1. Let $u'(t) = 2t(1+u)$, $u(0) = 0$.

$$\begin{aligned}
u_1(t) &= \int_0^t f(s, u(0))ds \\
&= \int_0^t f(s, 0)ds \\
&= \int_0^t 2sds \\
&= t^2
\end{aligned}$$

   We can then derive $u_2$ and $u_3$:

$$\begin{aligned}
u_2(t) &= \int_0^t f(s, u_1(s))ds \\
&= \int_0^t f(s, s^2)ds \\
&= \int_0^t 2s(1+s^2)ds \\
&= \int_0^t 2s + \int_0^t 2s^3 ds \\
&= t^2 + \frac{t^4}{2} \\
u_3(t) &= \int_0^t f(s, u_2(s))ds \\
&= \int_0^t f(s, s^2 + \frac{s^4}{2})ds \\
&= \int_0^t 2s(1 + s^2 + \frac{s^4}{2})ds \\
&= \int_0^t 2s + \int_0^t 2s^3 ds + \int_0^t s^5 ds \\
&= t^2 + \frac{t^4}{2} + \frac{t^6}{6}
\end{aligned}$$

   It seems that the Picard iteration for this function takes the following pattern:

$$u_k(t) = \sum_{i=1}^{k} \frac{t^{2i}}{i!}$$

2.

3.

4. See the source code at `https://github.com/codeandkey/math481-iastate-sp2020`.

5.

6. See the graph: