ECS303

(Following Roll No. to be filled by candidate)

Roll No. | | | | | | | | | 3 | 4 |

B.Tech.

THIRD SEMESTER EXAMINATION 2015-16

ECS303

DISCRETE MATHEMATICAL STRUCTURES

Time: 3 hours **Max Mark: 100**

Note

- Attempt all questions.
- Marks and number of question to attempt from the section is mentioned before each section.
- Assume missing data suitably .Illustrate the answer with suitable sketch.

1. Attempt any FOUR parts of the following: [4x5]

a. Define relation, Domain and range of a relation. How relation is represented pictorially. Explain giving examples

b. Prove by mathematical induction

$$1^2+3^2+5^2+....+(2n-1)^2=\frac{n(2n+1)(2n-1)}{3}=\frac{1}{3}n(4n^2-1)$$

c. Show that $(P\cap Q)\times(R\cap S)=(P\times R)\cap(Q\times S)$ for some arbitrary sets of P,Q,R&S.

d. A is a set of just ten natural numbers 1 to 10. A relation R is defined as follows $xRy \Leftrightarrow x+2y=10$ where $x,y\in A$.
Then evaluate the following (i) domain of R, (ii) Range of R (iii)R^{-1}

e. Differentiate between mapping and relation?

2. Attempt any TWO parts of the following [4x5]

a. Let $(A,*)$ be a group, show that $(A,*)$ is an ablican group if and only if $a^3\times b^3=(a\times b)^3 \quad \forall a,b\in A$

b. State & prove Lagrange's theorem.

c. Explain Homomorphism of groups. Also describe properties of Homomorphism.

d. Define an integral domain. Prove that every field is an integral domain.

e. Show that the intersection of two subgroups of a group G, is a subgroup of G.

3. Attempt any TWO parts of the following [2x10]

a. What is Hasse Diagram? Let $(P,\leq)$ be a poset. Show that the Hasse diagram of $(P,\leq)$ is not unique. Also draw Hasse diagram for $D_{105}=\{1,3,5,7,15,21,35,105\}$ where D_{105} denotes positive integral divisors of 105.

b. Discuss properties of lattice. Explain Distributive lattice. Prove that in a distributive lattice, if an element has a compliment then this is a compliment unique.

c. What is difference between lattice & partial order is a lattice. In a Boolean algebra $(B,\vee,\wedge)$ is a relation $\leq$ is defined by $a\leq b$ is $a\vee b=b$ or $a\wedge b=a$
Prove that the relation $\leq$ is a partial order in B and $(B,\leq)$ is a lattice.

4. Attempt any TWO parts of the following [2x10]

a. Prove that following statements are logically equivalent.

i. $(p\Rightarrow q)\vee r=(p\vee r)\Rightarrow(q\vee r)$

ii. $(p\uparrow q)\oplus(p\uparrow q)=(p\vee q)\wedge(p\downarrow r)$

iii. $p\vee(q\wedge r)=(p\vee q)\wedge(p\vee r)$

b. If A pq means $p\wedge q$ and Np means $\sim p$, then rewrite the following statement using A and N for $\wedge$ and $\sim$ respectively.

i. $\sim(p\wedge\sim q)\wedge(\sim q\wedge\sim r)$

ii. $(\sim p\wedge q)\wedge\sim(\sim p\wedge\sim q)$

iii. $(\sim p\wedge\sim q)\wedge\sim[(p\wedge q)\wedge(\sim q\wedge p)]$

iv. $p\wedge\sim q$

c. Explain quantities and types of quantifier. Negate the statements

i. $\exists x P(x)\vee\forall y Q(y)$

ii. $\forall x(p(x)\wedge\exists y Q(y)$

iii. $\exists x\forall y[P(x)\vee\sim Q(y)]$

5. Attempt any TWO parts of the following [2x10]

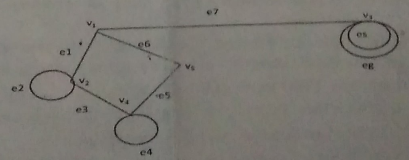
a. Explain graph coloring. Prove that a graph with at least one edge is 2-Chromatic is and only if it has no circuits of odd length. Name the graph whose chromatic polynomial is
$P_n(\lambda)=\lambda^4-3\lambda^3+3\lambda^2-\lambda$ Explain your answer.

b. What is generating function? Write generating function of the sequence $\{a_r\}$ defined by

i. $a_r=\frac{(-1)^r(r+2)(r+1)}{2}$

ii. $a_r=(r+1)3^r$

c. Write the adjacency and incidence matrix of the graph given below.



Prove that the sum of the degrees of all the vertices in a graph is equal to twice the number of edges.