DC Networks

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Syllabus

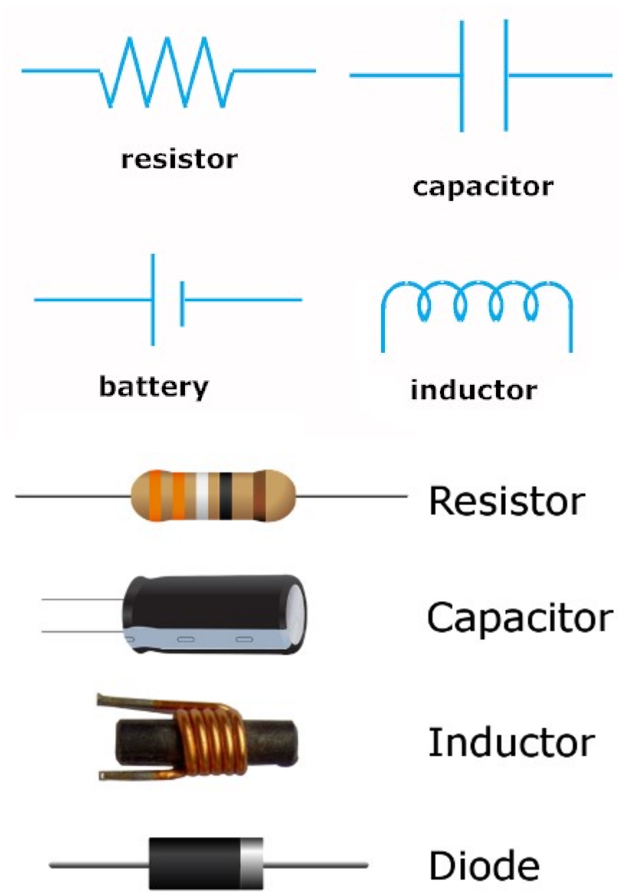
DC Networks:

- Kirchhoff’s laws
- Node voltage and mesh current methods
- Delta-star and star-delta conversions
- Superposition principle
- Thevenin’s and Norton’s theorems
- Maximum Power Transfer Theorem

Basic Elements & Introductory Concepts

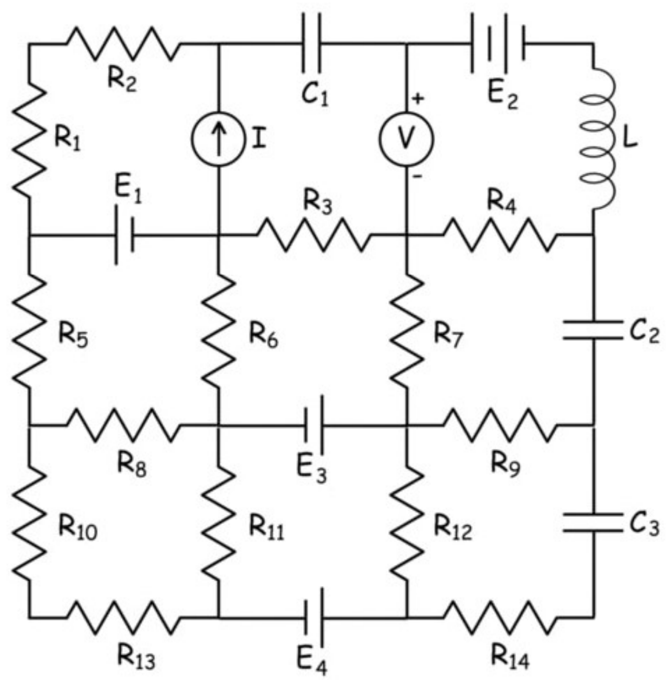
Basic Elements & Introductory Concepts

Circuit Elements: Any individual circuit component (resistor, inductor, capacitor, voltage source, current source, generator, etc.) with two terminals by which it can be connected to other circuit components is called a circuit element.



Basic Elements & Introductory Concepts

Electrical Network: A combination of various circuit elements (resistor, inductor, capacitor, voltage source, current source) connected in any manner what so ever is called an electrical network.



We may classify circuit elements in two categories, *passive* and *active* elements.

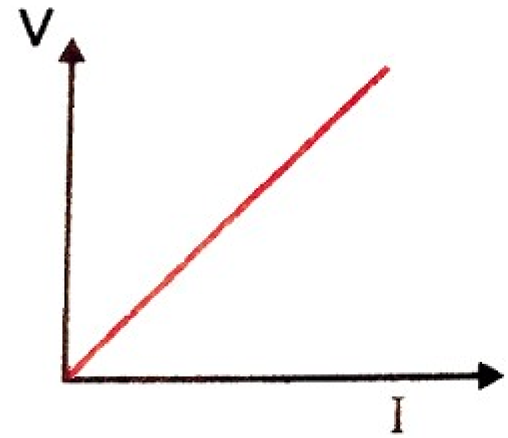
Basic Elements & Introductory Concepts

Passive Element: The element which receives energy (or absorbs energy) and then either converts it into heat (R) or stored it in an electric (C) or magnetic (L) field is called passive element.

Active Element: The elements that supply energy to the circuit is called active element. Examples of active elements include voltage and current sources, generators, and electronic devices that require power supplies.

Basic Elements & Introductory Concepts

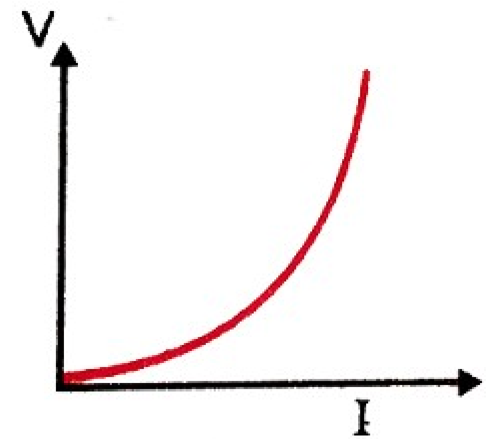
Linear Circuit: A circuit or network whose parameters that is elements like resistors, inductors and capacitors are always constant irrespective of the change in time, voltage, temperature, etc. is known as linear circuit.



- The *Ohm's law* can be applied to such network.
- The mathematical equations of such network can be obtained by using the *law of superposition*.

Basic Elements & Introductory Concepts

Non-linear Circuit: A circuit or network whose parameters change their values with change in time, voltage, temperature, etc. is known as non-linear circuit.

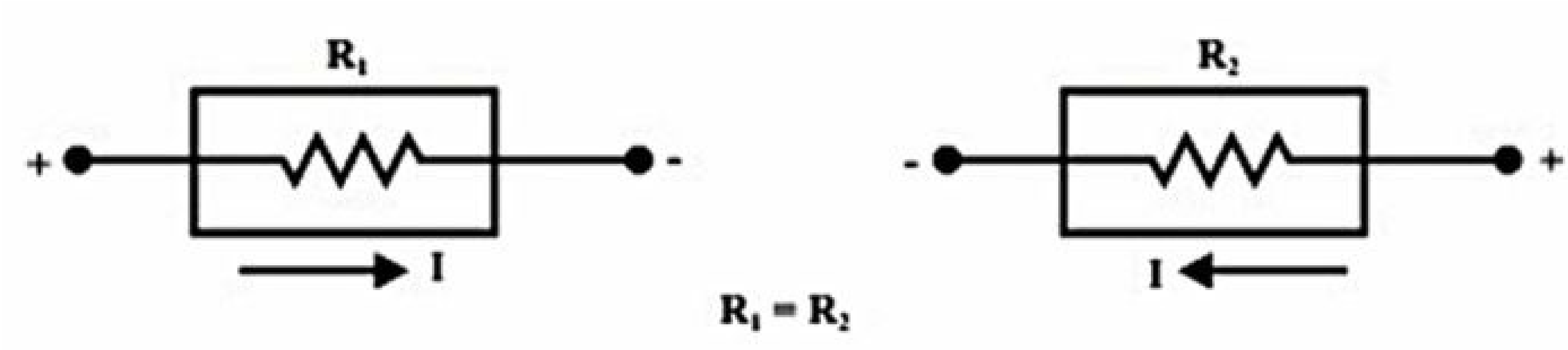


- The *Ohm's law* may not be applied to such network.
- Such network does not follow the *law of superposition*.

A circuit consisting of a diode is a non-linear circuit.

Basic Elements & Introductory Concepts

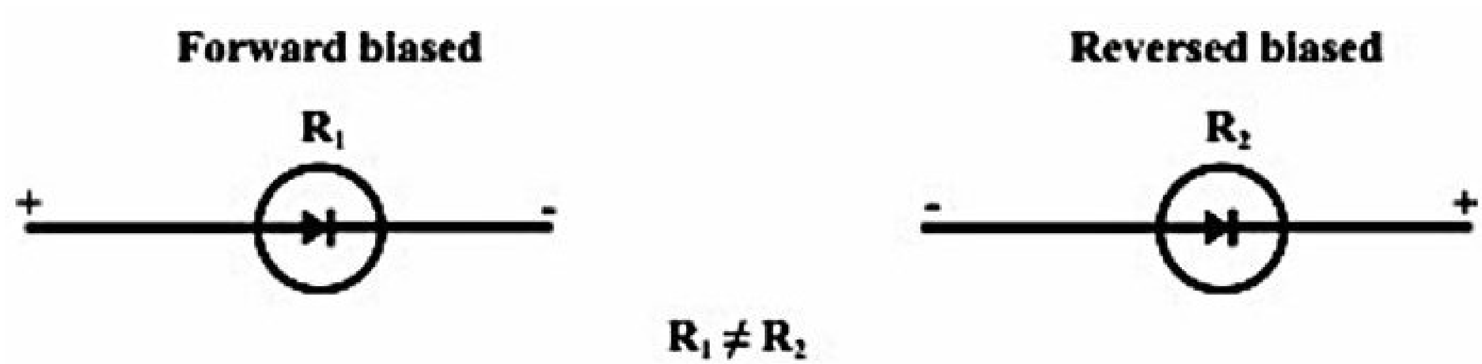
Bilateral Circuit: A circuit whose characteristics, behaviour is same irrespective of the direction of current through various elements of it, is called a bilateral circuit.



A circuit consisting only resistors is a good example of bilateral circuit.

Basic Elements & Introductory Concepts

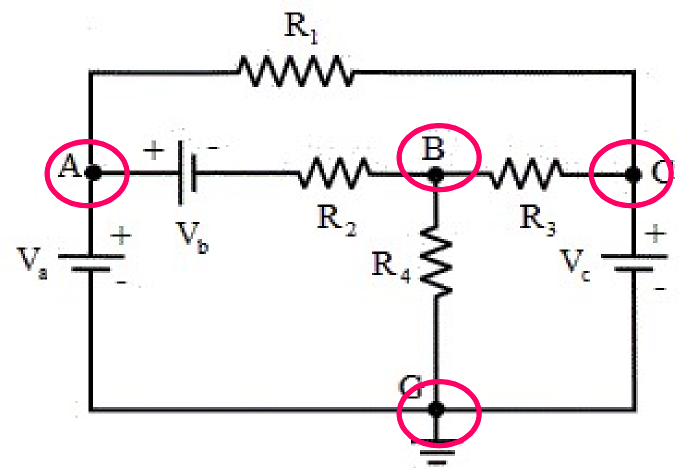
Unilateral Circuit: A circuit whose operation, behaviour is dependent on the direction of current through various elements of it, is called a unilateral circuit.



Circuit consisting diodes, which allows flow of current only in one direction is a good example of unilateral circuit.

Basic Elements & Introductory Concepts

Node: A node in an electric circuit is a point where two or more components are connected together.

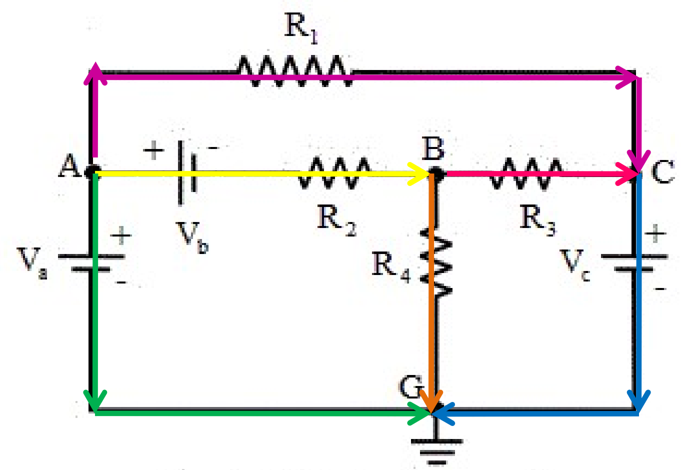


Generally, a point, or a node in an circuit specifies a certain voltage level with respect to a reference point or node.

A, B, C and G are nodes.

Basic Elements & Introductory Concepts

Branch: A branch is a conducting path between two nodes in a circuit containing the circuit elements.



A-C

A-B

B-C

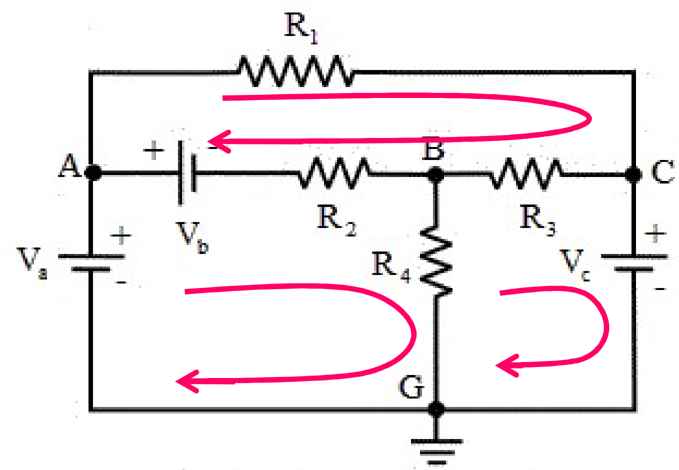
A-G

B-G

C-G

Basic Elements & Introductory Concepts

Loop: A loop is any closed path in an electric circuit i.e., a closed path or loop in a circuit is a continuous sequence of branches which starting and end points for tracing the path are, in effect, the same node and touches no other node more than once.



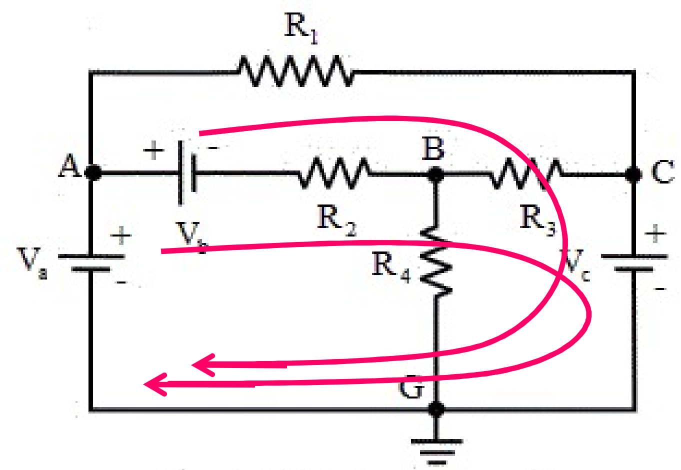
ABGA,

BCGB,

ACBA,

Basic Elements & Introductory Concepts

Loop: A loop is any closed path in an electric circuit i.e., a closed path or loop in a circuit is a continuous sequence of branches which starting and end points for tracing the path are, in effect, the same node and touches no other node more than once.



ABCGA,

ACGA.

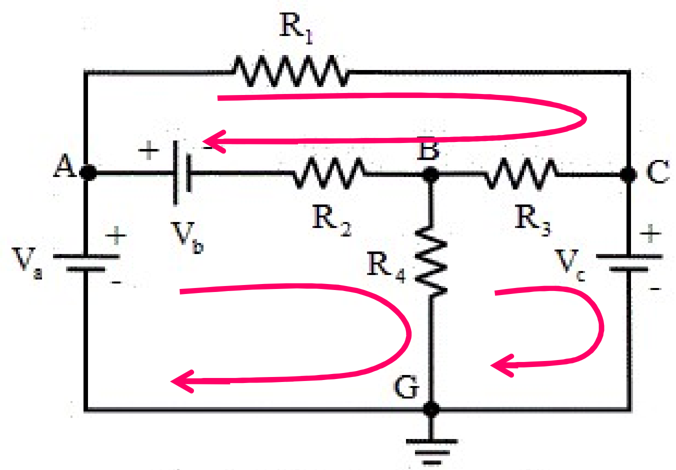
Basic Elements & Introductory Concepts

Mesh: A mesh is a special case of loop that does not have any other loops within it or in its interior.

ABGA,

BCGB,

ACBA.



"All meshes are loops but all loops are not meshes".

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Ohm’s Law

Ohm’s law states that the voltage *v* across a resistor is directly proportional to the current *i* flowing through the resistor.

That is,

$$v \propto i$$

$$v = iR$$

R is called the *resistance* of the resistor.

The resistance is a material property which can change if the internal or external conditions of the element are altered, e.g., if there are changes in the temperature.

The resistance R of an element denotes its ability to resist the flow of electric current; it is measured in ohms (Ω).

Kirchhoff’s Laws

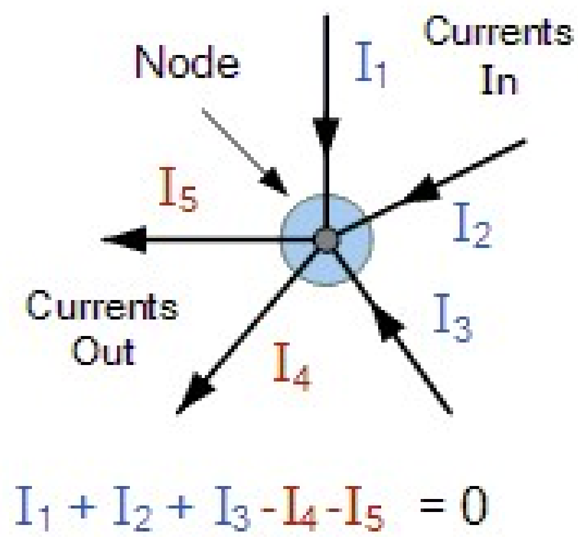
Kirchhoff's Laws

Kirchhoff's laws are basic analytical tools in order to obtain the solutions of currents and voltages for any electric circuit; whether it is supplied from a direct-current system or an alternating current system.

Kirchhoff's Laws

Kirchhoff's Current Law (KCL): KCL states that at any node (junction) in a circuit the algebraic sum of currents entering and leaving a node at any instant of time must be equal to zero.

Currents *entering(+ve sign)* and currents *leaving (-ve sign)* the node must be assigned opposite algebraic signs.

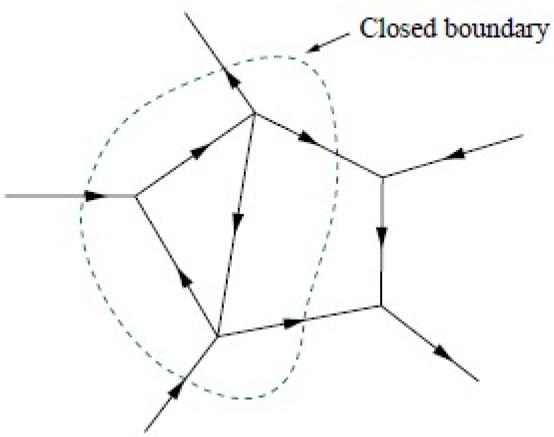


Kirchhoff’s Laws

Kirchhoff’s Current Law (KCL):

KCL also applies to a closed boundary. This may be regarded as a generalized case, because a node may be regarded as a closed surface shrunk to a point.

In two dimensions, a closed boundary is the same as a closed path.



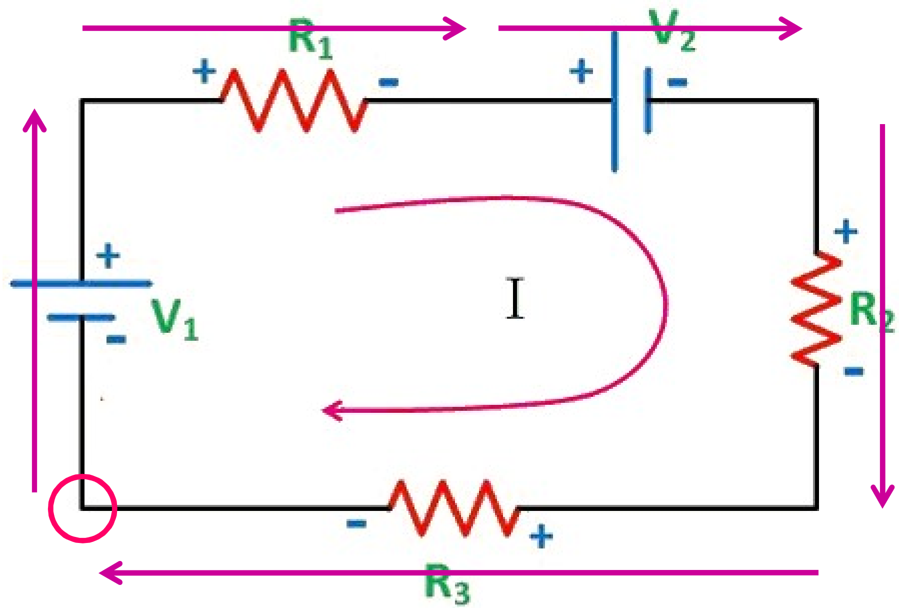
Kirchhoff's Laws

Kirchhoff's Voltage Law (KVL): It states that in a closed circuit, the algebraic sum of all source voltages must be equal to the algebraic sum of all the voltage drops.

Voltage drop is encountered when current flows in an element (resistance or load) from the *higher-potential* terminal toward the *lower potential* terminal.

Voltage rise is encountered when current flows in an element (voltage source) from *lower potential* terminal (or negative terminal of voltage source) toward the *higher potential* terminal (or positive terminal of voltage source).

Kirchhoff's Laws



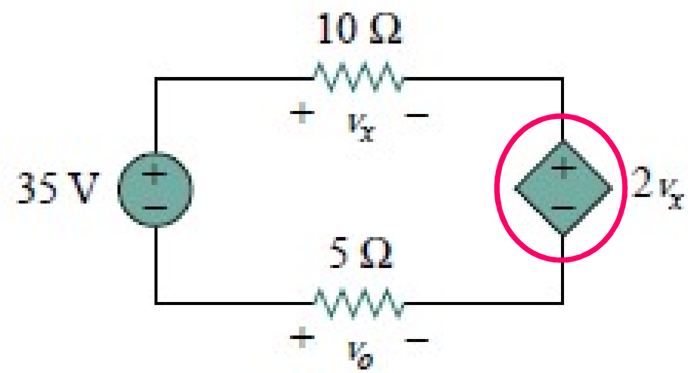
KVL equation for the circuit:

$$V_1 - IR_1 - V_2 - IR_2 - IR_3 = 0$$

Or, $V_1 - V_2 = IR_1 + IR_2 + IR_3$

Example1

Find v_x and v_0 in the given circuit.



- ➢What is a dependent source?

Dependent Sources

These type of sources (either voltage source or current source) depend on a voltage across or a current through some other element elsewhere in the circuit.

Both voltage and current types of sources may be dependent and either may be controlled by a voltage or a current.

Dependent Sources

Dependent sources are of four types:

(i) Voltage-controlled voltage source (VCVS)

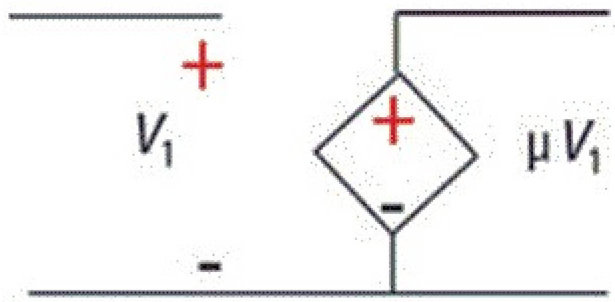
(ii) Current-controlled voltage source (CCVS)

(iii) Voltage-controlled current source(VCCS)

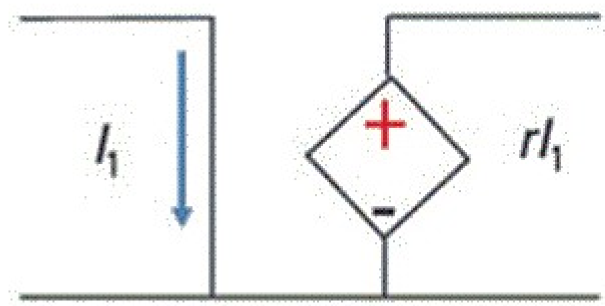
(iv) Current-controlled current source(CCCS)

Dependent Sources

Voltage-controlled voltage source (VCVS):

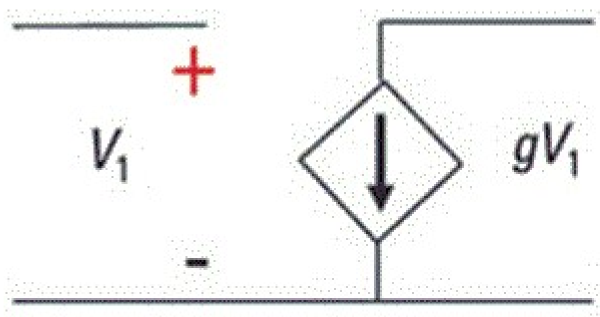


Current-controlled voltage source (CCVS):

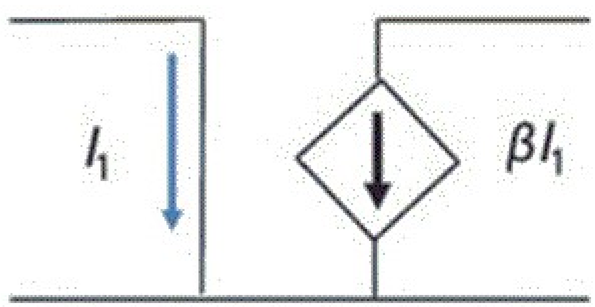


Dependent Sources

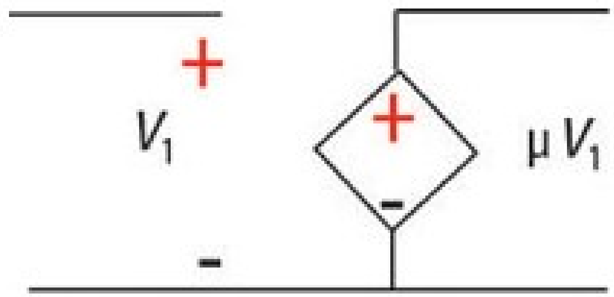
Voltage-controlled current source(VCCS):



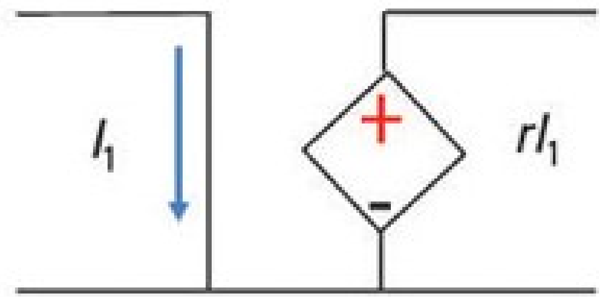
Current-controlled current source(CCCS):



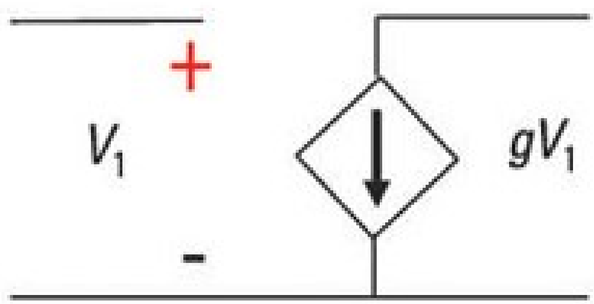
Dependent Sources



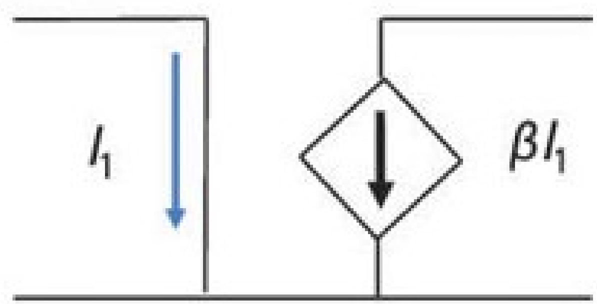
Voltage-controlled voltage source (VCVS)



Current-controlled voltage source (CCVS)



Voltage-controlled current source (VCCS)



Current-controlled current source (CCCS)

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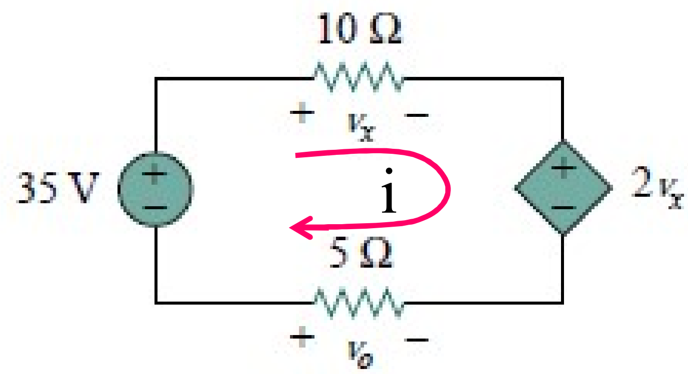
Find v_x and v_0 in the given circuit.



Applying KVL across the loop,

$$35 - v_x - 2v_x + v_0 = 0$$

$$v_x = 10i$$

$$v_0 = -5i$$

$$\text{Or,} \quad 35 - 10i - 2(10i) - 5i = 0$$

$$\text{Or,} \quad 35 - 35i = 0$$

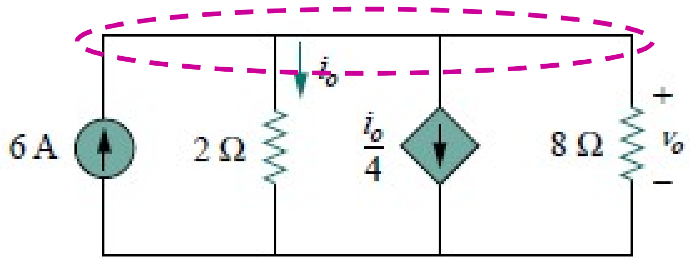
$$\text{Or,} \quad i = 1 \text{ A}$$

$$\therefore \quad v_x = 10 \times 1 \text{ V} = 10 \text{ V}$$

$$v_0 = -5 \times 1 \text{ V} = -5 \text{ V}$$

Example2

Find v_0 and i_0 in the given circuit.



Applying KCL,

$$6 - i_0 - \frac{i_0}{4} - \frac{v_0}{8} = 0$$

$$\frac{v_0}{2} = i_0$$

$$\frac{v_0}{8} = \frac{i_0}{4}$$

Or, $6 - i_0 - \frac{i_0}{4} - \frac{i_0}{4} = 0$

Or, $i_0 = 4\ \text{A}$

$\therefore\quad v_0 = 2 \times 4\ \text{V} = 8\ \text{V}$

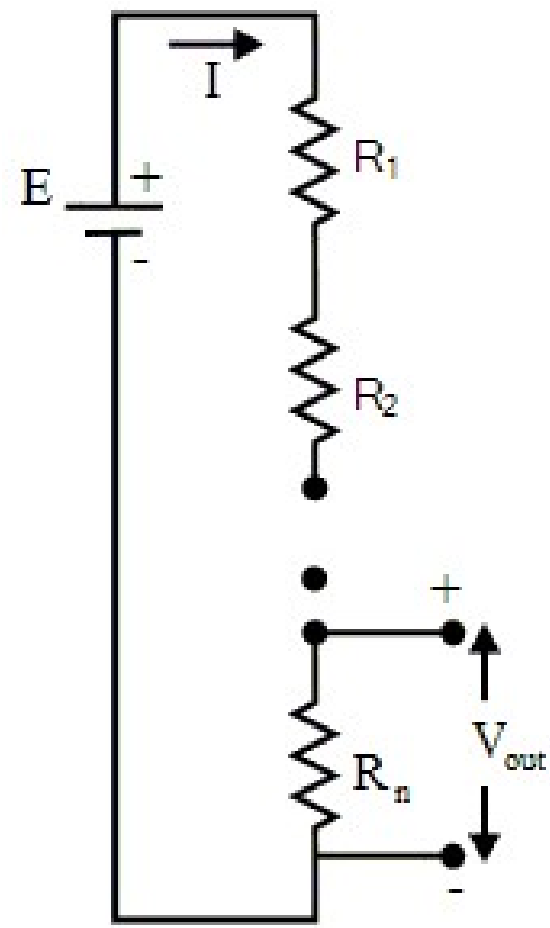
Voltage Divider Rule

The basic idea behind the voltage divider rule is to assign a portion of the total voltage to each resistor.

$$R_{eq} = R_1 + R_2 + \ldots + R_n$$

$$I = \frac{E}{R_{eq}} = \frac{E}{R_1 + R_2 + \ldots + R_n}$$

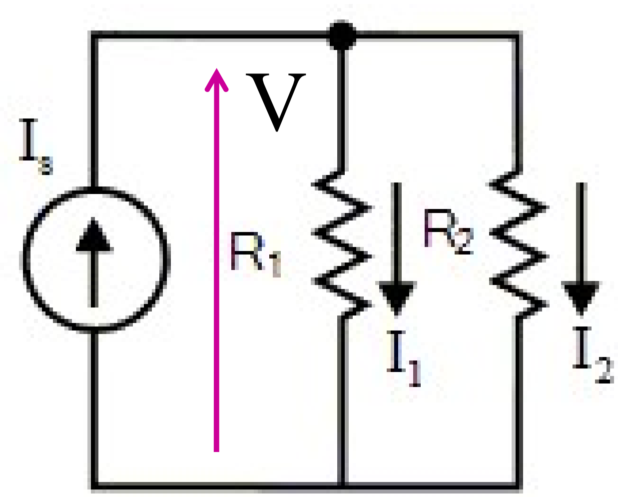
$$V_{out} = IR_n = \frac{E}{R_1 + R_2 + \ldots + R_n} \bullet R_n$$



Current Divider Rule

Here the supply current is distributed between the two branches of the circuit.

Let the voltage across the branch be V.



$$I_1 = \frac{V}{R_1}; \qquad I_2 = \frac{V}{R_2}$$

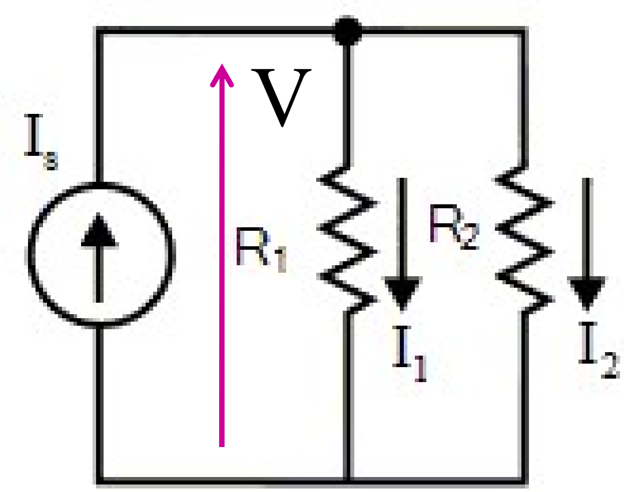
$$I_s = I_1 + I_2 = \frac{V}{R_1} + \frac{V}{R_2} = \frac{(R_1 + R_2)}{R_1 R_2} \cdot V$$

Current Divider Rule

Therefore,

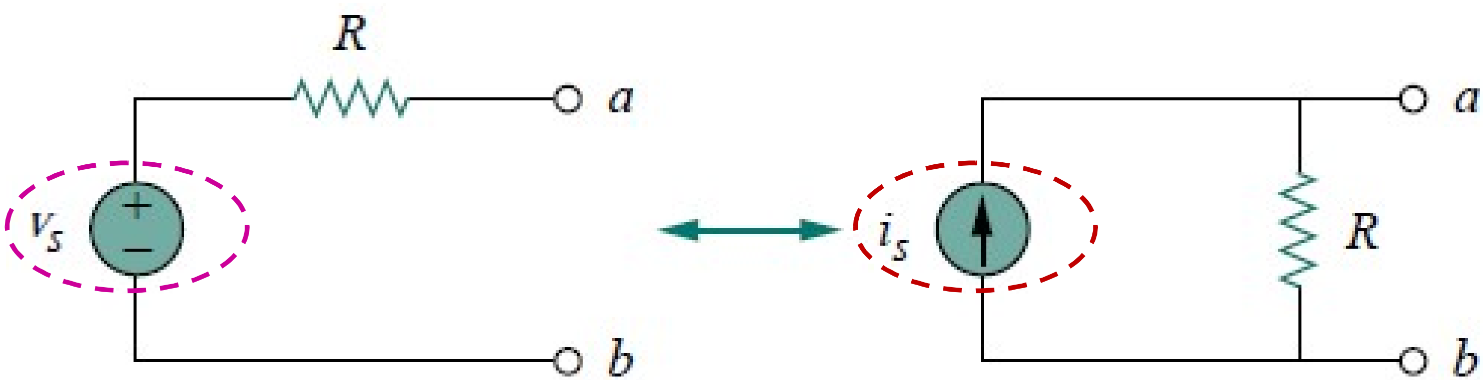
$$I_1 = \frac{R_2}{R_1 + R_2} \cdot I_s$$

$$I_2 = \frac{R_1}{R_1 + R_2} \cdot I_s$$



Source Transformation

A source transformation is the process of replacing a voltage source v_s in series with a resistor R by a current source i_s in parallel with a resistor R, or vice versa.



$$v_s = i_s R \qquad\qquad i_s = \frac{v_s}{R}$$

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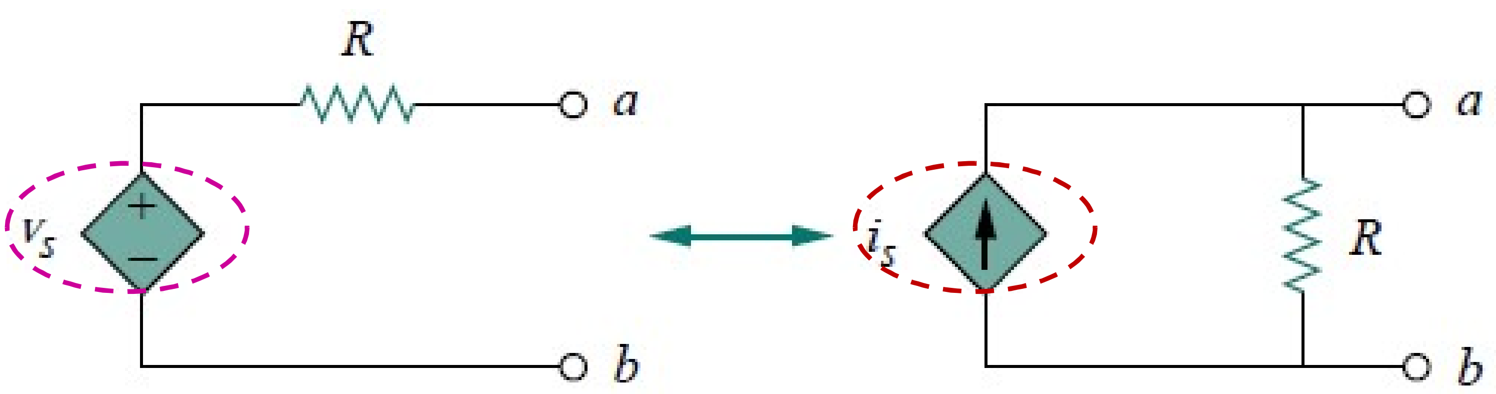
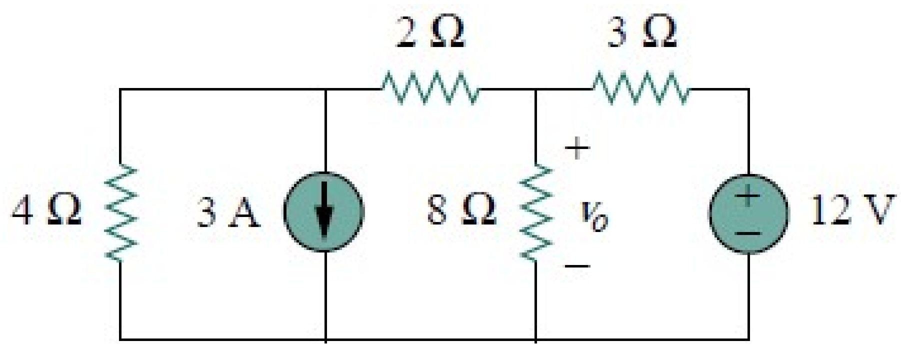


$v_s = i_s R$

$i_s = \dfrac{v_s}{R}$

Example3

Using source transformation, find v_0 in the given circuit.



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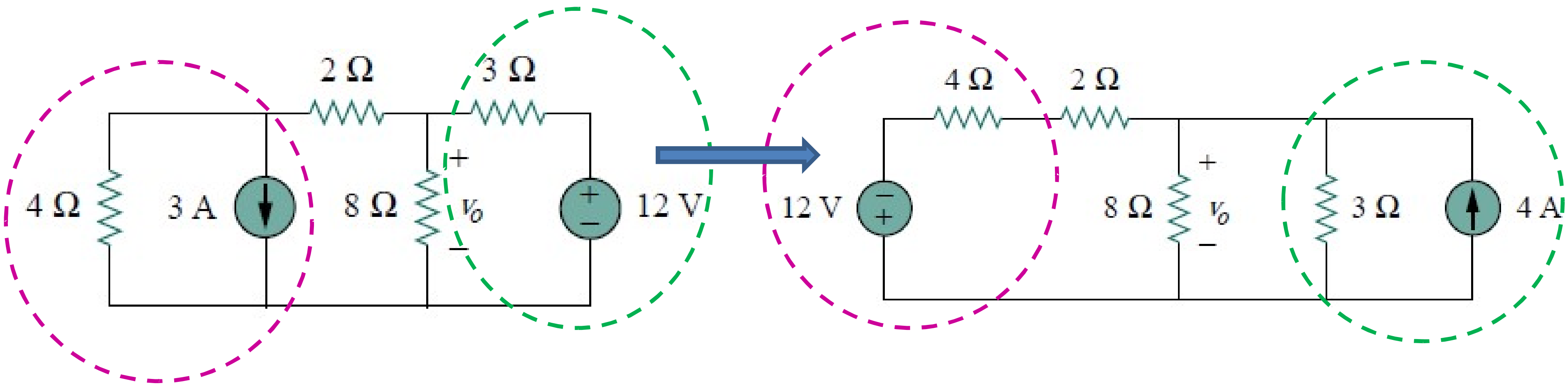
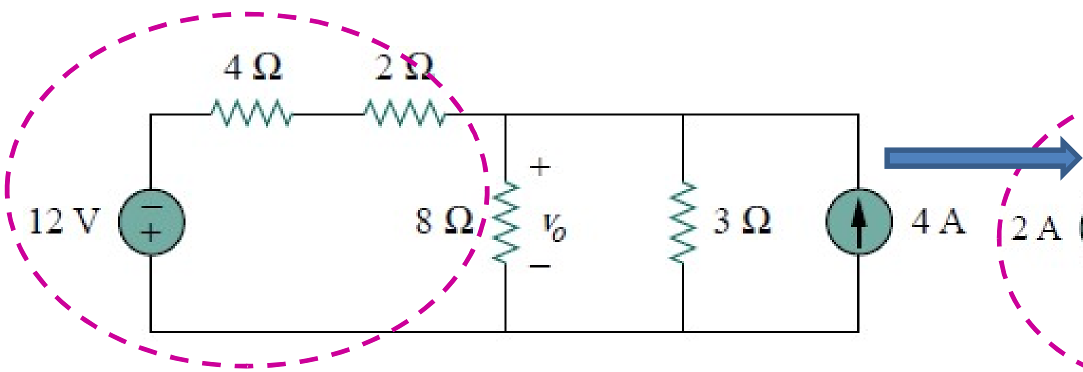


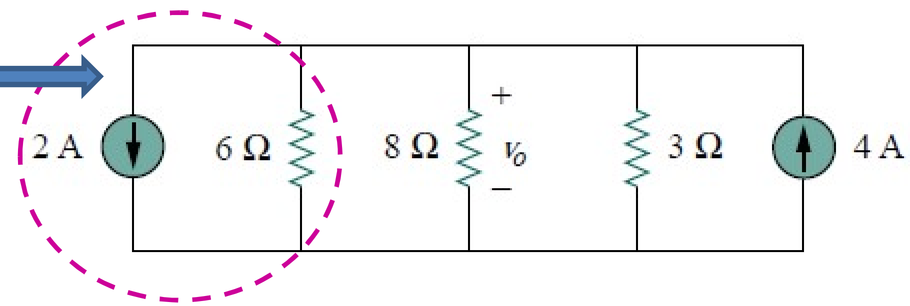
$$\boxed{4\times 3\text{ V}=12\text{ V}}$$

$$\boxed{\frac{12}{3}\text{ A}=4\text{ A}}$$

Example3

Using source transformation, find v_0 in the given circuit.

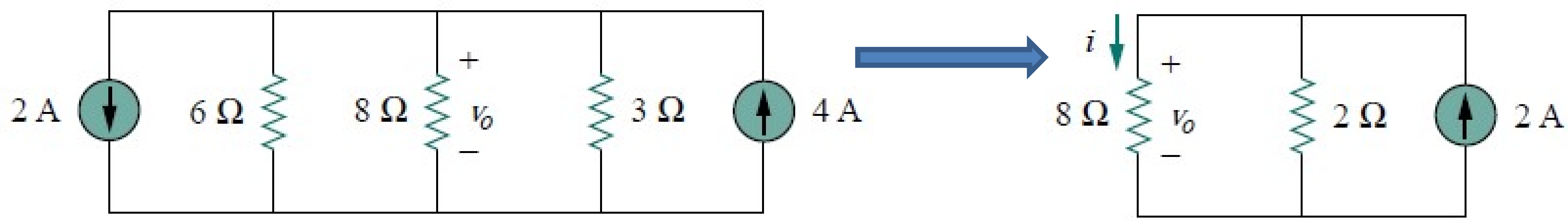




$$\boxed{\frac{12}{(4+2)}\ \text{A} = 2\ \text{A}}$$

Example3

Using source transformation, find v_0 in the given circuit.

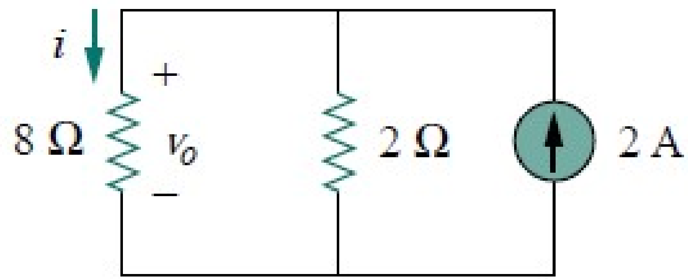


$$(4-2)\ \text{A} = 2\ \text{A}$$

$$\frac{6\times 3}{6+3}\ \Omega = 2\ \Omega$$

Example3

Using source transformation, find v_0 in the given circuit.



Using current division rule,

$$i = \frac{2}{2+8} \times 2\ \text{A} = 0.4\ \text{A}$$

$$\therefore\ v_0 = 8 \times 0.4\ \text{V} = 3.2\ \text{V}$$

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Node voltage and Mesh current methods

Circuit Analysis

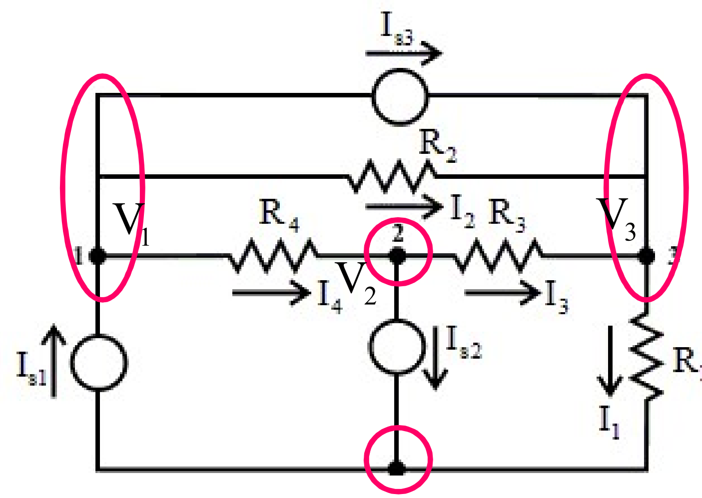
Circuit Analysis: The method by which one can determine a variable (either a voltage or a current) of a circuit is called circuit analysis.

Mesh Analysis: Circuit analysis based on the fundamental principles of circuit laws, namely, Ohm’s law and Kirchhoff’s voltage law(KVL) is known as mesh analysis.

Node Voltage Analysis: Circuit analysis based on the fundamental principles of circuit laws, namely, Ohm’s law and Kirchhoff’s current law(KCL) is known as node voltage analysis.

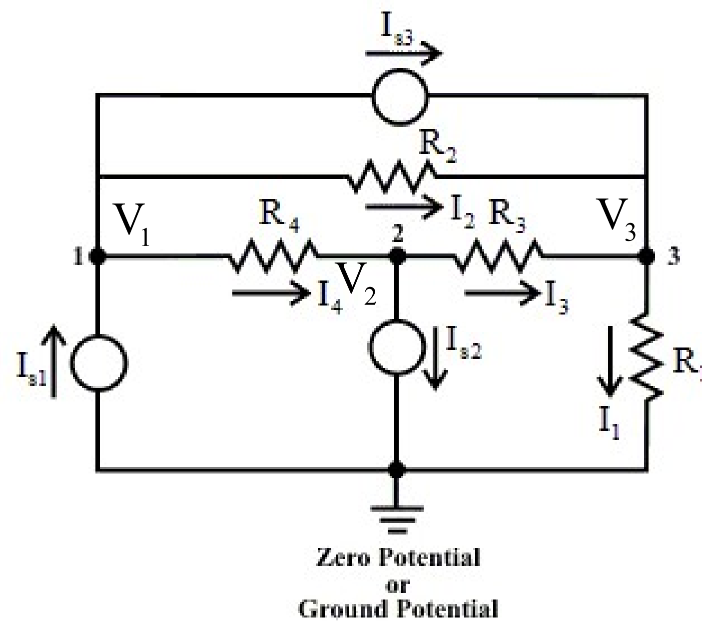
Node Voltage Analysis

- In the node voltage method, we identify all the nodes on the circuit.
- Choosing one of them as the reference voltage (i.e., zero potential) and subsequently assign other node voltages (unknown) with respect to a reference voltage.



Node Voltage Analysis

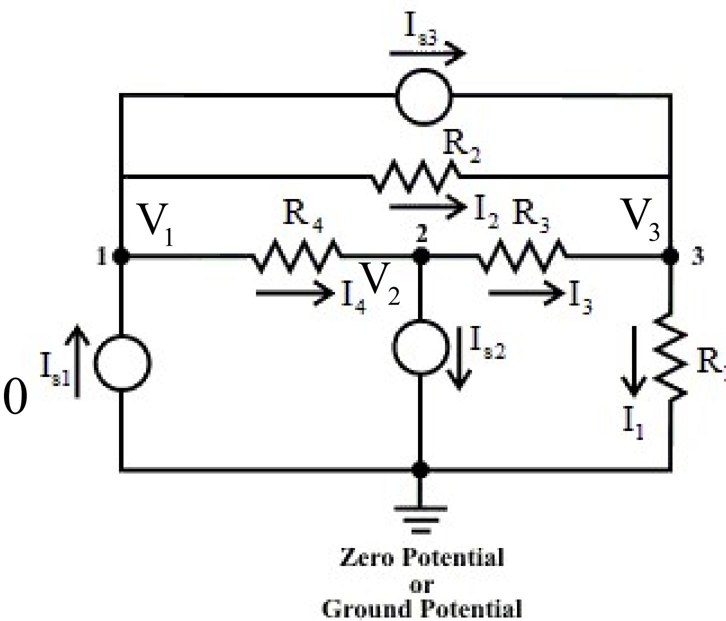
- If the circuit has "n" nodes there are "n-1" node voltages are unknown.
- At each of these "n-1" nodes, we can apply KCL equation.
- The unknown node voltages become the independent variables of the problem and the solution of node voltages can be obtained by solving a set of simultaneous equations.



Node Voltage Analysis

KCL equation at "Node-1":

$$I_{s1} - I_{s3} - \left(\frac{V_1 - V_2}{R_4}\right) - \left(\frac{V_1 - V_3}{R_2}\right) = 0$$



$$I_{s1} - I_{s3} - \left(\frac{1}{R_2} + \frac{1}{R_4}\right)V_1 + \frac{1}{R_4}V_2 + \left(\frac{1}{R_2}\right)V_3 = 0$$

$$I_{s1} - I_{s3} = \left(\frac{1}{R_2} + \frac{1}{R_4}\right)V_1 - \frac{1}{R_4}V_2 - \left(\frac{1}{R_2}\right)V_3$$

$$\text{........}(1)$$

$$I_{s1} - I_{s3} = G_{11}V_1 - G_{12}V_2 - G_{13}V_3$$

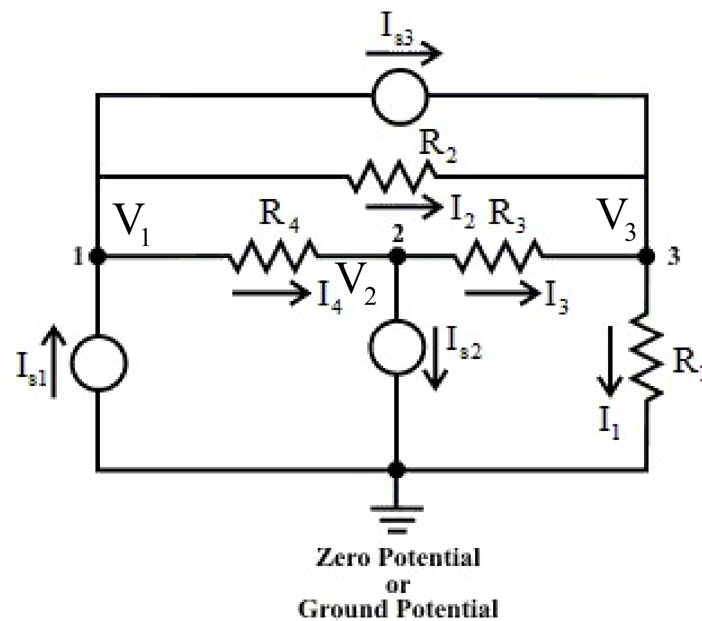
Node Voltage Analysis

KCL equation at "Node-2":

$$\left(\frac{V_1 - V_2}{R_4}\right) - \left(\frac{V_2 - V_3}{R_3}\right) - I_{s2} = 0$$

$$-I_{s2} = -\left(\frac{1}{R_4}\right)V_1 + \left(\frac{1}{R_3} + \frac{1}{R_4}\right)V_2 - \left(\frac{1}{R_3}\right)V_3$$

........(2)



$$-I_{s2} = -G_{21}V_1 + G_{22}V_2 - G_{23}V_3$$

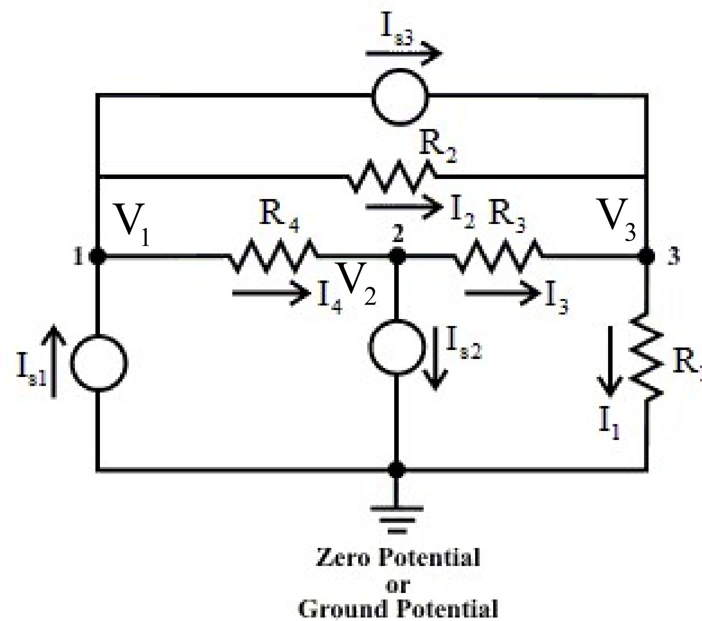
Node Voltage Analysis

KCL equation at "Node-3":

$$I_{s3} + \left(\frac{V_2 - V_3}{R_3}\right) + \left(\frac{V_1 - V_3}{R_2}\right) - \left(\frac{V_3}{R_1}\right) = 0$$

$$I_{s3} = -\left(\frac{1}{R_2}\right)V_1 - \left(\frac{1}{R_3}\right)V_2 + \left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right)V_3$$

……..(3)



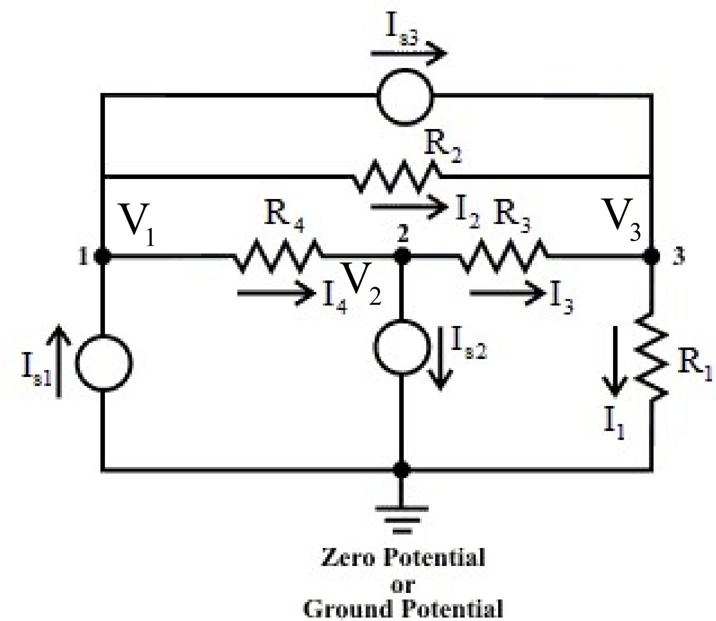
$$\boxed{I_{s3} = -G_{31}V_1 - G_{32}V_2 + G_{33}V_3}$$

Node Voltage Analysis

$$I_{s1} - I_{s3} = G_{11}V_1 - G_{12}V_2 - G_{13}V_3$$

$$-I_{s2} = -G_{21}V_1 + G_{22}V_2 - G_{23}V_3$$

$$I_{s3} = -G_{31}V_1 - G_{32}V_2 + G_{33}V_3$$



Node Voltage Analysis

In general, for i-th node, the KCL equation can be written as,

$$\sum I_{ii} = -G_{i1}V_1 - G_{i2}V_2 ..... + G_{ii}V_i - G_{ii+1}V_{i+1} - .....G_{iN}V_N$$

$\sum I_{ii} =$ the algebraic sum of all the current sources connected to node-i

$G_{ii} =$ the sum of the values of conductance connected to the node-i

$G_{ij} =$ the sum of the values of conductance connected between the nodes i and j

$V_i =$ the unknown node voltages

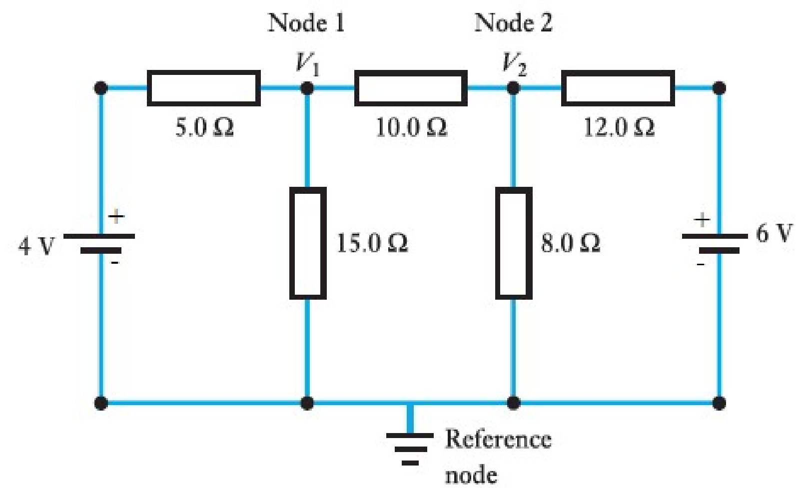
$$i = 1, 2, 3.........N$$

Node Voltage Analysis

Summarize:

Step-1: Identify all nodes in the circuit. Select one node as the reference node (assign as ground potential or zero potential) and label the remaining nodes as unknown node voltages with respect to the reference node.

Step-2: Assign branch currents in each branch. (The choice of direction is arbitrary).



Node Voltage Analysis

Summarize:

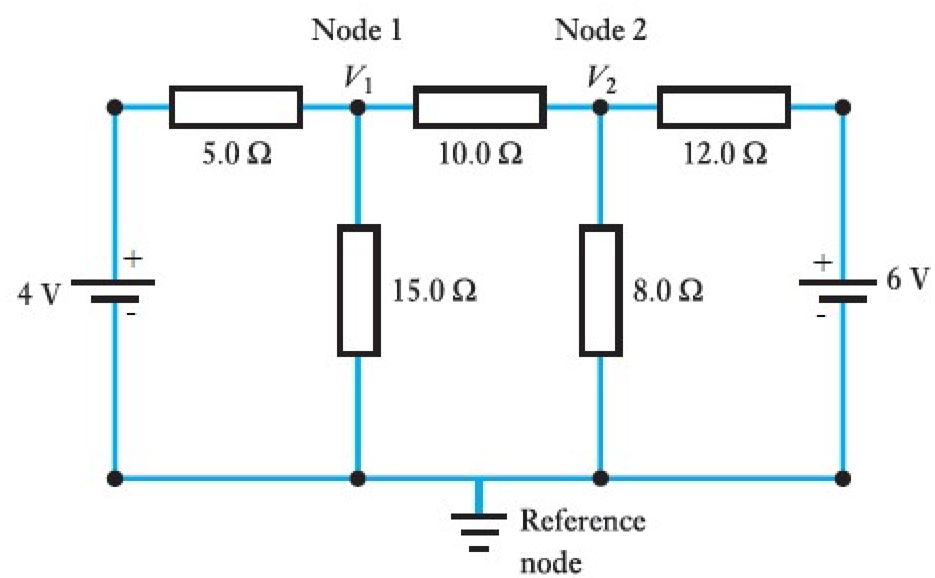
Step-3: Express the branch currents in terms of node assigned voltages.

Step-4: Write the standard form of node equations by inspecting the circuit. (No of node equations = No of nodes (N) – 1).

Step-5: Solve a set of simultaneous algebraic equation for node voltages and ultimately the branch currents.

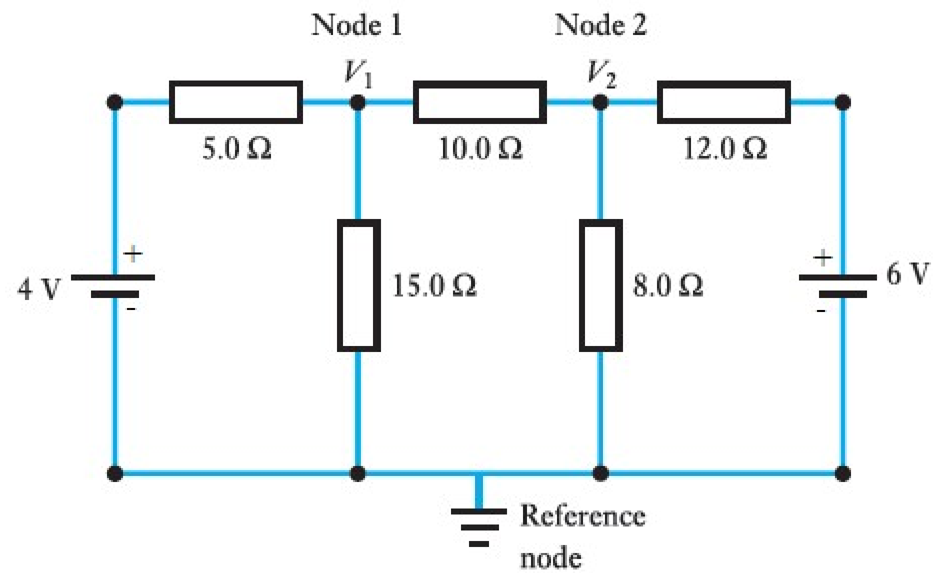
Example 4

Using the Node Voltage method calculate the voltages V_1 and V_2 in the figure and hence calculate the currents in the 8 Ω resistor.



Example 4

Step1: Reference node shown. Voltages V_1 and V_2 assigned.



Example 4

Step2: Assign currents in each connection to each node.

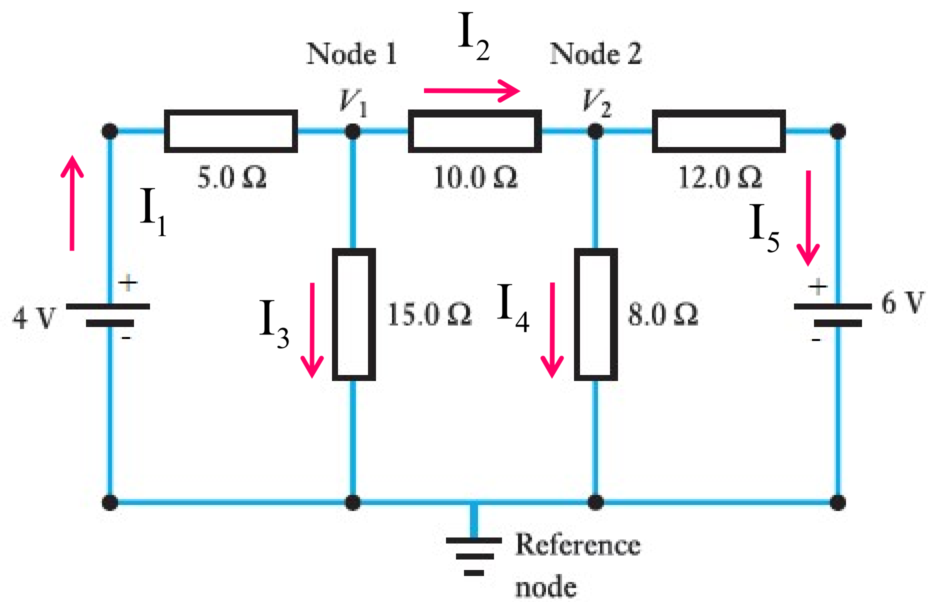
$$I_1 = \frac{4 - V_1}{5}$$

$$I_2 = \frac{V_1 - V_2}{10}$$

$$I_3 = \frac{V_1 - 0}{15}$$

$$I_4 = \frac{V_2 - 0}{8}$$

$$I_5 = \frac{V_2 - 6}{12}$$



Example 4

Step3: Apply Kirchhoff 's current law at each node.

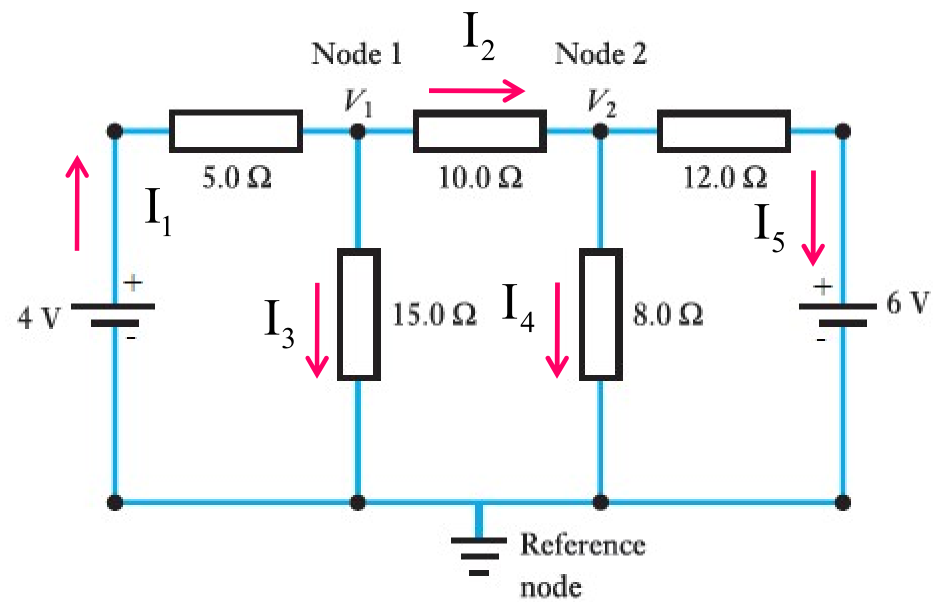
At node1

$$I_1 = I_2 + I_3$$

$$\frac{4 - V_1}{5} = \frac{V_1 - V_2}{10} + \frac{V_1 - 0}{15}$$

$$11V_1 - 3V_2 = 24$$

……(1)



Example 4

Step3: Apply Kirchhoff 's current law at each node.

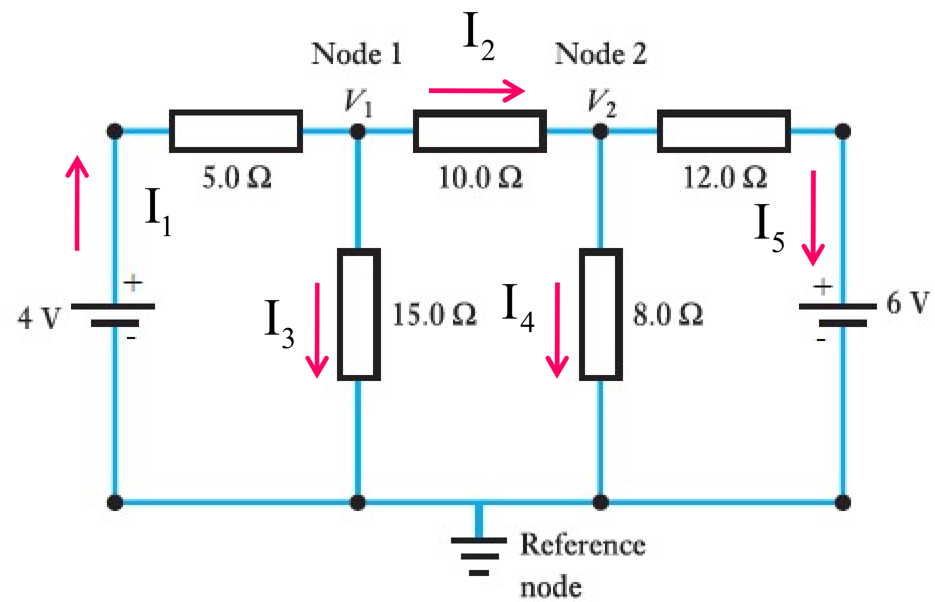
At node2

$$I_2 = I_4 + I_5$$

$$\frac{V_1 - V_2}{10} = \frac{V_2 - 0}{8} + \frac{V_2 - 6}{12}$$

$$12V_1 - 37V_2 = -60$$

……(2)



Example 4

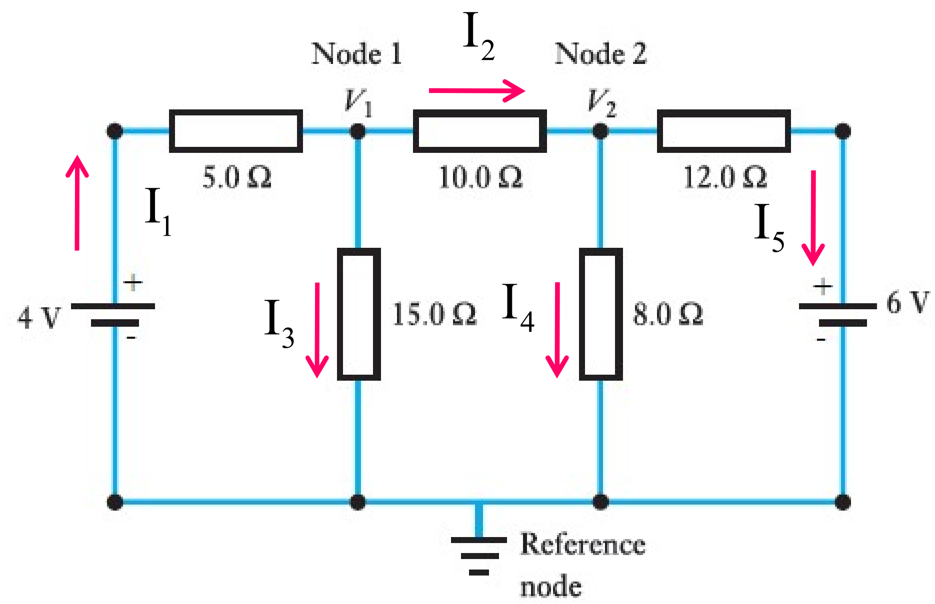
Step4: Solve for V_1 and V_2.

$V_1 = 2.88$ V

$V_2 = 2.55$ V

Hence current through 8 ohm resistor is

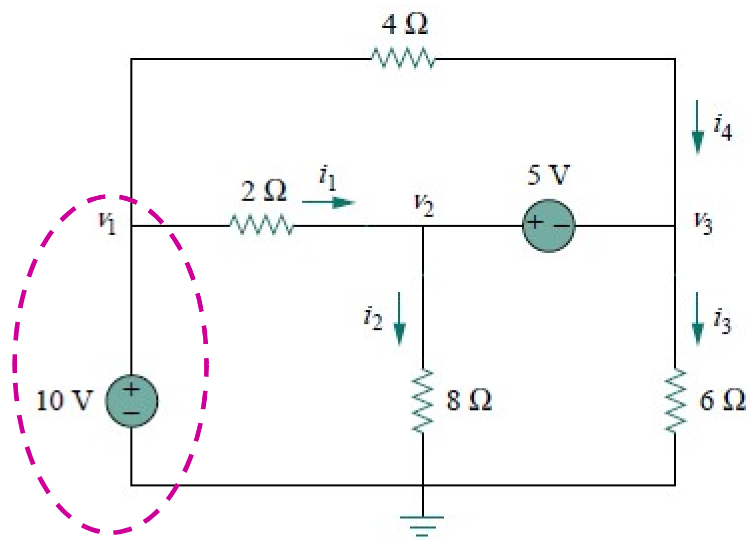
$$\frac{V_2}{8} = 0.32 \text{ A}$$



Nodal Analysis with Voltage Source

Case-I:

If a voltage source is connected between the reference node and a non-reference node, we simply set the voltage at the non-reference node equal to the voltage of the voltage source.

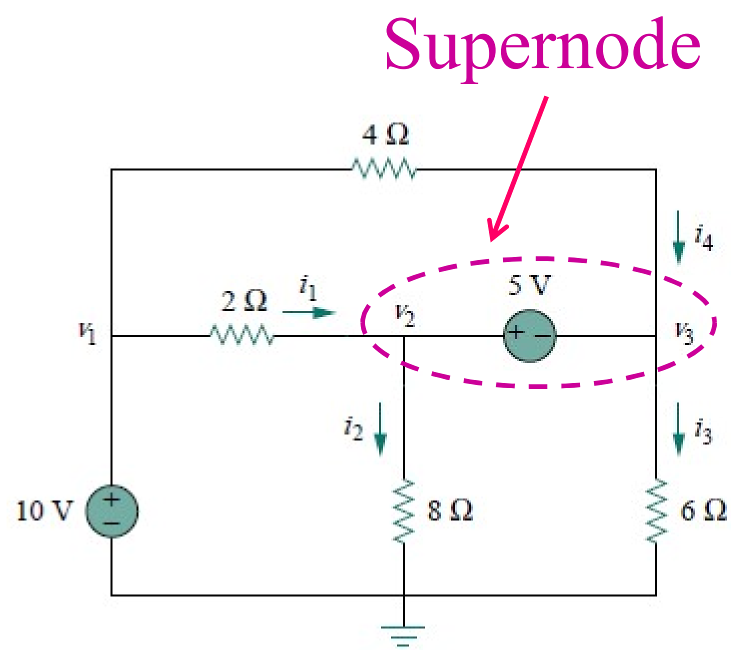


$$V_1 = 10 \text{ V} \qquad \ldots\ldots\ldots(1)$$

Nodal Analysis with Voltage Source

Case-II:

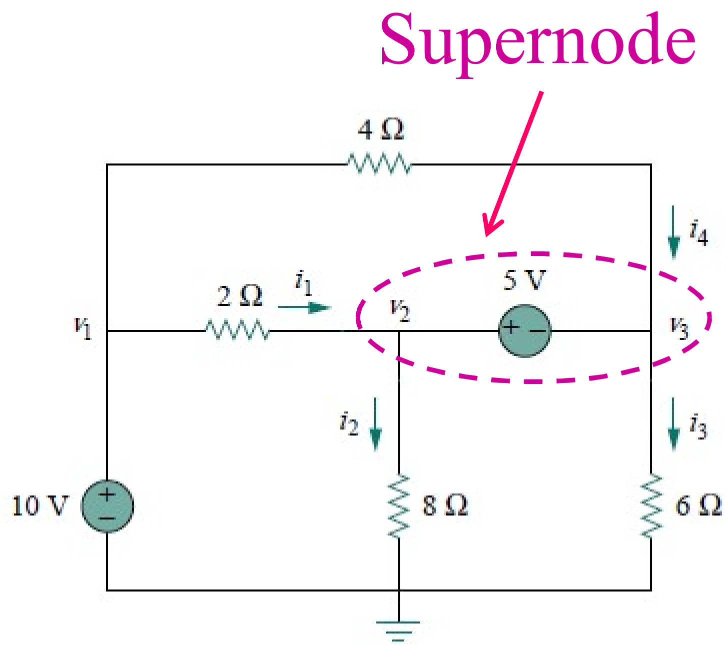
If the voltage source (dependent or independent) is connected between two non-reference nodes, the two non-reference nodes form a generalized node or supernode; we apply both KCL and KVL to determine the node voltages.



Nodal Analysis with Voltage Source

Supernode:

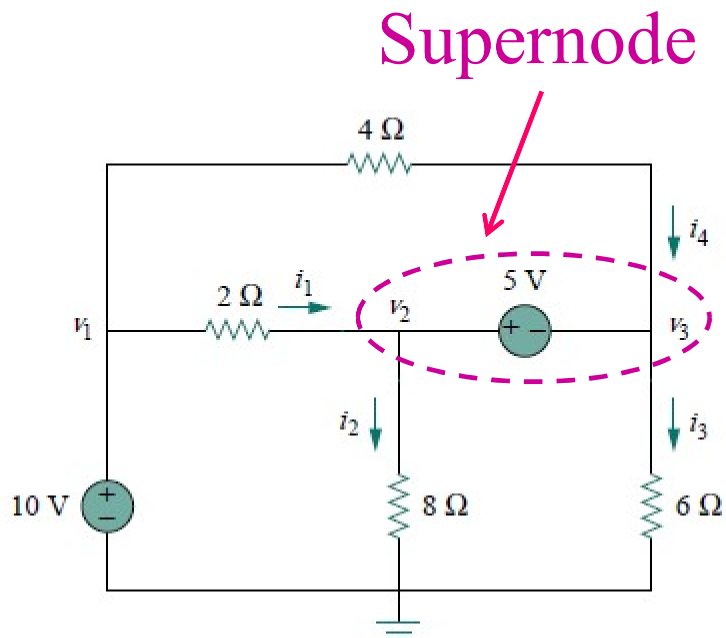
A supernode is formed by enclosing a (dependent or independent) voltage source connected between two non-reference nodes and any elements connected in parallel with it.



Nodal Analysis with Voltage Source

Why supernodes are treated differently?

Because an essential component of nodal analysis is applying KCL, which requires knowing the current through each element. There is no way of knowing the current through a voltage source in advance.



Nodal Analysis with Voltage Source

Applying KCL at the supernode,

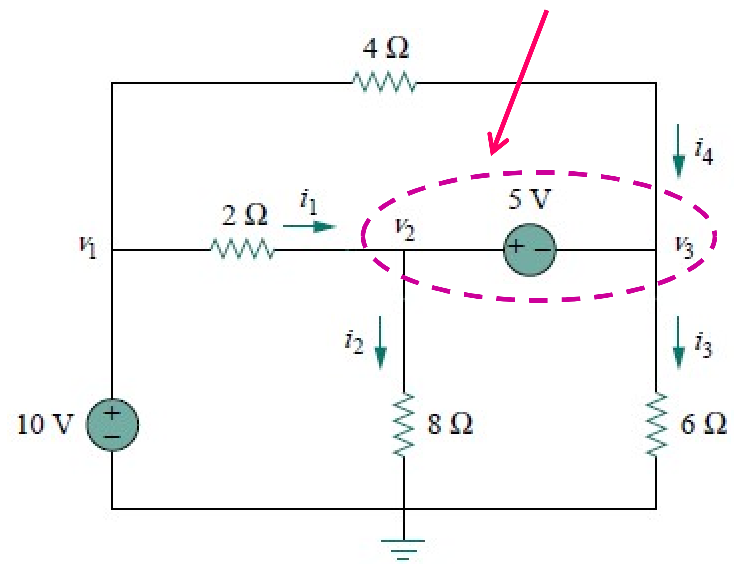
$$i_1 - i_2 - i_3 + i_4 = 0$$

Or, $$\frac{V_1 - V_2}{2} - \frac{V_2 - 0}{8} - \frac{V_3 - 0}{6} + \frac{V_1 - V_3}{4} = 0$$

Or, $$18V_1 - 15V_2 - 10V_3 = 0$$

…….(2)

Supernode

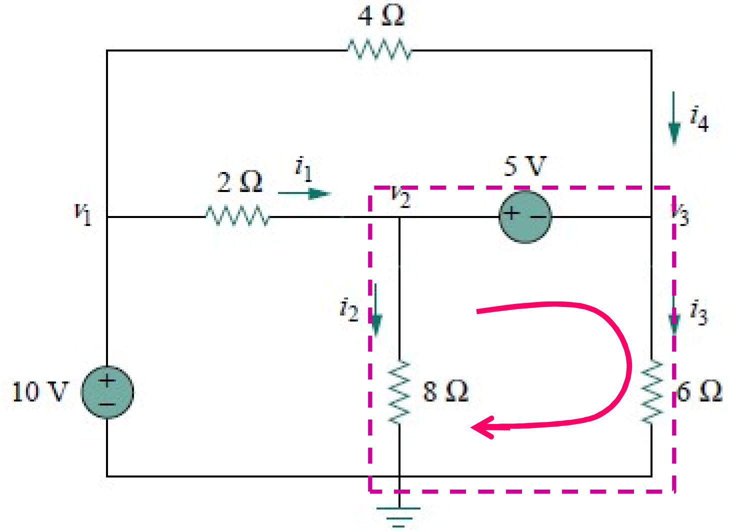


Nodal Analysis with Voltage Source

Applying KVL around the loop,

$$V_2 - 5 - V_3 = 0$$

Or, $V_2 - V_3 = 5$ ....(3)



Solving equations (1), (2) and (3),

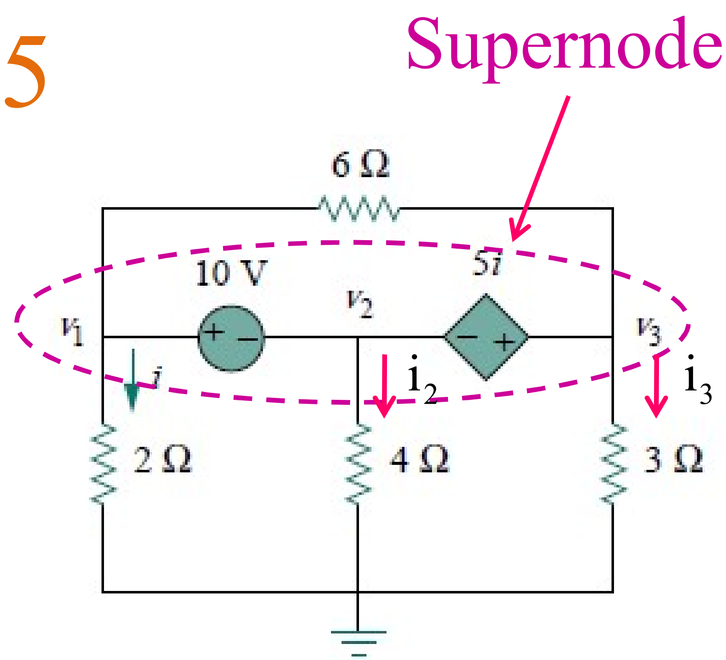
$$V_1 = 10 \text{ V}$$

$$V_2 = 9.2 \text{ V}$$

$$V_3 = 4.2 \text{ V}$$

Example 5

Find the voltages at the three non-reference nodes in the given circuit.



Applying KCL at the supernode,

$$i + i_2 + i_3 = 0$$

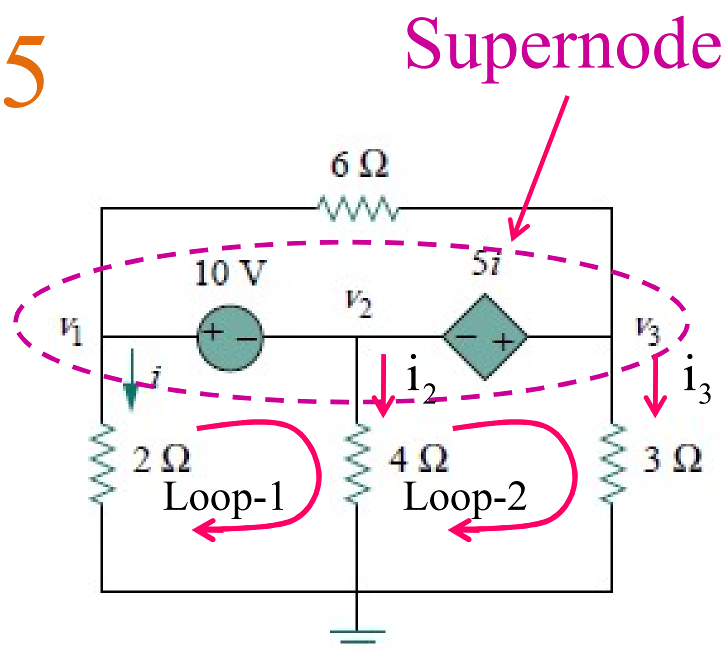
$$\text{Or,} \quad \frac{V_1 - 0}{2} + \frac{V_2 - 0}{4} + \frac{V_3 - 0}{3} = 0$$

$$\text{Or,} \quad 6V_1 + 3V_2 + 4V_3 = 0$$

…….(1)

Example 5

Find the voltages at the three non-reference nodes in the given circuit.



Applying KVL around loop-1,

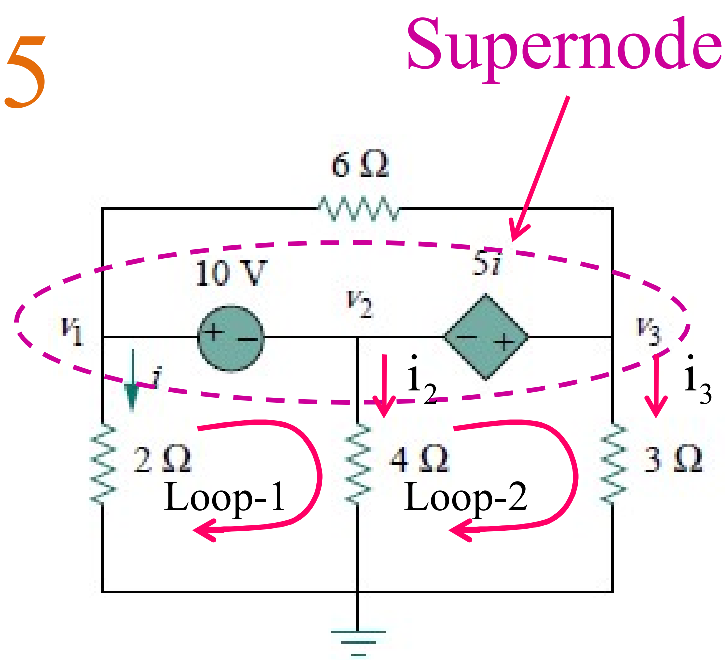
$$V_1 - 10 - V_2 = 0$$

$$\text{Or,} \quad V_1 - V_2 = 10$$

…….(2)

Example 5

Find the voltages at the three non-reference nodes in the given circuit.



Applying KVL around loop-2,

$$V_2 + 5i - V_3 = 0$$

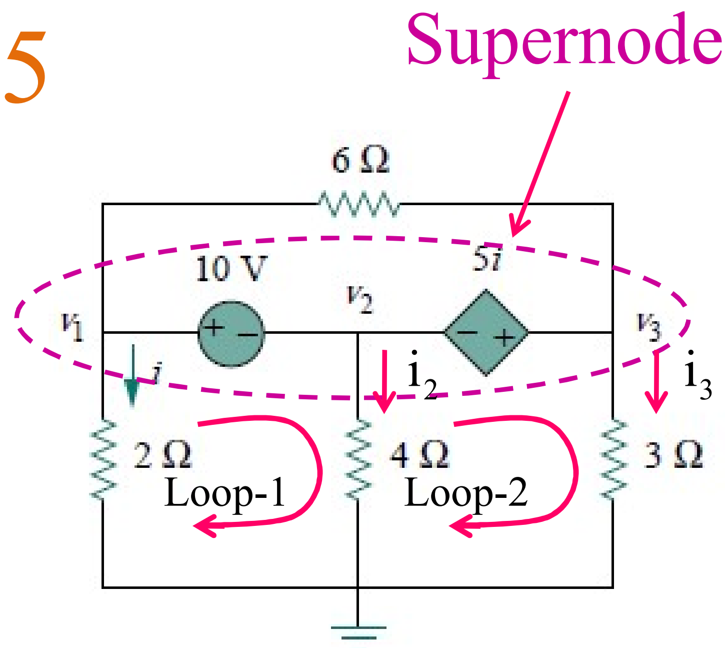
$$\text{Or,} \quad V_2 + 5\frac{V_1}{2} - V_3 = 0$$

$$\text{Or,} \quad 5V_1 + 2V_2 - 2V_3 = 0 \quad \ldots\ldots.(3)$$

$$\boxed{i = \frac{V_1 - 0}{2}}$$

Example 5

Find the voltages at the three non-reference nodes in the given circuit.



Solving equations (1), (2) and (3),

$$V_1 = 3.043 \text{ V}$$

$$V_2 = -6.956 \text{ V}$$

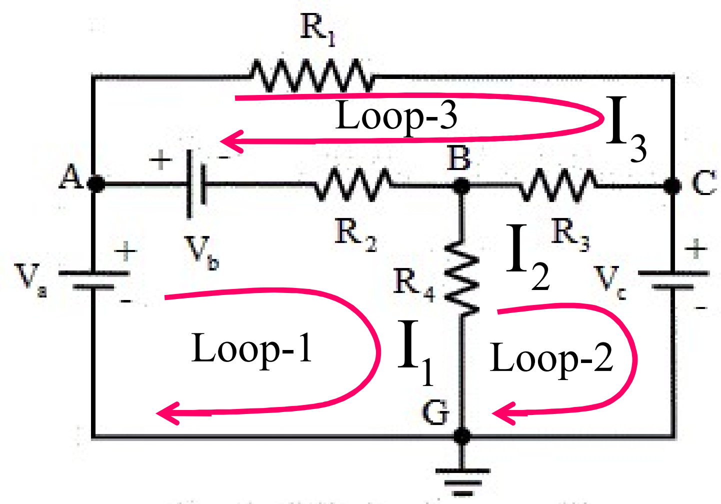
$$V_3 = 0.652 \text{ V}$$

Mesh or Loop Analysis

A mesh has the following properties

(i) every node in the closed path is exactly formed with two branches

(ii) no other branches are enclosed by the closed path.



Mesh or Loop Analysis

Steps to determine mesh currents:

Step-1: Draw the circuit on a flat surface with no conductor crossovers.

Step-2: Assign mesh currents $i_1, i_2, \ldots, i_n$ to the n meshes.

Step-3: Apply KVL to each of the n meshes. Use Ohm's law to express the voltages in terms of the mesh currents.

Step-4: Solve the resulting n simultaneous equations to get the mesh currents.

Mesh or Loop Analysis

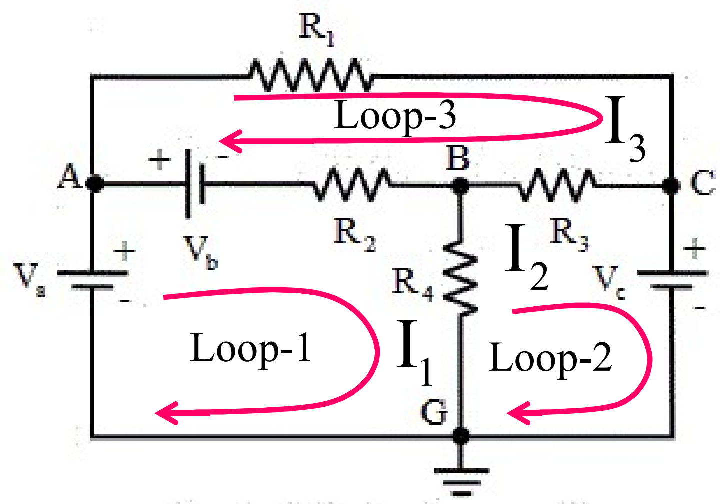
Applying KVL around mesh (loop)-1:

$$V_a - V_b - (I_1 - I_3)R_2 - (I_1 - I_2)R_4 = 0$$

$$V_a - V_b = (R_2 + R_4)I_1 - R_4 I_2 - R_2 I_3$$

$$V_a - V_b = R_{11}I_1 - R_{12}I_2 - R_{13}I_3$$

.........(1)



Mesh or Loop Analysis

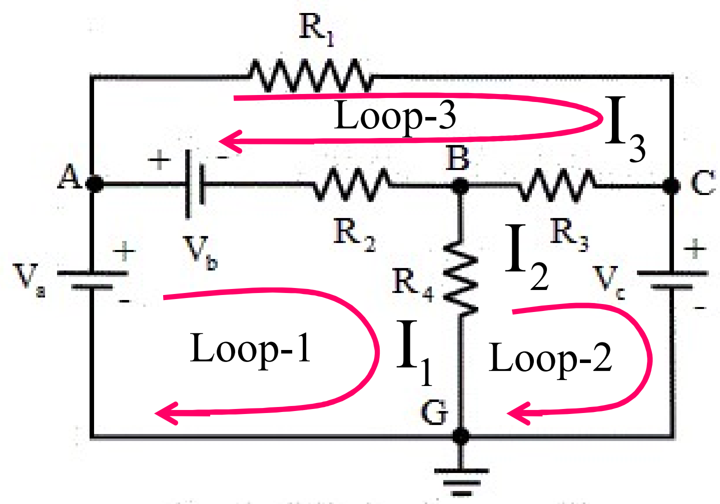
Applying KVL around mesh (loop)-2:

$$-V_c - (I_2 - I_1)R_4 - (I_2 - I_3)R_3 = 0$$

$$-V_c = -R_4 I_1 + (R_4 + R_3) I_2 - R_3 I_3$$

$$-V_c = -R_{21} I_1 + R_{22} I_2 - R_{23} I_3$$

.........(2)



Mesh or Loop Analysis

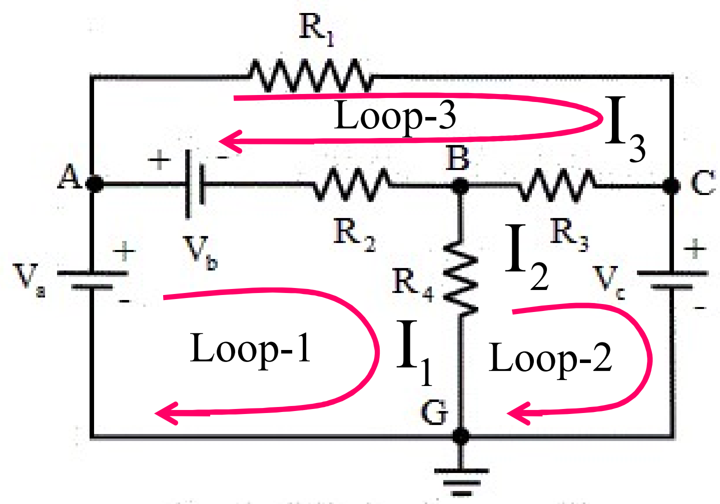
Applying KVL around mesh (loop)-3:

$$V_b - I_3R_1 - (I_3 - I_2)R_3 - (I_3 - I_1)R_2 = 0$$

$$V_b = -R_2I_1 - R_3I_2 + (R_1 + R_2 + R_3)I_3$$

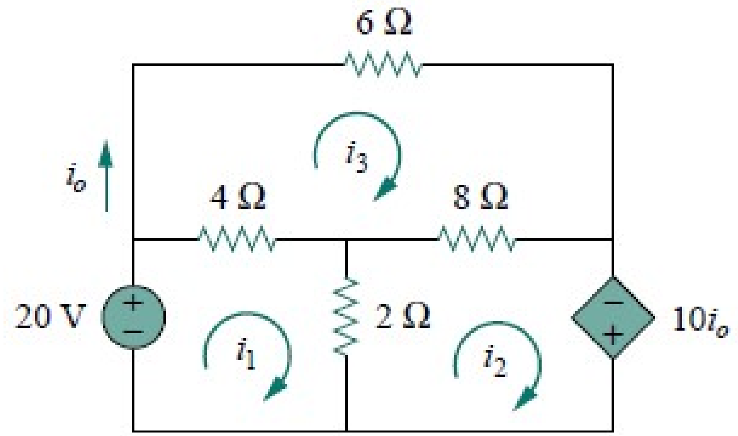
$$V_b = -R_{31}I_1 - R_{32}I_2 + R_{33}I_3$$

.........(3)



Example 6

Using mesh analysis, find i_0 in the given circuit.



From Loop3,

$i_3 = i_0$ …….(1)

Applying KVL around loop1,

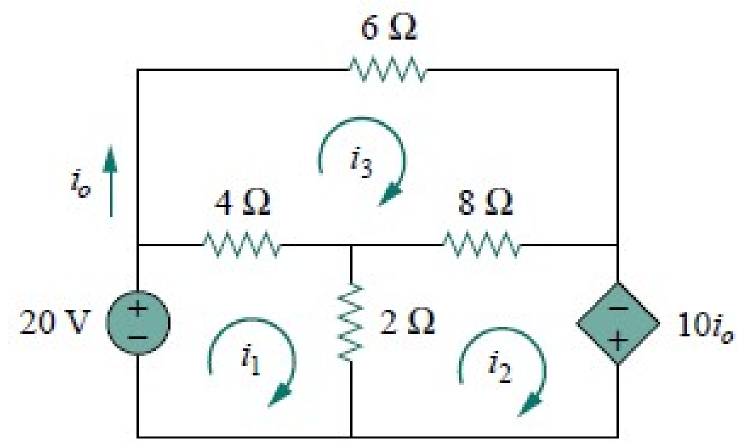
$$20 - (i_1 - i_3) \cdot 4 - (i_1 - i_2) \cdot 2 = 0$$

Or, $6i_1 - 2i_2 - 4i_3 = 20$

…….(2)

Example 6

Using mesh analysis, find i_0 in the given circuit.



Applying KVL around loop2,

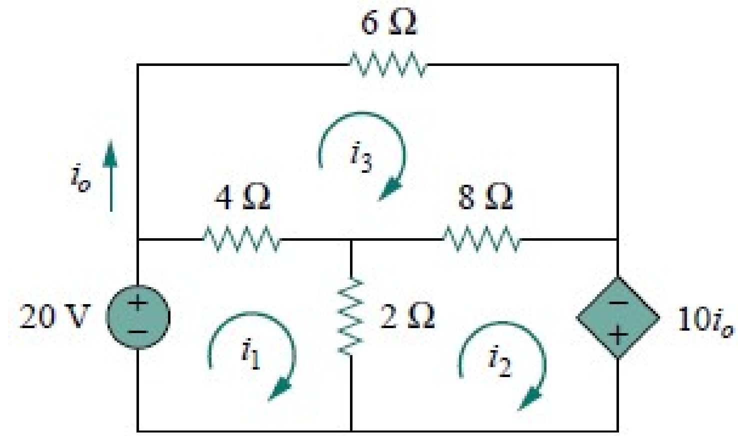
$$-(i_2 - i_1)\cdot 2 - (i_2 - i_3)\cdot 8 + 10i_0 = 0$$

Or, $2i_1 - 10i_2 + 18i_3 = 0$

…….(3)

Example 6

Using mesh analysis, find i_0 in the given circuit.



Applying KVL around loop3,

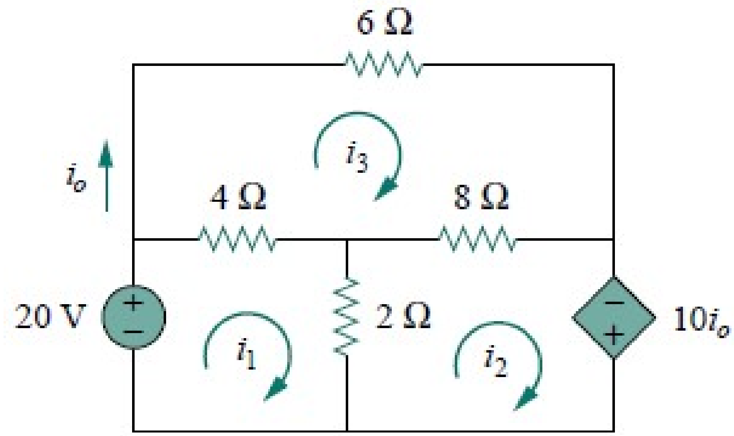
$$-6i_3-(i_3-i_2)\cdot 8-(i_3-i_1)\cdot 4=0$$

Or, $4i_1+8i_2-18i_3=0$

…….(4)

Example 6

Using mesh analysis, find i_0 in the given circuit.



Solving equations (2), (3) and (4),

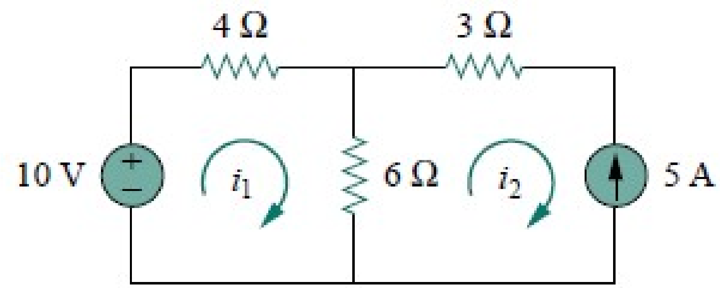
$$i_3 = -5 \text{ A}$$

$$\therefore\ i_0 = -5 \text{ A}$$

Mesh Analysis with Current Source

Case-I:

When a current source exists only in one mesh: Consider the circuit shown in the figure, for example.



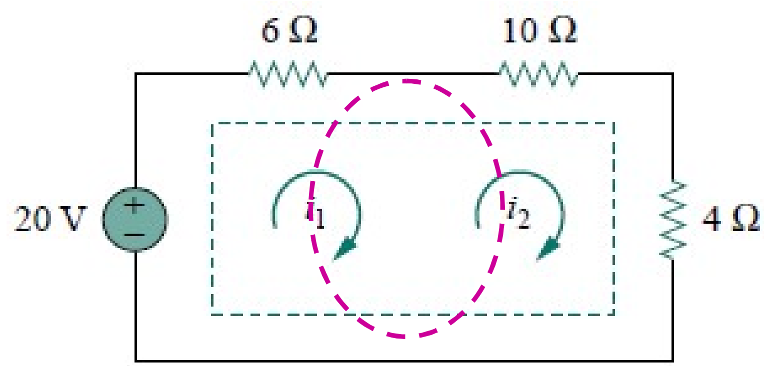
We set $i_2 = -5$ A and write a mesh equation for the other mesh in the usual way, that is

$$10 - 4i_1 - (i_1 - i_2) \cdot 6 = 0 \qquad \Rightarrow \; i_1 = -2\ \text{A}$$

Mesh Analysis with Current Source

Case-II:

When a current source exists between two meshes: Consider the circuit shown in the figure, for example.

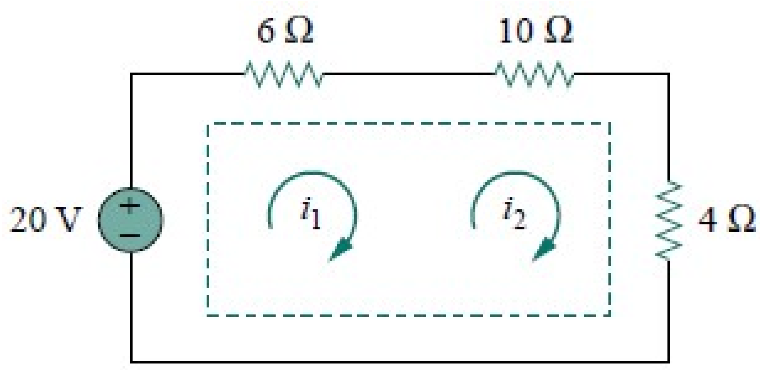


We create a supermesh by excluding the current source and any elements connected in series with it.

Mesh Analysis with Current Source

Case-II:

Thus, a supermesh results when two meshes have a (dependent or independent) current source in common.



However, a supermesh must satisfy KVL like any other mesh.

- Therefore, applying KVL to the supermesh gives

$$20 - 6i_1 - 10i_2 - 4i_2 = 0 \qquad \text{Or, } 6i_1 + 14i_2 = 20 \quad ....(1)$$

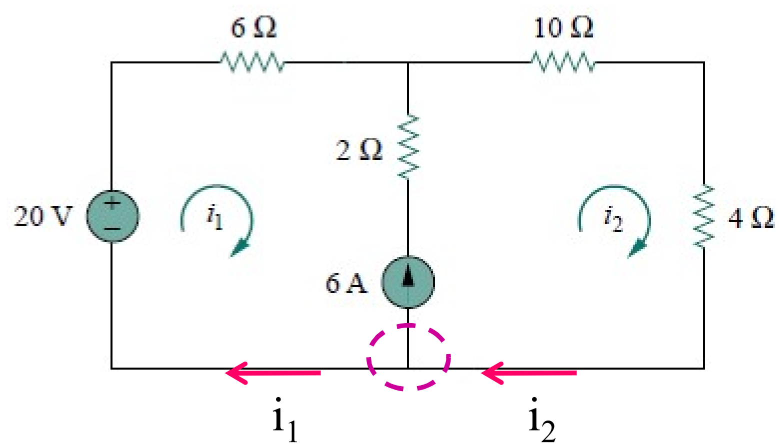
Mesh Analysis with Current Source

Case-II:

We apply KCL to a node in the branch where the two meshes intersect.

Applying KCL,

$$i_2 = i_1 + 6 \qquad \text{........(2)}$$



Solving equations (1) and (2),

$i_1 = -3.2\ \text{A}$ $\qquad$ $i_2 = 2.8\ \text{A}$

Syllabus

DC Networks:

- Kirchhoff’s laws
- Node voltage and mesh current methods
- Delta-star and star-delta conversions
- Superposition principle
- Thevenin’s and Norton’s theorems
- Maximum Power Transfer Theorem.

Syllabus

DC Networks:

- Kirchhoff's laws
- Node voltage and mesh current methods
- Delta-star and star-delta conversions
- Superposition principle
- Thevenin's and Norton's theorems
- Maximum Power Transfer Theorem.

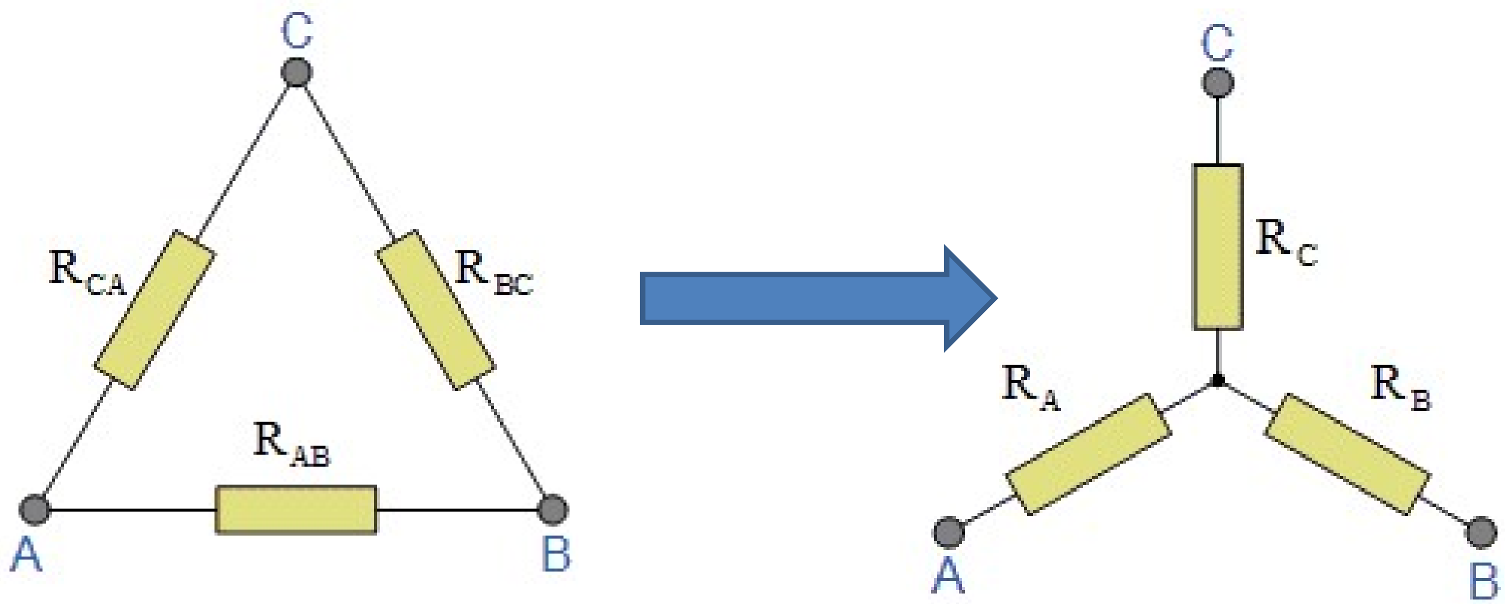
Delta-star and Star-delta conversions

Delta-Star and Star-Delta Conversions

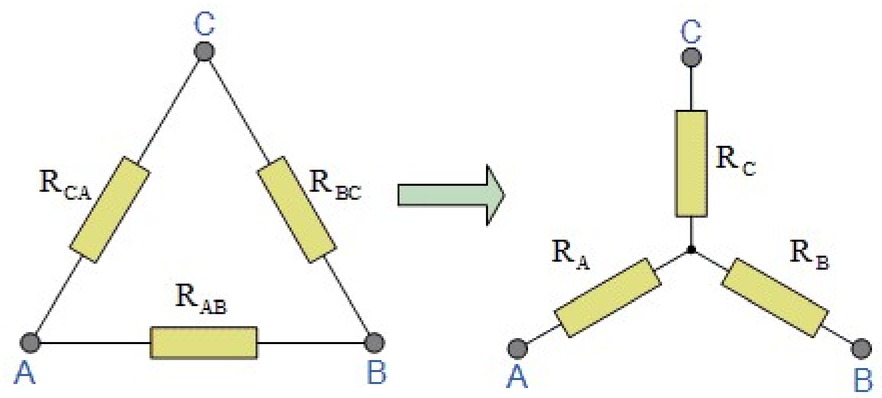
In the complicated networks involving large number of elements, Delta to Star and Star to Delta conversion considerably reduce the complexity of the network and network can be analyzed very quickly.

These transformations allow us to replace three star connected elements of network, by equivalent delta connected elements without affecting currents in other branches and vice versa.

Delta-Star Conversions



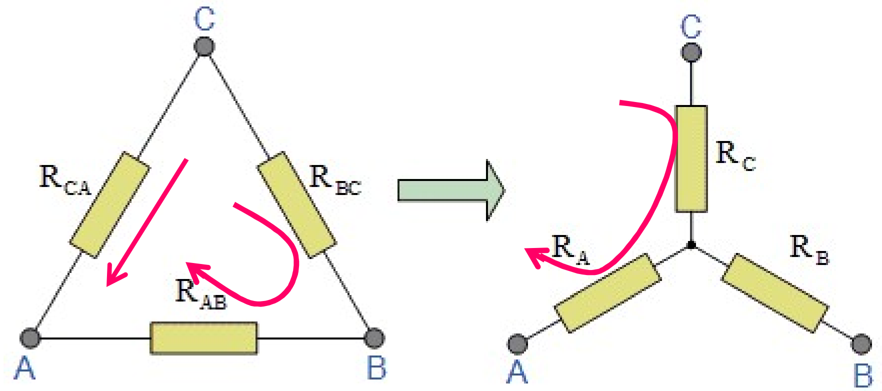
Delta-Star Conversions



It is assumed that the delta resistances (R_{AB}, R_{BC} and R_{CA}) are known.

Our problem is to find the values of R_A, R_B and R_C in star network that will produce the same resistance when measured between similar pairs of terminals.

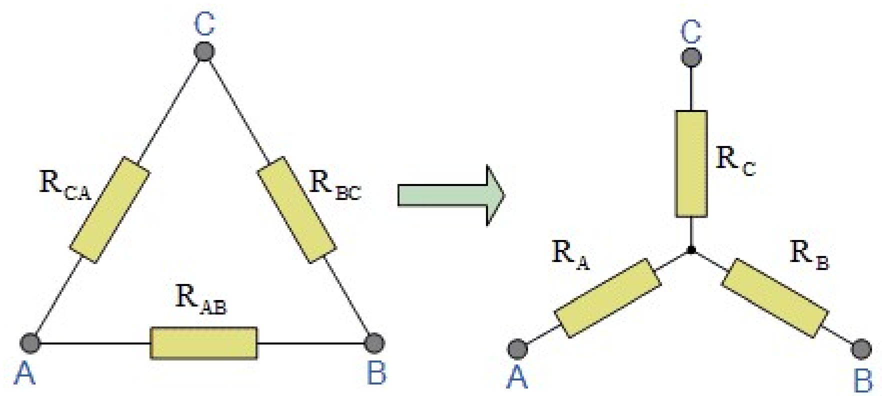
Delta-Star Conversions



Between A-C terminals:

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(1)$$

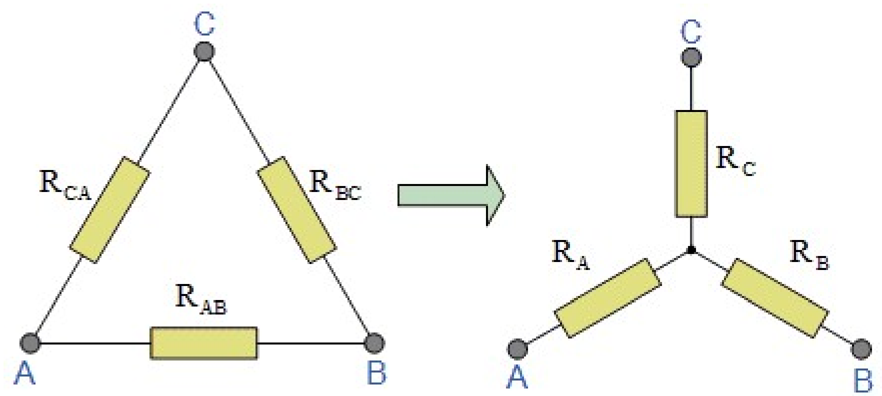
Delta-Star Conversions



Between C-B terminals:

$$R_C + R_B = \frac{R_{BC}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \dots\dots\dots\dots\dots\dots(2)$$

Delta-Star Conversions



Between B-A terminals:

$$R_B + R_A = \frac{R_{AB}\left(R_{BC} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \text{.................}(3)$$

Delta-Star Conversions

Adding equations 1,2 and 3

$$R_A + R_C = \frac{R_{CA}\left(R_{AB} + R_{BC}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A + R_B + R_C = \frac{\left(R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(4)$$

Subtracting equation 1 from 4,

$$R_B = \frac{R_{AB}R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(5)$$

Delta-Star Conversions

Adding equations 1,2 and 3

$$R_C + R_B = \frac{R_{BC}\left(R_{AB} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A + R_B + R_C = \frac{\left(R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(4)$$

Subtracting equation 2 from 4,

$$R_A = \frac{R_{AB}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(6)$$

Delta-Star Conversions

Adding equations 1,2 and 3

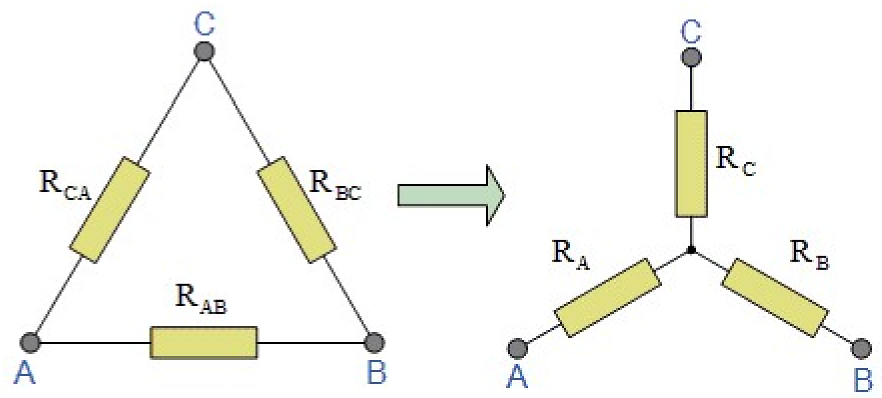
$$R_B + R_A = \frac{R_{AB}\left(R_{BC} + R_{CA}\right)}{R_{AB} + R_{BC} + R_{CA}}$$

$$R_A + R_B + R_C = \frac{\left(R_{AB}R_{BC} + R_{BC}R_{CA} + R_{CA}R_{AB}\right)}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(4)$$

Subtracting equation 3 from 4,

$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad \ldots\ldots\ldots(7)$$

Delta-Star Conversions

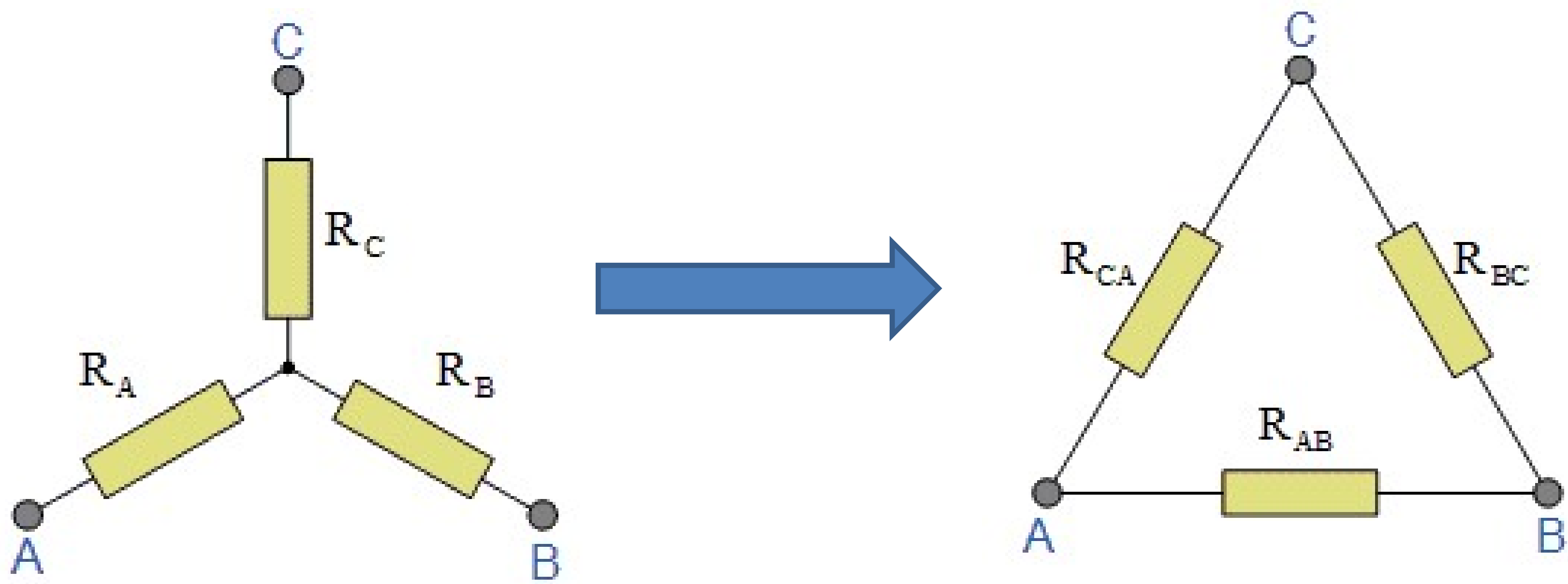


Observation:

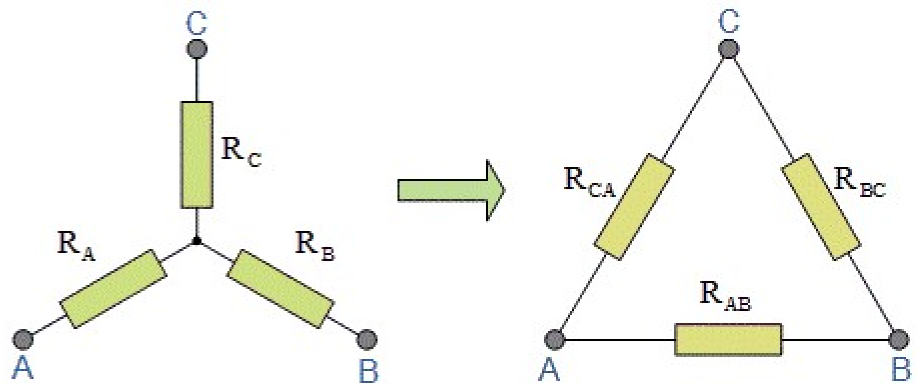
The equivalent Y-resistance connected to a given terminal is equal to the product of the two Δ-resistances connected to the same terminal divided by the sum of the Δ-resistances .

$$R_A = \frac{R_{AB} R_{CA}}{R_{AB} + R_{BC} + R_{CA}} \qquad R_B = \frac{R_{AB} R_{BC}}{R_{AB} + R_{BC} + R_{CA}} \qquad R_C = \frac{R_{BC} R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

Star-Delta Conversions



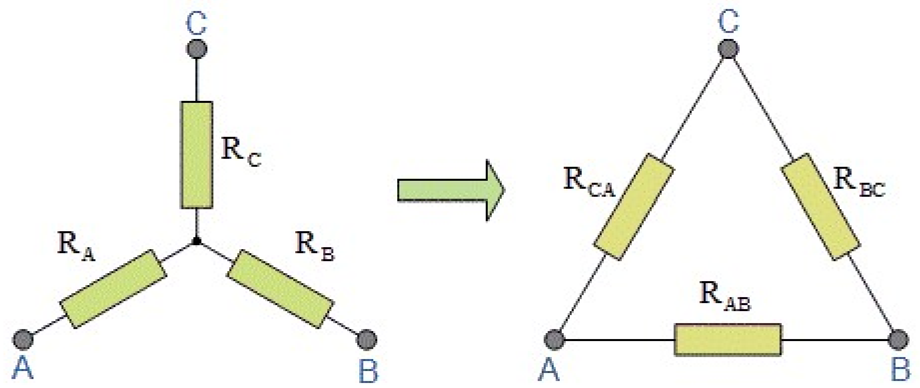
Star-Delta Conversions



It is assumed that the star resistances (R_A, R_B and R_C) are known.

Our problem is to find the values of R_{AB}, R_{BC} and R_{CA} in delta network that will produce the same resistance when measured between similar pairs of terminals.

Star-Delta Conversions

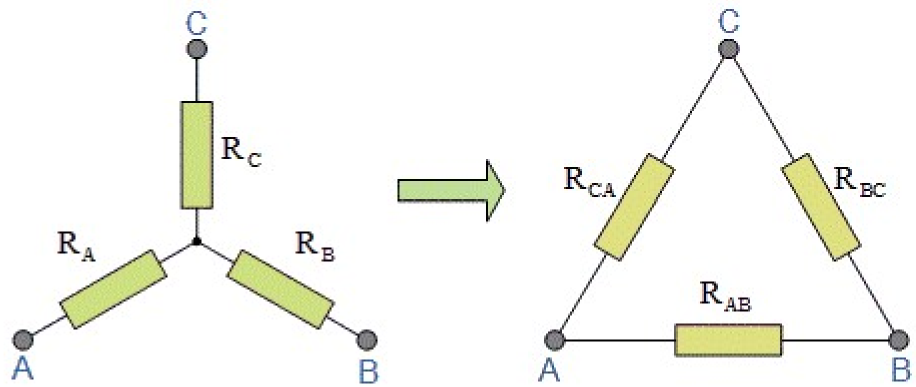


Using equations 5, 6 and 7,

$$R_A R_B + R_B R_C + R_C R_A = \frac{R_{AB} R_{BC} R_{CA}\left(R_{AB} + R_{BC} + R_{CA}\right)}{\left(R_{AB} + R_{BC} + R_{CA}\right)^2}$$

$$\text{Or, } R_A R_B + R_B R_C + R_C R_A = \frac{R_{AB} R_{BC} R_{CA}}{\left(R_{AB} + R_{BC} + R_{CA}\right)}$$

Star-Delta Conversions



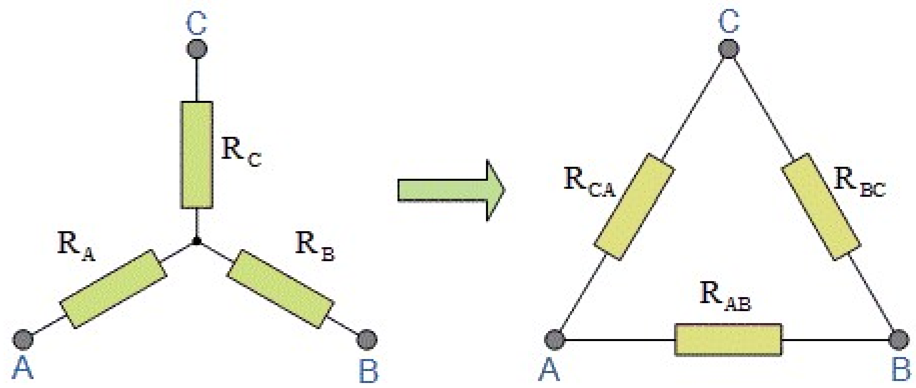
$$R_C = \frac{R_{BC}R_{CA}}{R_{AB} + R_{BC} + R_{CA}}$$

$$\text{Or, } R_{AB} \bullet \frac{R_{BC}R_{CA}}{\left(R_{AB} + R_{BC} + R_{CA}\right)} = R_A R_B + R_B R_C + R_C R_A$$

$$\text{Or, } R_{AB} \bullet R_C = R_A R_B + R_B R_C + R_C R_A$$

$$\text{Or, } R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C}$$

Star-Delta Conversions

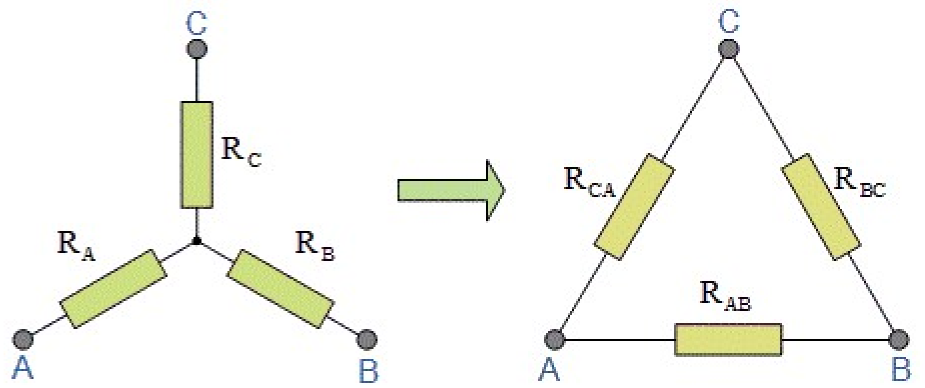


Similarly,

$$R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A}$$

$$R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

Star-Delta Conversions



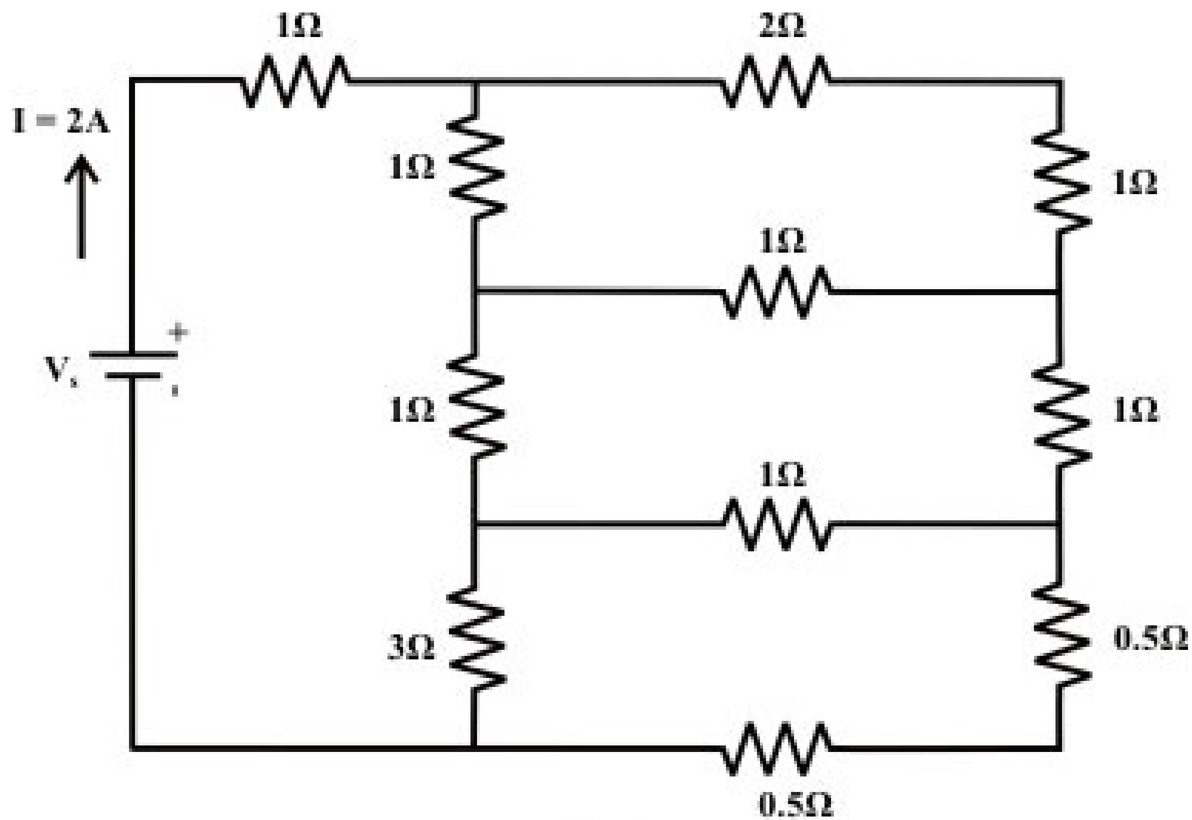
Observation:

The equivalent Δ-resistance between two terminals is the sum of the two Y-resistances connected to those terminals plus the product of the same two resistances divided by the third resistance.

$$R_{AB} = R_A + R_B + \frac{R_A R_B}{R_C} \qquad R_{BC} = R_B + R_C + \frac{R_B R_C}{R_A} \qquad R_{CA} = R_C + R_A + \frac{R_C R_A}{R_B}$$

Problem

Find the value of the voltage source (V_s) that delivers 2A current through the circuit as shown in the figure.



Syllabus

DC Networks:

- Kirchhoff's laws
- Node voltage and mesh current methods
- Delta-star and star-delta conversions
- Superposition principle
- Thevenin's and Norton's theorems
- Maximum Power Transfer Theorem.

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Superposition Principle

Superposition Principle

- If a circuit has two or more independent sources, one way to determine the value of a specific variable (voltage or current) is to use nodal or mesh analysis.
- Another way is to determine the contribution of each independent source to the variable and then add them up. The latter approach is known as the *superposition*.

Superposition Principle

The *superposition principle* states that the voltage across (or current through) an element in a linear circuit is the algebraic sum of the voltages across (or currents through) that element due to each independent source acting alone.

Superposition Principle

How to apply superposition principle?

- We consider one independent source at a time while all other independent sources are turned off. This implies that we replace every voltage source by 0 V (or a short circuit), and every current source by 0 A (or an open circuit).
- Dependent sources are left intact because they are controlled by circuit variables.

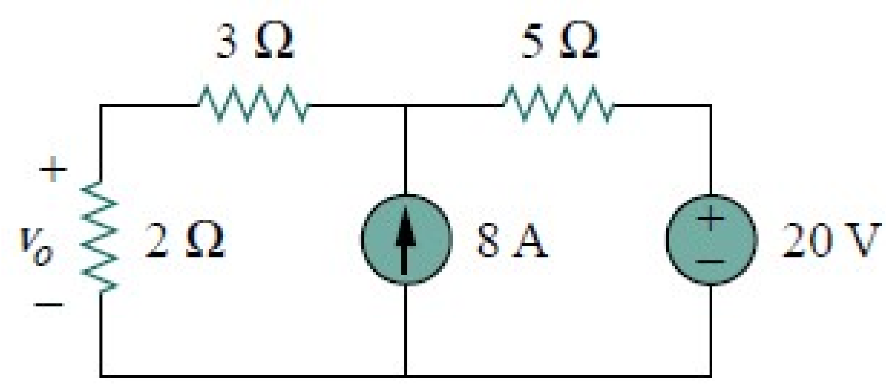
Superposition Principle

Steps to apply Superposition Principle:

1. Turn off all independent sources except one source. Find the output (voltage or current) due to that active source using nodal or mesh analysis.
2. Repeat step 1 for each of the other independent sources.
3. Find the total contribution by adding algebraically all the contributions due to the independent sources.

Example 7

Using the superposition theorem, find v_0 in the given circuit.



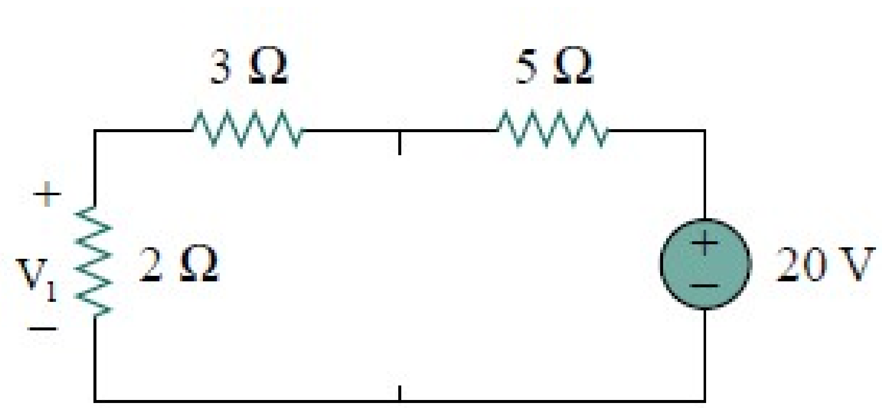
Let,

$$V_0 = V_1 + V_2$$

where, v_1 and v_2 are the contributions due to the 20V voltage source and the 8A current source, respectively.

Example 7

Using the superposition theorem, find v_0 in the given circuit.

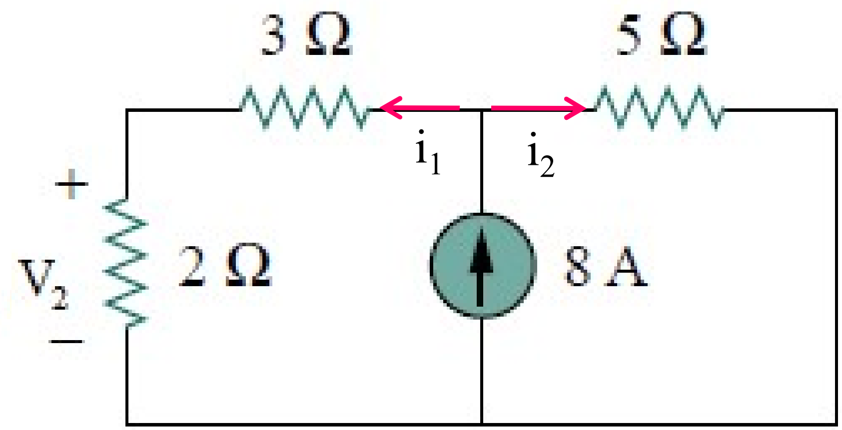


Deactivating the current source and applying voltage divider rule,

$$V_1 = \left(\frac{2}{2+3+5}\right)\times 20\ \text{V} \qquad = 4\ \text{V}$$

Example 7

Using the superposition theorem, find v_0 in the given circuit.



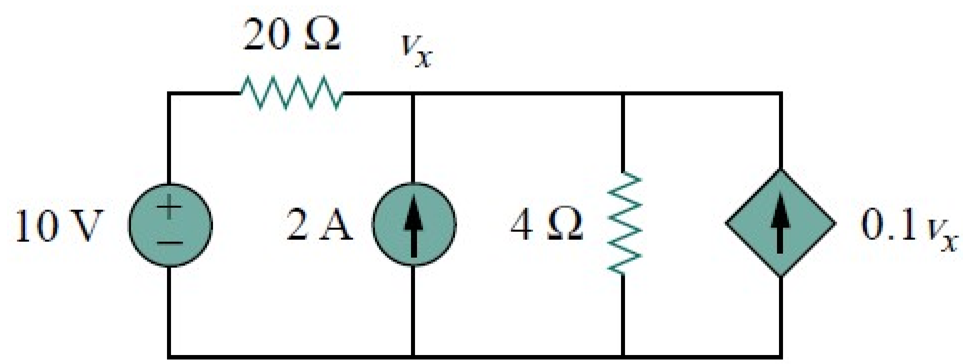
Deactivating the voltage source and applying current divider rule,

$$i_1 = \left(\frac{5}{2+3+5}\right)\times 8\ \text{A} \qquad = 4\ \text{A}$$

$$\therefore\ V_2 = 4\times 2\ \text{V} \quad = 8\ \text{V} \qquad\qquad \therefore\ V_0 = (4+8)\ \text{V} = 12\ \text{V}$$

Example 8

Using the superposition theorem, find v_x in the given circuit.



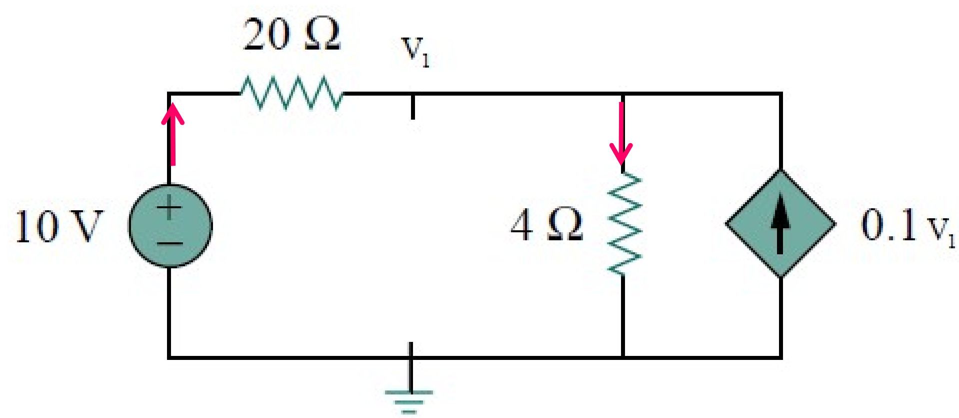
Let,

$$V_x = V_1 + V_2$$

where, v_1 and v_2 are the contributions due to the 10V voltage source and the 2A current source, respectively.

Example 8

Using the superposition theorem, find v_x in the given circuit.



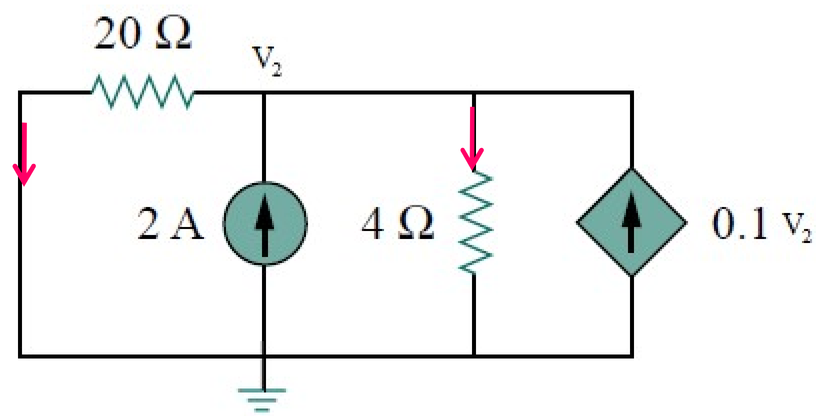
Deactivating the current source and using node analysis,

$$\frac{10 - V_1}{20} + 0.1V_1 = \frac{V_1 - 0}{4}$$

Or, $4V_1 = 10$ $\Rightarrow V_1 = 2.5\ \text{V}$

Example 8

Using the superposition theorem, find v_x in the given circuit.



Deactivating the voltage source and using node analysis,

$$2 + 0.1V_2 = \frac{V_2 - 0}{20} + \frac{V_2 - 0}{4}$$

Or, $4V_2 = 40$ $\Rightarrow V_2 = 10\ \text{V}$ $\therefore\ V_x = (2.5 + 10)\ \text{V} = 12.5\ \text{V}$

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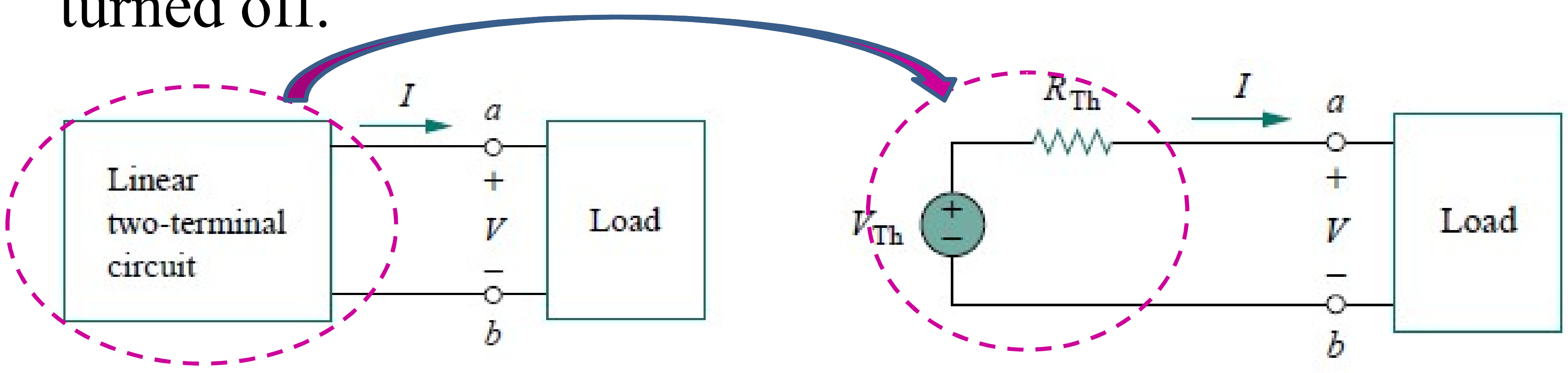
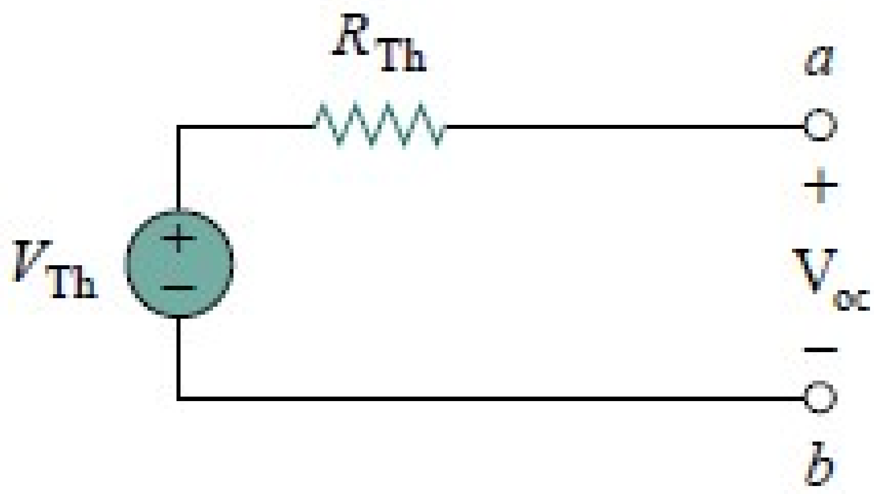
Thevenin’s Theorem

Thevenin's Theorem

Thevenin's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a voltage source V_{th} in series with a resistor R_{th}, where V_{th} is the open-circuit voltage at the terminals and R_{th} is the input or equivalent resistance at the terminals when the independent sources are turned off.



Thevenin's Theorem



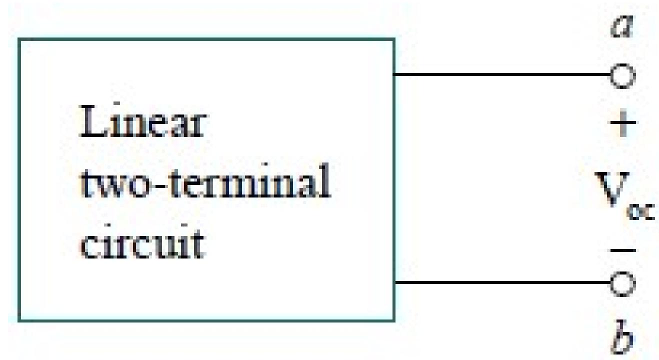
Thevenin's equivalent circuit

Thevenin’s Theorem

Steps to determine Thevenin’s equivalent circuit:

Step-1: Disconnect the load from the circuit.

Step-2: Calculate the open-circuit voltage (V_{oc}) at the load terminals applying mesh-current or node-voltage or superposition method.



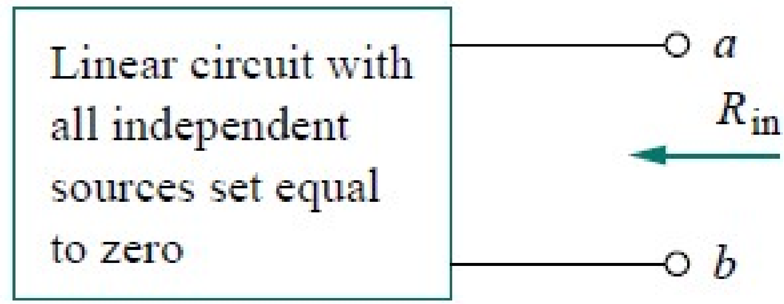
$$V_{th} = V_{oc}$$

Thevenin's Theorem

Steps to determine Thevenin's equivalent circuit:

Step-3: Calculation of internal resistance.

Case-I: If the network has no dependent sources, we turn off all independent sources. R_{th} is the input resistance of the network looking between terminals a and b.

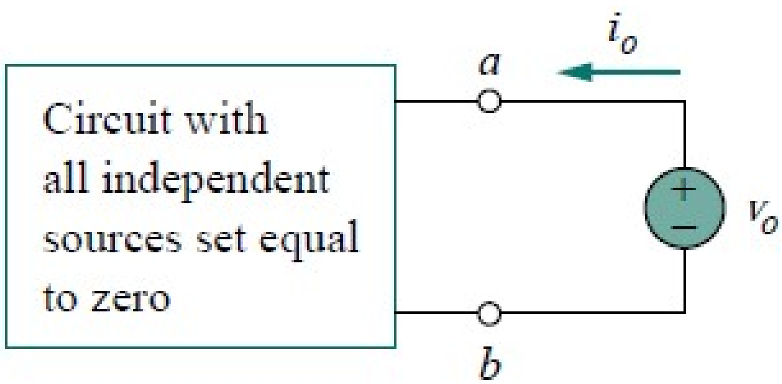


$$R_{th} = R_{in}$$

Thevenin's Theorem

Steps to determine Thevenin's equivalent circuit:

Step-3: Calculation of internal resistance.



Case-II: If the network has dependent sources, we turn off all independent sources.

We apply a voltage source v_0 at terminals a and b and determine the resulting current i_0.

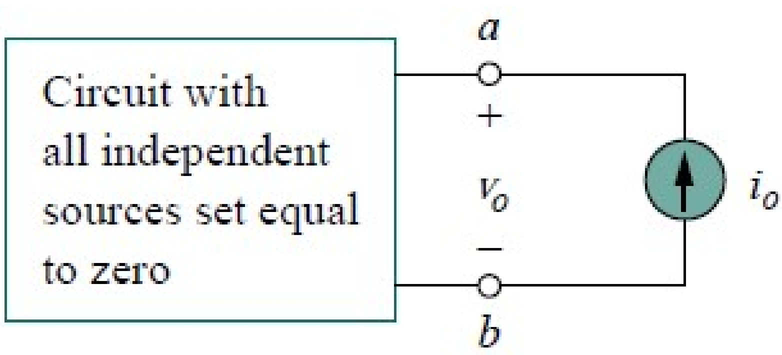
Then,

$$R_{th} = \frac{v_0}{i_0}$$

Thevenin's Theorem

Steps to determine Thevenin's equivalent circuit:

Step-3: Calculation of internal resistance.



Case-II: If the network has dependent sources, we turn off all independent sources.

Alternatively, we apply a current source i_0 at terminals a and b and determine the terminal voltage v_0.

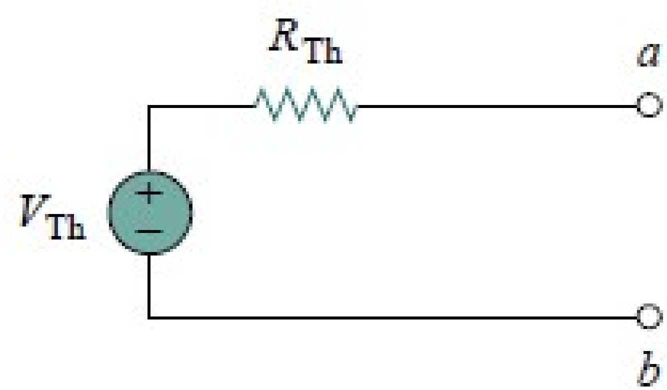
Then,

$$R_{th} = \frac{v_0}{i_0}$$

Thevenin's Theorem

Steps to determine Thevenin's equivalent circuit:

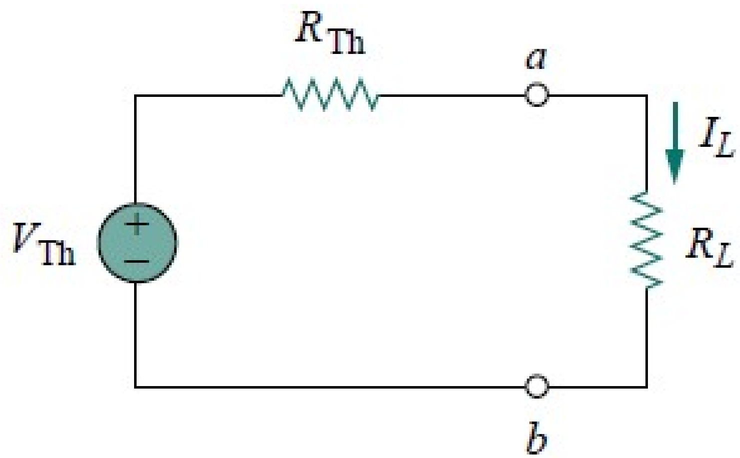
Step-4: Place R_{th} in series with V_{th} to form the Thevenin's equivalent circuit.



Thevenin's Theorem

Calculation of load current and load voltage:

Reconnect the original load to the Thevenin's equivalent circuit.



Therefore, load current,

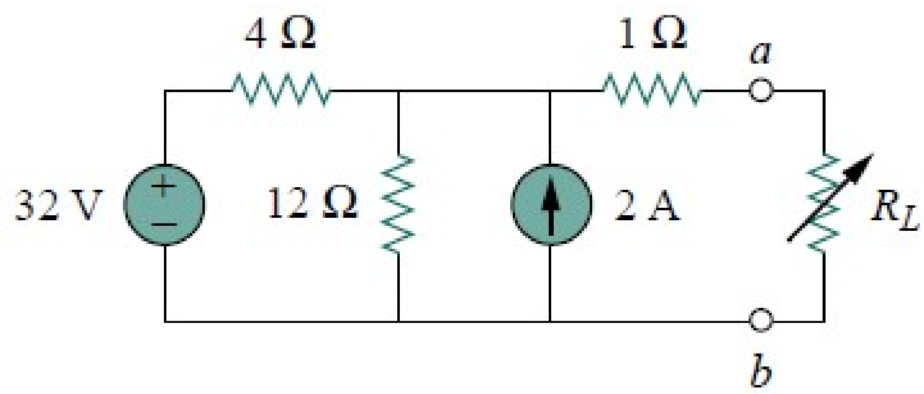
$$I_L = \frac{V_{th}}{R_{th} + R_L}$$

Voltage across the load,

$$V_L = I_L R_L = \frac{R_L}{R_{th} + R_L} V_{th}$$

Example 9

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b. Then find the current through R_L = 6, 16 and 36 ohm.



Example 9

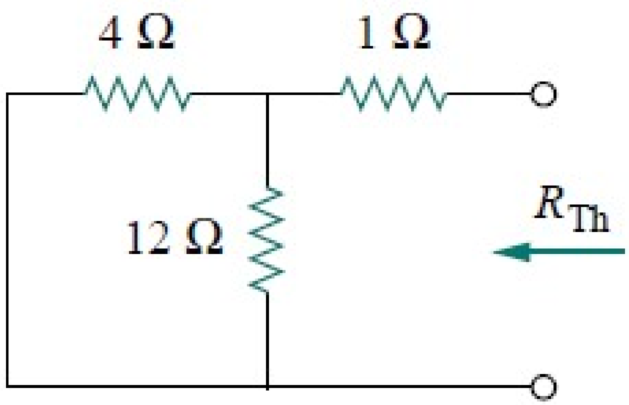
Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b. Then find the current through R_L = 6, 16 and 36 ohm.

Deactivating all the independent sources,

$$R_{Th} = \left[(4 \parallel 12) + 1 \right] \Omega$$

$$= \left[\left(\frac{4 \times 12}{4 + 12} \right) + 1 \right] \Omega$$

$$= [3 + 1] \Omega \quad = 4 \Omega$$



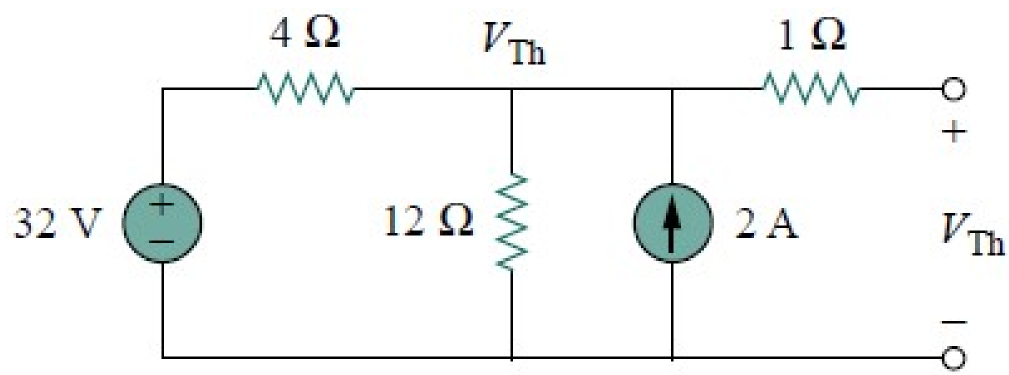
Example 9

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b. Then find the current through R_L = 6, 16 and 36 ohm.

Applying node analysis,

$$\frac{32 - V_{Th}}{4} + 2 = \frac{V_{Th}}{12}$$

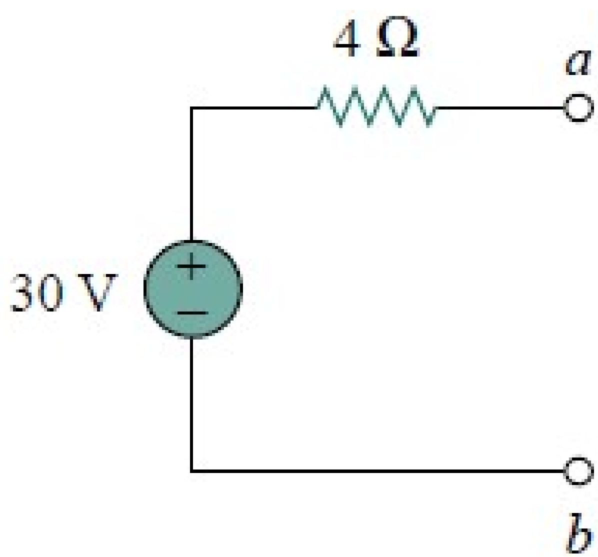
$$\therefore\ V_{Th} = 30\ V$$



Example 9

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b. Then find the current through R_L = 6, 16 and 36 ohm.

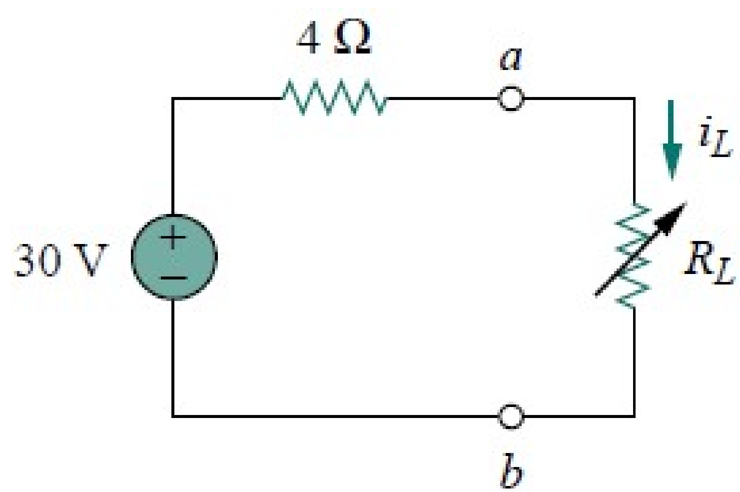
Therefore, the Thevenin's equivalent circuit is



Example 9

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b. Then find the current through R_L = 6, 16 and 36 ohm.

Now, reconnecting the load,



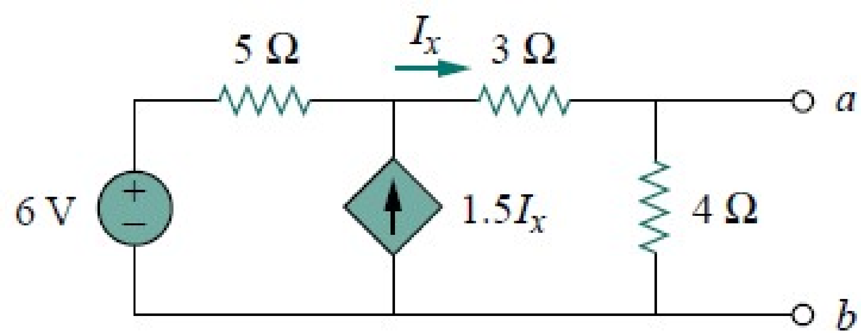
$$i_L\big|_{R_L=6} = \frac{30}{4+6}\ \text{A} = 3\ \text{A}$$

$$i_L\big|_{R_L=16} = \frac{30}{4+16}\ \text{A} = 1.5\ \text{A}$$

$$i_L\big|_{R_L=36} = \frac{30}{4+36}\ \text{A} = 0.75\ \text{A}$$

Example 10

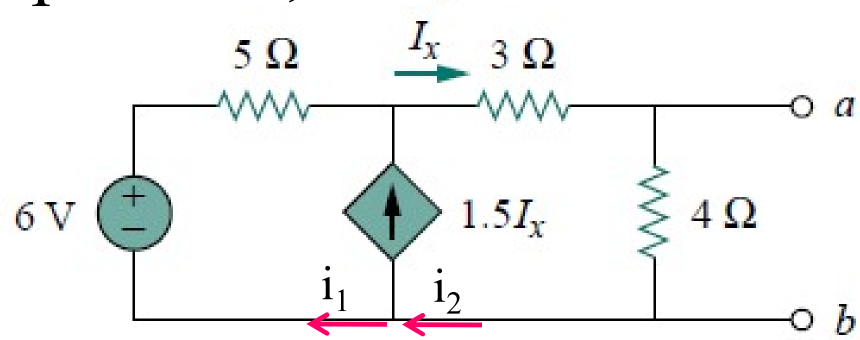
Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.



Example 10

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Applying KVL around the supermesh,



$$6-5i_1-3i_2-4i_2=0$$

$$\text{Or,}\ \ 5i_1+7i_2=6 \qquad ....(1)$$

Applying KCL,

$$i_2=i_1+1.5I_x \qquad ....(2)$$

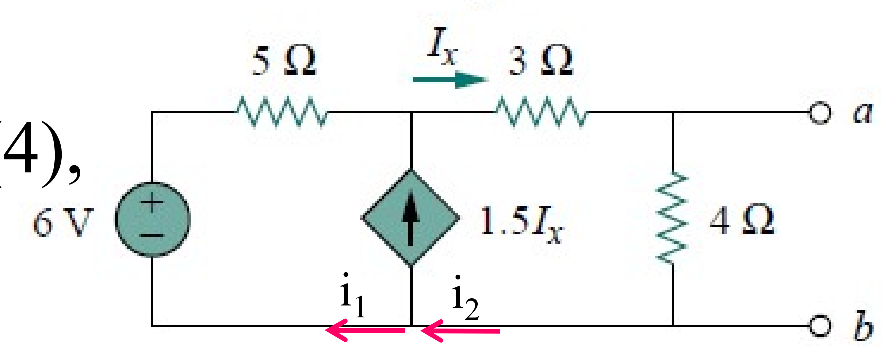
Also, $I_x=i_2 \qquad ....(3)$

Example 10

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Therefore equation (2) becomes,

$$i_1 + 0.5i_2 = 0 \quad ....(4)$$



Solving equations (1) and (4),

$$i_2 = \frac{12}{9} \text{ A}$$

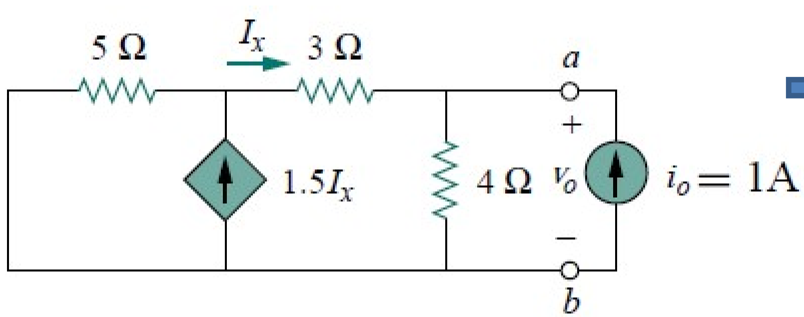
Therefore,

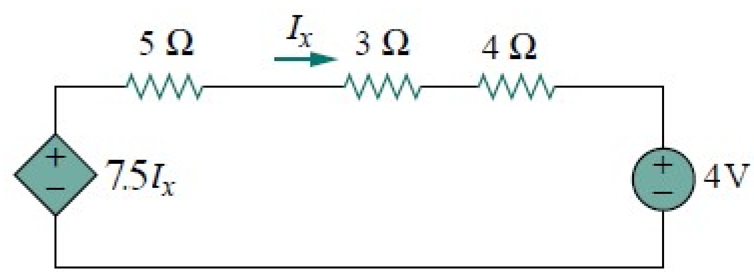
$$V_{Th} = V_{ab} = 4 \times \frac{12}{9} \text{ V} = 5.33 \text{ V}$$

Example 10

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Deactivating 6V source and connecting a 1A current source across a-b terminals,





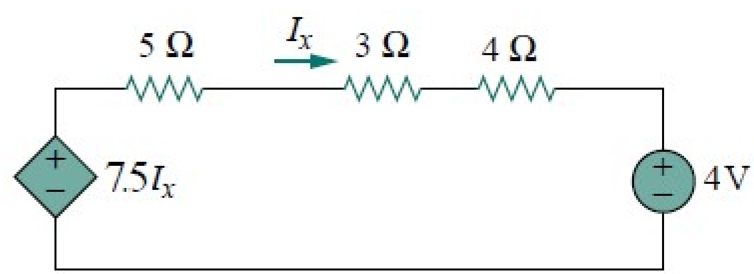
Example 10

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Applying KVL around the loop,

$$7.5I_x - (5+3+4)I_x - 4 = 0$$

$$\text{Or, } I_x = -\frac{4}{4.5}\text{ A}$$



Example 10

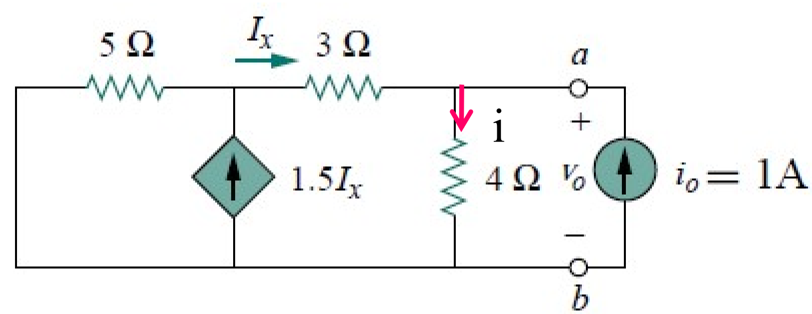
Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Now, current through 4 Ω resistor

$$i = i_0 + I_x$$

$$= \left(1 - \frac{4}{4.5}\right) \text{ A} = \frac{0.5}{4.5} \text{ A}$$

$$\therefore\ v_0 = 4 \times \frac{0.5}{4.5} \text{ V} = \frac{4}{9} \text{ V}$$

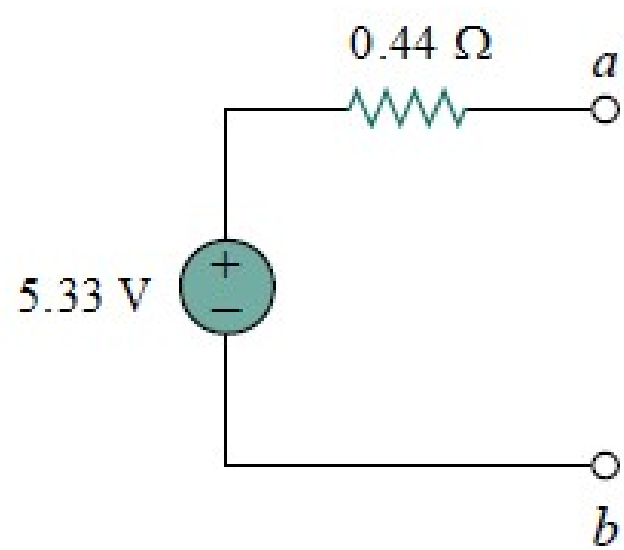


$$R_{Th} = \frac{v_0}{i_0} = \frac{\left(4/9\right)}{1}\ \Omega = 0.44\ \Omega$$

Example 10

Find the Thevenin equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

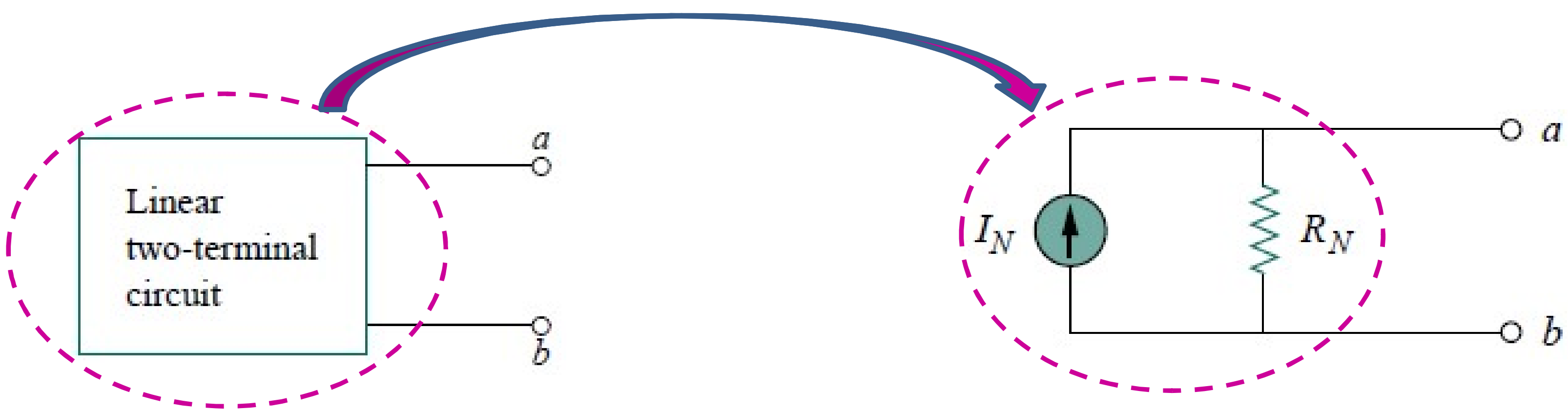
Therefore, the Thevenin's equivalent circuit is



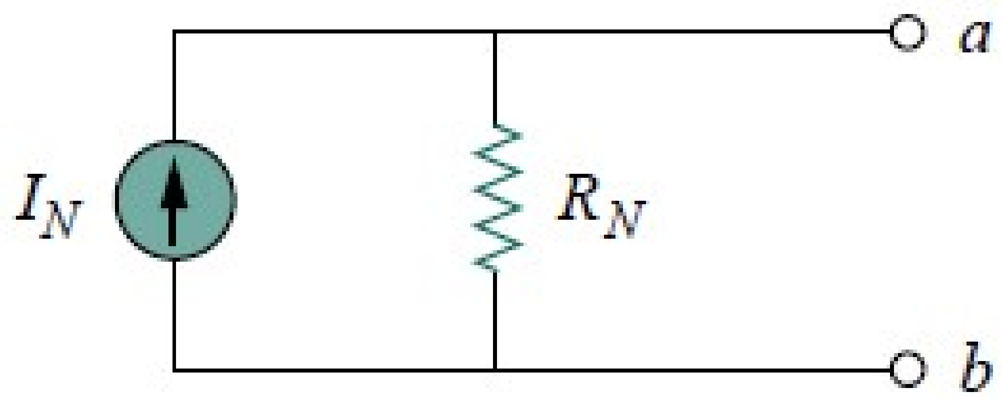
Norton’s Theorem

Norton's Theorem

Norton's theorem states that a linear two-terminal circuit can be replaced by an equivalent circuit consisting of a current source I_N in parallel with a resistor R_N, where I_N is the short-circuit current through the terminals and R_N is the input or equivalent resistance at the terminals when the independent sources are turned off.



Norton's Theorem

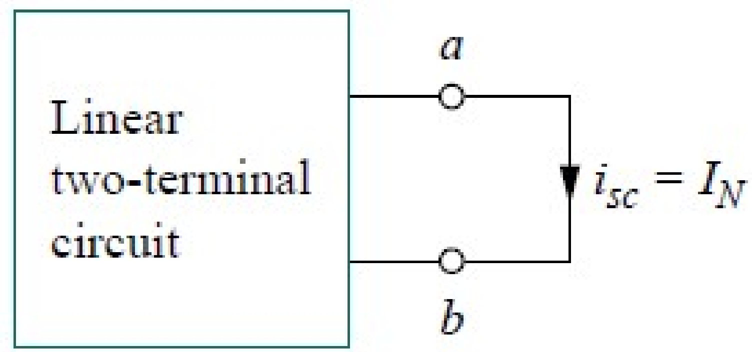


Norton's equivalent circuit

Norton's Theorem

Steps to determine Norton's equivalent circuit:

Step-1: Short the output terminals after disconnecting the load resistance (R_L) from the terminals a-b and then calculate the short circuit current I_N.



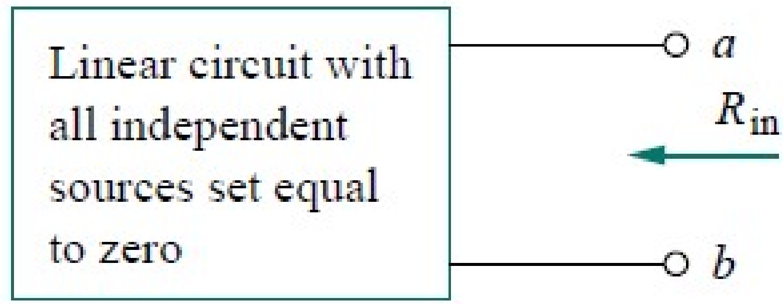
$$I_N = i_{sc}$$

Norton's Theorem

Steps to determine Norton's equivalent circuit:

Step-2: Calculation of internal resistance.

Calculation of R_N is same as that of R_{Th}.

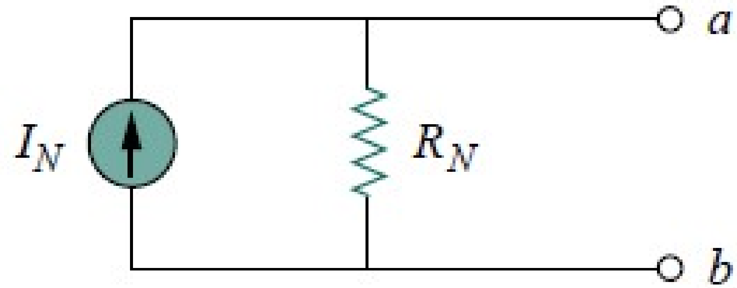


$$R_{Th} = R_N$$

Norton's Theorem

Steps to determine Norton's equivalent circuit:

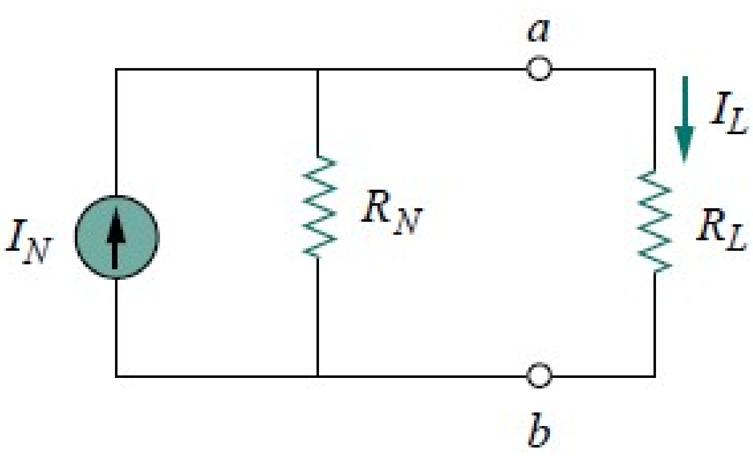
Step-3: Place R_N in parallel with current I_N to form the Norton's equivalent circuit.



Norton's Theorem

Calculation of load current and load voltage:

Reconnect the original load to the Norton's equivalent circuit.



Therefore, load current,

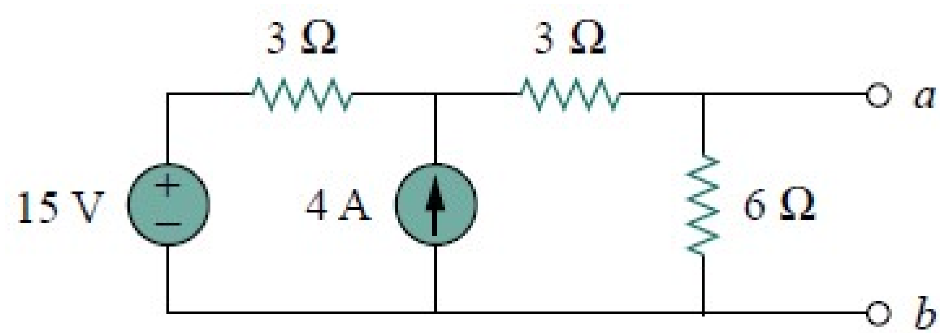
$$I_L = \frac{R_N}{R_N + R_L} \cdot I_N$$

Voltage across the load,

$$V_L = I_L R_L$$

Example 11

Find the Norton equivalent circuit of the circuit shown in the given figure.



Example 11

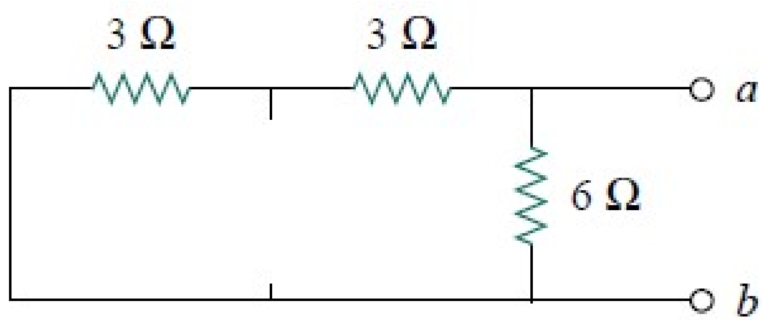
Find the Norton equivalent circuit of the circuit shown in the given figure.

Deactivating all the independent sources,

$$R_N = \left[(3+3) \parallel 6 \right] \ \Omega$$

$$= \left[\frac{6 \times 6}{6+6} \right] \ \Omega$$

$$= 3 \ \Omega$$



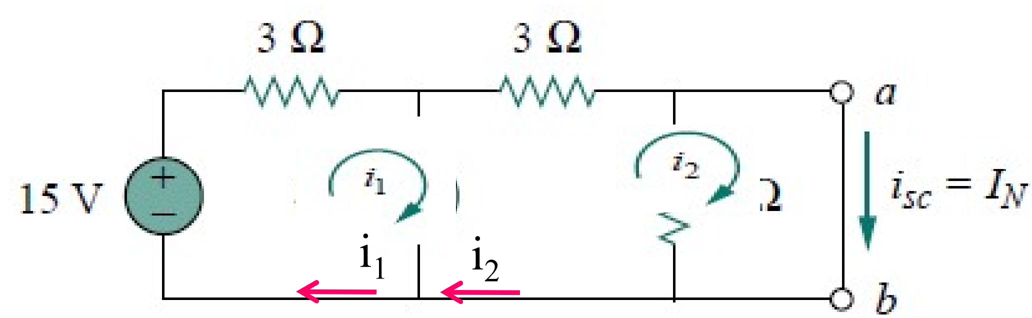
Example 11

Find the Norton equivalent circuit of the circuit shown in the given figure.

Applying KVL around the supermesh,

$$15 - 3i_1 - 3i_2 = 0$$

Or, $i_1 + i_2 = 5 \quad ....(1)$



Applying KCL,

$$i_2 = i_1 + 4 \quad ....(2)$$

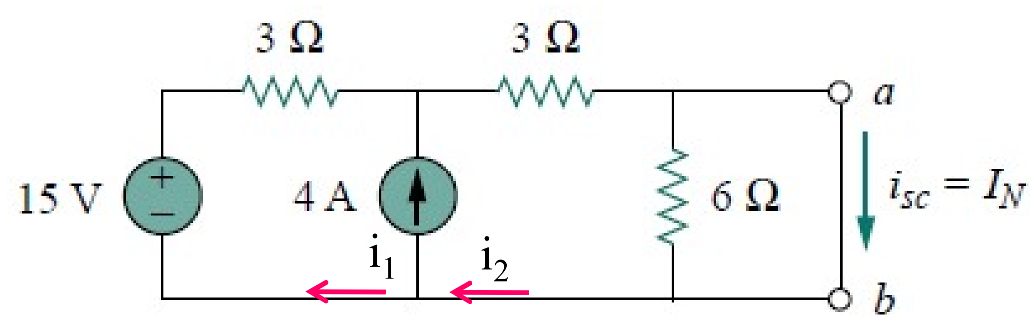
Example 11

Find the Norton equivalent circuit of the circuit shown in the given figure.

Solving equations (1) and (2),

$$i_1 = 0.5\ \text{A}$$

$$i_2 = 4.5\ \text{A}$$

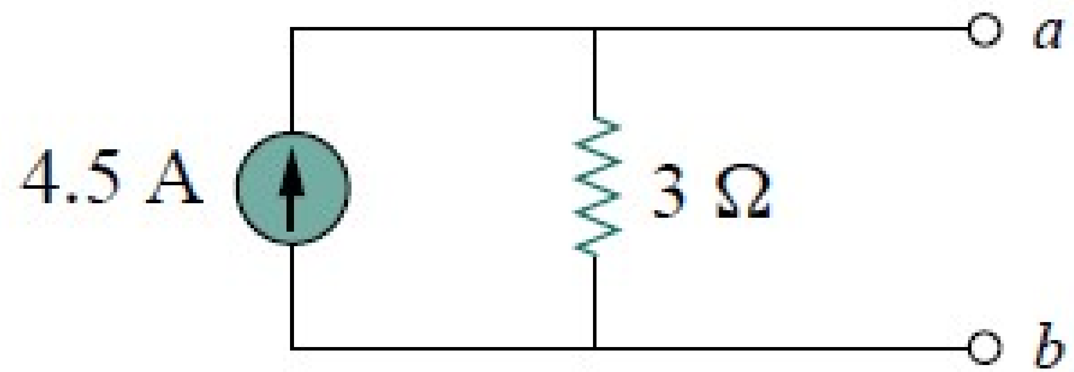


$$i_2 = i_{sc} = I_N = 4.5\ \text{A}$$

Example 11

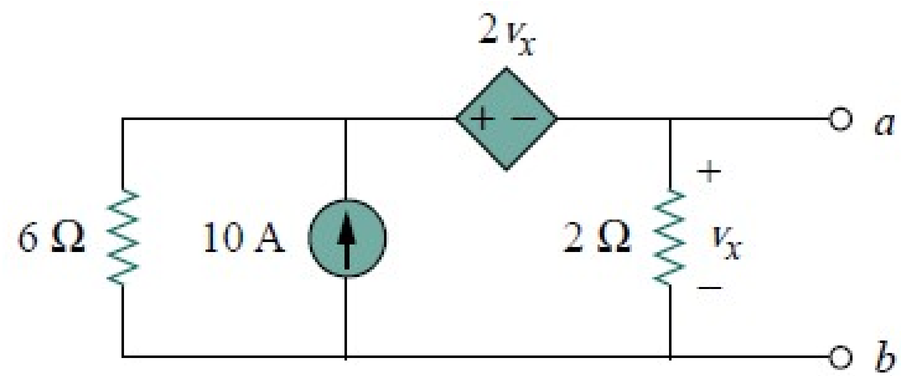
Find the Norton equivalent circuit of the circuit shown in the given figure.

Therefore, the Norton's equivalent circuit is



Example 12

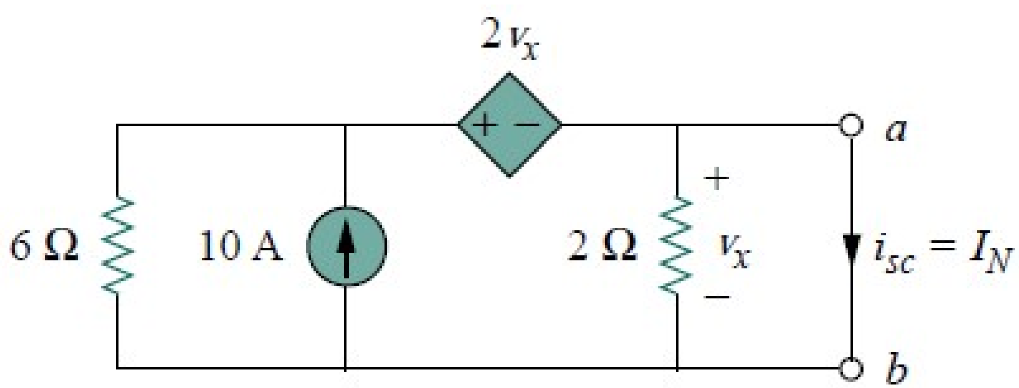
Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.



Example 12

Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

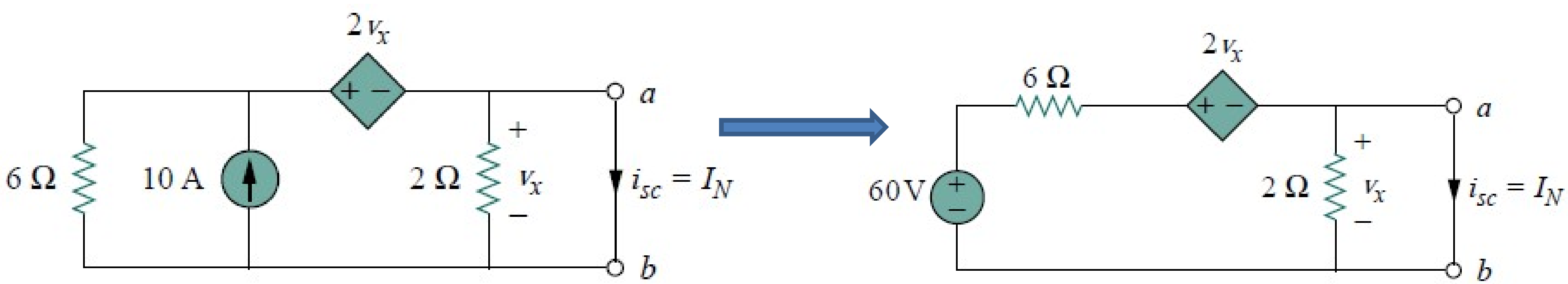
Making a short-circuit across the terminals a-b,



Example 12

Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Transforming the 10A current source into its equivalent voltage source,



Example 12

Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

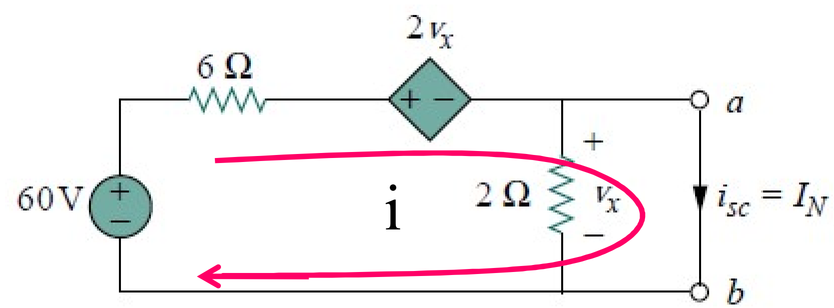
Applying KVL around the loop,

$$60 - 6i - 2v_x = 0$$

$$\text{Or, } i = \frac{30 - v_x}{3}$$

Now, $v_x = 0$

$$\therefore\ i = \frac{30}{3}\ \text{A} = 10\ \text{A} = I_N$$



Example 12

Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

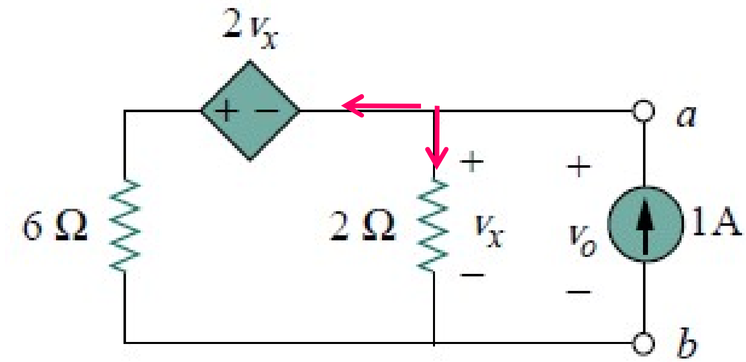
Deactivating the 10A current source and connecting a 1A current source across the terminals a-b,

Applying node analysis,

$$1 = \frac{v_x + 2v_x}{6} + \frac{v_x}{2}$$

Or, $v_x = 1$

$\therefore\ v_0 = 1\ \text{V}$



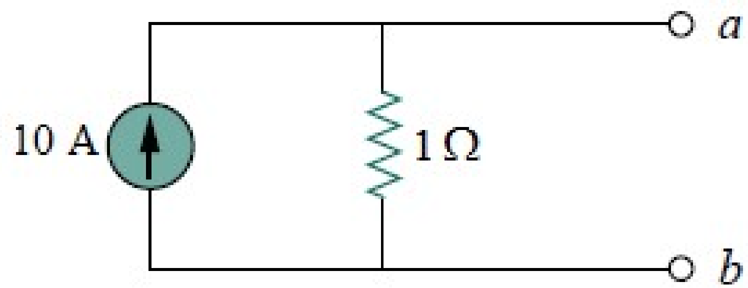
Example 12

Find the Norton equivalent circuit of the circuit shown in the given figure, to the left of the terminals a-b.

Therefore,

$$R_N = \frac{v_0}{i_0} = \frac{1}{1}\ \Omega = 1\ \Omega$$

Therefore, the Norton's equivalent circuit is



Syllabus

DC Networks:

- Kirchhoff's laws
- Node voltage and mesh current methods
- Delta-star and star-delta conversions
- Superposition principle
- Thevenin's and Norton's theorems
- Maximum Power Transfer Theorem.

Syllabus

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- Kirchhoff's laws
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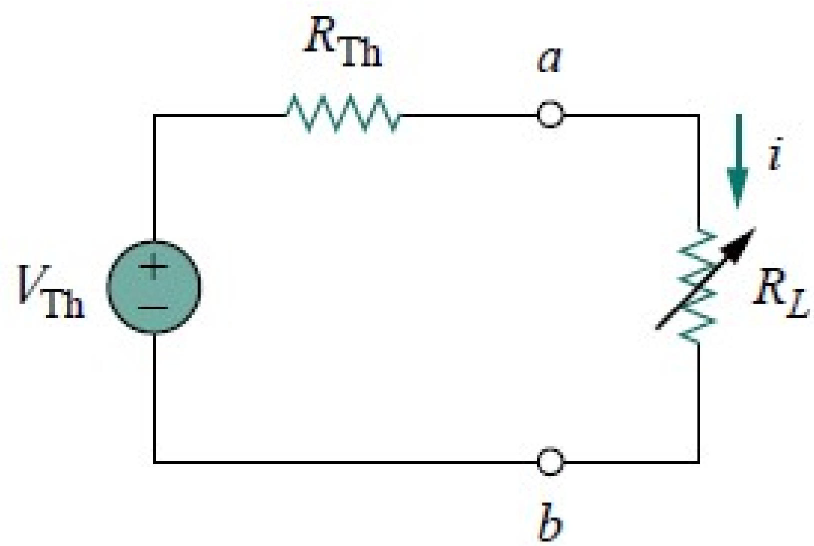
- Thevenin's and Norton's theorems
- Maximum Power Transfer Theorem.

Maximum Power Transfer Theorem

Maximum Power Transfer Theorem

Maximum power is transferred to the load when the load resistance equals the Thevenin resistance as seen from the load ($R_L = R_{Th}$).



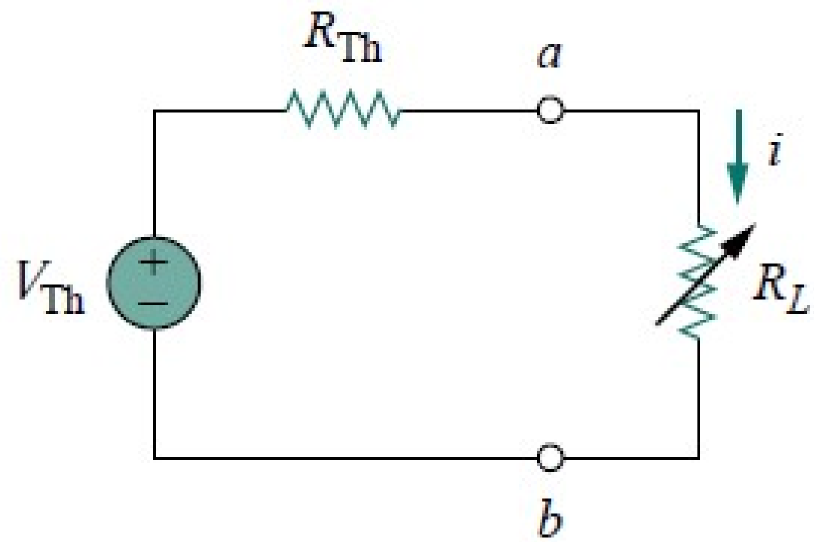
Maximum Power Transfer Theorem

Load current,

$$i = \frac{V_{Th}}{R_{Th} + R_L}$$

Power delivered to the load,

$$p = i^2 R_L = \left(\frac{V_{Th}}{R_{Th} + R_L} \right)^2 R_L$$



Maximum Power Transfer Theorem

For maximum power transfer,

$$\frac{dp}{dR_L} = 0$$

$$\text{Or,}\quad \frac{d}{dR_L}\left[\left(\frac{V_{Th}}{R_{Th} + R_L}\right)^2 R_L\right] = 0$$

$$\text{Or,}\quad V_{Th}^2\left[\frac{\left(R_{Th} + R_L\right)^2 - 2R_L\left(R_{Th} + R_L\right)}{\left(R_{Th} + R_L\right)^4}\right] = 0$$

$$\text{Or,}\quad V_{Th}^2\left[\frac{\left(R_{Th} + R_L - 2R_L\right)}{\left(R_{Th} + R_L\right)^3}\right] = 0$$

Maximum Power Transfer Theorem

For maximum power transfer,

Or, $R_{Th} - R_L = 0$

Or, $R_L = R_{Th}$

showing that the maximum power transfer takes place when the load resistance R_L equals the Thevenin resistance R_{Th}.

Maximum Power Transfer Theorem

For maximum power dissipation in the load, the condition needs to be satisfied is

$$\left.\frac{d^2p}{dR_L^2}\right|_{R_L=R_{Th}} = -\frac{V_{Th}^2}{8R_{Th}} < 0$$

The expression for maximum power dissipated to the load is given by

$$p_{max} = \left.\left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 R_L\right|_{R_L=R_{Th}} = \frac{V_{Th}^2}{4R_{Th}}$$

Maximum Power Transfer Theorem

Total power delivered by the source,

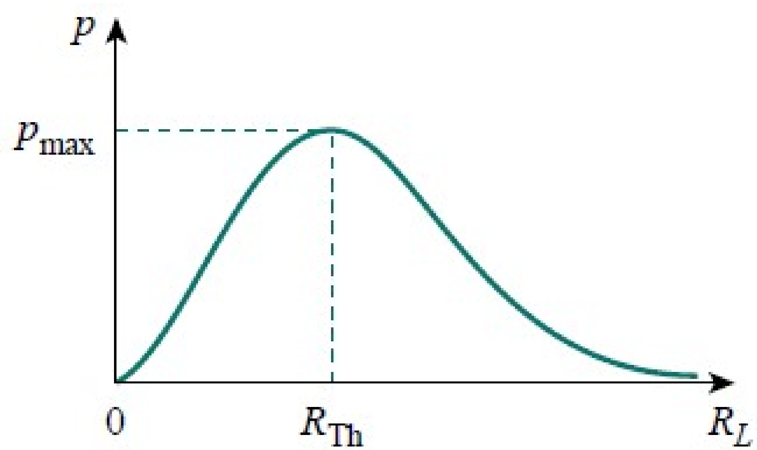
$$p_T = i^2\left(R_{Th} + R_L\right) = 2i^2R_L$$

This means that the Thevenin voltage source itself dissipates as much power in its internal resistance R_{Th} as the power absorbed by the load R_L.

Maximum Power Transfer Theorem

Therefore, efficiency under maximum power transfer condition is given by,

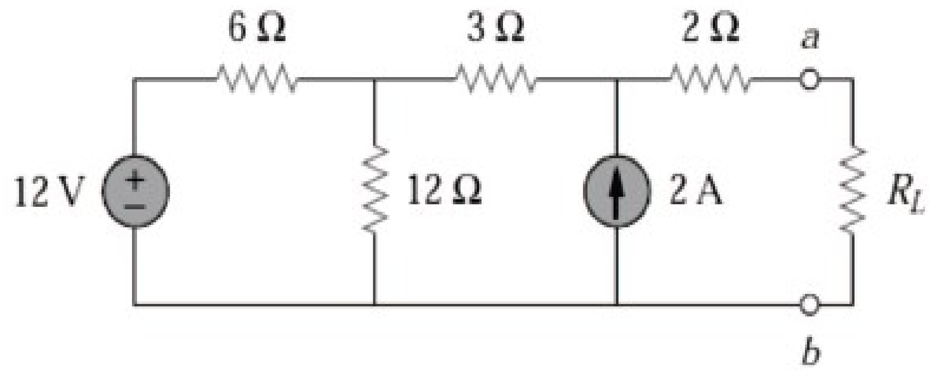
$$\eta = \frac{i^2 R_L}{2i^2 R_L} \times 100\% = 50\%$$



Power delivered to the load as a function of R_L.

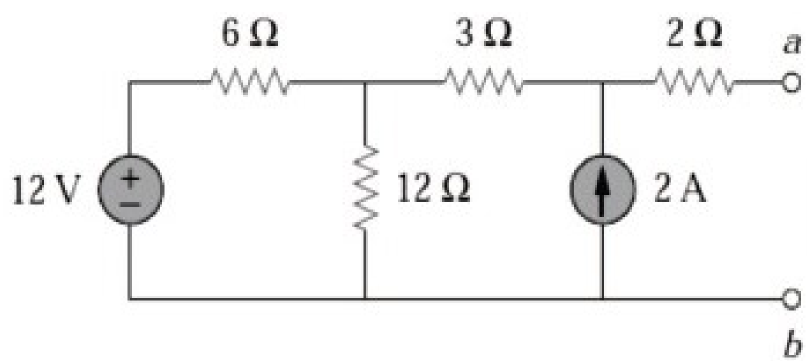
Example 13

Find out the value of load resistance R_L for maximum power transfer. Also find the corresponding maximum power.



Example 13

Find out the value of load resistance R_L for maximum power transfer. Also find the corresponding maximum power.



Example 13

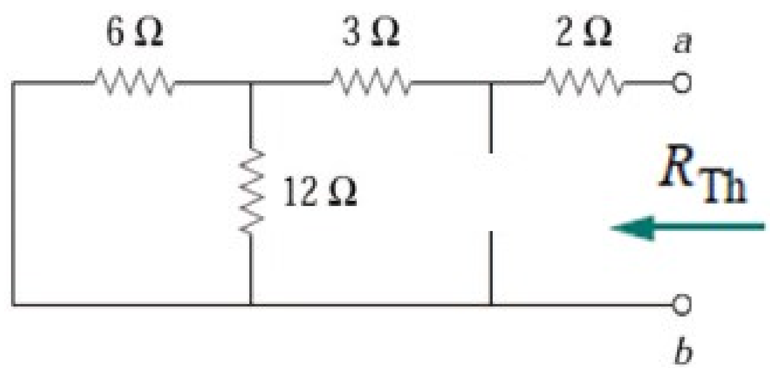
Find out the value of load resistance R_L for maximum power transfer. Also find the corresponding maximum power.

Deactivating all the independent sources,

$$R_{Th} = \left[(6 \| 12) + 3 + 2 \right] \, \Omega$$

$$= \left[\left(\frac{6 \times 12}{6 + 12} \right) + 5 \right] \, \Omega$$

$$= [4 + 5] \, \Omega \quad = 9 \, \Omega$$



Therefore, for maximum power transfer,

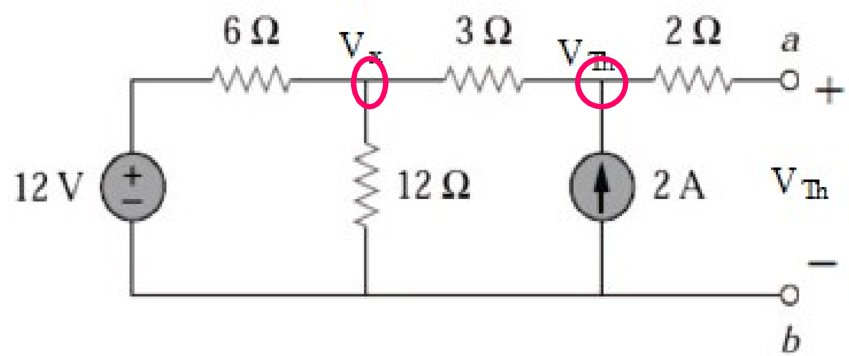
$$R_L = R_{Th} = 9 \, \Omega$$

Example 13

Find out the value of load resistance R_L for maximum power transfer. Also find the corresponding maximum power.

Now,

Applying node analysis,



$$\frac{V_x - 12}{6} + \frac{V_x - 0}{12} = 2$$

$$V_x = 16 \text{ V}$$

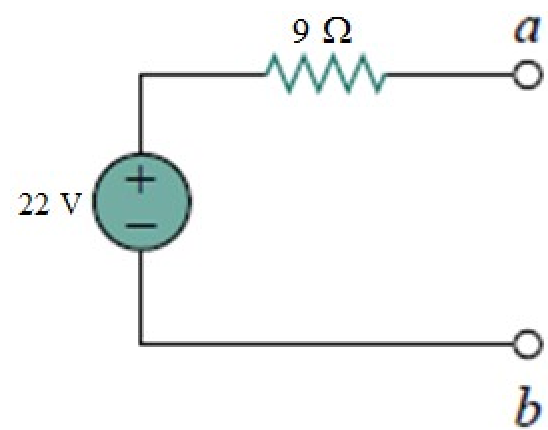
$$\therefore \quad \frac{V_{Th} - V_x}{3} = 2$$

$$V_{Th} = 22 \text{ V}$$

Example 13

Find out the value of load resistance R_L for maximum power transfer. Also find the corresponding maximum power.

Therefore, the Thevenin's equivalent circuit is

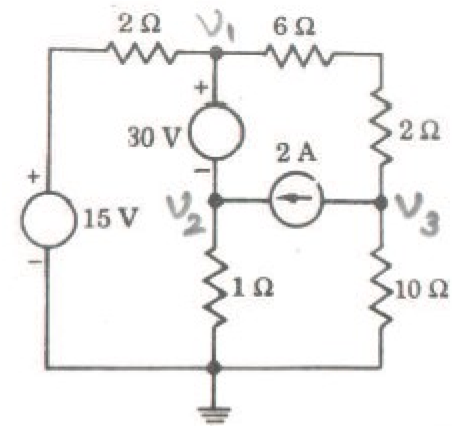


Therefore, maximum power transferred to load,

$$p_{max} = \frac{V_{Th}^2}{4R_L} = \frac{22^2}{4 \times 9} \text{ W}$$

$$= 13.44 \text{ W}$$

Problem1: Find out the node voltages v_1, v_2 and v_3 using nodal analysis.

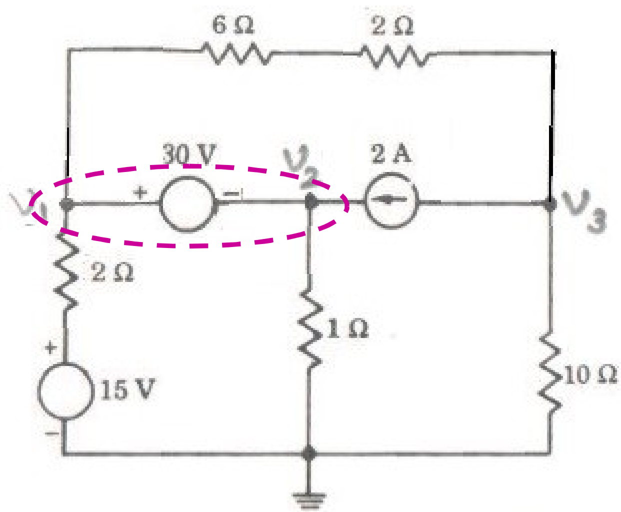


Redrawing the circuit as shown in the figure,

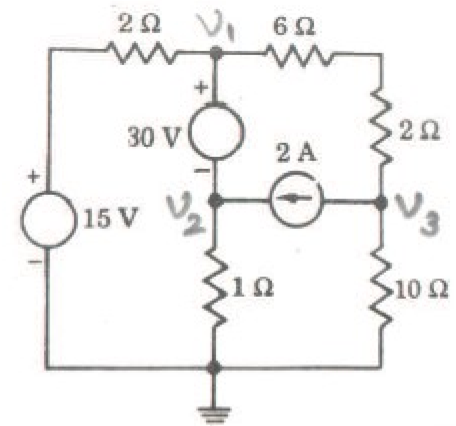
Applying KCL at the super-node,

$$\frac{v_1 - 15}{2} + \frac{v_1 - v_3}{8} - 2 + \frac{v_2}{1} = 0$$

$$5v_1 + 8v_2 - v_3 = 76 \qquad (1)$$

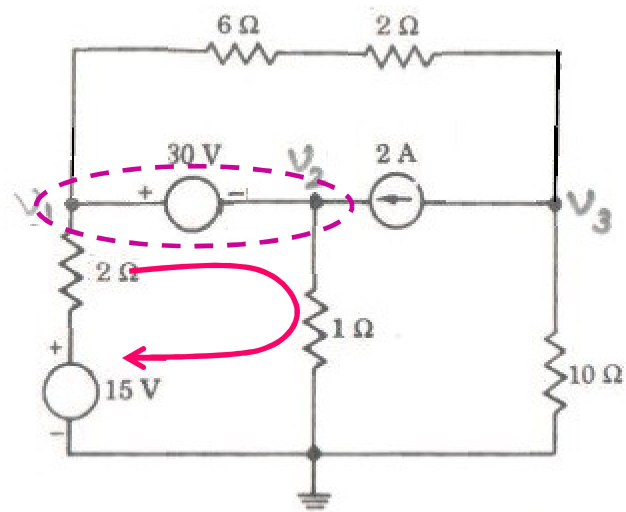


Problem1: Find out the node voltages v_1, v_2 and v_3 using nodal analysis.



Redrawing the circuit as shown in the figure,

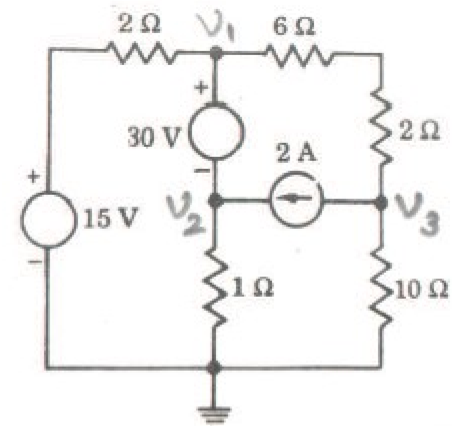
Applying KVL around the loop containing the super-node,



$$v_1 - 30 - v_2 = 0$$

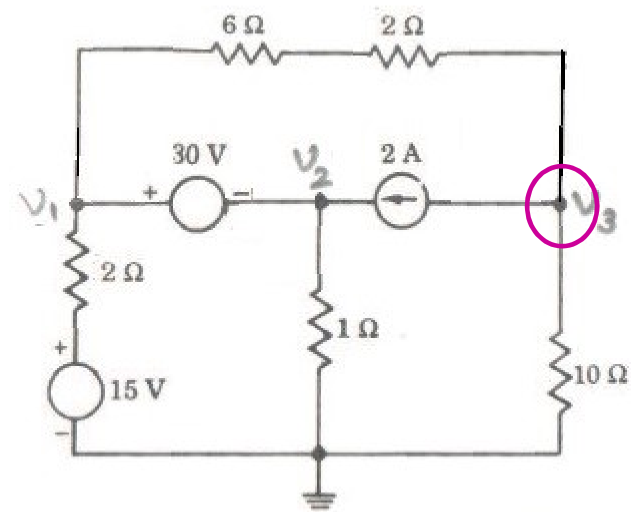
$$v_1 - v_2 = 30 \qquad (2)$$

Problem1: Find out the node voltages v_1, v_2 and v_3 using nodal analysis.



Redrawing the circuit as shown in the figure,

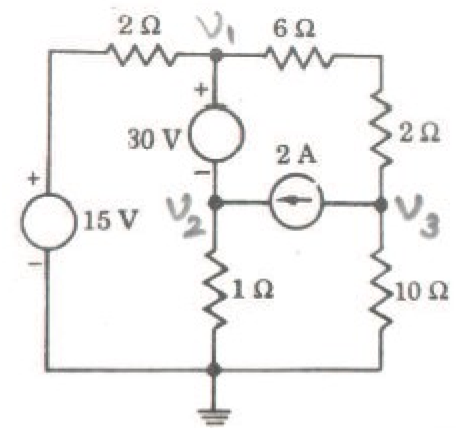
Applying KCL at node-3,



$$\frac{v_1 - v_3}{8} = 2 + \frac{v_3}{10}$$

$$5v_1 - 9v_3 = 80 \qquad (3)$$

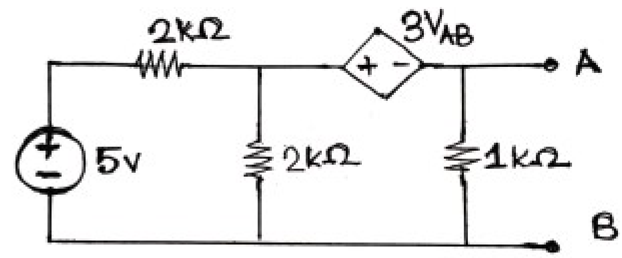
Problem1: Find out the node voltages v_1, v_2 and v_3 using nodal analysis.



Solving equations (1), (2) and (3),

$$v_1 = 24.68 \text{ V}$$
$$v_2 = -5.32 \text{ V}$$
$$v_3 = 4.82 \text{ V}$$

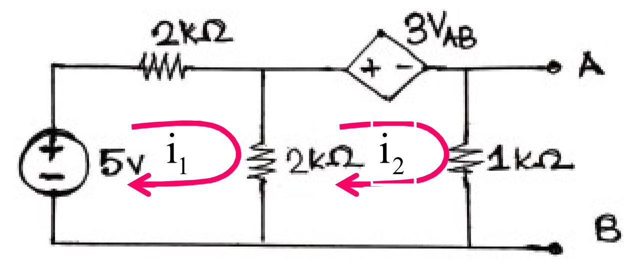
Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



KVL around loop-1,

$$5-2000i_1-2000(i_1-i_2)=0$$

Or, $800i_1-400i_2=1 \qquad (1)$

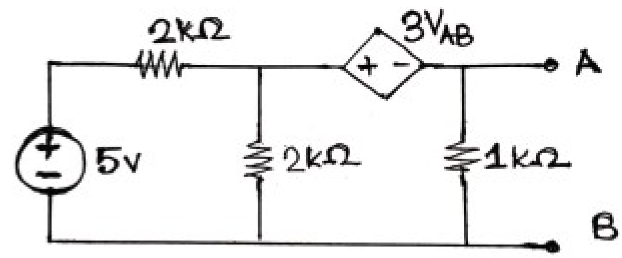


KVL around loop-2,

$$-2000(i_2-i_1)-3V_{AB}-1000i_2=0$$

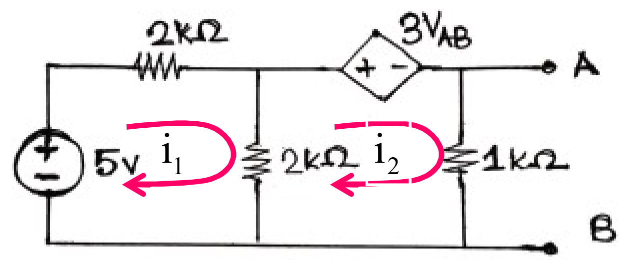
Or, $2000i_1-3000i_2=3V_{AB} \qquad (2)$

Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



Now,

$$V_{AB} = 1000 i_2 \qquad (3)$$



From equation (2),

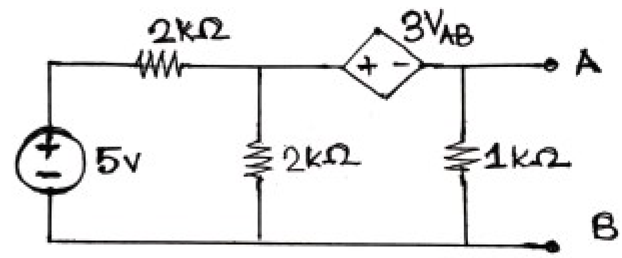
$$i_1 = 3 i_2 \qquad (4)$$

Solving equations (1) and (4),

$$i_1 = 1.5 \text{ mA}$$

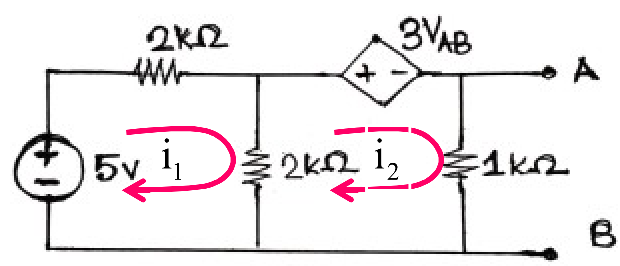
$$i_2 = 0.5 \text{ mA}$$

Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



Therefore,

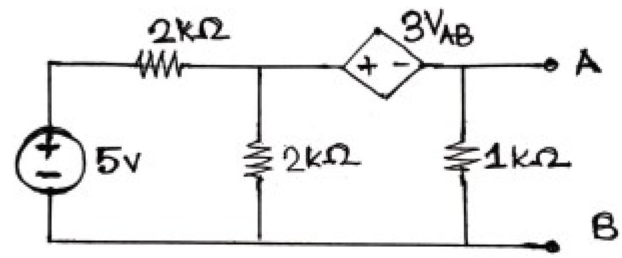
$$V_{AB} = 1000 \times 0.5 \times 10^{-3} \text{ V} = 0.5 \text{ V}$$



Hence,

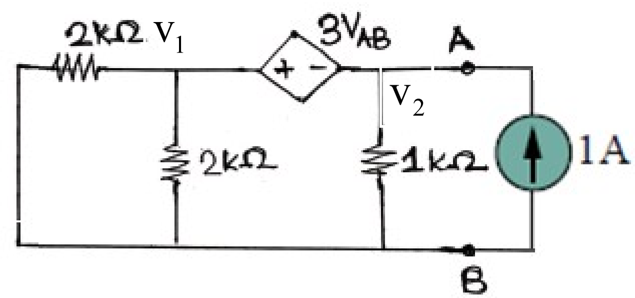
$$V_{Th} = V_{AB} = 0.5 \text{ V}$$

Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



Deactivating the 5V voltage source and connecting a 1A current source across the terminals A-B,

From ciruit diagram,



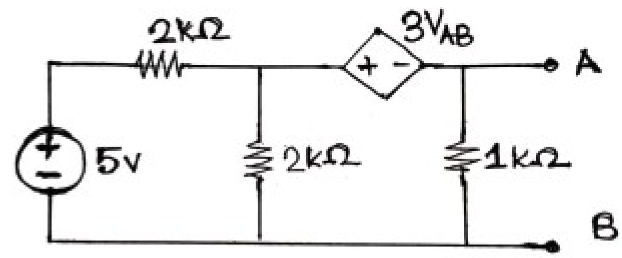
$$v_2 = V_{AB} \qquad (5)$$

$$v_1 - v_2 = 3V_{AB} \qquad (6)$$

From equation (5) and (6),

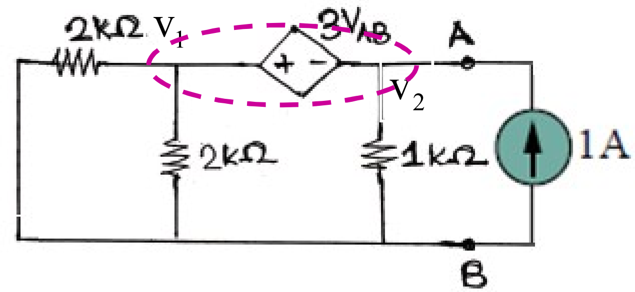
$$v_1 = 4V_{AB} \qquad (7)$$

Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



Deactivating the 5V voltage source and connecting a 1A current source across the terminals A-B,

Applying KCL at the super-node,



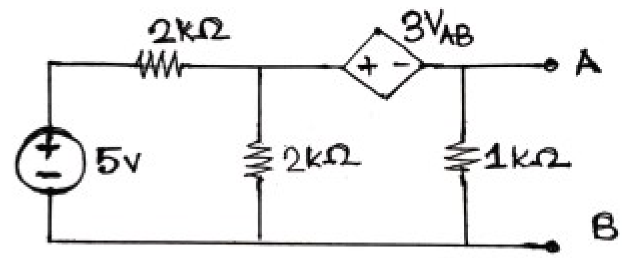
$$\frac{v_1}{2000}+\frac{v_1}{2000}+\frac{V_{AB}}{1000}-1=0$$

Or, $v_1+V_{AB}=1000 \qquad (8)$

Solving equations (7) and (8),

$$V_{AB}=200 \text{ V}$$

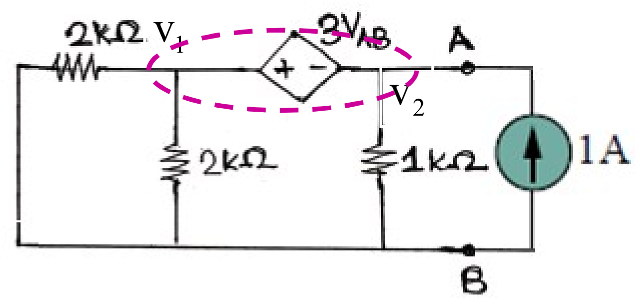
Problem2:Find V_{Th} and R_{Th} with respect to terminal A-B for the circuit given in figure.



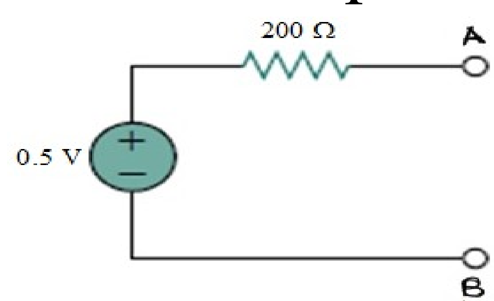
Deactivating the 5V voltage source and connecting a 1A current source across the terminals A-B,

Therefore,

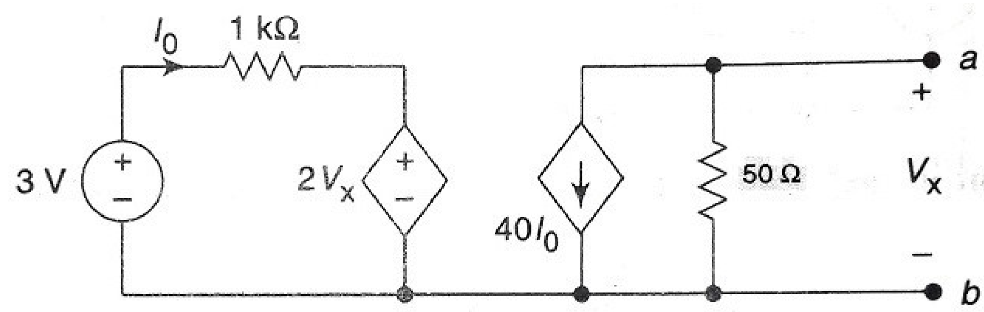
$$R_{Th} = \frac{200}{1}\ \Omega \ = 200\ \Omega$$



Therefore, the Thevenin's equivalent circuit is

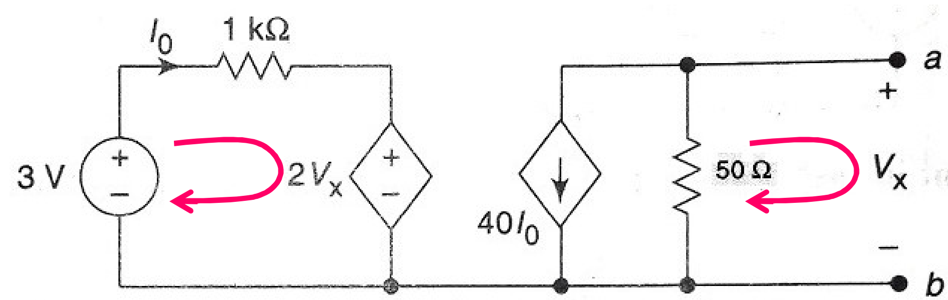


Problem3: Find the Thevenin's equivalent between terminals a and b of the circuit shown in the figure.



By KVL for the right-hand side mesh,

$$V_{Th} = V_x = (-40I_0) \times 50 = -2000I_0$$



Or, $\quad V_x = -2000I_0 \qquad (1)$

From the left-hand side mesh,

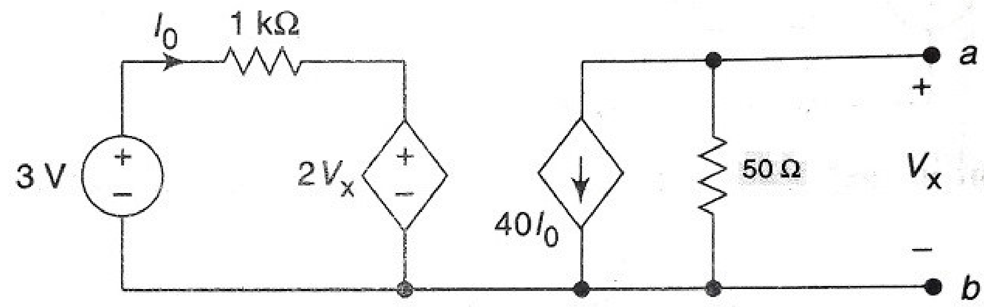
$$3 - 1000I_0 - 2V_x = 0$$

Or, $\quad I_0 = \dfrac{3 - 2V_x}{1000} \qquad (2)$

Solving equations (1) and (2),

$$V_x = 2\text{ V} = V_{Th}$$

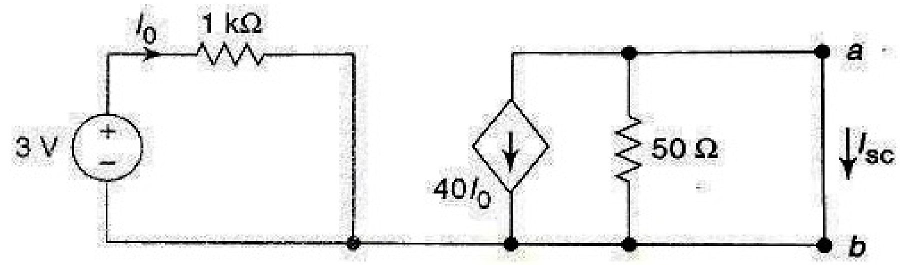
Problem3: Find the Thevenin's equivalent between terminals a and b of the circuit shown in the figure.



To determine the Thevenin's resistance, a-b terminals are short circuited.

From the left-hand side mesh,

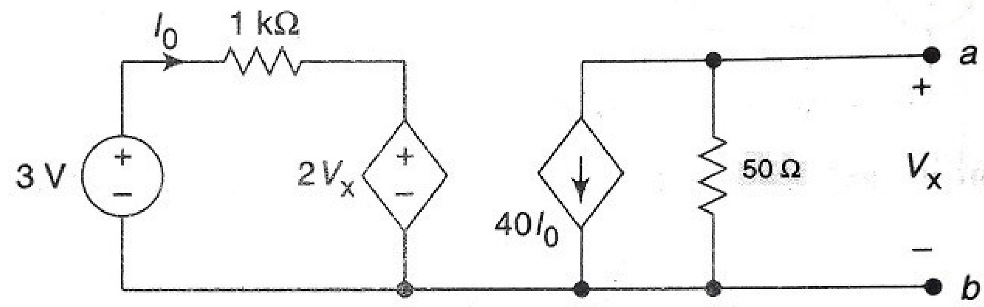
$$I_0 = \frac{3}{1000}\ \text{A}$$



From the right-hand side mesh,

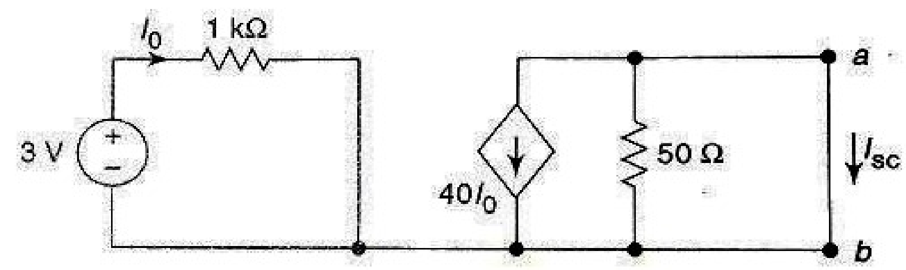
$$I_{sc} = -40I_0 \quad = (-40)\times\frac{3}{1000}\ \text{A} \quad = -0.12\ \text{A}$$

Problem3: Find the Thevenin's equivalent between terminals a and b of the circuit shown in the figure.



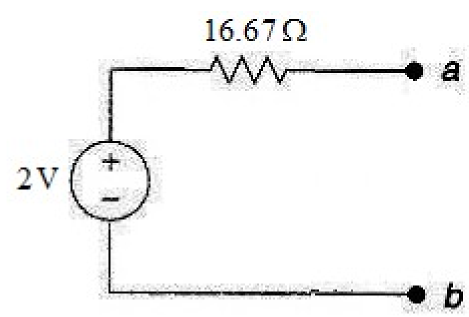
To determine the Thevenin's resistance, a-b terminals are short circuited.

Therefore,



$$R_{Th} = \frac{V_{Th}}{I_{sc}} = \frac{2}{0.12}\ \Omega \quad = 16.67\ \Omega$$

Therefore, the Thevenin's equivalent circuit is



Thank You