

2025 Troy Integration Bee Mock Qualifying Exam - ANSWER KEY

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You have 15 minutes to complete as many of the following integrals as possible. The only allowed materials are a pencil, eraser, and scratch paper—no calculators. For indefinite integrals, the $+C$ term need not be included. Both $\ln(x)$ and $\log(x)$ will be, by default, interpreted as being in base e . \Re denotes the real part of a complex number. The denominators of fractions need not be rationalized, but otherwise, answers must be in simplest form. Scratch work will not be considered and there is no partial credit; only your boxed final answer on this sheet matters.

$$1. \int \left(\frac{x^{-\frac{19}{4}}}{\sqrt[8]{x^2} \sqrt{x^8}} \right)^{\frac{1}{10}} dx = \boxed{10 \sqrt[10]{x} + C}$$

$$2. \int \frac{\ln x}{x \ln(x) + x} dx = \boxed{\ln \left(\left| \frac{x}{\ln|x| + 1} \right| \right) + C}$$

$$3. \int (\sin^2(x) + \cos^2(x) + \tan^2(x) + \cot^2(x)) dx = \boxed{\tan(x) - \cot(x) - x + C}$$

$$4. \int \left| \begin{array}{cc} -1 & 1 \\ 1 & \log(x) \end{array} \right| dx = \boxed{-x \ln(x) + C}$$

$$5. \int e^x \sin(x) \cos(x) dx = \boxed{\frac{e^x \sin(2x) - 2e^x \cos(2x)}{10} + C}$$

$$6. \int_1^{16} \{\sqrt{x}\} dx = \boxed{8}$$

$$7. \int_0^9 [x] \lceil x \rceil \{x\} dx = \boxed{120}$$

$$8. \int_{-1}^1 \frac{\arctan(x) \cos(x)}{1+x^2} dx = \boxed{0}$$

$$9. \int x \sqrt{x \sqrt{x \sqrt{x} \dots}} dx = \boxed{\frac{x^3}{3} + C}$$

$$10. \int \frac{2x^3 - 1}{x^4 + x} dx = \boxed{\ln(x^2 + \frac{1}{x}) + C}$$

$$11. \int_{\pi/2}^{\pi} \sqrt{1 - \cos(x)} dx = \boxed{2}$$

$$12. \int_0^1 x d(x^p) = \boxed{\frac{p}{p+1}}$$

$$13. \Re \left(\int \frac{1}{1 + e^{i\pi} + e^{ix}} dx \right) = \boxed{\sin(x) + C}$$

$$14. \int_{-\infty}^{\infty} \frac{16}{x^4 + 4} dx = \boxed{4\pi}$$

$$15. \int_0^{\pi} \frac{256}{(5 + 3 \cos(x))^2} dx = \boxed{20\pi}$$