ENGINEERING MATHEMATICS

ALL BRANCHES

Differential Equation
Solution of first order linear
DE - 1

DPP-02 Solution

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Question - 01

The solution of the first order differential equation $x'(t) = -3x(t), x(0) = x_0$ is

A $x(t) = x_0 e^{-3t}$

B $x(t) = x0e-3$

C $x(t) = x_0 e^{-t/3}$

D $x(t) = x_0 e^{-t}$

$x'(t) = -3x(t)$

$\frac{dx}{dt} = -3x$

$\int \frac{dx}{x} = \int -3\,dt$

$\ln x = -3t + c$ —①

At $t = 0$, $x = x_0$

$\ln x_0 = -3 \times 0 + c$

$c = \ln x_0$

$\ln x = -3t + \ln x_0$

$\ln x/x_0 = -3t$

$x(t) = x_0 e^{-3t}$

Question - 02

The solution of the differential equation $\frac{dy}{dx} + 2xy = e^{-x^2}$ with $y(0) = 1$ is

A $(1+x)e^{+x^2}$

B $(1+x)e^{-x^2}$

C $(1-x)e^{+x^2}$

D $(1-x)e^{-x^2}$

$\frac{dy}{dx} + Py = Q$

$\therefore P = 2x$

$\therefore Q = e^{-x^2}$

$I.F. = e^{\int P dx} = e^{\int 2x dx} = e^{x^2}$

Soln :- $y \cdot e^{x^2} = \int Q \cdot e^{x^2}$

$y \cdot e^{x^2} = \int e^{-x^2} \cdot e^{x^2}$

$y \cdot e^{x^2} = x + c$

At $x = 0$, $y = 1$

$\therefore c = 1$

$y = e^{-x^2}(x+1)$

Question - 03

The solution of the differential equation $x^2\frac{dy}{dx}+2xy-x+1=0$ given that at $x=1, y=0$ is

A $\frac{1}{2}-\frac{1}{x}+\frac{1}{2x^2}$

B $\frac{1}{2}-\frac{1}{x}-\frac{1}{2x^2}$

C $\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$

D $-\frac{1}{2}+\frac{1}{x}+\frac{1}{2x^2}$

$\frac{dy}{dx}+\frac{2}{x}y=\frac{x-1}{x^2}$

I.F. $= e^{\int P\,dx} = e^{\int \frac{2}{x}dx} = e^{2\ln x} = x^2$

Soln:-

$y\cdot x^2 = \int \frac{(x-1)}{x^2}\,x^2\,dx$

$yx^2 = \frac{x^2}{2} - x + c$

$x=1$, $y=0$

$\therefore c = 1/2$

$y = \frac{1}{2} - \frac{1}{x} + \frac{1}{2x^2}$

Question - 04

Which of the following is a solution to the differential equation $\frac{dx(t)}{dt}+3x(t)=0$?

A $x(t) = 3e^{-t}$

B $x(t) = 2e^{-3t}$

C $x(t)=\frac{-3}{2}t^2$

D $x(t) = 3t^2$

$$\frac{dx}{dt} + 3x = 0$$

$$\frac{dx}{x} + 3\,dt = 0$$

$$\ln x + 3t = \log c$$

$$\ln x - \ln c = -3t$$

$$\frac{x}{c} = e^{-3t}$$

$$x = c\,e^{-3t}$$

Question - 05

Solution of $\frac{dy}{dx} = -\frac{x}{y}$ at $x = 1$ and $y = \sqrt{3}$ is

A $x - y^2 = -2$

B $x + y^2 = 4$

C $x^2 - y^2 = -2$

D $x^2 + y^2 = 4$

$$y\,dy = -x\,dx$$

$$\frac{y^2}{2} = -\frac{x^2}{2} + c$$

At $x = 1$, $y = \sqrt{3}$

$$\frac{3}{2} = -\frac{1}{2} + c$$

$$c = 2$$

$$y^2 + x^2 = 4$$

Question - 06

The order of the differential equation $\frac{d^2y}{dt^2}+\left(\frac{dy}{dt}\right)^3+y^4=e^{-t}$ is

Order = 2

Degree = 1

A 1

B 2

C 3

D 4

Question - 07

The solution of $x\frac{dy}{dx}+y=x^4$ with the condition $y(1)=\frac{6}{5}$ is

A $y=\frac{x^4}{5}+\frac{1}{x}$

B $y=\frac{4x^4}{5}+\frac{4}{5x}$

C $y=\frac{x^4}{5}+1$

D $y=\frac{x^5}{5}+1$

$$\frac{dy}{dx}+\left(\frac{1}{x}\right)y=\frac{x^4}{x}$$

$P=1/x$, $Q=x^3$

$$I.F.=e^{\int\frac{1}{x}dx}=e^{\ln x}=x$$

Soln :- $y\cdot x=\int x^3\cdot x$

$$y\cdot x=\frac{x^5}{5}+c$$

$x=1$, $y=6/5$

$$\frac{6}{5}\cdot 1=\frac{1}{5}+c$$

$c=1$

$$y=\frac{x^4}{5}+\frac{1}{x}$$

Question - 08

The solution of the differential equation $\frac{dy}{dx} - y^2 = 1$ satisfying the conditions $y(0) = 1$ is

A $y = e^{x^2}$

B $y = \sqrt{x}$

C $y = \cot\left(x + \frac{\pi}{4}\right)$

D $y = \tan\left(x + \frac{\pi}{4}\right)$

$\frac{dy}{dx} = 1 + y^2$

$\int \frac{dy}{1+y^2} = \int dx$

$\tan^{-1} y = x + c$

At $x = 0$, $y = 1$

$\tan^{-1} 1 = 0 + c$

$c = \pi/4$

$y = \tan\left(x + \frac{\pi}{4}\right)$

Question - 09

Which one of the following differential equations has a solution given by the function $y = 5\sin\left(3x + \frac{\pi}{3}\right)$

$\sin(A+B) = \sin A \cos B + \cos A \sin B$

A $\frac{dy}{dx} - \frac{5}{3}\cos(3x) = 0$

B $\frac{dy}{dx} + \frac{5}{3}\cos(3x) = 0$

C $\frac{d^2y}{d^2x} + 9y = 0$

D $\frac{d^2x}{d^2y} - 9y = 0$

$y = 5\left(\sin 3x \cos\frac{\pi}{3} + \cos 3x \sin\frac{\pi}{3}\right)$

$= \left(5 \times \frac{1}{2} \sin 3x + 5\frac{\sqrt{3}}{2} \cos 3x\right)$

$y = (C_1 \sin 3x + C_2 \cos 3x)$

$y' = (3C_1 \cos 3x - 3 C_2 \sin 3x)$

$y'' = -9C_1 \sin 3x - 9 C_2 \cos 3x)$

$y'' = -9(C_1 \sin 3x + C_2 \cos 3x)$

$y'' = -9y$ $\quad y'' + 9y = 0$

Question - 10

Consider the differential equation $\frac{dy}{dx} + y = e^x$ with $y(0) = 1$. Then the value of $y(1)$ is

A $e + e^{-1}$

B $\frac{1}{2}\left[e - e^{-1}\right]$

C $\frac{1}{2}\left[e + e^{-1}\right]$

D $2\left[e - e^{-1}\right]$

$P = 1 \quad Q = e^x$

$I.F. = e^{\int P\,dx} = e^{\int 1\,dx} = e^x$

Soln :- $y \cdot e^x = \int e^x \cdot e^x\,dx$

$y\,e^x = \frac{e^{2x}}{2} + c$

At $x = 0$, $y = 1$

$1 \cdot 1 = \frac{1}{2} + c \Rightarrow c = 1/2$

$y = \frac{e^x}{2} + \frac{1}{2}e^{-x}$

$y = \frac{1}{2}\left(e + e^{-1}\right)$

Thank you

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Soldiers !