Homework 1

Xiaojun Peng

March 26, 2022

1. P45 5: We have 3 points: $(x_i, f(x_i)) = (81,9), (100,10), (121,11), i = 0,1,2$. So we construct a quadratic polynomial (Lagrange):

$$L_2(x) = l_0(x)f(x_0) + l_1(x)f(x_1) + l_2(x)f(x_2),$$

where

$$\begin{aligned} l_0(x) &= \frac{(x-100)(x-121)}{(81-100)(81-121)} = \frac{(x-100)(x-121)}{760}, \\ l_1(x) &= \frac{(x-81)(x-121)}{(100-81)(100-121)} = -\frac{(x-81)(x-121)}{399}, \\ l_2(x) &= \frac{(x-81)(x-100)}{(121-81)(121-100)} = \frac{(x-81)(x-100)}{840}. \end{aligned}$$

Let x = 105, we can calculate:

$$\begin{aligned} f(105) \approx L_2(105) = 9 \cdot \frac{(105-100)(105-121)}{760} - 10 \cdot \frac{(105-81)(105-121)}{399} + 11 \cdot \frac{(105-81)(105-100)}{840} \\ = 10.24812. \end{aligned}$$

According to the error in polynomial interpolation :

$$f(x) - L_2(x) = \frac{f^{(3)}(\xi_x)}{3!}(x-81)(x-100)(x-121),$$

and $f(x) = \sqrt{x} \Rightarrow f^{(3)}(x) = \frac{3}{8}x^{-\frac{5}{2}}$,which is monotonically decreasing in [81,121], the maximum is obtained when x=81, which is $6.350658 \cdot 10^{-6}$.

As a result, the bound of absolute error is

$$|f(x) - L_2(x)| \leq |\frac{6.350658 \cdot 10^{-6}}{6} \cdot (105-81) \cdot (105-100) \cdot (105-121)| = 0.002032211.$$

We know that $\sqrt{105} = 10.24695$, the real error is $10.24695 - 10.24812 = -0.00117$.

2. P45 7

   - (1) The interpolation polynomial in Newton's form is

     $$N_3(x) = f(4) + f[1,4](x-4) + f[1,3,4](x-4)(x-1) + f[1,2,3,4](x-4)(x-1)(x-3).$$

     To be specific,

     $$\begin{aligned} N_3(x) &= 1 + 2(x-4) + (x-4)(x-1) - (x-4)(x-1)(x-3) \\ &= -x^3 + 9x^2 - 22x + 9 \end{aligned}$$

   - (2)We know that $f(2) = N_3(2) = -7$ and $f[1,2,3,4] = \frac{f[1,3,4]-f[2,3,4]}{1-2}$, so we have

     $$f[2,3,4] = f[1,3,4] + f[1,2,3,4] = 0$$

     .

3. P45 12 The divided-difference table used for constructing the Hermite interpolation polynomial is as follow:

z_i	$f(z_i)$	$f[z_{i-1}, z_i]$	$f[z_{i-2}, z_{i-1}, z_i]$
3	5		
5	15	5	
5	15	7	1

The polynomial is:

$$H(x) = 5 + 5(x-3) + (x-3)(x-5) = x^2 - 3x + 5.$$

So $f(3.7) \approx H(3.7) = 7.59$.

And the error term is $f(x) - H(x) = \frac{f^{(3)}(\xi_x)}{3!}(x-3)(x-5)^2, \xi_x \in [3,5]$.