Probability Theory

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Abstract

Notes of Probability Theory mainly from the book *Introduction to Probability Theory by Hoel, Port, Stone.*

1 Probability Spaces

2 Discrete Random Variables

3 Expectation of Discrete Random Variables

- Let n be the number of trials and $N_n(x_i)$ be the number of times you observed x_i, then for large n

$$f(x_i) = \frac{N_n(x_i)}{n}.$$

- A random variable X has *finite* expectation if and only if

$$\sum_{i=0}^{i=\infty} |x_i| f(x_i) < \infty,$$

  then the expectation is

$$EX = \sum_{i=0}^{i=\infty} x_i f(x_i).$$

  Otherwise EX is undefined.

- Quick Expectations to remember: Binomial - np, Poisson - λ, Geometric - $\frac{1-p}{p}$.

- Expectation of a function (if it is finite):

$$E\phi(X) = \sum_x \phi(x) f(x)$$

- Properties

  $E(c_1 X_1 + c_2 X_2 + ...) = c_1 EX_1 + c_2 EX_2 + ...$

  $|EX| \leq E|X|$

  $X \leq Y \implies EX \leq EY$

  If $P(|X| \leq M) = 1$ then $EX \leq M$

  if X and Y are independent, $E(XY) = (EX)(EY)$. The converse is not true.

  if X is a non negative, integer valued random variable, X has a finite expectation if and only if $\sum_{x=0}^{x=\infty} P(X \geq x)$ converges. Then $EX = \sum_{x=0}^{x=\infty} P(X \geq x)$.

References

References follow the acknowledgments. Use unnumbered first-level heading for the references. Any choice of citation style is acceptable as long as you are consistent. It is permissible to reduce the font size to `small` (9 point) when listing the references. **Remember that you can use more than eight pages as long as the additional pages contain *only* cited references.**

[1] Alexander, J.A. & Mozer, M.C. (1995) Template-based algorithms for connectionist rule extraction. In G. Tesauro, D.S. Touretzky and T.K. Leen (eds.), *Advances in Neural Information Processing Systems 7*, pp. 609–616. Cambridge, MA: MIT Press.

[2] Bower, J.M. & Beeman, D. (1995) *The Book of GENESIS: Exploring Realistic Neural Models with the GEneral NEural SImulation System.* New York: TELOS/Springer–Verlag.

[3] Hasselmo, M.E., Schnell, E. & Barkai, E. (1995) Dynamics of learning and recall at excitatory recurrent synapses and cholinergic modulation in rat hippocampal region CA3. *Journal of Neuroscience* **15**(7):5249-5262.