ALL BRANCHES

ENGINEERING MATHEMATICS

Lecture No.-03

Probability

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Topics to Be Covered

- FUNDAMENTAL COUNTING
- ADDITION THEOREM
- CONDITIONAL PROBABILITY
- TOTAL PROBABILITY THEOREM
- BAYE'S THEOREM
- STATISTICS – I (PROBABILITY DISTRIBUTIONS)
- STATISTICS – II (CORRELATION AND REGRESSION)

PROBABILITY BASICS

Ex: 50 students

$\rightarrow DS \Rightarrow 25$

$\rightarrow CN \Rightarrow 15$

$\rightarrow TOC \Rightarrow 20$

$\rightarrow DS \cap CN \Rightarrow 10$

$\rightarrow CN \cap TOC \Rightarrow 7$

$\rightarrow DS \cap TOC \Rightarrow 6$

$\rightarrow (DS \cap CN \cap TOC) \Rightarrow 5$

i) , ii) , iii) , iv), v), vi), vii)

i) $n(\underline{DS} \cup \underline{CN} \cup \underline{TOC}) = 25+15+20-10-7-6+5 = 42$

$$P(DS \cup CN \cup TOC) = \frac{42}{50} = 0.84$$

[PROBABILITY BASICS]

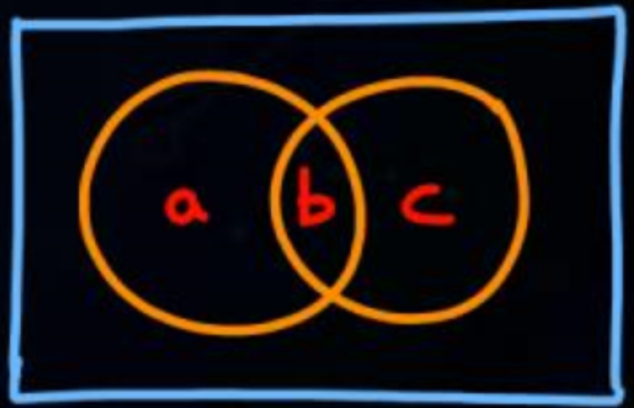
$P(A\cap\bar{B}) = P(A) - P(A\cap B)$

$P(\bar{A}\cap B) = P(B) - P(A\cap B)$

Addition Theorem of Probability

a) For two events

$$\boxed{P(A\cup B) = P(A) + P(B) - P(A\cap B)}$$



$n(A) = a+b$

$n(B) = b+c$

$n(A\cap B) = b$

$a+2b+c-b$

$n(A)+n(B)-n(A\cap B)$

$$\boxed{P(A\cup B) = P(A) + P(B)}$$ For mutually exclusive

b) For three events

$$P(A\cup B\cup C) = P(A)+P(B)+P(C)-P(A\cap B)-P(B\cap C)-P(C\cap A)+P(A\cap B\cap C)$$

$$\boxed{P(A\cup B\cup C) = P(A)+P(B)+P(C)}$$ For mutually exclusive

Ex:- A & B are two events

$P(A)=\frac{1}{3}$

$P(A\,U\,B)=\frac{3}{4}, P\,(A\,\cap\,B)=\frac{1}{4}, \overbrace{P(\bar{A})=\frac{2}{3}}$,find P(B)?

$P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{3}{4}=\frac{1}{3}+P(B)-\frac{1}{4}$

$\boxed{P(B)=\frac{2}{3}}$

$P(\bar{A})=2/3$

$P(A)=1-2/3$

$P(A)+P(\bar{A})=1$

PROBABILITY BASICS

Ex:- Aman wins $= \frac{2}{3}$, Suresh wins $= \frac{1}{5}$

What is the probability that

i) Both of them wins

ii) At least one of them wins

$$\underbrace{AS,\ \bar{A}S,\ A\bar{S}}\ ,\ \bar{A}\bar{S}\ . = 1$$

$$\frac{2}{15} \quad \frac{1}{15} \quad \frac{8}{15} \quad \frac{4}{15} \quad = 1$$

i) $p(A \cap S) = p(A) \cdot p(S) = \frac{2}{3} \cdot \frac{1}{5} = \frac{2}{15}$

ii) At least one $\Rightarrow p(\bar{A} \cap S) + p(A \cap \bar{S}) + p(A \cap S)$

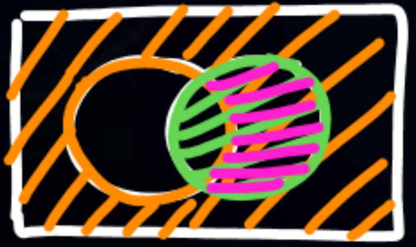
$= \frac{1}{3} \cdot \frac{1}{5} + \frac{2}{3} \cdot \frac{4}{5} + \frac{2}{3} \cdot \frac{1}{5} = \frac{11}{15}$

$= 1 - p(\text{None of them})$

$1 - p(\bar{A} \cap \bar{S}) = 1 - \frac{1}{3} \cdot \frac{4}{5} = 1 - \frac{4}{15} = \frac{11}{15}$

Ex:- $P(A) = \frac{1}{4}$ $P(A/B) = \frac{1}{2}$ $P(B/A) = \frac{2}{3}$, $P(B) = ?$

Soln:- $P(A/B) = \dfrac{P(A\cap B)}{P(B)}$ $\qquad P(B/A) = \dfrac{P(A\cap B)}{P(A)}$



$$\boxed{P(B).P(A/B) = P(A).P(B/A)}$$

$P(B).\ 1/2 = 1/4 \cdot 2/3 \qquad \Rightarrow P(B) = 1/3$

Ex:- $P(X) = 1/3$; $P(X/Y) = 1/2$; $P(Y/X) = 2/5$

✓ A) $P(Y) = 4/15$

✓ B) $P(X'/Y) = 1/2$

× C) $P(X\cap Y) = 1/5$

× D) $P(X\cup Y) = 2/5$

Check if X and Y are independent.

Soln:- i) $P(X).P(Y/X) = P(Y).P(X/Y)$

$\frac{1}{3}\cdot\frac{2}{5} = P(Y)\cdot\frac{1}{2} \Rightarrow \boxed{P(Y) = 4/15}$

ii) $P(X'/Y) = \dfrac{P(X'\cap Y)}{P(Y)} = \dfrac{P(Y) - P(X\cap Y)}{P(Y)}$

$= 1 - P(X/Y) = 1 - 1/2 = 1/2$

NOTE :- $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$P(\bar{A} \cap B) = P(B) - P(A \cap B)$

iii) $P(X/Y) = \dfrac{P(X \cap Y)}{P(Y)}$

$P(X \cap Y) = P(Y) . P(X/Y) = \frac{1}{2} \cdot \frac{4}{15} = \frac{2}{15}$

iv) $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$

$= \frac{1}{3} + \frac{4}{15} - \frac{2}{15}$

$= \cancel{\frac{21}{45}} = \frac{7}{15}$

v) $P(X \cap Y) = P(X) . P(Y)$

$\frac{2}{15} \neq \frac{1}{3} \cdot \frac{4}{15} = \frac{4}{45}$ (not independent)

Ex:- Three shooters $X, Y, Z \to 1/3, 1/4, 1/5$ (Independent)

E) ← i) What is the prob. that one of them misses the target?

ii) If only one misses the target what is the probability that he is Y ?

Soln:- $= P(\bar{A} \cap B \cap C) + P(A \cap \bar{B} \cap C) + P(A \cap B \cap \bar{C})$

$$= \left(1-\frac{1}{3}\right)\cdot\frac{1}{4}\cdot\frac{1}{5} + \frac{1}{3}\cdot\left(1-\frac{1}{4}\right)\cdot\frac{1}{5} + \frac{1}{3}\cdot\frac{1}{4}\left(1-\frac{1}{5}\right)$$

$$P(E) = \frac{2}{60} + \frac{3}{60} + \frac{4}{60} = \frac{9}{60} = \frac{3}{20}$$

$$P\left(\frac{Y}{E}\right) = \frac{P(Y \cap E)}{P(E)} = \frac{3/60}{9/60} = 1/3$$

$Y \to$ Prob. that only B misses

$E \to$ Prob. that one of them misses

→ P(At least one of them hits) = 1 − P(None of them)

$1 - P(\bar{X} \cap \bar{Y} \cap \bar{Z})$

$$P\left(\frac{X}{E}\right) = \frac{P(X \cap E)}{P(E)} = \frac{2/60}{9/60} = 2/9$$

$$P\left(\frac{Y}{E}\right) = \frac{P(Y \cap E)}{P(E)} = \frac{3/60}{9/60} = 3/9$$

$$P\left(\frac{Z}{E}\right) = \frac{P(Z \cap E)}{P(E)} = \frac{4/60}{9/60} = 4/9$$

$$\left.X\ Y\ Z\right\} - 3$$

$$\left.\begin{array}{l} X\ Y\ \bar{Z} \\ X\ \bar{Y}\ Z \\ \bar{X}\ Y\ Z \end{array}\right\} - 2$$

$$\left.\begin{array}{l} X\ \bar{Y}\ \bar{Z} \\ \bar{X}\ Y\ \bar{Z} \\ \bar{X}\ \bar{Y}\ Z \end{array}\right\} - 1$$

$$\left.\bar{X}\ \bar{Y}\ \bar{Z}\right\} - 0$$

$$1$$

Ex:- A dice rolled twice & sum is observed as 7. What is conditional prob. that number 2 has appeared at least once?

$P(S) = 6/36$

$(1,6)(2,5)(3,4)(4,3)(5,2)(6,1)$

S → Sum is 7.

$P(N) = 11/36$

$(2,5)(5,2)$

N → Number 2 at least once.

$$P(N/S) = \frac{P(N \cap S)}{P(S)} = \frac{2/36}{6/36} = 1/3$$

Ex:- Two number is selected from

$\boxed{1, 2, \ldots 11}$

If sum of integers is even. Find prob. that selected integers are odd.

Sum Even → E + E, O + O

Even = 5 {2, 4, 6, 8, 10}
odd = 6 {1, 3, 5, 7, 9, 11}

O → Selected integers are odd

E → Sum of integers is even.

$P(O/E) =$

PROBABILITY BASICS

$$P\left(\begin{matrix}\text{Normal year}\\ \text{53 Sundays}\end{matrix}\right) = \frac{1}{7}$$

Ex:- What is the probability that a leap year selected at random will contain 53 Sundays?

Normal year → 365 days → 52 weeks + 1 day

Leap year → 366 days → 52 weeks + 2 day

S M T W T F S

2 day → {SM, MT, TW, WT, TF, FS, SS} (7)

E → {SM, SS} = (2)

$$P\left(\begin{matrix}\text{leap year}\\ \text{53 Sundays}\end{matrix}\right) = \frac{2}{7}$$

PROBABILITY BASICS

Cards

	♠ Spades	♣ Club	♦ Diamond	♥ Heart
4 Ace {	A	A	A	A
	2	2	2	2
	3	3	3	3
	⋮	⋮	⋮	⋮
	10	10	10	10
	J	J	J	J
	Q	Q	Q	Q
	K	K	K	K
	13	13	13	13

J, Q, K } Face cards (12)

52 cards → 26 Red (♦, ♥)

52 cards → 26 Black (♠ ♣)

Ex:- i) 2 cards at random find prob. both are King.

ii) 2 card is drawn find prob. that it is King or a red card.

Soln:- $p(E) = p(K_1 \cap K_2) = p(K_1) \cdot p(K_2/K_1) = \frac{4}{52} \cdot \frac{3}{51}$

without replacement

$p(E) = p(K_1 \cap K_2) = p(K_1) \cdot p(K_2) = \frac{4}{52} \cdot \frac{4}{52}$

w/o replacement

1 card → $P(K \cup R) = p(K) + p(R) - p(K \cap R) = \frac{4}{52} + \frac{26}{52} - \frac{2}{52} = \frac{28}{5}$

2 cards → $P(K \cup R) =$

Permutation & Combination

Combination

Selection of r things out on n things.

$$= \boxed{{}^{n}C_{r} = \frac{n!}{r!(n-r)!}}$$

Permutation

Selection & arrangement of r things out on n things.

$$\boxed{{}^{n}P_{r} = {}^{n}C_{r} \cdot r! = \frac{n!}{n-r!}}$$

$0! = 1$

$1! = 1$

$2! = 2$

$3! = 6$

$4! = 24$

A B C

A, B, C → 3

AB, BC, CA → 3

$^{3}C_{2}$

ABC → 1

AB BA

BC CB

CA AC

$(^{3}P_{2})$

Permutation & Combination

NOTE :- (1) If all items are different and all taken at a time, then all items can be arranged in $\underline{n!}$.

Ex:- 3 letters → A B C   ABC ACB BAC BCA CAB CBA

$3! = 6$ ways

Ex:- AYUSH ⇒ $5!$ $\underline{5} \times \underline{4} \times \underline{3} \times \underline{2} \times \underline{1} = 120$

(2) No. of arra.. in n things
- → P_1 are alike
- → P_2 are alike
- → P_3 are alike
- → $n-(P_1+P_2+P_3)$ are diff

$$= \frac{n!}{P_1!\,P_2!\,P_3!}$$

PROBABILITY BASICS

Permutation & Combination

Ex:- MISSISSIPPI →

$n = 11 \to 1M, 4I, 4S, 2P$

$= \dfrac{11!}{1!\,4!\,4!\,2!}$

$\underline{4} \times \underline{3} \times \underline{2} \times \underline{1}$

ABCD

$4!$

ABBD

$\dfrac{4!}{2!}$

③ When n diff. things are arranged with repetitions allowed when r things are taken at a time

$= n \times n \times n \ldots r \text{ times} = n^r$.

$\underline{n}\ \underline{n}\ \underline{n}\ \underline{n} \ldots$

Ex:- ABCD → All taken → 4^4

2 taken at time → $\underline{4} \times \underline{4} = 4^2$

Permutation & Combination

(4) No. of selection of r things out of n alike/identical things.

= 1

= 1 + 1 (if 0 things are selected)

(5) Total no. of selections of zero or more things out of n identical things = n + 1

(zero or more things)

$n+1 \rightarrow$ 0 things are selected.

0 ball
1 ball
2 ball
3 ball
4 ball

$\Big\}$ 5 ways of selection

Permutation & Combination

(6) Total no. of selections of zero or more things out n different things:-

$$^nC_0 + {^nC_1} + {^nC_2} + {^nC_3} + \dots\dots {^nC_n} = 2^n$$

$\downarrow$ 0 things $\quad\downarrow$ 1 thing $\quad\downarrow$ 2 thing

25

The no. of selections of 1 or more things:-

$\boxed{10}$

$$^nC_1 + {^nC_2} + {^nC_3} + \dots {^nC_n} = 2^n - 1$$

(7) No. of ways of distributing n identical things among r persons when each person may get any no. of things $^{n+r-1}C_{r-1}$

PROBABILITY BASICS

Fundamental Theorem of Counting

If for every m ways there are n ways.

Total no. of ways $= m \times n$

If for every m ways there are n ways and for every n ways there are p ways.

Total no. of ways $= m \times n \times p$

Ex:- What is the probability that two boys share same birth month ?

$\underline{12}$ $\underline{12}$

Ex:- 5 cards are drawn from pack of 52 cards P (that it has only one ace)

Ex:- What is the probability that the chosen triangle is equilateral if 3 of 6 vertices of a regular hexagon are chosen at random?

Thank you

GW

Soldiers !