Functional Analysis

陈辉

Contents

1 Metric Spaces

1.1 Metric Space

(P18) **Definition 1.1.1 (Metric space,metric)** A *metric space* is a pair (X, d) where X is a set and $d : X \times X \to R$

1. d is real-valued, finite and nonnegative.
2. $d(x, y) = 0$ iff x=y.
3. $d(x, y) = d(y, x)$
4. $d(x, y) \leq d(x, z) + d(z, y)$ (**Triangle inequality**).

Examples

Example 1.1.6 (Sequence space l^∞)

- $x = (\xi_j)$
- $|\xi_j| \leq c_x$, where c_x is a real number may depend on x, but not on j.
- $d(x, y) = \sup_{j \in N} |\xi_j - \eta_j|$

Example 1.1.7 (Function space C[a,b]) X is the set of all real-valued functions defined on closed interval $J = [a, b]$ and

$$d(x, y) = \max_{t \in J} |x(t) - y(t)|,$$

1.2 Further Examples of Metric Spaces

(P24)

Example 1.2.1 (Sequence space s) In contrast with l^∞

1.3 Open Set, Closed Set, Neighborhood

Definition 1.3.1 (Open Set) Given a point $x_0 \in X$ and a real number $r \geq$.