ENGINEERING MATHEMATICS

ALL BRANCHES

System of Equations

Linear Algebra

DPP-07 Solution

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Question 1

The system of equation $3x - y + z = 0, 15x - 6y + 5z = 0,$
$\lambda x - 2y + 2z = 0$ has a non-zero solution, if λ is

A 6

B -6

C 2

D -2

$$3x - y + z = 0$$
$$15x - 6y + 5z = 0$$
$$\lambda x - 2y + 2z = 0$$

Homogenous system

① $\rho(A) = n$
→ Trivial soln.
(Zero soln.)

② $\rho(A) < n$
→ Non-trivial soln.
(Infinite soln.)

$\therefore$ For non-zero solution
$\rho(A) < 3$ (No. of unknowns)

$$\therefore |A|_{3\times3} = \begin{vmatrix} 3 & -1 & 1 \\ 15 & -6 & 5 \\ \lambda & -2 & 2 \end{vmatrix} = 0$$

$$\begin{vmatrix} 3 & -1 & 1 \\ 15 & -6 & 5 \\ \lambda & -2 & 2 \end{vmatrix} = 0$$

$$3(-12+10) + 1(30 - 5\lambda) + 1(-30 + 6\lambda) = 0$$

$$-6 + \cancel{30} - 5\lambda - \cancel{30} + 6\lambda = 0$$

$$\boxed{\lambda = 6}$$

Question 2

The system of equation $x - 2y + z = 0, 2x - y + 3z = 0,$ $\lambda x + y - z = 0$ has the trivial solution as the only solution, if λ is

A $\lambda \neq -\frac{4}{5}$

B $\lambda = \frac{4}{3}$

C $\lambda \neq 2$

D None of these

$$x - 2y + z = 0$$
$$2x - y + 3z = 0$$
$$\lambda x + y - z = 0$$

$$\begin{vmatrix} 1 & -2 & 1 \\ 2 & -1 & 3 \\ \lambda & 1 & -1 \end{vmatrix} \neq 0$$

$$1(1-3) + 2(-2-3\lambda) + 1(2+\lambda) \neq 0$$
$$-\cancel{2} - 4 - 6\lambda + \cancel{2} + \lambda \neq 0$$
$$\lambda \neq -4/5$$

Homogenous system

→ For trivial soln (zero soln)

$\rho(A)$ = No. of unknowns

$\therefore \rho(A) = 3$

$\Rightarrow |A|_{3\times3} \neq 0$

Question 3

The system equations $x + y + z = 6, x + 2y + 3z = 10,$
$x + 2y + \lambda z = 12$ is inconsistent, if λ is

A 3

B -3

C 0

D None of these

$$x + y + z = 6$$
$$x + 2y + 3z = 10$$
$$x + 2y + \lambda z = 12$$

Non-homogenous system

- $\rho(A) = \rho(A:B) \leq n$
  $\Rightarrow$ Consistent soln.
- $\rho(A) \neq \rho(A:B)$
  $\Rightarrow$ Inconsistent soln.

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 2 & 3 & 10 \\ 1 & 2 & \lambda & 12 \end{array}\right]$$

$$\xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 1 & \lambda-1 & 6 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 1 & 2 & 4 \\ 0 & 0 & \lambda-3 & 2 \end{array}\right]$$

$\therefore \rho(A:B) = 3$ ($\because$ No. of non-zero rows $= 3$)

$\Rightarrow \rho(A) \neq \rho(A:B)$ $\therefore \rho(A) = 2 =$ No. of non-zero rows.

$\therefore \lambda - 3 = 0 \Rightarrow \boxed{\lambda = 3}$

Question 4

The system of equations $5x + 3y + 7z = 4, 3x + 26y + 2z = 9,$ $7x + 2y + 10z = 5$ has

A a unique solution

B no solution

C an infinite number of solutions

D none of these

$$(A:B) = \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right] \xrightarrow[R_3 \to R_3 - \frac{7}{5}R_1]{R_2 \to R_2 - \frac{3}{5}R_1} \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & \frac{121}{5} & -\frac{11}{5} & \frac{33}{5} \\ 0 & -\frac{11}{5} & \frac{1}{5} & -\frac{3}{5} \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + \frac{R_2}{11}} \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & \frac{121}{5} & -\frac{11}{5} & \frac{33}{5} \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$\therefore \rho(A) = 2$

$\therefore \rho(A|B) = 2$

$\Rightarrow \rho(A) = \rho(A:B) <$ no. of unknowns

$\Rightarrow$ Infinite solutions.

Question 5

The system of equations $x - 4y + 7z = 14, 3x + 8y - 2z = 13,$
$7x - 8y + 26z = 5$ has

A a unique solution

B no solution

C an infinite number of solution

D none of these

$$(A|B)=\left[\begin{array}{ccc|c}1 & -4 & 7 & 14\\ 3 & 8 & -2 & 13\\ 7 & -8 & 26 & 5\end{array}\right] \xrightarrow[R_3 \to R_3 - 7R_1]{R_2 \to R_2 - 3R_1} \left[\begin{array}{ccc|c}1 & -4 & 7 & 14\\ 0 & 20 & -23 & -29\\ 0 & 20 & -23 & -93\end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c}1 & -4 & 7 & 14\\ 0 & 20 & -23 & -29\\ 0 & 0 & 0 & -64\end{array}\right]$$

$\therefore \rho(A) = 2$

$\therefore \rho(A:B) = 3$

$\therefore \rho(A) \neq \rho(A|B)$

$\Rightarrow$ No solution (Inconsistent)

Question 6

Consider the following system of equations

$$\left.\begin{aligned} 2x_1 + x_2 + x_3 &= 0 \\ x_2 - x_3 &= 0 \\ x_1 + x_2 &= 0 \end{aligned}\right\}$$ Homogenous system

This system has

A a unique solution

B no solution

C infinite number of solutions

D five solutions

Coefficient matrix $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & -1 \\ 1 & 1 & 0 \end{bmatrix}$

Check $|A|_{3\times 3} = 2(0+1) + 0 + 1(-1-1)$

$= 2 + 0 - 2 = 0$

$\therefore \rho(A) < 3$

Check $|A|_{2\times 2} = \begin{vmatrix} 2 & 1 \\ 0 & 1 \end{vmatrix} = 2 \neq 0$

$\therefore \rho(A) = 2$

$\Rightarrow \rho(A) = 2 <$ No. of unknowns (3)

$\Rightarrow$ Infinite solutions

① $\rho(A) = n.$

$\Rightarrow$ Zero soln.

② $\rho(A) < n$

$\Rightarrow$ Infinite soln.

Question 7

For what value of a, if any will the following system of equation in x, y and z has a solution?

$$2x + 3y = 4$$
$$x + y + z = 0$$
$$3x + 2y - z = a$$

A Any real number

B 0

C 1

D There is no such value

$$(A:B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 2 & 3 & 0 & 4 \\ 3 & 2 & -1 & a \end{array}\right] \xrightarrow[R_3 \to R_3 - 3R_1]{R_2 \to R_2 - 2R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 4 \\ 0 & -1 & -4 & a \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 + R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 0 \\ 0 & 1 & -2 & 4 \\ 0 & 0 & -6 & a+4 \end{array}\right]$$

$\rho(A) = 3$

$\rho(A:B) = 3$

$\therefore \rho(A) = \rho(A:B)$

$a+4$ can take any value then also $\rho(A) = \rho(A:B) = 3$

$\therefore$ a can take any real value.

Question 8

The system of equations

$$x + y + z = 6$$
$$x + 4y + 6z = 20$$
$$x + 4y + \lambda z = \mu$$

has No solution for values of λ and μ given by

A $\lambda = 6, \mu = 20$

C $\lambda \neq 6, \mu = 20$

B $\lambda = 6, \mu \neq 20$

D $\lambda \neq 6, \mu = 20$

$$(A:B) = \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 1 & 4 & 6 & 20 \\ 1 & 4 & \lambda & \mu \end{array}\right] \xrightarrow[R_3 \to R_3 - R_1]{R_2 \to R_2 - R_1} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 3 & \lambda - 1 & \mu - 6 \end{array}\right]$$

$$\xrightarrow{R_3 \to R_3 - R_2} \left[\begin{array}{ccc|c} 1 & 1 & 1 & 6 \\ 0 & 3 & 5 & 14 \\ 0 & 0 & \lambda - 6 & \mu - 20 \end{array}\right]$$

For no solution

$\rho(A) \neq \rho(A:B)$

$\therefore \lambda - 6 = 0$

and $\mu - 20 \neq 0$

$\therefore \lambda = 6 \; ; \mu \neq 20$

✓ → $\lambda - 6 = 0 \; ; \; \mu - 20 = 0$ $\quad \rho(A) = \rho(A:B) = 2$

✓ → $\lambda - 6 \neq 0 \; ; \; \mu - 20 = 0$ $\quad \rho(A) = \rho(A:B) = 3$

$\rho(A) = 3 \qquad \rho(A:B) = 3$

✗ → $\lambda - 6 = 0 \; ; \; \mu - 20 \neq 0 \Rightarrow \rho(A) \neq \rho(A:B)$

$\rho(A) = 2 \qquad \rho(A:B) = 3$

✓ → $\lambda - 6 \neq 0 \; ; \; \mu - 20 \neq 0$ $\quad \rho(A) = \rho(A:B) = 3$

Thank you

GW

Soldiers !