ENGINEERING MATHEMATICS

ALL BRANCHES

Calculus
Continuity and Differentiability of Function

DPP-04 Solution

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Question 1

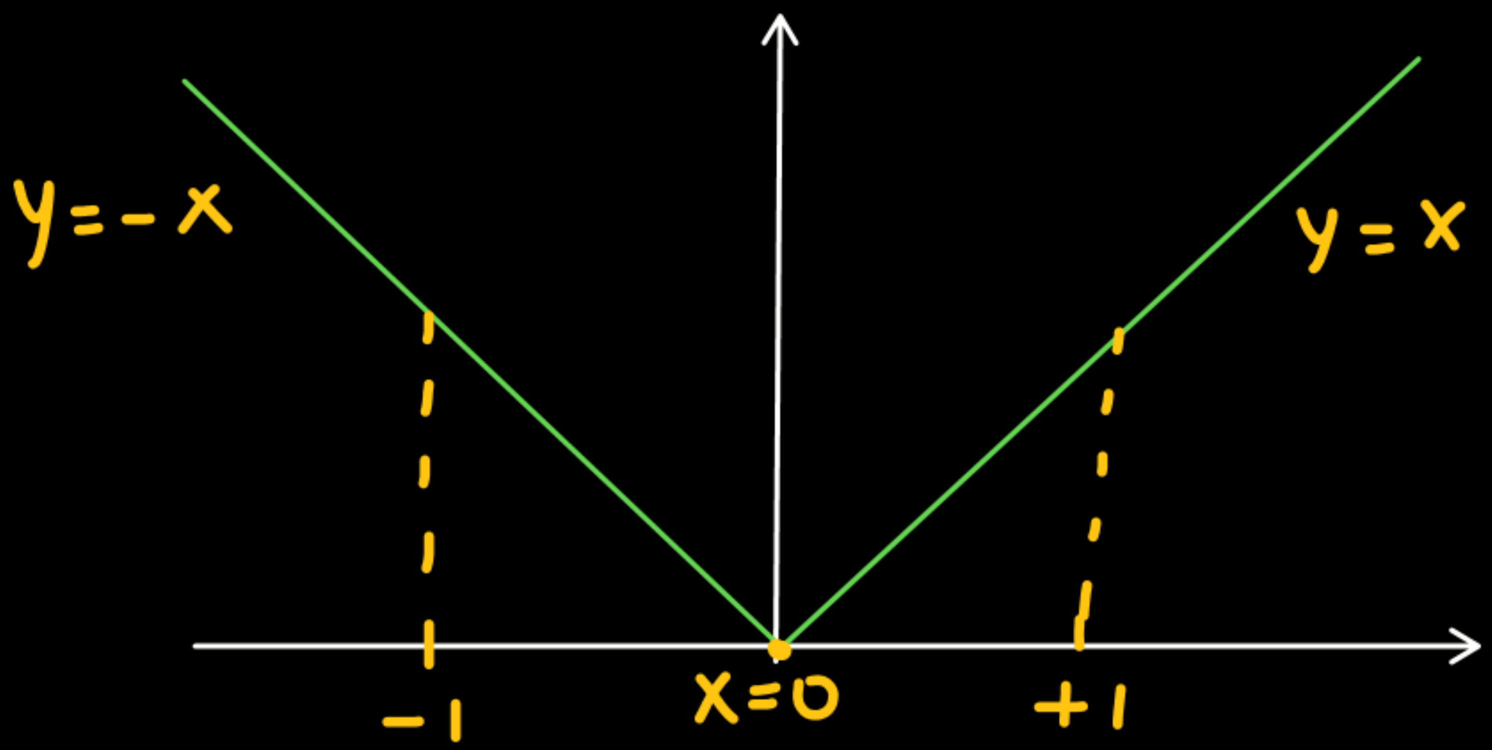
Consider the function $f(x) = |x|$ in the interval $-1 \leq x \leq 1$. At the point $x = 0$, $f(x)$ is

A Continuous and differentiable

B Non-continuous and differentiable

C Continuous and non-differentiable

D Neither continuous nor differentiable



$$|x| = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$

At $x = 0$ $|x|$ is continuous but $|x|$ is not differentiable

L.H.D. $= -1$

R.H.D. $= +1$

$\because$ LHD $\neq$ RHD

Question 2

The function $y = |2 - 3x|$

A. is continuous $\forall\, x \in R$ and differentiable $\forall\, x \in R$

B. is continuous $\forall\, x \in \mathrm{R}$ and differentiable $\forall\, x \in \mathrm{R}$ except at $x = 3/2$

C. is continuous $\forall\, x \in \mathrm{R}$ and differentiable $\forall\, x \in \mathrm{R}$ except at $x = 2/3$

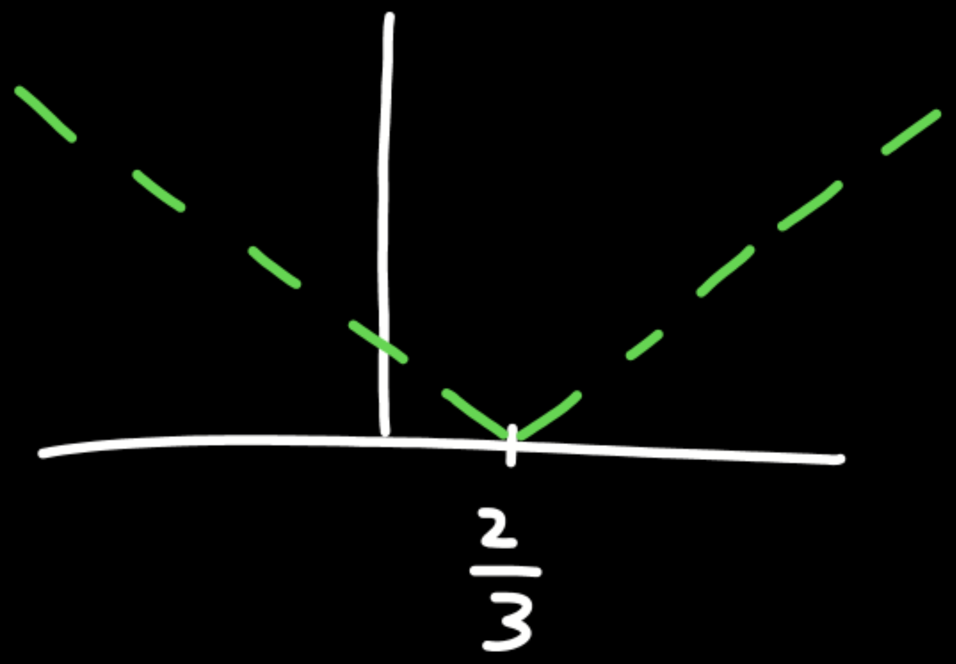
D. is continuous $\forall\, x \in \mathrm{R}$ except $x = 3$ and differentiable $\forall\, x \in \mathrm{R}$

$$y = |2-3x| = \begin{cases} 2-3x & ; 2-3x \geq 0 \\ -(2-3x) & ; 2-3x < 0 \end{cases}$$

$$y = |2-3x| = \begin{cases} 2-3x & ; x \leq \frac{2}{3} \\ -2+3x & ; x > \frac{2}{3} \end{cases}$$

At $x = \frac{2}{3}$, $LHL = RHL = 0$

$\therefore$ At $x = 2/3$; $|2-3x|$ is continuous.



At $x = \frac{2}{3}$; $LHD = -3$

$RHD = +3$

Since $LHD \neq RHD$ $\therefore$ it $|2-3x|$ is not differentiable at $x = 2/3$.

Question 3

Consider the function $f(x) = |x^3|$, where x is real. Then the function $f(x)$ at $x = 0$ is

A Continuous but not differentiable

B Once differentiable but not twice

C Twice differentiable but not thrice

D Three differentiable

$$f(x) = \begin{cases} x^3 & ; x \geq 0 \\ -x^3 & ; x < 0 \end{cases}$$

Since LHL = RHL $= 0$ $\therefore$ it is continuous

$$f'(x) = \begin{cases} 3x^2 & ; x \geq 0 \\ -3x^2 & ; x < 0 \end{cases}$$

Since LHD = RHD $= 0$ $\therefore$ it is differentiable.

$$f''(x) = \begin{cases} 6x & ; x \geq 0 \\ -6x & ; x < 0 \end{cases}$$

Since LHD = RHD $= 0$ $\therefore$ it is twice differentiable.

$$f'''(x) = \begin{cases} 6 & ; x \geq 0 \\ -6 & ; x < 0 \end{cases}$$

Since LHD $\neq$ RHD $\therefore$ it is not differentiable thrice.

$-6 \neq 6$

Question 4

The value of x for which the function $f(x) = \frac{x^2-3x-4}{x^2+3x-4}$ is NOT continuous are

For discontinuity,

$$x^2+3x-4=0$$
$$(x+4)(x-1)=0$$
$$x=-4, 1$$

A 4 and –1

B 4 and 1

C –4 and 1

D –4 and –1

Question 5

If $y = |x|$ for $x < 0$ and $y = x$ for $x \geq 0$, then

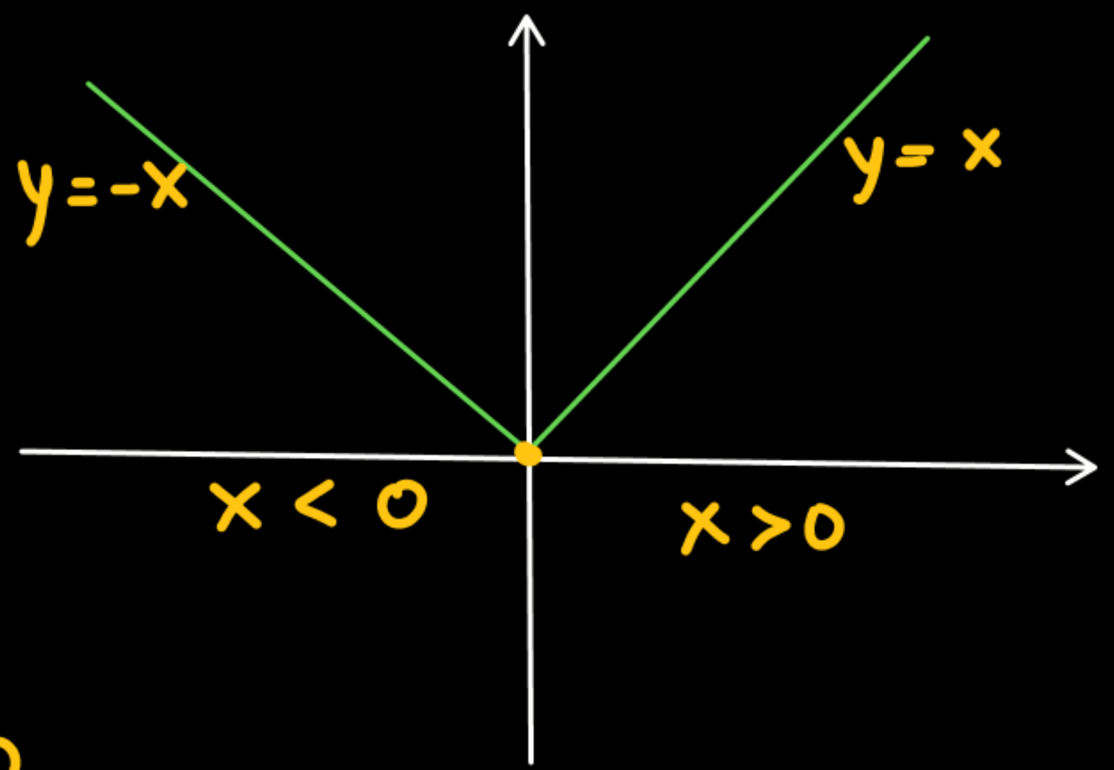
A $\dfrac{dy}{dx}$ is discontinuous at $x = 0$

B y is discontinuous at $x = 0$

C y is not defined at $x = 0$

D None of these

$$y = \begin{cases} x & ; x \geq 0 \\ -x & ; x < 0 \end{cases}$$



Continuous

LHL = RHL = 0

Not differentiable at x = 0

LHD ≠ RHD

(-1) ≠ (1)

Question 6

The function $f(x) = |x + 1|$ on the interval $[-2, 0]$ is

A Continuous and differentiable

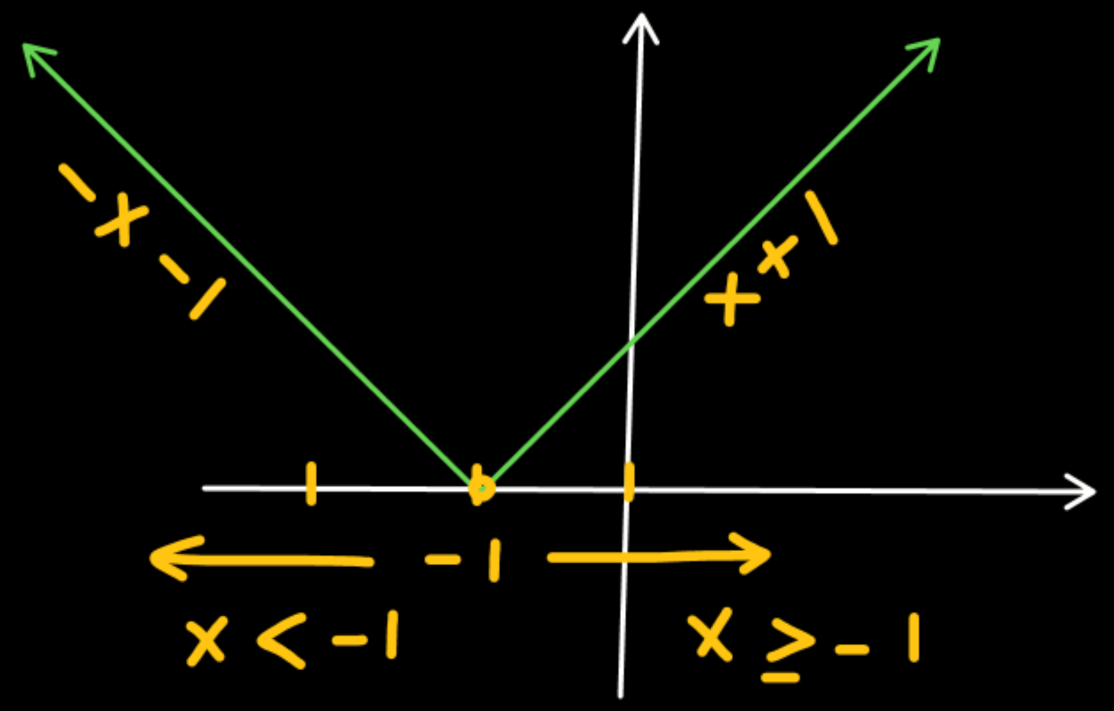
B Continuous on the integers but not differentiable at all points

C Neither continuous nor differentiable

D Differentiable but not continuous

$$y = \begin{cases} x+1 & ; \ x+1 \geq 0 \\ -(x+1) & ; \ x+1 < 0 \end{cases}$$

$$y = \begin{cases} x+1 & ; \ x \geq -1 \\ -x-1 & ; \ x < -1 \end{cases}$$



From graph $f(x)$ is continuous at all points in $[2, 0]$ but at $x = -1$ $f(x)$ is not differentiable

$\because$ LHD $\neq$ RHD (corner point)

$(-1) \neq (1)$

Question 7

If a function is continuous at a point, its first derivative

A. May or may not exist

B. Exists always

C. Will not exist

D. Has unique value

Continuity
↳ may or may not be differentiable.

Differentiable
↳ always continuous.

Question 8

If function f(x) is defined as $f(x) = \dfrac{xe^{1/x}}{1+e^{1/x}}; \quad x \neq 0$

$$= 0; \quad x = 0$$

A $f(x)$ is continuous and differentiable

B $f(x)$ is not continuous but differentiable

C $f(x)$ is continuous but not differentiable

D None

$$f(x) = \frac{x e^{1/x}}{1+e^{1/x}} \quad ; \quad x \neq 0$$
$$= 0 \quad ; \quad x = 0$$

Now $\lim_{h\to 0} f(0+h) = \lim_{h\to 0} \frac{h\, e^{1/h}}{1+e^{1/h}} = \frac{h}{\frac{1}{e^{1/h}}+1} = \frac{0}{0+1} = 0$

$\lim_{h\to 0} f(0-h) = \lim_{h\to 0} \frac{-h\, e^{-1/h}}{1+e^{-1/h}} = \frac{0\times 0}{1+0} = 0$

Since $\lim_{h\to 0} f(0-h) = \lim_{h\to 0} f(0+h)$ $\therefore$ $f(x)$ is continuous at $x=0$

LHL = RHL = Value

$$Lf'(0) = \lim_{h\to 0} \frac{f(0-h)-f(0)}{0-h-0} = \frac{\frac{-h\, e^{-1/h}}{1+e^{-1/h}} - 0}{-h} = 0$$

$$Rf'(0) = \frac{f(0+h) - f(0)}{0+h-0} = \frac{\frac{h\,e^{1/h}}{1+e^{1/h}} - 0}{h} = \frac{e^{1/h}}{1+e^{1/h}}$$

$$\lim_{h \to 0} \frac{1}{\frac{1}{e^{1/h}} + 1} = \frac{1}{0+1} = 1$$

$$Lf'(0) \neq Rf'(0)$$

$$0 \neq 1$$

Hence $f(x)$ is continuous but not differentiable at $x=0$.

Thank you

GW Soldiers !