

A BRIEF LIST OF MATLAB COMMANDS

Some Basic Commands (Note command syntax is case-sensitive!)

```

matlab    loads the program matlab into your workspace.
quit      quits matlab, returning you to the operating system.
exit      same as quit.
who       lists all of the variables in your matlab workspace.
whos      list the variables and describes their matrix size.
clear     deletes all matrices from active workspace.
clear x   deletes the matrix  x  from active workspace.
...       the ellipsis defining a line continuation is three
          successive periods.
save      saves all the matrices defined in the current
          session into the file, matlab.mat.
load      loads contents of matlab.mat into current workspace.
save filename  saves the contents of workspace into
          filename.mat
save filename x y z
          saves the matrices x, y and z into the file titled
          filename.mat.
load filename  loads the contents of filename into current
          workspace; the file can be a binary (.mat) file
          or an ASCII file.
!           the ! preceding any unix command causes the unix
          command to be executed from matlab.

```

Commands Useful in Plotting.

```

plot(x,y)  creates an Cartesian plot of the vectors x & y.
plot(y)    creates a plot of y vs. the numerical values of the
          elements in the y-vector.
semilogx(x,y) plots log(x) vs y.
semilogy(x,y) plots x vs log(y)
loglog(x,y)  plots log(x) vs log(y).
grid        creates a grid on the graphics plot.
title('text') places a title at top of graphics plot.
xlabel('text') writes 'text' beneath the x-axis of a plot.
ylabel('text') writes 'text' beside the y-axis of a plot.
text(x,y,'text') writes 'text' at the location (x,y) .
text(x,y,'text','sc') writes 'text' at point x,y assuming
          lower left corner is (0,0) and upper
          right corner is (1,1).
gtext('text') writes text according to placement of mouse
hold on    maintains the current plot in the graphics window
          while executing subsequent plotting  commands.
hold off   turns OFF the 'hold on' option.
polar(theta,r) creates a polar plot of the vectors r & theta
          where theta is in radians.
bar(x)      creates a bar graph of the vector x. (Note also
          the command stairs(y).)

bar(x,y)    creates a bar-graph of the elements of the vector y,
          locating the bars according to the vector elements
          of 'x'. (Note also the command stairs(x,y).)
hist(x)     creates a histogram. This differs from the bargraph

```

in that frequency is plotted on the vertical axis.

`mesh(z)` creates a surface in xyz space where `z` is a matrix of the values of the function `z(x,y)`. `z` can be interpreted to be the height of the surface above some xy reference plane.

`surf(z)` similar to `mesh(z)`, only surface elements depict the surface rather than a mesh grid.

`contour(z)` draws a contour map in xy space of the function or surface `z`.

`meshc(z)` draws the surface `z` with a contour plot beneath it.

`meshgrid` `[X,Y]=meshgrid(x,y)` transforms the domain specified by vectors `x` and `y` into arrays `X` and `Y` that can be used in evaluating functions for 3D mesh/surf plots.

`print` sends the contents of graphics window to printer.

`print filename -dps` writes the contents of current graphics to 'filename' in postscript format.

Equation Fitting

`polyfit(x,y,n)` returns the coefficients of the n-degree polynomial for the vectors `x` and `y`. `n` must be at least 1 larger than the length of the vectors `x` and `y`. If `n+1 = length(x)` the result is an interpolating polynomial. If `n+1 > length(x)` the result is a least-squares polynomial fit. The coefficients are stored in order with that of the highest order term first and the lowest order last.

`polyval(c,x)` calculates the values of the polynomial whose coefficients are stored in `c`, calculating for every value of the vector `x`.

Data Analysis Commands

`max(x)` returns the maximum value of the elements in a vector or if `x` is a matrix, returns a row vector whose elements are the maximum values from each respective column of the matrix.

`min(x)` returns the minimum of `x` (see `max(x)` for details).

`mean(x)` returns the mean value of the elements of a vector or if `x` is a matrix, returns a row vector whose elements are the mean value of the elements from each column of the matrix.

`median(x)` same as `mean(x)`, only returns the median value.

`sum(x)` returns the sum of the elements of a vector or if `x` is a matrix, returns the sum of the elements from each respective column of the matrix.

`prod(x)` same as `sum(x)`, only returns the product of elements.

`std(x)` returns the standard deviation of the elements of a vector or if `x` is a matrix, a row vector whose elements are the standard deviations of each column of the matrix.

`sort(x)` sorts the values in the vector `x` or the columns of a matrix and places them in ascending order. Note that this command will destroy any association that may exist between the elements in a row of matrix `x`.

`hist(x)` plots a histogram of the elements of vector, `x`. Ten

bins are scaled based on the max and min values.
`hist(x,n)` plots a histogram with 'n' bins scaled between the max and min values of the elements.
`hist((x(:,2)))` plots a histogram of the elements of the 2nd column from the matrix x.
`fliplr(x)` reverses the order of a vector. If x is a matrix, this reverse the order of the columns in the matrix.
`flipud(x)` reverses the order of a matrix in the sense of exchanging or reversing the order of the matrix rows. This will not reverse a row vector!
`reshape(A,m,n)` reshapes the matrix A into an mxn matrix from element (1,1) working column-wise.

SPECIAL MATRICES

`zeros(n)` creates an nxn matrix whose elements are zero.
`zeros(m,n)` creates a m-row, n-column matrix of zeros.
`ones(n)` creates a n x n square matrix whose elements are 1's.
`ones(m,n)` creates a mxn matrix whose elements are 1's.
`ones(A)` creates an m x n matrix of 1's, where m and n are based on the size of an existing matrix, A.
`zeros(A)` creates an mxn matrix of 0's, where m and n are based on the size of the existing matrix, A.
`eye(n)` creates the nxn identity matrix with 1's on the diagonal.

Miscellaneous Commands

`length(x)` returns the number elements in a vector.
`size(x)` returns the size m(rows) and n(columns) of matrix x.
`rand` returns a random number between 0 and 1.
`randn` returns a random number selected from a normal distribution with a mean of 0 and variance of 1.
`rand(A)` returns a matrix of size A of random numbers.

ALGEBRAIC OPERATIONS IN MATLAB

Scalar Calculations.

+	addition
-	subtraction
*	multiplication
/	right division (a/b means $a \div b$)
\	left division (a\b means $b \div a$)
^	exponentiation

The precedence or order of the calculations included in a single line of code follows the below order:

Precedence	Operation
------------	-----------

- 1 parentheses
- 2 exponentiation, left to right
- 3 multiplication and division, left right
- 4 addition and subtraction, left right

MATRIX ALGEBRA

In matrix multiplication, the elements of the product, C, of two matrices A*B is calculated from

$$C_{ij} = \sum_k (A_{ik} * B_{kj}) \quad \{\text{summation over the double index } k\}$$

To form this sum, the number of columns of the first or left matrix (A) must be equal to the number of rows in the second or right matrix (B). The resulting product, matrix C, has an order for which the number of rows equals the number of rows of the first (left) matrix (A) and the product (C) has a number of columns equal to the number of columns in the second (right) matrix (B). It is clear that A*B IS NOT NECESSARILY EQUAL TO B*A!

The PRODUCT OF A SCALAR AND A MATRIX is a matrix in which every element of the matrix has been multiplied by the scalar.

ARRAY PRODUCTS

Sometimes it is desired to simply multiply or divide each element of an matrix by the corresponding element of another matrix. These are called 'array operations' in 'matlab'. Array or element-by-element operations are executed when the operator is preceded by a '.' (period). Thus

- a .* b multiplies each element of a by the respective element of b
- a ./ b divides each element of a by the respective element of b
- a .\ b divides each element of b by the respective element of a
- a .^ b raise each element of a by the respective b element

TRANSPOSE OF A MATRIX

- x' The transpose of a matrix is obtained by interchanging the rows and columns. The 'matlab' operator that creates the transpose is the single quotation mark, '.

INNER PRODUCT OF TWO VECTORS

The inner product of two row vectors G1 and G2 is G1*G2'.
The inner product of two column vectors H and J is H'*J.

OUTER PRODUCT OF TWO VECTORS

If two row vectors exist, G1 and G2, the outer product is simply

$$G1' * G2 \quad \{\text{Note } G1' \text{ is } nx1 \text{ and } G2 \text{ is } 1xn\}$$

and the result is a square matrix in contrast to the scalar result for the inner product. DON'T CONFUSE THE OUTER PRODUCT WITH THE VECTOR PRODUCT IN MECHANICS! If the two vectors are column vectors, the outer product must be formed by the product of one vector times the transpose of the second!

SOLUTION TO SIMULTANEOUS EQUATIONS

Using the Matrix Inverse

`inv(a)` returns the inverse of the matrix `a`.
If $ax=b$ is a matrix equation and `a` is the coefficient matrix, the solution `x` is $x=inv(a)*b$.

Using Back Substitution

`a\b` returns a column vector solution for the matrix equation $ax=b$ where `a` is a coefficient matrix.
`b/a` returns a row vector solution for the matrix equation $xa=b$ where `a` is a coefficient matrix.

Some basic commands you will need:

<code>matlab</code>	loads the program matlab into your workspace
<code>quit</code>	quits matlab, returning you to the operating system
<code>exit</code>	same as quit
<code>who</code>	lists all of the variables in your matlab workspace
<code>whos</code>	list the variables and describes their matrix size

NOTE - When using the workstations, clicking on UP ARROW will recall previous commands. If you make a mistake, the DELETE key OR the backspace key may be used to correct the error; however, one of these two keys may be inoperable on particular systems.

'matlab' uses variables that are defined to be matrices. A matrix is a collection of numerical values that are organized into a specific configuration of rows and columns. The number of rows and columns can be any number, for example, 3 rows and 4 columns define a 3 x 4 matrix which has 12 elements in total. A scalar is represented by a 1 x 1 matrix in matlab. A vector of `n` dimensions or elements can be represented by a `n x 1` matrix, in which case it is called a column vector, or a vector can be represented by a `1 x n` matrix, in which case it is called a row vector of `n` elements. The matrix name can be any group of letters and numbers up to 19, but always beginning with a letter. Thus 'x1' can be a variable name, but '1x' is illegal. 'supercalafragilesticexpealladotious' can be a variable name; however, only the first 19 characters will be stored! Understand that 'matlab' is "case sensitive", that is, it treats the name 'C' and 'c' as two different variables. Similarly, 'MID' and 'Mid' are treated as two different variables. Here are examples of matrices that could be defined in 'matlab'. Note that the set of numerical values or elements of the matrix are bounded by brackets[].

```

c = 5.66   or   c = [5.66]           c is a scalar or
                                         a 1 x 1 matrix

x = [ 3.5, 33.22, 24.5 ]             x is a row vector or
                                         1 x 3 matrix

x1 = [ 2
       5
       3
      -1]                             x1 is column vector or
                                         4 x 1 matrix

A = [ 1  2  4
      2 -2  2
      0  3  5
      5  4  9 ]                       A is a 4 x 3 matrix

```

An individual element of a matrix can be specified with the notation $A(i,j)$ or $A_{i,j}$ for the generalized element, or by $A(4,1)=5$ for a specific element.

When 'matlab' prints a matrix on the monitor, it will be organized according to the size specification of the matrix, with each row appearing on a unique row of the monitor screen and with each column aligned vertically and right-justified.

The numerical values that are assigned to the individual elements of a matrix can be entered into the variable assignment in a number of ways. The simplest way is by direct keyboard entry; however, large data sets may be more conveniently entered through the use of stored files or by generating the element values using matlab expressions. First, we will look at the use of the keyboard for direct entry.

KEYBOARD DEFINITION OR ENTRY FOR A MATRIX

A matrix can be defined by a number of matlab expressions. Examples are listed below for a 1 x 3 row vector, x , whose elements are $x(1) = 2$, $x(2) = 4$ and $x(3) = -1$.

```

x = [ 2  4  -1 ]   or   x=[2 4 -1]   or   x = [ 2,4,-1 ]

```

(A keystroke 'enter' follows each of the above matlab statements.) Notice that brackets must be used to open and close the set of numbers, and notice that commas or blanks may be used as delimiters between the fields defining the elements of the matrix. Blanks used around the = sign and the brackets are superfluous; however, they sometimes make the statement more readable.

A 2x4 matrix, y , whose elements are $y(1,1)=0$, $y(1,2) = y(1,3) = 2$, $y(1,4) = 3$, $y(2,1) = 5$, $y(2,2) = -3$, $y(2,3) = 6$ and $y(2,4) = 4$ can be defined

```

y = [ 0 2 2 3
      5 -3 6 4 ]

```

or

```
y = [ 0 2 2 3 ; 5 -3 6 4 ]
```

The semicolon ";" is used to differentiate the matrix rows when they appear on a single line for data entry.

The elements of a matrix can be defined with algebraic expressions placed at the appropriate location of the element. Thus

```
a = [ sin(pi/2) sqrt(2) 3+4 6/3 exp(2) ]
```

defines the matrix

```
a = [ 1.0000 1.4142 7.0000 2.0000 7.3891 ]
```

A matrix can be defined by augmenting previously defined matrices. Recalling the matrix, x, defined earlier

```
x1 = [ x 5 8 ] creates the result
```

```
x1 = [ 2 4 -1 5 8 ]
```

The expression

```
x(5) = 8
```

creates

```
x = [ 2 4 -1 0 8 ]
```

Notice that the value "0" is substituted for x(4) which has not been explicitly defined.

Recalling the definition of matrix, y, above, the expressions

```
c = [ 4 5 6 3 ]
z = [ y;c ]
```

creates

```
z = [ 0 2 2 3
      5 -3 6 4
      4 5 6 3 ]
```

Note that every time a matrix is defined and an 'enter' keystroke is executed, matlab echoes back the result. TO CANCEL THIS ECHO, THE MATLAB COMMAND LINE CAN INCLUDE A SEMICOLON AT THE END OF THE LINE BEFORE THE KEYSTROKE 'ENTER'.

```
z = [ y ; c ] ;
```

LINE CONTINUATION

Occasionally, a line is so long that it can not be expressed in the 80 spaces available on a line, in which case a line continuation is needed. In matlab, the ellipsis defining a line continuation is three successive periods, as in "...". Thus

```

4 + 5 + 3 ...
   + 1 + 10 + 2 ...
   + 5

```

gives the result

```
ans = 30
```

Notice that in this simple arithmetic operation, no matrix was defined. When such an operation is executed in matlab, the result is assigned to the matrix titled "ans". A subsequent operation without an assignment to a specific matrix name will replace the results in 'ans' by the result of the next operation. In the above, 'ans' is a 1x1 matrix, but it need not be so in general.

BEFORE YOU QUIT THIS SESSION !!!!!

If this is your first lesson using matlab, execute the matlab commands 'who' and whos' before you 'quit'. Note that each of these commands lists the matrices you have defined in this session on the computer. The command 'whos' also tells you the properties of each matrix, including the number of elements, the row and column size (row x column) and whether the elements are complex or real.

IMPORTANT! If you execute the matlab command 'save' before you quit, all of the matrices that have been defined will be saved in a file titled matlab.mat stored in your workspace. Should you desire to save specific matrices during any session, the command 'save' followed by the name of the matrix can be executed. More detail on how to save and recall your matrices is discussed in Lesson 2.

PRACTICE PROBLEMS

Determine the size and result for the following matrices. Subsequently, carry out the operations on matlab that define the matrices, and check your results using the 'whos' statement.

1. `a = [1,0,0,0,0,1]`
2. `b = [2;4;6;10]`
3. `c = [5 3 5; 6 2 -3]`
4. `d= [3 4
5 7
9 10]`
5. `e = [3 5 10 0; 0 0 ...
0 3; 3 9 9 8]`
6. `t = [4 24 9]
q = [t 0 t]`
7. `x = [3 6]`


```

y = [d;x]
z = [x;d]

8.  r = [ c; x,5]

9.  v = [ c(2,1); b ]

10. a(2,1) = -3 (NOTE: Recall matrix "a" was defined in (1)
               above.)

```

New commands in this lesson:

```

save          saves all the matrices defined in the current
               session into the file, matlab.mat, located in
               the directory from which you executed matlab.

load          loads contents of matlab.mat into current workspace

save filename x y z
               save the matrices x, y and z into the file titled
               filename.mat.

load filename  loads the contents of filename into current
               workspace; the file can be a binary (.mat) file
               or an ASCII file.

clear x       erases the matrix 'x' from your workspace

clear         erases ALL matrices from your workspace

```

NOTE - When using PROMATLAB on a workstation, files are stored in the directory from which you invoked the 'matlab' command. When using the workstation, create a matlab subdirectory titled 'matlab' or some similar name. Thereafter, store all files and conduct all matlab sessions in that subdirectory.

ASCII FILES

An ASCII file is a file containing characters in ASCII format, a format that is independent of 'matlab' or any other executable program. Using ASCII format, you can build a file using a screen editor or a wordprocessing program, for example, that can be read and understood by 'matlab'. The file can be read into 'matlab' using the "load" command described above.

Using a text editor or using a wordprocessor that is capable of writing a file in ASCII format, you simply prepare a matrix file for which EACH ROW OF THE MATRIX IS A UNIQUE LINE IN THE FILE, with the elements in the row separated by blanks. An example is the 3 x 3 matrix

```

2  3  6
3 -1  0
7  0 -2

```

If these elements are formatted as described above and stored in a filename titled, for example, x.dat, the 3 x 3 matrix 'x' can be

loaded into 'matlab' by the command

```
load x.dat
```

Open the Text Editor window on your workstation. Build a 3x3 matrix in the editor that follows the format explained above. Store this in your matlab directory using the command

```
save ~/matlab/x.dat
```

The suffix, .dat, is not necessary; however, it is strongly recommended here to distinguish the file from other data files, such as the .mat files which will be described below. Of course if you define the file with the .dat or any other suffix, you must use that suffix when downloading the file to 'matlab' with the 'load' command. Remember, the file must be stored in the directory in which you are executing matlab. In the above example, it is assumed that this directory is titled 'matlab'.

Now go to a window in which matlab is opened. We desire to load the matrix x into matlab. We might wonder if the file x.dat is actually stored in our matlab directory. To review what files are stored therein, the unix command 'ls' can be evoked in matlab by preceding the command with an exclamation mark, ! . Type the command

```
! ls
```

Matlab will list all of the files in your directory. You should find one titled "x.dat".

Now, type the command

```
load x.dat
```

The 3x3 matrix defined above is now down-loaded into your working space in matlab. To ensure that is so, check your variables by typing 'who' or 'whos'.

FILES BUILT BY MATLAB....THE .mat FILE

If at any time during a matlab session you wish to store a matrix that has been defined in the workspace, the command

```
save filename
```

will create a file in your directory that is titled filename.mat. The file will be in binary format, understandable only by the 'matlab' program. Note that 'matlab' appends the .mat suffix and this can be omitted by the user when describing the filename.

If the matrix 'x' has been defined in the workspace, it can be stored in your directory in binary matlab format with the command

```
save x
```

The filename need not be the same as the matrix name. The file

named, for example, 'stuff' can be a file that contains the matrix 'x'. Any number of matrices can be stored under the filename. Thus the command string

```
save stuff x y z
```

will store the matrices x, y and z in the file titled 'stuff'. The file will automatically have the suffix .mat added to the filename. If, during a session, you type the command 'save' with no other string, all of the current matrices defined in your workspace will be stored in the file 'matlab.mat'.

The load command when applied to .mat files follows the same format as discussed above for ASCII files; however, you can omit the file suffix '.mat' when loading .mat files to your workspace.

COLON OPERATOR

The colon operator ' : ' is understood by 'matlab' to perform special and useful operations. If two integer numbers are separated by a colon, 'matlab' will generate all of the integers between these two integers.

```
a = 1:8
```

generates the row vector, $a = [1 \ 2 \ 3 \ 4 \ 5 \ 6 \ 7 \ 8]$.

If three numbers, integer or non-integer, are separated by two colons, the middle number is interpreted to be a "range" and the first and third are interpreted to be "limits". Thus

```
b = 0.0 : .2 : 1.0
```

generates the row vector $b = [0.0 \ .2 \ .4 \ .6 \ .8 \ 1.0]$

The colon operator can be used to create a vector from a matrix. Thus if

```
x = [ 2  6  8
      0  1  7
     -2  5 -6 ]
```

The command

```
y = x(:,1)
```

creates the column vector

```
y = [ 2
      0
     -2 ]
```

and

```
yy = x(:,2)
```

creates

```
yy = [ 6
```

```
1
5 ]
```

The command

```
z = x(1,:)
```

creates the row vector

```
z = [ 2 6 8 ]
```

The colon operator is useful in extracting smaller matrices from larger matrices. If the 4 x 3 matrix *c* is defined by

```
c = [ -1  0  0
       1  1  0
       1 -1  0
       0  0  2 ]
```

Then

```
d1 = c(:,2:3)
```

creates a matrix for which all elements of the rows from the 2nd and third columns are used. The result is a 4 x 2 matrix

```
d1 = [  0  0
        1  0
       -1  0
         0  2 ]
```

The command

```
d2 = c(3:4,1:2)
```

creates a 2 x 2 matrix in which the rows are defined by the 3rd and 4th row of *c* and the columns are defined by the 1st and 2nd columns of the matrix, *c*.

```
d2 = [ 1 -1
        0  0 ]
```

USING THE CLEAR COMMANDS

Before quitting this session of 'matlab', note the use of the commands 'clear' and 'clc'. Note that 'clc' simply clears the screen, but does not clear or erase any matrices that have been defined. The command 'clear' erase or removes matrices from your workspace. Use this command with care. NOTE THE COMMAND 'CLEAR' WITHOUT ANY MODIFYING STRING WILL ERASE ALL MATRICES IN YOUR WORKSPACE.

PROBLEMS

Define the 5 x 4 matrix, *g*.

```
g = [ 0.6  1.5  2.3 -0.5
```

```

8.2  0.5 -0.1 -2.0
5.7  8.2  9.0  1.5
0.5  0.5  2.4  0.5
1.2 -2.3 -4.5  0.5 ]

```

Determine the content and size of the following matrices and check your results for content and size using 'matlab'.

1. `a = g(:,2)`
2. `b = g(4,:)`
3. `c = [10:15]`
4. `d = [4:9;1:6]`
5. `e = [-5,5]`
6. `f = [1.0:-.2:0.0]`
7. `t1 = g(4:5,1:3)`
8. `t2 = g(1:2:5,:)`

New commands in this lesson:

`plot(x,y)` creates a Cartesian plot of the vectors `x` & `y`

`plot(y)` creates a plot of `y` vs. the numerical values of the elements in the `y`-vector.

`semilogx(x,y)` plots `log(x)` vs `y`

`semilogy(x,y)` plots `x` vs `log(y)`

`loglog(x,y)` plots `log(x)` vs `log(y)`

`grid` creates a grid on the graphics plot

`title('text')` places a title at top of graphics plot

`xlabel('text')` writes 'text' beneath the x-axis of a plot

`ylabel('text')` writes 'text' beside the y-axis of a plot

`text(x,y,'text')` writes 'text' at the location `(x,y)`

`text(x,y,'text','sc')` writes 'text' at point `x,y` assuming lower left corner is `(0,0)` and upper right corner is `(1,1)`.

`polar(theta,r)` creates a polar plot of the vectors `r` & `theta` where `theta` is in radians.

`bar(x)` creates a bar graph of the vector `x`. (Note also the command `stairs(x)`.)

`bar(x,y)` creates a bar-graph of the elements of the vector `y`, locating the bars according to the vector elements of `'x'`. (Note also the command `stairs(x,y)`.)

CARTESIAN OR X-Y PLOTS

One of 'matlab' most powerful features is the ability to create graphic plots. Here we introduce the elementary ideas for simply presenting a graphic plot of two vectors. More complicated and powerful ideas with graphics can be found in the 'matlab' documentation.

A Cartesian or orthogonal `x,y` plot is based on plotting the `x,y` data pairs from the specified vectors. Clearly, the vectors `x` and `y` must have the same number of elements. Imagine that you wish to plot the function `exp` for values of `x` from 0 to 2.

```
x = 0:.1:2;  
y = exp(x);  
plot(x,y)
```

NOTE: The use of selected functions such as `exp()` and `sin()` will be used in these tutorials without explanation if they take on an unambiguous meaning consistent with past experience. Here it is observed that operating on a matrix with these functions simply creates a matrix in which the elements are based on applying the function to each corresponding element of the argument.

Of course the symbols `x,y` are arbitrary. If we wanted to plot temperature on the ordinate and time on the abscissa, and vectors for temperature and time were loaded in 'matlab', the command would be

```
plot(time,temperature)
```

Notice that the command `plot(x,y)` opens a graphics window. If you now execute the command `'grid'`, the graphics window is redrawn. (Note you move the cursor to the command window before typing new commands.) To avoid redrawing the window, you may use the line continuation ellipsis. Consider

```
plot(x,y),...  
grid,...  
title('Exponential Function'),...  
xlabel('x'),...  
ylabel('exp(x)'),  
text(.6,.4,' y = exp(x) ','sc')
```

Note that if you make a mistake in typing a series of lines of code, such as the above, the use of line continuation can be frustrating. In a future lesson, we will learn how to create batch files (called '.m' files) for executing a series of 'matlab' commands. This approach gives you better opportunity to return to the command string and edit errors that have been created.

Having defined the vectors `x` and `y`, construct semilog and log-log plots using these or other vectors you may wish to create. Note

in particular what occurs when a logarithmic scale is selected for a vector that has negative or zero elements. Here, the vector `x` has a zero element (`x(1)=0`). The logarithm of zero or any negative number is undefined and the `x,y` data pair for which a zero or negative number occurs is discarded and a warning message provided.

Try the commands `plot(x)` and `plot(y)` to ensure you understand how they differ from `plot(x,y)`.

Notice that 'matlab' draws a straight line between the data-pairs. If you desire to see a relatively smooth curve drawn for a rapidly changing function, you must include a number of data points. For example, consider the trigonometric function, `sin(x1)` (`x1` is selected as the argument here to distinguish it from the vector '`x`' that was defined earlier.)

Plot `sin(x1)` over the limits $0 \leq x1 \leq \pi$. First define `x1` with 5 elements,

```
x1 = 0 : pi/4 : pi
y1 = sin(x1)
plot(x1,y1)
```

Now, repeat this after defining a new vector for `x1` and `y1` with 21 elements. (Note the use of the semicolon here to prevent the printing and scrolling on the monitor screen of a vector or matrix with a large number of elements!)

```
x1 = 0 : .05*pi : pi ;
y1 =sin(x1);
plot(x1,y1)
```

POLAR PLOTS

Polar plots are constructed much the same way as are Cartesian `x-y` plots; however, the arguments are the angle '`theta`' in radians measured CCW from the horizontal (positive-`x`) axis, and the length of the radius vector extended from the origin along this angle. Polar plots do not allow the labeling of axes; however, notice that the scale for the radius vector is presented along the vertical and when the '`grid`' command is used the angles-grid is presented in 15-degree segments. The '`title`' command and the '`text`' commands are functional with polar plots.

```
angle = 0:.1*pi:3*pi;
radius = exp(angle/20);
polar(angle,radius),...
title('An Example Polar Plot'),...
grid
```

Note that the angles may exceed one revolution of 2π .

BAR GRAPHS

To observe how 'matlab' creates a bargraph, return to the vectors `x,y` that were defined earlier. Create bar and stair graphs using these or other vectors you may define. Of course the '`title`' and '`text`' commands can be used with bar and stair graphs.

```
bar(x,y) and bar(y)
stair (x,y) and stair(y)
```

MULTIPLE PLOTS

More than a single graph can be presented on one graphic plot. One common way to accomplish this is hold the graphics window open with the 'hold' command and execute a subsequent plotting command.

```
x1=0:.05*pi:pi;
y1=sin(x1);
plot(x1,y1)
hold
y2=cos(x1);
plot(x1,y2)
```

The "hold" command will remain active until you turn it off with the command 'hold off'.

You can create multiple graphs by using multiple arguments. In addition to the vectors x,y created earlier, create the vectors a,b and plot both vector sets simultaneously as follows.

```
a = 1 : .1 : 3;
b = 10*exp(-a);
plot(x,y,a,b)
```

Multiple plots can be accomplished also by using matrices rather than simple vectors in the argument. If the arguments of the 'plot' command are matrices, the COLUMNS of y are plotted on the ordinate against the COLUMNS of x on the abscissa. Note that x and y must be of the same order! If y is a matrix and x is a vector, the rows or columns of y are plotted against the elements of x. In this instance, the number of rows OR columns in the matrix must correspond to the number of elements in 'x'. The matrix 'x' can be a row or a column vector!

Recall the row vectors 'x' and 'y' defined earlier. Augment the row vector 'y' to create the 2-row matrix, yy.

```
yy=[y;exp(1.2*x)];
plot(x,yy)
```

PLOTTING DATA POINTS & OTHER FANCY STUFF

Matlab connects a straight line between the data pairs described by the vectors used in the 'print' command. You may wish to present data points and omit any connecting lines between these points. Data points can be described by a variety of characters (. , + , * , o and x .) The following command plots the x,y data as a "curve" of connected straight lines and in addition places an 'o' character at each of the x1,y1 data pairs.

```
plot(x,y,x1,y1,'o')
```


Lines can be colored and they can be broken to make distinctions among more than one line. Colored lines are effective on the color monitor and color printers or plotters. Colors are useless on the common printers used on this network. Colors are denoted in 'matlab' by the symbols r(red), g(green), b(blue), w(white) and i(invisible). The following command plots the x,y data in red solid line and the r,s data in broken green line.

```
plot(x,y,'r',r,s,'--g')
```

PRINTING GRAPHIC PLOTS

Executing the 'print' command will send the contents of the current graphics widow to the local printer.

You may wish to save in a graphics file, a plot you have created. To do so, simply append to the 'print' command the name of the file. The command

```
print filename
```

will store the contents of the graphics window in the file titled 'filename.ps' in a format called postscript. You need not include the .ps suffix, as matlab will do this. When you list the files in your matlab directory, it is convenient to identify any graphics files by simply looking for the files with the .ps suffix. If you desire to print one of these postscript files, you can conveniently "drag and drop" the file from the file-manager window into the printer icon.

Commands introduced in this lesson:

max(x)	returns the maximum value of the elements in a vector or if x is a matrix, returns a row vector whose elements are the maximum values from each respective column of the matrix.
min (x)	returns the minimum of x (see max(x) for details).
mean(x)	returns the mean value of the elements of a vector or if x is a matrix, returns a row vector whose elements are the mean value of the elements from each column of the matrix.
median(x)	same as mean(x), only returns the median value.
sum(x)	returns the sum of the elements of a vector or if x is a matrix, returns the sum of the elements from each respective column of the matrix.
prod(x)	same as sum(x), only returns the product of elements.
std(x)	returns the standard deviation of the elements of a vector or if x is a matrix, a row vector whose elements are the standard deviations of each column of the matrix.

`sort(x)` sorts the values in the vector `x` or the columns of a matrix and places them in ascending order. Note that this command will destroy any association that may exist between the elements in a row of matrix `x`.

`hist(x)` plots a histogram of the elements of vector, `x`. Ten bins are scaled based on the max and min values.

`hist(x,n)` plots a histogram with '`n`' bins scaled between the max and min values of the elements.

`hist((x(:,2)))` plots a histogram of the elements of the 2nd column from the matrix `x`.

The 'matlab' commands introduced in this lesson perform simple statistical calculations that are mostly self-explanatory. A simple series of examples are illustrated below. Note that 'matlab' treats each COLUMN OF A MATRIX as a unique set of "data", however, vectors can be row or column format. Begin the exercise by creating a 12x3 matrix that represents a time history of two stochastic temperature measurements.

Load these data into a matrix called 'timtemp.dat' using an editor to build an ASCII file.

Time(sec)	Temp-T1 (K)	Temp-T2 (K)
0.0	306	125
1.0	305	121
2.0	312	123
3.0	309	122
4.0	308	124
5.0	299	129
6.0	311	122
7.0	303	122
8.0	306	123
9.0	303	127
10.0	306	124
11.0	304	123

Execute the commands at the beginning of the lesson above and observe the results. Note that when the argument is 'timtemp', a matrix, the result is a 1x3 vector. For example, the command

```
M = max(timtemp)
gives
M = 11  312  129
```

If the argument is a single column from the matrix, the command identifies the particular column desired. The command

```
M2 = max(timtemp(:,2))
gives
M2 = 312
```

If the variance of a set of data in a column of the matrix is desired, the standard deviation is squared. The command

```

T1_var = (std(timtemp(:,2)))^2
gives
T1_var = 13.2727

```

If the standard deviation of the matrix of data is found using

```

STDDEV = std(timtemp)
gives
STDDEV = 3.6056  3.6432  2.3012

```

Note that the command

```
VAR = STDDEV^2
```

is not acceptable; however, the command

```
VAR = STDDEV.^2
```

is acceptable, creating the results,

```
VAR = 13.0000  13.2727  5.2955
```

USING m-FILES - SCRATCH and FUNCTION FILES

Sometimes it is convenient to write a number of lines of 'matlab' code before executing the commands. We have seen how this can be accomplished using the line continuation ellipsis; however, it was noted in this approach that any mistake in the code required the entire string to be entered again. Using an m-file we can write a number of lines of 'matlab' code and store it in a file whose name we select with the suffix '.m' added. Subsequently the command string or file content can be executed by invoking the name of the file while in 'matlab'. This application is called a SCRATCH FILE.

On occasion it is convenient to express a function in 'matlab' rather than calculating or defining a particular matrix. The many 'matlab' stored functions such as sin(x) and log(x) are examples of functions. We can write functions of our definition to evaluate parameters of our particular interest. This application of m-files is called FUNCTION FILES.

SCRATCH FILES

A scratch file must be prepared in a text editor in ASCII format and stored in the directory from which you invoked the command to download 'matlab'. The name can be any legitimate file name with the '.m' suffix.

As an example, suppose we wish to prepare a x-y plot of the function

$$y = e^{-x/10} \sin(x) \quad 0 \leq x \leq 10$$

To accomplish this using a scratch ".m-file", we will call the file 'explot.m'. Open the file in a text editor and type the code below. (Note the use of the '%' in line 1 below to create a

comment. Any text or commands typed on a line after the '%' will be treated as a comment and ignored in the executable code.)

```
% A scratch m-file to plot exp(-x/10)sin(x)
x = [ 0:.2:10 ];
y = exp(-x/10) .* sin(x);
plot(x,y),...
title('EXPONENTIAL DAMPED SINE FUNCTION'),...
xlabel('x'),...
ylabel('y'),...
text(.6,.7,'y = exp(-x/10)*sin(x)', 'sc')
```

Store this file under the name 'explot.m' in your 'matlab' directory. Now when you are in 'matlab', any time you type the command 'explot' the x-y plot of the damped exponential sine function will appear.

FUNCTION FILES

Suppose that you wish to have in your 'matlab' workspace a function that calculates the sine of the angle when the argument is in degrees. The matlab function sin(x) requires that x be in radians. A function can be written that use the 'matlab' sin(x) function, where x is in radians, to accomplish the objective of calculating the sine of an argument expressed in degrees. Again, as in the case of the scratch file, we must prepare the file in ASCII format using a text editor, and we must add the suffix '.m' to the file. HERE THE NAME OF THE FILE MUST BE THE NAME OF THE FUNCTION.

The following code will accomplish our objective. Note that the first line must always begin with the word "function" followed by the name of the function expressed as " y = function-name". Here the function name is "sind(x)". This is the name by which we will call for the function once it is stored.

```
function y = sind(x)
% This function calculates the sine when the argument is degrees
% Note that array multiplication and division allows this to
% operate on scalars, vectors and matrices.
y = sin( x .* pi ./ 180 )
```

Again, note the rule in writing the function is to express in the first line of the file the word 'function', followed by
y = function call,
here 'sind' with the function argument in parentheses, 'x' is the function call. Thus,

```
function y=sind(x)
```

Now every time we type sind(x) in 'matlab', it will return the value of the sine function calculated under the assumption that x is in degrees. Of course any matrix or expression involving a matrix can be written in the argument.

ALGEBRAIC OPERATIONS IN MATLAB

Scalar Calculations. The common arithmetic operators used in spreadsheets and programming languages such as BASIC are used in 'matlab'. In addition a distinction is made between right and left division. The arithmetic operators are

+	addition
-	subtraction
*	multiplication
/	right division (a/b means $a \div b$)
\	left division (a\b means $b \div a$)
^	exponentiation

When a single line of code includes more than one of these operators the precedence or order of the calculations follows the below order:

Precedence	Operation
1	parentheses
2	exponentiation, left to right
3	multiplication and division, left right
4	addition and subtraction, left right

These rules are applied to scalar quantities (i.e., 1x1 matrices) in the ordinary manner. (Below we will discover that nonscalar matrices require additional rules for the application of these operators!!) For example,

```
3*4      executed in 'matlab' gives
ans=12
```

```
4/5      executed in 'matlab' gives
ans=.8000
```

```
4\5      executed in 'matlab' gives
ans=1.2500
```

```
x = pi/2; y = sin(x)      executed in 'matlab' gives
y = 1
```

```
z = 0; w = exp(4*z)/5      executed in 'matlab' gives
z= .2000
```

Note that many programmers will prefer to write the expression for w above in the format

$$w = (\exp(4*x))/5$$

which gives the same result and is sometimes less confusing when the string of arithmetic operations is long. Using 'matlab', carry out some simple arithmetic operations with scalars you define. In doing so utilize the 'matlab' functions sqrt(x), abs(s), sin(x), asin(x), atan(x), atan2(x), log(x), and log10(x) as well as exp(x). Many other functions are available in 'matlab' and can be found in the documentation.

Matrix Calculations. Because matrices are made up of a number of elements and not a single number (except for the 1x1 scalar matrix), the ordinary rules of commutative, associative and distributive operations in arithmetic do not always follow.

Moreover, a number of important but common-sense rules will prevail in matrix algebra and 'matlab' when dealing with nonscalar quantities.

Addition and Subtraction of Matrices. Only matrices of the SAME ORDER can be added or subtracted. When two matrices of the same order are added or subtracted in matrix algebra, the individual elements are added or subtracted. Thus the distributive rule applies.

$$A + B = B + A \quad \text{and} \quad A - B = B - A$$

If $C = A + B$ then each element $C_{ij} = A_{ij} + B_{ij}$.

Define A and B as follows:

```
A=[1 2 3; 3 3 3; 5 3 1]
B=[2 -3 4; 2 -2 2; 0 4 0]
```

Then note that

$$C = A + B \quad \text{and} \quad C = B + A$$

gives

$$C = \begin{bmatrix} 3 & -1 & 7 \\ 5 & 1 & 5 \\ 5 & 7 & 1 \end{bmatrix}$$

Now define the row vector

$$x = [3 \quad 5 \quad 7]$$

and the column vector

$$y = \begin{bmatrix} 4 \\ -1 \\ -3 \end{bmatrix}$$

Note that the operation

$$z = x + y$$

is not valid! It is not valid because the two matrices do not have the same order. (x is a 1x3 matrix and y is a 3x1 matrix.) Adding any number of 1x1 matrices or scalars is permissible and follows the ordinary rules of arithmetic because the 1x1 matrix is a scalar. Adding two vectors is permissible so long as each is a row vector (1xn matrix) or column vector (nx1 matrix). Of course any number of vectors can be added or subtracted, the result being the arithmetic sum of the individual elements in the column or row vectors. Square matrices can always be added or subtracted so long as they are of the same order. A 4x4 square matrix can not be added to a 3x3 square matrix, because they are not of the same order, although both matrices are square.

Multiplication of Matrices. Matrix multiplication, though straightforward in definition, is more complex than arithmetic multiplication because each matrix contains a number of elements. Recall that with vector multiplication, the existence of a number

of elements in the vector resulted in two concepts of multiplication, the scalar product and the vector product. Matrix multiplication has its array of special rules as well.

In matrix multiplication, the elements of the product, C , of two matrices $A*B$ are calculated from

$$C_{ij} = \sum_k A_{ik} * B_{kj}$$

To form this sum, the number of columns of the first or left matrix (A) must be equal to the number of rows in the second or right matrix (B). The resulting product, matrix C , has an order for which the number of rows equals the number of rows of the first (left) matrix (A) and the product (C) has a number of columns equal to the number of columns in the second (right) matrix (B). It is clear that $A*B$ IS NOT NECESSARILY EQUAL TO $B*A$! It is also clear that $A*B$ and $B*A$ only exist for square matrices!

Consider the simple product of two square 2x2 matrices.

```
a = [ 1 2; 3 4];
b = [ 8 7; 6 5];
```

Calling the product $c = a*b$

```
c11 = a11*b11 + a12*b21
c12 = a11*b12 + a12*b22
c21 = a21*b11 + a22*b21
c22 = a21*b12 + a22*b22
```

Carry out the calculations by hand and verify the result using 'matlab'. Next, consider the following matrix product of a 3x2 matrix X and a 2x4 matrix Y .

```
X = [2  3 ; 4 -1 ; 0  7];
Y = [5  -6  7  2 ; 1  2  3  6];
```

First, note that the matrix product $X*Y$ exists because X has the same number of columns (2) as Y has rows (2). (Note that $Y*X$ does NOT exist!) If the product of $X*Y$ is called C , the matrix C must be a 3x4 matrix. Again, carry out the calculations by hand and verify the result using 'matlab'.

Note that the PRODUCT OF A SCALAR AND A MATRIX is a matrix in which every element of the matrix has been multiplied by the scalar. Verify this by recalling the matrix X defined above, and carry out the product $3*X$ on 'matlab'. (Note that this can be written $X*3$ as well as $3*X$, because quantity '3' is scalar.)

ARRAY PRODUCTS

Recall that addition and subtraction of matrices involved addition or subtraction of the individual elements of the matrices. Sometimes it is desired to simply multiply or divide each element of an matrix by the corresponding element of another matrix. These are called 'array operations' in 'matlab'. Array or element-by-element operations are executed when the operator is preceded by a

'.' (period). Thus

```

a .* b    multiplies each element of a by the respective
           element of b
a ./ b    divides each element of a by the respective element
           of b
a .\ b    divides each element of b by the respective element
           of a
a .^ b    raise each element of a by the respective b element

```

For example, if matrices G and H are defined

```

G = [ 1  3  5; 2  4  6];
H = [-4  0  3; 1  9  8];

G .* H = [ -4   0  15
           2  36  48 ]

```

TRANSPOSE OF A MATRIX

The transpose of a matrix is obtained by interchanging the rows and columns. The 'matlab' operator that creates the transpose is the single quotation mark, '. Recalling the matrix G

```

G' = [ 1  2
       3  4
       5  6 ]

```

Note that the transpose of a $m \times n$ matrix creates a $n \times m$ matrix. One of the most useful operations involving the transpose is that of creating a column vector from a row vector, or a row vector from a column vector.

INNER (SCALAR) PRODUCT OF TWO VECTORS

The scalar or inner product of two row vectors, G1 and G2 is found as follows. Create the row vectors for this example by decomposing the matrix G defined above.

```

G1 = G(1,:)
G2 = G(2,:)

```

Then the inner product of the 1x3 row vector G1 and the 1x3 row vector G2 is

```

G1 * G2' = 44

```

Verify this result by carrying out the operations on 'matlab'.

If the two vectors are each column vectors, then the inner product must be formed by the matrix product of the transpose of a column vector times a column vector, thus creating an operation in which a $1 \times n$ matrix is multiplied with a $n \times 1$ matrix.

In summary, note that the inner product must always be the product of a row vector times a column vector.

OUTER PRODUCT OF TWO VECTORS

If two row vectors exist, G1 and G2 as defined above, the outer product is simply

```
G1' * G2      {Note G1' is 3x1 and G2 is 1x3}
```

and the result is a square matrix in contrast to the scalar result for the inner product. DON'T CONFUSE THE OUTER PRODUCT WITH THE VECTOR PRODUCT IN MECHANICS! If the two vectors are column vectors, the outer product must be formed by the product of one vector times the transpose of the second!

OTHER OPERATIONS ON MATRICES

As noted above, many functions that exist can be applied directly to a matrix by simply allowing the function to operate on each element of the matrix array. This is true for the trigonometric functions and their inverse. It is true also for the exponential function, `exp()`, and the logarithmic functions, `log()` and `log10()`.

Exponential operations (using `^`) on matrices are quite different from their use with scalars. If W is a square matrix W^2 implies $W*W$ which is quite different from $W.*W$. Be certain of your intent before raising a matrix to an exponential power. Of course W^2 only exists if W is a square matrix.

SPECIAL MATRICES

A number of special functions exist in 'matlab' to create special or unusual matrices. For example, matrices with element values of zero or 1 are sometimes useful.

```
zeros(4)
```

creates a square matrix (4 x 4) whose elements are zero.

```
zeros(3,2)
```

creates a 3 row, 2 column matrix of zeros.

Similarly the commands `ones(4)` creates a 4 x 4 square matrix whose elements are 1's and `'ones(3,2)'` creates a 3x2 matrix whose elements are 1's.

If a m x n matrix, say A, already exists, the command `'ones(A)'` creates an m x n matrix of 1's. The command `'zeros(A)'` creates an mxn matrix of 0's. Note that these commands do not alter the previously defined matrix A, which is only a basis for the size of the matrix created by commands `zeros()` and `ones()`.

The identity matrix is a square matrix with 1's on the diagonal and 0's off the diagonal. Thus a 4x4 identity matrix, a, is

```
a=[1 0 0 0
   0 1 0 0
```

```
0 0 1 0
0 0 0 1]
```

This matrix can be created with the 'eye' function. Thus,

```
a = eye(4)
```

The 'eye' function will create non-square matrices. Thus

```
creates      eye(3,2)
              1  0
              0  1
              0  0
```

```
and
creates      eye(2,3)
              1  0  0
              0  1  0
```

PRACTICE

1. The first four terms of the fourier series for the square wave whose amplitude is 5 and whose period is 2π is

$$y = (20/\pi) [\sin x + (1/3)\sin 3x + (1/5)\sin 5x + (1/7)\sin 7x]$$

Calculate this series, term by term, and plot the results for each partial sum.