ENGINEERING MATHEMATICS

ALL BRANCHES

Vector Calculus
Divergence & Curl of Vector Function

DPP-03 Solution

By- CHETAN SIR

Question 1

Curl of vector $\vec{v}(x,y,z) = 2x^2\hat{\imath} + 3z^2\hat{\jmath} + y^3\hat{k}$ at $x = y = z = 1$ is

A $-3i$

B $3i$

C $3i - 4j$

D $3i - 6k$

$$\text{Curl } \vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ 2x^2 & 3z^2 & y^3 \end{vmatrix}$$

$$= (3y^2 - 6z)\hat{i} - (0-0)\hat{j} + (0-0)\hat{k}$$

$$(3.1^2 - 6.1)\hat{i}$$

$$= -3\hat{i}$$

Question 2

If $\vec{r} = x\hat{a}_x + y\hat{a}_y + z\hat{a}_z$ and $|\vec{r}| = r$, then div $(r^2\nabla(ln\, r)) =$ __3__.

$$\nabla \ln r = \frac{1}{r} \cdot \frac{\vec{r}}{r}$$

$$r^2 \nabla \ln r = \cancel{r^2} \cdot \frac{\vec{r}}{\cancel{r^2}} = \vec{r}$$

$$x\hat{i} + y\hat{j} + z\hat{k}$$

$$\text{div}(r^2 \nabla \ln r) = 3$$

Question 3

A vector $\vec{P}$ is given by $\vec{P} = x^3y\vec{a}_x - x^2y^2\vec{a}_y - x^2yz\vec{a}_z$. Which one of the following statements is TRUE?

A. $\vec{P}$ is solenoidal, but not irrotational

B. $\vec{P}$ is irrotational, but not solenoidal

C. $\vec{P}$ is neither solenoidal nor irrotational

D. $\vec{P}$ is both solenoidal and irrotational

$$\text{div}\,\vec{P} = 3x^2y - 2x^2y - x^2y = 0$$

$\therefore$ $\vec{P}$ is solenoidal.

$$\text{curl}\,\vec{P} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^3y & -x^2y^2 & -x^2yz \end{vmatrix}$$

$$= -x^2z\,\hat{i} + 2xyz\,\hat{j} - (2xy^2 + x^3)\,\hat{k} \neq 0$$

$\therefore$ $\vec{P}$ is rotational.

(not irrotational)

Question 4

The velocity field of an incompressible flow is given by

$$V = (a_1x + a_2y + a_3z)\hat{\imath} + (b_1x + b_2y + b_3z)j + (c_1x + c_2y + c_3z)\hat{k}$$

and $a_1 = 2$ & $c_3 = -4$. The value of b_2 is 2.

$$\text{div}\,\vec{V} = 0$$

$$\frac{\partial V_1}{\partial x} + \frac{\partial V_2}{\partial y} + \frac{\partial V_3}{\partial z} = 0$$

$$a_1 + b_2 + c_3 = 0$$

$$2 + b_2 - 4 = 0$$

$$\boxed{b_2 = 2}$$

Question 5

$\nabla \times \nabla \times$ P (where P is a vector) is equal to

A $P \times \nabla \times P - \nabla^2 P$

B $\nabla^2 P + \nabla(\nabla \times P)$

C $\nabla^2 P + \nabla \times P$

D $\nabla(\nabla \bullet P) - \nabla^2 P$

$\nabla \times \nabla \times P$ = grad div $P - \nabla^2 P$
(curl curl P)

$= (\nabla . P)\nabla - (\nabla . \nabla)P$

$= \nabla(\nabla . P) - \nabla^2 P$

$a \times b \times c$
$= (a \cdot c)\vec{b} - (a.b)\vec{c}$

Question 6

The curl of vector $A = e^{xy}\, i + \sin xyj + \cos^2 xzk$ is

A $ye^{xy}\, i + x\cos xy\, j - 2x\sin 2xz\, k$

B $z\sin 2xzj + (y\cos xy - xe^{xy})k$

C $z\sin 2xzi + (x\cos xy - xe^{xy})k$

D $xye^{xy}\, i + xy\cos xyj - 2xz\sin 2xzk$

$$\text{Curl } \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ e^{xy} & \sin xy & \cos^2 xz \end{vmatrix}$$

$$(0-0)\hat{i} - \left(2\cos xz\,(-\sin xz).z\right)\hat{j} + \left(y\cos xy - e^{xy}.x\right)\hat{k}$$

$$(z \sin 2xz)\hat{j} + \left(y\cos xy - xe^{xy}\right)\hat{k}$$

Question 7

If $A = (3y^2 - 2z)i - 2x^2 zj + (x + 2y)k$, the value of $\nabla \times \underbrace{\nabla \times A}$ at $P(-2, 3, -1)$ is

$$\nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 3y^2-2z & -2x^2z & x+2y \end{vmatrix}$$

$$= (2 + 2x^2)\hat{i} - (1 - (-2))\hat{j} + (-4xz - 6y)\hat{k}$$

$$\nabla \times \nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ 2+2x^2 & -3 & -4xz-6y \end{vmatrix}$$

$$= -6\hat{i} + 4z\hat{j} \quad (-2,3,-1) = -6\hat{i} - 4\hat{j} = -(6\hat{i} + 4\hat{j})$$

A $-(6i + 4j)$ ✓

B $8(i+j)$

C $-8(i+j)$

D 0

Question 8

The directional derivative of function $\Phi = xy + yz + zx$ at point $P(3, -3, -3)$ in the direction toward point $Q(4, -1, -1)$ is

A -3

B 1

C -2

D 0

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = \hat{i} + 2\hat{j} + 2\hat{k}$$

$$D.D. = \vec{\nabla}\phi \cdot \hat{PQ}$$

$$= (y+z)\hat{i} + (x+z)\hat{j} + (y+x)\hat{k} \cdot \frac{[\hat{i}+2\hat{j}+2\hat{k}]}{\sqrt{1^2+2^2+2^2}}$$

$$= (-6\hat{i} + 0\hat{j} + 0\hat{k})\frac{(\hat{i}+2\hat{j}+2\hat{k})}{3}$$

$$\frac{-6+0+0}{3} = -\frac{6}{3} = -2$$

Question 9

The maximum value of the directional derivative of the function $\phi = 2x^3 + 3y^2 + 5z^2$ at a point $(1, 1, -1)$ is

A 10

B -4

C $\sqrt{152}$

D 152

Question 10

The grad. $\nabla \times A$ of a vector field $A = x^2 yi + y^2 zj - 2xzk$ is

curl grad $A = 0$

A $2xy + 2yz - 2x$

B $x^2 y + y^2 z - 2xz$

C $2x^2 y + 2y^2 z - 2xz$

D 0

$$\nabla \times A = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 y & y^2 z & 2xz \end{vmatrix}$$

$$= (-y^2)\hat{i} - (2z)\hat{j} - x^2 \hat{k}$$

$$\text{grad}(\nabla \times A) = 0$$

Thank you

GW Soldiers !