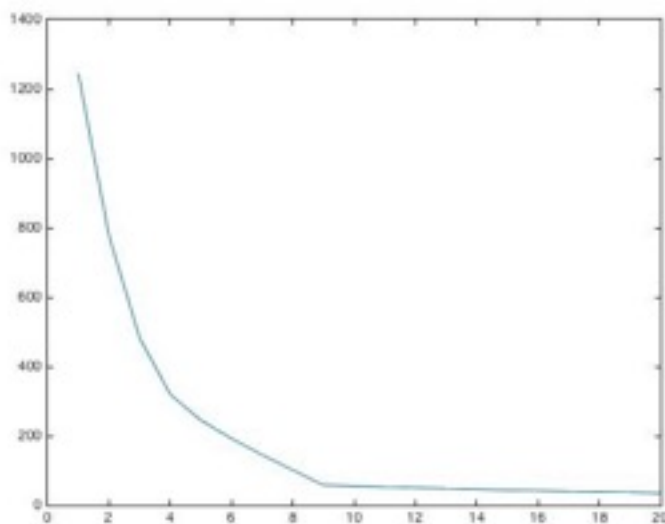


**Problem1: k-means Clustering**

- (a) There are total  $k^n$  partitionings. Compute the optimal center of each sets and their corresponding objective values. The algorithm's running time is exponential in the number of data points.

Experiment 1: The effect of k on Lloyd's method

(f)



- (g) Choose  $k = 9$ . The results agrees with the intuitive clusters of 2D dataset.

Experiment 2: The effect of initialization on Lloyd's method

(h)

obj\_kmeans++ = 77.1705

obj\_random = 88.6968

(i)

k-means picks the initial centers randomly. Since they're based on pure luck, they can be selected really badly. The K-means++ algorithm tries to solve this problem, by spreading the initial centers evenly.

**Problem2: Disagreement-based Active Learning**

(a)

True, because labeling function  $h^*$  will always be in our hypothesis class  $H$ .

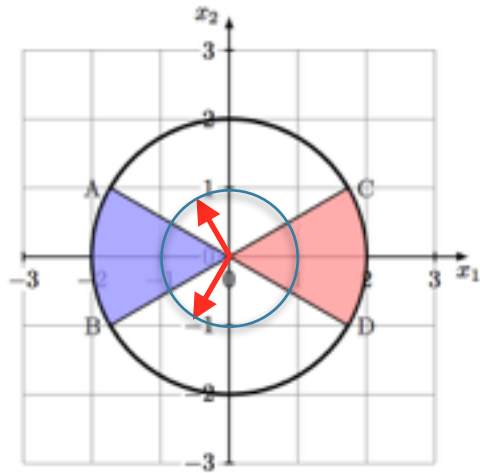
(b)

False,  $IH_{tl} \geq 1$ .  $\hat{H} \subset H'$

(c)

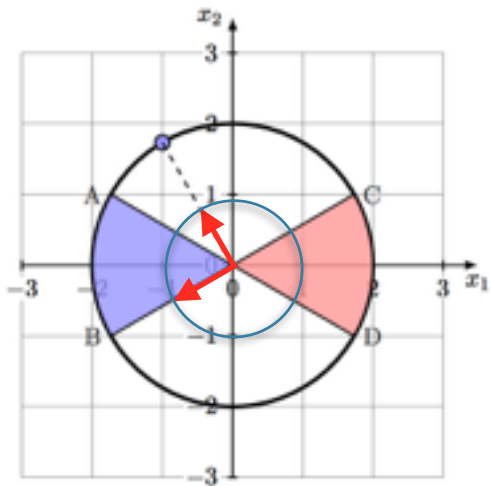
True,  $H_{\text{hat}}$  is always included in  $H'$ .

(d)



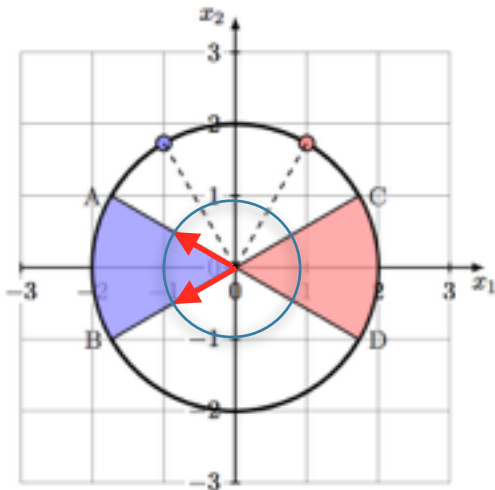
The version space  $H_1$  is between the red arrows ( $120^\circ$ ,  $240^\circ$ ),  $|w| = 1$ .

(e)



The version space  $H_1$  is between the red arrows ( $120^\circ$ ,  $210^\circ$ ),  $|w| = 1$ .

(f)



The new version space  $H_3$  is between the red arrows ( $150^\circ, 210^\circ$ ),  $|w| = 1$ .

### Problem3: Parity Functions

(a)

To prove that  $\text{VCdim}(H_{\text{parity}}) \leq n$ , we need to prove that  $n+1$  points can not be shattered. For  $n+1$  points, we need  $2^{n+1}$  combinations of  $x$  to shatter; however, we have only  $2^n$  combinations. Therefore, the VC dimension of  $H_{\text{parity}} \leq n$ .

(b)

$$x = \{1, 0\}$$

$$\text{pts1} = (1, 0)$$

$$\text{pts2} = (0, 1)$$

for case: '1,1':  $S = \{1, 2\}$

$$h_s(\text{pts1}) = 1$$

$$h_s(\text{pts2}) = 1$$

for case: '1,0':  $S = \{1\}$

$$h_s(\text{pts1}) = 1$$

$$h_s(\text{pts2}) = 0$$

for case: '0,1':  $S = \{2\}$

$$h_s(\text{pts1}) = 0$$

$$h_s(\text{pts2}) = 1$$

for case: '0,0':  $S = \{ \}$

$$h_s(\text{pts1}) = 0$$

$$h_s(\text{pts2}) = 0$$

$$\text{VCdim}(\mathcal{H}_{\text{parity}}) = n$$