Assignment 2: Naive Bayes and Text Classification

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1 Task 1: Simple Bayes (20 Points)

1.1 Task (10 Points)

Define:

- $P(\text{Box x}) :=$ probability that Box x is chosen)

- $P(\text{Apple}) :=$ probability that an apple is chosen

- $P(\text{Orange}) :=$ probability that an orange is chosen

Given: $P(\text{Box 1}) = P(\text{Box 2})$

	Apples	Oranges
Box 1	4	10
Box 2	6	8

We assume $P(\text{Box 1}) + P(\text{Box 2}) = 1$.
$\implies P(\text{Apple} \mid \text{Box 1}) = 4/14 = 2/7$, $P(\text{Orange} \mid \text{Box 1}) = 10/14 = 5/7$,
$P(\text{Apple} \mid \text{Box 2}) = 6/14 = 3/7$ and $P(\text{Orange} \mid \text{Box 2}) = 8/14 = 4/7$.
$P(\text{Box 1}) = P(\text{Box 2}) = 1/2$.

What is the probability of choosing an apple?

$$
\begin{aligned}
P(\text{Apple}) &= P(\text{Apple, Box 1}) + P(\text{Apple, Box 2}) \\
&= P(\text{Box 1}) \cdot P(\text{Apple} \mid \text{Box 1}) + P(\text{Box 2}) \cdot P(\text{Apple} \mid \text{Box 2}) \\
&= \frac{1}{2} \cdot \frac{2}{7} + \frac{1}{2} \cdot \frac{3}{7} \\
&= \frac{5}{14}
\end{aligned}
$$

If an apple is chosen, what is the probability that it came from box 1?

$$
\begin{aligned}
P(\text{Box 1} \mid \text{Apple}) \quad &= \quad \frac{P(\text{Box 1, Apple})}{P(\text{Apple})} \\
&= \quad \frac{P(\text{Apple, Box 1})}{P(\text{Apple})} \\
&= \quad \frac{\frac{1}{2} \cdot \frac{2}{7}}{\frac{5}{14}} \\
&= \quad \frac{1 \cdot 2 \cdot 14}{2 \cdot 7 \cdot 5} \\
&= \quad \frac{28}{70} \\
&= \quad \frac{2}{5}
\end{aligned}
$$

1.2   Task (10 Points)

Given: Given are two M&M bags from 1994 and 1996 and the probabilities of finding a specific colour in the two different bags. The probabilities are as follows:

	Yellow	Green	Other
1994	0.3	0.2	0.5
1996	0.16	0.24	0.6

Now scenario $A :=$"one M&M of each bag is taken out, one is green and the other is yellow." occurs.
What is the probability of scenario $B :=$ "the yellow M&M came from the 1994 bag"? So it's asked for the probability

$$
P(B \mid A) = \frac{P(B, A)}{P(A)}
$$

First take a look at $P(B, A)$, notice that A and B implies that not only is the 1994 M&M yellow, but the 1996 M&M is green.

$$
\begin{aligned}
P(B, A) \quad &= \quad P(\text{1994 Bag was picked}) * P(\text{yellow M\&M}) \\
&\quad + P(\text{1996 Bag was picked}) * P(\text{green M\&M}) \\
&= \quad 0.5 * 0.3 + 0.5 * 0.24 \\
&= \quad 0.27
\end{aligned}
$$

Notice that because of statement A we have to take one out of each bag, so we have to treat the probabilities for each bag to be 0.5. Now $P(A)$ can also be computed by summing up all the allowed combinations of M&M that fulfil scenario A.

A.1 1994 M&M is yellow $\implies$ 1996 M&M is green.

A.2 1996 M&M is green $\implies$ 1996 M&M is yellow.

This means there are only scenario $A.1$ and $A.2$ that fulfil A.

$$[A.1 \Leftrightarrow B \text{ and } A] \implies [P(A.1) = P(B, A) = 0.27]$$

Analogously $P(A.2)$ can be computed to be

$$
\begin{aligned}
P(A.2) &= 0.5 * 0.2 + 0.5 * 0.16 \\
&= 0.18
\end{aligned}
$$

resulting in

$$
\begin{aligned}
P(A) &= P(A.1) + P(A.2) \\
&= 0.27 + 0.18 \\
&= 0.45
\end{aligned}
$$

Therefore the answer to our previous question is

$$
\begin{aligned}
P(B \mid A) &= \frac{P(B, A)}{P(A)} \\
&= \frac{0.27}{0.45} \\
&= 0.6
\end{aligned}
$$

2 Task 2: Fake News Classification with Naive Bayes (20 Points)

This task was answered in the jupyter-notebook.

3 Task 3: kNN for Text Classification (30 Points)

3.1 How do you represent the text?

These are just some thoughts and can be deleted.
We could add one dimension for each term, which represents the frequency of this specific term. E.g.

- point $[0, 0]$ represents a text with 0 'hi's and 0 'bye's (empty text []).

- point $[2, 1]$ represents a text with 2 'hi's and 1 'bye' (['hi', 'hi', 'bye'], ['hi', 'bye', 'hi'], ['bye', 'hi', 'hi']).

We should use a min-max scaling on this data.

3.2 What distance function do you use?