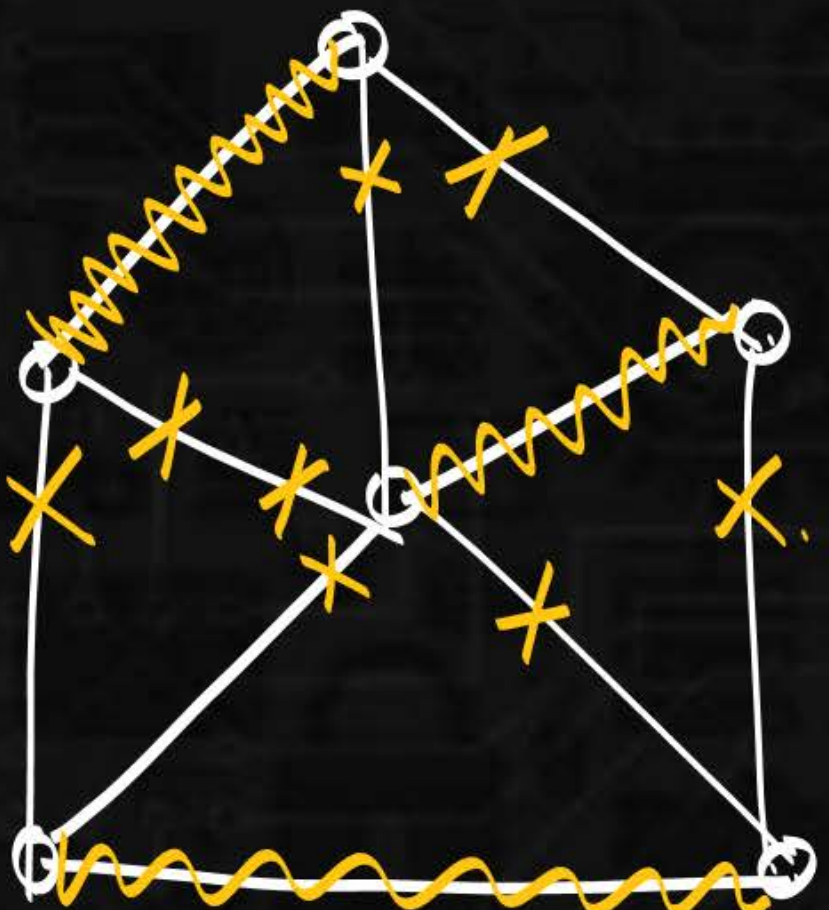
$m(W4) = 2.$

W5



$m(W5) = 2.$

W6.

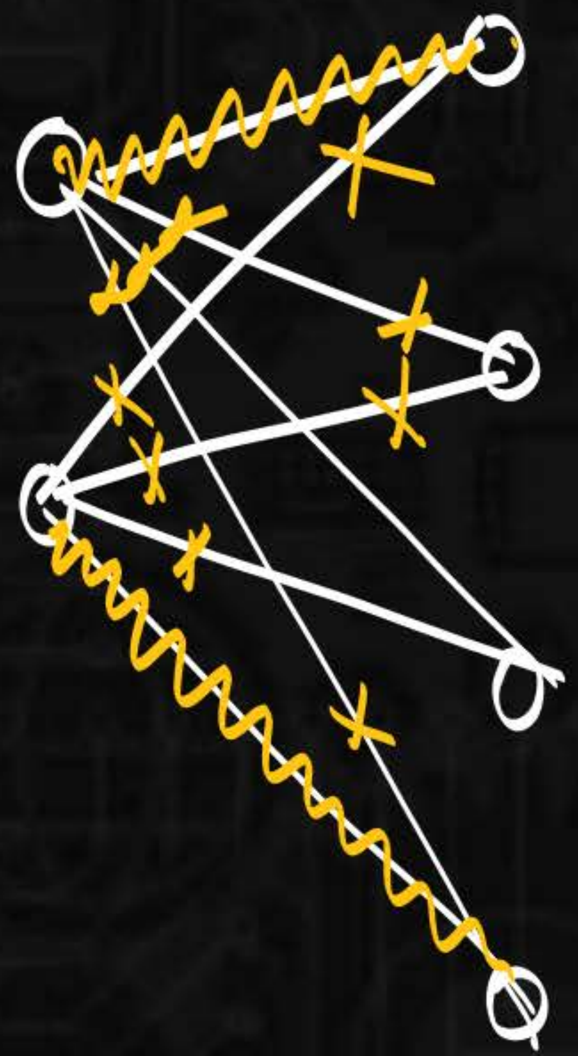


$m(W6) = 3.$

$$m(W_n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$$m(C_n) = m(K_n) = m(W_n) = \left\lfloor \frac{n}{2} \right\rfloor$$

$K_{2,4}$.



$$m(K_{2,4}) = \min(2,4) = 2.$$

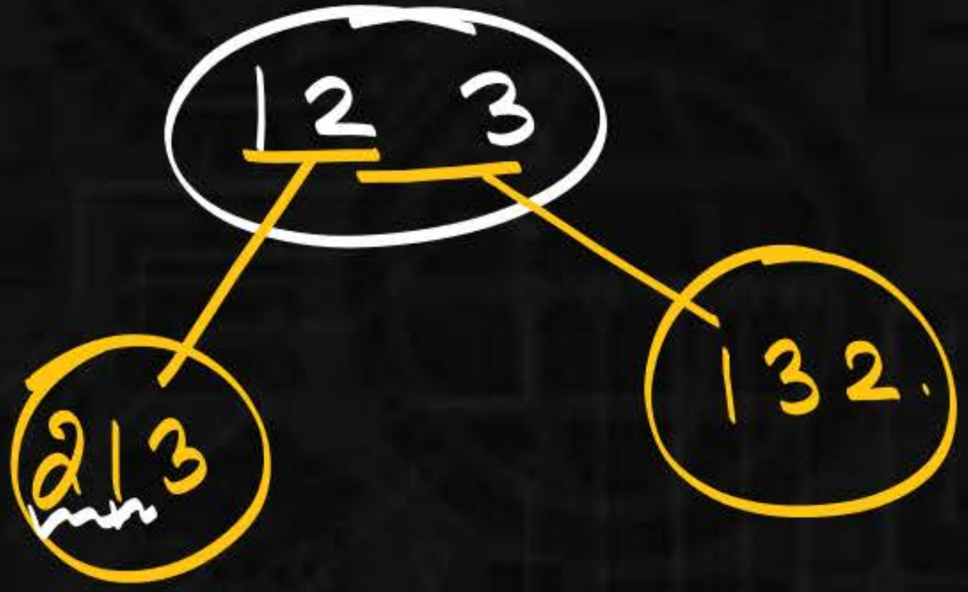
$$\boxed{m(K_{m,n}) = \min(m,n)}$$

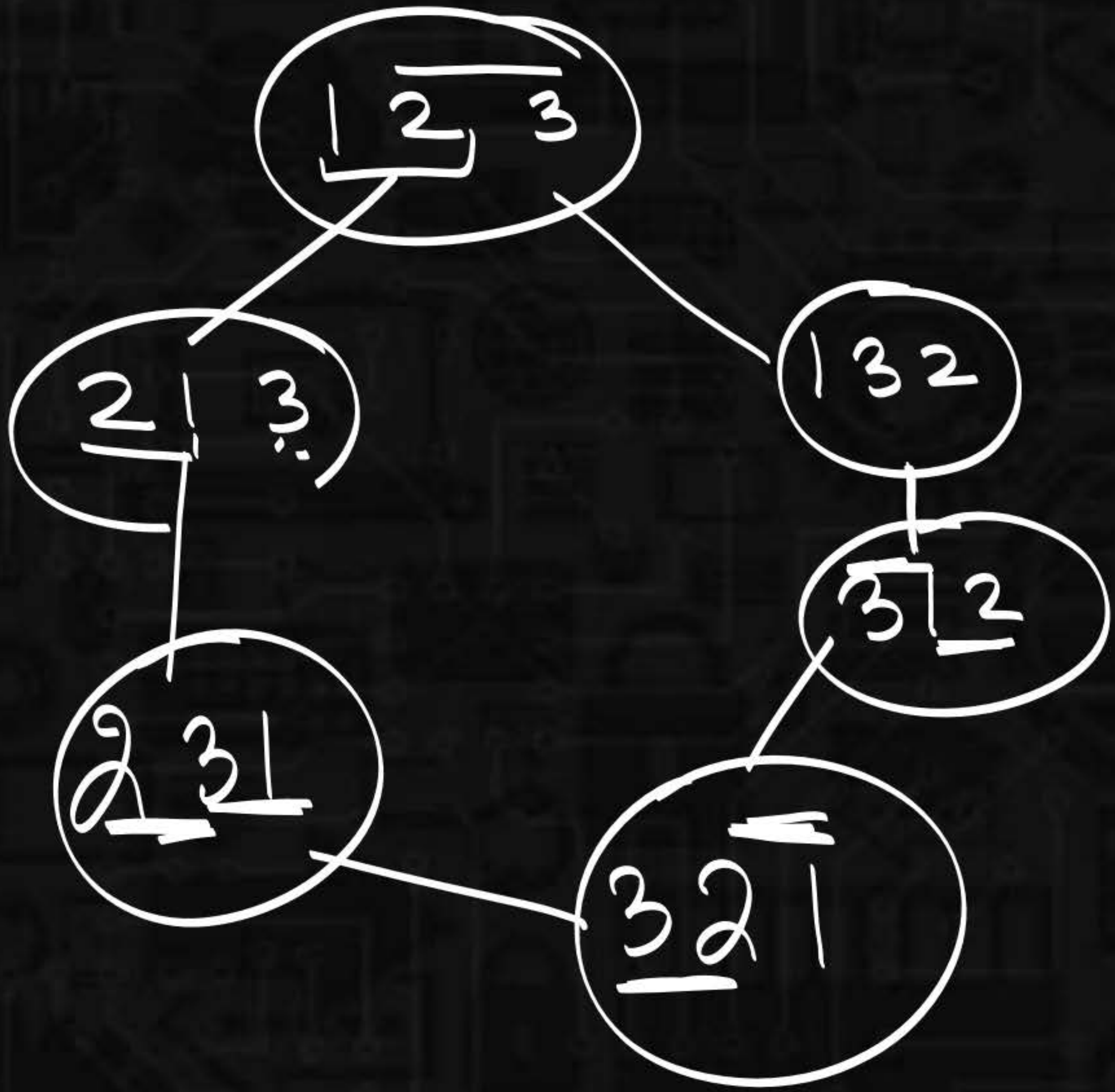
Consider graph vertices are represented as permutations of a given set $\{1, 2, 3\}$. 2 vertices are connected if there adjacent elements are interchanges, $m(G) = ?$

$S = \{1, 2, 3\}$

$3!$

$$\left\{\begin{array}{c} 1\,2\,3 \\ 1\,3\,2 \\ 2\,1\,3 \\ 2\,3\,1 \\ 3\,1\,2 \\ 3\,2\,1. \end{array}\right.$$



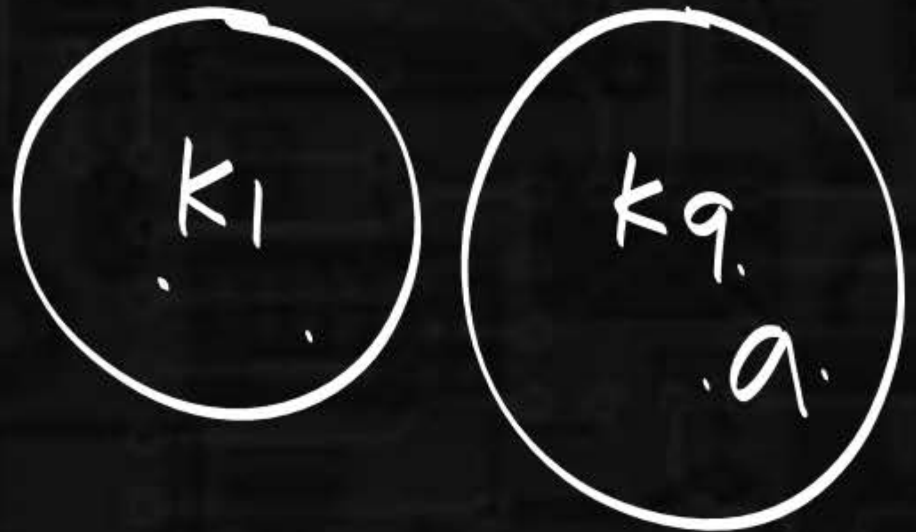


C_6

$m(C_6) = \lfloor 6/2 \rfloor = 3.$

Consider a disconnected graph of 10 vertices & maximum edges $M(G)+\chi(G) = ?$

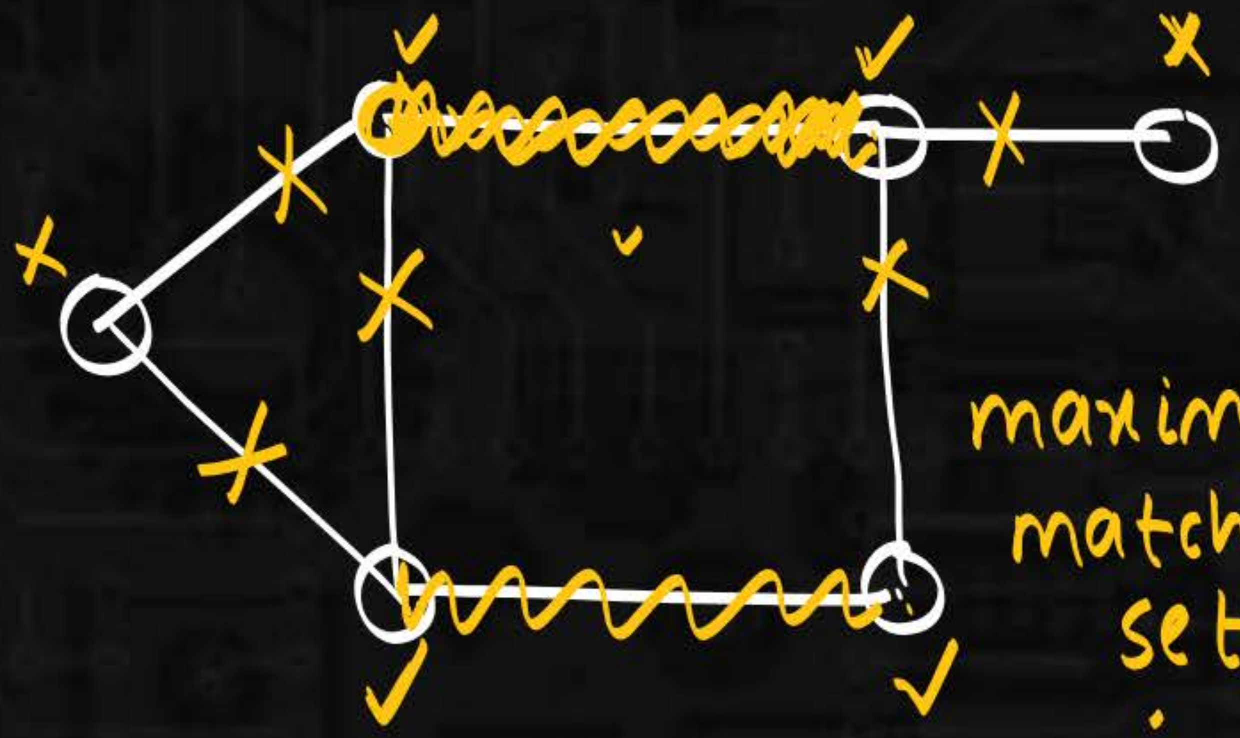
$$\begin{cases} 10v \\ k=2 \end{cases}$$



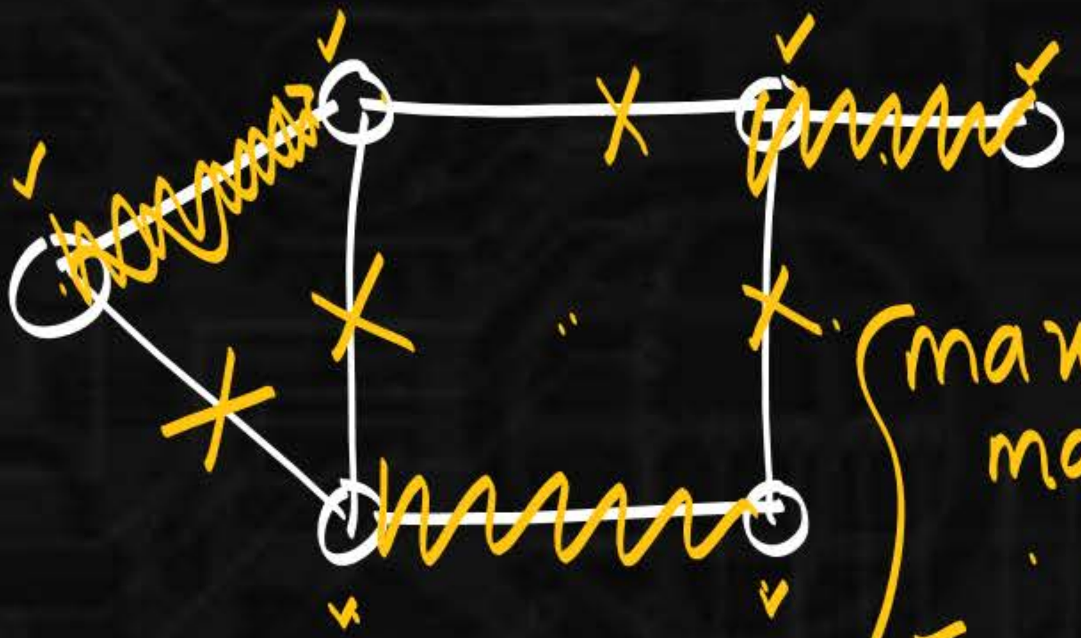
$\chi(K_9) = 9$

$M(K_9) = \left\lfloor \frac{9}{2} \right\rfloor = 4$

Perfect matching : maximal matching set
+ which saturates all the vertices.



maximal matching set.



P.M. maximal matching set + covers all vertices

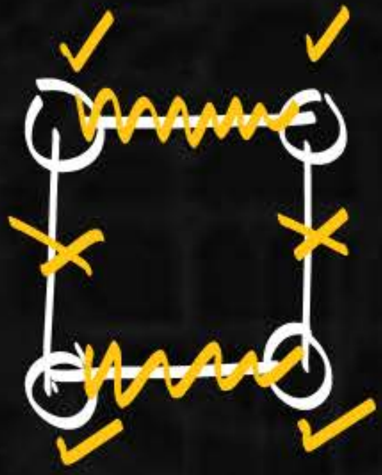
mms

mms

mms.

P.M will not exist.

if Perfect matching exist then no. of vertices will be even.



p $\qquad\qquad$ q

if (Perfect matching exist) then (no. of vertices will be even.)

if odd no. of vertices then P.M will not exist.

$p \rightarrow q$

viceversa.

$q \rightarrow p$

Contra:

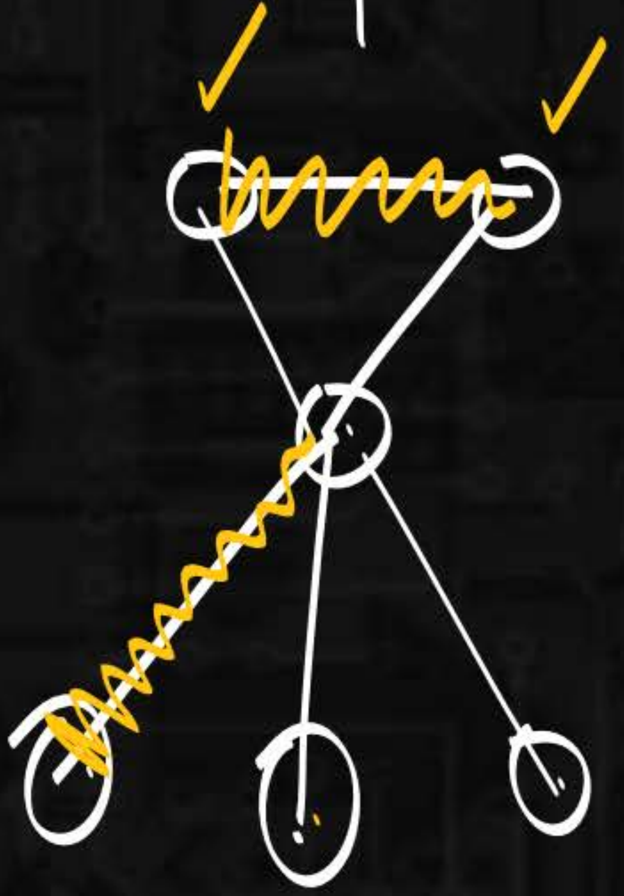
$\neg q \rightarrow \neg p$

P Q

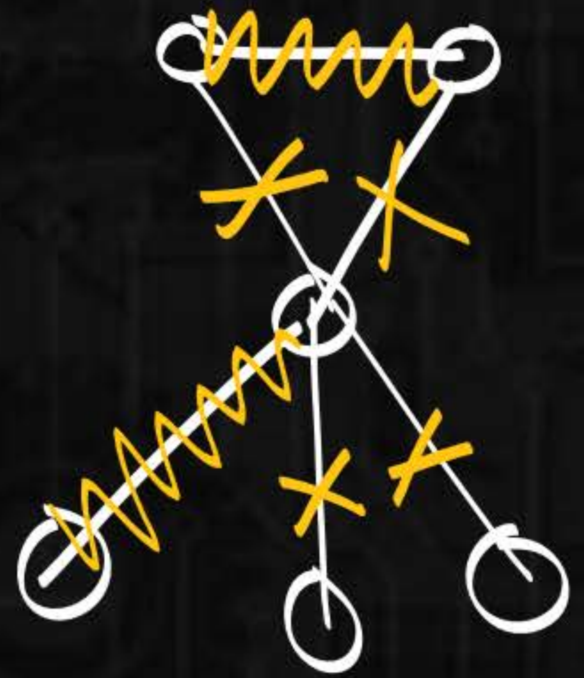
if (Perfect matching exist) then (no. of vertices will be even)

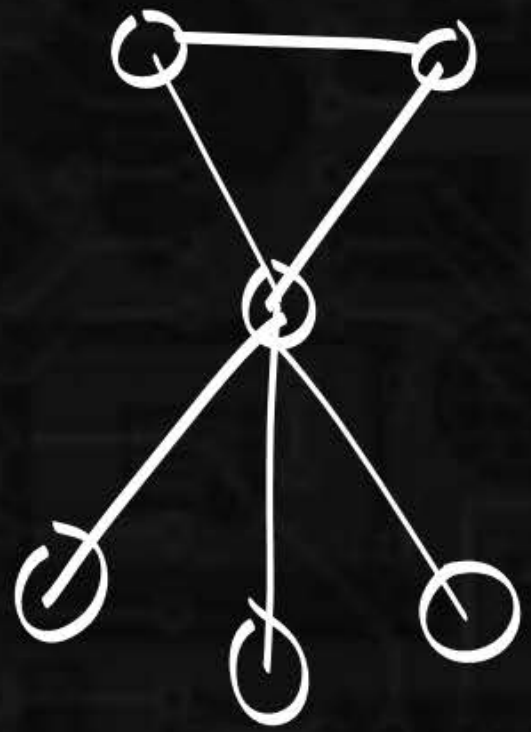
viceversa is not true.

if Graph is having even no. of vertices then Perfect matching exist (false)



n = 6





$n = 4.$

$K_{1,3}.$


THANK YOU GW
SOLDIERS !