HUL315 Assignment 2

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February 13,2015

1 Q1: Find E [X] for Lognormal and Logistic distribution

1.1 Lognormal

A Lognormal distribution is defined such that:-

$$X \sim LN(\mu, \sigma^2) \tag{1}$$

if

$$X = e^Y \tag{2a}$$

$$Y \sim N(\mu, \sigma^2) \tag{2b}$$

To derive the Expected Value, we need to first figure out the probability distribution function.

$$\begin{aligned} Prob(X < k) &= Prob(e^Y < k) \\ &= Prob(Y < log(K)) \\ &= \int_{-\infty}^{log(k)} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(y-\mu)^2}{2\sigma^2}} dy \\ &= \int_{-\infty}^{k} \frac{1}{x\sqrt{2\pi}\sigma} e^{\frac{-(log(x)-\mu)^2}{2\sigma^2}} dx \end{aligned}$$

Now, for the expected value. We will use a substitution of $y = x - \mu, dy = dx$

$$\begin{aligned} E(Y) = E(e^X) &= \int_{-\infty}^{\infty} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-(x-\mu)^2}{2\sigma^2}} dx \\ &= \int_{-\infty}^{\infty} e^{y+\mu} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-y^2}{2\sigma^2}} dy \\ &= e^{\mu} \int_{-\infty}^{\infty} e^{y} \frac{1}{\sqrt{2\pi}\sigma} e^{\frac{-y^2}{2\sigma^2}} dy \end{aligned}$$

This integration is fairly trivial and can be performed by converting $y - \frac{y^2}{2\sigma^2}$ to the closest square. So, we end up with

$$e^{\mu}e^{\frac{\sigma^2}{2}} = e^{\mu+\frac{\sigma^2}{2}}$$

Now, for $\mathrm{E}\big[X^2\big]$, which, after subtracting from the expected value, will return the Variance. We will continue to use the same substitutions and the same integration results.

$$\begin{aligned} E(Y^2) = E(e^{2X}) &= \int_{-\infty}^{\infty} e^{2x}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(x-\mu)^2}{2\sigma^2}}dx \\ &= \int_{-\infty}^{\infty} e^{2(y+\mu)}\frac{1}{\sqrt{2\pi}\sigma}e^{\frac{-(y)^2}{2\sigma^2}}dx \\ &= e^{2\mu+2\sigma^2} \end{aligned}$$

$$\begin{aligned} Var(Y) &= E\big[X^2\big] - E\big[X\big]^2 \\ &= e^{2\mu+2\sigma^2} - e^{-2\mu+\sigma^2} \\ &= e^{2\mu+\sigma^2}(e^{\sigma^2} - 1) \end{aligned}$$

2 Q2:Give an example where covariance of 2 random variables but not independant

I found a great example of this on crossValidated, the statistics StackExchange forum. It goes as follows:-

Take a random variable X with $E\big[X\big] = 0$ and $E\big[X^3\big] = 0$, for example a normal distribution with zero mean. Take $Y = X^2$. Clearly, X and Y are dependant. But,

$$cov(X,Y) = E\big[XY\big] - E\big[X\big]E\big[Y\big] = E\big[X^3\big] = 0 \tag{3}$$

Praise be the gentle humans of StackExchange!

3 Q3: