Homework 1

Summary

This report presents a pattern recognition approach to segment a cheetah image into foreground (cheetah) and background (grass) components using discrete cosine transform (DCT) features extracted from 8×8 image blocks.

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1. Introduction

Problem Statement

The objective is to classify each 8×8 block in the cheetah image as either foreground (cheetah) or background (grass) using frequency-domain features derived from the DCT.

Methodology Overview

- **Feature Space**: 8×8 DCT coefficient blocks converted to 64-dimensional vectors
- **Feature Selection**: Index of the 2nd largest DCT coefficient (by absolute value)
- **Classification**: Minimum probability of error rule using Bayesian decision theory
- **Training Data**: Provided in `TrainingSamplesDCT_8.mat`

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2. Prior Probability Estimation (Part a)

Approach

The prior probabilities P(Y = cheetah) and P(Y = grass) were estimated from the training dataset by computing the proportion of samples in each class.

Results

Training Set Statistics:

- Number of foreground (cheetah) samples: **250**
- Number of background (grass) samples: **1053**

Estimated Prior Probabilities:

- P(Y = cheetah) = **0.1919**
- P(Y = grass) = **0.8081**

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3. Class-Conditional Probability Distributions (Part b)

Methodology

For each training sample, the index of the DCT coefficient with the 2nd largest energy (absolute value) was computed. Histograms were constructed for each class to estimate P(X|Y=cheetah) and P(X|Y=grass).

Visualization

Histogram:

Foreground (Cheetah):

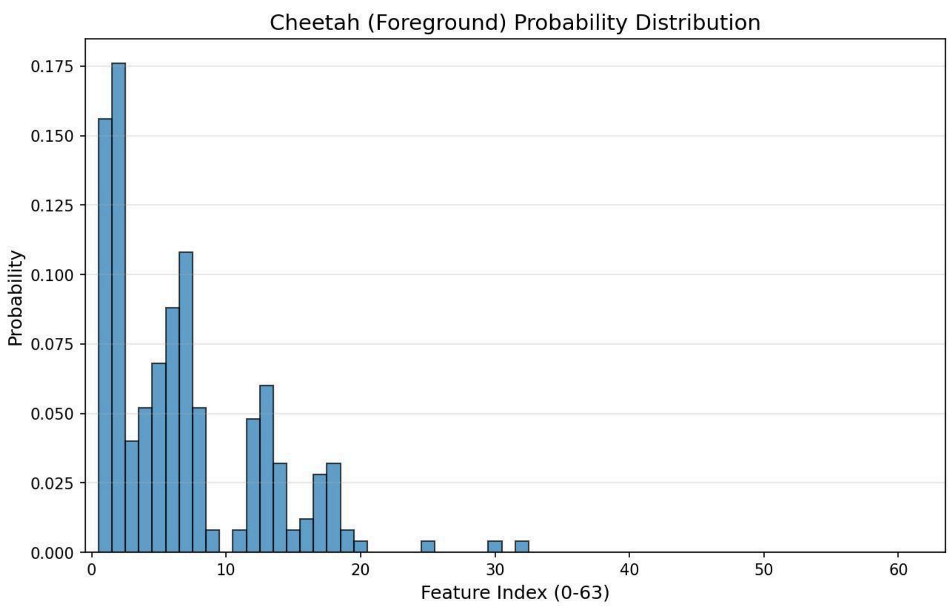

Figure 1 (a)

Background (Grass):

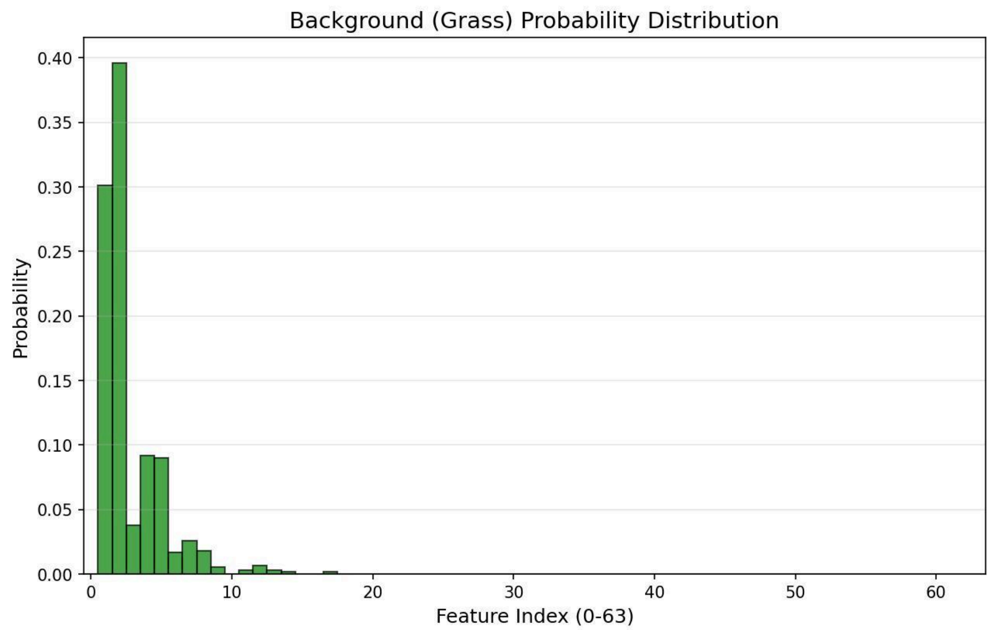

Figure 1 (b)

Figure 1 (a) & 1 (b): Class-conditional probability distributions P(X|Y=cheetah) and P(X|Y=grass) showing the frequency of DCT coefficient indices for each class.

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4. Image Segmentation Results (Part c)

Classification Procedure

Using the Bayes minimum probability of error rule:

Decision Rule:

Classify block as "cheetah" if: P(Y=cheetah|X) > P(Y=grass|X)

Using Bayes' theorem:
P(Y|X) = P(X|Y) × P(Y) / P(X)

For each 8×8 block in `cheetah.bmp`:

1. Computed DCT coefficients
2. Extracted feature X (index of 2nd largest coefficient)
3. Applied decision rule using estimated probabilities

4. Stored classification in array A

Segmentation Results

Image Processing Details:

A padding of 4 pixels on the borders was applied on the image for making sure that the edges are captured properly

- Original image size: **255 x 270**
- Padded image size: **263 x 278**

Classification Distribution:

- Pixels classified as cheetah: **7493**
- Pixels classified as grass: **61357**

Visualization

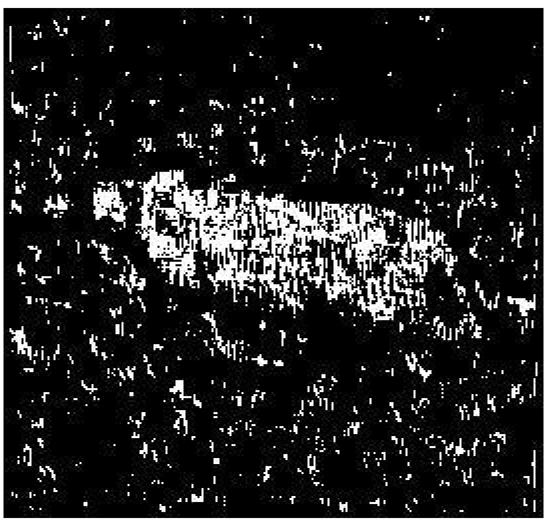

Figure 2: Segmentation mask showing predicted classification for each 8×8 block (white = cheetah, black = grass).

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5. Performance Evaluation (Part d)

Error Analysis

The segmentation mask was compared against the ground truth provided in `cheetah_mask.bmp` to evaluate classification performance.

Results

Confusion Matrix:

	Predicted Cheetah	Predicted Grass	Total
Actual Cheetah	4,537	8,672	13,209
Actual Grass	2,956	52,685	55,641
Total	7,493	6,1357	68,850

Performance Metrics:

Probability of Error (Weighted by Priors):

P(error | cheetah) × P(cheetah) = 0.125964

P(error | background) × P(background) = 0.042933

Total Probability of Error = 0.168897

- **Total Probability of Error = 0.168897**
- **Accuracy**: **83.11%**

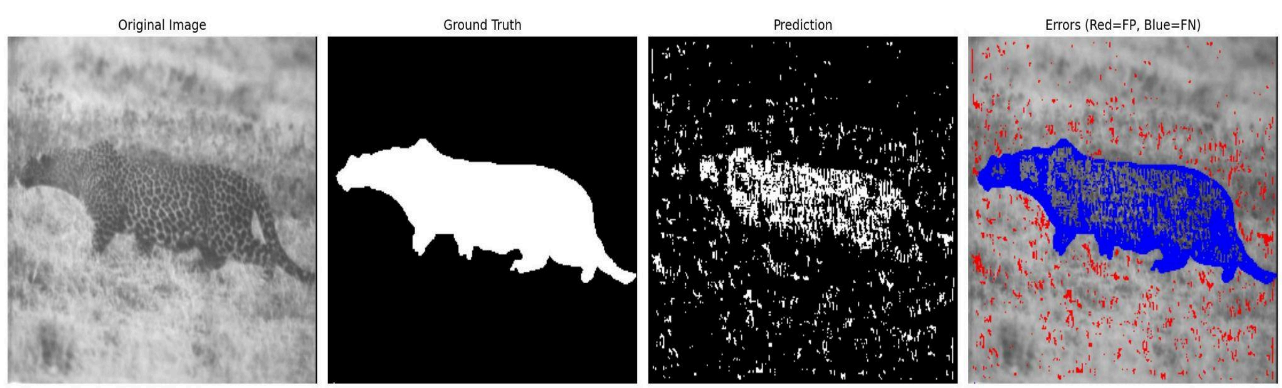
True Positives (TP) = 4,537

False Negatives (FN) = 8,672

False Positives (FP) = 2,956

True Negatives (TN) = 52,685

Error Distribution



Appendix

Code

```
# -*- coding: utf-8 -*- #
import scipy.io
import numpy as np
import matplotlib.pyplot as plt
from PIL import Image
from scipy.fftpack import dct

### Utility functions

def second_largest_index(row):
   # Find the index of the second largest value in the row.
   sorted_indices = np.argsort(row)[::-1]
   return sorted_indices[1]

def dct2(block):
   # 2D Discrete Cosine Transform
   return dct(dct(block.T, norm='ortho').T, norm='ortho')

def load_zigzag_pattern(filename):
   # Load zigzag pattern from text file
   pattern = np.loadtxt(filename, dtype=int)
   return pattern

def zigzag_scan(matrix, pattern):
   # Apply zigzag scan using the provided pattern
   result = np.zeros(64)
   for i in range(8):
       for j in range(8):
           zigzag_index = pattern[i, j]
           result[zigzag_index] = matrix[i, j]
   return result

### Question 1

# Load the .mat file
mat_data = scipy.io.loadmat('TrainingSamplesDCT_8.mat')

# Extract the datasets (each row is a sample, each column is a feature)
```

```python
background_data = mat_data['TrainsampleDCT_BG']
cheetah_data = mat_data['TrainsampleDCT_FG']

# Get the number of datapoints
background_count = background_data.shape[0]
cheetah_count = cheetah_data.shape[0]
total_count = background_count + cheetah_count

print(f"Number of background samples: {background_count}")
print(f"Number of cheetah samples: {cheetah_count}")
print(f"Feature dimension: {background_data.shape[1]}")

# Calculate prior probabilities
prior_cheetah = cheetah_count / total_count
prior_background = background_count / total_count

print(f"Prior probability of cheetah: {prior_cheetah:.4f}")
print(f"Prior probability of background: {prior_background:.4f}")

# Question 2

# Foreground (Cheetah) distribution
cheetah_indices = []
for row in range(cheetah_data.shape[0]):
   second_max_idx = second_largest_index(cheetah_data[row, :])
   cheetah_indices.append(second_max_idx)

# Create histogram with 64 bins (indices 0-63)
counts_cheetah, _ = np.histogram(cheetah_indices, bins=np.arange(-0.5, 64.5, 1))
prob_cheetah = counts_cheetah / counts_cheetah.sum()

# Plot foreground histogram
plt.figure(figsize=(10, 6))
plt.bar(range(64), prob_cheetah, width=1.0, edgecolor='black', alpha=0.7)
plt.xlabel('Feature Index (0-63)', fontsize=12)
plt.ylabel('Probability', fontsize=12)
plt.title('Cheetah (Foreground) Probability Distribution', fontsize=14)
plt.xlim(-0.5, 63.5)
plt.grid(axis='y', alpha=0.3)
plt.savefig('foreground_hist.jpg', dpi=150, bbox_inches='tight')
plt.close()
```

```python
# Background (Grass) distribution
background_indices = []
for row in range(background_data.shape[0]):
   second_max_idx = second_largest_index(background_data[row, :])
   background_indices.append(second_max_idx)

# Create histogram with 64 bins (indices 0-63)
counts_background, _ = np.histogram(background_indices, bins=np.arange(-0.5, 64.5, 1))
prob_background = counts_background / counts_background.sum()

# Plot background histogram
plt.figure(figsize=(10, 6))
plt.bar(range(64), prob_background, width=1.0, edgecolor='black', alpha=0.7,
color='green')
plt.xlabel('Feature Index (0-63)', fontsize=12)
plt.ylabel('Probability', fontsize=12)
plt.title('Background (Grass) Probability Distribution', fontsize=14)
plt.xlim(-0.5, 63.5)
plt.grid(axis='y', alpha=0.3)
plt.savefig('background_hist.jpg', dpi=150, bbox_inches='tight')
plt.close()

### Question 3

zigzag_pattern = load_zigzag_pattern('Zig-Zag Pattern.txt')
print("Zigzag pattern loaded successfully")
img = np.array(Image.open('cheetah.bmp').convert('L'), dtype=float) / 255.0
original_height, original_width = img.shape
print(f"Original image size: {original_height} x {original_width}")

# Add uniform padding of 4 pixels on all sides
# This allows us to extract 8x8 blocks starting from position (0,0) in the padded
image
padding = 4
img_padded = np.pad(img, ((padding, padding), (padding, padding)),
                   mode='constant', constant_values=0)
padded_height, padded_width = img_padded.shape
print(f"Padded image size: {padded_height} x {padded_width}")

# The output will have the same dimensions as the original image
# Each pixel (i,j) in the output corresponds to the 8x8 block centered around
# pixel (i,j) in the original image
```

```python
output_height = original_height
output_width = original_width

# Initialize array to store the feature index for each block
feature_indices = np.zeros((output_height, output_width), dtype=int)

# Process each 8x8 block
print("Processing 8x8 blocks...")
for i in range(output_height):
   for j in range(output_width):
       # Extract 8x8 block from padded image
       # Block starts at (i, j) in padded coordinates
       block = img_padded[i:i+8, j:j+8]

       # Apply 2D DCT and take absolute values
       dct_block = np.abs(dct2(block))

       # Apply zigzag scan
       zigzag_vector = zigzag_scan(dct_block, zigzag_pattern)

       # Find index of second largest coefficient (0-indexed)
       feature_idx = second_largest_index(zigzag_vector)
       feature_indices[i, j] = feature_idx

print(f"Feature extraction complete. Output shape: {feature_indices.shape}")

# Bayesian classification
print("Performing Bayesian classification...")
prediction = np.zeros((output_height, output_width), dtype=int)

for i in range(output_height):
   for j in range(output_width):
       feature_idx = feature_indices[i, j]

       # Calculate posterior probabilities (up to normalization constant)
       posterior_cheetah = prob_cheetah[feature_idx] * prior_cheetah
       posterior_background = prob_background[feature_idx] * prior_background

       # Classify based on maximum posterior probability
       if posterior_cheetah >= posterior_background:
           prediction[i, j] = 1  # Cheetah
       else:
```

```python
            prediction[i, j] = 0  # Background

print(f"Classification complete.")
print(f"Number of cheetah pixels predicted: {np.sum(prediction)}")
print(f"Number of background pixels predicted: {prediction.size -
np.sum(prediction)}")

# Save prediction
plt.figure(figsize=(10, 8))
plt.imshow(prediction, cmap='gray')
plt.axis('off')
plt.title('Prediction (White=Cheetah, Black=Background)', fontsize=14)
plt.savefig('prediction.jpg', dpi=150, bbox_inches='tight', pad_inches=0)
plt.close()

# Save as binary image
prediction_img = Image.fromarray((prediction * 255).astype(np.uint8))
prediction_img.save('prediction_binary.jpg')

# Display comparison
plt.figure(figsize=(15, 5))
plt.subplot(1, 3, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image', fontsize=12)
plt.axis('off')

plt.subplot(1, 3, 2)
plt.imshow(img_padded, cmap='gray')
plt.title('Padded Image', fontsize=12)
plt.axis('off')

plt.subplot(1, 3, 3)
plt.imshow(prediction, cmap='gray')
plt.title('Prediction', fontsize=12)
plt.axis('off')

plt.tight_layout()
plt.savefig('comparison.jpg', dpi=150, bbox_inches='tight')
plt.show()

### Question 4
```

```python
# Load ground truth mask
ground_truth = np.array(Image.open('cheetah_mask.bmp').convert('L'), dtype=float) /
255.0
ground_truth = (ground_truth > 0.5).astype(int)  # Binarize

# Verify dimensions match
assert ground_truth.shape == prediction.shape, \
   f"Dimension mismatch: ground_truth {ground_truth.shape} vs prediction
{prediction.shape}"

height, width = ground_truth.shape

# Initialize counters
false_negatives = 0  # Cheetah classified as background
false_positives = 0  # Background classified as cheetah
total_cheetah = 0
total_background = 0

# Compare prediction with ground truth
for i in range(height):
   for j in range(width):
       if ground_truth[i, j] == 1:  # True cheetah
           total_cheetah += 1
           if prediction[i, j] == 0:  # Predicted as background
               false_negatives += 1
       else:  # True background
           total_background += 1
           if prediction[i, j] == 1:  # Predicted as cheetah
               false_positives += 1

# Calculate error rates
false_negative_rate = false_negatives / total_cheetah if total_cheetah > 0 else 0
false_positive_rate = false_positives / total_background if total_background > 0 else
0

# Calculate probability of error (Bayes error weighted by priors)
error_cheetah = false_negative_rate * prior_cheetah
error_background = false_positive_rate * prior_background
total_error = error_cheetah + error_background

# Print results
print(f"Ground Truth Statistics:")
```

```
print(f"  Total cheetah pixels: {total_cheetah}
({100*total_cheetah/(height*width):.2f}%)")
print(f"  Total background pixels: {total_background}
({100*total_background/(height*width):.2f}%)")
print()
print(f"Classification Errors:")
print(f"  False Negatives (missed cheetah): {false_negatives}")
print(f"  False Positives (false alarms): {false_positives}")
print(f"  Total misclassified: {false_negatives + false_positives}")
print()
print(f"Error Rates:")
print(f"  False Negative Rate: {false_negative_rate:.6f}
({100*false_negative_rate:.4f}%)")
print(f"  False Positive Rate: {false_positive_rate:.6f}
({100*false_positive_rate:.4f}%)")
print()
print(f"Probability of Error (Weighted by Priors):")
print(f"  P(error | cheetah) × P(cheetah) = {error_cheetah:.6f}")
print(f"  P(error | background) × P(background) = {error_background:.6f}")
print(f"  Total Probability of Error = {total_error:.6f}")
print()

# Visualize results
plt.figure(figsize=(18, 5))

plt.subplot(1, 4, 1)
plt.imshow(img, cmap='gray')
plt.title('Original Image', fontsize=12)
plt.axis('off')

plt.subplot(1, 4, 2)
plt.imshow(ground_truth, cmap='gray')
plt.title('Ground Truth', fontsize=12)
plt.axis('off')

plt.subplot(1, 4, 3)
plt.imshow(prediction, cmap='gray')
plt.title('Prediction', fontsize=12)
plt.axis('off')

# Show errors
error_map = np.zeros((height, width, 3))
```

```python
error_map[:, :, 0] = img  # Red channel
error_map[:, :, 1] = img  # Green channel
error_map[:, :, 2] = img  # Blue channel

# False positives in red
for i in range(height):
   for j in range(width):
       if ground_truth[i, j] == 0 and prediction[i, j] == 1:
           error_map[i, j] = [1, 0, 0]  # Red
       elif ground_truth[i, j] == 1 and prediction[i, j] == 0:
           error_map[i, j] = [0, 0, 1]  # Blue

plt.subplot(1, 4, 4)
plt.imshow(error_map)
plt.title('Errors (Red=FP, Blue=FN)', fontsize=12)
plt.axis('off')

plt.tight_layout()
plt.savefig('error_analysis.jpg', dpi=150, bbox_inches='tight')
plt.show()
```