(Following Roll No. to be filled by candidate)

Roll No. 1504310007

B. TECH.

FIRST SEMESTER EXAMINATION, 2015-16

EAS 103

MATHEMATICS - I

Time:3 Hours **Max. Marks: 100**

Note:

- Attempt all questions.
- Marks and number of questions to be attempted from the section is mentioned before each section.

1. Attempt any *Two* parts of the following: [2×10]

   a. Find the value of y_n when $x=0$ if $y=\sin\left[m\log(x+\sqrt{1+x^2})\right]$

   b. (i)Find the value of $\left(\frac{\partial^2}{\partial z^2}-\frac{\partial^2}{\partial x\partial y}\right)u$, where $u=\frac{1}{\sqrt{4xy-z^2}}$.

   (ii) If $f(x,y)=\sin^{-1}\left(\frac{y}{x}\right)+x\sin^{-1}\left(\frac{x}{y}\right)$, then find $x\frac{\partial f}{\partial x}+y\frac{\partial f}{\partial y}$.

   Hence or otherwise evaluate $x^2\frac{\partial^2 f}{\partial x^2}+2xy\frac{\partial^2 f}{\partial x\partial y}+y^2\frac{\partial^2 f}{\partial y^2}$

   c. (i)If $w=\sqrt{x^2+y^2+z^2}$, where $x=u\cos v, y=u\sin v, z=uv$, then prove that

   $$u\frac{\partial w}{\partial u}-v\frac{\partial w}{\partial v}=\frac{u}{\sqrt{1+v^2}}$$

   (ii) Obtain the second degree Taylor's series approximation to the function $f(x,y)=e^y\log(x+y)$ about the point $(1,0)$.

2. Attempt any *Two* parts of the following: [2×10]

   a. If u,v,w are the roots of the equation $(\lambda-x)^3+(\lambda-y)^3+(\lambda-z)^3=0$ in λ then find $\frac{\partial(u,v,w)}{\partial(x,y,z)}$.

   b. The work that must be done to propel a ship of displacement D, for a distance S in time t is proportional to $\frac{S^2D^{2/3}}{t^2}$. Find approximately the percentage increase of work necessary when the displacement is increased by 1%, time is diminished by 1% and the distance diminished by 3%.

   c. Find the dimensions of a rectangular box of maximum capacity whose surface is given when the box is open at the top.

3. Attempt any *Two* parts of the following: [2×10]

   a. (i) Find the rank of AB by reducing into Echelon form where

   $$A=\begin{bmatrix}2 & 1 & -3 & -6\\ 3 & -3 & 1 & 2\\ 1 & 1 & 1 & 2\end{bmatrix}, B=\begin{bmatrix}1 & 2 & -1 & 4\\ 2 & 4 & 3 & 4\\ 1 & 2 & 3 & 4\\ -1 & -2 & 6 & 7\end{bmatrix}$$

   (ii) If A be a square matrix of order 3 and eigen vectors of A corresponding to eigen value 1, 1, 3 are $[1, 0, -1]^T$, $[0,1,-1]^T$ and $[1,1,0]^T$ respectively. Find A.

   b. For what values of λ the following system of equations

   $2x_1-2x_2+x_3=\lambda x_1$, $2x_1-3x_2+2x_3=\lambda x_2$, $-x_1+2x_2=\lambda x_3$

   can possess a non-trivial solution. Obtain the general solution in each case.

   c. Verify Cayley-Hamilton theorem for the matrix and hence find A^{-2}. Also find the eigen values.

   $$A=\begin{bmatrix}2 & -1 & 1\\ -1 & 2 & -1\\ 1 & -1 & 2\end{bmatrix}.$$

4. Attempt any ***Four*** parts of the following: [4×5]

   a. Prove that $\Gamma(m)\,\Gamma\left(\frac{1-m}{2}\right)=\frac{\sqrt{\pi}\,\Gamma(m/2)}{2^{1-m}\cos\frac{m\pi}{2}}$.

   b. Evaluate $\iint_R (x^2+y^2)\,dxdy$, where R is the region bounded $x^2+y^2-2ax=0$

   c. Change the order of integration of the following integral and hence evaluate it

   $$\int_0^3\int_1^{\sqrt{4-y}}(x+y)\,dxdy$$

   d. Find the area common to the circle $x^2+y^2=64$ and the parabola $y^2=12x$.

   e. Evaluate $\iiint_V xyz\,dxdydz$, where V is the region bounded by the coordinate planes and the sphere $x^2+y^2+z^2=a^2$.

   f. Find the volume enclosed between the two surfaces $z=2x^2+3y^2$ and $z=16-2x^2-y^2$.

5. Attempt any ***Four*** parts of the following: [4×5]

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a. Prove that $\nabla^2 f(r) = f''(r) + \frac{1}{r} f'(r)$

b. Find the values of a, b, c so that the directional derivative of $f = axy^2 + byz + cz^2x^3$ at (1, 2,–1) has maximum magnitude 16 in a direction parallel to x–axis.

c. Show that the vector field **F** given by

$$\mathbf{F} = (x^2 - yz)\,\mathbf{i} + (y^2 - xz)\,\mathbf{j} + (z^2 - xy)\,\mathbf{k}$$

is conservative. Find its scalar potential. Also find the work done in moving a particle from (1, 0, 0) to (3, 1, 2).

d. Prove that $\operatorname{grad}(\mathbf{f} \cdot \mathbf{g}) = \mathbf{f} \times \operatorname{curl} \mathbf{g} + \mathbf{g} \times \operatorname{curl} \mathbf{f} + (\mathbf{f}.\nabla)\mathbf{g} + (\mathbf{g}.\nabla)\mathbf{f}$

e. Evaluate $\oint_C (3x^2 - 8y^2)dx + (4y - 6xy)dy$, where C is the boundary of the closed region bounded by $y = \sqrt{x}, y = x^2$.

f. Evaluate $\iint \mathbf{F}.\hat{\mathbf{n}}dS$ where $\mathbf{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$ and S is the region bounded by $y^2 = 4x,\ x = 1, z = 0$ to $z = 3$.