ENGINEERING MATHEMATICS

ALL BRANCHES

Calculus

Mean Value Theorem

DPP-05 Solution

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Question 1

If $f(x) = x^3 - 6x^2 + 11x - 6$ is on [1, 3], then the point $c \leftrightarrow (1, 3)$ such that $f'(c) = 0$ is given by

A $c = 2 \pm \frac{1}{\sqrt{2}}$

B $c = 2 \pm \frac{1}{\sqrt{3}}$

C $c = 2 \pm \frac{1}{2}$

D None

$f(1) = 1^3 - 6(1)^2 + 11(1) - 6 = 0$

$f(3) = 3^3 - 6(3)^2 + 11(3) - 6 = 0$

$f(x)$ is continuous in $[1, 3]$

$f(x)$ is differentiable in $(1, 3)$

$f(1) = f(3) = 0$

There is at least one point $c \in (1,3)$ such that

$f'(c) = 0$

$3x^2 - 12x + 11 = 0$

$3c^2 - 12c + 11 = 0$ $\quad ax^2 + bx + c = 0$

$\therefore a = 3;\ b = -12;\ c = 11$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{12 \pm \sqrt{144 - 4(3)(11)}}{2 \times 3}$$

$$= 2 \pm \frac{1}{\sqrt{3}}$$

$$1 < 2 + \frac{1}{\sqrt{3}},\ 2 - \frac{1}{\sqrt{3}} < 3$$

Question 2

Let $f(x) = \sin 2x$, $0 \leq x \leq \frac{\pi}{2}$ and $f'(c) = 0$ for $c \leftrightarrow \left(0, \frac{\pi}{2}\right)$ Then, c is equal to

$f(x)$ is continuous & differentiable in $(0, \pi/2)$

$$f(0) = f(\pi/2) = 0$$

Rolle's theorem $\rightarrow$ There is at least one point $c \in (0, \pi/2)$ such that $f'(c) = 0$

$$2\cos 2c = 0$$

$$\cos 2c = \cos\frac{\pi}{2}$$

$$c = \frac{\pi}{4} \; (0, \pi/2)$$

A $\frac{\pi}{4}$

B $\frac{\pi}{3}$

C $\frac{\pi}{6}$

D None

Question 3

Let $f(x) = x(x+3)\, e^{-x/2}\ (-3 \le x \le 0)$ Let $c \leftrightarrow\ ]-3, 0[$ such that $f'(c) = 0$.

(x^2+3x)

Then, the value of c is

A 3

B –3

C –2

D –1/2

$f(x) = \underbrace{x(x+3)}_{A}\ \underbrace{e^{-x/2}}_{E}$

$\therefore$ Since algebric & exponential are both continuous & differentiable. $\therefore$ $f(x)$ is also cont. & diff.

$f(-3) = f(0) = 0$

Rolle's theorem $[\, f'(c) = 0 \,]$

$-\frac{1}{2}(x^2+3x)\, e^{-x/2} + (2x+3)\, e^{-x/2} = 0$

$e^{-x/2}\left[-\frac{x^2}{2} - \frac{3x}{2} + 2x + 3\right] = 0$

$$\therefore \quad -x^2 + x + 6 = 0$$

$$c^2 - c - 6 = 0$$

$$(c-3)(c+2) = 0$$

$$c = 3, -2$$

Only $c = -2 \in (-3, 0)$

Question 4

If Rolle's theorem holds for $f(x) = x^3 - 6x^2 + kx + 5$ on $[1, 3]$ with $c = 2 + \frac{1}{\sqrt{3}}$, then value of k is

A -3

B 3

C 7

D 11

If Rolle's theorem;

$f'(c) = 0$ $\qquad$ $f(1) = f(3)$

$3c^2 - 12c + K = 0$

$$\frac{12 \pm \sqrt{144 - 12K}}{6} = 0$$

$2 + \frac{\sqrt{144 - 12K}}{6}$ $\qquad$ On comparing,

$2 + \frac{1}{\sqrt{3}}$

$$\frac{\sqrt{144-12K}}{6} = \frac{1}{\sqrt{3}}$$

$$\frac{144-12K}{36} = \frac{1}{3}$$

$$144-12K = 12$$

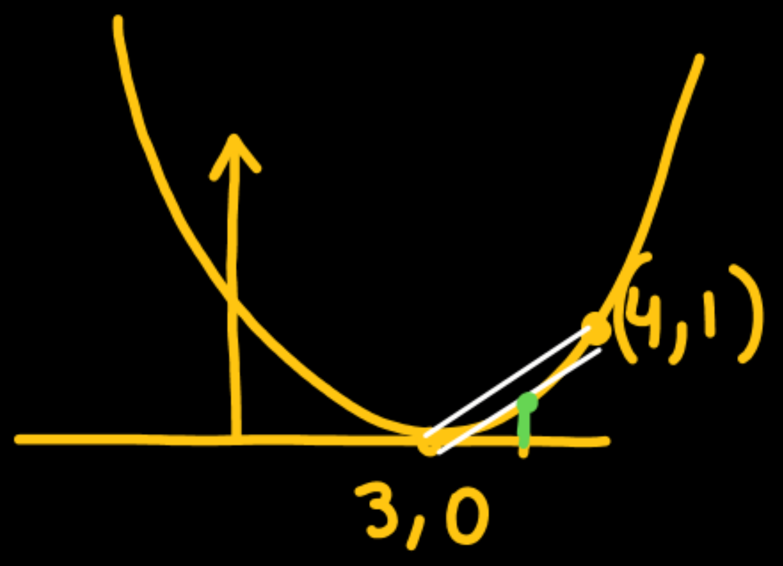
$$132 = 12K$$

$$\boxed{K = 11}$$

Question 5

A point on the parabola $y = (x - 3)^2$, where the tangent is parallel to the chord joining $\underbrace{A\ (3, 0) \text{ and } B\ (4, 1)}$ is

$$\text{Slope} = \frac{1-0}{4-3} = 1$$



A $(7, 1)$

B $\left(\frac{3}{2}, \frac{1}{4}\right)$

C $\left(\frac{7}{2}, \frac{1}{4}\right)$

D $\left(-\frac{1}{2}, \frac{1}{2}\right)$

$$\frac{dy}{dx} = 2(x-3) = 1$$

$$x = 3 + 1/2 = 7/2$$

$$\therefore y = \frac{1}{4}$$

At $\left(\frac{7}{2}, \frac{1}{4}\right)$ tangent is parallel to chord.

Question 6

A point on the curve $y = \sqrt{x-2}$ on [2, 3] where the tangent is parallel to the chord joining the end points of the curve is

A $\left(\frac{9}{4}, \frac{1}{2}\right)$

B $\left(\frac{7}{2}, \frac{1}{4}\right)$

C $\left(\frac{7}{4}, \frac{1}{2}\right)$

D $\left(\frac{9}{2}, \frac{1}{4}\right)$

$x = 2;\ y = 0$

$x = 3;\ y = 1$

Slope $= \frac{0-1}{2-3} = 1$

$\frac{dy}{dx} = \frac{1}{2\sqrt{x-2}} = 1$

$2\sqrt{x-2} = 1$

$4(x-2) = 1$

$4x - 8 = 1$

$x = 9/4 \quad \therefore y = 1/2$

$c = \frac{9}{4} \in [2, 3]$

$\left(\frac{9}{4}, \frac{1}{2}\right)$

Question 7

Let $f(x) = x(x-1)(x-2)$ be defined in $\left[0, \frac{1}{2}\right]$. Then, the value of c of the mean value theorem is

A 0.16

B 0.20

C 0.24

D None ✓

$f'(c) = \frac{f(b) - f(a)}{b - a}$ [By mean value theorem]

$f(x) = x^3 - 3x^2 + 2x$

$f'(x) = 3x^2 - 6x + 2$

$f\left(\frac{1}{2}\right) = \frac{1}{2}\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)$

$= 3/8$

$f(0) = 0(-1)(-2)$

$= 2$

$3c^2 - 6c + 2 = \frac{3/8 - 2}{1/2 - 0} = -\frac{13}{4}$

$12c^2 - 24c + 8 = -13$

$12c^2 - 24c + 21 = 0$

$4c^2 - 8c + 7 = 0$

$c = \frac{8 \pm \sqrt{64 - 112}}{2 \times 4}$ = Complex roots

Question 8

Let $f(x) = \sqrt{x^2 - 4}$ be defined in [2, 4]. Then, the value of c of the mean value theorem is

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$ By LMVT

A $-\sqrt{6}$

B $\sqrt{6}$ ✓

C $\sqrt{3}$

D $2\sqrt{3}$

$$f'(x) = \frac{2x}{2\sqrt{x^2 - 4}}$$

$$f(4) = \sqrt{12}$$

$$f(2) = 0$$

$$\frac{c}{\sqrt{c^2 - 4}} = \frac{\sqrt{12} - 0}{4 - 2}$$

$$2c = \sqrt{12(c^2 - 4)}$$

$$4c^2 = 12c^2 - 48$$

$$48 = 8c^2$$

$$c^2 = 6 \qquad c = \pm\sqrt{6}$$

$\because c = \sqrt{6}$ only lies in [2, 4]

Thank you

GW

Soldiers !