SNM_HW03

PB19111713钟颖康

3.2

$$
\begin{aligned}
&①x_{n+1}=\varphi_1(x_n)=\frac{2x_n^3-5x_n^2+42}{19},\\
&\quad\varphi_1'(x)=\frac{6x^2-10x}{19},\\
&\quad|\varphi_1(3)|=\frac{24}{19}>1,\text{故当}x\in[2.5,3.5]\text{时不保证收敛}\\
&②x_{n+1}=\varphi_2(x_n)=\sqrt{\frac{2x_n^3-19x+42}{5}},\\
&\quad\varphi_2'(x)=\frac{1}{2}\cdot(\frac{2x^3-19x+42}{5})^{-\frac{1}{2}}\cdot(6x^2-19),\\
&\quad|\varphi_2(3)|=\frac{7}{2}\cdot\sqrt{\frac{5}{39}}>1,\text{故当}x\in[2.5,3.5]\text{时不保证收敛}\\
&③x_{n+1}=\varphi_3(x_n)=\varphi_3(x_n)=\sqrt[3]{\frac{5x_n^2+19x-42}{2}},\\
&\quad\varphi_3'(x)=\frac{1}{3}\cdot(\frac{5x_n^2+19x-42}{2})^{-\frac{2}{3}}\cdot\frac{10x+19}{x},\\
&\quad|\varphi_3(3)|=\frac{49}{6}\cdot\sqrt[3]{\frac{1}{900}}<1,\text{故当}x\in[2.5,3.5]\text{时收敛}
\end{aligned}
$$

5.1.(2)

观察易知

$$||A||_1=7,||A||_\infty=8$$

由于

$$A^TA=\begin{pmatrix}26&4&-1\\4&11&7\\-1&7&37\end{pmatrix}$$

解得三个特征值分别为$\lambda_1\approx8.2760,\lambda_2\approx38.7648,\lambda_3\approx26.9591$

故

$$||A||_2=\sqrt{38.7684}=6.226$$

5.2.(2)

$$B=\begin{pmatrix}5&2&2\\2&6&0\\2&0&4\end{pmatrix}$$

计算易知B的三个特征值分别为$\lambda_1=2,\lambda_2=8,\lambda_3=5$

故谱半径$\rho(B)=8$,

$$B^TB=\begin{pmatrix}33&22&18\\22&40&4\\18&4&20\end{pmatrix}$$

解得三个特征值分别为$\lambda_1=64,\lambda_2=25,\lambda_3=4$

2-范数$||B||_2=\sqrt{64}=8$