

(Following Roll No. to be filled by candidate)

Roll No.

--	--	--	--	--	--	--	--	--	--

B.Tech.
THIRD SEMESTER EXAMINATION 2015-16
EOF038

DISCRETE MATHEMATICS

Time: 3 hours

Max Mark: 100

Note

Attempt all questions.

Marks and number of question to attempt from the section is mentioned before each section.

1. Attempt any Four parts of the following:

[4x5]

- If A, B, C be sets, then prove that $A - (B \cup C) = (A - B) \cap (A - C)$
- In a class of 25 students, 12 have taken Mathematics, 8 have taken Mathematics but not biology. Find the number of students who have taken Mathematics and Biology and those who have taken Biology but not Mathematics.
- Let $N = \{1, 2, 3, \dots\}$ and a relation is defined in $N \times N$ as $(a, b) R (c, d) \Rightarrow ad = bc$ hence find whether R is an equivalence relation or not?
- Let R be a relation from set A to the set B and S be a relation from set B to set C then prove that $(S \circ R)^{-1} = R^{-1} \circ S^{-1}$
- Let $f: R \rightarrow R$ be a function defined by $f(x) = px + q \forall x \in R$. Also $f \circ f = I_R$, find the value of p and q.
- Show that the $f(x) = x^3$ and $g(x) = x^{1/3}$ for all $x \in R$ are inverse to one another.

2. Attempt any four parts of the following:

[4x5]

- Show that $n^3 + 2n$ is divisible by 3 for $n \geq 1$.
- Show that $[(p \wedge q) \Rightarrow p] \Rightarrow (q \wedge \sim q)$ is a contradiction.
- Show that the statement is a tautology using the truth table $[(-q \Rightarrow \sim p) \wedge (q \Rightarrow p)] \Rightarrow (p \Leftrightarrow q)$
- Prove the validity of the following argument "If I get a job and work hard, I will get promoted. If I get promoted then I will be happy, I will not be happy, Therefore, either I will not get the job or I will not work hard"
- Obtain the principal disjunctive normal form of $q \vee (p \vee \sim q)$
- Write down the negation of the following proposition for every number x there is a number y such that $y < x$.

3. Attempt any two parts of the following:

[2x10]

EOE038

- (i) In how many ways can 7 boys and 5 girls be seated in a row so that no two girls may sit together.
(ii) Out of 5 men and 2 women, a committee of 3 is to be formed. In how many ways can this be done so as to include exactly one woman?
- Using generating function, evaluate the sum of $1^2 + 2^2 + 3^2 + \dots$
- Solve the recurrence relation $a_r - 7a_{r-1} + 10a_{r-2} = 0 \quad \forall a_0 = 3, a_1 = 3$

4. Attempt any two parts of the following:

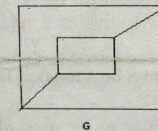
[2x10]

- Show that a set $G = \{a + 2\sqrt{b} : a, b \in \mathbb{Q}\}$ is a group w.r.t addition.
- (i) Find the order of an each element of a multiplicative group $G = \{1, -1, i, -i\}$
(ii) Prove that if a is a generator of a cyclic group G then a^{-1} is also a generator of G.
- If in a ring R with unity $(xy)^2 = x^2y^2$ for all $x, y \in R$ then prove that R is a commutative.

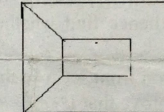
5. Attempt any four parts of the following:

[4x5]

- Show that the maximum number of edges in a simple graph with n vertices is $\frac{n(n-1)}{2}$
- Determine whether the following graph are isomorphic



G



G1

- Prove that a graph G is a tree iff it is minimally connected.
- Construct a tree with a given preorder and inorder traversal of a binary tree
Preorder: g b q a c p d e r
Inorder: q b c a g p e d r
- Find a chromatic polynomial and chromatic number for a graph $K_{3,3}$.
- Define Deterministic Finite Automation. Determine whether the string 101, 110001, 1001, 1111 are accepted or rejected by the transition graph given in the following figure

