20F-MATH168-1 Frontiers article

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TOTAL POINTS

100 / 100

QUESTION 1

1 Essay 100 / 100

✓ + 0 pts **Graded (see point adjustment for score)**

+ 100 **Point adjustment**

This was an engaging read and you did a great job of illustrating the concepts using Mean Girls! Thanks for making the minor changes I suggested.

gradescope

What networks can tell us about Mean Girls

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1 Abstract

In the real world, people tend to naturally divide into their own groups and communities. Whether it is the groups of friends you make in school or the clubs you join for after-school activities, these communities can often give someone a lot of information about you. The same can be said for networks! Mathematics and computers are used to study various aspects of the structure of these networks as they can tell us a lot about how things are connected. Scientists have developed some sophisticated ways to do these but this paper will discuss simpler notions of how to describe these networks, using methods called k-cores and cliques.

2 The movie *Mean Girls* and Networks

One of the best movies that shows us these ideas is *Mean Girls*. It is the story about a girl, Cady Heron, who moves to Evanston, Illinois and starts attending North Shore High School, after being homeschooled for all her life. As expected, Cady has a hard time making new friends until she meets Janis and Damian, who tell her all about the different social groups in the school. The most infamous of these groups are dubbed the "Plastics", and it is made up of Gretchen, Karen and the most popular girl in school, Regina. The Plastics find out about Cady and become interested in befriending her. Learning about this invitation, Janis convinces Cady to become friends with the "Plastics" to spy on them for her. Along the way, Cady befriends Aaron, Regina's boyfriend, and many other people. She finds herself on a journey trying to fit in and find the right friends for herself, even joining the Mathletes and becoming friends with its leader, Kevin. The story is a great example of how small, localized groups can arise in a big network such as the one that forms in a high school.

In any high school, including North Shore High school, the people are connected in a network, specifically a social network. A social network is a collection of nodes where the nodes are people and the edges between them are a social connection [1]. The people in North Shore High represent nodes and their connections are based on social relationships such as friendship, being in the same after-school club, playing the same sport, being in the same class, teacher-student relationships, etc. These connections can be drawn on a graph as a line between the nodes called an edge.

3 The k-core analysis of *Mean Girls*

Although we could look at the whole social network of North Shore High, studying a smaller social network such as the one in Figure 1 can help us determine the k-cores and cliques more easily. Here, each person is considered a node by mathematicians, and the number of connections each person has is called a degree. Regina is friends with Gretchen, Karen, Cady and Aaron, which is why we see edges between them. Regina therefore has a degree of 4. Similarly, Cady has a degree of seven, since she is friends with all of the people in our network. The degree of a node gives us an idea of where information can flow from one person to another in a social network. The higher the degree of a person, the more directions knowledge can spread from one person to another, meaning the degree of a node can be seen as a measure of its importance. Although this gives us some idea of the role each person plays in the network, it does not tell us much about the groups

this person is a part of.

To resolve this problem, mathematicians often study the k-cores of a network. A k-core is a connected set of nodes in which every node is connected to at least k other nodes in the set. This means that to be in a k-core, each person has to know at least a k number of people in the network. For example, a person can only be in a 2-core if they know at least two or more people in the network.

Scientists have developed a technique to calculate the k-cores in a network easily. For each number k, we identify the number of nodes that have a degree of k or higher, keep those nodes and remove the rest, until we reach a number k for which there are no such nodes with degree k or higher in the network. Once we reach this number, we no longer remove nodes and stop our calculation there. This is called a stopping condition.

We can calculate k-cores for the network in Figure 1. First, we look for the people in the network that have degree 1 or higher. These are the people that have 1 or more friends. Since everyone in the network has one or more friends, we don't remove anyone. Therefore, everyone in this network is part of the 1-core. Moving on to k=2, we want to keep the people who have atleast degree 2 and remove those who have a degree less than 2. Since Kevin is the only one with less than two friends, we remove him and all the connections he has to the other people in the network. This means that our 2-core contains everyone except Kevin. Next, we have k = 3. Going back to the original network, we remove the people who have a degree of less than 3 and their edges, until we have a remaining network where everyone has at least three friends. The only people who have less than three friends are Kevin, Aaron, Janis and Damian, so we remove them from the network and all their connections. Therefore, our 3-core contains Cady, Regina, Gretchen and Karen. Next, we identify our 4-core. We remove members from the original network who have less than four friends, and their connections, step by step until we are left with a network that only has members with at least 4 friends. In the first iteration, we remove Kevin, Janis, Damian, Aaron, Gretchen and Karen since they have degree less than 4 to begin with. However, once we remove them and their connections, we're only left with Regina and Cady, with one connection between them. Since at this stage Regina and Cady both have degree less than 4, we end up removing them as well. Therefore, while trying to find the 4-core, we have ended up removing everyone in the network! This means that we have reached the stopping condition for our network. We identify that the maximum core for our network is a 3-core. Figure 2 shows the people of North Shore High and the cores they belong to.

4 The mathematical cliques of *Mean Girls*

We can have directed and undirected networks. Directed networks are networks in which all the connections between the nodes point in a specific direction, from one node to another. In this case, if one person considers the other a friend, it is a sufficient condition for an edge to form between them. Suppose person A considers person B a friend, then a connection forms pointing from person A towards person B. Vice versa, if person B considers person A a friend, a connection forms from person B and pointing towards person A. If both people consider each other friends, there are two connections that form, one pointing from person A to B and one pointing from person B to A.

In life, we encounter cases where one person may consider you a friend but you may not consider them your friend. If one was to map that relationship with a network, then both you and the other person would represent nodes, and there would only be a connection pointing from that person towards you. This is called a directed edge. In contrast, undirected networks are those in which the connections are undirected, meaning two people both have to consider each other to be friends in order for there to be a connection between them. Figure 1 is an example of an undirected network.

We all know about cliques like high schools, in which people with similar interests tend to befriend and spend time with each other, and are typically reluctant to let others join easily. The same can be said about North Shore High, where there are cliques of cheerleaders, jocks, burnouts and even the Plastics. There is a

similar notion of cliques in mathematics. A clique is described as a set of nodes in an undirected network such that every member of the set is connected by an edge to every other member in the set [1]. This means that people are members of a clique if and only if each person is friends with all the other people in the clique.

In many ways, the mathematical notion of cliques is similar to our understanding of cliques in a social context. Since we characterize them to be a close-knit group of people who share similar interests, two members cannot be in the same clique if only one of them considers the other a friend and not vice versa; friendships are bidirectional in a clique. Similarly, mathematicians only define cliques on undirected networks.

To be in a clique, a member has to have connections with all the other members in a clique, which is also what it means to be in a close-knit group. The Plastics are considered a clique, both in the mathematical and non-mathematical sense. Each member of the Plastics is best friends with the other two members so there is a non-directional connection between all of them. In fact, when Cady joins the group, she becomes part of their on-screen clique. From Figure 2, one can see that Cady also becomes part of their mathematical clique, as she forms friendships with all 3 members. Notably, Aaron isn't part of the clique because he is only friends with two out of four members of the clique. In the context of the movie, he isn't considered part of the Plastics either.

5 Why study the structure of networks?

Scientists study the structure of networks to gather information about how nodes interact with each other. However, networks can be very large and can have thousands of nodes, making it difficult to easily determine the most important members. Even though the degree of a node tells us how important a member is in the network, one may want to determine which members are the most closely connected. Cliques are also very common in large networks and indicate highly cohesive subgroups, where members are closely connected to each other. These give an idea about how closely connected members of a network are. One can also determine groups that are the most influential, especially in large networks. In some sense, this is similar to the notion of popularity; individual nodes with high degree indicate which members of the network are popular, but groups of nodes in higher k-cores indicate which groups of people are the most popular. From Figure 2, we see that Regina, Gretchen, Karen and Cady are in the highest k-core and therefore compose the most popular group, which is mirrored on-screen. Such ideas of the localized structure can give us a better understanding of how information can flow in smaller subgroups of a large network and through this illustration with the narrative of *Mean Girls*, we gave you some intuition about the role smaller groups play in larger networks.

6 Glossary

Network: A collection of nodes and the connections between nodes.
Node: The things in a network that are connected to other things. For instance, in the *Mean Girls* social network, the characters in the movie are the nodes.
Edge: Connections between nodes.
Directed Network: Networks in which each edge has a direction, pointing from one node to another.
Undirected Network: Networks in which edges don't have a direction.
Degree: The total number of nodes a node is connected to.
K-core: A connected set of nodes where each node is connected to atleast k other nodes in the set.
Clique: Set of nodes within an undirected network such that every node in the set is connected by an edge to every other node in the set.

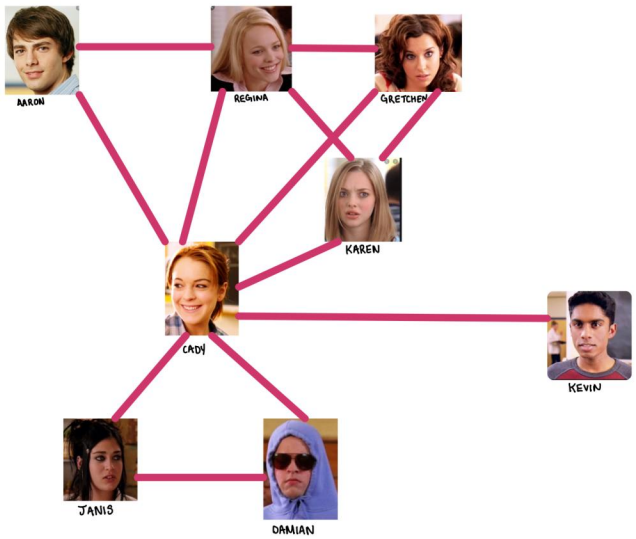

Figure 1: The social network representation of the movie *Mean Girls*

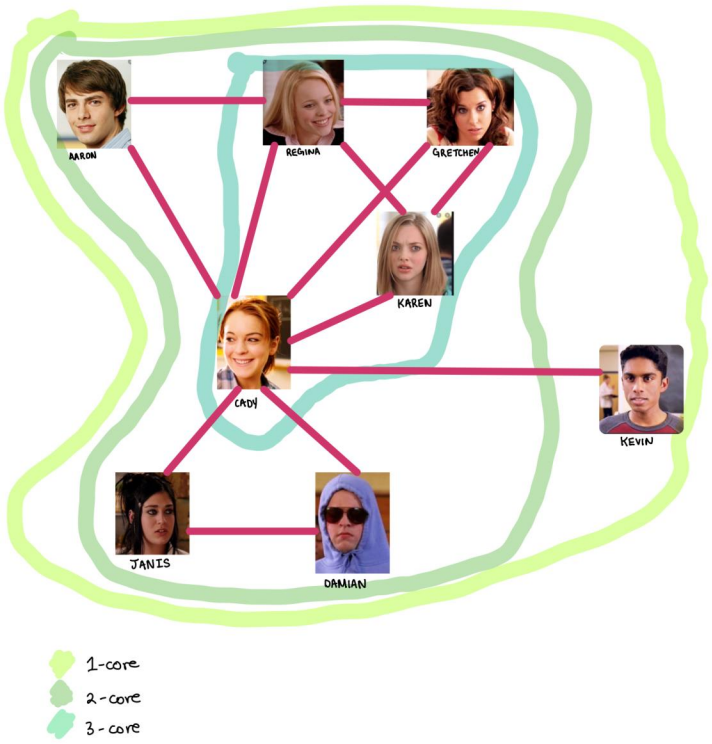

Figure 2: K-core analysis of the *Mean Girls* social network

7 Conflict of Interest Statement

The author declares that the research was conducted in the absence of any commercial or financial relationships that could be construed as a potential conflict of interest.

8 Acknowledgements

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9 References

[1] Newman, M. E. J. 2018. Networks, 2nd Edn. Oxford: Oxford University Press.