TROY HIGH

Integration Bee Finals

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Example

$$\int 1 \, dx$$

Example

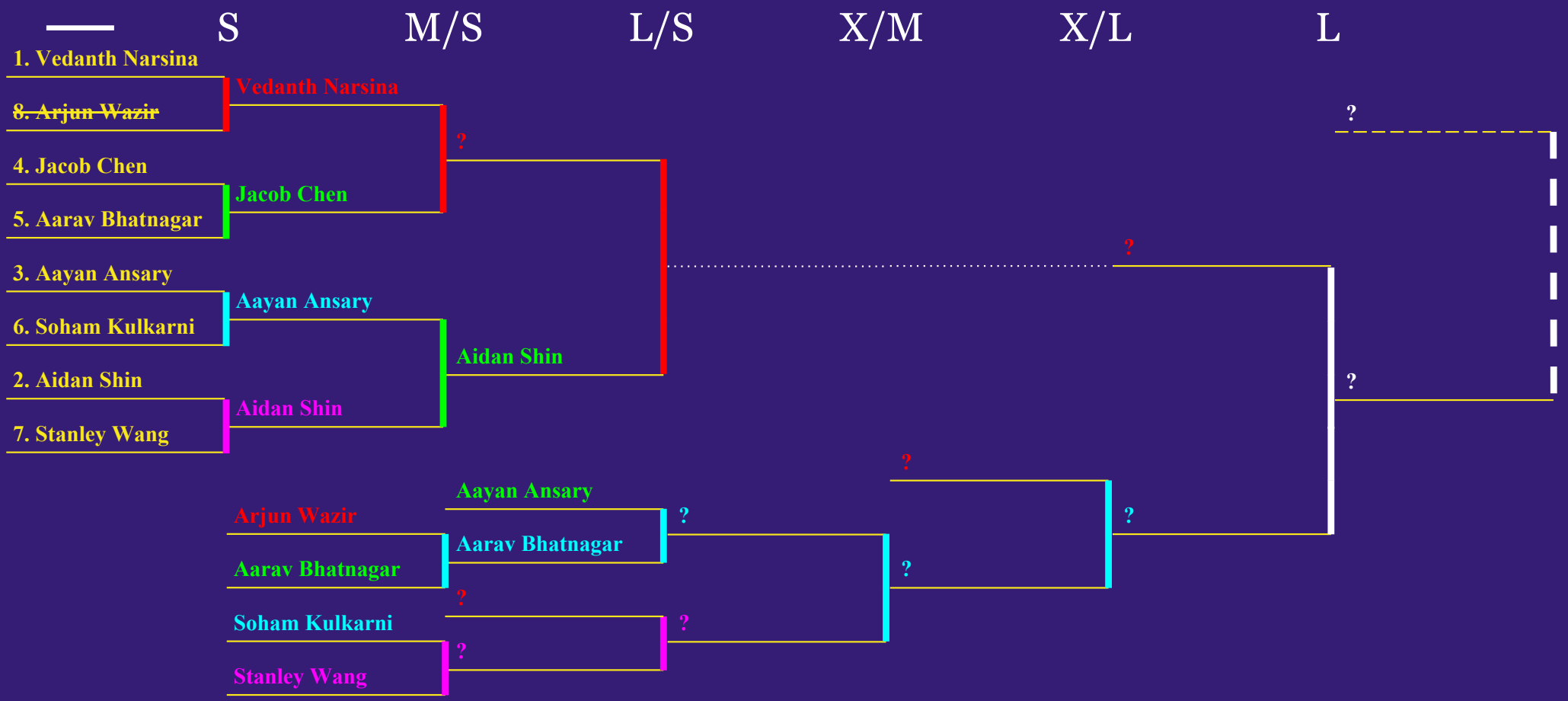
$$x + C$$

Tournament Design

Once a participant has lost exactly two games, they are eliminated.

- Short Questions: best of two, then sudden death
  - Sudden death will have overtime of 30 seconds
  - Ties will be given to the contestant with the higher qualifying exam score
- Medium Questions: best of four, then sudden death
  - Sudden death will have overtime of 60 seconds
  - Ties will be given to the contestant with the higher qualifying exam score
- Long Questions: best of six, then sudden death
  - Sudden death will have overtime of 90 seconds
  - Ties will be given to the contestant with the higher qualifying exam score

Double Elimination Bracket



Question Clarifications

$\lfloor n \rfloor$ = floor(n) rounds a real number down to the nearest integer.

$\lceil n \rceil$ = ceiling(n) rounds a real number up to the nearest integer.

$\{n\} = n - \lfloor n \rfloor$ finds the non-integer part of a real number.

|n| denotes the magnitude of a complex number, absolute value of a real number, magnitude of a vector, and determinant of a matrix.

n_b denotes that a number's representation is in base ***b***. All numbers are in base 10 unless otherwise specified.

sgn(n) = n/|n| finds the sign of a number. Note that sgn(0) = 0.

$\mathcal{U}$(a,b), the uniform distribution, randomly outputs a real number between ***a*** and ***b***.

Pr(X) finds the probability of event X. E(Y) finds the expected value of expression Y.

W(x) is the inverse function of xe^x, such that $W(xe^x) = x$. For example, W(e) = 1.

Π denotes a repeated product (the multiplicative equivalent of ∑)

Answer Format

When finished, box your answer, shout “Done!”, and stand back from the board. The judges will then immediately declare it correct or incorrect. If correct, you receive credit; if not, your competitor has the remaining time to declare their own answer.

+C need not be written for indefinite integrals.

Expressions inside logarithms need not be written with absolute value. ln() and log() are both assumed to be in base ***e*** unless otherwise specified.

Simplify your answers. While at the judges’ discretion, it is generally recommended to use common denominators, reduce fractions, evaluate simple expressions, and arithmetically calculate integers with absolute value under 5000. If an expression is insufficiently simplified, you will receive a “prompt” rather than verdict.

All answers are real, defined, and in terms of x, functions of x, and various constants.

SHORT QUESTIONS

60 seconds per question

1 attempt per question

0

Short

$$\int \frac{d\left(\tan^{-1}\left(\frac{1}{x}\right)\right)}{d\left(\frac{1}{x}\right)}\,dx$$

0

X

$$x - \tan^{-1} x + C$$

0 **Short**

$$\int \left(\sin^6 x \cos^8 x - \sin^8 x \cos^6 x\right)\, dx$$

0

X

$$\frac{\sin^7(2x)}{896} + C$$

0 **Short**

$$\int \frac{dx}{\sqrt[5]{20x+25}}$$

0

X

$$\frac{(20x+25)^{\frac{4}{5}}}{16}+C$$

0 **Short**

$$\int \cos^2 x \, d(\sin x)$$

0

X

$$\sin x - \frac{\sin^3 x}{3} + C$$

0 **Short**

$$\int \frac{dx}{x\left(\ln^2 x + 4\right)}$$

0

X

$$\frac{1}{2}\tan^{-1}\left(\frac{\ln x}{2}\right)+C$$

0 **Short**

$$\int_{-\frac{\pi}{4}}^{\frac{\pi}{4}} \ln\left|\sec x + \tan x\right| dx$$

0 **X**

0

0 **Short**

$$\int_{-2}^{2} |x+1||x||x-1|\,dx$$

0 **X**

5

0 **Short**

$$\int \frac{dx}{\log_x\left(e^x\right)}$$

0

X

$$\frac{1}{2}\ln^2 x + C$$

0 **Short**

$$\int_{1}^{2025} \Pr\left(\mathcal{U}(1, x) < 2\right)\ dx$$

0

X

$$1 + \ln 2024$$

0 **Short**

$$\int_{1-e^{2025}}^{1-e^{-2025}} \frac{dx}{1+|x|}$$

0

X

$$\ln(2e^{2025} - 1) + C$$

0 **Short**

$$\int_0^1 \frac{x^4 - 1}{x - 1}\, dx$$

0

X

$$\frac{25}{12}$$

0 **Short**

$$\int e^{2025x}\, d|2025x|$$

0

X

$$\frac{x}{|x|}e^{2025x} + C$$

0 **Short**

$$\int \frac{dx}{x \ln x - x}$$

0

X

$$\ln(\ln x - 1) + C$$

START HERE

0 **Short**

$$\int \sqrt{x^{42}}\, dx$$

0

X

$$\frac{x^{21}|x|}{22} + C$$

0 **Short**

$$\int_1^\infty \frac{1}{x\sqrt{x^8-1}}\,dx$$

0

X

$$\frac{\pi}{8}$$

0

Short

$$\int \begin{vmatrix} x & 3x & xe^x \\ 1 & 2 & e^x \\ 0 & 1 & e^x \end{vmatrix} dx$$

0

X

$$(1-x)e^x + C$$

0 **Short**

$$\lim_{n\to\infty} \int_0^1 (x^n + x)\ dx$$

0 **X**

$$\frac{1}{2}$$

0 **Short**

$$\int_{\ln 2}^{\ln 3} \frac{1}{1-e^x}\,dx$$

0

X

$$\ln 3 - 2 \ln 2$$

0 **Short**

$$\int \left(\frac{\sin x}{x^2} - \frac{\cos x}{x} \right) \, dx$$

0

X

$$-\frac{\sin x}{x} + C$$

0 **Short**

$$\int_0^3 \min\{x, x^2, x^3\}\, dx$$

0

X

$$\frac{17}{4}$$

0 **Short**

$$\int_0^{2\pi} \sqrt{1 - \cos^2(x)}\, dx$$

0

X

4

0 **Short**

$$\int \frac{d\left(\sqrt{x}\right)}{d\left(x^2\right)}\,dx$$

0

X

$$-\frac{1}{2\sqrt{x}}+C$$

MEDIUM QUESTIONS

120 seconds per question

2 attempt per question

0 **Medium**

$$\int_{e^2}^{e^3} \ln(x) \ln\left(\ln(x) - 1\right) dx$$

0

X

$$e^3(2\ln 2 - 1) + e^2$$

0 **Medium**

$$\int_0^3 \left(\sqrt{1+\sqrt{1+\sqrt{1+\sqrt{1+\ldots}}}} \right)^{\lceil x \rceil} dx$$

0

X

$$4 + 2\sqrt{5}$$

0 **Medium**

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{3}} \left(\tan^{-1}(x) + \tan^{-1}\left(\frac{1}{x}\right) \right) \, dx$$

0

X

$$\frac{\pi^2}{12}$$

0 **Medium**

$$\int \frac{x}{\sqrt{2x - x^2}}\, dx$$

0

X

$$\sin^{-1}(x-1) - \sqrt{2x - x^2} + C$$

0

Medium

$$\int_0^{\pi} \frac{dx}{\sqrt{\sec x \tan x}}$$

0

X

4

0 **Medium**

$$\int_4^6 \left(\sqrt[3]{x-5}+5\right)^3 \, dx$$

0

X

268

START HERE

0 **Medium**

$$\int_0^{\frac{\pi}{2}} \frac{x}{\cos\left|x-\frac{\pi}{4}\right|}\,dx$$

0

X

$$\frac{\pi}{2} \ln\left(\sqrt{2}+1\right)$$

0 **Medium**

$$\int e^{x^2}(2x\sin x + \cos x)\,dx$$

0

X

$$e^{x^2} \sin x + C$$

0 **Medium**

$$\int_0^\infty \frac{1}{\lfloor x \rfloor !}\, dx$$

0 **X**

e

0 **Medium**

$$\int_0^1 e^x \prod_{n=0}^{\infty} \left(\sqrt[n!]{e^{x^n}} \right) \, dx$$

0

X

$$e^e - e$$

0 **Medium**

$$\int_{-1}^{2025} i^x \, dx$$

0

X

$$\frac{4}{\pi}$$

0

Medium

$$\int_0^2 \left\{x^2\right\}\, dx$$

0

X

$$\sqrt{2} + \sqrt{3} - \frac{7}{3}$$

0 **Medium**

$$\int \frac{x^2+1}{x^3-x}\,dx$$

0

X

$$\ln|x-1| - \ln|x| + \ln|x+1| + C$$

0 **Medium**

$$\int_0^{\infty} \frac{d}{dx}\left(\frac{e^x \sin\left(xe^x\right)}{e^{2x}-\cos x}\right)\, dx$$

0

X

$$-\frac{1}{2}$$

0 **Medium**

$$\int \left(\cos^{-1} x\right)^2 \, dx$$

0

X

$$x\left(\cos^{-1}x\right)^2-2\sqrt{1-x^2}\left(\cos^{-1}x\right)-2x$$

0 **Medium**

$$\int \mathrm{E}\left(\sqrt{\mathcal{U}\left(0, x^2\right)}\right)\, dx$$

0

X

$$\frac{x^2}{3} + C$$

0 **Medium**

$$\int_0^1 x^5 \left(1 - x^2\right)^3 \, dx$$

0

X

$$\frac{1}{120}$$

0 **Medium**

$$\int (\sin x + \cos x)^4 \, dx$$

0

X

$$\frac{3x}{2} - \cos(2x) - \frac{\sin(4x)}{8} + C$$

0 **Medium**

$$\int \left| e^{\sqrt{x}+ix^2} \right| \, dx$$

0

X

$$2e^{\sqrt{x}}\left(\sqrt{x}-1\right)$$

LONG QUESTIONS

180 seconds per question

3 attempts per question

0 **Long**

$$\int e^{\mathrm{W}(x)}\, dx$$

0

X

$$\frac{(2\mathrm{W}(x)+1)\,e^{2\mathrm{W}(x)}}{4}+C$$

0 **Long**

$$\int_1^2 \sec^{-1}(\sqrt{x})\, dx$$

0

X

$$\frac{\pi}{2} - 1$$

0 **Long**

$$\int_0^2 (x^3 + x^2 + x + 1)\, d\lceil x \rceil$$

0 **X**

5

0 **Long**

$$\lim_{n\to\infty} \int_0^\infty e^{-x^n}\, dx$$

0 **X**

1

0 **Long**

$$\int_0^{2\pi} (\sin(x) + \cos(2x) + \sin(3x) + \cos(4x))^2 \, dx$$

0 **X**

4π

0 **Long**

$$\int e^{2x}(\sin x + \cos x)\, dx$$

0

X

$$\frac{e^{2x}(3\sin x + \cos x)}{5} + C$$

0 **Long**

$$\int \tan^{-1}\left(\frac{1}{\sqrt{x}}\right)\, dx$$

0

X

$$x\tan^{-1}\left(\frac{1}{\sqrt{x}}\right)+\sqrt{x}-\tan^{-1}\left(\sqrt{x}\right)+C$$

0

Long

$$\int \sum_{n=0}^{\infty} 2^{-n} \cos\left(x + \frac{n\pi}{2}\right)\, dx$$

0

X

$$\frac{4\sin x + 2\cos x}{5}$$

0 **Long**

$$\int_{-\frac{1}{2}}^{\frac{1}{4}} \frac{d}{dx} \left(\left(x + \left(x + \left(x + (x + \ldots)^2 \right)^2 \right)^2 \right)^2 \right) \, dx$$

0

X

$$\frac{2\sqrt{3}-3}{4}$$

0 **Long**

$$\int_0^{2025\pi} \left| \sin x + \sqrt{-|\cos(2x)|} \right| \, dx$$

0

X

4050

0 **Long**

$$\int_1^{\sqrt{2}} \sqrt{x^2 - 1}\, dx$$

0

X

$$\frac{\sqrt{2} - \ln\left(\sqrt{2} + 1\right)}{2}$$

0 **Long**

$$\int_0^\infty \frac{xe^{-x}\sin\left(1-e^{-x}\right)-2+2\cos\left(1-e^{-x}\right)}{x^3}\,dx$$

0

X

$$-\frac{1}{2}$$

0

Long

$$\int \left(0.\overline{x}_{x+1} - 0.\overline{1}_x\right)\ dx$$

0

X

$$x - \ln(x - 1)$$

0 **Long**

$$\int \sum_{n=0}^{\infty} \left(\sum_{k=0}^{\infty} \frac{(-1)^k x^{2k+1}}{(2k+1)!} \right)^{2n} dx$$

0

X

$$\tan x + C$$

0 **Long**

$$\lim_{n\to\infty} \int_0^{\frac{\pi}{3}} \frac{1}{5+\tan^n(x)}\,dx$$

0

X

$$\frac{\pi}{20}$$

0 **Long**

$$\int_0^\infty \frac{\sin \lfloor x \rfloor}{3^{\lfloor x \rfloor}} \, dx$$

0

X

$$\frac{3 \sin 1}{10 - 6 \cos 1}$$

0 **Long**

$$\lim_{n\to\infty} \int_0^{2\pi} \sqrt[n]{\sin^n(x) + \cos^n(x)}\, dx$$

0

X

$$2\sqrt{2}$$

0 **Long**

$$\int_1^3 \left\lfloor \sqrt{\frac{2x}{x+1}} \right\rfloor dx$$

0 **X**

2

0 **Long**

$$\int_0^1 e^x \prod_{k=0}^{\infty} \left(\frac{1}{1+x^{2^k}} \right) \, dx$$

0

X

$$e - 2$$

0 **Long**

$$\int_0^{\cos^{-1}\left(\frac{3}{5}\right)} \frac{128}{\left(1+\frac{3}{5}\cos x\right)^2}\,dx$$

0

X

$$125\pi - 150$$

0 **Long**

$$\int_0^{900} \{\sqrt{x}\}\, dx$$

0 **X**

455

0 **Long**

$$\int_{-e}^{-1} \frac{\ln|x||\ln x|}{x}\,dx$$

0

X

$$\frac{\pi^3 - (\pi^2 + 1)^{3/2}}{3}$$

0 **Long**

$$\int_1^{2025} i \left(\frac{1+i\sqrt{3}}{2} \right)^{\lfloor x \rfloor} dx$$

0

X

$$-\sqrt{3}$$

0 **Long**

$$\int_0^{2025} \frac{\lfloor \sqrt{x} \rfloor}{2 \lfloor \sqrt{x} \rfloor + 1} \, dx$$

0 **X**

990

0 **Long**

$$\int (xe^x(\cos x - \sin x) + e^x \sin x)\, dx$$

0

X

$$(xe^x - e^x)\cos x + C$$

Thank You For Playing!

Come Again Next Year!