ENGINEERING MATHEMATICS

ALL BRANCHES

Calculus
Multiple Integral

DPP-10 Solution

By- CHETAN SIR

Question 1

The value of the integral $\int_0^{\pi} \underset{x}{} \underset{f(x)}{x\cos^2 x\,dx}$ is

A $\pi^2/8$

B $\pi^2/4$

C $\pi^2/2$

D π^2

$$\pi \int_0^{\pi/2} \cos^2 x\,dx = \pi \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi^2}{4}$$

$$I = \int_0^{\pi} x\cos^2 x = \int_0^{\pi} (\pi - x)\cos^2(\pi - x) \quad \text{[Prop 4]}$$

$$I = \int_0^{\pi} \pi\cos^2 x - \int_0^{\pi} x\cos^2 x \qquad \pi \to 2a$$

$$2I = \pi \cdot 2\int_0^{\pi/2} \cos^2 x \quad \text{[Prop 6]}$$

$f(x) = f(\pi - x)$

$$I = \pi \cdot \frac{1}{2} \cdot \frac{\pi}{2} = \frac{\pi^2}{4}$$

Question 2

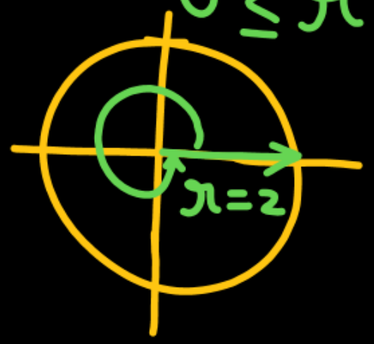
$dx\,dy \rightarrow r\,dr\,d\theta$

The integral $\frac{1}{2\pi}\int\int_D (x+y+10)dxdy$, where D denotes the disc:

$x^2+y^2 \leq 4$, evaluates to 20

r^2 $0 \leq \theta \leq 2\pi$

$0 \leq r \leq 2$



$x = r\cos\theta$

$y = r\sin\theta$

$x^2+y^2 \leq 4$

$; r \leq 2$

$$\frac{1}{2\pi}\int_{r=0}^{r=2}\int_{\theta=0}^{\theta=2\pi}(r\cos\theta + r\sin\theta + 10)\,r\,dr\,d\theta$$

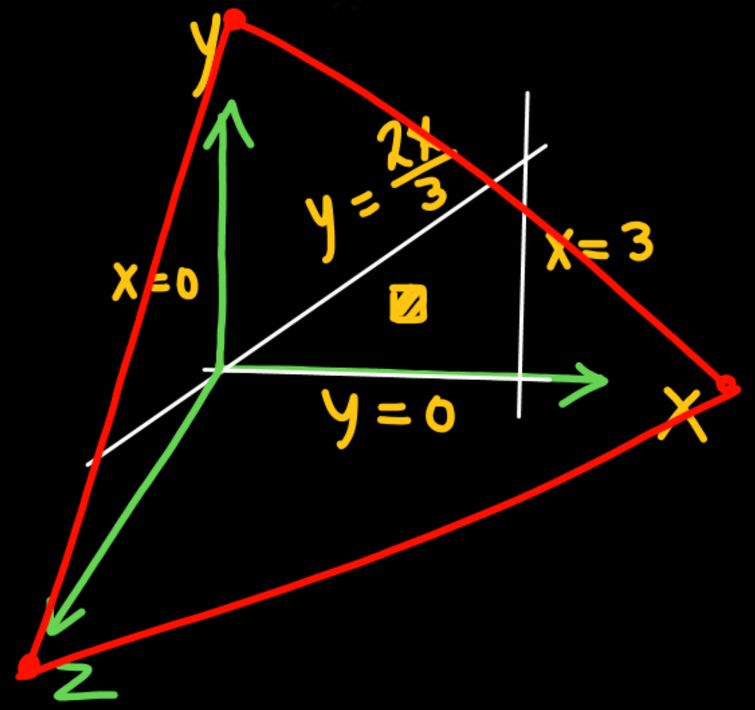
$$\frac{1}{2\pi}\int_{r=0}^{r=2}\left[r\sin\theta - r\cos\theta + 10\theta\right]_0^{2\pi} r\,dr$$

$$\frac{1}{2\pi} \cancel{20}^{10}\pi \left[\frac{r^2}{2}\right]_0^2 = 20$$

Question 3

A triangle in the xy-plane is bounded by the straight lines $2x = 3y$, $y = 0$ and $x = 3$. The volume above the triangle and under the plane $x + y + z = 6$ is ____.

$$\frac{x}{6} + \frac{y}{6} + \frac{z}{6} = 1$$



$$0 \le z \le 6 - x - y$$
$$0 \le y \le 2x/3$$
$$0 \le x \le 3$$

$$\text{Vol.} = \int_{x=0}^{x=3} \int_{y=0}^{y=2x/3} \int_{z=0}^{z=6-x-y} dz\, dy\, dx$$

$$\iint (6 - x - y)\, dy\, dx$$

$$\int_0^3 \left[6y - xy - \frac{y^2}{2}\right]_0^{2x/3} dx$$

$$\int_0^3 6\left(\frac{2x}{3}\right) - x\left(\frac{2x}{3}\right) - \frac{(2x/3)^2}{2} dx$$

$$\int_0^3 4x - \frac{2}{3}x^2 - \frac{2}{9}x^2 \, dx$$

$$\left[4\frac{x^2}{2} - \frac{2}{3}\frac{x^3}{3} - \frac{2}{9}\frac{x^3}{3}\right]_0^3$$

$$4\frac{(3)^2}{2} - \frac{2}{3}\frac{(3)^3}{3} - \frac{2}{9}\frac{(3)^3}{3}$$

$$18 - 6 - 2 = \boxed{10}$$

Question 4

The area of the region bounded by the curve $y(x^2 + 2) = 3x$ and $4y = x^2$ is given by

A $\int_0^1 \int_{y=0}^{\frac{x^2}{4}} dxdy$

B $\int_0^1 \int_{y=0}^{\frac{2}{4}} dydx$

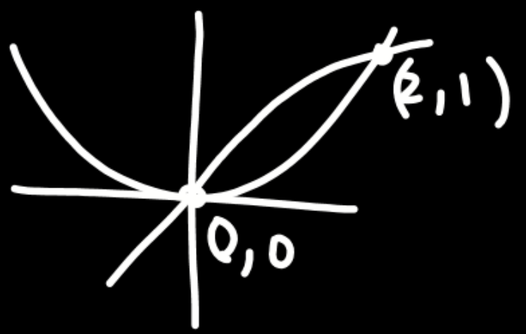
C $\int_0^2 \int_{y=\frac{x^2}{4}}^{\frac{3x}{(x^2+2)}} dydx$

D $\int_{y=0}^1 \int_{y=\frac{x^2}{4}}^{\frac{3x}{(x^2+2)}} dxdy$

$y = \frac{3x}{x^2+2}$ $\quad y = \frac{x^2}{4}$

Point of intersection (0,0) & (2,1)

$$\int_{x=0}^{x=2} \int_{y=x^2/4}^{y=\frac{3x}{x^2+2}} dy\, dx$$



Question 5

The area enclosed between the straight line $y = x$ and the parabola $y = x^2$ in the $x - y$ plane is

A 1/6

B 1/4

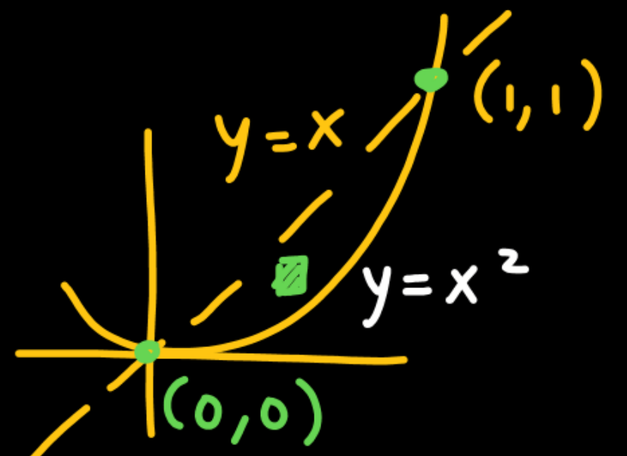
C 1/3

D 1/2

$$\int_{x=0}^{x=1} (x - x^2)\, dx$$

$$\left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \frac{1}{2} - \frac{1}{3}$$

$$= \frac{1}{6}$$



Question 6

The parabolic arc $y = \sqrt{x}, 1 \leq x \leq 2$ is revolved around the x – axis. The volume of the solid of revolution is

A $\pi/4$

B $\pi/2$

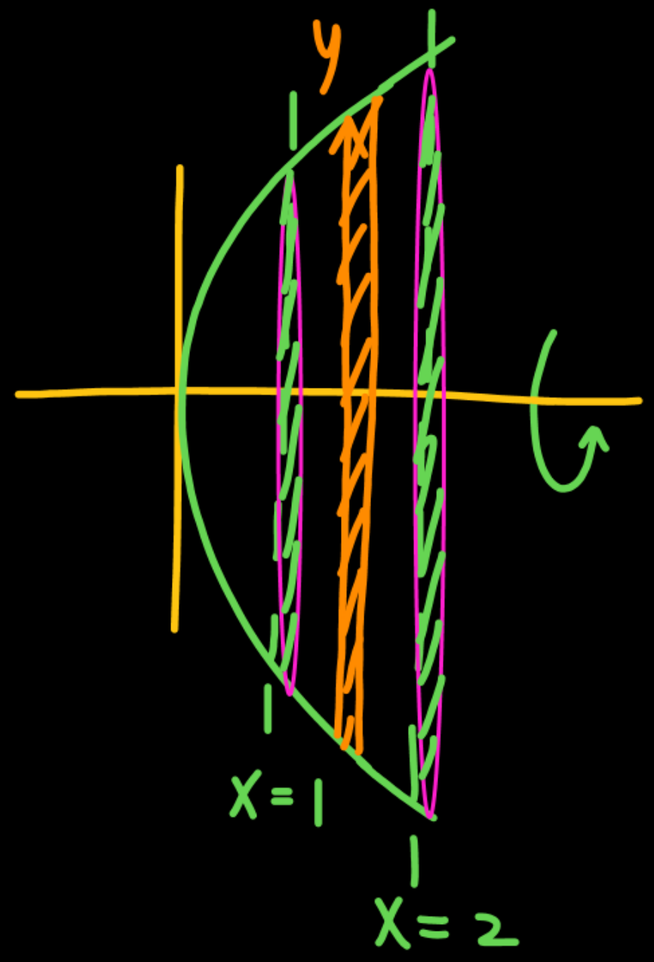
C $3\pi/4$

D $3\pi/2$

$$\text{Vol.} = \int_{x=1}^{x=2} \pi y^2 \, dx$$

$$\int_1^2 \pi (\sqrt{x})^2 \, dx$$

$$= \pi \left[\frac{x^2}{2}\right]_1^2 = \frac{3}{2}\pi$$



Question 7

The value of the integral $\int_{-\infty}^{\infty} \frac{dx}{1+x^2}$ is

A $-\pi$

B $-\pi/2$

C $\pi/2$

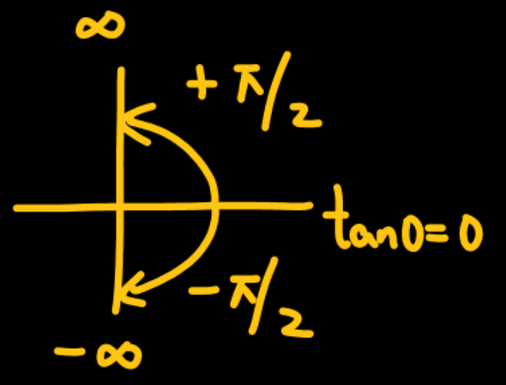
D π ✓

$$\left[\tan^{-1} x\right]_{-\infty}^{\infty}$$

$$= \tan^{-1}(\infty) - \tan^{-1}(-\infty)$$

$$\frac{\pi}{2} - \left(-\frac{\pi}{2}\right)$$

$$= \pi$$



Question 8

The length of the curve $y = \frac{2}{3}x^{\frac{3}{2}}$ between $x = 0$ and $x = 1$ is

A 0.27

B 0.67

C 1

D 1.22

$$L = \int_{x_1}^{x_2} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$\frac{dy}{dx} = \frac{2}{3} \cdot \frac{3}{2} x^{3/2 - 1}$$

$$= \int_0^1 \sqrt{1 + (x^{1/2})^2}\, dx$$

$$\int_0^1 \sqrt{1+x}\, dx = \left[\frac{(1+x)^{3/2}}{3/2}\right]_0^1 = \frac{2}{3}\left[2^{3/2} - 1^{3/2}\right]$$

$$= \frac{2}{3}\left[\sqrt{8} - 1\right] = 1.22 \text{ units}$$

Question 9

The value of $\int_0^1 \int_0^{\sqrt{1+x^2}} dydx$ is

A $\frac{\pi}{4} log(\sqrt{2}+1)$

B $\frac{\pi}{4} log(\sqrt{2}-1)$

C $\frac{\pi}{2} log(\sqrt{2}+1)$

D None of these

$$\int_0^1 [y]_0^{\sqrt{1+x^2}} dx$$

$$\int_0^1 \sqrt{1+x^2}\, dx$$

$$\frac{1}{2}\left[x\sqrt{1+x^2} + \log\left(x+\sqrt{1+x^2}\right)\right]_0^1$$

$$= \frac{1}{2}\left[\sqrt{2} + \log\left(1+\sqrt{2}\right)\right]$$

Question 10

If A is the region bounded by the parabolas $y^2 = 4x$ and $x^2 = 4y$, then $\iint_A y\,dx\,dy$ is equal to

A 48/5

B 36/5

C 32/5

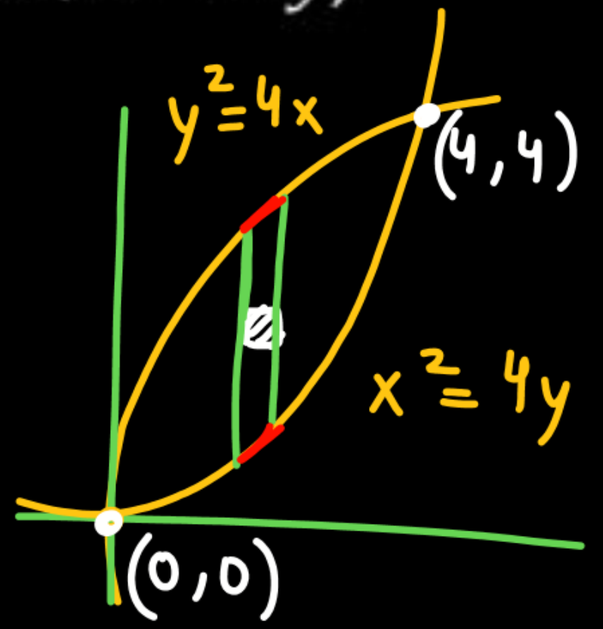
D None of these

$$\int_{x=0}^{x=4} \int_{y=x^2/4}^{y=2\sqrt{x}} y\,dy\,dx$$

$$\int_0^4 \left[\frac{y^2}{2}\right]_{x^2/4}^{2\sqrt{x}} dx$$

$$\frac{1}{2}\left[4x - \frac{x^4}{16}\right]$$

$$\int_0^4 2x - \frac{x^4}{32}\,dx$$



$$\left(\frac{x^2}{4}\right)^2 = 4x$$

$$x^4 - 64x = 0$$

$$x(x^3 - 4^3) = 0$$

$$x = 0, 4$$

$$\left[2 \cdot \frac{x^2}{2} - \frac{x^5}{32.5}\right]_0^4$$

$$16 - \frac{4^5}{32 \times 5}$$

$$= 16 - \frac{16 \times 2}{5}$$

$$= \frac{3}{5} \times 16 = \frac{48}{5}$$

Thank you

GW

Soldiers !