Game Theory

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Abstract

My attempt at solving the Problem Sets for Game Theory at IISC 2019.

1 Introduction

1.1 warm-up

1. A game in my understanding is an *interaction* between agents or players. They take some *actions* to achieve some goal (usually the maximization of their expected utility). These actions can be simultaneous or sequential or mixed?, actions can be one step or multi step, their actions and action histories may/may not be visible to others, the players themselves may/may not be rational and/or intelligent.

   In a non-cooperative game, players will not form a coalition with one another and will not play *together* whereas in cooperative GT, players form coalitions and decide to play a defined strategy (to increase their utility).

2. In game theory, we are given a game and we analyze various types of equilibria in the game whereas in mechanism design we are given a social choice function that we want to implement and we design a game that implements it in the optimal case.

   e.g. Fair utilization of computational resources of a lab. Given a set of resources at your disposal and players wanting to use them (their utilities can be based on the proximity to any upcoming deadlines that they are targeting), how do yo fairly divide resources among all the members.

3. Intelligence assumes that players are capable of computing the *optimal strategy* (for rational players this strategy is maximizing their expected utility). Rationality assumes that players will play to maximize their expected utility.

4. In the matching pennies problem, Player 1 wins on a match and Player 2 wins on a mismatch. Since the probabilities of matching and not matching are equally likely $(\frac{2}{4}, \frac{2}{4})$, the expected utility of both the players should be zero. very doubtful on this.

1.2 Workhorse

1. Need to get the book

2. The strategic form game here $G = (N, (S_i)_{i\in N}, (u_i)_{i\in N})$ consists of $N = \{1, 2, ..., n\}$, $S_i = \{1, 2, ..., m\}$, the strategy vector $s \in S$ will be $(x_1, x_2, ..., x_n)$ where each $x_i \in \{1, 2, ..., m\}$.

   $$u_i(x_1, x_2, ..., x_n) = \begin{cases} \frac{1}{|\{k: x_k = argmin\ (|x_k - (2/3)\bar{x}|)\}|} & \text{if} \quad x_i = argmin\ (|x_i - (2/3)\bar{x}|) \\ 0 \quad \text{otherwise} \end{cases}$$

   Can we get a PSNE analysis for this?

32nd Conference on Neural Information Processing Systems (NIPS 2018), Montréal, Canada.

3. Pigou network. need to study.
4. Don't know In every case that I can think of, the players who get to pick first and second can form a coalition and leave the third player unhappy.

1.3 Thought Provoking

2 Dominant Strategy Equilibria

2.1 Warm-up

1. By definition, a strongly dominant strategy s_i^* is such that $u_i(s_i^*, s_{-i}) > u_i(s_i^*, s_{-i})$ for all s_i. If there is another such strategy let's say $s_i^{(2*)}$ then it is dominated by s_i^* which means that another strictly dominant strategy cannot exist.
2. If the system is in a dominant strategy equilibrium then for player i playing strategy s_i, $u_i(s_i, s_{-i}) \geq u_i(t_i, s_{-i})$ for any other strategy t_i of player i for every strategy s_{-i} of other players. This means that $u_i(s_i, s_{-i}) = \underset{t_i \in S_i}{max}\ u_i(t_i, s_{-i})$ which means that s_i is the best response for player i for every strategy s_{-i} of other players. This means that s_i is the Nash Equilibrium strategy for player i. Since this is true for all players, they are in Nash Equilibrium.
3. A simple game (two players and two strategies each) with a strictly dominant strategy.

1/2	A	B
A	10,10	10, 1
B	0, 10	1, 1

4. Same as above.