

# SNM\_Report\_Project1

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## Program2

## 2.1 Introduction

**程序 2** 下面给出美国 1920~1970 年的人口表:

年份	1920	1930	1940	1950	1960	1970
人口 (千人)	105711	123203	131669	150697	179323	203212

用表中数据构造一个 5 次 Lagrange 插值多项式, 并用此估计 1910, 1965 和 2002 年的人口. 1910 年的实际人口数约为 91772000, 请判断插值计算得到的 1965 年和 2002 年的人口数据准确性是多少?

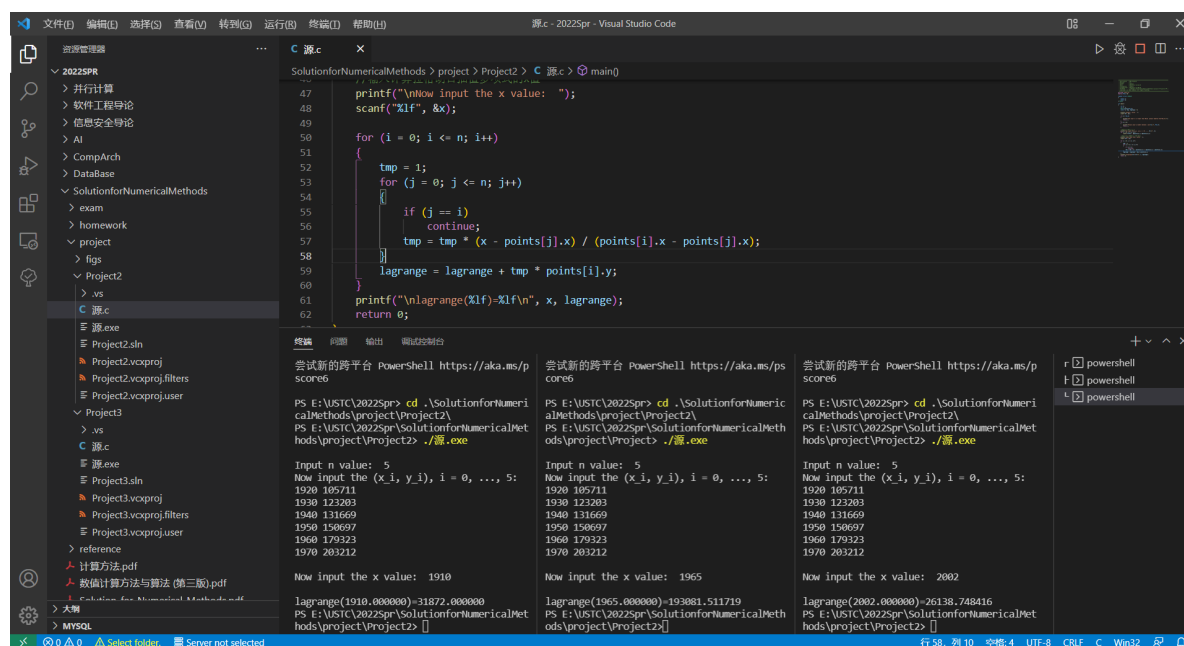
## 2.2 Method

题目要求构造Lagrange插值多项式，根据课本上的公式

$$L_n(x) = \sum_{i=0}^n \prod_{0 \leq i \leq n, i \neq j} \frac{x - x_j}{x_i - x_j} f(x_i)$$

可直接写出程序求出1910, 1965和2002年的人口。

## 2.3 Result



根据程序运行结果,

$$\begin{array}{rcl} L_5(1910) & = & 31872 \\ L_5(1965) & = & 193081.51 \\ L_5(2002) & = & 26138.75 \end{array}$$

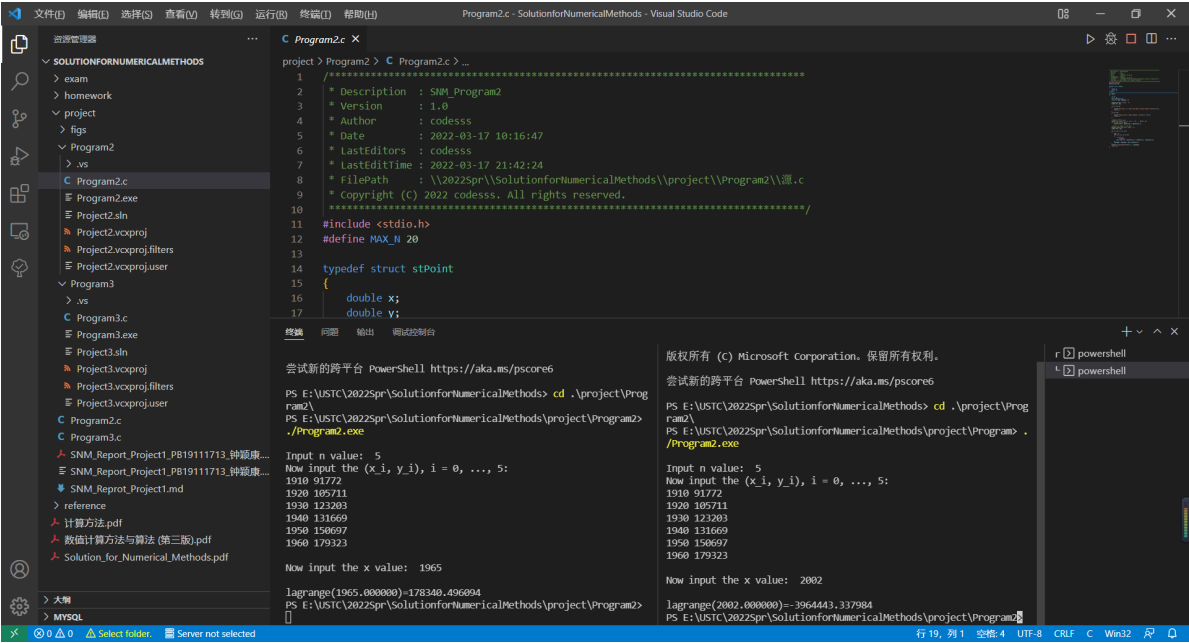
即所求三个年份的人口约为31872, 193081.51和26138.75。

## 2.4 Discussion

现给出1910年的实际人口为91772000, 故选取1910, 1920, 1930, 1940, 1950和1960这6个插值点构造一个新的插值多项式, 记为 $\tilde{L}_5(x)$ , 根据事后估计法, 误差即可表示为

$$f(x) - L_n(x) \approx \frac{x - x_0}{x_0 - x_{n+1}}(L_n(x) - \tilde{L}_n(x))$$

利用之前的程序计算 $\tilde{L}_5(x)$



```
project > Program2 > C: Program2 > ...
1  /*****
2  * Description : SNM_Program2
3  * Version : 1.0
4  * Author : codesss
5  * Date : 2022-03-17 10:16:47
6  * LastEditors : codesss
7  * LastEditTime : 2022-03-17 21:42:24
8  * Filepath : \\2022Spr\\SolutionforNumericalMethods\\project\\Program2\\源.c
9  * Copyright (C) 2022 codesss. All rights reserved.
10 *****/
11 #include <stdio.h>
12 #define MAX_N 20
13
14 typedef struct stPoint
15 {
16     double x;
17     double y;
18 }
19
20 int main()
21 {
22     int n;
23     double x, y;
24     double *x_array, *y_array;
25     x_array = (double *)malloc(sizeof(double) * MAX_N);
26     y_array = (double *)malloc(sizeof(double) * MAX_N);
27     printf("Input n value: ");
28     scanf("%d", &n);
29     printf("Now input the (x_i, y_i), i = 0, ..., %d:\n", n);
30     for (int i = 0; i <= n; i++)
31     {
32         scanf("%lf%lf", &x_array[i], &y_array[i]);
33     }
34     printf("Now input the x value: ");
35     scanf("%lf", &x);
36     double result = Lagrange(x, x_array, y_array, n);
37     printf("Lagrange(%lf, %lf) = %lf\n", x, result);
38     return 0;
39 }
```

尝试新的跨平台 PowerShell https://aka.ms/powershell

PS E:\USTC\2022Spr\SolutionforNumericalMethods> cd .\project\Program2\

PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Program2> ./Program2.exe

Input n value: 5

Now input the (x\_i, y\_i), i = 0, ..., 5:

1910 91772

1920 105711

1930 123203

1940 131669

1950 150697

1960 179323

Now input the x value: 1965

Lagrange(1965.000000) = 178340.496994

PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Program2>

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PS E:\USTC\2022Spr\SolutionforNumericalMethods> cd .\project\Program2\

PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Program2> ./Program2.exe

Input n value: 5

Now input the (x\_i, y\_i), i = 0, ..., 5:

1910 91772

1920 105711

1930 123203

1940 131669

1950 150697

1960 179323

Now input the x value: 2002

Lagrange(2002.000000) = -3964443.337984

PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Program2>

可知

$$\begin{aligned}\tilde{L}_5(1965) &= 178340.50 \\ \tilde{L}_5(2002) &= -3964443.34\end{aligned}$$

相应地, 可知误差分别为

$$\begin{aligned}f(1965) - L_5(1965) &\approx -1228.42 \\ f(2002) - L_5(2002) &\approx 2128310.45\end{aligned}$$

肉眼可见误差相当巨大, 故插值计算得到的1965年和2002年人口数据不太准确。

## Program3

### 3.1 Introduction

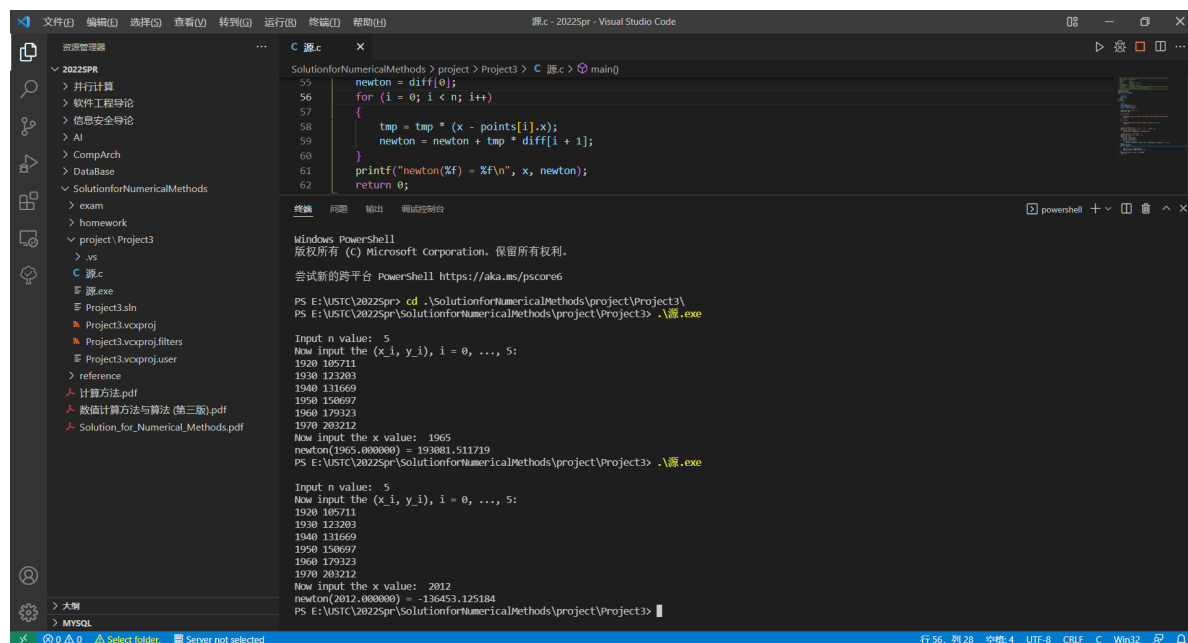
**程序 3** 数据同上表, 用 Newton 插值估计:

- (1) 1965 年的人口数;
- (2) 2012 年的人口数.

## 3.2 Method

思路参考课本附录2中的程序 1。

## 3.3 Result



The screenshot shows the Visual Studio Code interface. The left sidebar displays the file explorer with a project structure including '2022SPR', '并行计算', '软件工程导论', '信息安全导论', 'AI', 'CompArch', 'SolutionforNumericalMethods', 'exam', 'homework', 'project', 'Project3', '.vs', '源.c', '源.exe', 'Project3.sln', 'Project3.vcxproj', 'Project3.vcxproj.filters', 'Project3.vcxproj.user', 'reference', '计算方法.pdf', '数值计算方法与算法 (第三版).pdf', 'Solution\_for\_Numerical\_Methods.pdf', '大纲', and 'MYSQL'. The main editor window shows a C program named '源.c' with the following code:

```
55 newton = diff[0];
56 for (i = 0; i < n; i++)
57 {
58     tmp = tmp * (x - points[i].x);
59     newton = newton + tmp * diff[i + 1];
60 }
61 printf("newton(%f) = %f\n", x, newton);
62 return 0;
```

The bottom panel shows the '终端' (Terminal) output, which is a Windows PowerShell session. The user has navigated to the project directory and executed the program. The output shows the results of the Newton-Raphson method for two different input values of x:

```
Input n value: 5
Now input the (x_i, y_i), i = 0, ..., 5:
1920 185711
1930 123203
1940 131669
1950 150697
1960 179323
1970 203212
Now input the x value: 1965
newton(1965.000000) = 193081.511719
PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Project3> .\源.exe

Input n value: 5
Now input the (x_i, y_i), i = 0, ..., 5:
1920 185711
1930 123203
1940 131669
1950 150697
1960 179323
1970 203212
Now input the x value: 2012
newton(2012.000000) = -136453.125184
PS E:\USTC\2022Spr\SolutionforNumericalMethods\project\Project3>
```

根据程序运行结果，

$$\begin{aligned} N_5(1965) &= 193081.51 \\ N_5(2002) &= -136453.13 \end{aligned}$$

即所求两个年份的人口约为193081.51和-136453.13。