A STATE SPACE MODEL FOR BERLIN HOUSE PRICES*

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Abstract

How risky are investments in residential real estate? To answer this question, information is needed about the behavior of house prices. The hedonic methodology has become a standard approach for modelling the prices of heterogeneous assets. Although intuitively appealing, it is often criticized that this approach has no sound theoretical background. We have developed a model that partly circumvents this criticism. Based on an approximation for the present value, our model delivers a state space form for the determination of house prices. Thus,

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we can incorporate in an economically meaningful way other economic variables like the inflation rate, mortgage rates and returns of other assets. Under some restrictive conditions, our model reduces to the standard hedonic approach. We use the EM algorithm with a final scoring step to estimate our model with monthly data of single-family house sales from the four South-West districts of Berlin for the years 1982:7 to 1999:12.

JEL Codes: C32, C43, G12
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1 Introduction

In most industrial countries real estate is the greatest component of private household's wealth. For example, in Germany real estate's share of total wealth is about 53% (Deutsche Bundesbank 1999, p.43). For many households, owner-occupied housing is the single most important asset in their portfolios. As a consequence, private households' real estate investments are at center stage in the ongoing discussion about private pension schemes and optimal portfolio composition. Questions arising in this discussion are: How risky are investments in residential real estate? How is this risk related to the risk of other assets like stocks and bonds? Does real estate provide a hedge against inflation? Potential house buyers, sellers and developers of new houses are all interested in answers to these questions. Also banks want to know more about the risk of real estate because they use houses as collateral for mortgages. To answer these questions, a careful analysis of the time series properties of real estate prices is needed.

In this paper, we study the movement of house prices in Berlin, Germany, during a twenty year span. Our primary data source is a data base consisting of all transactions of single-family homes in Berlin between January 1980 and December 1999. Studying the development of house prices, though, is complicated by the fact that houses are heterogeneous assets. Indeed, it has been said that no two houses are ever identical. It is thus imperative in the empirical model to include variables that measure house characteristics. We therefore make ample use of the rich information in our data set describing the properties of each unit sold.

The standard approach for constructing a model of the prices of heterogeneous assets is hedonic regression (e.g. Shiller 1993, Cho 1996, Sheppard 1997, Hill, Knight, and Sirmans 1997). Due to the fact that the so-called repeat-sales approach derives from the hedonic methodology, we incorporate it under this heading (Dombrow, Knight, and Sirmans 1997). A hedonic model starts with the assumption that on average the observed price can be explained by some function $f(I_t, \mathbf{x}_{n,t}, \boldsymbol{\beta}_t)$. Here, I_t is a common price component that "drives" the prices of all houses, the vector $\mathbf{x}_{n,t}$ comprises the characteristics of house n and the vector $\boldsymbol{\beta}_t$ contains all—possible time variable—coefficients of the functional form. Most studies assume a log-log functional form and that I_t is just a period specific constant term. However, although there is some theoretical work on the derivation of functional forms (see the seminal paper of Rosen 1974), it is sometimes difficult to interpret the hedonic coefficients in a plausible way. We build on this work but go beyond the conventional specification in several important respects.

In our paper, we propose the well-known present value relation as a means to explain the behavior of house prices. Our approach is quite similar to the one used in Engle, Lilien, and Watson (1985). We generalize their approach. First, we derive a hedonic regression equation from the well-known present value model of asset prices, thus providing theoretical motivation and aiding interpretation of the hedonic model. Still, present value theory does not exactly pin down the functional form of the hedonic regression equation. We therefore use a cross-validation criterion to choose between various possible transformations of the continuous explanatory variables in the empirical work. Moreover we augment the hedonic equation by a model of the unobservable component of house prices reflecting the general tendency of the market for residential real estate. This component is assumed to be common to all prices in a certain period after controlling for the heterogeneity of house attributes. It is specified as an autoregressive process that also depends on financial variables such as the spreads between mortgage and interest rates with the same maturity or the returns of other assets such as stocks and bonds. Our economic model associates this component of house prices with (expected) deviations from the long run rate of return of single-family homes.

The key to handling an hedonic model that has been augmented by an equation with an unobservable dependent variable is to write the model in state space form. Once the model has been put into this form, the Kalman filter can be used to estimate the unobservable price component and the EM algorithm can be used to calculate maximum likelihood estimates of the unknown coefficients of the model. Finally, we perform a scoring step to improve the efficiency of the EM algorithm estimates and to obtain estimated standard errors.

We estimate the augmented hedonic model using a subsample of the data base of all transactions that contains 4410 sales of single-family houses in the four South-West districts of Berlin between July 1982 and December 1999. Our estimates of the coefficients of the hedonic equation provide plausible and easily interpretable values of the premiums or rebates that different house characteristics command. The estimated process of the common price component is highly persistent and sluggish.

The remainder of this paper is organized as follows: we motivate and derive a hedonic regression model in Section 2 with the help of present value theory. Because the present value formulae are highly non-linear, we use a well-known approximation. We also propose an equation for the unobservable price component. In the following section the empirical model is put into state space form and the estimation strategy is laid out. It basically consists of combining the Kalman filter and the EM algorithm to get estimates of the unknowns in our model. Section 4 contains the empirical part of the paper. Section 5 interprets the results. The last Section concludes. An Appendix contains some derivations of used expressions.

2 Present Value Relation

We start with the assumption that the sales price of a house is equal to the sum of the discounted net proceeds that the investor expects in the future. The economic reasoning for the relation is as follows: the buyer of a house accrues a "dividend" from holding the house whereas the seller incurs opportunity costs of not receiving this "dividend". The dividend is simply the rent of the object. The process of the rent gives the cost of "shelter". This process must be discounted with a rate that compensates for the risk of holding a house. The discount rate is identical to the marginal return of other investments with the same level of risk. Due to this fact, we use "discount rate" and "return rate" as synonyms. That is the standard framework of the well-known *present value relation* (see Cochrane 2001)

$$P_{n,t} = \sum_{j=1}^{\infty} \mathrm{E}_t \left[\frac{D_{n,t+j}}{\prod_{i=1}^{j}(1+R_{t+i})} \right] . \tag{1}$$

Here $P_{n,t}$ denotes the sales price of house n that is sold in period t. $\mathrm{E}_t[\cdot]$ is a shorthand notation for the expectation taken conditional on the information available at time t. The net proceeds are given by the net rents $D_{n,t}$ for the house. Given the gross rents, one can derive the net rents by accounting for maintenance and running costs. The net rents are discounted with time-varying rates R_{t+j}. Due to the last assumption, the above stated relation can be seen as a pure identity. Later on, we have to put structure on the process of the discount rate.

Instead of working with equation (1) directly, we use a log-linearized version of it (cf. Campbell, Lo, and MacKinlay 1997, Cochrane 2001). Let r_t denote the log of one plus the return rate and $d_{n,t}$ the log rent. The first order approximation for the log price is

$$p_{n,t} = \frac{k}{1-\rho} + d_{n,t} + \sum_{j=0}^{\infty} \rho^j \Big(\mathrm{E}_t[\Delta d_{n,t+1+j}] - \mathrm{E}_t[r_{t+1+j}] \Big) \tag{2}$$

with $\rho \equiv 1/(1+\theta)$, $k \equiv \ln(1+\theta) - \theta \ln \theta/(1+\theta)$. Here, θ is the geometric average of the rent-price ratio during the sample period. We have $\theta > 0$ and $\rho < 1$. It is easy to see from (2) that ρ can be interpreted as a discount factor. The discount rate is given by θ. As usual, Δ denotes the difference operator. So, $\Delta d_{n,t}$ gives approximately the growth rate of the rents.

Under the assumptions of the well known *Gordon growth model* (cf. Campbell, Lo, and MacKinlay 1997, p.256) the approximation of the log price is exact. The assumptions of this model are that the discount rate and the growth rate of the expected rents are constant. Thus, (1) boils down to

$$P_{n,t} = \frac{(1+G)D_{n,t}}{R-G} \tag{3}$$

with the rent growth rate G and the discount rate R. With the constant rent-price ratio $\theta = (R-G)/(1+G)$ we obtain

$$p_{n,t} = d_{n,t} - \ln\theta \; . \tag{4}$$

It is easy to check that (2) reduces to this equation with $r_{t+1+j} = \ln(1+R)$ and $\Delta d_{n,t+1+j} = \ln(1+G)$. Here, θ is the inverse of the so-called *capitalization rate*. To derive the price one merely needs to capitalize—that is: to multiply—the rent with this rate.

Because we have data on owner-occupied houses it is impossible for us to observe the rents for the different objects. However, we have data on the rent index for Berlin. We will refer to the notional object that corresponds to this index as *reference house*. Let d_t^0 denote this rent index. We assume that there exists a close connection between the unobservable rents of house n and the rent for the reference house. This connection is given by

$$d_{n,t} = \delta + d_t^0 + (\mathbf{x}_{n,t} - \mathbf{x}^0)^T \boldsymbol{\beta} + \varepsilon_{n,t} \tag{5}$$

where $\varepsilon_{n,t}$ is white noise. The constant δ absorbs the normalization of the rent index. $\mathbf{x}$ comprises the—possibly transformed—characteristics for every object such as its age or its floor size. The characteristics of the reference dwelling, $\mathbf{x}^0$, are unobservable whereas the characteristics of house n, $\mathbf{x}_{n,t}$, are observable. The rent for house n is thus given by the rent for the reference dwelling plus a premium for differences in characteristics. The differences are evaluated with the implicit prices $\boldsymbol{\beta}$. If we assume that the differences remain constant over time or that the characteristics switch to the characteristics of the reference object immediately after the sale we obtain with (5)

$$\mathrm{E}_t[\Delta d_{n,t+j}] = \mathrm{E}_t[\Delta d_{t+j}^0] \quad \text{for} \quad j > 0 \; . \tag{6}$$

The assumption that the differences in characteristics remain constant is problematic if the reference object does not age. In that case, the valuation coefficient for the age appears as a constant in (6). An example for a switch in characteristics is easily given: the house is vacant at the date of sale but the reference house is an occupied one. However, immediately after the sale the new owner will move in.

We derive now for (2) with (5) and (6)

$$p_{n,t} = \kappa + p_t^0 - \sum_{j=0}^{\infty} \rho^j \mathrm{E}_t[r_{t+1+j}] + \mathbf{x}_{n,t}^T \boldsymbol{\beta} + \varepsilon_{n,t} \tag{7}$$

where κ absorbs all constants. Here,

$$p_t^0 \equiv d_t^0 + \sum_{j=0}^{\infty} \rho^j \mathrm{E}_t[\Delta d_{t+1+j}^0] \tag{8}$$

is up to a constant equal to the *fundamental value* of the reference house. This value would equal the price if the return rate deviation is zero. As such, it is just the sum of expected future rents, discounted at a constant rate. With a slight abuse of terminology, we will designate p_t^0 directly as fundamental value.

To make the model empirically applicable we need to find expressions for the unobservable conditional expectations in expression (8). We propose that the growth rate of the rent can be modelled with a VAR(1) that incorporates lagged growth rates of the rent and perhaps other variables (building activity, income development etc.). Let $\mathbf{v}_t$ contain at least the current and some lagged observations of the rent growth rate. Then we have

$$\mathbf{v}_{t+1} = \mathbf{c} + \mathbf{A}\mathbf{v}_t + \mathbf{u}_{t+1} \ , \tag{9}$$

where $\mathbf{c}$ and $\mathbf{A}$ contain unknown coefficients, and $\mathbf{u}_{t+1}$ is noise. The first element in $\mathbf{v}_t$ is the current observation of the rent growth rate. Thus we get with the unit vector $\mathbf{e}_1^T = [1 \quad 0 \quad \cdots \quad 0]$

$$\mathrm{E}_t\left[\Delta d_{t+1+j}^0\right] = \mathbf{e}_1^T \left(\sum_{i=0}^{j} \mathbf{A}^i \right) \mathbf{c} + \mathbf{e}_1^T \mathbf{A}^{j+1} \mathbf{v}_t \quad \text{for} \quad j \geqslant 0 \ . \tag{10}$$

If the roots of $\rho\mathbf{A}$ are inside the unit circle we obtain

$$\sum_{j=0}^{\infty} \rho^j \mathrm{E}_t\left[\Delta d_{t+1+j}^0\right] = \frac{1}{1-\rho} \mathbf{e}_1^T (\mathbf{I} - \rho\mathbf{A})^{-1} \mathbf{c} + \mathbf{e}_1^T \mathbf{A} (\mathbf{I} - \rho\mathbf{A})^{-1} \mathbf{v}_t \ . \tag{11}$$

The expression for the constant term is derived in Appendix A.1. To replace $\mathbf{A}$ and $\mathbf{c}$, we estimate (9) for the whole sample period. After that, we are able to calculate the value of the discounted expected rent growth rates.

Thus far we have not considered the pure benefit of being the *owner* of a house. We have only controlled for the the differences in characteristics with respect to the reference house. It can be argued that house-ownership generates "value" per se, because it gives the owner the right to model the object in accordance to her own taste. But ownership also means incurring costs like transaction or property taxes. Furthermore, if the house is rented out the owner has to expend maintenance cost. In addition to that, there exists a principal agent problem between lessor and lessee. The unobservable renter will handle the dwelling with less care than the owner. However, the lessor commands a remuneration for this adverse effect and this will increase the rent relative to the notional rent for owner occupied housing (Homburg 1993). If all those influences remain constant during our sample period, they are captured in the constant κ. But, if they change during the sample period—for example, because of changes in tax rates—we have to control for them explicitly. Furthermore, there might be unusual circumstances—e.g. personal relationship between buyer and seller, annuity payments—that influence the price. We will consider such changes and unusual circumstances explicitly through dummy variables in the vectors $\mathbf{x}_{n,t}$.

Finally, we must make assumptions about the behavior of the unobservable return rate r_t. One possible specification for the process of the return rate is

$$r_{t+1+j} = \phi r_{t+j} + (1-\phi)r^* + \mathbf{s}_{t+j}^T \boldsymbol{\gamma} + \nu_{t+1+j} \; . \tag{12}$$

The random component ν_t is white noise. The required return depends on its own lagged values and on the long run rate r^*. Furthermore, the return is influenced by shocks $\mathbf{s}_t$ of some financial indicators. These indicators are spreads between mortgage and interest rates, the inflation rate, changes in interest or tax rates, and returns of stock indexes. We assume that these shocks are incorporated immediately into the return rate and thus $\mathrm{E}_t[\mathbf{s}_{t+j}] =$

$\mathbf{0}$ for $j > 0$. We obtain after some manipulations

$$\mathrm{E}_t[r_{t+1+j}] = r^* + \phi^j \mathrm{E}_t[r_{t+1} - r^*] \quad \text{for} \quad j \geqslant 0\ . \tag{13}$$

It is easy to see that the long run required rate is equal to r^* for $|\phi| < 1$. If we substitute (13) into the present value (7), define $r^e_{t+1} \equiv \mathrm{E}_t[r_{t+1} - r^*]$ and assume $|\phi| < 1/\rho$ we get (where—once again—all constants are absorbed by κ)

$$p_{n,t} = \kappa + p^0_t - \frac{1}{1-\rho\phi} r^e_{t+1} + \mathbf{x}^T_{n,t}\boldsymbol{\beta} + \varepsilon_{n,t}\ . \tag{14}$$

The expected changes in the return rate, r^e, are unobservable. However, rewriting (12) in deviation form for $j = 0$, taking expectations at t and using $r_t - r^* = r^e_t + \nu_t$, we derive

$$r^e_{t+1} = \phi r^e_t + \mathbf{s}^T_t \boldsymbol{\gamma} + \tilde{\nu}_t \tag{15}$$

with $\tilde{\nu}_t \equiv \phi\nu_t$.

Let ψ denote $1/(1-\rho\phi)$ and multiply the above equation with this term, one obtains eventually

$$\Delta^0 p_{n,t} = \kappa - r^e_{\psi,t+1} + \mathbf{x}^T_{n,t}\boldsymbol{\beta} + \varepsilon_{n,t}\ . \tag{16a}$$

and

$$r^e_{\psi,t+1} = \phi r^e_{\psi,t} + \mathbf{s}^T_t \boldsymbol{\gamma}_\psi + \tilde{\nu}_{\psi,t} \tag{16b}$$

Here, $\Delta^0 p_{n,t}$ denotes $p_{n,t} - p^0_t$. The subscript in the return equation indicates the transformation. The first equation is easy to interpret: the deviation between the current price and the fundamental price for the reference house is a linear function of the characteristics of the object, and the cumulative effect of the current return rate deviation. The second equation shows that the cumulated return deviations are influenced by their previous value and the shocks in the financial indicators. Because $r^e_{\psi,t}$ is unobservable, we can not use OLS to estimate the price equation. However by writing down the system (16) as a state space model, we can apply the Kalman filter to estimate $r^e_{\psi,t}$.

3 State Space Form and Estimation Algorithm

The general state space form (SSF) is given as (with *s*tate and *m*easurement)

$$\boldsymbol{\alpha}_t = \mathbf{T}_t \boldsymbol{\alpha}_{t-1} + \boldsymbol{\varepsilon}_t^s \tag{17a}$$

$$\mathbf{y}_t = \mathbf{Z}_t \boldsymbol{\alpha}_t + \boldsymbol{\varepsilon}_t^m \ . \tag{17b}$$

with $\boldsymbol{\varepsilon}_t^s \sim \mathcal{N}(\mathbf{0}, \mathbf{R}_t)$ and $\boldsymbol{\varepsilon}_t^m \sim \mathcal{N}(\mathbf{0}, \mathbf{H}_t)$ (this notation mainly follows Harvey 1989). The disturbance vectors are distributed independently.

If the disturbance terms in (16) satisfy the above stated distributional assumptions our model is easily arranged into SSF. Let N_t denote the number of all houses sold at time t. There are K_β house characteristics, and K_γ short run influence variables. $K = K_\beta + K_\gamma + 1$ is the number of constant state variables and $S = K + 1$ is the number of all state variables. We obtain

$$\boldsymbol{\alpha}_t \equiv \begin{bmatrix} r^e_{\psi,t+1} \\ \boldsymbol{\gamma} \\ \kappa \\ \boldsymbol{\beta} \end{bmatrix} \ , \quad \mathbf{T}_t \equiv \left[\begin{array}{c:c:c} \phi & \mathbf{s}_t^T & \mathbf{0}_{1\times(K_\beta+1)} \\ \hdashline \mathbf{0}_{K\times 1} & \multicolumn{2}{c}{\mathbf{I}_K} \end{array}\right] \ , \quad \boldsymbol{\varepsilon}_t^s \equiv \begin{bmatrix} \tilde{\nu}_{\psi,t} \\ \mathbf{0}_{K\times 1} \end{bmatrix} \tag{18a}$$

$$\mathbf{y}_t \equiv \begin{bmatrix} \Delta^0 p_{1,t} \\ \vdots \\ \Delta^0 p_{N_t,t} \end{bmatrix} \ , \quad \mathbf{Z}_t \equiv \begin{bmatrix} -\mathbf{i}_{N_t} & \mathbf{0}_{N_t\times K_\gamma} & \mathbf{i}_{N_t} & \mathbf{X}_t \end{bmatrix} \ , \quad \boldsymbol{\varepsilon}_t^m \equiv \begin{bmatrix} \varepsilon_{1,t} \\ \vdots \\ \varepsilon_{N_t,t} \end{bmatrix} \ . \tag{18b}$$

Thus, whereas the number of state variables per period is equal to S and fixed, the number of observations per period—i.e. N_t—varies.

We are primarily interested in calculating the unobserved state vectors $\boldsymbol{\alpha}_t$. They contain the cumulated discount rate deviations $r^e_{\psi,t+1}$, the coefficients of the financial indicators $\boldsymbol{\gamma}$, and the influences of the characteristics $\boldsymbol{\beta}$. If we knew all parameters of the SSF (17), we could use the Kalman smoother to figure out the state vectors. On the other hand if we knew $\boldsymbol{\alpha}_t$ the parameters could be readily estimated by maximum likelihood. In our model the variances of the disturbances and ϕ are unknown. To estimate these coefficients we use the EM algorithm (Dempster, Laird, and Rubin 1977) and one subsequent step of scoring.

There is a vast literature about both methods and about some efficient ways to combine both methods (cf. Engle and Watson 1983). Normally, one should start with the EM algorithm and iterate until the parameter estimates converge. After this is done, these estimates can be used as starting values for the scoring algorithm. This algorithm delivers as a by-product an estimate of the information matrix. Both algorithms start with the log-likelihood function of the state space form (17) and make extensive use of the Kalman filter and the Kalman smoother. The filter equations are given in Appendix A.2. However, we need some start values to initialize the estimation algorithm. We use OLS for this task. Furthermore, we carry out the necessary model selection for the rent equation (5) in the OLS framework. In the next Subsection we discussion the estimation algorithm for the SSF. Subsection 3.2 presents our model selection procedure.

3.1 The Estimation Algorithm for the SSF

To set up the log-likelihood we multiply the system of the state equations with the S dimensional unit vector $\mathbf{e}_1$. The log-likelihood is, up to a constant (cf. Wu, Pai, and Hosking 1996)

$$\begin{aligned}\ln L(\boldsymbol{\psi}) = &- \frac{1}{2}\ln|\boldsymbol{\Sigma}| - \frac{1}{2}\boldsymbol{\varepsilon}_0^T\boldsymbol{\Sigma}^{-1}\boldsymbol{\varepsilon}_0 \\ &- \frac{1}{2}\sum_{t=1}^{T}\ln|\tilde{\mathbf{R}}_t| - \frac{1}{2}\sum_{t=1}^{T}\tilde{\boldsymbol{\varepsilon}}_t^{sT}\tilde{\mathbf{R}}_t^{-1}\tilde{\boldsymbol{\varepsilon}}_t^{s} \\ &- \frac{1}{2}\sum_{t=1}^{T}\ln|\mathbf{H}_t| - \frac{1}{2}\sum_{t=1}^{T}\boldsymbol{\varepsilon}_t^{mT}\mathbf{H}_t^{-1}\boldsymbol{\varepsilon}_t^{m}\end{aligned} \tag{19}$$

with $\boldsymbol{\varepsilon}_0 = \boldsymbol{\alpha}_0 - \boldsymbol{\mu}$, $\tilde{\mathbf{R}}_t \equiv \mathbf{e}_1^T\mathbf{R}_t\mathbf{e}_1$, $\tilde{\boldsymbol{\varepsilon}}_t^s = \mathbf{e}_1^T(\boldsymbol{\alpha}_t - \mathbf{T}_t\boldsymbol{\alpha}_{t-1})$ and $\boldsymbol{\varepsilon}_t^m = \mathbf{y}_t - \mathbf{Z}_t\boldsymbol{\alpha}_t$. However, we do not observe the state vectors. The idea of the EM algorithm is to maximize instead the expected value of the log-likelihood function. To derive the expected value of (19), let us define for $t \leqslant T$

$$\mathbf{a}_{t|T} \equiv \mathrm{E}_T[\boldsymbol{\alpha}_t] \tag{20a}$$

$$\mathbf{P}_{t|T} \equiv \mathrm{E}_T[(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})^T] \tag{20b}$$

$$\mathbf{P}_{t,t-1|T} \equiv \mathrm{E}_T[(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})(\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t-1|T})^T]\ . \tag{20c}$$

Furthermore we rewrite

$$\boldsymbol{\varepsilon}_0 = (\boldsymbol{\alpha}_0 - \mathbf{a}_{0|T}) + (\mathbf{a}_{0|T} - \boldsymbol{\mu})\ ,$$

$$\tilde{\varepsilon}_t^s = \mathbf{e}_1^T\left((\boldsymbol{\alpha}_t - \mathbf{a}_{t|T}) - \mathbf{T}_t(\boldsymbol{\alpha}_{t-1} - \mathbf{a}_{t-1|T}) + (\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T})\right)$$

and

$$\boldsymbol{\varepsilon}_t^m = (\mathbf{y}_t - \mathbf{Z}_t\mathbf{a}_{t|T}) + \mathbf{Z}_t(\boldsymbol{\alpha}_t - \mathbf{a}_{t|T})\ .$$

We have for our model $\mathbf{H}_t = \sigma_\varepsilon^2\mathbf{I}_{N_t}$ and $\tilde{\mathbf{R}}_t = \sigma^2_{\tilde{\nu}_\psi}$. The assumption of uncorrelated errors in the discount rate and the price equation allows identification of the two variances (see Schwann 1998). After all, we obtain for (19) with $\mathrm{E}[\boldsymbol{\varepsilon}^T\boldsymbol{\Omega}^{-1}\boldsymbol{\varepsilon}] = \mathrm{tr}\{\boldsymbol{\Omega}^{-1}\mathrm{E}[\boldsymbol{\varepsilon}\boldsymbol{\varepsilon}^T]\}$

$$\begin{aligned}\mathrm{E}_T[\ln L(\boldsymbol{\psi})] = &-\frac{1}{2}\ln|\boldsymbol{\Sigma}| - \frac{1}{2}\mathrm{tr}\big\{\boldsymbol{\Sigma}^{-1}(\mathbf{P}_{0|T} + (\mathbf{a}_{0|T} - \boldsymbol{\mu})(\mathbf{a}_{0|T} - \boldsymbol{\mu})^T)\big\}\\ &-\frac{T}{2}\ln\sigma^2_{\tilde{\nu}} - \frac{1}{2\sigma^2_{\tilde{\nu}}}\sum_{t=1}^{T}\mathbf{e}_1^T\mathbf{S}_t\mathbf{e}_1 - \frac{1}{2}\ln\sigma^2_\varepsilon\sum_{t=1}^{T}N_t\\ &-\frac{1}{2\sigma^2_\varepsilon}\sum_{t=1}^{T}\mathrm{tr}\{\mathbf{M}_t\}\end{aligned} \tag{21}$$

where

$$\begin{aligned}\mathbf{S}_t \equiv \mathrm{E}_T[\boldsymbol{\varepsilon}_t^s\boldsymbol{\varepsilon}_t^{sT}] = &\ \mathbf{P}_{t|T} - \mathbf{P}_{t,t-1|T}\mathbf{T}_t^T - \mathbf{T}_t\mathbf{P}_{t,t-1|T} + \mathbf{T}_t\mathbf{P}_{t-1|T}\mathbf{T}_t^T\\ &+ (\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T})(\mathbf{a}_{t|T} - \mathbf{T}_t\mathbf{a}_{t-1|T})^T\end{aligned}$$

and

$$\mathbf{M}_t \equiv \mathrm{E}_T[\boldsymbol{\varepsilon}_t^m\boldsymbol{\varepsilon}_t^{mT}] = \mathbf{Z}_t\mathbf{P}_{t|T}\mathbf{Z}_t^T + (\mathbf{y}_t - \mathbf{Z}_t\mathbf{a}_{t|T})(\mathbf{y}_t - \mathbf{Z}_t\mathbf{a}_{t|T})^T\ .$$

Due to the fact that the number of houses sold per period varies through time the filter procedure has to handle missing values. Generally, the Kalman filter is well suited for handling missing observations (e.g. Harvey 1989, p. 144). One can either replace the missing observations with zeros and adjust the covariance matrix accordingly (see Shumway and Stoffer 2000, 4.4) or one can cancel out the missing observations from all matrices (Koopman, Shephard, and Doornik 1999). It is possible to show that both methods deliver equivalent results. We use the second method in our algorithms.

The unknown parameters—collected in $\boldsymbol{\psi}$—are $\boldsymbol{\mu}$, vech$\boldsymbol{\Sigma}$, ϕ, σ_ε^2 and $\sigma_{\tilde{\nu}_\psi}^2$. We have to choose these parameters in such a manner that the value of the expected log-likelihood is maximized. It is easy to see that $\hat{\boldsymbol{\mu}} = \mathbf{a}_{0|T}$ and that there is no way to derive an optimal choice vech$\hat{\boldsymbol{\Sigma}}$. So we use the covariance matrix derived for the OLS estimates. For the other unknown coefficients we obtain with the help of the first order conditions

$$\hat{\sigma}_{\tilde{\nu}_\psi}^2 = \frac{1}{T}\sum_{t=1}^{T} \mathbf{e}_1^T \mathbf{S}_t \mathbf{e}_1 \tag{22a}$$

$$\hat{\sigma}_\varepsilon^2 = \frac{1}{\sum_{t=1}^T N_t}\sum_{t=1}^{T} \operatorname{tr}\{\mathbf{M}_t\} \tag{22b}$$

$$\hat{\phi} = \frac{\sum_{t=1}^T \mathbf{e}_1^T(\mathbf{P}_{t,t-1|T} + \mathbf{a}_{t|T}\mathbf{a}_{t|T}^T - \mathbf{T}_{t,-\phi}(\mathbf{P}_{t-1|T} + \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T))\mathbf{e}_1}{\sum_{t=1}^T \mathbf{e}_1^T(\mathbf{P}_{t-1|T} + \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T)\mathbf{e}_1}\ , \tag{22c}$$

where $\mathbf{T}_{t,-\phi}$ is $\mathbf{T}_t$, but ϕ is replaced by a zero. The derivation of the last expression is given in Appendix A.3. The EM algorithm consists of the following iterative procedure: start with some reasonable values for the unknown coefficients (see Subsection 3.2), evaluate the matrices in the expected log-likelihood function with the Kalman smoother, and estimate the unknown coefficients. Use these estimates for a new evaluation of the expected log-likelihood and so on. Our algorithm stops if the relative change of the log-likelihood is below some prescribed convergence level. As Harvey (1989, p. 126) shows, it is possible to rewrite the log-likelihood (19) function in the *prediction error decomposition* form

$$\ln L(\boldsymbol{\psi}) = -\frac{1}{2}\sum_{t=1}^{T} \ln|\mathbf{F}_t| - \frac{1}{2}\sum_{t=1}^{T} \mathbf{v}_t^T \mathbf{F}_t^{-1} \mathbf{v}_t \tag{23}$$

with $\mathbf{v}_t \equiv \mathbf{y}_t - \mathbf{Z}_t \mathbf{a}_{t|t-1}$. The matrix $\mathbf{F}_t$ is a by-product of the Kalman filter. In the above log-likelihood function we have omitted the expression for $t = 0$ and a constant term that depends solely on the number of observations. The EM algorithm guarantees that the value of the likelihood increases for every iteration. However, it is a drawback of the algorithm that it does not deliver an estimate of the information matrix. This matrix is necessary to calculate standard errors for the estimated coefficients. Thus, we complete the estimation procedure with a final scoring step for (23) evaluated at the estimates of the EM algorithm. As Engle and Watson (1981) have shown, the elements of the information matrices are given by (with $i, j = 1, \ldots 3$)

$$\boldsymbol{\mathcal{I}}_{ij} = \sum_{t=1}^{T} \left(\frac{1}{2} \operatorname{tr} \left\{ \mathbf{F}_t^{-1} \frac{\partial \mathbf{F}_t}{\partial \psi_i} \mathbf{F}_t^{-1} \frac{\partial \mathbf{F}_t}{\partial \psi_j} \right\} + \left(\frac{\partial \mathbf{v}_t}{\partial \psi_i} \right)^T \mathbf{F}_t^{-1} \frac{\partial \mathbf{v}_t}{\partial \psi_j} \right) . \tag{24}$$

The derivatives are evaluated numerically with *forward differences* in the following way (see Fletcher 1987, p.23): run the filter with the estimated coefficients of the EM algorithm. Then rerun the filter three times, where in every pass one of the coefficients is perturbed slightly. We label such a pass for coefficient i with the superscript (i). For example, assume that the change of every coefficient is given by $\delta\%$ (that is, $\psi_i^{(i)} = (1 + \delta)\psi_i$). Then one obtains

$$\frac{\partial \mathbf{F}_t}{\partial \psi_i} \approx (\delta \psi_i)^{-1} \left(\mathbf{F}_t^{(i)} - \mathbf{F}_t \right) \quad \text{and} \quad \frac{\partial \mathbf{v}_t}{\partial \psi_i} \approx (\delta \psi_i)^{-1} \left(\mathbf{v}_t^{(i)} - \mathbf{v}_t \right) . \tag{25}$$

3.2 Model Selection and Initial Values

Economic theory does not suggest a particular functional form for the dependency of the rent on the explanatory characteristics of the respective house. Most variables in (5) are dummies representing various qualitative characteristics of the houses such as their location or the presence of a swimming pool. These discrete explanatory variables naturally enter the model in a linear way. For the continuous variables the following *Box-Cox type* transformations are considered

$$T_\lambda(x) = \begin{cases} \lambda^{-1} \left\{ \left(s^{-1}(x^\lambda + a_\lambda) \right)^\lambda - 1 \right\} & \text{for } \lambda \in \Lambda, \\ \ln\{s^{-1}(x + a_0)\} & \text{for } \lambda = 0 \end{cases} \tag{26}$$

with $\Lambda = \{-2, -1, -0.5, 0.5, 1, 2\}$. Here x denotes any of the continuous explanatory variables, a_λ is a constant depending on λ, s is the sample standard deviation of variable x and λ is the parameter that determines the transformation. A particular value of λ implies a value of the constant a_λ. These constants are computed according to the suggestions made in Bunke, Droge, and Polzehl (1999) and aim to make, for any given λ, the transformation as nonlinear as possible.

If we rewrite the price equation (16a) with $I_{t+1} \equiv \kappa - r^e_{\psi,t+1}$ we obtain

$$\Delta^0 p_{n,t} = I_{t+1} + \mathbf{x}_{it}^T \boldsymbol{\beta} + \varepsilon_{it} \; . \tag{27}$$

We choose λ_j for each of the J variables simultaneously by the following cross-validation criterion

$$\boldsymbol{\lambda}^* = \arg\min_{\boldsymbol{\lambda}} \sum_{t=1}^{T} \sum_{n=1}^{N_t} \left(\Delta^0 p_{n,t} - \widehat{\Delta^0 p_{-n,t}}(\boldsymbol{\lambda}) \right)^2 , \tag{28}$$

where $\boldsymbol{\lambda}$ is the vector comprised of the λ_j for the different variables. Here, $\widehat{\Delta^0 p_{-n,t}}(\boldsymbol{\lambda})$ denotes the predicted value of $\Delta^0 p_{n,t}$ from an OLS fit of regression (27) using the transformations of the continuous explanatory variables according to the value of $\boldsymbol{\lambda}$ under consideration but omitting the observation indexed (n, t) from the regression fit. By omitting an observation from the regression used for predicting that very observation the cross validated choice of $\boldsymbol{\lambda}^*$ is optimal in the sense of minimizing an estimate of the expected squared prediction error (see Bunke, Sommerfeld, and Stehle 1997, Bunke 1998). Given the best transformations, we can estimate the series for I_t, $\boldsymbol{\beta}$, σ^2_ε with OLS and use them and their covariances for the initialization of our estimation algorithm. Furthermore, we can regress $\hat{I}_{t+1}$ on own lagged realizations and other financial indicators to derive start values of the unknown coefficients of the discount rate equation.

4 Data and Estimation

The data sets are provided by the *Gutachterausschuß für Grundstückswerte in Berlin*. This commission collects information on all real estate transactions in Berlin. The main data set contains about 22000 observations of all transactions of single-family houses in Berlin between January 1980 and December 1999. Besides the price, we observe about 100 characteristics of each house such as the size of the lot, floor space, age of the house, location, availability and numerous qualitative variables indicating specific conditions of the house, the neighborhood or the transaction (e.g. transaction between relatives). We also have data for 5065 sales of apartment houses for the years 1980 to 2000. For every sold object we know the price and the yearly rent of the object. We use these data to calculate a proxy for the discount factor ρ that is used in the approximation (2) of the present value.

Before we characterize the sample we use for estimation, we want to give a brief description about Berlin's market for real estate. According to the figures of Berlin's bureau of statistics (*Statistisches Landesamt Berlin*, StaLa), Berlin had 1.82 million dwellings in 1998. Here, dwellings comprise apartments, single family houses—detached, semi-detached, and row houses—, and condominiums. 11.04% of all non-vacant dwellings were privately owned. The ratio between the floor space of the privately owned dwellings and rented dwellings was 1.55, where the average floor space for a rented apartment was 66.6 square meters in 1998. About 71% of all privately owned dwellings were condominiums (Statistisches Landesamt Berlin 1999).

For the estimation of our model, we take the observations of the four South-West districts *Zehlendorf*, *Wilmersdorf*, *Steglitz*, and *Charlottenburg*. These districts cover 19% of Berlin's area. In 1998 they accounted for 17% of Berlin's total population of about 3.4 million. The ratio of inhabitants to area lies a little bit above the average for all districts, but is much lower than the ratio for the inner city districts. 20% of all Berlin dwellings lay in the four South-West districts. 15% of the dwellings there—that is an absolute number of 48 600 dwellings—were privately owned. The average floor space in 1998 was about 81 square meters and was thus 15% higher than the average for the whole city. The unemployment rate in these districts is lower than the average for the whole city. All four districts are of high-quality and relatively homogeneous. Especially Wilmersdorf (Grunewald) and Zehlendorf (Wannsee) have very nice sections with forests and lakes. It is quite reasonable that houses in these districts share the same *market risk*, so that yields of house ownership will be discounted by the same rate.

We measure the rent of the reference house, d_t^0, by the monthly rent subaggregate of the consumer price index for Berlin, provided by the StaLa. However, the construction principle of this index changed slightly in the year 1995. All values of this index before 1995 are calculated for four person households with middle income living in the Western part of Berlin. Thereafter, the values are calculated for all households. We assume, that this change does not influences the rent index as a measure of the opportunity cost.

Furthermore, in our model some information is not specific to the house but rather describe the opportunities of the investor. We have collected information about tax rates and government housing programs during the relevant time period. As financial indicators we have different monthly mortgages rates (with varying degrees of interest rate fixedness), the range of these rates offered by different banks, the monthly consumer price index for Berlin West, monthly interest rates given by returns on bonds, the return of the DAX stock index (a performance index) and the return of the CDAX stock index (a price index). The different mortgage rates and the ranges are available only since June 1982. Before that date, the *Deutsche Bundesbank* has calculated merely an average mortgage rate. Because we want to include the subdivided rates and also some lags, we let our sample begin in August 1982. After that, our sample contains 4410 observations for the four South-West districts and covers 209 months. There are at least 6 observations per month, at most 43 observations, and on average 21 observations. The median price for the whole period is 600000.- Deutsche Mark and the average price is about 757163.- Deutsche Mark.

4.1 The Fundamental Value

To calculate the time series of the fundamental value that is defined in (8), we need an estimation of the right-hand-side of (11). To estimate this expression, we take the following steps: First, take the logarithm of the rent index and calculate the first differences in the transformed variables. The new variable Δd_t^0 approximates the growth rate of the rent index. After that, we fit the following regression

$$\Delta d_t^0 = \delta_0 + \delta_1 \Delta d_{t-1}^0 + \delta_2 \Delta d_{t-12}^0 + u_t \tag{29}$$

to the data. The results are given in Table 1. The Q-test shows that the

Table 1: *Regression results for the process of the rent index*

	Coefficient	t-Statistic	Prob.
δ_0	0.0012	3.0857	0.0023
δ_1	0.1448	2.5993	0.0099
δ_2	0.5056	8.9812	0.0000
	Regression Diagnostics		
R^2	0.3005	mean of Δd^0	0.0038
$\overline{R}^2$	0.2946	F-statistic	50.6940
DW	2.0231	Prob(F-statistic)	0.0000

Note: data are 251 monthly observations for the growth rate of the rent index from 1979:2 to 1999:12. Due to the lags, the estimated series starts at 1980:2. DW is the Durbin Watson statistic.

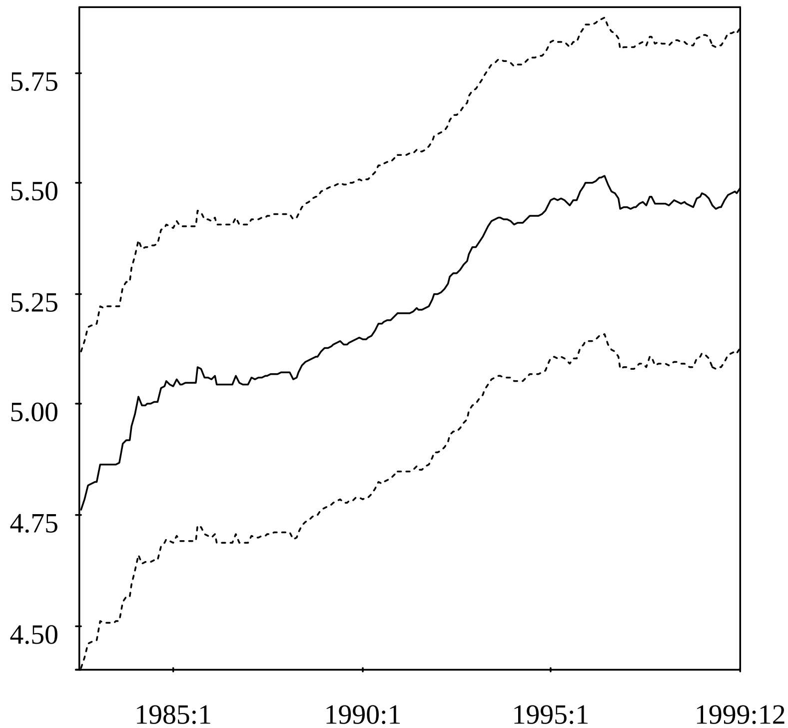
simple and the squared residuals are uncorrelated. We can rewrite (29) as

$$\mathbf{v}_t = \mathbf{c} + \mathbf{A}\mathbf{v}_{t-1} + \mathbf{u}_t \tag{30}$$

where the (13×1) vector $\mathbf{v}_t$ contains the observations of Δd_t^0 from t to $t-12$. For $\mathbf{c}$ we have $c_1 = \delta_0$ and all other elements are zero. Furthermore, $a_{1,1} = \delta_1$, $a_{1,13} = \delta_2$, $a_{j,j-1} = 1$ for $j = \{2, .., 13\}$ and all other elements are zero. Finally, the first element in $\mathbf{u}_t$ is the noise term u_t and all other elements are zero. The matrix $\mathbf{A}$ has 13 distinct eigenvalues which all have modulus less than 1.

To calculate ρ, we use our data set on apartment houses. We have information on the rent receipts for the different houses. However, we have to adjust these receipts in several ways to get the net rent payments that accrues to the owner of the house. We calculate that about 35% of the gross rents are maintenance costs. Furthermore, we check the sensitivity of ρ with respect to different figures of administration costs. We calculate the monthly

Figure 1: *Fundamental value*. Plot of the fundamental value of the reference house, $\hat{p}^0_t$, from 1982:7 to 1999:12. The series is calculated according to the fundamental value relationship given in equation (8). Confidence intervals are calculated with the delta method (see for example Greene 2000, p.357).



inverse capitalization rate θ with different relative administration costs that range from 0% up to 12.5%. According to these figures, the inverse capitalization rate lies between 0.39% and 0.44%. If we round up to the third digit we obtain for any of these values

$$\hat{\rho} = 0.996 \ . \tag{31}$$

With this result at hand, we can calculate the fundamental value p^0_t with d^0_t and the relationship given in equation (11). The series from 1982:7 to 1999:12 is plotted in Figure 1. We see immediately that the value soars in the first years of the Eighties and in the first half of the Nineties. It reaches its peak in 1995. After then, the value remains on at a relatively constant level. Starting in 1985 the value resembles roughly the shape of the yearly single-family house price index of the *Ring Deutscher Makler* (RDM). The

RDM is an association of German realtors and valuers that conducts every year an inquiry of its members about the situation of the real estate market. The index can be used as an rough market indicator. For the first years of the Eighties, this index shows a behavior that is different from the fundamental value. Whereas this index remains on a plateau, the fundamental value tightens during that period. However, this behavior of the fundamental value is in accordance with the RDM rent index, which was increasing during that period.

4.2 Model Selection for the Data

The size of the lot, the size of the floor space and the age of the building are the continuous variables in our model selection procedure. $\boldsymbol{\lambda}^*$ chosen for our data consists of $\lambda_1 = 1$ (size of the lot), $\lambda_2 = -2$ (size of the floor space), and $\lambda_3 = -0.5$ (age of the building). The value of the cross-validation criterion in (28) for these transformations is 0.7583. Furthermore, we obtain for our regression—where we have included an overall constant— a degree of determination of $R^2 = 0.7852$ and an adjusted degree of $\bar{R}^2 = 0.7736$.

The final model contains, in addition to the three continuous characteristics, sixteen characteristics. The additional characteristics are: dummies for *detached house* and *row house* (excluded category is semi-detached house), dummies for *Wilmersdorf*, *Zehlendorf*, and *Steglitz* (Charlottenburg is excluded), dummies for houses in *good condition* or in *bad condition* (excluded category is normal condition), a dummy for *noise* in the environs of the house (e.g. the object lies in the air lane or near a railway track), a dummy for a *indoor pool*, a dummy for houses with valuable *inventory* (e.g. built-in kitchen, furniture, sauna), a dummy if the house is *vacant* and not occupied by the seller, a dummy for houses still *under construction* at the date of the sale, a dummy if the object is *rented out* (and thus, the buyer is an investor who wants to accrue rent payments), a dummy if the house is *purchased by former tenant*, a dummy if *personal circumstances* exist (e.g. sale between relatives or a divorced couple), and eventually a dummy if the transaction shows *unusual*—legal or financial—*circumstances* (e.g. payment by installments, right of residence for the former owner).

We want shortly explain the different tax and assistant dummies that we have incorporated in the selection procedure. Before 1987, the notional rent of owner-occupied housing was taxed—just like ownership of rented objects—through the income tax. On the other hand, it was possible to deduct depreciation cost from the tax bill. In 1987, the taxation of the notional rent for owner-occupied housing was repealed. However, the deduction possibilities were modified only slightly. To catch up possible effects of this change in taxation we have generated a dummy for all owner-occupied houses that are sold before 1987. But the estimated coefficient is not significant at the 5% level. A plausible explanation is that the value of the notional rent was a (low) flat sum and that the owner had many possibilities to decrease his tax bill. So, in most cases the net effect was zero or even positive and the repealing of the tax in 1987—combined with the slight modification in the deduction possibilities—had no positive effect at all on the present value of a owner-occupied house.

In 1993, the maximal amount of purchase cost that is deductible from the income tax was halved for objects that were older than 3 years. We have captured this effect with a dummy for all objects that were sold after 1992 and were older than three years at the date of the purchase. The coefficient for this dummy is also insignificant. One possible explanation is that the overall effect is only marginal or is not identifiable because most sellers had not the right to claim for the deduction (because their income was too high or because they have already claimed the deduction in former years).

In 1996, the whole system to promote owner-occupied housing was changed. Instead of assisting through deduction possibilities, the law *Eigenheimzulagengesetz* introduced direct allowance for owner-occupied houses. However, it was the intention of that law to continue the pre-existing rules. We have generated a dummy for all owner-occupied houses that were sold after 1995. The coefficient for the dummy is insignificant. That shows that the new law really continues the old arrangements.

Eventually, the rate of the sales tax—*Grunderwerbssteuer*—was increased in 1997 from 2% to 3.5%. Due to the fact that sales between direct relatives and couples are exempted from this tax, we have generated a dummy for this change in taxation. But even for this dummy, the respective coefficient is insignificant. A possible explanation is that we have used the variable personal circumstances as indicator for sales between relatives and couples. This variable contains also sales between companies and employees and we are unable to distangle such sales.

Furthermore, there are some other taxes that influence the net rent of a house. An example is the *Grundsteuer*—real estate tax—that is collected by the Federal State. However, there were no large changes of this tax and there are only few exemptions from this tax, so that we neglect it.

Perhaps, there is also a generalized explanation for the failure to identify effects of taxes and subsidies. The amount of assistance depends on the specific characteristics of the household (for example the number of kids) and not on the house per se. It is impossible to identify any effect without detailed information on sellers and buyers.

To select the financial indicators, we run a regression of the estimated coefficients of the time dummies from (27). Let $\hat{I}_t$ denote the estimated coefficient multiplied with minus one (recall that $I_t \equiv \kappa - r^e_{\psi,t}$). Then we fit

$$\hat{I}_{t+1} = c + \phi \hat{I}_t + \mathbf{s}_t^T \boldsymbol{\gamma} + \nu_t \tag{32}$$

and select the significant financial indicators. The vector of the financial indicators contains lagged values of the inflation rate for Berlin, the return of the DAX, the range of mortgage rates from different banks, and spreads between mortgage and interest rates with different interest rate fixedness and—respectively—maturities (for a study, which explores the behavior of the spreads in detail, see Nautz and Wolters 1996). For the selected model the p-value of the F-test is 0.000 and $R^2 = 0.7835$. The estimated value of ϕ is 0.788 and that of σ^2_ν is 0.0048. The only indicators with significant coefficients at the 5% level are the spread with a fixedness of two years and the range with interest rate fixedness of five years. We have tested both series for a unit root with the augmented Dickey-Fuller test. For the test, we have included a constant and the one period lag. We can reject the hypothesis of a unit root for the spread at the 10% level. The test statistic is -2.81 and thus very close to critical value for the 5% level, -2.87. For the range, we can reject the unit root hypothesis at the 5% level.

4.3 Results from the Estimation Procedure

We use the selected transformed variables, the two financial indicators and the estimated coefficients to initialize the EM algorithm. After each iteration, the value of the log-likelihood function in the prediction error decomposition form (23) is calculated. The results are given in Table 2. If we compare these

Table 2: *Estimation output for the coefficients in the system matrices*

	Coefficient	t-Statistic	Prob.
ϕ	0.9408	35.526	0.0000
$\sigma^2_{\tilde{\nu}}$	0.0002	0.61131	0.3309
σ^2_{ε}	0.0560	45.985	0.0000

Note: convergence of the EM algorithm is reached after 5 iterations. Results are calculated with a final scoring step. The value of the log-likelihood is 1.5% higher compared with the value evaluated at the OLS estimates.

result with the OLS estimates we see immediately that the AR-coefficient of the return equation has increased substantially. On the other hand, the variance of the expected return deviations is not different from zero. It seems as if the AR-coefficient has soared much of the variance. The estimated variance of the price equation is almost unchanged. This is also true for the other coefficients of the price equation that are reported in Table 3.

Table 3: *Hedonic coefficients of the price equation*

	Coefficient	t-Statistic	Prob.
T_1(lot size)	0.1770	51.0822	0.0000
T_{-2}(floor space)	26.1231	403.6657	0.0000
$T_{-0.5}$(age)	-0.0423	-17.2832	0.0000
row house	-0.0333	-4.5329	0.0000
detached house	0.0772	10.5098	0.0000
Wilmersdorf	0.2563	17.0500	0.0000

—*continued*—

Table 3: *Hedonic coefficients, continued*

	Coefficient	t-Statistic	Prob.
Zehlendorf	0.0883	7.3835	0.0000
Steglitz	-0.0969	-8.0506	0.0000
good condition	0.1159	19.8721	0.0000
bad condition	-0.2264	-9.4732	0.0000
noise	-0.2582	-4.5287	0.0000
indoor pool	0.1040	5.1511	0.0000
inventory	0.0831	5.9382	0.0000
vacant	0.0955	7.6472	0.0000
under construction	-0.2163	-6.6520	0.0000
rented out	-0.1466	-6.3490	0.0000
purchased by former tenant	-0.0995	-5.8745	0.0000
personal circumstances	-0.1609	-12.8832	0.0000
unusual circumstances	-0.1595	-7.7570	0.0000
κ	-3.5348	-157.6076	0.0000

Note: estimated coefficients of the price equation (16a). The variables are explained in Subsection 4.2.

Eventually, Table 4 reports the estimated coefficients from the return equation. Compared with the results for the OLS regression of the return equation, the signs of the coefficients stay the same, but they decrease in magnitude. We will give economic interpretation for all coefficients in the next section.

Table 4: *Estimated coefficients of the return equation*

	Coefficient	t-Statistic	Prob.
spread2	2.0376	7.0883	0.0000
range5	-0.0238	-3.1840	0.0015

Note: estimated coefficients of the return equation (16b). spread2 is the difference between the mortgage rate with rate fixedness of two years and the interest rate with same maturity; range5 is the range of mortgage rates with interest rate fixedness of five years offered by different banks.

5 Interpretation of the Results

Starting with the hedonic coefficients $\hat{\boldsymbol{\beta}}$, given in Table 3, we find that the rent for a house increases both with the size of the lot and the size of the living area and decreases with the age of the dwelling. If we calculate the elasticities—evaluated at the sample mean of the respective variable—, we obtain values of about 0.29% for lot size (mean is 600 square meters), 0.6% for floor space (mean is 170 square meters), and -0.03% for age (mean is 42 years).

Since the dependent variable is the log ratio of price and fundamental value, the coefficients of a dummy variable is approximately the percentage premium for the respective characteristic. The rent of a house with otherwise the same characteristics decreases by 3.3% if it is a row house and increases by 7.7% if it is a detached house. Here, the excluded category is a semi-detached house. As such, people are willing to pay a premium for "privacy". They will also pay a premium if the house lies in the districts Wilmersdorf or Zehlendorf. As we have already mentioned, there are very nice parts in these districts. Especially Grunewald in Wilmersdorf is very attractive. The hedonic coefficient reveals that the premium is about 25%. On the other hand—compared with the reference district Charlottenburg—Steglitz charges a rebate of 9.7%.

If the house is in good condition, the rent increases by 11.6% compared with a house in normal condition. If the house is in bad condition, the rent decreases by 22.6%. The rent decreases by 25.8% if the house is located in a noisy environment in the vicinity of rail tracks, highways, or airports.

The rent increases by 10.4% if the object has an indoor pool. There is some information in the text files of our data set about the cost for constructing an indoor pool. The cost can go up to 100000.- Deutsche Mark. The average price for houses with indoor pool is 1.1 million Deutsche Mark, so that the hedonic coefficient is quite reasonable. If the house has inventory—in most cases in-built kitchen and some in-built furniture—the rent increases by 8.3%. This is reasonable because such equipment is a necessary part of a house. Calculated with the average price of about 778000.- Deutsche Mark for houses with inventory, the average value of the inventory is about 65000.-Deutsche Mark.

If the house is vacant, the rent increases by 9.5%. This is really a high number. Even the lower bound of the 0.95 confidence band is 7.1%. That is still a large premium for the instant availability of a house.

Next we turn to the dummies that can not be interpreted as part of the rent $D_{n,t}$. They describe special circumstances of the deal or the use of the house that are only relevant for house buyers—and not for tenants. The first of these is the fact that the house is still under construction when the deal is struck. The "risk premia" for buying an unfinished house is about 22% of the price for an otherwise identical object.

If the house is rented, the price decreases. In that case, the buyer is an investor who wants to accrue the rent payments. It is common practice of valuers to assume that the additional relative cost are about 2% for rent default risk, and 5% for administration cost. Furthermore, it is quite reasonable to assume that maintenance cost are higher for houses that are not owner-occupied. So, the figure of 14.7% is reasonable.

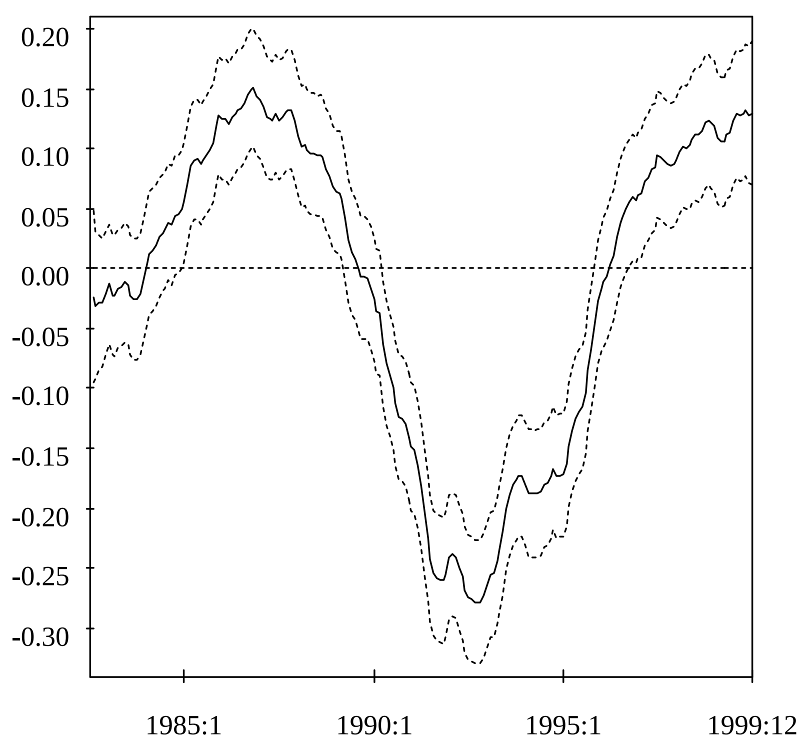
The purchase of the house by the former tenant decreases the price by about 10%. The seller has not to search for a buyer of the house and he is well-informed about the soundness of the buyer. This explains the rebate. On the other hand, the former tenant might have a special interest in buying the house because he had modelled it according to his own taste. This "inflexibility" might increase the price. Nevertheless, it is possible that the house is in poor condition or the taste of the tenant is a little bit idiosyncratic. Whereas others would command a large rebate for the condition of the house, the tenant does not care much about it and is as such the preferred buyer from the view of the owner.

If personal circumstances exist, the price decreases by 16%. This category comprises sales between relatives—especially between parents and their kids—, where bequest motifs might explain the rebate. In addition, it contains sales between divorced couples and partition of an estate, where there might not be enough time and patience for getting a good deal. However, it contains also sales between neighbors. Here, the buyer has a special interest that might increase the price. However, as already mentioned by discussing the rebate for the purchase of a former tenant, in such a case the seller has no search and information cost.

Finally, unusual circumstances with respect to the business dealing command a rebate of about 16%. This is reasonable for sales where the former owner has obtained the right of residence in some part of the house. It is not so obvious for deals where the payments are by installments. However, in most cases of payment by installment the buyer who has to repay with payments in kind—e.g. conceding the right of residence, sometimes combined with nursing care for the former owner.

The estimated series of the expected return deviations is plotted in Figure 2. The confidence intervals are calculated with the first element of the smoothed covariance matrix and suggest that the return deviation was zero

Figure 2: *Smoothed deviations of the expected return.* Plot of the estimated series $r^e_{\psi,t+1}$ from 1982:7 to 1999:12. Confidence intervals are calculated with the first element of the smoothed covariance matrix.



for the first years of our sample period. Beginning in the year 1985, the discount rate was increasing and the price—for the reference house—was lower than the corresponding fundamental value. This down-weighting process reached its peak in 1987. Thereafter, the prices—compared with the fundamental value—increased steadily. Starting in 1990, investor's confidence reached very high levels and prices increased substantially. There are at least three—complementary—explanations for this surge in confidence: the economic, currency and social union in July 1990, the German reunification in October 1990, and the decision of the German parliament in June 1991 for Berlin as the Capital of the unified country. If we compare this with the behavior of the fundamental value in Figure 1, we see that the rents reacted obviously slower to the new situation. In 1996, the average return was reached once again.

The plotted series gives the cumulated effect of a return deviation on the price of the object—see Equation (16b). To evaluate the one period return deviation, we calculate $\psi = 16$ and assume that r^* is about 0.8%. To motivate the last number, recall that in the Gordon growth model $\ln(1+R) = \ln(1+G) + \ln(1+\theta)$. For our sample period, the average monthly growth rate of the rent index is 0.38% (see Table 1). If we use the average monthly capitalization rate of 0.415%, we obtain $r^* \approx 0.8\%$. To guarantee plausible return deviations, we should have $r_t^e + r^* > 0$ and thus $r_{\psi,t}^e > -0.128$. However, even if we use the upper limit of the the confidence bands, the return deviations are below that critical value from April 1991 to August 1993. The minimum upper bound is -0.226 and thus the long run discount rate should be at least 0.14%—or 17% on a yearly basis—to guarantee that the discount rate is always positive. In this case, the inverse capitalization rate θ will be about 1%. We see that the confidence effect during the reunion "boom" was very high and our procedure has some problems to capture this fact.

Eventually, the return deviations are influenced by the spread of mortgage and interest rates with a interest rate fixedness of two years. This effect is positive and could be interpreted as a "risk premium". Here, we assume that banks "finance" the mortgages by deposits with the same maturity. In periods where the real estate market is riskier, the banks claim a higher interest premium. The discount rate reacts very sensible to a change in this implicit premium. One reason could be that banks have always the possibility to diversify their real estate risk—a chance, that the ordinary house owner has not. The range of the mortgage rates with interest rate fixedness of five years has a small negative effect on the discount rate. The range might be a measure for the consensus of the different banks about the current risk on the real estate market. If the range is high, the banks assess the risk differently, and if the range is small, they assess the risk similarly. Given this interpretation, the discount rate is corrected for the degree of consent.

6 Conclusion

We have used the present value relation to derive a model that explains the formation and movement of real estate prices with movements of the rent level, the characteristics of a house and some financial indicators. Our estimates reveal the implicit hedonic prices for the different characteristics. The values of the coefficients are plausible and in accordance with the assessment of professional valuers. Furthermore, we have seen that investors were overconfident after the German reunification and after the decision to make Berlin the capital of the reunified Germany. However, our model with a constant long run discount rate has problems to capture this speculative period. Perhaps we have to find some indicators that allow for catching up this effect.

In addition to that, there is another direction on which we could concentrate in the future: given the well-known pricing kernel for assets, where the discount factor is governed also by consumption or wealth, we should incorporate some income measures in our return equation. However, it is not easy to find such a measure on a monthly basis. Furthermore, we should try to find more—or better—financial indicators that influence the deviations of the discount rate. Future work will concentrate on such topics.

A Appendix

A.1 Constant Term

The constant in (11) is given as

$$\mathbf{e}_1^T \left(\sum_{j=0}^{\infty} \rho^j \sum_{i=0}^{j} \mathbf{A}^i \right) \mathbf{c} \; . \tag{33}$$

We obtain for the double sum in the brackets

$$\mathbf{I}(1+\rho+\rho^2+\ldots)+\rho\mathbf{A}(1+\rho+\rho^2+\ldots)+(\rho\mathbf{A})^2(1+\rho+\rho^2+\ldots)+\ldots \tag{34}$$

So, the constant is

$$\frac{1}{1-\rho}\mathbf{e}_1^T \left(\sum_{j=0}^{\infty} (\rho\mathbf{A})^j \right) \mathbf{c} \tag{35}$$

and thus equal to the constant term in (11).

A.2 Kalman Filter Recursions

Here we reproduce the calculation procedure of the Kalman filter and the Kalman smoother. For a derivation of the recursions see Harvey (1989).

A.2.1 Calculation Procedure for the Kalman Filter

Start at $t = 1$: using an initial guess for $\boldsymbol{\mu}$ and $\boldsymbol{\Sigma}$ to calculate

$$\mathbf{a}_{1|0} = \mathbf{T}_1\boldsymbol{\mu} \; , \quad \mathbf{P}_{1|0} = \mathbf{T}_1\boldsymbol{\Sigma}\mathbf{T}_1^T + \mathbf{R}_1 \; , \quad \mathbf{F}_1 = \mathbf{Z}_1\mathbf{P}_{1|0}\mathbf{Z}_1^T + \mathbf{H}_1 \tag{36a}$$

$$\mathbf{a}_1 = \mathbf{a}_{1|0} + \mathbf{P}_{1|0}\mathbf{Z}_1^T\mathbf{F}_1^{-1}(\mathbf{y}_1 - \mathbf{Z}_1\mathbf{a}_{1|0}) \tag{36b}$$

$$\mathbf{P}_1 = \mathbf{P}_{1|0} - \mathbf{P}_{1|0}\mathbf{Z}_1^T\mathbf{F}_1^{-1}\mathbf{Z}_1\mathbf{P}_{1|0} \tag{36c}$$

Step at $t \leqslant T$: calculate with $\mathbf{a}_{t-1}$ and $\mathbf{P}_{t-1}$

$$\mathbf{a}_{t|t-1} = \mathbf{T}_t\mathbf{a}_{t-1} \tag{37a}$$

$$\mathbf{P}_{t|t-1} = \mathbf{T}_t\mathbf{P}_{t-1}\mathbf{T}_t^T + \mathbf{R}_t \; , \quad \mathbf{F}_t = \mathbf{Z}_t\mathbf{P}_{t|t-1}\mathbf{Z}_t^T + \mathbf{H}_t \tag{37b}$$

$$\mathbf{a}_t = \mathbf{a}_{t|t-1} + \mathbf{P}_{t|t-1}\mathbf{Z}_t^T\mathbf{F}_t^{-1}(\mathbf{y}_t - \mathbf{Z}_t\mathbf{a}_{t|t-1}) \tag{37c}$$

$$\mathbf{P}_t = \mathbf{P}_{t|t-1} - \mathbf{P}_{t|t-1}\mathbf{Z}_t^T\mathbf{F}_t^{-1}\mathbf{Z}_t\mathbf{P}_{t|t-1} \tag{37d}$$

A.2.2 Calculation Procedure for the Kalman Smoother

To run the Kalman smoother, one needs $\mathbf{a}_t$, $\mathbf{P}_t$ and $\mathbf{P}_{t|t-1}$ for $t = 1 \ldots T$ from the previous procedure.

Start at $t = T$: $\mathbf{a}_{T|T} = \mathbf{a}_T$ and $\mathbf{P}_{T|T} = \mathbf{P}_T$

Step at $t \leqslant T-1$: calculate with $\mathbf{a}_{t+1|T}$ and $\mathbf{P}_{t+1|T}$

$$\mathbf{P}_t^* = \mathbf{P}_t\mathbf{T}_{t+1}^T\mathbf{P}_{t+1|t}^{-1} \tag{38a}$$

$$\mathbf{a}_{t|T} = \mathbf{a}_t + \mathbf{P}_t^*(\mathbf{a}_{t+1|T} - \mathbf{T}_{t+1}\mathbf{a}_t) \tag{38b}$$

$$\mathbf{P}_{t|T} = \mathbf{P}_t + \mathbf{P}_t^*(\mathbf{P}_{t+1|T} - \mathbf{P}_{t+1|t})\mathbf{P}_t^{*T} \tag{38c}$$

Some of the state variables in our model are constant by definition. We show in Appendix A.4 that the Kalman smoother delivers constant estimates for these variables for all t.

We need furthermore a smoothed series for $\mathbf{P}_{t,t-1|T}$. The recursions are (see Shumway and Stoffer 2000, Shumway and Stoffer 1982)

Start at $t = T$:

$$\mathbf{P}_{T,T-1|T} = (\mathbf{I} - \mathbf{P}_{T|T-1}\mathbf{Z}_T^T(\mathbf{Z}_T\mathbf{P}_{T|T-1}\mathbf{Z}_T^T + \mathbf{H}_T)^{-1}\mathbf{Z}_T)\mathbf{T}_T\mathbf{P}_{T-1} \tag{39}$$

Step at $t < T-1$: calculate

$$\mathbf{P}_{t,t-1|T} = (\mathbf{P}_t + \mathbf{P}_t^*(\mathbf{P}_{t+1,t|T} - \mathbf{T}_{t+1}\mathbf{P}_t))\mathbf{P}_{t-1}^{*T} \tag{40}$$

A.3 Matrix Differentiation

In this appendix, we differentiate $\mathbf{e}_1^T \mathbf{S}_t \mathbf{e}_1$ with respect to ϕ. We use some results of vector and matrix differentiation (cf. Lütkepohl 1996, p.208).

$\mathbf{S}_t$—see (21)—is equal to a sum of scalars. We obtain for the relevant scalars with $d\mathbf{T}_t/d\phi = d\mathbf{T}_t^T/d\phi = \mathbf{e}_1\mathbf{e}_1^T$

$$\frac{d\mathbf{e}_1^T \mathbf{T}_t \mathbf{P}_{t,t-1|T}\mathbf{e}_1}{d\phi} = \mathbf{e}_1^T \mathbf{P}_{t,t-1|T}\mathbf{e}_1 \ , \tag{41a}$$

$$\frac{d\mathbf{e}_1^T \mathbf{T}_t \mathbf{P}_{t-1|T}\mathbf{T}_t^T\mathbf{e}_1}{d\phi} = 2\mathbf{e}_1^T \mathbf{T}_t \mathbf{P}_{t-1|T}\mathbf{e}_1 \ , \tag{41b}$$

$$\frac{d\mathbf{e}_1^T \mathbf{T}_t \mathbf{a}_{t-1|T}\mathbf{a}_{t|T}^T\mathbf{e}_1}{d\phi} = \mathbf{e}_1^T \mathbf{a}_{t|T}\mathbf{a}_{t-1|T}^T\mathbf{e}_1 \ , \tag{41c}$$

$$\frac{d\mathbf{e}_1^T \mathbf{T}_t \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T\mathbf{T}_t^T\mathbf{e}_1}{d\phi} = 2\mathbf{e}_1^T \mathbf{T}_t \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T\mathbf{e}_1 \ . \tag{41d}$$

Thus, we obtain eventually

$$\frac{d\mathbf{e}_1^T \mathbf{S}_t \mathbf{e}_1}{d\phi} = 2\mathbf{e}_1^T(\mathbf{T}_t\mathbf{P}_{t-1|T} + \mathbf{T}_t\mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T - \mathbf{P}_{t,t-1|T} - \mathbf{a}_{t|T}\mathbf{a}_{t-1|T}^T)\mathbf{e}_1 \ . \tag{42}$$

Finally, we can rewrite the half of the right-hand-side of (42) with

$$\begin{aligned} \mathbf{e}_1^T\mathbf{T}_t &= \mathbf{e}_1^T(\phi\mathbf{e}_1\mathbf{e}_1^T + \mathbf{T}_{t,-\phi}) \\ &= \phi\mathbf{e}_1^T + \mathbf{e}_1^T\mathbf{T}_{t,-\phi} \end{aligned}$$

as

$$\begin{aligned} &\phi\mathbf{e}_1^T(\mathbf{P}_{t-1|T} + \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T)\mathbf{e}_1 + \mathbf{e}_1^T(\mathbf{T}_{t,-\phi}(\mathbf{P}_{t-1|T} + \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T) \\ &\qquad - \mathbf{P}_{t,t-1|T} - \mathbf{a}_{t|T}\mathbf{a}_{t-1|T}^T)\mathbf{e}_1 \end{aligned} \tag{43}$$

and use this for the derivation of the third equation in (22).

One can derive that the second-order cross partial derivatives of the expected log likelihood function are zero at the stationary point $(\hat{\sigma}^2_{\tilde{\nu}_\psi}, \hat{\sigma}^2_\varepsilon, \hat{\phi})$. One obtains furthermore with

$$\frac{d^2\mathbf{e}_1^T \mathbf{S}_t \mathbf{e}_1}{d\phi^2} = 2\mathbf{e}_1^T(\mathbf{P}_{t-1|T} + \mathbf{a}_{t-1|T}\mathbf{a}_{t-1|T}^T)\mathbf{e}_1 \geqslant 0 \tag{44}$$

so that the own partial derivatives are all negative. Thus, the values (22) fulfill also the second order condition for a local maximum.

A.4 Constant State Variables and the Smoother

We want to show that the Kalman smoother produces constant estimates through time for all state variables that are constant by definition. Firstly, we make the following partition of the transition matrix

$$\mathbf{T}_{t+1} = \begin{bmatrix} \mathbf{T}_{11,t+1} & \mathbf{T}_{12,t+1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} . \tag{45}$$

The matrix has the dimension $S \times S$ and the identity matrix has the dimsion $K \times K$ with $S \geqslant K$. Furthermore, we define with the same partition

$$\tilde{\mathbf{P}}_t \equiv \mathbf{T}_{t+1} \mathbf{P}_t \mathbf{T}_{t+1}^T = \begin{bmatrix} \tilde{\mathbf{P}}_{11,t} & \tilde{\mathbf{P}}_{12,t} \\ \tilde{\mathbf{P}}_{12,t} & \tilde{\mathbf{P}}_{22,t} \end{bmatrix} \tag{46}$$

and $\tilde{\tilde{\mathbf{P}}}_{11,t} \equiv \tilde{\mathbf{P}}_{11,t} + \tilde{\mathbf{R}}_{t+1}$, where $\tilde{\mathbf{R}}_{t+1}$ contains the variance of the state variables that are stochastic. Finally, $\mathbf{P}_{t+1|t}$ is equal to the matrix $\tilde{\mathbf{P}}_t$ with $\tilde{\tilde{\mathbf{P}}}_{11,t}$ instead of $\tilde{\mathbf{P}}_{11,t}$ (see (37b)).

Our goal is to show for (38a)

$$\mathbf{P}_t^* = \begin{bmatrix} \mathbf{M}_{11,t} & \mathbf{M}_{12,t} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} , \tag{47}$$

where both $\mathbf{M}$s stand for some complicated matrices. With this result at hand, we obtain immediately from (38b) for the K constant state variables

$$\mathbf{a}_{t|T}^K = \mathbf{a}_{t+1|T}^K = \mathbf{a}_T^K \tag{48}$$

for all t ($\mathbf{a}_{t|T}^K$ contains the last K elements of the smoothed predictor $\mathbf{a}_{t|T}$).

Now we derive (47): We assume that the inverse of $\mathbf{T}_{t+1}$ and $\mathbf{T}_{11,t+1}$ exist. Because a SSF of our model makes only sense if $\phi \neq 0$, we should assume that this condition is fulfilled. For the partitioned matrix (cf. Sydsæter, Strøm, and Berck 2000, 19.48) we derive

$$\mathbf{T}_{t+1}^{-1} = \begin{bmatrix} \mathbf{T}_{11,t+1}^{-1} & -\mathbf{T}_{11,t+1}^{-1} \mathbf{T}_{12,t+1} \\ \mathbf{0} & \mathbf{I} \end{bmatrix} . \tag{49}$$

Now, it is easy to see that

$$\mathbf{P}_t^* = \mathbf{T}_{t+1}^{-1} \tilde{\mathbf{P}}_t \mathbf{P}_{t+1|t}^{-1} . \tag{50}$$

We have (cf. Sydsæter, Strøm, and Berck 2000, 19.49)

$$\mathbf{P}_{t+1|t}^{-1} = \begin{bmatrix} \boldsymbol{\Delta}_t & -\boldsymbol{\Delta}_t \tilde{\mathbf{P}}_{12,t} \tilde{\mathbf{P}}_{22,t}^{-1} \\ -\tilde{\mathbf{P}}_{22,t}^{-1} \tilde{\mathbf{P}}_{12,t} \boldsymbol{\Delta}_t & \tilde{\mathbf{P}}_{22,t}^{-1} + \tilde{\mathbf{P}}_{22,t}^{-1} \tilde{\mathbf{P}}_{12,t} \boldsymbol{\Delta}_t \tilde{\mathbf{P}}_{12,t} \tilde{\mathbf{P}}_{22,t}^{-1} \end{bmatrix} \tag{51}$$

with $\boldsymbol{\Delta}_t$ as a known function of the partial matrices. If we multiply this matrix with the lower partition of $\tilde{\mathbf{P}}_t$ we obtain immediately $[\mathbf{0} \quad \mathbf{I}]$. With this result and (49) we derive (47).

Furthermore, it is possible to show with the same results that the lower right partition of $\mathbf{P}_{t|T}$ is equal to the lower right partition of $\mathbf{P}_T$ for all t. Just write with (38a)

$$\mathbf{P}_{t|T} = \mathbf{P}_t(\mathbf{I} - \mathbf{T}_{t+1}^T \mathbf{P}_t^{*T}) + \mathbf{P}_t^* \mathbf{P}_{t+1|T} \mathbf{P}_t^{*T} \ . \tag{52}$$

Then check with (45) and (47) that the lower-right partition of the first matrix on the right hand side is a $K \times K$ matrix of zeros. The lower-right partition of the second matrix is given by the the lower-right partition of $\mathbf{P}_{t+1|T}$.

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