IND4-5

Problem: 5

Number of pages: 9

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Contestant Code IND 4

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Solution 2

Obviously, k is more than 2016 bcoz we need to erase each $(x-1)\ldots,(x-2016)$ on either side.

We will show that $k=2016$ by presenting the foll construction.

In LHS, erase ~~(x-1), (x-4)(x-5)~~ all factors $(x-k)$ such that $k\equiv 1 \pmod 4$ or $k\equiv 0 \pmod 4$. Then retain exactly these factors on RHS. Then the equation is

$$\underbrace{(x-2)(x-3)(x-6)(x-7)\ldots(x-2015)}_{f(x)}$$
$$-\underbrace{(x-1)(x-4)(x-5)(x-8)\ldots(x-2016)}_{g(x)}=0.$$

We will show that this pol^n is greater than ~~or equal to~~ 0 for all ~~$\mathbb{Z}$~~ $\mathbb{R}$. Hence this wont have real roots.

1) ~~$x\leq 1$~~ $a\leq 2$

$\Rightarrow$ $a\leq 2 \Rightarrow (a-2),(a-3)\ldots(a-2015)$ is positive. for $1\leq a\leq 2$, $(a-1)(a-4)\ldots$ is negative hence $P(a)$ is positive.

for $a\leq 1$, see that $(x-2)(x-3)\geq (x-1)(x-4)$ and so on. Hence this case is done.

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~~Case 2 : $a \geq 11$~~

for ~~$12 \geq a \geq 11$~~, $|a-2$

Case 2 : $a \geq 2015$

for $2016 \geq a \geq 2015$, $(a-2016)(a-2015)\dots(a-1)$ is negative. So $P(a)$ is positive.

for $a \geq 2016$, note again that $(a-2015)(a-2014) \geq (a-2016)(a-2013)$ and so on. □.

~~Case 3 $a \in [4k+3, 4k+6]$ ($[3k, 3k+3]$) for $503 \geq k \geq 0$)~~

~~in this case, if $a \in [4k+4, 4k+5]$ then $g(a)$ is negative, $f(a)$ is positive. so done.~~

~~if $a \in$ then we again use $(a-(4k+4))(a-(4k+1)) \leq (a-$~~

~~$(4k+4-a)(4k+1-a) \geq (4k+3$~~

Case 3 $a \in [4k, 4k+1]$, $503 \geq k \geq 1$

here we again use $(a-4k)(a-(4k-3)) \geq (a-(4k-1))(a-(4k-2))$

& $(4k+1-a)(4k+4-a) \geq (4k+2-a)(4k+3-a)$

and so on.

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$\boxed{\text{Case 4}}$ $a \in [4k+1, 4k+2)$ or $a \in [4k+3, 4k+4]$

here simply $g(a)$ is negative and $f(a)$ is positive.

$\boxed{\text{Case 5}}$ $a \in [4k+2, 4k+3]$

This is the most difficult case.

This is the most difficult case too

Hoping it holds in this case too

$\cancel{0.5\times 0.5\times 5.5}\times 2.5$

$\cancel{0.5\times 0.5}\times \cancel{4}3.5\times 4.5\times 3.5\times 4.5$

$\leq \cancel{1.5\times 1.5}\times 2.5\times 5.5\times 2.5\times 5.5\times 9$

$1+\frac{2}{5}$

$\cancel{2016}\frac{(2015-2.5)(2014-2.5)}{(2016-2.5)(2013-2.5)}$

$2 \quad \cancel{2012.5}\frac{(2012.5)(2011.5)}{(2013.5)(2010.5)}\leq \sqrt[w]{9}$

$\cancel{6}\cancel{2.5}\frac{4.5\times 3.5}{2.5\times 5.5}=\frac{45\times 35}{25\times 55}=\frac{\cancel{1225}\;1575}{1375}\leq \sqrt[51]{9}$

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$2016 = 1024 + 512$

$2 \times 6 - 4 \times 1 = 5$

$f(2) \le f_1(2)$

$f_1'(2) \le f_2(2)$

$f_2'(2)$

$P(x) - Q(x)$

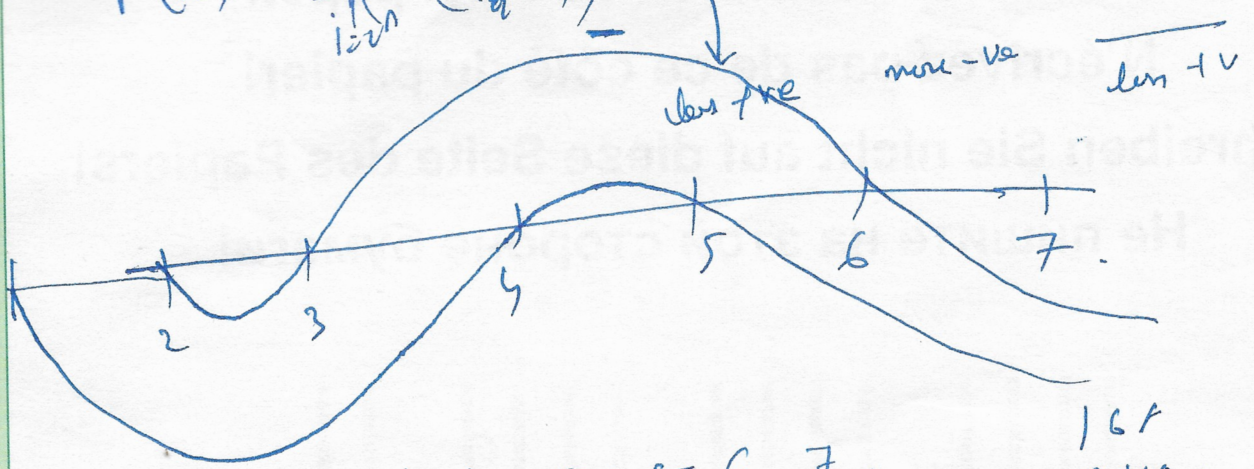
$P_{1024}(x) - Q_{1024}(x) = c$

$5 \times 8 \times 10 \times 11$

440.

$\downarrow$

$P(x) \quad 2\prod_{i=1}^{n}(c + (Q_{\alpha}))$ — +ve -ve +ve



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240.

1, 4, ~~5, 8~~ 6, 7.

$\cancel{(x-1)}$

$(x-1)(x-4)(x-6)(x-7) - (x-2)(x-3)(x-5)(x-8) \le 0.$

$(x-1)(x-4)(x-6)(x-7)(x-9)(x-13)(x-15)(x-16)$

$\left[\frac{S_1+2}{S_1} \quad \frac{S_2+S_2}{S}\right] \cdot \frac{(x-2)(x-3)}{(x-1)(x-4)} \le 1$

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$(x-1)(x-4) = (x-2)(x-3) - 2$

$\cancel{(x-5)(x-6) = (x-6)(x-7) - 2}$

$(x-6)(x-7) = (x-5)(x-8) + 2$

$2(x-2)(x-3) - 2(x-5)(x-8)$

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$f(x) - g(x) \geqslant 0.$

$f(1.5) \geqslant \cancel{g(1.5)}\ 0 \geqslant g(1.5)$

5.5 7.5

$f(3.5) \geqslant 0 \geqslant g(3.5)$

$(x-6)\ (x-7)$

$(x-2)$ $(x-2)(x-3)\cancel{(x-5)}\cancel{(x-6)}\ldots(x-8)\ldots$

$f(2015.5) \geqslant 0.$ $g(x) = (x-1)(x-4)(x-8)$

6, 10 ...

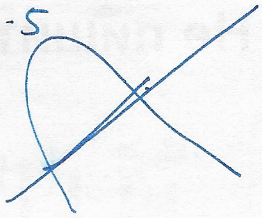
$(x-2)\ (x-3)\ (x-6)\ (x-7)\ (x-10)\ (x-11)$

$-(x-1)\ (x-4)\ (x-5)\ (x-8)\ (x-9)\ (x-12) \geqslant 0.$

5.5 6.5 9.5

$1.5 \times \cancel{2.5}\ 1.5 \times 2.5 \times \cancel{3.5 \times 4.5 \times 7}$

$- 0.5 \times 0.5 \times 3.5 \times 4.5 \times 7.5 \times 8.5$



$144 \times 12.$

$36 \times 770.$

$12 \times 9 \times 8 \times 5 \times 4 \times 1 \leq (6+x)\ (7+x)$

$\geqslant (5+x)\ (8+x)$

~~44~~ ~~169~~ 5×5.5

$1.5 \times 1.5 \times 2.5$ 144×4.5

$\geqslant 0.5\ 0.5 \times 3.5$

$42 \geqslant 46$

81 121

84 42 64 100

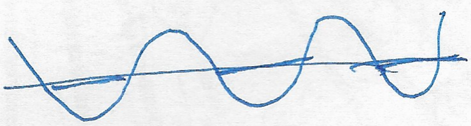
$36 + 126 + \cancel{42} + \cancel{156}$ 25 49

16 36

$20 + 32 + 40 + 160$ 9

1 4

252 331 319



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$p(x) - g(x) \geq 0.$

$x_1 \vee x_2.$

$x_1 \_\_\_\_ x_2$ $p(a) \leq 0.$ $g(a) = 0.$

$(x-a)(x-b)$

$x_1, a, b, x_2.$

$\Rightarrow \cancel{g(x) \leq 0}.$

$g(x) \leq 0.$

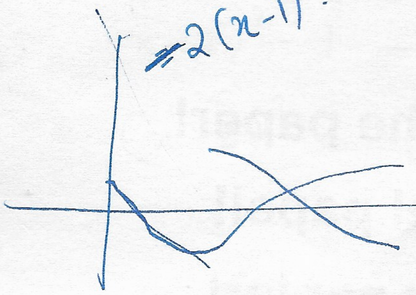
$g(y) \leq 0.$ $x^2 - 3x + 2$

$-(x-3)$

$f^2(x) + g^2(x)$

$-(x-2016) \geq 0$

$\cancel{-}2(x-1).$

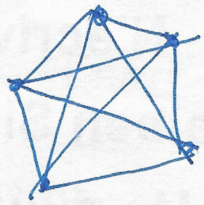


$x_3 x_4$

$x_1 x_2$

$\cancel{12 \quad 45} \quad 68$

$12\,34\,56\,\cancel{7}8\,9\,10\,11\,12$



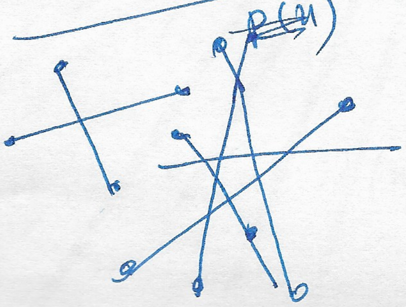
$\cancel{(x-1)(x-2)(x-5)(x-6)(x-9)(x-10)}$

$\cancel{g(x-3)(x-4)(x-7)(x-8)(x-11)}(x-12)$

$-(x-1)(x-2)(x-5)(x-6)(x-9)(x-10) \leq 0.$

$-(x-3)(x-4)(x-7)(x-8)(x-11)(x-12) \geq 0$

$(x-1)(x-2 \ldots)(x-2016)\left[\frac{1}{p^2(x)} - \frac{1}{g(x)}\right) \geq 0.$



$f(1) \leq g(1)$

$\frac{g(1)-f(1)}{g(1)f(1)} \geq 0$

$\frac{1}{f(1)} - \frac{1}{g(1)} \geq 0.$

$-\cancel{f(x)} \leq g(1) \leq 0 \leq f(1)$

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$(x+1)(x+2)-(x-3)=0.$

$4032-\frac{2048}{1024}$

$x^2-3x+2-x+3$

$1,2,3,4,5,6,7,8,9,10,11,12.$

x^2-4x+5

$\frac{2016\times 2017}{2}$

$(x-1)(x-2)-(x-3)(x-4).$

$(x-1)(x-3)-(x-2)(x-4)$

$1,2,7,8.$

$(x-2)^2-1-[(x-3)^2-1]$

$18.$

$(x-2)^2=(x-3)^2.$

$\cancel{2+14+56}$

$2+7+8+14+16+56$

$x-2=-x+3$

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$\frac{(x-1)\cdots(x-2016)}{p(x)}-p(x)=g^2(x)$

$\cancel{16}\ 379$

$(x-4)(x-1)-(x-3)(x-2)$

$\cancel{(x-4)(x-1)}$

$(x-2)(x-3)-(x-4)(x-1)$

$x^2-5x+4-(x^2-5x-6)$

$(x-x_1)\ldots(x-x_k)-(x-y_1)(x-\ldots y_k)\geq 0$

$(x-1)(x-3)(x-5)(x-$

$\Rightarrow k$ ~~odd.~~

$(x-1)(x-2)(x-3)(x-4)-(x-5)(x-6\ldots x-18)\geq 0.$

$(x+1)(x+2)-(x-3)=0.$ $4032-\frac{2048}{1024}$

$x^2-3x+2-x+3$ $1,2,3,4,5,6,7,8,9,10,11,12.$

x^2-4x+5

$(x-1)(x-2)-(x-3)(x-4)$ $\frac{2016\times 2017}{2}$

$(x-1)(x-3)-(x-2)(x-4)$ $1,2,7,8.$

$(x-2)^2-1-[(x-3)^2-1]$ $18.$

$(x-2)^2=(x-3)^2.$ $\cancel{2+14+56}$

$x-2=-x+3$ $2+7+8+14+16+56$

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$\frac{(x-1)\cdots(x-2016)}{p(x)}-p(x)=g^2(x)$

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$(x-4)(x-1)-(x-3)(x-2)$ $\cancel{(x-4)(x-1)}$

$x^2-5x+4-(x^2-5x-6)$ $(x-2)(x-3)-(x-4)(x-1)$

$(x-x_1)\ldots(x-x_k)-(x-y_1)(x-+x y_k)\geq 0$

$(x-1)(x-3)(x-5)(x-$ $\Rightarrow k$

$(x-1)(x-2)(x-3)(x-4)-(x-5)(x-6\ldots x-18)\geq 0.$