Functional Analysis

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Contents

1 Metric Spaces **2**
1.1 Metric Space 2
1.2 Further Examples of Metric Spaces 2
1.3 Open Set, Closed Set, Neighborhood 2

1 Metric Spaces

1.1 Metric Space

(P18) **Definition 1.1.1 (Metric space,metric)** A *metric space* is a pair (X, d) where X is a set and $d : X \times X \to R$

1. d is real-valued, finite and nonnegative.
2. $d(x, y) = 0$ iff x=y.
3. $d(x, y) = d(y, x)$
4. $d(x, y) \leq d(x, z) + d(z, y)$ (**Triangle inequality**).

Examples

Example 1.1.6 (Sequence space l^∞)

- $x = (\xi_j)$
- $|\xi_j| \leq c_x$, where c_x is a real number may depend on x, but not on j.
- $d(x, y) = \sup_{j \in N} |\xi_j - \eta_j|$

Example 1.1.7 (Function space C[a,b]) X is the set of all real-valued functions defined on closed interval $J = [a, b]$ and

$$d(x, y) = \max_{t \in J} |x(t) - y(t)|,$$

1.2 Further Examples of Metric Spaces

(P24)

Example 1.2.1 (Sequence space s) In contrast with 6,

1.3 Open Set, Closed Set, Neighborhood

Definition 1.3.1 (Ball and sphere)

Definition 1.3.2 (Open set, closed set) A subset M of a metric space is *open* if it contains a ball about each of its points. A subset K is closed if $K^c = X - K$ is open.

Remark (Topological Space) For the collection of all the open subsets of X called $\mathcal{T}$:

- (T1) $\emptyset \in \mathcal{T}, X \in \mathcal{T}$.
- (T2) The union of **any** members of $\mathcal{J}$ is a member of $\mathcal{J}$.
- (T3) The intersection of **finitely** many members of $\mathcal{J}$ is a member of it.

Definition 1.3.3 (Continuous mapping) For $X = (X, d)$ *and* $Y = (Y, \tilde{d})$, $T : X \longrightarrow Y$ is continuous at point x_0 if for every $\epsilon > 0$ there's a $\delta > 0$ such that

Theorem 1.3.4 (Continuous mapping) A mapping $T : X \longrightarrow Y$ is continuous iff **the inverse image of any open subset of Y is an open subset of X**.