ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-04

Probability

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Topics to be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

PROBABILITY BASICS

v) At least two of the events (E)

$n(A \cap B) + n(B \cap C) + n(C \cap A) + n(A \cap B \cap C)$

$$n(E) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2n(A \cap B \cap C)$$

$(f+g) + (e+g) + (d+g) - 2g$

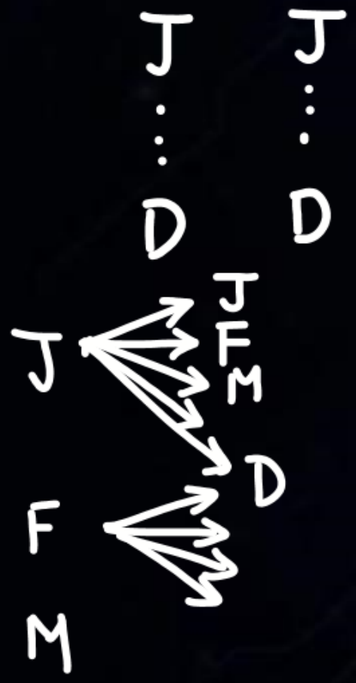
vi) At least one events $\Rightarrow n(A) + n(B) + n(C) + n(A \cap B) + n(B \cap C) + n(C \cap A) + n(A \cap B \cap C)$

$$n(E) = n(A \cup B \cup C)$$

vii) At least 3 events (All 3 of them) $\quad n(E) = n(A \cap B \cap C)$

viii) Neither A nor B nor C $= n - n(A \cup B \cup C)$

Ex:- What is the probability that two boys share same birth month ?



Total no. of ways = 12×12 = 144 ways

Favourable cases = {JJ, FF, MM, ... DD}

$$P(E) = \frac{n(E)}{n(S)} = \frac{12}{144}$$

PROBABILITY BASICS

Ex:- 5 cards are drawn from pack of 52 cards P (that it has only one ace)

52 cards.

↓

5 cards

1 Ace (4 Ace) | 4 Non-Ace (48 Non-Ace)

$n \rightarrow r$

${}^{n}C_{r}$

$$P(E) = \frac{n(E)}{n(S)} = \frac{{}^{4}C_1 \times {}^{48}C_4}{{}^{52}C_5}$$

— — — — — — —

H H

7 places

2H | 5T

$= {}^{7}C_2$

$\frac{7!}{2!\,5!}$

Ex:- What is the probability that the chosen triangle is equilateral if 3 of 6 vertices of a regular hexagon are chosen at random?

Total no. of triangles $= {}^{6}C_{3}$

Total no. equilateral Δ's $= 2$

$$P(E) = \frac{2}{20} = \frac{1}{10}$$

${}^{6}C_{3} = \frac{6 \times 5 \times 4}{1 \times 2 \times 3}$

PROBABILITY BASICS

Ex:- Two dice are tossed , one dice is regular and other dice is biased having P(1)=P(6) = 1/6, P(2)=P(4)=0, P(3)=P(5)=1/3, then Find the probability of obtaining a sum of 4.

1st Dice 2nd Dice

1, 6 → 1/6
2, 4 → 0
3, 5 → 1/3

Sum = 4 $E = \{\overset{I}{(1,3)}\ \overset{II}{(2,2)}\ \overset{III}{(3,1)}\}$

$$p(1).p(3) + p(2).p(2) + p(3).p(1)$$

$$\frac{1}{6} \times \frac{1}{3} + \frac{1}{6} \times \frac{0}{6} + \frac{1}{6} \times \frac{1}{6} = \frac{1}{12}$$

Odd in Favour and Odd Against Favour

$$P(E) = \frac{a}{a+b} \qquad P(\bar{E}) = \frac{b}{a+b}$$

- Odd in favour $= \dfrac{P(E)}{P(\bar{E})} = \dfrac{\frac{a}{a+b}}{\frac{b}{a+b}} = \dfrac{a}{b}$

- Odd against favour $= \dfrac{P(\bar{E})}{P(E)} = \dfrac{\frac{b}{a+b}}{\frac{a}{a+b}} = \dfrac{b}{a}$

PROBABILITY BASICS

Ex:- Probability of Cracking GATE ? $P = \frac{1}{1000}$, $q = \frac{999}{1000}$

i) Find the odd in favour of cracking GATE

ii) Find the odd in against of cracking GATE

i) Odd in favour $= \frac{P}{q} = \frac{1/1000}{999/1000} = \frac{1}{999}$

ii) Odd against favour $= \frac{q}{P} = \frac{999/1000}{1/1000} = \frac{999}{1}$

PROBABILITY BASICS

Total Probability Theorem

Let $E_1, E_2, E_3 \ldots E_n$ are partitions of S which are mutually exclusive and exhaustive event.

- $E_i \cap E_j = \phi$
- $E_1 \cup E_2 \cup E_3 \ldots E_n = S$

$$P(A) = P(E_1).P(A/E_1) + P(E_2).P(A/E_2) + \ldots$$
$$\ldots\ldots \quad \ldots \quad P(E_n).P(A/E_n)$$

A can be taken from $E_1, E_2, \ldots E_n$.

PROBABILITY BASICS

Ex:- A factory (Bajaj) have 3 Machines X, Y & Z having probability of manufacturing defective products are given below:

X : 2 % Defective
Y : 3 % Defective
Z : 4 % Defective

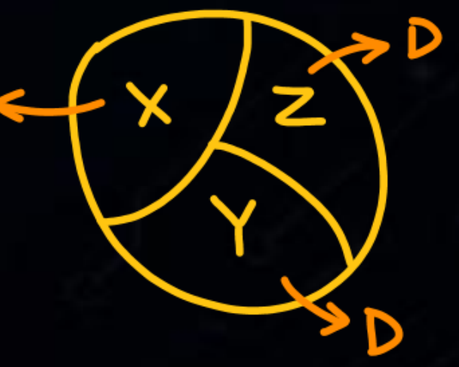
X → 95 $P(X) = 95/300$
Y → 200 $P(Y) = 200/300$
Z → 5 $P(Z) = 5/300$

Find the probability that product is defective it selected from Bajaj

$$P(D) = P(X).P(D/X) + P(Y).P(D/Y) + P(Z).P(D/Z)$$

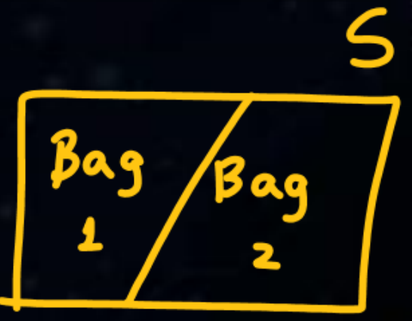
Product is defective

$$= \frac{1}{3} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{3} \times \frac{4}{100}$$



Bag 1

Bag 2

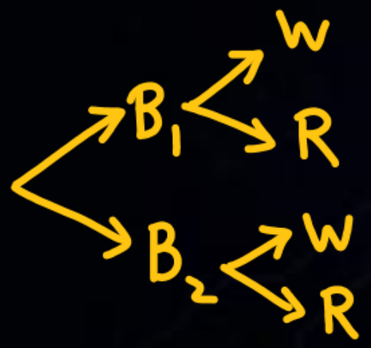


Find the probability that drawn ball is white.

W

$$P(W) = P(\text{Bag } 1) \cdot P\left(W/_{\text{Bag } 1}\right) + P(\text{Bag } 2) \cdot P\left(W/_{\text{Bag } 2}\right)$$

$$= \frac{1}{2} \times \frac{5}{8} + \frac{1}{2} \times \frac{6}{10}$$



BAYE'S THEOREM (inverse probability theorem)

It is given/known that A has already occured then we have to probability that it has occured from the occurence of Event E_i.

$$P\left(\frac{E_i}{A}\right) = \frac{P(E_i \cap A)}{P(A)}$$

$$= \frac{P(E_i) \cdot P(A/E_i)}{\sum P(E_i) \cdot P(A/E_i)}$$

PROBABILITY BASICS

Ex:- A factory (Bajaj) have 3 Machines X, Y & Z having probability of manufacturing defective products are given below:

X : 2 % Defective
Y : 3 % Defective
Z : 4 % Defective

if it known that the bulb is defective, then find the probability

i) It is manufactured from Machine X
ii) It is manufactured from Machine Y
iii) It is manufactured from Machine Z

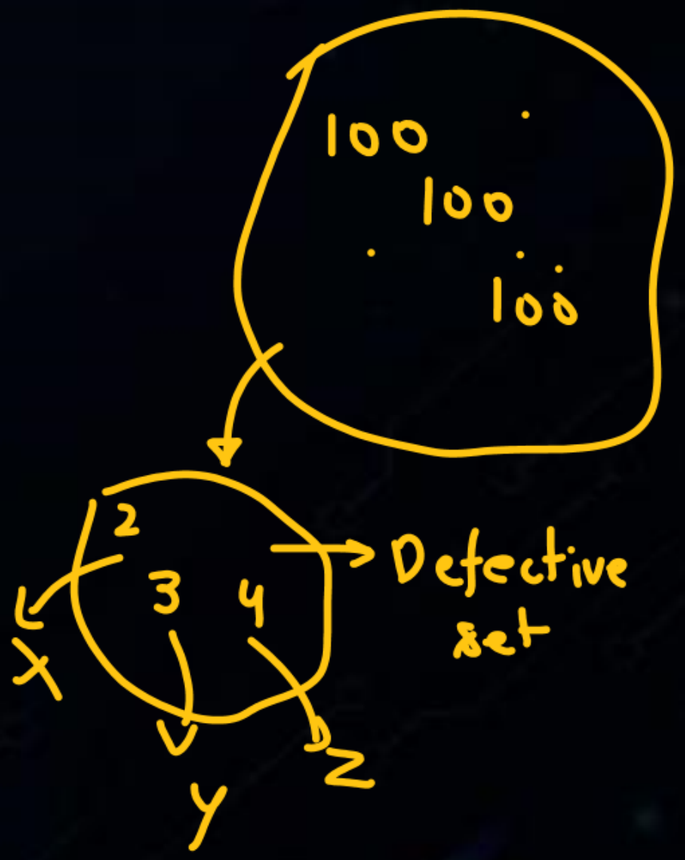
$$P(D) = \frac{1}{3} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{3} \times \frac{4}{100}$$

PROBABILITY BASICS

$$P\left(\frac{X}{D}\right) = \frac{P(X \cap D)}{P(D)} = \frac{\frac{1}{3} \times \frac{2}{100}}{\frac{1}{3} \times \frac{2}{100} + \frac{1}{3} \times \frac{3}{100} + \frac{1}{3} \times \frac{4}{100}} = \frac{2}{9}$$

$$P\left(\frac{Y}{D}\right) = \frac{P(Y \cap D)}{P(D)} = \frac{\frac{1}{3} \times \frac{3}{100}}{\frac{1}{3} \times \frac{9}{100}} = \frac{3}{9}$$

$$P\left(\frac{Z}{D}\right) = \frac{P(Z \cap D)}{P(D)} = \frac{\frac{1}{3} \times \frac{4}{100}}{\frac{1}{3} \times \frac{9}{100}} = \frac{4}{9}$$



PROBABILITY BASICS

Ex:- There are 2 bags , bag1 contains 4 white balls and 5 black balls and bag2 contains 3 white balls and 6 black balls.

i) Find the probability of selecting white ball?

ii) If it already known that ball is white, what is the probability that it is selected from Bag 1?

iii) If it already known that ball is white, what is the probability that it is selected from Bag 2?

i) $P(W) = P(B_1).P(W/B_1) + P(B_2).P(W/B_2) = \frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{3}{9} = \frac{7}{18}$

ii) $P(B_1/W) = \frac{P(B_1 \cap W)}{P(W)} = \frac{\frac{1}{2} \times \frac{4}{9}}{\frac{1}{2} \times \frac{4}{9} + \frac{1}{2} \times \frac{3}{9}} = \frac{4}{7}$

iii) $P(B_2/W) = \frac{P(B_2 \cap W)}{P(W)} = \frac{\frac{1}{2} \times \frac{3}{9}}{P(W)} = \frac{3}{7}$

BINOMIAL THEOREM ON PROBABILITY :-

Random exp. → H, T

2 outcomes → p, q

$p + q = 1$

P(probability that we get exactly r successes out of n trials)

$= {}^{n}C_{r}\, p^{r}\, q^{n-r}$

Total no. of ways = ____ ____ ____ ____ .... n trials = 2^n

① Exactly first 2H in 5 tosses = $\underbrace{\frac{1}{2} \times \frac{1}{2}}_{} \times \frac{1}{2} \times \frac{1}{2} \times \frac{1}{2}$ {HHTTT}

② Exactly 2H in 5 tosses = ${}^{5}C_{2} \cdot \left(\frac{1}{2}\right)^{2} \left(\frac{1}{2}\right)^{3}$ {HHTTT, HTHTT, 10 ways

$= \frac{5!}{2!\,3!}$

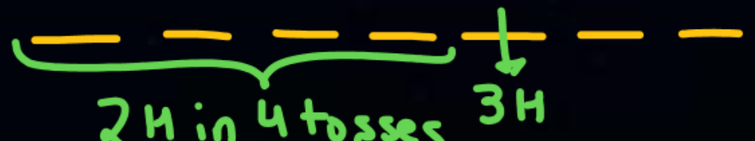
③ Exactly 3rd Head in 5th toss in total of 5 tosses

Total no. of ways.
$= 2 \times 2 \times 2 \times \ldots$
$= 2^5$

$\frac{6 \times 1}{2^5}$

$${}^4C_2 \cdot \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \times \left(\frac{1}{2}\right)$$

④ Exactly 3rd Head in 5th toss in total of 7 tosses.



$\frac{{}^4C_2 \times 1 \times 2 \times 2}{2^7}$

$${}^4C_2 \cdot \left(\frac{1}{2}\right)^2\left(\frac{1}{2}\right)^2 \cdot \frac{1}{2} \times \left\{\frac{1}{2}+\frac{1}{2}\right\} \times \left\{\frac{1}{2}+\frac{1}{2}\right\}$$

PROBABILITY BASICS

Q. Out of 15 record players, 4 are defective. One by one it is tested. Find the probability that 9th one examined is last defective?

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3D in 8 testing (3D, 5 ND) — 4D (9th testing)

15 players → 4D, 11ND

8 players → 3D, 5 ND

After 8 testing → 7 items (1-D)

$$\frac{{}^{4}C_{3} \times {}^{11}C_{5}}{{}^{15}C_{8}} \times \frac{1}{7} \quad \text{(Method I)}$$

$$\left\{\underbrace{\left(\frac{4}{15} \times \frac{3}{14} \times \frac{2}{13}\right)}_{\text{Defective}} \times \underbrace{\left(\frac{11}{12} \times \frac{10}{11} \times \frac{9}{10} \times \frac{8}{9} \times \frac{7}{8}\right)}_{\text{Non-defective}} \times {}^{8}C_{3}\right\} \times \frac{1}{7} \quad \text{(Method 2)}$$

PROBABILITY BASICS

Ex:- The probability that it will rain today is 0.5. The probability that it will rain tomorrow is 0.6. The probability that it will rain today or tomorrow is 0.7. What is the probability that it will rain today & tomorrow?

A. 0.3 B. 0.25

C. 0.35 D. 0.4

Thank you

GW

Soldiers !