1

Arrays and Strings

Hopefully, all readers of this book are familiar with arrays and strings, so we won't bore you with such details. Instead, we'll focus on some of the more common techniques and issues with these data structures.

Please note that array questions and string questions are often interchangeable. That is, a question that this book states using an array may be asked instead as a string question, and vice versa.

▸ Hash Tables

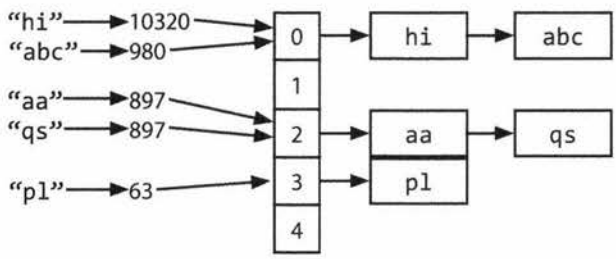
A hash table is a data structure that maps keys to values for highly efficient lookup. There are a number of ways of implementing this. Here, we will describe a simple but common implementation.

In this simple implementation, we use an array of linked lists and a hash code function. To insert a key (which might be a string or essentially any other data type) and value, we do the following:

1. First, compute the key's hash code, which will usually be an `int` or `long`. Note that two different keys could have the same hash code, as there may be an infinite number of keys and a finite number of ints.
2. Then, map the hash code to an index in the array. This could be done with something like `hash(key) % array_length`. Two different hash codes could, of course, map to the same index.
3. At this index, there is a linked list of keys and values. Store the key and value in this index. We must use a linked list because of collisions: you could have two different keys with the same hash code, or two different hash codes that map to the same index.

To retrieve the value pair by its key, you repeat this process. Compute the hash code from the key, and then compute the index from the hash code. Then, search through the linked list for the value with this key.

If the number of collisions is very high, the worst case runtime is `O(N)`, where `N` is the number of keys. However, we generally assume a good implementation that keeps collisions to a minimum, in which case the lookup time is `O(1)`.



Alternatively, we can implement the hash table with a balanced binary search tree. This gives us an `O(log N)` lookup time. The advantage of this is potentially using less space, since we no longer allocate a large array. We can also iterate through the keys in order, which can be useful sometimes.

▸ ArrayList & Resizable Arrays

In some languages, arrays (often called lists in this case) are automatically resizable. The array or list will grow as you append items. In other languages, like Java, arrays are fixed length. The size is defined when you create the array.

When you need an array-like data structure that offers dynamic resizing, you would usually use an ArrayList. An ArrayList is an array that resizes itself as needed while still providing `O(1)` access. A typical implementation is that when the array is full, the array doubles in size. Each doubling takes `O(n)` time, but happens so rarely that its amortized insertion time is still `O(1)`.

```
1  ArrayList<String> merge(String[] words, String[] more) {
2    ArrayList<String> sentence = new ArrayList<String>();
3    for (String w : words) sentence.add(w);
4    for (String w : more) sentence.add(w);
5    return sentence;
6  }
```

This is an essential data structure for interviews. Be sure you are comfortable with dynamically resizable arrays/lists in whatever language you will be working with. Note that the name of the data structure as well as the "resizing factor" (which is 2 in Java) can vary.

Why is the amortized insertion runtime O(1)?

Suppose you have an array of size N. We can work backwards to compute how many elements we copied at each capacity increase. Observe that when we increase the array to K elements, the array was previously half that size. Therefore, we needed to copy $K/2$ elements.

```
final capacity increase   : n/2 elements to copy
previous capacity increase: n/4 elements to copy
previous capacity increase: n/8 elements to copy
previous capacity increase: n/16 elements to copy
...
second capacity increase  : 2 elements to copy
first capacity increase   : 1 element to copy
```

Therefore, the total number of copies to insert N elements is roughly $N/2 + N/4 + N/8 + \ldots + 2 + 1$, which is just less than N.

> If the sum of this series isn't obvious to you, imagine this: Suppose you have a kilometer–long walk to the store. You walk 0.5 kilometers, and then 0.25 kilometers, and then 0.125 kilometers, and so on. You will never exceed one kilometer (although you'll get very close to it).

Therefore, inserting N elements takes `O(N)` work total. Each insertion is `O(1)` on average, even though some insertions take `O(N)` time in the worst case.

▸ StringBuilder

Imagine you were concatenating a list of strings, as shown below. What would the running time of this code be? For simplicity, assume that the strings are all the same length (call this x) and that there are n strings.

```
1  String joinWords(String[] words) {
2    String sentence = "";
3    for (String w : words) {
4      sentence = sentence + w;
5    }
6    return sentence;
7  }
```

On each concatenation, a new copy of the string is created, and the two strings are copied over, character by character. The first iteration requires us to copy x characters. The second iteration requires copying 2x characters. The third iteration requires 3x, and so on. The total time therefore is $O(x + 2x + ... + nx)$. This reduces to $O(xn^2)$.

> Why is it $O(xn^2)$? Because $1 + 2 + ... + n$ equals $n(n+1)/2$, or $O(n^2)$.

`StringBuilder` can help you avoid this problem. `StringBuilder` simply creates a resizable array of all the strings, copying them back to a string only when necessary.

```
1  String joinWords(String[] words) {
2    StringBuilder sentence = new StringBuilder();
3    for (String w : words) {
4      sentence.append(w);
5    }
6    return sentence.toString();
7  }
```

A good exercise to practice strings, arrays, and general data structures is to implement your own version of `StringBuilder`, `HashTable` and `ArrayList`.

Additional Reading: Hash Table Collision Resolution (pg 636), Rabin-Karp Substring Search (pg 636).

Interview Questions

1.1 **Is Unique:** Implement an algorithm to determine if a string has all unique characters. What if you cannot use additional data structures?

Hints: #44, #117, #132

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1.2 **Check Permutation:** Given two strings, write a method to decide if one is a permutation of the other.

Hints: #1, #84, #122, #131

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1.3 **URLify:** Write a method to replace all spaces in a string with '%20'. You may assume that the string has sufficient space at the end to hold the additional characters, and that you are given the "true" length of the string. (Note: If implementing in Java, please use a character array so that you can perform this operation in place.)

EXAMPLE

Input: `"Mr John Smith    ", 13`

Output: `"Mr%20John%20Smith"`

Hints: #53, #118

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1.4 **Palindrome Permutation:** Given a string, write a function to check if it is a permutation of a palindrome. A palindrome is a word or phrase that is the same forwards and backwards. A permutation is a rearrangement of letters. The palindrome does not need to be limited to just dictionary words.

EXAMPLE

Input: `Tact Coa`

Output: `True (permutations: "taco cat", "atco cta", etc.)`

Hints: #106, #121, #134, #136

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1.5 **One Away:** There are three types of edits that can be performed on strings: insert a character, remove a character, or replace a character. Given two strings, write a function to check if they are one edit (or zero edits) away.

EXAMPLE

```
pale,  ple  -> true
pales, pale -> true
pale,  bale -> true
pale,  bake -> false
```

Hints: #23, #97, #130

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1.6 **String Compression:** Implement a method to perform basic string compression using the counts of repeated characters. For example, the string `aabcccccaaa` would become `a2b1c5a3`. If the "compressed" string would not become smaller than the original string, your method should return the original string. You can assume the string has only uppercase and lowercase letters (a - z).

Hints: #92, #110

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1.7 **Rotate Matrix:** Given an image represented by an `NxN` matrix, where each pixel in the image is 4 bytes, write a method to rotate the image by 90 degrees. Can you do this in place?

Hints: #51, #100

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1.8 **Zero Matrix:** Write an algorithm such that if an element in an `MxN` matrix is 0, its entire row and column are set to 0.

Hints: #17, #74, #102

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1.9 **String Rotation:** Assume you have a method `isSubstring` which checks if one word is a substring of another. Given two strings, `s1` and `s2`, write code to check if `s2` is a rotation of `s1` using only one call to `isSubstring` (e.g., "`waterbottle`" is a rotation of "`erbottlewat`").

Hints: #34, #88, #104

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Additional Questions: Object-Oriented Design (#7.12), Recursion (#8.3), Sorting and Searching (#10.9), C++ (#12.11), Moderate Problems (#16.8, #16.17, #16.22), Hard Problems (#17.4, #17.7, #17.13, #17.22, #17.26).

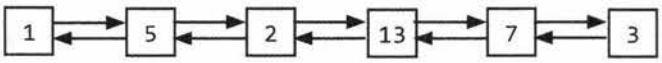
Hints start on page 653.

2

Linked Lists

A linked list is a data structure that represents a sequence of nodes. In a singly linked list, each node points to the next node in the linked list. A doubly linked list gives each node pointers to both the next node and the previous node.

The following diagram depicts a doubly linked list:



Unlike an array, a linked list does not provide constant time access to a particular "index" within the list. This means that if you'd like to find the Kth element in the list, you will need to iterate through K elements.

The benefit of a linked list is that you can add and remove items from the beginning of the list in constant time. For specific applications, this can be useful.

▸ Creating a Linked List

The code below implements a very basic singly linked list.

```
1   class Node {
2     Node next = null;
3     int data;
4
5     public Node(int d) {
6       data = d;
7     }
8
9     void appendToTail(int d) {
10      Node end = new Node(d);
11      Node n = this;
12      while (n.next != null) {
13        n = n.next;
14      }
15      n.next = end;
16    }
17  }
```

In this implementation, we don't have a `LinkedList` data structure. We access the linked list through a reference to the head `Node` of the linked list. When you implement the linked list this way, you need to be a bit careful. What if multiple objects need a reference to the linked list, and then the head of the linked list changes? Some objects might still be pointing to the old head.

We could, if we chose, implement a `LinkedList` class that wraps the `Node` class. This would essentially just have a single member variable: the head `Node`. This would largely resolve the earlier issue.

Remember that when you're discussing a linked list in an interview, you must understand whether it is a singly linked list or a doubly linked list.

▸ Deleting a Node from a Singly Linked List

Deleting a node from a linked list is fairly straightforward. Given a node `n`, we find the previous node `prev` and set `prev.next` equal to `n.next`. If the list is doubly linked, we must also update `n.next` to set `n.next.prev` equal to `n.prev`. The important things to remember are (1) to check for the null pointer and (2) to update the head or tail pointer as necessary.

Additionally, if you implement this code in C, C++ or another language that requires the developer to do memory management, you should consider if the removed node should be deallocated.

```
1  Node deleteNode(Node head, int d) {
2    Node n = head;
3
4    if (n.data == d) {
5      return head.next; /* moved head */
6    }
7
8    while (n.next != null) {
9      if (n.next.data == d) {
10       n.next = n.next.next;
11       return head; /* head didn't change */
12     }
13     n = n.next;
14   }
15   return head;
16 }
```

▸ The "Runner" Technique

The "runner" (or second pointer) technique is used in many linked list problems. The runner technique means that you iterate through the linked list with two pointers simultaneously, with one ahead of the other. The "fast" node might be ahead by a fixed amount, or it might be hopping multiple nodes for each one node that the "slow" node iterates through.

For example, suppose you had a linked list $a_1->a_2->...->a_n->b_1->b_2->...->b_n$ and you wanted to rearrange it into $a_1->b_1->a_2->b_2->...->a_n->b_n$. You do not know the length of the linked list (but you do know that the length is an even number).

You could have one pointer `p1` (the fast pointer) move every two elements for every one move that `p2` makes. When `p1` hits the end of the linked list, `p2` will be at the midpoint. Then, move `p1` back to the front and begin "weaving" the elements. On each iteration, `p2` selects an element and inserts it after `p1`.

▸ Recursive Problems

A number of linked list problems rely on recursion. If you're having trouble solving a linked list problem, you should explore if a recursive approach will work. We won't go into depth on recursion here, since a later chapter is devoted to it.

However, you should remember that recursive algorithms take at least O(n) space, where n is the depth of the recursive call. All recursive algorithms *can* be implemented iteratively, although they may be much more complex.

Interview Questions

2.1 **Remove Dups:** Write code to remove duplicates from an unsorted linked list.

FOLLOW UP

How would you solve this problem if a temporary buffer is not allowed?

Hints: #9, #40

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2.2 **Return Kth to Last:** Implement an algorithm to find the kth to last element of a singly linked list.

Hints: #8, #25, #41, #67, #126

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2.3 **Delete Middle Node:** Implement an algorithm to delete a node in the middle (i.e., any node but the first and last node, not necessarily the exact middle) of a singly linked list, given only access to that node.

EXAMPLE

Input: the node c from the linked list a->b->c->d->e->f

Result: nothing is returned, but the new linked list looks like a->b->d->e->f

Hints: #72

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2.4 **Partition:** Write code to partition a linked list around a value x, such that all nodes less than x come before all nodes greater than or equal to x. If x is contained within the list, the values of x only need to be after the elements less than x (see below). The partition element x can appear anywhere in the "right partition"; it does not need to appear between the left and right partitions.

EXAMPLE

Input: 3 -> 5 -> 8 -> 5 -> 10 -> 2 -> 1 [partition = 5]

Output: 3 -> 1 -> 2 -> 10 -> 5 -> 5 -> 8

Hints: #3, #24

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2.5 **Sum Lists:** You have two numbers represented by a linked list, where each node contains a single digit. The digits are stored in *reverse* order, such that the 1's digit is at the head of the list. Write a function that adds the two numbers and returns the sum as a linked list.

EXAMPLE

Input: (7-> 1 -> 6) + (5 -> 9 -> 2). That is, 617 + 295.

Output: 2 -> 1 -> 9. That is, 912.

FOLLOW UP

Suppose the digits are stored in forward order. Repeat the above problem.

EXAMPLE

Input: (6 -> 1 -> 7) + (2 -> 9 -> 5). That is, 617 + 295.

Output: 9 -> 1 -> 2. That is, 912.

Hints: #7, #30, #71, #95, #109

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2.6 **Palindrome:** Implement a function to check if a linked list is a palindrome.

Hints: #5, #13, #29, #61, #101

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2.7 **Intersection**: Given two (singly) linked lists, determine if the two lists intersect. Return the intersecting node. Note that the intersection is defined based on reference, not value. That is, if the kth node of the first linked list is the exact same node (by reference) as the jth node of the second linked list, then they are intersecting.

Hints: #20, #45, #55, #65, #76, #93, #111, #120, #129

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2.8 **Loop Detection:** Given a circular linked list, implement an algorithm that returns the node at the beginning of the loop.

DEFINITION

Circular linked list: A (corrupt) linked list in which a node's next pointer points to an earlier node, so as to make a loop in the linked list.

EXAMPLE

Input: A -> B -> C -> D -> E -> C [the same C as earlier]

Output: C

Hints: #50, #69, #83, #90

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Additional Questions: Trees and Graphs (#4.3), Object-Oriented Design (#7.12), System Design and Scalability (#9.5), Moderate Problems (#16.25), Hard Problems (#17.12).

Hints start on page 653.

3

Stacks and Queues

Questions on stacks and queues will be much easier to handle if you are comfortable with the ins and outs of the data structure. The problems can be quite tricky, though. While some problems may be slight modifications on the original data structure, others have much more complex challenges.

▸ Implementing a Stack

The stack data structure is precisely what it sounds like: a stack of data. In certain types of problems, it can be favorable to store data in a stack rather than in an array.

A stack uses LIFO (last-in first-out) ordering. That is, as in a stack of dinner plates, the most recent item added to the stack is the first item to be removed.

It uses the following operations:

- `pop()`: Remove the top item from the stack.
- `push(item)`: Add an item to the top of the stack.
- `peek()`: Return the top of the stack.
- `isEmpty()`: Return true if and only if the stack is empty.

Unlike an array, a stack does not offer constant-time access to the `i`th item. However, it does allow constant-time adds and removes, as it doesn't require shifting elements around.

We have provided simple sample code to implement a stack. Note that a stack can also be implemented using a linked list, if items were added and removed from the same side.

```
1  public class MyStack<T> {
2    private static class StackNode<T> {
3      private T data;
4      private StackNode<T> next;
5
6      public StackNode(T data) {
7        this.data = data;
8      }
9    }
10
11   private StackNode<T> top;
12
13   public T pop() {
14     if (top == null) throw new EmptyStackException();
15     T item = top.data;
```

```
16        top = top.next;
17        return item;
18     }
19
20     public void push(T item) {
21        StackNode<T> t = new StackNode<T>(item);
22        t.next = top;
23        top = t;
24     }
25
26     public T peek() {
27        if (top == null) throw new EmptyStackException();
28        return top.data;
29     }
30
31     public boolean isEmpty() {
32        return top == null;
33     }
34  }
```

One case where stacks are often useful is in certain recursive algorithms. Sometimes you need to push temporary data onto a stack as you recurse, but then remove them as you backtrack (for example, because the recursive check failed). A stack offers an intuitive way to do this.

A stack can also be used to implement a recursive algorithm iteratively. (This is a good exercise! Take a simple recursive algorithm and implement it iteratively.)

▸ Implementing a Queue

A queue implements FIFO (first-in first-out) ordering. As in a line or queue at a ticket stand, items are removed from the data structure in the same order that they are added.

It uses the operations:

- `add(item)`: Add an item to the end of the list.
- `remove()`: Remove the first item in the list.
- `peek()`: Return the top of the queue.
- `isEmpty()`: Return true if and only if the queue is empty.

A queue can also be implemented with a linked list. In fact, they are essentially the same thing, as long as items are added and removed from opposite sides.

```
1  public class MyQueue<T> {
2     private static class QueueNode<T> {
3        private T data;
4        private QueueNode<T> next;
5
6        public QueueNode(T data) {
7           this.data = data;
8        }
9     }
10
11    private QueueNode<T> first;
12    private QueueNode<T> last;
13
14    public void add(T item) {
```

```
15      QueueNode<T> t = new QueueNode<T>(item);
16      if (last != null) {
17         last.next = t;
18      }
19      last = t;
20      if (first == null) {
21         first = last;
22      }
23   }
24
25   public T remove() {
26      if (first == null) throw new NoSuchElementException();
27      T data = first.data;
28      first = first.next;
29      if (first == null) {
30         last = null;
31      }
32      return data;
33   }
34
35   public T peek() {
36      if (first == null) throw new NoSuchElementException();
37      return first.data;
38   }
39
40   public boolean isEmpty() {
41      return first == null;
42   }
43 }
```

It is especially easy to mess up the updating of the first and last nodes in a queue. Be sure to double check this.

One place where queues are often used is in breadth-first search or in implementing a cache.

In breadth-first search, for example, we used a queue to store a list of the nodes that we need to process. Each time we process a node, we add its adjacent nodes to the back of the queue. This allows us to process nodes in the order in which they are viewed.

Interview Questions

3.1 **Three in One:** Describe how you could use a single array to implement three stacks.

Hints: #2, #12, #38, #58

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3.2 **Stack Min:** How would you design a stack which, in addition to `push` and `pop`, has a function `min` which returns the minimum element? `Push`, `pop` and `min` should all operate in `O(1)` time.

Hints: #27, #59, #78

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3.3 **Stack of Plates:** Imagine a (literal) stack of plates. If the stack gets too high, it might topple. Therefore, in real life, we would likely start a new stack when the previous stack exceeds some threshold. Implement a data structure `SetOfStacks` that mimics this. `SetOfStacks` should be composed of several stacks and should create a new stack once the previous one exceeds capacity. `SetOfStacks.push()` and `SetOfStacks.pop()` should behave identically to a single stack (that is, `pop()` should return the same values as it would if there were just a single stack).

FOLLOW UP

Implement a function `popAt(int index)` which performs a pop operation on a specific sub-stack.

Hints: #64, #81

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3.4 **Queue via Stacks:** Implement a `MyQueue` class which implements a queue using two stacks.

Hints: #98, #114

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3.5 **Sort Stack:** Write a program to sort a stack such that the smallest items are on the top. You can use an additional temporary stack, but you may not copy the elements into any other data structure (such as an array). The stack supports the following operations: `push`, `pop`, `peek`, and `isEmpty`.

Hints: #15, #32, #43

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3.6 **Animal Shelter:** An animal shelter, which holds only dogs and cats, operates on a strictly "first in, first out" basis. People must adopt either the "oldest" (based on arrival time) of all animals at the shelter, or they can select whether they would prefer a dog or a cat (and will receive the oldest animal of that type). They cannot select which specific animal they would like. Create the data structures to maintain this system and implement operations such as `enqueue`, `dequeueAny`, `dequeueDog`, and `dequeueCat`. You may use the built-in `LinkedList` data structure.

Hints: #22, #56, #63

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Additional Questions: Linked Lists (#2.6), Moderate Problems (#16.26), Hard Problems (#17.9).

Hints start on page 653.

4

Trees and Graphs

Many interviewees find tree and graph problems to be some of the trickiest. Searching a tree is more complicated than searching in a linearly organized data structure such as an array or linked list. Additionally, the worst case and average case time may vary wildly, and we must evaluate both aspects of any algorithm. Fluency in implementing a tree or graph from scratch will prove essential.

Because most people are more familiar with trees than graphs (and they're a bit simpler), we'll discuss trees first. This is a bit out of order though, as a tree is actually a type of graph.

> Note: Some of the terms in this chapter can vary slightly across different textbooks and other sources. If you're used to a different definition, that's fine. Make sure to clear up any ambiguity with your interviewer.

Types of Trees

A nice way to understand a tree is with a recursive explanation. A tree is a data structure composed of nodes.

- Each tree has a root node. (Actually, this isn't strictly necessary in graph theory, but it's usually how we use trees in programming, and especially programming interviews.)
- The root node has zero or more child nodes.
- Each child node has zero or more child nodes, and so on.

The tree cannot contain cycles. The nodes may or may not be in a particular order, they could have any data type as values, and they may or may not have links back to their parent nodes.

A very simple class definition for `Node` is:

```
1  class Node {
2    public String name;
3    public Node[] children;
4  }
```

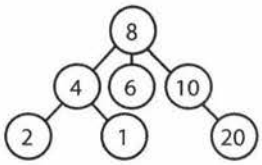
You might also have a `Tree` class to wrap this node. For the purposes of interview questions, we typically do not use a `Tree` class. You can if you feel it makes your code simpler or better, but it rarely does.

```
1  class Tree {
2    public Node root;
3  }
```

Tree and graph questions are rife with ambiguous details and incorrect assumptions. Be sure to watch out for the following issues and seek clarification when necessary.

Trees vs. Binary Trees

A binary tree is a tree in which each node has up to two children. Not all trees are binary trees. For example, this tree is not a binary tree. You could call it a ternary tree.



There are occasions when you might have a tree that is not a binary tree. For example, suppose you were using a tree to represent a bunch of phone numbers. In this case, you might use a 10-ary tree, with each node having up to 10 children (one for each digit).

A node is called a "leaf" node if it has no children.

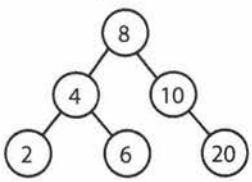
Binary Tree vs. Binary Search Tree

A binary search tree is a binary tree in which every node fits a specific ordering property: `all left descendents <= n < all right descendents`. This must be true for each node `n`.

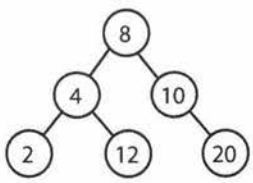
> The definition of a binary search tree can vary slightly with respect to equality. Under some definitions, the tree cannot have duplicate values. In others, the duplicate values will be on the right or can be on either side. All are valid definitions, but you should clarify this with your interviewer.

Note that this inequality must be true for all of a node's descendents, not just its immediate children. The following tree on the left below is a binary search tree. The tree on the right is not, since 12 is to the left of 8.

A binary search tree.



Not a binary search tree.



When given a tree question, many candidates assume the interviewer means a binary *search* tree. Be sure to ask. A binary search tree imposes the condition that, for each node, its left descendents are less than or equal to the current node, which is less than the right descendents.

Balanced vs. Unbalanced

While many trees are balanced, not all are. Ask your interviewer for clarification here. Note that balancing a tree does not mean the left and right subtrees are exactly the same size (like you see under "perfect binary trees" in the following diagram).

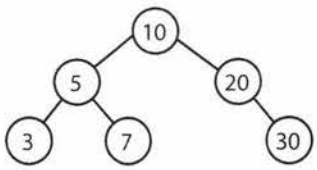
One way to think about it is that a "balanced" tree really means something more like "not terribly imbalanced." It's balanced enough to ensure `O(log n)` times for `insert` and `find`, but it's not necessarily as balanced as it could be.

Two common types of balanced trees are red-black trees (pg 639) and AVL trees (pg 637). These are discussed in more detail in the Advanced Topics section.

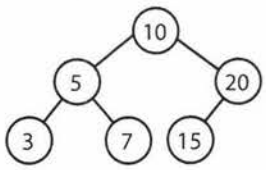
Complete Binary Trees

A complete binary tree is a binary tree in which every level of the tree is fully filled, except for perhaps the last level. To the extent that the last level is filled, it is filled left to right.

not a complete binary tree



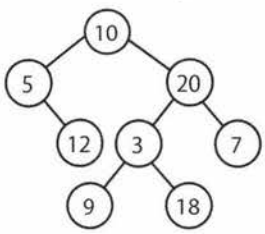
a complete binary tree



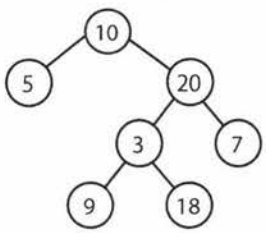
Full Binary Trees

A full binary tree is a binary tree in which every node has either zero or two children. That is, no nodes have only one child.

not a full binary tree

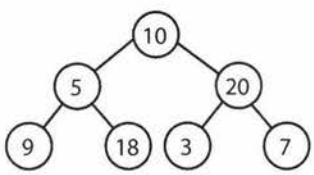


a full binary tree



Perfect Binary Trees

A perfect binary tree is one that is both full and complete. All leaf nodes will be at the same level, and this level has the maximum number of nodes.



Note that perfect trees are rare in interviews and in real life, as a perfect tree must have exactly $2^k - 1$ nodes (where k is the number of levels). In an interview, do not assume a binary tree is perfect.

▸ Binary Tree Traversal

Prior to your interview, you should be comfortable implementing in-order, post-order, and pre-order traversal. The most common of these is in-order traversal.

In-Order Traversal

In-order traversal means to "visit" (often, print) the left branch, then the current node, and finally, the right branch.

```
1  void inOrderTraversal(TreeNode node) {
2    if (node != null) {
3      inOrderTraversal(node.left);
4      visit(node);
5      inOrderTraversal(node.right);
6    }
7  }
```

When performed on a binary search tree, it visits the nodes in ascending order (hence the name "in-order").

Pre-Order Traversal

Pre-order traversal visits the current node before its child nodes (hence the name "pre-order").

```
1  void preOrderTraversal(TreeNode node) {
2    if (node != null) {
3      visit(node);
4      preOrderTraversal(node.left);
5      preOrderTraversal(node.right);
6    }
7  }
```

In a pre-order traversal, the root is always the first node visited.

Post-Order Traversal

Post-order traversal visits the current node after its child nodes (hence the name "post-order").

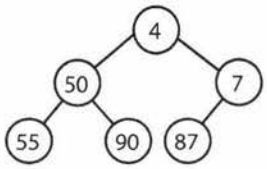
```
1  void postOrderTraversal(TreeNode node) {
2    if (node != null) {
3      postOrderTraversal(node.left);
4      postOrderTraversal(node.right);
5      visit(node);
6    }
7  }
```

In a post-order traversal, the root is always the last node visited.

▸ Binary Heaps (Min-Heaps and Max-Heaps)

We'll just discuss min-heaps here. Max-heaps are essentially equivalent, but the elements are in descending order rather than ascending order.

A min-heap is a *complete* binary tree (that is, totally filled other than the rightmost elements on the last level) where each node is smaller than its children. The root, therefore, is the minimum element in the tree.

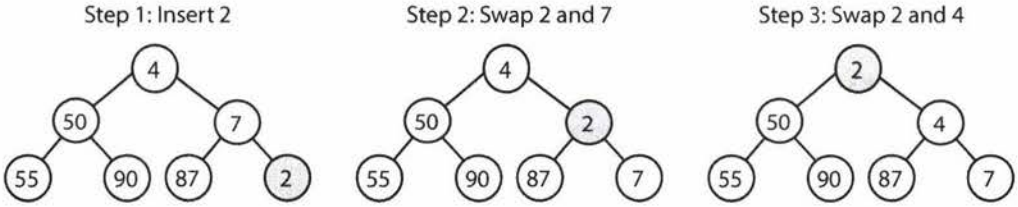


We have two key operations on a min-heap: `insert` and `extract_min`.

Insert

When we insert into a min-heap, we always start by inserting the element at the bottom. We insert at the rightmost spot so as to maintain the complete tree property.

Then, we "fix" the tree by swapping the new element with its parent, until we find an appropriate spot for the element. We essentially bubble up the minimum element.



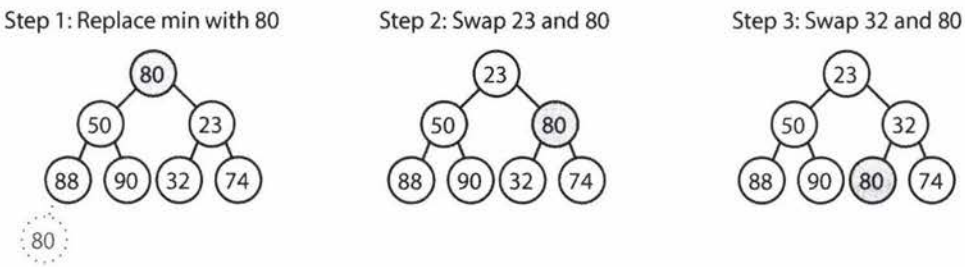
This takes `O(log n)` time, where n is the number of nodes in the heap.

Extract Minimum Element

Finding the minimum element of a min-heap is easy: it's always at the top. The trickier part is how to remove it. (In fact, this isn't that tricky.)

First, we remove the minimum element and swap it with the last element in the heap (the bottommost, rightmost element). Then, we bubble down this element, swapping it with one of its children until the min-heap property is restored.

Do we swap it with the left child or the right child? That depends on their values. There's no inherent ordering between the left and right element, but you'll need to take the smaller one in order to maintain the min-heap ordering.



This algorithm will also take `O(log n)` time.

▸ Tries (Prefix Trees)

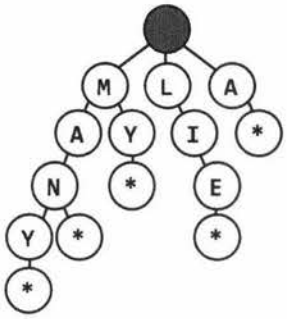
A trie (sometimes called a prefix tree) is a funny data structure. It comes up a lot in interview questions, but algorithm textbooks don't spend much time on this data structure.

A trie is a variant of an n-ary tree in which characters are stored at each node. Each path down the tree may represent a word.

The * nodes (sometimes called "null nodes") are often used to indicate complete words. For example, the fact that there is a * node under MANY indicates that MANY is a complete word. The existence of the MA path indicates there are words that start with MA.

The actual implementation of these * nodes might be a special type of child (such as a `TerminatingTrieNode`, which inherits from `TrieNode`). Or, we could use just a boolean flag `terminates` within the "parent" node.

A node in a trie could have anywhere from 1 through `ALPHABET_SIZE + 1` children (or, 0 through `ALPHABET_SIZE` if a boolean flag is used instead of a * node).



Very commonly, a trie is used to store the entire (English) language for quick prefix lookups. While a hash table can quickly look up whether a string is a valid word, it cannot tell us if a string is a prefix of any valid words. A trie can do this very quickly.

> How quickly? A trie can check if a string is a valid prefix in `O(K)` time, where K is the length of the string. This is actually the same runtime as a hash table will take. Although we often refer to hash table lookups as being `O(1)` time, this isn't entirely true. A hash table must read through all the characters in the input, which takes `O(K)` time in the case of a word lookup.

Many problems involving lists of valid words leverage a trie as an optimization. In situations when we search through the tree on related prefixes repeatedly (e.g., looking up M, then MA, then MAN, then MANY), we might pass around a reference to the current node in the tree. This will allow us to just check if Y is a child of MAN, rather than starting from the root each time.

▸ Graphs

A tree is actually a type of graph, but not all graphs are trees. Simply put, a tree is a connected graph without cycles.

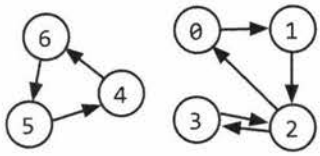
A graph is simply a collection of nodes with edges between (some of) them.

- Graphs can be either directed (like the following graph) or undirected. While directed edges are like a

one-way street, undirected edges are like a two-way street.

- The graph might consist of multiple isolated subgraphs. If there is a path between every pair of vertices, it is called a "connected graph."
- The graph can also have cycles (or not). An "acyclic graph" is one without cycles.

Visually, you could draw a graph like this:



In terms of programming, there are two common ways to represent a graph.

Adjacency List

This is the most common way to represent a graph. Every vertex (or node) stores a list of adjacent vertices. In an undirected graph, an edge like `(a, b)` would be stored twice: once in a's adjacent vertices and once in b's adjacent vertices.

A simple class definition for a graph node could look essentially the same as a tree node.

```
1  class Graph {
2    public Node[] nodes;
3  }
4
5  class Node {
6    public String name;
7    public Node[] children;
8  }
```

The Graph class is used because, unlike in a tree, you can't necessarily reach all the nodes from a single node.

You don't necessarily need any additional classes to represent a graph. An array (or a hash table) of lists (arrays, arraylists, linked lists, etc.) can store the adjacency list. The graph above could be represented as:

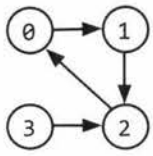
```
0: 1
1: 2
2: 0, 3
3: 2
4: 6
5: 4
6: 5
```

This is a bit more compact, but it isn't quite as clean. We tend to use node classes unless there's a compelling reason not to.

Adjacency Matrices

An adjacency matrix is an NxN boolean matrix (where N is the number of nodes), where a `true` value at `matrix[i][j]` indicates an edge from node `i` to node `j`. (You can also use an integer matrix with 0s and 1s.)

In an undirected graph, an adjacency matrix will be symmetric. In a directed graph, it will not (necessarily) be.



	0	1	2	3
0	0	1	0	0
1	0	0	1	0
2	1	0	0	0
3	0	0	1	0

The same graph algorithms that are used on adjacency lists (breadth-first search, etc.) can be performed with adjacency matrices, but they may be somewhat less efficient. In the adjacency list representation, you can easily iterate through the neighbors of a node. In the adjacency matrix representation, you will need to iterate through all the nodes to identify a node's neighbors.

Graph Search

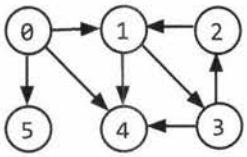
The two most common ways to search a graph are depth-first search and breadth-first search.

In depth-first search (DFS), we start at the root (or another arbitrarily selected node) and explore each branch completely before moving on to the next branch. That is, we go deep first (hence the name *depth*-first search) before we go wide.

In breadth-first search (BFS), we start at the root (or another arbitrarily selected node) and explore each neighbor before going on to any of their children. That is, we go wide (hence *breadth*-first search) before we go deep.

See the below depiction of a graph and its depth-first and breadth-first search (assuming neighbors are iterated in numerical order).

Graph



Depth-First Search

```
1  Node 0
2    Node 1
3      Node 3
4        Node 2
5        Node 4
6  Node 5
```

Breadth-First Search

```
1  Node 0
2  Node 1
3  Node 4
4  Node 5
5  Node 3
6  Node 2
```

Breadth-first search and depth-first search tend to be used in different scenarios. DFS is often preferred if we want to visit every node in the graph. Both will work just fine, but depth-first search is a bit simpler.

However, if we want to find the shortest path (or just any path) between two nodes, BFS is generally better. Consider representing all the friendships in the entire world in a graph and trying to find a path of friendships between Ash and Vanessa.

In depth-first search, we could take a path like `Ash -> Brian -> Carleton -> Davis -> Eric -> Farah -> Gayle -> Harry -> Isabella -> John -> Kari`... and then find ourselves very far away. We could go through most of the world without realizing that, in fact, Vanessa is Ash's friend. We will still eventually find the path, but it may take a long time. It also won't find us the shortest path.

In breadth-first search, we would stay close to Ash for as long as possible. We might iterate through many of Ash's friends, but we wouldn't go to his more distant connections until absolutely necessary. If Vanessa is Ash's friend, or his friend-of-a-friend, we'll find this out relatively quickly.

Depth-First Search (DFS)

In DFS, we visit a node a and then iterate through each of a's neighbors. When visiting a node b that is a neighbor of a, we visit all of b's neighbors before going on to a's other neighbors. That is, a exhaustively searches b's branch before any of its other neighbors.

Note that pre-order and other forms of tree traversal are a form of DFS. The key difference is that when implementing this algorithm for a graph, we must check if the node has been visited. If we don't, we risk getting stuck in an infinite loop.

The pseudocode below implements DFS.

```
1  void search(Node root) {
2    if (root == null) return;
3    visit(root);
4    root.visited = true;
5    for each (Node n in root.adjacent) {
6      if (n.visited == false) {
7        search(n);
8      }
9    }
10 }
```

Breadth-First Search (BFS)

BFS is a bit less intuitive, and many interviewees struggle with the implementation unless they are already familiar with it. The main tripping point is the (false) assumption that BFS is recursive. It's not. Instead, it uses a queue.

In BFS, node a visits each of a's neighbors before visiting any of *their* neighbors. You can think of this as searching level by level out from a. An iterative solution involving a queue usually works best.

```
1  void search(Node root) {
2    Queue queue = new Queue();
3    root.marked = true;
4    queue.enqueue(root); // Add to the end of queue
5
6    while (!queue.isEmpty()) {
7      Node r = queue.dequeue(); // Remove from the front of the queue
8      visit(r);
9      foreach (Node n in r.adjacent) {
10       if (n.marked == false) {
11         n.marked = true;
12         queue.enqueue(n);
13       }
14     }
15   }
16 }
```

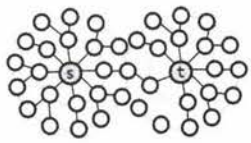
If you are asked to implement BFS, the key thing to remember is the use of the queue. The rest of the algorithm flows from this fact.

Bidirectional Search

Bidirectional search is used to find the shortest path between a source and destination node. It operates by essentially running two simultaneous breadth-first searches, one from each node. When their searches collide, we have found a path.

Breadth-First Search
Single search from s to t that collides after four levels.

Bidirectional Search
Two searches (one from s and one from t) that collide after four levels total (two levels each).



To see why this is faster, consider a graph where every node has at most k adjacent nodes and the shortest path from node s to node t has length d.

- In traditional breadth-first search, we would search up to k nodes in the first "level" of the search. In the second level, we would search up to k nodes for each of those first k nodes, so k^2 nodes total (thus far). We would do this d times, so that's $O(k^d)$ nodes.
- In bidirectional search, we have two searches that collide after approximately $^d/_2$ levels (the midpoint of the path). The search from s visits approximately $k^{d/2}$, as does the search from t. That's approximately $2\,k^{d/2}$, or $O(k^{d/2})$, nodes total.

This might seem like a minor difference, but it's not. It's huge. Recall that $(k^{d/2})*(k^{d/2}) = k^d$. The bidirectional search is actually faster by a factor of $k^{d/2}$.

Put another way: if our system could only support searching "friend of friend" paths in breadth-first search, it could now likely support "friend of friend of friend of friend" paths. We can support paths that are twice as long.

Additional Reading: Topological Sort (pg 632), Dijkstra's Algorithm (pg 633), AVL Trees (pg 637), Red-Black Trees (pg 639).

Interview Questions

4.1 **Route Between Nodes:** Given a directed graph, design an algorithm to find out whether there is a route between two nodes.

Hints: #127

pg 241

4.2 **Minimal Tree:** Given a sorted (increasing order) array with unique integer elements, write an algorithm to create a binary search tree with minimal height.

Hints: #19, #73, #116

pg 242

4.3 **List of Depths:** Given a binary tree, design an algorithm which creates a linked list of all the nodes at each depth (e.g., if you have a tree with depth D, you'll have D linked lists).

Hints: #107, #123, #135

pg 243

4.4 **Check Balanced:** Implement a function to check if a binary tree is balanced. For the purposes of this question, a balanced tree is defined to be a tree such that the heights of the two subtrees of any node never differ by more than one.

Hints: #21, #33, #49, #105, #124

pg 244

4.5 **Validate BST:** Implement a function to check if a binary tree is a binary search tree.

Hints: #35, #57, #86, #113, #128

pg 245

4.6 **Successor:** Write an algorithm to find the "next" node (i.e., in-order successor) of a given node in a binary search tree. You may assume that each node has a link to its parent.

Hints: #79, #91

pg 248

4.7 **Build Order:** You are given a list of projects and a list of dependencies (which is a list of pairs of projects, where the second project is dependent on the first project). All of a project's dependencies must be built before the project is. Find a build order that will allow the projects to be built. If there is no valid build order, return an error.

EXAMPLE

Input:

```
projects: a, b, c, d, e, f
dependencies: (a, d), (f, b), (b, d), (f, a), (d, c)
```

Output: `f, e, a, b, d, c`

Hints: #26, #47, #60, #85, #125, #133

pg 250

4.8 **First Common Ancestor:** Design an algorithm and write code to find the first common ancestor of two nodes in a binary tree. Avoid storing additional nodes in a data structure. NOTE: This is not necessarily a binary search tree.

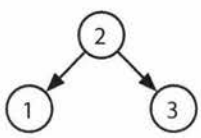
Hints: #10, #16, #28, #36, #46, #70, #80, #96

pg 257

4.9 **BST Sequences:** A binary search tree was created by traversing through an array from left to right and inserting each element. Given a binary search tree with distinct elements, print all possible arrays that could have led to this tree.

EXAMPLE

Input:



Output: `{2, 1, 3}, {2, 3, 1}`

Hints: #39, #48, #66, #82

pg 262

4.10 Check Subtree: T1 and T2 are two very large binary trees, with T1 much bigger than T2. Create an algorithm to determine if T2 is a subtree of T1.

A tree T2 is a subtree of T1 if there exists a node n in T1 such that the subtree of n is identical to T2. That is, if you cut off the tree at node n, the two trees would be identical.

Hints: #4, #11, #18, #31, #37

pg 265

4.11 Random Node: You are implementing a binary tree class from scratch which, in addition to `insert`, `find`, and `delete`, has a method `getRandomNode()` which returns a random node from the tree. All nodes should be equally likely to be chosen. Design and implement an algorithm for `getRandomNode`, and explain how you would implement the rest of the methods.

Hints: #42, #54, #62, #75, #89, #99, #112, #119

pg 268

4.12 Paths with Sum: You are given a binary tree in which each node contains an integer value (which might be positive or negative). Design an algorithm to count the number of paths that sum to a given value. The path does not need to start or end at the root or a leaf, but it must go downwards (traveling only from parent nodes to child nodes).

Hints: #6, #14, #52, #68, #77, #87, #94, #103, #108, #115

pg 272

Additional Questions: Recursion (#8.10), System Design and Scalability (#9.2, #9.3), Sorting and Searching (#10.10), Hard Problems (#17.7, #17.12, #17.13, #17.14, #17.17, #17.20, #17.22, #17.25).

Hints start on page 653.

5

Bit Manipulation

Bit manipulation is used in a variety of problems. Sometimes, the question explicitly calls for bit manipulation. Other times, it's simply a useful technique to optimize your code. You should be comfortable doing bit manipulation by hand, as well as with code. Be careful; it's easy to make little mistakes.

▸ Bit Manipulation By Hand

If you're rusty on bit manipulation, try the following exercises by hand. The items in the third column can be solved manually or with "tricks" (described below). For simplicity, assume that these are four-bit numbers.

If you get confused, work them through as a base 10 number. You can then apply the same process to a binary number. Remember that ^ indicates an XOR, and ~ is a NOT (negation).

`0110 + 0010`	`0011 * 0101`	`0110 + 0110`
`0011 + 0010`	`0011 * 0011`	`0100 * 0011`
`0110 - 0011`	`1101 >> 2`	`1101 ^ (~1101)`
`1000 - 0110`	`1101 ^ 0101`	`1011 & (~0 << 2)`

Solutions: line 1 (1000, 1111, 1100); line 2 (0101, 1001, 1100); line 3 (0011, 0011, 1111); line 4 (0010, 1000, 1000).

The tricks in Column 3 are as follows:

1. `0110 + 0110` is equivalent to `0110 * 2`, which is equivalent to shifting `0110` left by 1.
2. `0100` equals 4, and multiplying by 4 is just left shifting by 2. So we shift `0011` left by 2 to get `1100`.
3. Think about this operation bit by bit. If you XOR a bit with its own negated value, you will always get 1. Therefore, the solution to `a^(~a)` will be a sequence of 1s.
4. `~0` is a sequence of 1s, so `~0 << 2` is 1s followed by two 0s. ANDing that with another value will clear the last two bits of the value.

If you didn't see these tricks immediately, think about them logically.

▸ Bit Facts and Tricks

The following expressions are useful in bit manipulation. Don't just memorize them, though; think deeply about why each of these is true. We use "1s" and "0s" to indicate a sequence of 1s or 0s, respectively.

```
x ^ 0s = x      x & 0s = 0      x | 0s = x
x ^ 1s = ~x     x & 1s = x      x | 1s = 1s
 x ^ x = 0       x & x = x       x | x = x
```