2024 Troy Integration Bee Qualifying Exam KEY

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You have 20 minutes to complete as many of the following integrals as possible. The only allowed materials are a pencil, eraser, and scratch paper – no calculators. For indefinite integrals, the $+C$ term need not be included. Both $\ln(x)$ and $\log(x)$ will be, by default, interpreted as being in base e. The denominators of fractions need not be rationalized, but otherwise, answers must be in simplest form. Scratch work will not be considered and there is no partial credit; only your final answer on this sheet matters. Ties will be broken via reverse sudden death.

1. $\displaystyle\int \frac{dx}{\sqrt[10]{x^9}} = \boxed{10\sqrt[10]{x}+C}$

2. $\displaystyle\int_0^{2\sqrt{22}} \sqrt{2024-23x^2}\,dx = \boxed{22\sqrt{23}\pi}$

3. $\displaystyle\int_{-2024}^{0} |x| + \lfloor x \rfloor\,dx = \boxed{-1012}$

4. $\displaystyle\int \frac{x+2}{x^3+8}\,dx = \boxed{\tfrac{1}{\sqrt{3}}\tan^{-1}\left(\tfrac{x-1}{\sqrt{3}}\right)+C}$

5. $\displaystyle\int_{-\pi}^{0} \cos^5(x)\,dx = \boxed{0}$

6. $\displaystyle\int_0^{\infty} xe^{-x/3}\,dx = \boxed{9}$

7. $\displaystyle\int_0^{1/2} \cos^{-1}(x)\,dx = \boxed{\tfrac{1}{2}}$

8. $\displaystyle\int_{\pi/4}^{\pi/6} \tan^2(x) - \cot^2(x)\,dx = \boxed{2-\tfrac{4}{\sqrt{3}}}$

9. $\displaystyle\int_{-\pi/4}^{0} \frac{d}{dx}\left(\frac{\sin(x)+\cos(x)}{\cos(2x)}\right) = \boxed{1-\tfrac{1}{\sqrt{2}}}$

10. $\displaystyle\int x(e-x)^{2024}\,dx = \boxed{\tfrac{(x-e)^{2026}}{2026}+\tfrac{ex^{2025}}{2025}+C}$

11. $\displaystyle\int_0^3 \left(6x^2+2x+1\right)\left(2x^3+x^2\right)\,dx = \boxed{2034}$

12. $\displaystyle\int_{-\pi/2}^{\pi/2} e^{2x}x\sin(x)\,dx = \boxed{\tfrac{\pi}{4}\left(e^{\pi}+e^{-\pi}\right)}$

13. $\displaystyle\int_{-1}^{0} \frac{x+1}{(x^2-1)\sqrt{x^2-2x}}\,dx = \boxed{-\tfrac{\pi}{3}}$

14. $\displaystyle\int x\sqrt[3]{x\sqrt[3]{x\sqrt[3]{\ldots}}}\,dx = \boxed{\tfrac{2}{5}x^{5/2}+C}$

15. $\displaystyle\int_{-\infty}^{\infty} \sqrt{2}xe^{-2x^4}\,dx = \boxed{\tfrac{\sqrt{\pi}}{4}}$

16. $\displaystyle\int e^{(1+i)x}+e^{(1-i)x}\,dx = \boxed{e^x(\sin(x)+\cos(x))+C}$

17. $\displaystyle\int_{-\infty}^{0} \begin{bmatrix} 24 & 0 \\ 0 & 24 \end{bmatrix}^x dx = \boxed{\tfrac{1}{\log(24)}\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}}$

18. $\displaystyle\int_{-\infty}^{\infty} \frac{16}{x^4+4}\,dx = \boxed{4\pi}$

19. $\displaystyle\int_0^{2024} 2024^{\lfloor x/2024 \rfloor}\,dx = \boxed{\tfrac{4096576}{2025}}$

20. $\displaystyle\int_0^{\pi} \frac{256}{(5+3\cos(x))^2}\,dx = \boxed{20\pi}$