CS & IT ENGINEERING

DISCRETE MATHS

Mathematical Logic

Lecture No. 1

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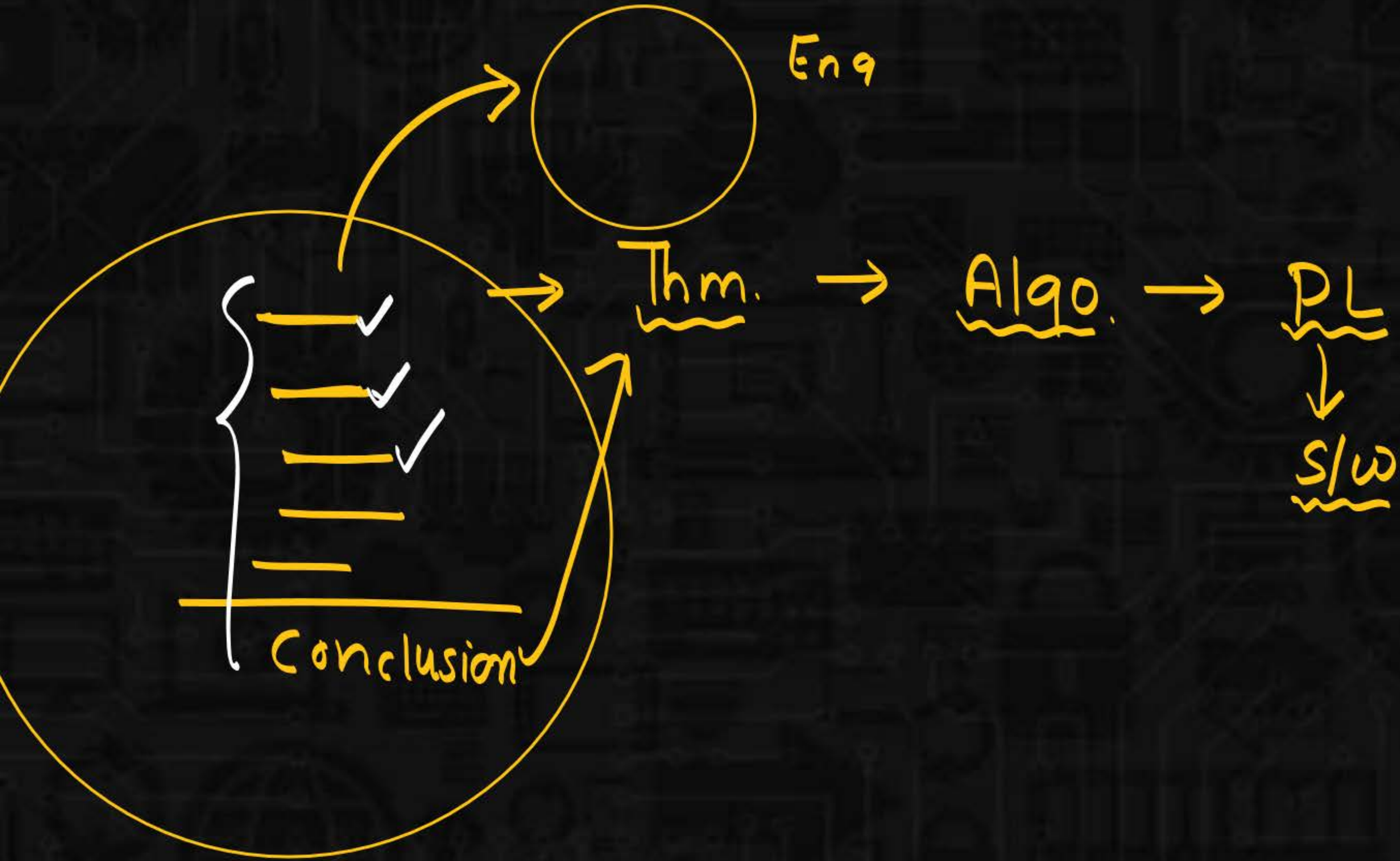
TOPICS

01 Connectives

02 Type 1

03 Type 2


Eng
Thm.
Algo.
PL
S/W
Conclusion

English stmt

Factual: → propositional stmt: → True / false.

Simple

Compound:

eg: $5 > 3$ $\boxed{S}$

$\boxed{C}$

$\boxed{S_1}$ $\boxed{S_2}$

$5 \geq 3$

$\boxed{5>3}$ OR $\boxed{5=3}$

Connective::

conjunction

disjunction

single Implication

double Implication.

modifier::

not / negation.

conjunction ($\wedge$) (AND/but)

S_1	S_2	C.
T	T	T
T	F	F
F	T	F
F	F	F.

$(\underline{F} \wedge \underline{\quad}) \equiv F.$

AND hates false.

$(\underline{F} \wedge \underline{\quad} \wedge \underline{\quad}) \equiv F.$

OR : → Inclusive OR ($\vee$)

→ Exclusive OR ($\oplus$)

OR LOVES True.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

$(T \vee \_) \equiv T$

$(T \vee \_ \vee \_) \equiv T$

Conditional stmt : ($\rightarrow$)

$p \rightarrow q$

if p then q

if p, q

* q if p

q when p

q whenever p

* q unless $\neg p$

p implies q

* p only if q

$$\underbrace{\text{if you win the match}}_{p}\ \text{then}\ \underbrace{\text{i will give pizza}}_{q}.$$

$F \longrightarrow \ \equiv T$

		$p \to q$
win(T)	pizza(T)	T
win(T)	~~pizza~~(F)	F
→ ~~win~~(F)	pizza(T)	T
→ ~~win~~(F)	~~pizza~~(F)	T

$T \to T \equiv T$

$T \to F \equiv F$

$F \to T \equiv T$

$F \to F \equiv T$

→ if Perfect matching exist then no of vertices would be even.

		Thm:
P.m(T)	Even(T)	T.
→ P.m(T)	~~Even~~(F)	F.
~~P.m~~(F)	Even(T)	True.
P.~~m~~	~~Even~~	True.

$$\begin{cases} T \rightarrow T \equiv T. \\ T \rightarrow F \equiv F. \\ F \rightarrow T \equiv T. \\ F \rightarrow F \equiv T. \end{cases}$$

$p \to q$

Pm $\to$ Even

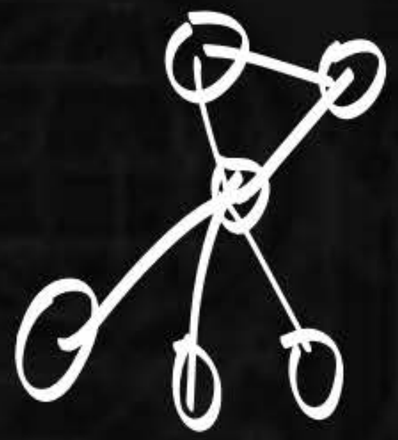
$p \to q \neq q \to p$

$p \to q \neq \neg p \to \neg q$.

$\boxed{p \to q = \neg q \to \neg p.}$

(viceversa)

Inverse	Converse	Contrapositive:
$\neg p \to \neg q$ ×	$q \to p$	$\neg q \to \neg p$.
not(p.m) $\to$ not(Even)	Even $\to$ p.m	not(even) $\to$ not(p.m)



$T \to F \equiv F$.

if $\underbrace{\text{G is planar}}_{p}$ then $\underbrace{e \leq 3n-6}_{q}$.

$p \to q = \neg q \to \neg p$.

not $(e \leq 3n-6)$ then G is not planar.

if $e > 3n-6$ then G is nonplanar.

D.			A			B.	C.
$q \to p$	p	q	$p \to q$	$\neg p$	$\neg q$	$\neg q \to \neg p$	$\neg p \to \neg q$
T	T	T	T	F	F	T	T
T	T	F	F	F	T	F	T
F	F	T	T	T	F	T	F
T	F	F	T	T	T	T	T

A

B.

exp A is logically equivalent to exp B.

$A \equiv B$

OR.

2. A, B are having same behaviour (

Double Implication ($\leftrightarrow$)

p	q	$p \leftrightarrow q$
T	T	T
F	F	T
T	F	F
F	T	F

E.G $\leftrightarrow$ Degree of all vertices are even.

$A \equiv B$ 1. same column.

2. same behaviour.

tautology: $(1 \equiv 2 \equiv 3)$

A	B	$A \Leftrightarrow B$
T	T	T
F	F	T
T	T	T
F	F	T

1. $A \Leftrightarrow B$ tautology.

   $A \equiv B$.

2. A, B are having same column.

3. they are having same behaviour.

T
T
T
T

→ tautology/valid.

all valid's are satisfiable

→ all contingency are satisfiable.

F
F
F
F

→ contradiction.

Satisfiable.

T ← atleast 1 True

Contingency:

not tautology.
not contradiction.

T
F

→ atleast.

$$\overline{\left[\overset{T}{p} \wedge \overset{T}{(p \to q)} \wedge \overset{T}{(q \to m)}\right]} \to \overset{F}{m}$$

$(T \to T)$

$(T \to F)$

F

$\{T \to F \equiv F$

$m = F$

$p = T$

$q = T$

F ⟶ True

$$\overbrace{\left(\overset{T}{p} \wedge \overbrace{(p \to q)}^{T} \wedge \overbrace{(q \to m)}^{T}\right)}^{T} \to \overset{F}{m}$$

$$T \to F \equiv F$$

$$(T \to T) \wedge \underbrace{(T \to F)}_{F}$$

$m = F$
$p = T$
$q = T$

$$F \longrightarrow$$

True

$$\left[a \wedge (a \to b) \wedge (\neg b \vee c) \wedge (c \to d)\right] \longrightarrow d.$$

a) $\neg(p \vee \neg q) \to \neg p$

b) $p \to (q \to r)$

c) $(p \to q) \to r$

d) $(p \to q) \to (q \to p)$

e) $[p \wedge (p \to q)] \to q$

f) $(p \wedge q) \to p$

g) $q \leftrightarrow (\neg p \vee \neg q)$

h) $[(p \to q) \wedge (q \to r)] \to (p \to r)$

$$\left[\overset{T}{a} \wedge \overset{T}{(a \to b)} \wedge \overset{T}{(\neg b \vee c)} \wedge \overset{T}{(c \to d)}\right] \to \overset{F}{d} \quad \text{(tautology)}$$

$(T \to T)$ $(F \vee T)$ $\underline{(T \to F)}$

F

$d = F$

$a = T$

$b = T$

$c = T$

F $\longrightarrow$

TM

$$[p \rightarrow (q \rightarrow r)] \rightarrow [(p \rightarrow q) \rightarrow (p \rightarrow r)]$$

a) $[(p \rightarrow q) \wedge (q \rightarrow r)] \rightarrow (p \rightarrow r)$

b) $[(p \rightarrow q) \wedge \neg q] \rightarrow \neg p$

c) $[(p \vee q) \wedge \neg p] \rightarrow q$

d) $[(p \rightarrow r) \wedge (q \rightarrow r)] \rightarrow [(p \vee q) \rightarrow r]$

a) $[p \wedge (p \rightarrow q) \wedge r] \rightarrow [(p \vee q) \rightarrow r]$

b) $[[(p \wedge q) \rightarrow r] \wedge \neg q \wedge (p \rightarrow \neg r)] \rightarrow (\neg p \vee \neg q)$

c) $[[p \vee (q \vee r)] \wedge \neg q] \rightarrow (p \vee r)$


THANK YOU GW
SOLDIERS !