10601 Machine Learning HW4
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1. Support Vector Machines

1.1 Feature Maps

1. No, because we can't use one line to seperate blue and red points. Therefore, it's not linear separable.

2. Yes, $x_2 - 2.5 \geq 0$, the corresponding margin is 1.5.

3. $K(x,z) = x\,z + x^2 z^2$

1.2 Constructing Kernels

1. $K(x,z) = c_1 k_1(x,z) + c_2 k_2(x,z)$ with $c_1, c_2 \geq 0$

$\phi(x) = (\sqrt{c_1}\,\phi_1(x), \sqrt{c_2}\,\phi_2(x))$

$\phi(x)\cdot\phi(z) = c_1 \underbrace{\phi_1(x)\cdot\phi_1(z)}_{k_1(x,z)} + c_2 \underbrace{\phi_2(x)\cdot\phi_2(z)}_{k_2(x,z)}$

2. $K(x,z) = k_1(x,z)\cdot k_2(x,z)$

$\phi(x) = (\phi_{1,i}(x)\ \phi_{2,j}(x))_{i\in\{1\ldots n\},\ j\in\{1\ldots m\}}$

$\phi(x)\cdot\phi(z) = \sum_{i,j} \phi_{1,i}(x)\,\phi_{2,j}(x)\,\phi_{1,i}(z)\,\phi_{2,j}(z)$

$= \sum_i \phi_{1,i}(x)\,\phi_{1,i}(z)\left(\sum_j \phi_{2,j}(x)\,\phi_{2,j}(z)\right)$

$= \sum_i \phi_{1,i}(x)\,\phi_{1,i}(z)\,k_2(x,z) = k_1(x,z)\,k_2(x,z)$

3.

$q(t)=\sum_{i=0}^{p} c_i t^i$, prove $K(x,z)=q(k_1(x,z))$

$$\sum_{i=0}^{p} c_i k_1(x,z)^i = c_0 k_1(x,z)^0 + c_1 k_1(x,z)^1 + \dots + c_p k_1(x,z)^p$$

Use $K(x,z)=k_1(x,z)k_2(x,z)$, we get:

$$\sum_{i=0}^{p} c_i k_1(x,z)^i = c_0 k^{(0)}(x,z) + c_1 k^{(1)}(x,z) + \dots + c_p k^{(p)}(x,z)$$

Then use $K(x,z)=c_1 k_1(x,z)+c_2 k_2(x,z)$, we get:

$$K(x,z)=q(k_1(x,z)) = \sum_{i=0}^{p} c_i k^{(i)}(x,z) = \sum_{i=0}^{p} c_i k_1(x,z)^i \ \#$$

4.

$K(x,z)=\exp(k_1(x,z))$

Since: $\exp(x)=\lim_{i\to\infty}\left(1+x+\dots+\frac{x^i}{i!}\right)$

From the fact that $k(x,z)=f(k_1(x,z))$,

$$K(x,z)=\lim_{i\to\infty} k_i(x,z)$$

5. $K(x,z)=x^T A z$

1.3 Support Vectors

1. $\sum_{i}^{N} \ell(w,x,y)=\max(0,1-y(w\cdot x))$

2. C controls the relative weighting between the twin goals of making the $\|w\|^2$ small (margin is large) and ensuring that most examples have functional margin ≥ 1

If $C \to 0$, r will be very large

$C \to \infty$, r will be very small

3. Classifier: $f(x)=\sum_{i}^{N}\alpha_i y_i \phi(x_i)\phi(x)+b$

$\Rightarrow f(x)=\sum_{i}^{N}\alpha_i y_i K(x_i,x)+b$

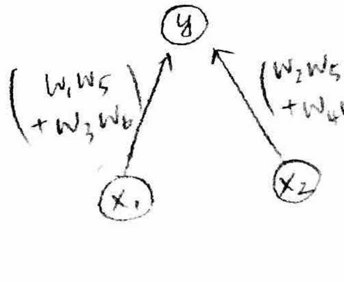
Learning $\max_{\alpha_i \geq 0} \sum \alpha_i - \frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j \phi(x_i)^T\phi(x_j)$

$\Rightarrow \max_{\alpha_i\geq 0} \sum\alpha_i - \frac{1}{2}\sum_{i,j}\alpha_i\alpha_j y_i y_j K(x_i,x_j)$

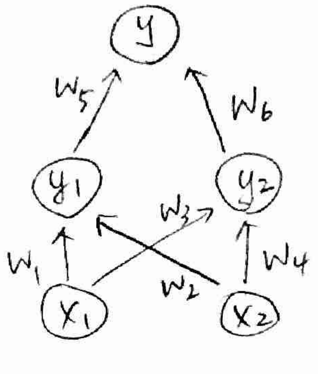
2. Feed-forward Neural Networks

1. Yes

$y=(w_1w_5+w_3w_6)x_1+(w_2w_5+w_4w_6)x_2+w_0$



2.



for (1,1)

$$(W_1+W_2)W_5+(W_3+W_4)W_6+W_0=1 \quad (1)$$

for (-1,1)

$$(-W_1+W_2)W_5+(-W_3+W_4)W_6+W_0=-1. \quad (2)$$

for (-1,-1)

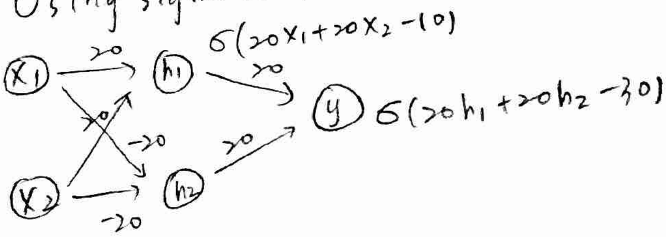
$$(-W_1-W_2)W_5+(-W_3-W_4)W_6+W_0=1 \quad . (3)$$

for (1,-1).

$$(W_1-W_2)W_5+(W_3-W_4)W_6+W_0=-1 \quad (4)$$

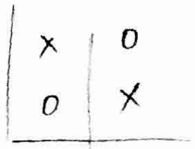
From (1),(3), we conclude that $W_0=1$
(2),(4) we conclude that $W_0=-1$] conflicts!

3.
Using sigmoid function, we can solve XOR problem.



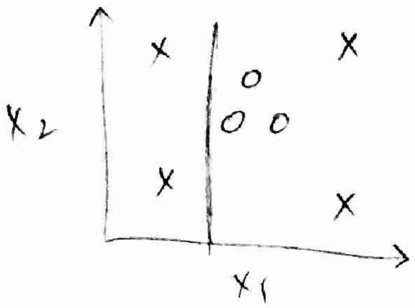
3. AdaBoost

1.



the minimum error rate is 0.5

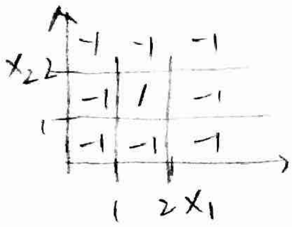
2、



$$h_1(x) = \begin{cases} 1 & \text{if } x_1 \leq 1 \\ -1 & \text{otherwise} \end{cases}$$

error rate $= \frac{2}{7}$

3、



① $x_1 < 1$, $x_2 > 2 \Rightarrow h_1 = -1$ $h_2 = 1$, $h_3 = 1$, $h_4 = -1$

② $2 \geq x_1 \geq 1$, $x_2 > 2 \Rightarrow h_1 = 1$, $h_2 = 1$, $h_3 = 1$, $h_4 = -1$

③ $x_1 > 2$, $x_2 > 2 \Rightarrow h_1 = 1$, $h_2 = -1$, $h_3 = 1$, $h_4 = -1$

④ $x_1 < 1$, $1 \leq x_2 \leq 2 \Rightarrow h_1 = -1$, $h_2 = 1$, $h_3 = 1$, $h_4 = 1$

⑤ $1 \leq x_1 \leq 2$, $1 \leq x_2 \leq 2 \Rightarrow h_1 = 1$, $h_2 = 1$, $h_3 = 1$, $h_4 = 1$

⑥ $x_1 > 2$, $1 \leq x_2 \leq 2 \Rightarrow h_1 = 1$, $h_2 = -1$, $h_3 = 1$, $h_4 = 1$

⑦ $x_1 < 1$, $x_2 < 1 \Rightarrow h_1 = -1$, $h_2 = 1$, $h_3 = -1$, $h_4 = 1$

⑧ $1 \leq x_1 \leq 2$, $x_2 < 1 \Rightarrow h_1 = 1$, $h_2 = 1$, $h_3 = -1$, $h_4 = 1$

⑨ $x_1 > 2$, $x_2 < 1 \Rightarrow h_1 = 1$, $h_2 = -1$, $h_3 = -1$, $h_4 = 1$

for ① $y_i f(x_i) = -1 \cdot \frac{1}{7}(0-3) = \frac{3}{7}$

② $y_i f(x_i) = -1 \cdot \frac{1}{7}(2-3) = \frac{1}{7}$

③ $y_i f(x_i) = -1 \cdot \frac{1}{7}(0-3) = \frac{3}{7}$

④ $y_i f(x_i) = -1 \cdot \frac{1}{7}(2-3) = \frac{1}{7}$

⑤ $y_i f(x_i) = 1 \cdot \frac{1}{7}(4-3) = \frac{1}{7}$

⑥ $y_i f(x_i) = -1 \cdot \frac{1}{7}(2-3) = \frac{1}{7}$

⑦ $y_i f(x_i) = -1 \cdot \frac{1}{7}(0-3) = \frac{3}{7}$

⑧ $y_i f(x_i) = -1 \cdot \frac{1}{7}(2-3) = \frac{1}{7}$

⑨ $y_i f(x_i) = -1 \cdot \frac{1}{7}(0-3) = \frac{3}{7}$

$\Rightarrow y_i f(x_i) \geq \frac{1}{7}$ #

3.2 Complexity and Overfitting

1. 4 params — dimension, threshold, direction, weight

2. $5 \times T$ params.

3. No. because the testing error doesn't increase.

3.3 Margin

1. $\text{Margin}_k(x) = \sum_{t: y = h_t(x)} a_t - \sum_{t: y \neq h_t(x)} a_t = y \cdot \sum a_t h_t(x) = y \cdot f_k(x)$

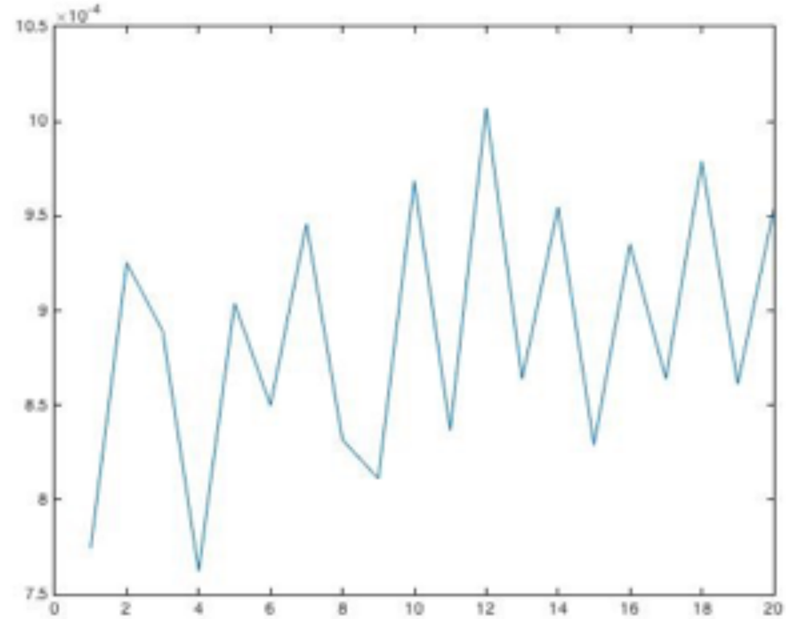
if $y = h_t(x) \Rightarrow y \cdot h_t(x) = 1$

$y \neq h_t(x) \Rightarrow y \cdot h_t(x) = -1$

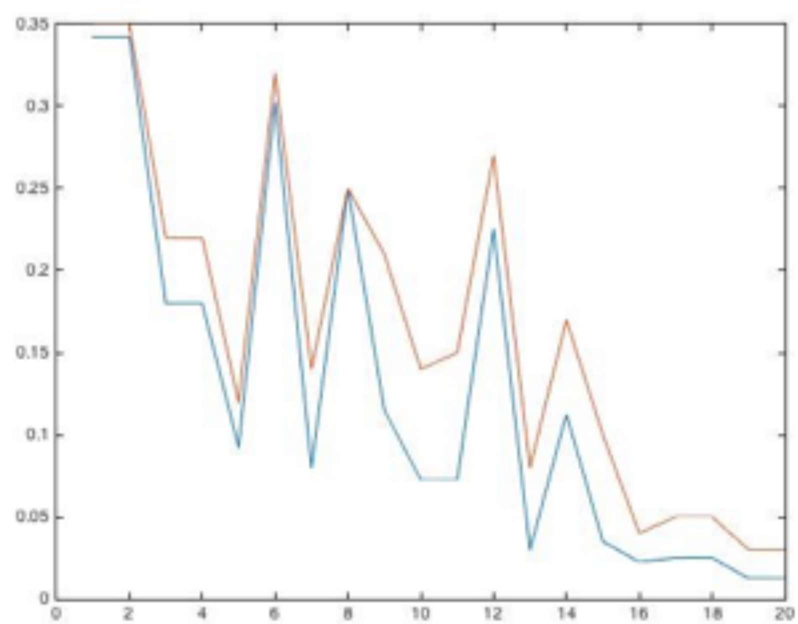
2.

3.5.1 Report Questions

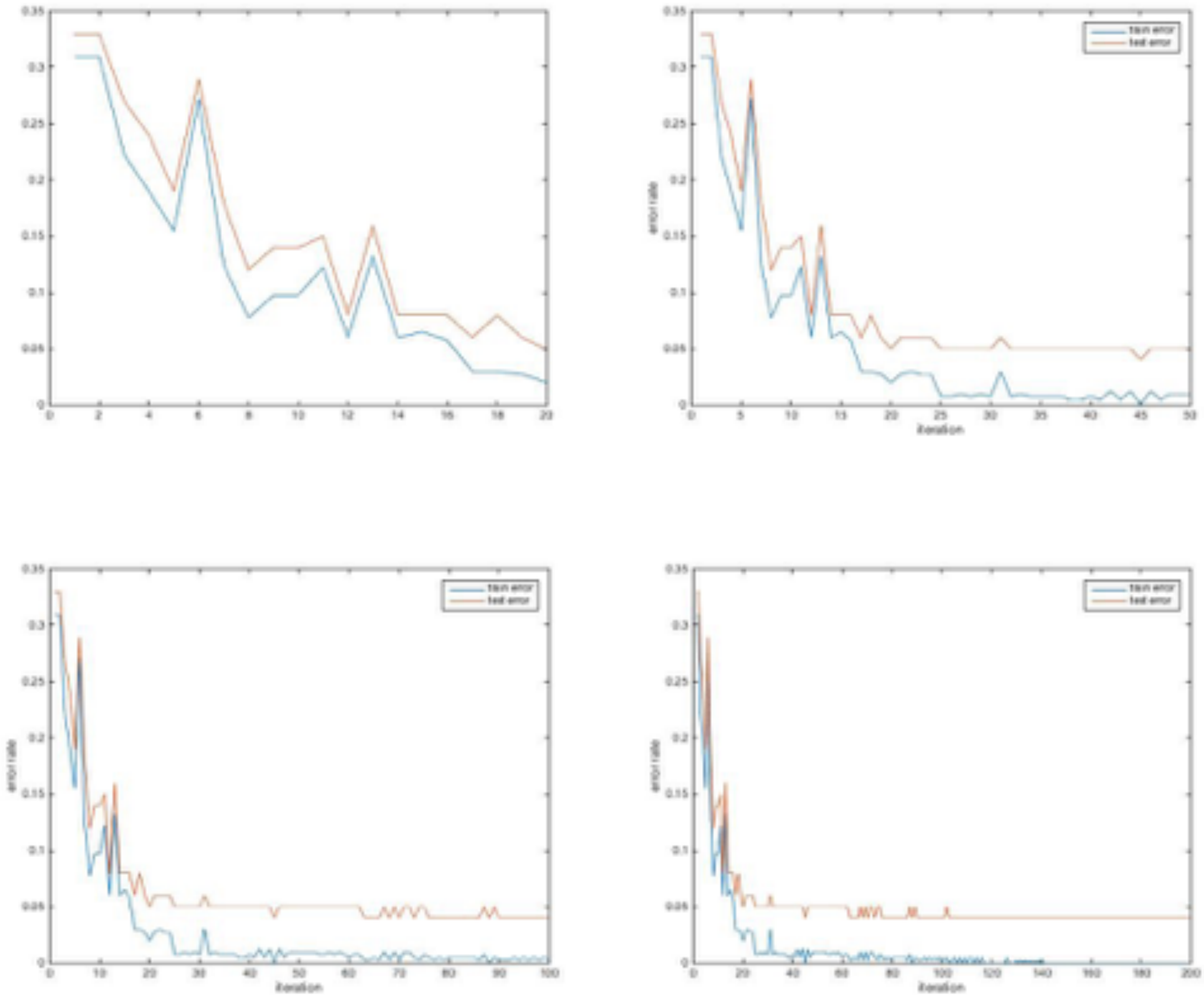
1. the highest weighted error rate: [h12, error rate = 0.0010]



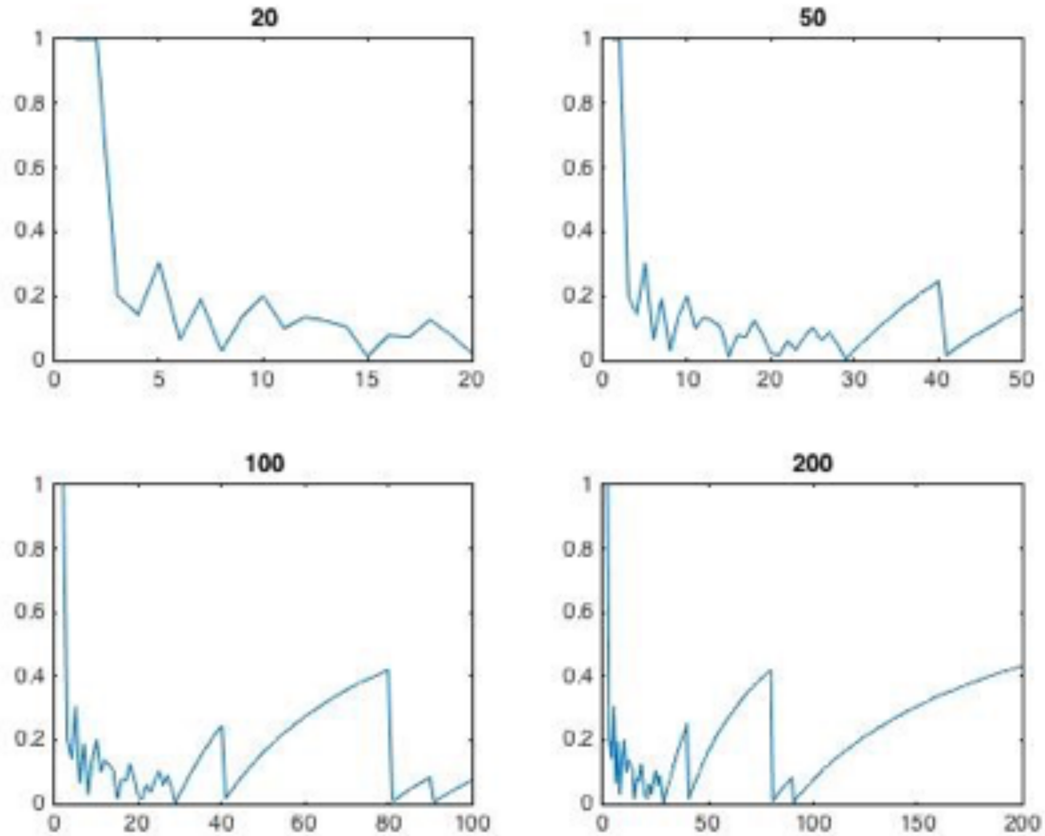
2. & 3.
T = 20



4.



5.



20
50
100
200