ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-02

Probability

By- Chetan Sir

Topics to Be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM

} Basic probability (Part-I)

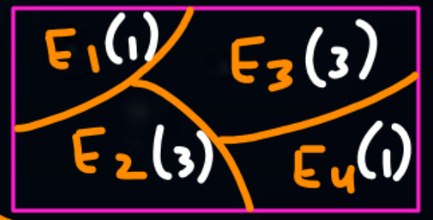
- STATISTICS – I (PROBABILITY DISTRIBUTIONS) } Part-II
- STATISTICS – II (CORRELATION AND REGRESSION) } Part-III

Ex:- Find the probability that the difference of number shown is 1 (on 2 dice)

$$E \rightarrow \{(1,2)(2,1)(2,3),(3,2)(3,4),(4,3)(4,5)(5,4)(5,6),(6,5)\}$$

$$P(E) = \frac{n(E)}{n(S)} = \frac{10}{36} = \frac{5}{18}$$

PROBABILITY BASICS



mutually exclusive & exhaustive.

E_1	E_2	E_3	E_4
0T	1T	2T	3T
↓	↓	↓	↓
1	3	3	1

Ex:- 3 coins tossed, find P(At least 2 tails)

Soln:- $p(2T \text{ or } 3T)$

$$= \frac{3}{8} + \frac{1}{8} = \frac{4}{8} = \frac{1}{2}$$

$E \rightarrow (TTH, THT, HTT, TTT)$

$E_1 \rightarrow (0H, 1H)$

$E_1 \rightarrow (TTT, HTT, THT, TTH)$

$E_2 \rightarrow (2T)$

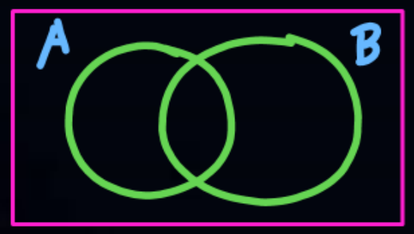
(HTT, THT, TTH)

$S =$ { HHH, HHT, HTH, HTT, THH, THT, TTH, TTT }

E_1 & E_2 are not mutually exclusive & not exhaustive

General Venn-Diagram :-

2 events A, B



S

vi) $P(B/A) = \frac{P(A \cap B)}{P(A)}$

i) Either A or B or both of them
$(A \cup B)$ ← At least A or B

ii) Both A and B (Elements occuring in A & B simultaneously)
$(A \cap B)$

iii) Neither A nor B $= P(\overline{A \cup B}) = 1 - P(A \cup B)$
$= P(\bar{A} \cap \bar{B})$

iv) $P(\bar{A}) = 1 - P(A)$ v) $P(\bar{B}) = 1 - P(B)$

E_1 ↓ Coin toss

S

E_2 ↓ Single card drawn from 52 cards.

S

$P(H \cap King)$
$= P(H) \cdot P(King)$
$\frac{1}{2} \cdot \frac{4}{52}$

PROBABILITY BASICS

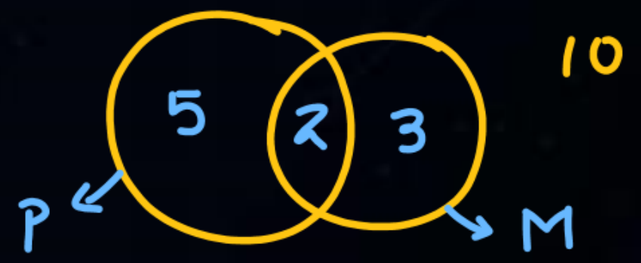
vii) Exactly one of the events → Exactly A / Exactly B

viii) Exactly A or B occurs

$= P(A \cap \bar{B})$

$= P(\bar{A} \cap B)$

Ex:- In a class 10 students → 7 Physics / 5 Maths



i) No. of those who Knows at least P or M
(Students who Know either P or M)

$n(P \cup M) = 10$

ii) Students who Knows both P and M $n(P \cap M) = 2$

$n(P \cup M) = n(P) + n(M) - n(P \cap M)$

$10 = 7 + 5 - n(P \cap M)$

iii) Students who don't Know $P = n(\bar{P}) = 3$

iv) Students who don't Know $M = n(\bar{M}) = 5$

v) Those who Know only $P = n(P \cap \bar{M}) = 5$

vi) Those who Know only $M = n(\bar{P} \cap M) = 3$

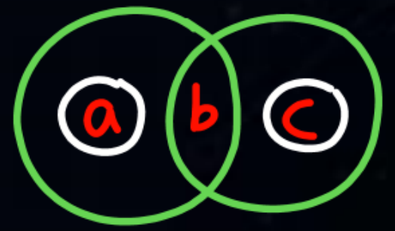
PROBABILITY BASICS

vii) Exactly Physics or Maths (E) $\rightarrow n(E) = 8\ (5+3)$

$$n(E) = n(A) + n(B) - 2\,n(A \cap B)$$

Exactly A or B occurs

$$a+b \quad b+c \quad -2b \quad = a+c$$



$$n(A) + n(B) - n(A \cap B) - n(A \cap B)$$

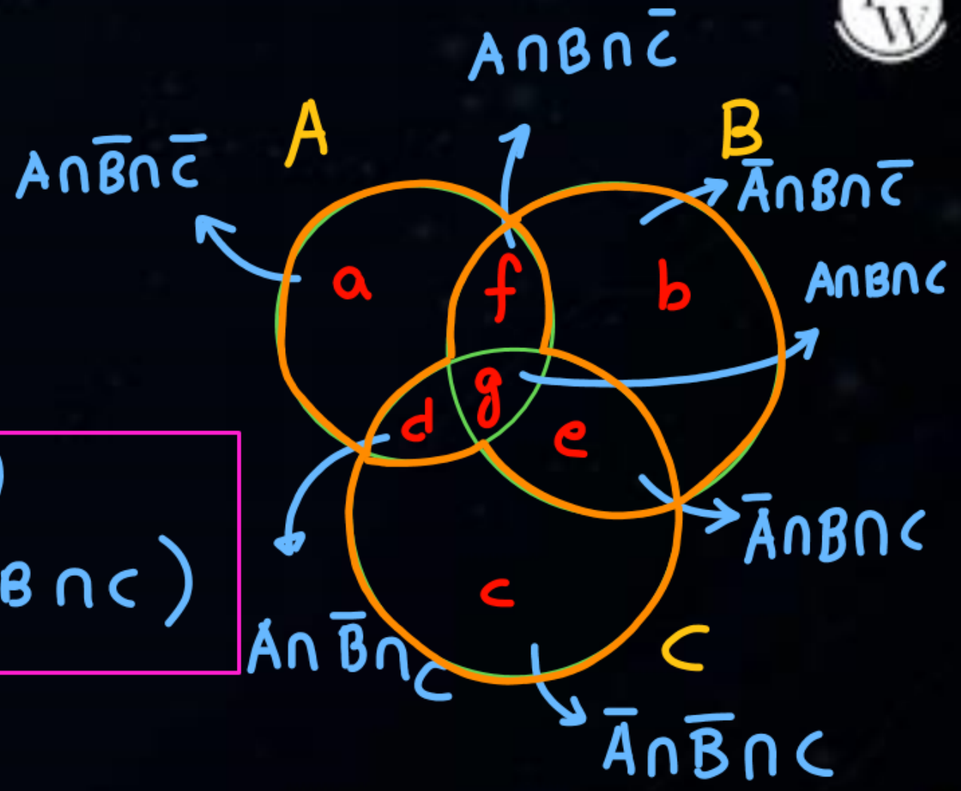
$$n(E) = n(A \cup B) - n(A \cap B)$$

PROBABILITY BASICS

3 event A, B, C :-

i) Either A or B or C or all of them
(At least A or B or C)

$$n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C)$$



$$(a + d + f + g) + (b + f + g + e) + (c + e + d + g) - (f + g) - (e + g) - (d + g) + (g)$$

ii) Exactly 2 of A, B, C occurs $= n(A \cap B \cap \bar{C}) + n(A \cap \bar{B} \cap C) + n(\bar{A} \cap B \cap C)$
↳ (A and B), (B and C), (A and C) → E

$$n(E) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$$

$$(f+g) + (e+g) + (d+g) - 3g$$

iii) All A and B and C occurs simultaneously (Exactly 3)

$n(A \cap B \cap C)$

iv) Exactly one of A or B or C occurs (E) $= n(A \cap \bar{B} \cap \bar{C}) + n(\bar{A} \cap B \cap \bar{C}) + n(\bar{A} \cap \bar{B} \cap C)$

$$n(E) = n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(C \cap A) + 3n(A \cap B \cap C)$$

$$= (a+d+f+g) + (b+e+f+g) + (c+d+e+g) - 2(f+g) - 2(e+g) - 2(d+g)$$

$$3g - 6g + 3g$$

PROBABILITY BASICS

v) At least two of the events (E)

$n(A \cap B) + n(B \cap C) + n(C \cap A) + n(A \cap B \cap C)$

$$n(E) = n(A \cap B) + n(B \cap C) + n(C \cap A) - 2n(A \cap B \cap C)$$

$(f+g) + (e+g) + (d+g) - 2g$

vi) At least one events $\Rightarrow n(A) + n(B) + n(C) + n(A \cap B) + n(B \cap C) + n(C \cap A) + n(A \cap B \cap C)$

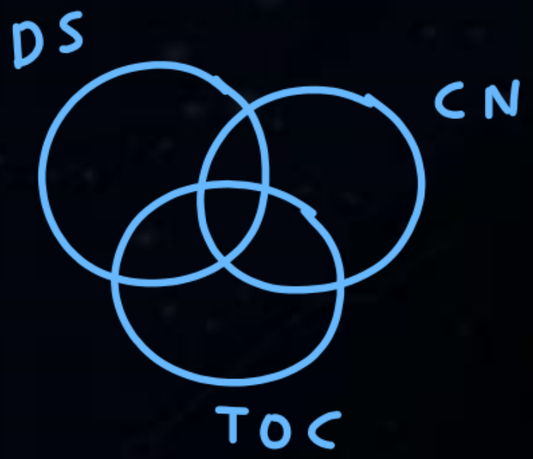
$$n(E) = n(A \cup B \cup C)$$

vii) At least 3 events (All 3 of them) $\quad n(E) = n(A \cap B \cap C)$

PROBABILITY BASICS

Ex: 50 students

→ DS ⇒ 25
→ CN ⇒ 15
→ TOC ⇒ 20
→ DS ∩ CN ⇒ 10
→ CN ∩ TOC ⇒ 7
→ DS ∩ TOC ⇒ 6
→ (DS ∩ CN ∩ TOC) ⇒ 5



i) , ii) , iii) , iv) , v) , vi) , vii)

PROBABILITY BASICS

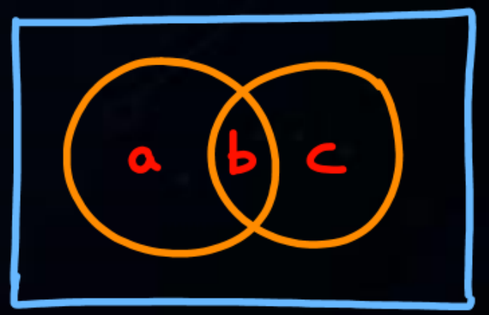
$P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

Addition Theorem of Probability

a) For two events

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$



$n(A) = a + b$

$n(B) = b + c$

$n(A \cap B) = b$

$a + 2b + c - b$

$n(A) + n(B) - n(A \cap B)$

$$P(A \cup B) = P(A) + P(B)$$ For mutually exclusive

b) For three events

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C)$$ For mutually exclusive

PROBABILITY BASICS

Ex:- A & B are two events

$$P(A \cup B) = \frac{3}{4}, P(A \cap B) = \frac{1}{4}, P(\bar{A}) = \frac{2}{3}$$,find P(B)?

PROBABILITY BASICS

Ex:- Aman wins $= \frac{2}{3}$, Suresh wins $= \frac{1}{5}$

What is the probability that

i) Both of them wins

ii) At least one of them wins

Thank you

GW

Soldiers !