2025 Troy Integration Bee Qualifying Exam - ANSWER KEY

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NAME: _____ SCORE: _____

You have 20 minutes to complete as many of the following integrals as possible. The only allowed materials are a pencil, eraser, and scratch paper—no calculators. For indefinite integrals, the $+C$ term need not be included. The denominators of fractions need not be rationalized, but otherwise, answers must be in simplest form. i will denote the imaginary unit. Scratch work will not be considered and there is no partial credit; only your final answer on this sheet matters.

BOX YOUR ANSWERS!

1. $\int (20-25x)^{-\sqrt{\frac{2025}{25}}}\,dx = \boxed{\frac{1}{200}(20-25x)^{-8}+C}$
2. $\int \ln\left((xe)^{2025}\right)\,dx = \boxed{2025x\ln|x|+C}$
3. $\int_0^5 2\pi x\sqrt{25-x^2}\,dx = \boxed{\frac{250\pi}{3}}$
4. $\int_{-45/2}^{45}\sqrt{1+\left(\frac{x}{\sqrt{2025-x^2}}\right)^2}\,dx = \boxed{30\pi}$
5. $\int_0^{\pi/3}\left(\tan^3(x)+\tan(x)\right)e^{\tan^2(x)}\,dx = \boxed{\frac{e^3-1}{2}}$
6. $\int \frac{2\sec^4(x)}{\tan(x)}\,dx = \boxed{\sec^2(x)+2\ln|\tan(x)|+C}$
7. $\int_0^{\sqrt{e}}(-2\ln(x)+1)\,x^{-\ln(x)}\,dx = \boxed{\sqrt[4]{e}}$
8. $\int_0^{\infty}\frac{d}{dx}\left(\frac{x+\sin x}{x}\right)\,dx = \boxed{-1}$
9. $\int \left|x^2+ix\right|\,dx = \boxed{\frac{1}{3}(x^2+1)^{\frac{3}{2}}+C}$
10. $\int\left(\begin{bmatrix}3x\\4x\\0\end{bmatrix}\times\begin{bmatrix}-4x^3\\3x^3\\0\end{bmatrix}\cdot\begin{bmatrix}\sqrt{2}\\1\\1\end{bmatrix}\right)\,dx = \boxed{5x^5+C}$
11. $\int \sqrt{1-\sin(x)}\,dx = \boxed{2\sqrt{1+\sin(x)}+C}$
12. $\int \begin{bmatrix}0 & x\\4 & 0\end{bmatrix}^{14}\,dx = \boxed{2048x^8\begin{bmatrix}1 & 0\\0 & 1\end{bmatrix}+\begin{bmatrix}C_1 & C_2\\C_3 & C_4\end{bmatrix}}$
13. $\int x\sqrt{x\sqrt[3]{x\sqrt[4]{x\sqrt[5]{\ldots}}}}\,dx = \boxed{\frac{x^e}{e}+C}$
14. $\int_0^{2025}\left(\left\lceil\frac{x+1}{2}\right\rceil-\left\lfloor\frac{x}{2}\right\rfloor\right)\,dx = \boxed{3037}$
15. $\int_{\pi}^{45\pi}\left(\sin^5(x)+\sin^6(x)-\sin^7(x)\right)\,dx = \boxed{\frac{55\pi}{4}}$
16. $\int_0^{\pi/2}\frac{dx}{1+\tan^{2025}(x)} = \boxed{\frac{\pi}{4}}$
17. $\int_{-1}^{1}x^2\cos^{-1}(x)\,dx = \boxed{\frac{\pi}{3}}$
18. $\int \frac{\sin(x)}{\sin(x)+\cos(x)}\,dx = \boxed{\frac{x-\ln|\sin(x)+\cos(x)|}{2}+C}$
19. $\int_4^{9/2}\left(x^4+\frac{1}{x^4}\right)\,d\left(\left(x+\frac{1}{x}\right)^2\right) = \boxed{\frac{37}{24}}$
20. $\int_{-\infty}^{\infty}\cos\left(x^2\right)\,dx = \boxed{\sqrt{\frac{\pi}{2}}}$