

ENGINEERING MATHEMATICS

ALL BRANCHES



Function and its Graph,
Domain and range of function
Calculus

DPP-1 Solution



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Question 1

What is the form of the function $f(x)$ for the following data?

| | | | | |
|--------|---|---|----|----|
| x | 0 | 1 | 2 | 3 |
| $f(x)$ | 3 | 6 | 11 | 18 |

A $x^2 + 2x + 3$

Option a) $f(x) = x^2 + 2x + 3$

$$x = 0 \quad f(0) = 3$$

$$x = 1 \quad f(1) = 6$$

$$x = 2 \quad f(2) = 11$$

Alternatively, $x = 3 \quad f(3) = 18$

Let $f(x) = ax^2 + bx + c$

Put the points & form at least 3 eqns. and solve a, b and c simultaneously.

B $x^2 - 2x + 3$

C $x^2 + 2x - 3$

D $x^2 - 2x - 3$

Question 2

Which one of the following functions is strictly bounded?

A $1/x^2$

$$\{-\infty < x < \infty\}$$

a) $0 < x^2 < \infty$

$$0 < \frac{1}{x^2} < \infty$$

\therefore it is not bounded.

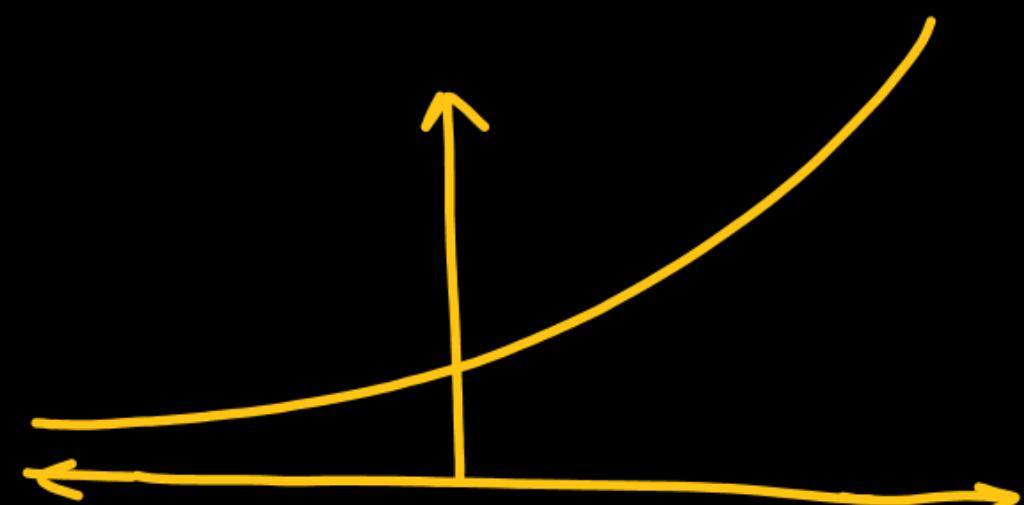
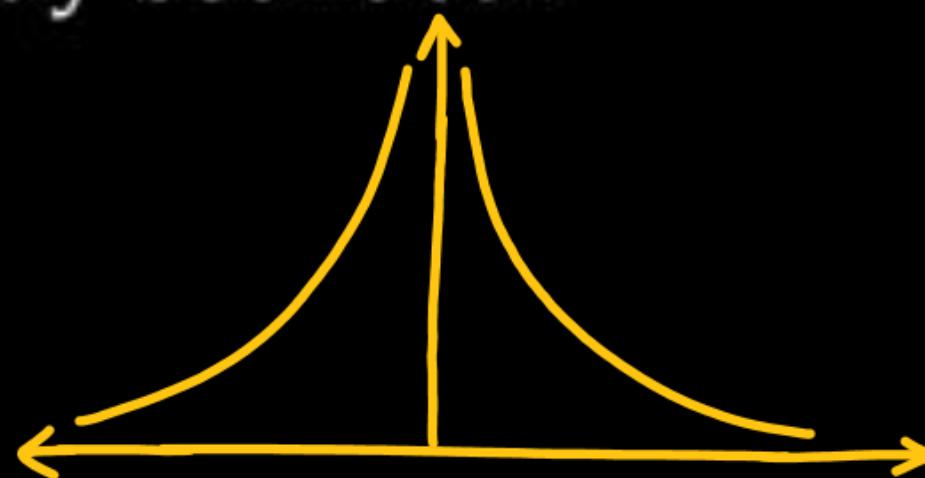
C x^2

b) $e^{-\infty} < e^x < e^{\infty}$

$$0 < e^x < \infty$$

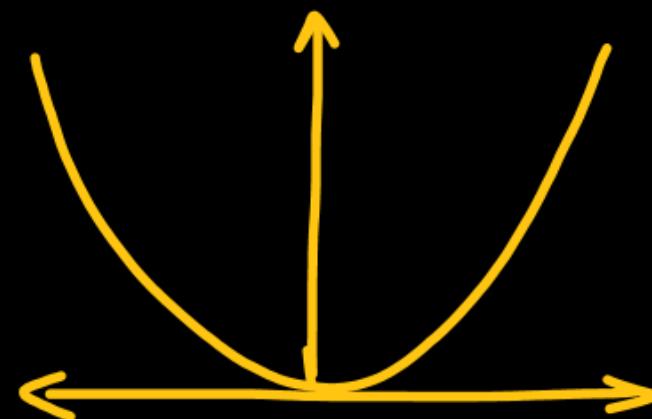
\therefore it is not bounded.

D e^{-x^2}



c) $0 < x^2 < \infty$

$\therefore x^2$ is not bounded



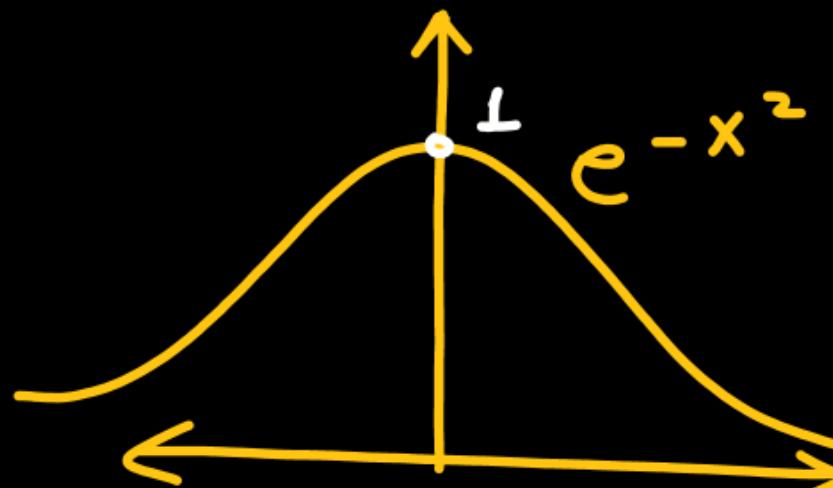
d) $0 < x^2 < \infty$

$$-\infty > -x^2 > 0$$

$$e^{-\infty} < e^{-x^2} < e^0$$

$$0 < e^{-x^2} < 1$$

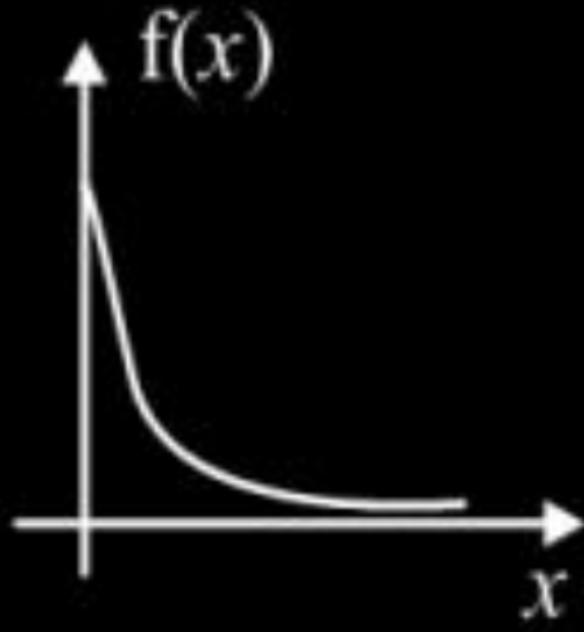
$\therefore e^{-x^2}$ is bounded.



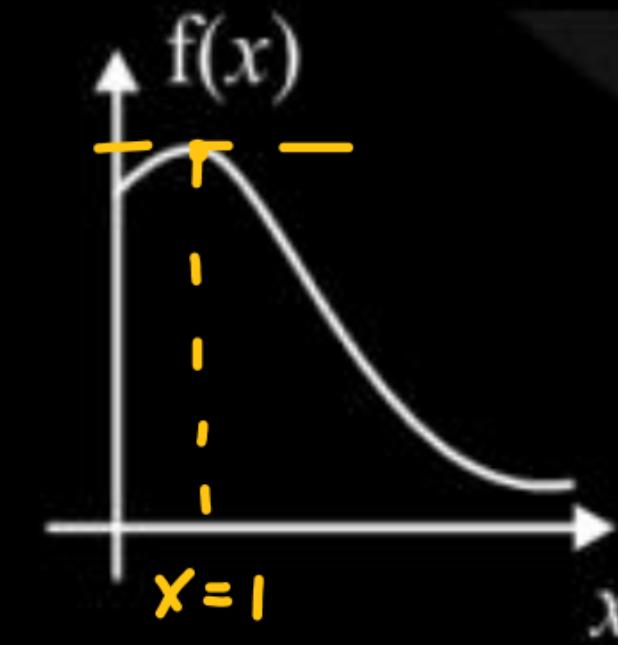
Question 3

Which one of the following graphs describes the function $f(x) = e^{-x} (x^2 + x + 1)$?

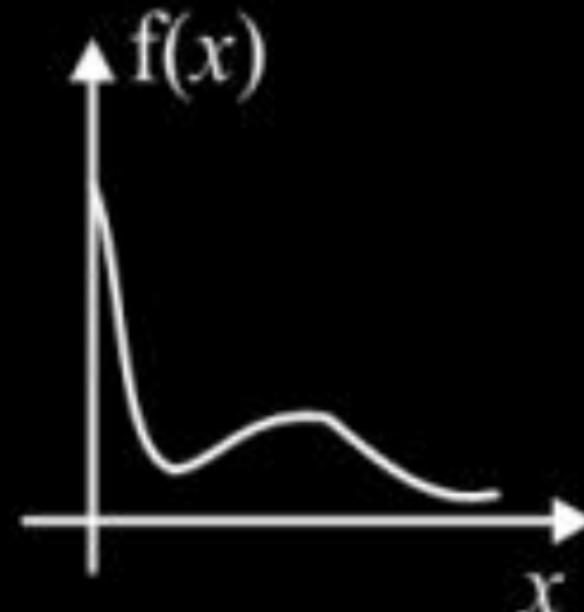
A



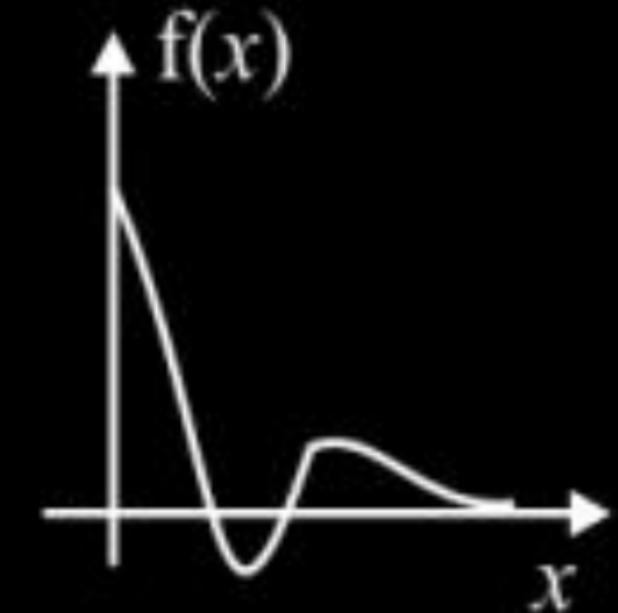
B



C



D



$$f(x) = e^{-x}(x^2 + x + 1)$$

$$f'(x) = e^{-x}(2x+1) - e^{-x}(x^2 + x + 1)$$

$$f'(x) = e^{-x}(x - x^2) = 0$$

$x(1-x) = 0 \quad \therefore x=0, 1$ are stationary points.

$$f''(x) = e^{-x}(1-2x) - e^{-x}(x - x^2)$$

$$f''(x) = e^{-x}(x^2 - 3x + 1)$$

At $x=0$; $f''(0) = e^{-0}(0^2 - 3.0 + 1) = 1 > 0 \Rightarrow \text{minima}$

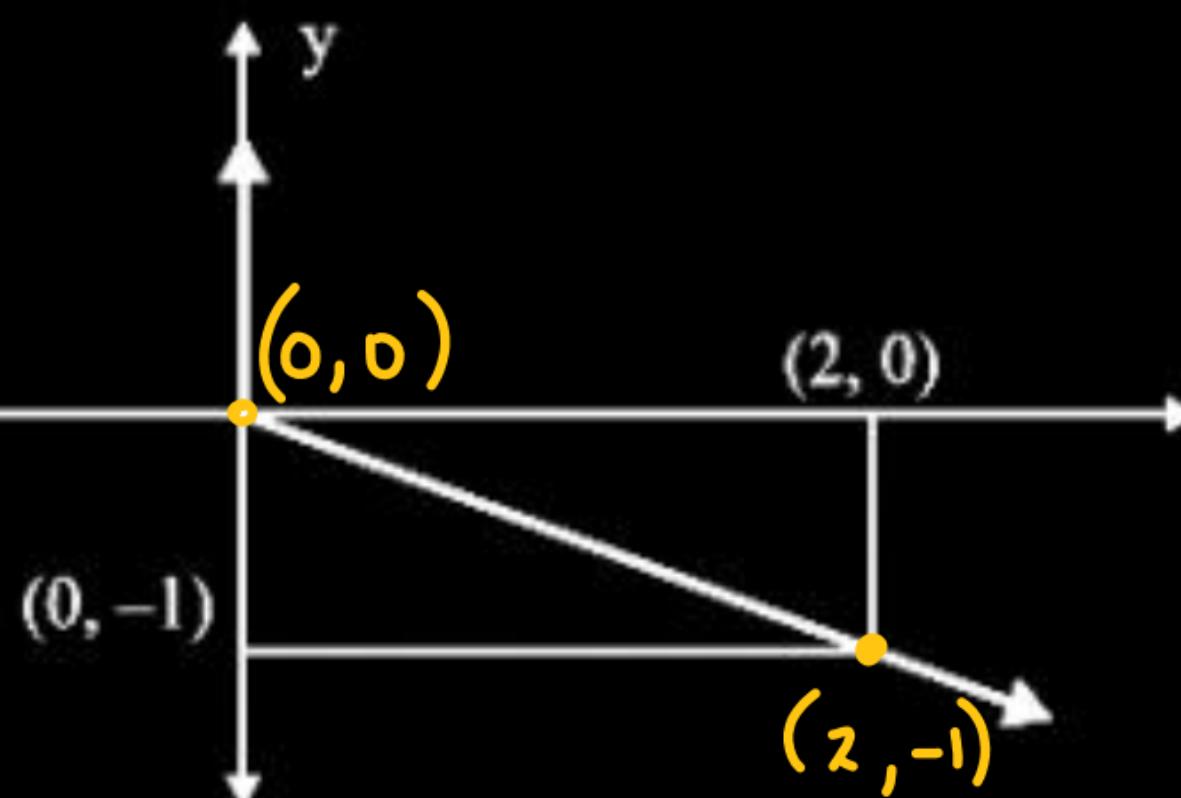
At $x=1$; $f''(1) = e^{-1}(1^2 - 3(1) + 1) = -e^{-1} < 0 \Rightarrow \text{maxima}$

$\Rightarrow f(x)$ has one local maxima at $x=1$
 \therefore Option b is correct.

Question 4

Choose the most appropriate equation for the function drawn as a thick line, in the plot below.

- A $x = y - |y|$
 $-1 - |-1| = -2$
- B $x = - (y - |y|)$
 $-(-1 - |-1|) = +2$
- C $x = y + |y|$
 $-1 + |-1| = 0$
- D $x = - (y + |y|)$
 $-(-1 + |-1|) = 0$



At $y = -1$; x should be 2 a/t this graph

Question 5

$f(x) = 2x^3 - 15x^2 + 36x + 1$ is increasing in the interval

- A $[2, 3]$ $\rightarrow f(x)$ is decreasing

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

$$f'(x) = 6x^2 - 30x + 36$$

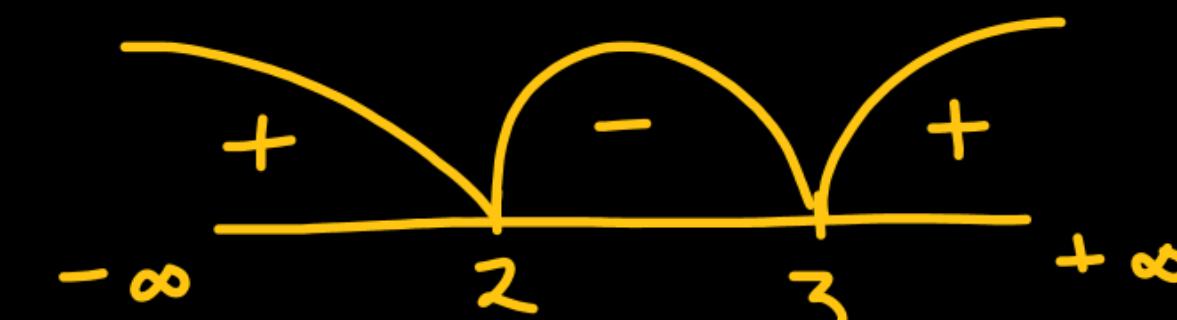
- B $[-\infty, 3]$

$$f'(x) = 6(x^2 - 5x + 6)$$

$$f'(x) = 6(x - 2)(x - 3)$$

- C $[-\infty, 2] \cup [3, \infty]$ $\rightarrow f(x)$ is increasing

- D None of these



$$f'(x) > 0 \Rightarrow x < 2 \text{ and } x > 3$$

Question 6

$f(x) = x^9 + 3x^7 + 6$ is increasing for

- A All positive real values of x
- B All negative real values of x
- C All non-zero real values of x
- D None of these

$$f(x) = x^9 + 3x^7 + 6$$

$$f'(x) = 9x^8 + 21x^6$$

$$f'(x) > 0$$

$$\Rightarrow \begin{cases} x < 0 \\ x > 0 \end{cases} f'(x) > 0 \Rightarrow \text{↑ ing}$$

$$x = 0 \quad f'(x) = 0$$

Question 7

$f(x) = x^2 e^{-x}$ is increasing in the interval

A $[-\infty, \infty]$

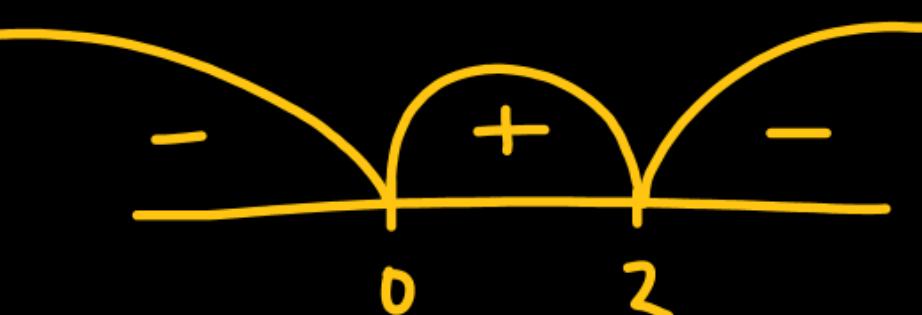
B $[-2, 0]$

C $[2, \infty]$

D $[0, 2]$

$$\begin{aligned}f(x) &= x^2 e^{-x} \\f'(x) &= e^{-x}(2x) - e^{-x}(x^2) \\&= e^{-x}(2x - x^2)\end{aligned}$$

$$f'(x) = e^{-x} \times (2-x)$$



$$f'(x) > 0 \Rightarrow [0 < x < 2]$$

Question 8



The least value of a for which $f(x) = x^2 + ax + 1$ is increasing on $[1, 2]$ is

A 2

B -2

C 1

D -1

$$f(x) = x^2 + ax + 1$$
$$f'(x) = (2x + a)$$

① $f'(x) > 0$
 $1 < x < 2$ $1 < x < 2$

$$2+a < f'(x) < 4+a$$

if $a = -2$

then $0 < f'(x) < 2$

if $a > -2$

Thank you
GW
Soldiers !

