

HOLIDAY HOME WORK (2021-22)

1 Is $f: [0, 2\pi] \rightarrow \mathbb{R}$, given by $f(x) = \cos x$, one-one? 1

2 If f is an invertible function defined as $f(x) = \frac{3x-4}{5}$, write $f^{-1}(x)$. 2

3. Show that the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \frac{x}{x^2+1}$, $\forall x \in \mathbb{R}$ is neither one-one nor onto. 4

4. The domain of the function $f: \mathbb{R} \rightarrow \mathbb{R}$ defined by $f(x) = \sqrt{4-x^2}$ is _____.

5. Given a function 'f' as $f(x) = 5x + 4$, $x \in \mathbb{R}$. If $g: \mathbb{R} \rightarrow \mathbb{R}$ is inverse of function 'f' then (a) g

5 (b) $g(x) = \frac{5}{4x-5}$ (c) $g(x) = \frac{x-4}{5}$ (d) $g(x) = 5x - 4$

6. Prove that $\sin^{-1}\left(\frac{4}{5}\right) + \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{16}{65}\right) = \frac{\pi}{2}$

7. Prove the following: $\cos\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right) = \frac{6}{5\sqrt{13}}$

8. Simplify: $\tan^{-1}\left[\frac{a \cos x - b \sin x}{b \cos x + a \sin x}\right]$, if $\frac{a}{b} \tan x > -1$.

9. Solve the following equation: $\tan^{-1}\left(\frac{x+1}{x-1}\right) + \tan^{-1}\left(\frac{x-1}{x}\right) = \tan^{-1}(-7)$.

10. Prove that $\cot^{-1}\left(\frac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}}\right) = \frac{x}{2}, x \in \left(0, \frac{\pi}{4}\right)$

11. A trust caring for handicapped children gets ₹30,000 every month from its donors. The trust uses the funds received for medical and educational care of the children and for that it charges some amount from them and deposits the balance amount in a private bank to get the money multiplied. In the future the trust goes on functioning regularly. What per cent of interest should the trust get from the bank to get a total of ₹1,800 every month? Use matrix method, to find the rate of interest. Do you think people should donate to such trust?

12. Three schools X, Y and Z organised a fete(mela) for collecting funds for flood victims in the form of hand-held fans, mats and toys made from recycled material, the sale price of each being ₹20, ₹15 and ₹10 respectively. The following table shows the number of articles of each type sold:

School / Articles	X	Y	Z
Hand-held fans	30	40	35
Mats	12	15	20
Toys	70	55	75

Using matrices, find the funds collected by each school by selling the above articles and the total funds collected. Also, write any one value generated by the above situation.

13. If $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$, find $A^2 - 5A + 4I$ and hence find a matrix X such that $A^2 - 5A + 4I + X = O$.

14. Obtain the inverse of the following matrix, using elementary operations:

$$A = \begin{bmatrix} 3 & 0 & -1 \\ 2 & 3 & 0 \\ 0 & 4 & 1 \end{bmatrix} \quad 6$$

15. By using elementary transformations, find the inverse of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} \quad 6$$

16. If $A = \begin{bmatrix} 0 & -\tan \frac{\alpha}{2} \\ \tan \frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2, show that $I + A = (I - A) \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$ 6

ACTIVITY – 1

To demonstrate a function which is one –one but not on to .