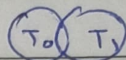
# ASSIGNMENT (Week 1)




1.1 Q1

Soln:

$P(\text{Today}) = .6 = P(T_0)$

$P(\text{Tomorrow}) = .5 = P(T_1)$

$P(\overline{T_0 \cup T_1}) = 1 - P(T_0 \cup T_1) = .3$

$\longrightarrow \quad P(T_0 \cup T_1) = .7$

$P(T_0) + P(T_1) - P(T_0 \cap T_1) = .7$

$P(T_0 \cap T_1) = .4$

(a) Today or tomorrow

$= P(T_0 \cup T_1) = \underline{0.7}$ Ans

(b) $P(T_0 \cap T_1) = \underline{0.4}$ Ans

(c) $P(T_0 \cap \overline{T_1}) + P(T_0 \cap T_1) = P(T_0)$

$P(T_0 \cap \overline{T_1}) = .6 - .4 = \underline{.2}$ Ans

(d) $X$ = today or tomorrow, but not both

$P(X) = P(T_0) + P(T_1) - 2P(T_0 \cap T_1)$

$= 1.1 - 2 \times .4$

$= \underline{0.3}$ Ans

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1.2 Q2

Soln

Sample space (S) =

(1,1) (1,2) (1,3) (1,4) (1,5) (1,6)
(2,1) (2,2) (2,3) (2,4) (2,5) (2,6)
(3,1) (3,2) (3,3) (3,4) (3,5) (3,6)
(4,1) (4,2) (4,3) (4,4) (4,5) (4,6)
(5,1) (5,2) (5,3) (5,4) (5,5) (5,6)
(6,1) (6,2) (6,3) (6,4) (6,5) (6,6)

For $X_1 + X_2 = 8$

(2,6) (5,3) (6,2) (3,5) (4,4)

$$P(A) = \frac{\text{Favourable outcomes}}{\text{Total possible}}$$

$$= \frac{5}{36} \text{ Ans}$$

1.3 Q3

1.4 Q4

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1.4 Que 4

1. CDF of $X$, $(F_X(x))$

$$F_X(x) = P(X \le x) = \underbrace{p\,F_d(x)}_{\text{if it lands heads}} + \underbrace{(1-p)F_c(x)}_{\text{if it lands tails}}$$

2. PDF $[f_X(x)]$

$$f_X(x) = p\,f_d(x) + (1-p)\,f_c(x)$$

3. $E[X] = p\,E[X_d] + (1-p)\,E[X_c]$

$$E[X_d] = \sum x\,f_d(x)$$

$$E[X_c] = \int_{-\infty}^{\infty} x\,f_c(x)\,dx$$

$$\boxed{E[X] = p\sum x\,f_d(x) + (1-p)\int_{-\infty}^{\infty} x\,f_c(x)\,dx}$$

4. $\text{Var}(X) = E[(X-\mu)^2] = E[X^2] - (E[X])^2$

$$= p\left[\sum_x x^2 f_d(x) - \left(\sum_x x f_d(x)\right)^2\right]$$

$$+ (1-p)\left[\int_{-\infty}^{\infty} x^2 f_c(x)\,dx - \left(\int_{-\infty}^{\infty} x f_c(x)\,dx\right)^2\right]$$

$$+ p(1-p)\left(\sum_x x f_d(x) - \int_{-\infty}^{\infty} x f_c(x)\,dx\right)^2$$

5. given

$X, Y \sim N(0,1)$ independent

$Z = 1 + X + XY^2 \qquad W = 1 + X$

We want

$\text{cov}(Z, W) = \text{Cov}(1 + X + XY^2, 1 + X)$

→ Linearity of covariance

$\text{cov}(Z, W) = \text{Cov}(X + XY^2, X)$

$= \text{cov}(X, X) + \text{Cov}(XY^2, X)$

$= \text{Var}(X) = 1$

$\text{cov}(XY^2, X) = E[X^2Y^2] - E[XY^2]E[X]$

$X$ & $Y$ → independent $E[X] = 0$ → $0$

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$$E(X^2Y^2) = E[X^2]\,E[Y^2] = 1\cdot 1 = 1$$

$$\mathrm{cov}(XY^2, X) = 1$$

$$\Rightarrow \mathrm{cov}(Z, W) = 1 + 1 = \underline{2} \text{ Ans}$$

Que 6.

Solⁿ: Let there be 4 companies A, B, C, D

Probability of receiving offer from ~~each~~ $x = P(x)$

~~$P(A) = P(B) = P$~~

~~$P(A \cap B \cap C \cap D) = 20\% = .2$~~

If $P(A \cap B \cap C \cap D) = .2$, then we don't have sufficient information.

If the chance of getting an offer from each company (not necessarily simultan-) $= \underline{.2}$

$$P(A) = P(B) = P(C) = P(D) = .2$$

Probability of not getting offer from any company $= (1 - .2)^4 = (.8)^4$

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$1-(.8)^4$ = Probability of getting an offer from atleast one company

$= .5904$

$= \underline{59.04\ \%}$

Hence, he is wrong.

<u>Que 7</u>

Probability of more than 120 errors

$= {}^{1000}C_{121}(.1)^{121} + {}^{1000}C_{122}(.1)^{122} \ldots + {}^{1000}C_{1000}(.1)^{1000}$

Que 8.

Let T be the total no. of sandwiches required for 95%.

Expectation of one guest's demand

$E[X] = 0 \cdot \frac{1}{4} + 1 \cdot \frac{1}{2} + 2 \cdot \frac{1}{4} = 1$

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$E[x^2] = 0^2 \cdot \frac{1}{4} + 1^2 \cdot \frac{1}{2} + 2^2 \cdot \frac{1}{4} = \frac{1}{2} + 1 = 1.5$

$Var(X) = E[X^2] - (E[X])^2 = 1.5 - 1 = 0.5$

$E[S] = 64 \times 1 = 64$

$Var(S) = 64 \times 0.5 = 32$

$\sigma_S = \sqrt{Var(S)} = \sqrt{32} = 5.66$

Using Central Limit Theorem

$S \approx N(64, 32)$

$P(T \le m) = 0.95$

$Z_{0.95} \approx 1.645$ (standard Normal)

$m = \mu + z\sigma$

$= 64 + 1.645(5.66) \approx 64 + 9.31$

$\approx 73.31$

74 sandwiches for 95% surity