Deep Q-Learning for Rubik's Cube

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Abstract

Deep Q-Learning is an algorithm which involves Q-Learning and Deep Learning. It has proved to be extremely succesfull for some applications [4]. The Rubik's Cube is a solved game as there exists an algorithm that solves any shuffled cube in the minimum number of moves, however we have applied a Deep Q-Learning algorithm to try to solve the Rubik's cube game. Because the reward is positive only when the cube is solved ans that the exploration grows exponentially, we have considered some tweaks for the algorithm such as curriculum learning. Mixed results have been obtain because of the slow convergence of the Q-learning algorithm.

1 Introduction

From a Reinforcement Learning (RL) perspective, the Rubik's Cube game is hard because whatever the action taken, the reward will always be zero, unless the cube is finished (same color on same face) which is a very improbable event if random moves are being taken from a suffled cube. Such situations are almost impossible to solve for classical Q-learning algorithms.

Here we consider a Deep Q-Learning algorithm that is a variant of the Q-Learning algorithm that uses a Feedforward Neural Network at the action-value function to be learned. Our hope is that the network learns patterns in the colors and the action in order to make possible the learning.

Moreover, we want to help the learning by showing easy examples first (cubes almost finished) and the increase the complexity of the examples until we reach the general problem of a completely randomly shuffled cube. Such an approach is knows as Curriculum Learning [1].

2 Deep Q-learning algorithm

In this section, we present the algorithm we have used and some theoretical considered aspects.

2.1 Q-Learning

Q-learning is a model-free reinforcement learning technique. It works by learning an action-value function that eventually gives the expected utility of taking a given action in a given state and following the optimal policy thereafter. In our case, the action-value function is a mapping between the combination of the state and the action taken, and the utility value, where the utility value is the sum of the present reward and the discounted future rewards. When the mapping is learned, the optimal policy is to simply select the action that has the highest value in each state.

2.2 Deep Learning

The main specificity about Deep Q-Learning is that the action-value function is a feedforward neural network. The input of the network is a vector representing the state and the action taken and the output is the utility value. Feedforward neural networks have several advantages. First, they can approximate arbitrarily well any continuous function thanks to the universal approximation theorem [2]. Moreover, the training on the parameters of feedforward neural networks has been made easy thanks to the backpropagation algorithm [5].

2.3 The Deep Q-Learning algorithm in [4]

2.3.1 Notations

For practical reasons, most of the notations used here are the same as in [4]. We note $\mathcal{D}$ the replay memory and N its capacity. The action-value mapping is noted Q. x_t denotes the state at time t, a_t the action taken at time t, r_t the observed reward and x_{t+1} the resulting state. The parametrization of the deep learning model (the feedforward neural network) is represented by θ.

2.3.2 A specificity : the Replay Memory

When interacting with the environment, the algorithm stores the quadruplet (x_t, a_t, r_t, x_{t+1}) in what is called a Replay Memory. Its purpose is to smooth the training over many past events and behaviors and not just what has just happened. This idea was first presented in [3]. At every step of the algorithm, when training in involved, a minibatch of quadruplet is taken from the replay memory. The gradient is computed on this minibatch. Thus, we see that every episode that is contained in the replay memory can be used many times for the training. Moreover, the correlations between close actions and states do not bias the algorithm anymore since the minibatch is taken from random elements of the replay memory ; our hope is that the variance of the updates will be lower with such a procedure. A small variance means that the algorithm avoid to fall in poor local minima and to oscillates between pseudo-stable states.

2.3.3 Algorithm

The algorithm presented here is the same as the one in [4]

```
 1: Initialize replay memory $\mathcal{D}$ to capacity $N$
 2: Initialize action-value function $Q$ with random weights
 3: for episode $= 1, M$ do
 4:     Initialise a shuffled cube $x_1$
 5:     for $t = 1, T$ do
 6:         With probability $\epsilon$ select a random action $a_t$
 7:         otherwise select $a_t = \max_a Q(x_t, a; \theta)$
 8:         Execute action $a_t$ in emulator and observe reward $r_t$ and new state $x_{t+1}$
 9:         Store transition $(x_t, a_t, r_t, x_{t+1})$ in the replay memory $\mathcal{D}$
10:         Sample random minibatch of transitions $(x_j, a_j, r_j, x_{j+1})$ from $\mathcal{D}$
11:         Set $y_j = \begin{cases} r_j & \text{for terminal } \phi_{j+1} \\ r_j + \gamma \max_{a'} Q(x_{j+1}, a'; \theta) & \text{for non-terminal } \phi_{j+1} \end{cases}$
12:         Perform a gradient descent step on $(y_j - Q(x_j, a_j; \theta))^2$ [1].
13:     end for
14: end for
```

algorithm 1: Deep Q-Learning algorithm

2.4 Curriculum Learning

The basic idea to start learning easy things and to move on harder things is the core of what is called Curriculum Learning. Formalised in [1], the article shows that some models can learn up to two times faster if the learning strategy follows some curriculum that show easy examples first, in comparison with a strategy that does not take care of the order of the examples.

In our case, the easy examples are cubes almost finished and hard examples are completely shuffled cubes. When we show harder and harder examples to the model, we keep showing easy ones so that the network does not forget them. The procedure of Curriculum learning goes as follow:

```
1: Initialize the network with random weights
2: for $c = 1, C$ do
3:     Train the network with intial cubes that are $1, 2, ..., c$ moves away from being finished.
4:     Save the parameters of the network
5: end for
```

algorithm 2: Curriculum Learning procedure

3 Experimentation

The code is written in *Python*. We have use the library *Theano* which allows automatic differenciation and symbolic optimization. We have also used the convenient library *Blocks* which is a Theano framework. One main feature about Theano is the fact that it can be run on Graphical Processing Units (GPU) in order to speed up the training. However, in our case we only used CPU computing because what takes the most time is not the computation of the gradient but rather the action that gives the highest utility value at a given state.

3.1 Practical considerations

3.1.1 Rubik's Cube

We have used the standard notations for the Rubik's cube. An action is a triplet of numbers (f, l, d). When such an action is performed, it moves the layer l parrallel to face l through d 90-degree turns in the clockwise direction. Layer 0 is the face itself, and higher l values are for layers deeper into the cube. Eventhough standard cubes have $N = 3$ small cubies on the edge of every face, our code works for any value of N.

3.1.2 Input data for the Q-function and feedforward neural network

The state of a cube is given by its stickers : 3D matrix of shape $(6, N, N)$ of color values (numbers from 0 to 5). We have used one-hot encoding for the color value so that every color is as far every other color and so the state of a cube come represented as a 4D matrix with shape $(6, N, N, 6)$ where the last dimension contains a 1 at the index of the color encoded and 0 everywhere else. We then flatten this 4D matrix to form a 1D vector and concatenate it with a flattened version of the action (described in 3.1.1). The final vector contains then all the information from the state of the cube (the stickers colors) and the action taken.

The vector described above is fed to a vanilla feedforward neural network. Our network has 1 hidden layer containing 50 units. The activation functions are Linear Rectifier ($f(x) = max(0, x)$).

3.1.3 Parameters of training and algorithm

For traning we have chosen a learning rate of 10^{-2}. The value of ϵ, the exploration parameter, is 0.2 (0.1 is more standard but, we have observed the in our case we need a lot of exploration in order to successfully train the model). The discount factor γ is equal to 0.9. We have set the maximum storage of the replay memory at 20000 and new examples replace randomly chosen old examples.

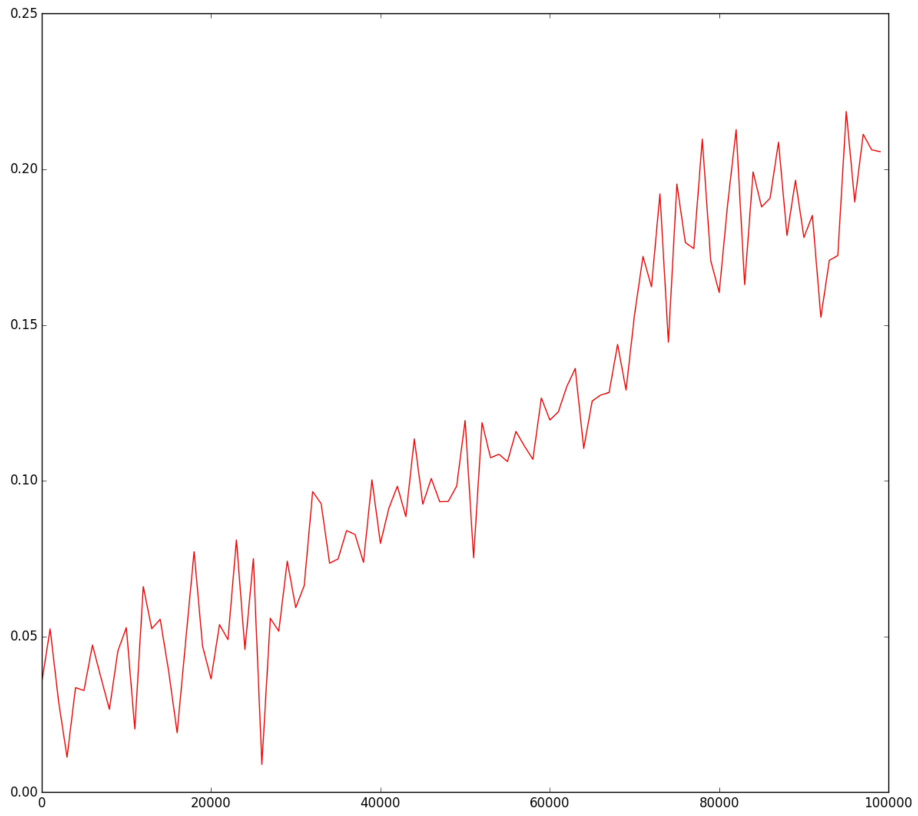
3.2 Experiments

The experiments follow obviously a coding phase of the environment and the algorithm.

3.2.1 Small Rubik's cube : $N = 2$

The first step of the curriculum learning procedure tells to start with cubes that are one move away from being finished. While we thought this would be an easy first step for the program, it turned out that this very first step is not easy at all for the computer. It takes 100000 iterations of the algorithm to go from 3% (initial full random) to about 20% of success as shown in the Figure 1.

Figure 1: Training of the algorithm on cubes with $N = 2$ that are 1 move away from being finished. X-axis represent the number of moves (and thus the number of updates of the parameters). Y-axis represent the moving average of success



4 Conclusion

This report presents the general Deep Q-learning algorithm and its adaptation for the Rubik's cube game. Despite the use of Curriculum Learning, the training of the model appears to be too slow ; it seems irealistic to have a Q-function that gives a good policy for the Rubik's cube with $N = 3$ or more.

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