RUTGERS

THE STATE UNIVERSITY
OF NEW JERSEY

Logic, Knowledge and Reasoning

Edgar Granados

Propositional Logic

- $\neg$
- $\wedge$
- $\vee$
- $\Rightarrow$
- $\Leftrightarrow$

Propositional Logic

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*					
false	*true*					
true	*false*					
true	*true*					

Propositional Logic

P	Q	$\neg P$	$P \wedge Q$	$P \vee Q$	$P \Rightarrow Q$	$P \Leftrightarrow Q$
false	*false*	*true*	*false*	*false*	*true*	*true*
false	*true*	*true*	*false*	*true*	*true*	*false*
true	*false*	*false*	*false*	*true*	*false*	*false*
true	*true*	*false*	*true*	*true*	*true*	*true*

Propositional Logic

Equivalences

$$
\begin{aligned}
(\alpha \wedge \beta) &\equiv (\beta \wedge \alpha) && \text{commutativity of } \wedge \\
(\alpha \vee \beta) &\equiv (\beta \vee \alpha) && \text{commutativity of } \vee \\
((\alpha \wedge \beta) \wedge \gamma) &\equiv (\alpha \wedge (\beta \wedge \gamma)) && \text{associativity of } \wedge \\
((\alpha \vee \beta) \vee \gamma) &\equiv (\alpha \vee (\beta \vee \gamma)) && \text{associativity of } \vee \\
\neg(\neg\alpha) &\equiv \alpha && \text{double-negation elimination} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\beta \Rightarrow \neg\alpha) && \text{contraposition} \\
(\alpha \Rightarrow \beta) &\equiv (\neg\alpha \vee \beta) && \text{implication elimination} \\
(\alpha \Leftrightarrow \beta) &\equiv ((\alpha \Rightarrow \beta) \wedge (\beta \Rightarrow \alpha)) && \text{biconditional elimination} \\
\neg(\alpha \wedge \beta) &\equiv (\neg\alpha \vee \neg\beta) && \text{De Morgan} \\
\neg(\alpha \vee \beta) &\equiv (\neg\alpha \wedge \neg\beta) && \text{De Morgan} \\
(\alpha \wedge (\beta \vee \gamma)) &\equiv ((\alpha \wedge \beta) \vee (\alpha \wedge \gamma)) && \text{distributivity of } \wedge \text{ over } \vee \\
(\alpha \vee (\beta \wedge \gamma)) &\equiv ((\alpha \vee \beta) \wedge (\alpha \vee \gamma)) && \text{distributivity of } \vee \text{ over } \wedge
\end{aligned}
$$

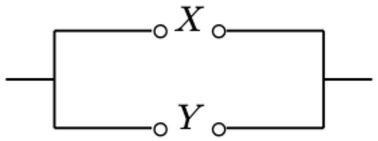
Propositional Logic

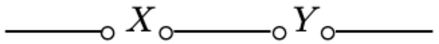
Some history...

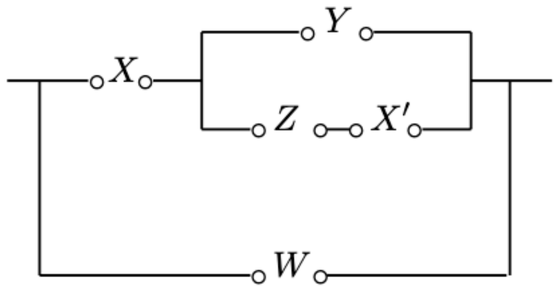
1936

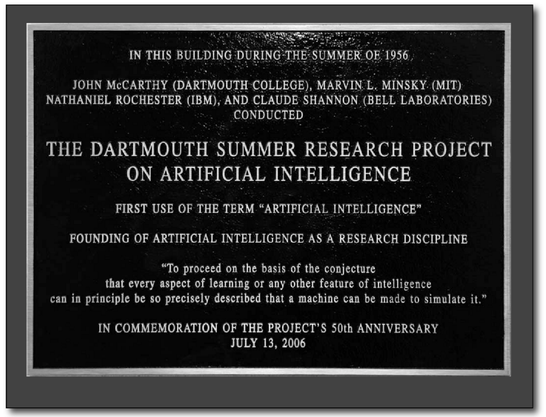
Shannon's thesis (1938) *"A Symbolic Analysis of Relay and Switching Circuits":*

In the control and protective circuits of complex electrical systems it is frequently necessary to make intricate interconnections of relay contacts and switches. Examples of these circuits occur in automatic telephone exchanges, industrial motor-control equipment, and in almost any circuits designed to perform complex operations automatically. In this paper a ***mathematical analysis of certain of the properties of such networks*** *will be made.*









Languages

BNF

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} &\rightarrow \textit{True} \mid \textit{False} \mid P \mid Q \mid R \mid \ldots \\
\textit{ComplexSentence} &\rightarrow (\ \textit{Sentence}\ ) \\
&\mid \neg\ \textit{Sentence} \\
&\mid \textit{Sentence} \wedge \textit{Sentence} \\
&\mid \textit{Sentence} \vee \textit{Sentence} \\
&\mid \textit{Sentence} \Rightarrow \textit{Sentence} \\
&\mid \textit{Sentence} \Leftrightarrow \textit{Sentence}
\end{aligned}
$$

OPERATOR PRECEDENCE : $\neg, \wedge, \vee, \Rightarrow, \Leftrightarrow$

Languages

BNF

Grammar → *Rule* | *Rule Grammar*

Rule → *LHS* → *RHS*

LHS → *Nonterminal*

RHS → *RhsSequence*

| *RhsSequence* | *RHS*

RhsSequence → *Nonterminal*

| *Terminal*

| *RhsSequence* *Nonterminal*

| *RhsSequence* *Terminal*

Languages

- What about English?
- Programming Languages?
- Other Languages?

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Languages

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Languages

Chomsky hierarchy

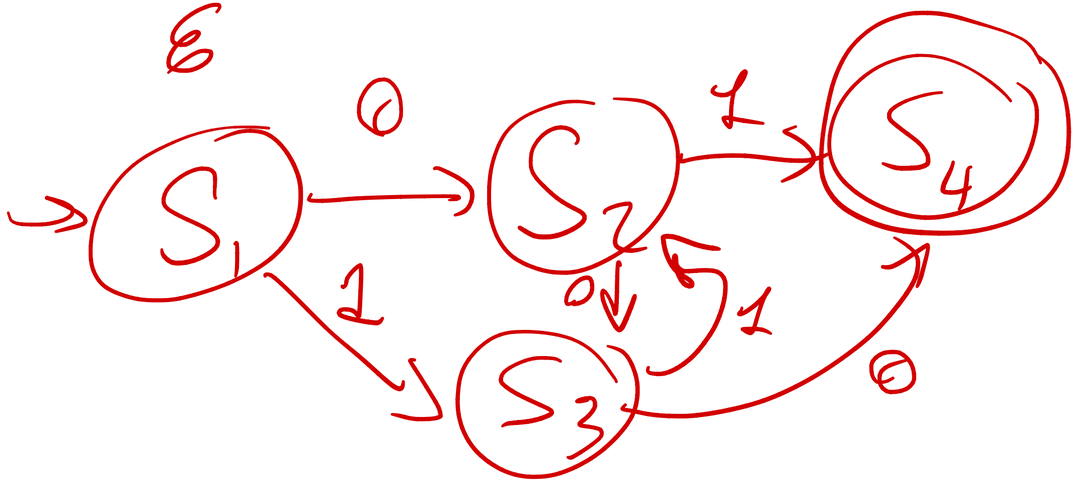
- RE - FSM

$\Sigma \cup \{+, \cdot, *, (\,), \varepsilon, \emptyset\}$

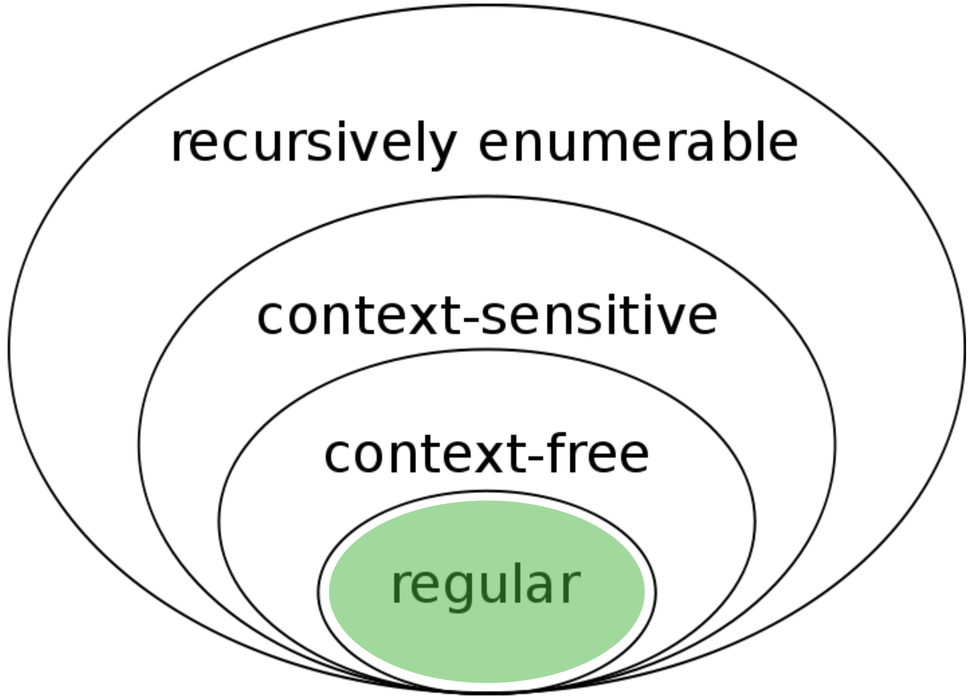
$\Sigma = \{0, 1\}$

$[01]^*$

ε



01
11
0010
010



Languages

Chomsky hierarchy

- LISP

```
s_expression = atomic_symbol \
               / "(" s_expression "."s_expression ")" \
               / list

list = "(" s_expression < s_expression > ")"

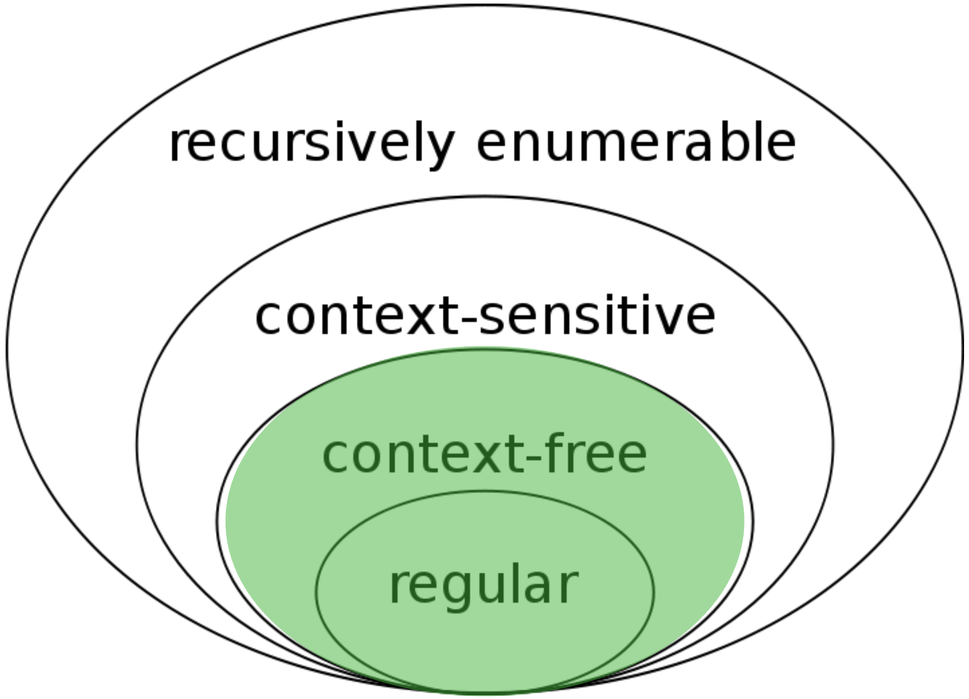
atomic_symbol = letter atom_part

atom_part = empty / letter atom_part / number atom_part

letter = "a" / "b" / " ..." / "z"

number = "1" / "2" / " ..." / "9"

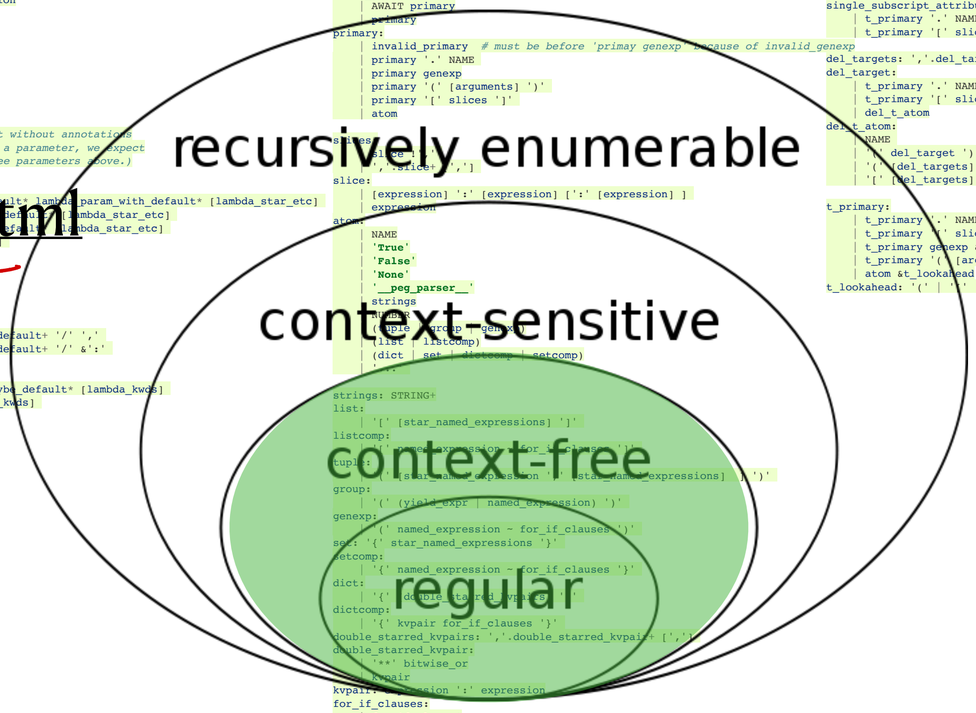
empty = " "
```



Languages

Chomsky hierarchy

- Python?
- https://docs.python.org/3/reference/grammar.html




```
# PEG grammar for Python


file: [statements] ENDMARKER
interactive: statement_newline
eval: expressions NEWLINE* ENDMARKER
func_type: '(' [type_expressions] ')' '->' expression NEWLINE* ENDMARKER
fstring: star_expressions

# type_expressions allow */** but ignore them
type_expressions:
    | ','.expression+ ',' '*' expression ',' '**' expression
    | ','.expression+ ',' '*' expression
    | ','.expression+ ',' '**' expression
    | '*' expression ',' '**' expression
    | '*' expression
    | '**' expression
    | ','.expression+

statements: statement+
statement: compound_stmt | simple_stmt
statement_newline:
    | compound_stmt NEWLINE
    | simple_stmt
    | NEWLINE
    | ENDMARKER
simple_stmt:
    | small_stmt !';' NEWLINE  # Not needed, there for speedup
    | ';'.small_stmt+ [';'] NEWLINE
# NOTE: assignment MUST precede expression, else parsing a simple assignment
# will throw a SyntaxError.
small_stmt:
    | assignment
    | star_expressions
    | return_stmt
    | import_stmt
    | raise_stmt
    | 'pass'
    | del_stmt
    | yield_stmt
    | assert_stmt
    | 'break'
    | 'continue'
    | global_stmt
    | nonlocal_stmt
compound_stmt:
    | function_def
    | if_stmt
    | class_def
    | with_stmt
    | for_stmt
    | try_stmt
    | while_stmt

# NOTE: annotated_rhs may start with 'yield'; yield_expr must start with 'yield'
assignment:
    | NAME ':' expression ['=' annotated_rhs ]
    | ('(' single_target ')'
         | single_subscript_attribute_target) ':' expression ['=' annotated_rhs ]
    | (star_targets '=' )+ (yield_expr | star_expressions) !'=' [TYPE_COMMENT]
    | single_target augassign ~ (yield_expr | star_expressions)
augassign:
    | '+='
    | '-='
    | '*='
    | '@='
    | '/='
    | '%='
    | '&='
    | '|='
    | '^='
    | '<<='
    | '>>='
    | '**='
    | '//='

global_stmt: 'global' ','.NAME+
nonlocal_stmt: 'nonlocal' ','.NAME+

yield_stmt: yield_expr

assert_stmt: 'assert' expression [',' expression ]

del_stmt:
    | 'del' del_targets &(';' | NEWLINE)
import_stmt: import_name | import_from
import_name: 'import' dotted_as_names
# note below: the ('.' | '...') is necessary because '...' is tokenized as ELLIPSIS
import_from:
    | 'from' ('.' | '...')* dotted_name 'import' import_from_targets
    | 'from' ('.' | '...')+ 'import' import_from_targets
import_from_targets:
    | '(' import_from_as_names [','] ')'
    | import_from_as_names !','
    | '*'
import_from_as_names:
    | ','.import_from_as_name+
import_from_as_name:
    | NAME ['as' NAME ]
dotted_as_names:
    | ','.dotted_as_name+
dotted_as_name:
    | dotted_name ['as' NAME ]
dotted_name:
    | dotted_name '.' NAME
    | NAME

if_stmt:
    | 'if' named_expression ':' block elif_stmt
    | 'if' named_expression ':' block [else_block]
elif_stmt:
    | 'elif' named_expression ':' block elif_stmt
    | 'elif' named_expression ':' block [else_block]
else_block: 'else' ':' block

while_stmt:
    | 'while' named_expression ':' block [else_block]

for_stmt:
    | 'for' star_targets 'in' ~ star_expressions ':' [TYPE_COMMENT] block [else_block]
    | ASYNC 'for' star_targets 'in' ~ star_expressions ':' [TYPE_COMMENT] block [else_block]
with_stmt:
    | 'with' '(' ','.with_item+ ','? ')' ':' block
    | 'with' ','.with_item+ ':' [TYPE_COMMENT] block
    | ASYNC 'with' '(' ','.with_item+ ','? ')' ':' block
    | ASYNC 'with' ','.with_item+ ':' [TYPE_COMMENT] block
with_item:
    | expression 'as' star_target &(',' | ')' | ':')
    | expression

try_stmt:
    | 'try' ':' block finally_block
    | 'try' ':' block except_block+ [else_block] [finally_block]
except_block:
    | 'except' expression ['as' NAME ] ':' block
    | 'except' ':' block
finally_block: 'finally' ':' block

return_stmt:
    | 'return' [star_expressions]

raise_stmt:
    | 'raise' expression ['from' expression ]
    | 'raise'

function_def:
    | decorators function_def_raw
    | function_def_raw
function_def_raw:
    | 'def' NAME '(' [params] ')' ['->' expression ] ':' [func_type_comment] block
    | ASYNC 'def' NAME '(' [params] ')' ['->' expression ] ':' [func_type_comment] block
func_type_comment:
    | NEWLINE TYPE_COMMENT &(NEWLINE INDENT)   # Must be followed by indented block
    | TYPE_COMMENT

params:
    | parameters

parameters:
    | slash_no_default param_no_default* param_with_default* [star_etc]
    | slash_with_default param_with_default* [star_etc]
    | param_no_default+ param_with_default* [star_etc]
    | param_with_default+ [star_etc]
    | star_etc

# Some duplication here because we can't write (',' | &')'),
# which is because we don't support empty alternatives (yet).
#
slash_no_default:
    | param_no_default+ '/' ','
    | param_no_default+ '/' &')'
slash_with_default:
    | param_no_default* param_with_default+ '/' ','
    | param_no_default* param_with_default+ '/' &')'

star_etc:
    | '*' param_no_default param_maybe_default* [kwds]
    | '*' ',' param_maybe_default+ [kwds]
    | kwds
kwds: '**' param_no_default

# One parameter.  This *includes* a following comma and type comment.
#
# There are three styles:
# - No default
# - With default
# - Maybe with default
#
# There are two alternative forms of each, to deal with type comments:
# - Ends in a comma followed by an optional type comment
# - No comma, optional type comment, must be followed by close paren
# The latter form is for a final parameter without trailing comma.
#
param_no_default:
    | param ',' TYPE_COMMENT?
    | param TYPE_COMMENT? &')'
param_with_default:
    | param default ',' TYPE_COMMENT?
    | param default TYPE_COMMENT? &')'
param_maybe_default:
    | param default? ',' TYPE_COMMENT?
    | param default? TYPE_COMMENT? &')'
param: NAME annotation?
annotation: ':' expression
default: '=' expression

decorators: ('@' named_expression NEWLINE )+

class_def:
    | decorators class_def_raw
    | class_def_raw
class_def_raw:
    | 'class' NAME ['(' [arguments] ')' ] ':' block

block:
    | NEWLINE INDENT statements DEDENT
    | simple_stmt
statements: statement+
star_expressions:
    | star_expression (',' star_expression )+ [',']
    | star_expression ','
    | star_expression
star_expression:
    | '*' bitwise_or
    | expression
star_named_expressions: ','.star_named_expression+ [',']
star_named_expression:
    | '*' bitwise_or
    | named_expression
named_expression:
    | NAME ':=' ~ expression
    | expression !':='
annotated_rhs: yield_expr | star_expressions

expressions:
    | expression (',' expression )+ [',']
    | expression ','
    | expression
expression:
    | disjunction 'if' disjunction 'else' expression
    | disjunction
    | lambdef

lambdef:
    | 'lambda' [lambda_params] ':' expression

lambda_params:
    | lambda_parameters

# lambda_parameters etc. duplicates parameters but without annotations
# or type comments, and if there's no comma after a parameter, we expect
# a colon, not a close parenthesis.  (For more, see parameters above.)
#
lambda_parameters:
    | lambda_slash_no_default lambda_param_no_default* lambda_param_with_default* [lambda_star_etc]
    | lambda_slash_with_default lambda_param_with_default* [lambda_star_etc]
    | lambda_param_no_default+ lambda_param_with_default* [lambda_star_etc]
    | lambda_param_with_default+ [lambda_star_etc]
    | lambda_star_etc
lambda_slash_no_default:
    | lambda_param_no_default+ '/' ','
    | lambda_param_no_default+ '/' &':'
lambda_slash_with_default:
    | lambda_param_no_default* lambda_param_with_default+ '/' ','
    | lambda_param_no_default* lambda_param_with_default+ '/' &':'

lambda_star_etc:
    | '*' lambda_param_no_default lambda_param_maybe_default* [lambda_kwds]
    | '*' ',' lambda_param_maybe_default+ [lambda_kwds]
    | lambda_kwds
lambda_kwds: '**' lambda_param_no_default

lambda_param_no_default:
    | lambda_param ','
    | lambda_param &':'
lambda_param_with_default:
    | lambda_param default ','
    | lambda_param default &':'
lambda_param_maybe_default:
    | lambda_param default? ','
    | lambda_param default? &':'
lambda_param: NAME

disjunction:
    | conjunction ('or' conjunction )+
    | conjunction
conjunction:
    | inversion ('and' inversion )+
    | inversion
inversion:
    | 'not' inversion
    | comparison
comparison:
    | bitwise_or compare_op_bitwise_or_pair+
    | bitwise_or
compare_op_bitwise_or_pair:
    | eq_bitwise_or
    | noteq_bitwise_or
    | lte_bitwise_or
    | lt_bitwise_or
    | gte_bitwise_or
    | gt_bitwise_or
    | notin_bitwise_or
    | in_bitwise_or
    | isnot_bitwise_or
    | is_bitwise_or
eq_bitwise_or: '==' bitwise_or
noteq_bitwise_or:
    | ('!=' ) bitwise_or
lte_bitwise_or: '<=' bitwise_or
is_bitwise_or: 'is' bitwise_or

bitwise_or:
    | bitwise_or '|' bitwise_xor
    | bitwise_xor
bitwise_xor:
    | bitwise_xor '^' bitwise_and
    | bitwise_and
bitwise_and:
    | bitwise_and '&' shift_expr
    | shift_expr
shift_expr:
    | shift_expr '<<' sum
    | shift_expr '>>' sum
    | sum

sum:
    | sum '+' term
    | sum '-' term
    | term
term:
    | term '*' factor
    | term '/' factor
    | term '//' factor
    | term '%' factor
    | term '@' factor
    | factor
factor:
    | '+' factor
    | '-' factor
    | '~' factor
    | power
power:
    | await_primary '**' factor
    | await_primary
await_primary:
    | AWAIT primary
    | primary
primary:
    | invalid_primary  # must be before 'primay genexp' because of invalid_genexp
    | primary '.' NAME
    | primary genexp
    | primary '(' [arguments] ')'
    | primary '[' slices ']'
    | atom
slices:
    | slice !','
    | ','.(slice | starred_expression)+ [',']
slice:
    | [expression] ':' [expression] [':' [expression] ]
    | expression
atom:
    | NAME
    | 'True'
    | 'False'
    | 'None'
    | '__peg_parser__'
    | strings
    | NUMBER
    | (tuple | group | genexp)
    | (list | listcomp)
    | (dict | set | dictcomp | setcomp)
    | '...'
strings: STRING+
list:
    | '[' [star_named_expressions] ']'
listcomp:
    | '[' named_expression ~ for_if_clauses ']'
tuple:
    | '(' [star_named_expression ~ ',' [star_named_expressions] ] ')'
group:
    | '(' (yield_expr | named_expression) ')'
genexp:
    | '(' named_expression ~ for_if_clauses ')'
set: '{' star_named_expressions '}'
setcomp:
    | '{' named_expression ~ for_if_clauses '}'
dict:
    | '{' [double_starred_kvpairs] '}'
dictcomp:
    | '{' kvpair for_if_clauses '}'
double_starred_kvpairs: ','.double_starred_kvpair+ [',']
double_starred_kvpair:
    | '**' bitwise_or
    | kvpair
kvpair: expression ':' expression
for_if_clauses:
    | for_if_clause+
for_if_clause:
    | ASYNC 'for' star_targets 'in' ~ disjunction ('if' disjunction )*
    | 'for' star_targets 'in' ~ disjunction ('if' disjunction )*
yield_expr:
    | 'yield' 'from' expression
    | 'yield' [star_expressions]

arguments:
    | args [','] &')'
args:
    | ','.(starred_expression | named_expression !'=')+ [',' kwargs ]
    | kwargs
kwargs:
    | ','.kwarg_or_starred+ ',' ','.kwarg_or_double_starred+
    | ','.kwarg_or_starred+
    | ','.kwarg_or_double_starred+
starred_expression:
    | '*' expression
kwarg_or_starred:
    | NAME '=' expression
    | starred_expression
kwarg_or_double_starred:
    | NAME '=' expression
    | '**' expression
# NOTE: star_targets may contain *bitwise_or, star_targets
star_targets:
    | star_target !','
    | star_target (',' star_target )* [',']
star_targets_list_seq: ','.star_target+ [',']
star_targets_tuple_seq:
    | star_target (',' star_target )+ [',']
    | star_target ','
star_target:
    | '*' (!'*' star_target)
    | target_with_star_atom
target_with_star_atom:
    | t_primary '.' NAME !t_lookahead
    | t_primary '[' slices ']' !t_lookahead
    | star_atom
star_atom:
    | NAME
    | '(' target_with_star_atom ')'
    | '(' [star_targets_tuple_seq] ')'
    | '[' [star_targets_list_seq] ']'

single_target:
    | single_subscript_attribute_target
    | NAME
    | '(' single_target ')'
single_subscript_attribute_target:
    | t_primary '.' NAME !t_lookahead
    | t_primary '[' slices ']' !t_lookahead
del_targets: ','.del_target+ [',']
del_target:
    | t_primary '.' NAME !t_lookahead
    | t_primary '[' slices ']' !t_lookahead
    | del_t_atom
del_t_atom:
    | NAME
    | '(' del_target ')'
    | '(' [del_targets] ')'
    | '[' [del_targets] ']'

t_primary:
    | t_primary '.' NAME &t_lookahead
    | t_primary '[' slices ']' &t_lookahead
    | t_primary genexp &t_lookahead
    | t_primary '(' [arguments] ')' &t_lookahead
    | atom &t_lookahead
t_lookahead: '(' | '[' | '.'
```

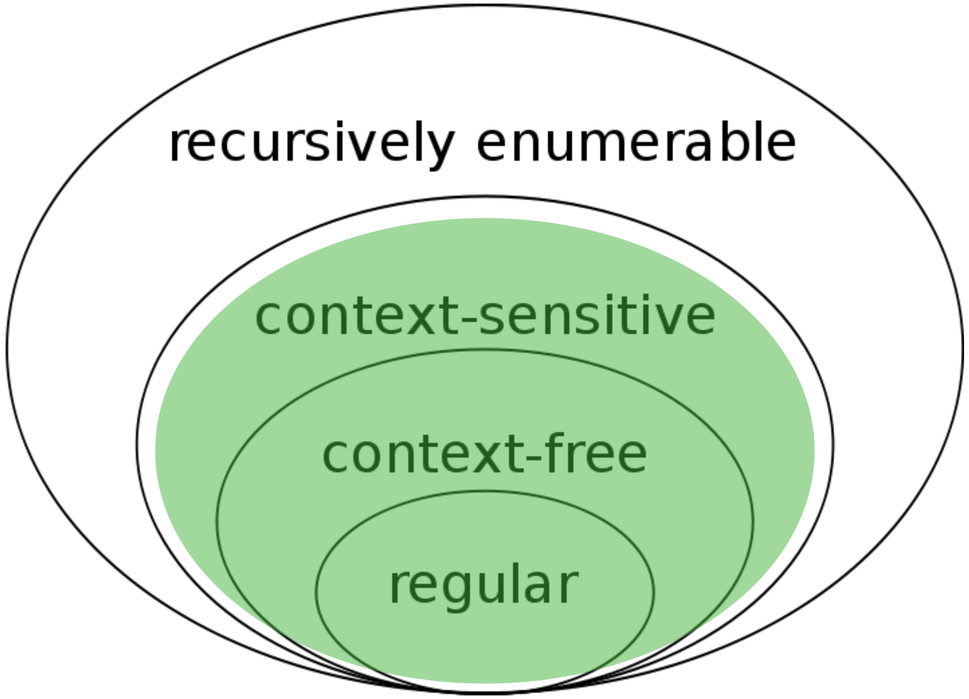
Languages

Chomsky hierarchy

- C++
  - ```
    int foo;
    typedef int foo;
    foo x;
    ```

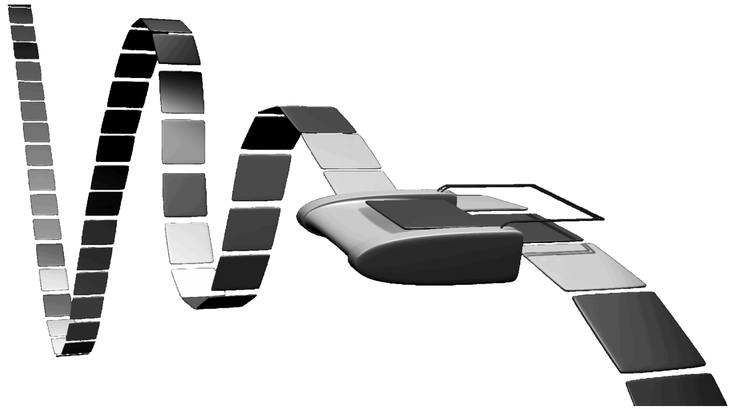
*bar

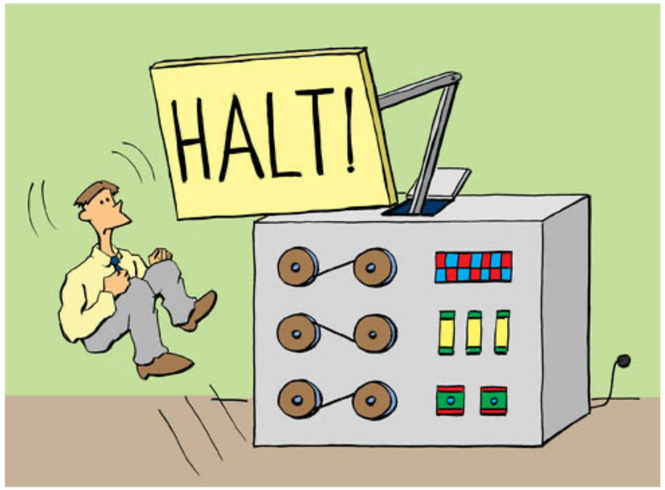
foo* bar;


    ```

Languages

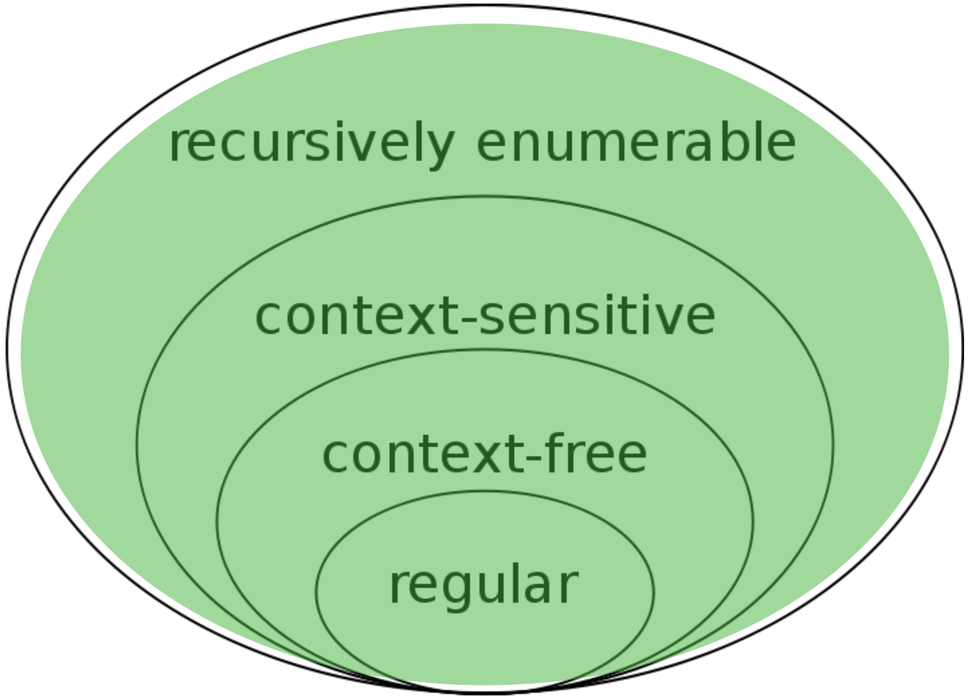
Chomsky hierarchy





Alan designed the perfect computer

Halt

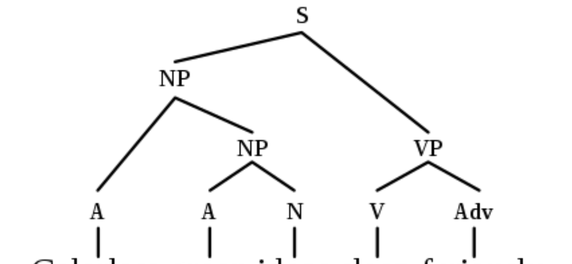


Languages

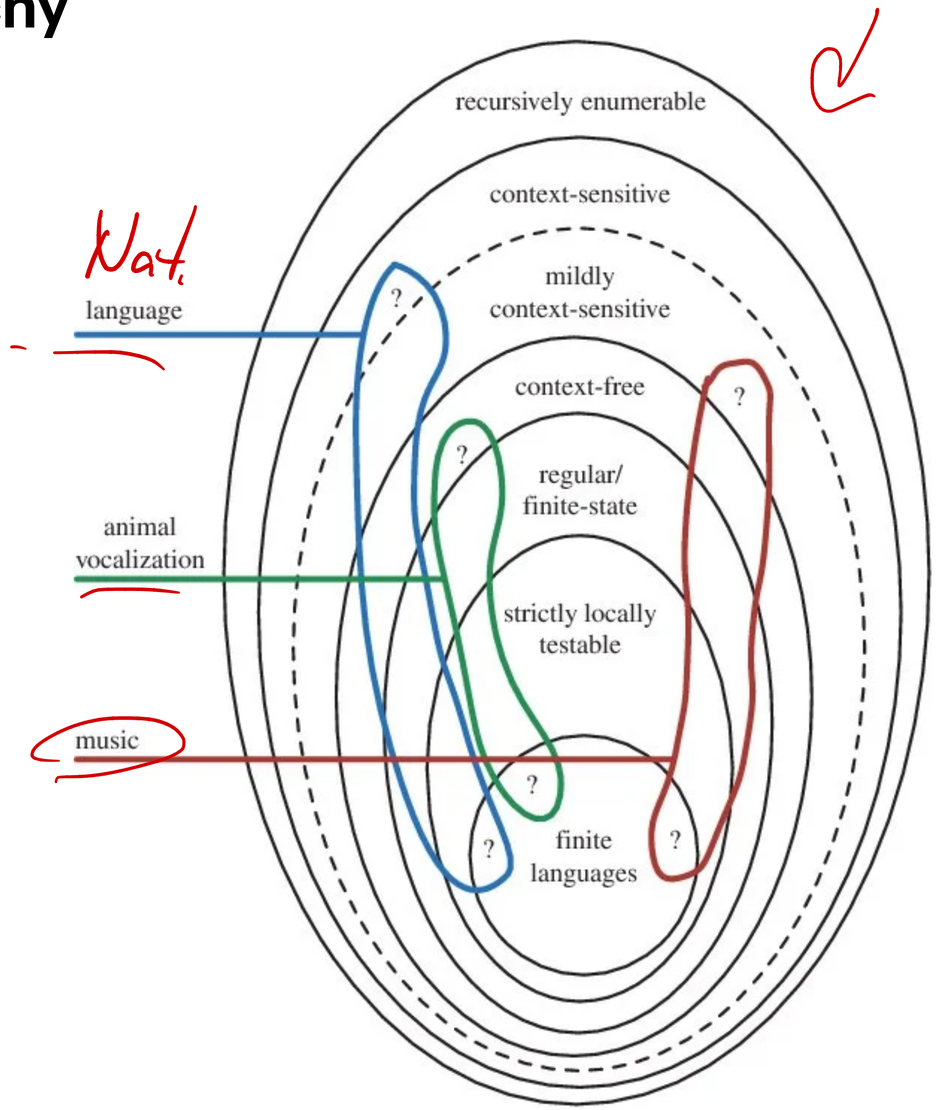
Chomsky hierarchy

- Natural Languages?

(1) S → NP + VP
(2) VP → Verb + NP
(3) NP → Det + N
(4) Verb → Aux + V
(5) Det → *the*, *a*, . . .
(6) N → *man*, *ball*, . . .
(7) Aux → *will*, *can*, . . .
(8) V → *hit*, *see*, . . .



adjectives in English absolutely have to be in this order: opinion-size-age-shape-colour-origin-material-purpose Noun. So you can have a lovely little old rectangular green French silver whittling knife. But if you mess with that word order in the slightest you'll sound like a maniac. It's an odd thing that every English speaker uses that list, but almost none of us could write it out. And as size comes before colour, green great dragons can't exist.



First Order Logic

First-order Predicate Calculus

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} &\rightarrow \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} &\rightarrow (\ \textit{Sentence}\ ) \\
&\mid \neg\ \textit{Sentence} \\
&\mid \textit{Sentence} \wedge \textit{Sentence} \\
&\mid \textit{Sentence} \vee \textit{Sentence} \\
&\mid \textit{Sentence} \Rightarrow \textit{Sentence} \\
&\mid \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
&\mid \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\textit{Term} &\rightarrow \textit{Function}(\textit{Term}, \ldots) \\
&\mid \textit{Constant} \\
&\mid \textit{Variable} \\
\textit{Quantifier} &\rightarrow \forall \mid \exists \\
\textit{Constant} &\rightarrow A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} &\rightarrow a \mid x \mid s \mid \cdots \\
\textit{Predicate} &\rightarrow \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} &\rightarrow \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\textsc{Operator Precedence} &: \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{aligned}
$$

First Order Logic

Example: Natural Numbers

- How to describe natural numbers?

$\mathbb{N} = \{0, 1, 2, \ldots\}$

First Order Logic

Example: Natural Numbers

- Peano Numbers!

$S(x)$

1. $\forall x \, (0 \neq S(x))$
2. $\forall x, y \, (S(x) = S(y) \Rightarrow x = y)$
3. $\forall x \, (x + 0 = x)$
4. $\forall x, y \, (x + S(y) = S(x + y))$
5. $\forall x \, (x \cdot 0 = 0)$
6. $\forall x, y \, (x \cdot S(y) = x \cdot y + x)$

First Order Logic

Prolog

$$
\begin{aligned}
\textit{Sentence} &\rightarrow \textit{AtomicSentence} \mid \textit{ComplexSentence} \\
\textit{AtomicSentence} &\rightarrow \textit{Predicate} \mid \textit{Predicate}(\textit{Term}, \ldots) \mid \textit{Term} = \textit{Term} \\
\textit{ComplexSentence} &\rightarrow (\ \textit{Sentence}\ ) \\
&\mid \neg\ \textit{Sentence} \\
&\mid \textit{Sentence} \wedge \textit{Sentence} \\
&\mid \textit{Sentence} \vee \textit{Sentence} \\
&\mid \textit{Sentence} \Rightarrow \textit{Sentence} \\
&\mid \textit{Sentence} \Leftrightarrow \textit{Sentence} \\
&\mid \textit{Quantifier}\ \textit{Variable}, \ldots\ \textit{Sentence} \\
\textit{Term} &\rightarrow \textit{Function}(\textit{Term}, \ldots) \\
&\mid \textit{Constant} \\
&\mid \textit{Variable} \\
\textit{Quantifier} &\rightarrow \forall \mid \exists \\
\textit{Constant} &\rightarrow A \mid X_1 \mid \textit{John} \mid \cdots \\
\textit{Variable} &\rightarrow a \mid x \mid s \mid \cdots \\
\textit{Predicate} &\rightarrow \textit{True} \mid \textit{False} \mid \textit{After} \mid \textit{Loves} \mid \textit{Raining} \mid \cdots \\
\textit{Function} &\rightarrow \textit{Mother} \mid \textit{LeftLeg} \mid \cdots \\
\text{OPERATOR PRECEDENCE} &: \neg, =, \wedge, \vee, \Rightarrow, \Leftrightarrow
\end{aligned}
$$

Prolog

```
<program>    → <clause> { <clause> }
<clause>     → <fact> | <rule>
<fact>       → <compound> .
<compound>   → <atom> ( <termlist> )
<rule>       → <head> :- <body> .
<head>       → <compound>
<body>       → <compound> { , <compound> }
<termlist>   → <term> { , <term> }
<term>       → <atom> | <num> | <var> | …
```

Notes:

- *{ <a> } means 0, 1 or more <a>'s*
- *This grammar is slightly more restrictive than the actual language*

First Order Logic

Example: Peano Numbers

$$\forall x \, (0 \neq S(x))$$
$$\forall x, y \, (S(x) = S(y) \Rightarrow x = y)$$
$$\forall x \, (x + 0 = x)$$
$$\forall x, y \, (x + S(y) = S(x + y))$$
$$\forall x \, (x \cdot 0 = 0)$$
$$\forall x, y \, (x \cdot S(y) = x \cdot y + x)$$

First Order Logic

Example: Sudoku

A B C D

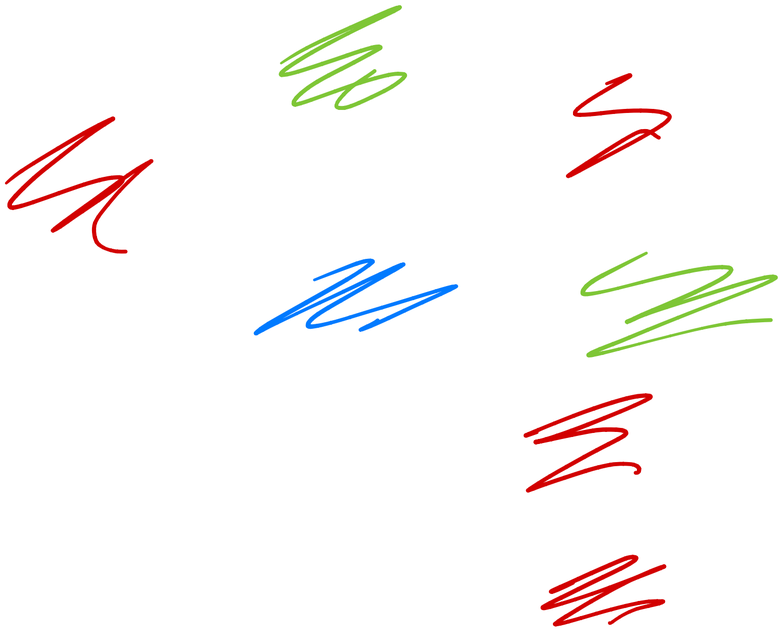
E F

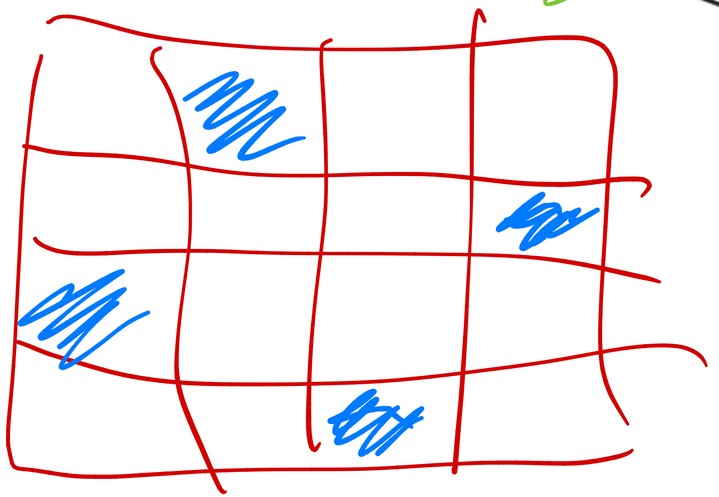
M N O P

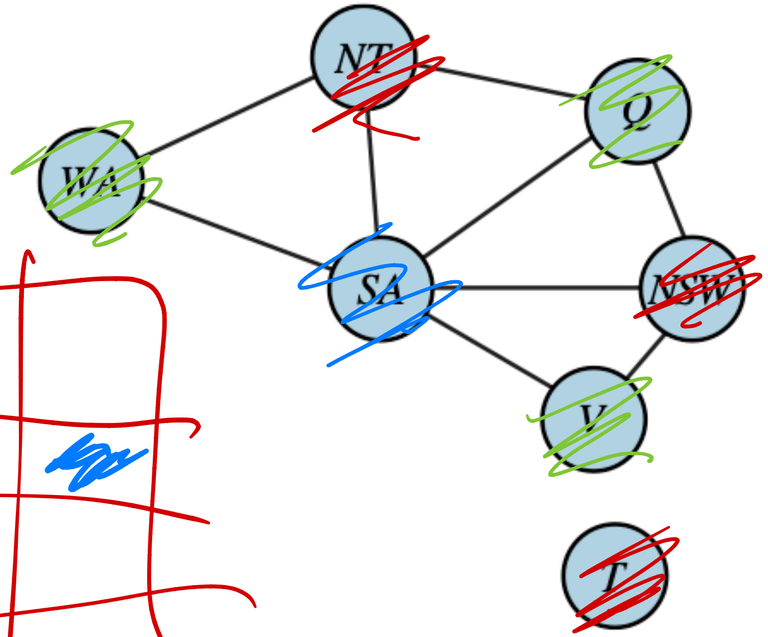
4	3	2	1
2	1	4	3
1	2	3	4
3	4	1	2

First Order Logic

Example: Sudoku







RUTGERS

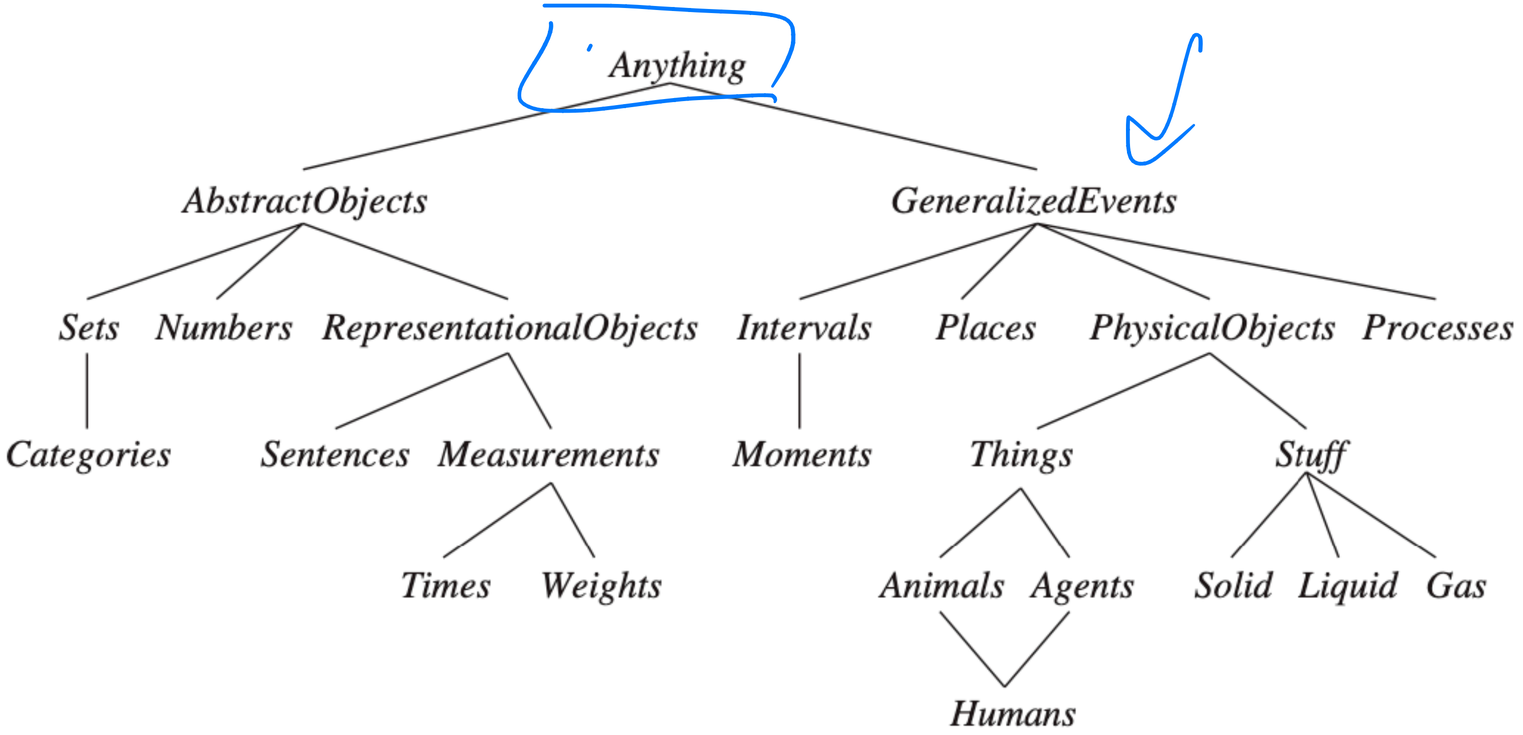
THE STATE UNIVERSITY
OF NEW JERSEY

Logic, Knowledge and Reasoning

Knowledge Representation

Edgar Granados

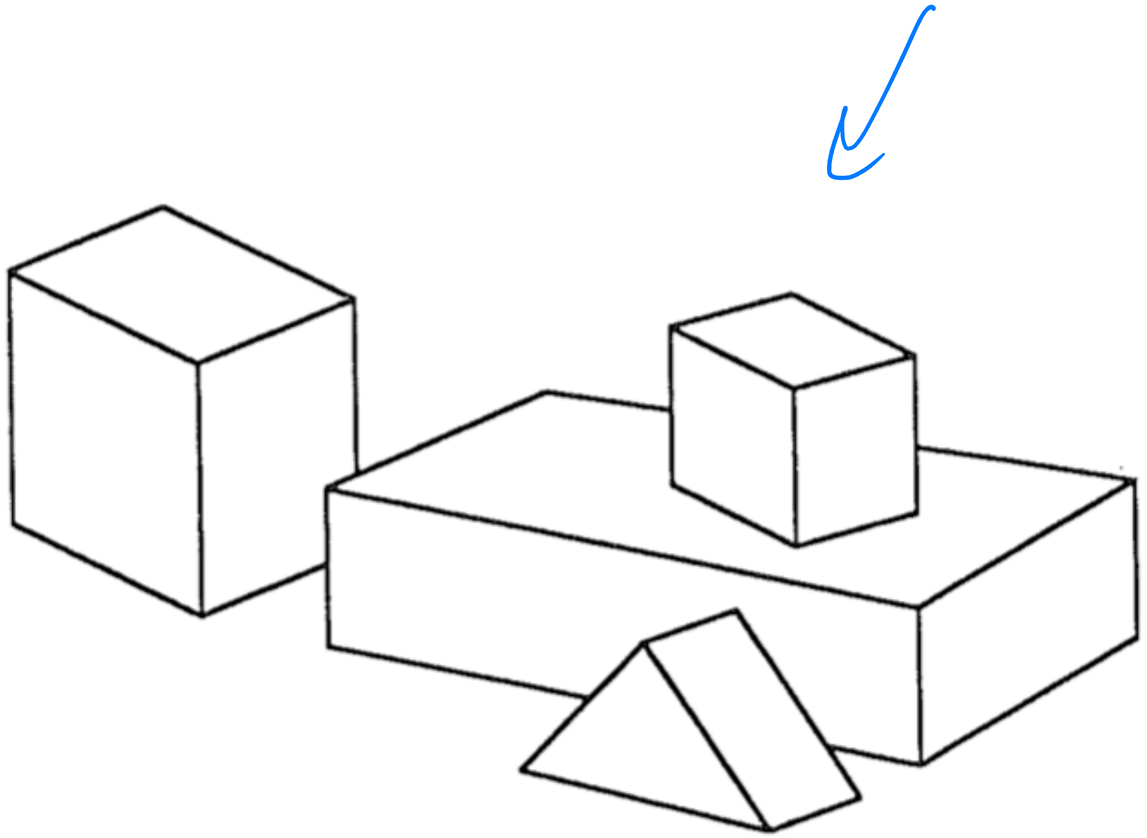
Knowledge Representation



Knowledge Representation

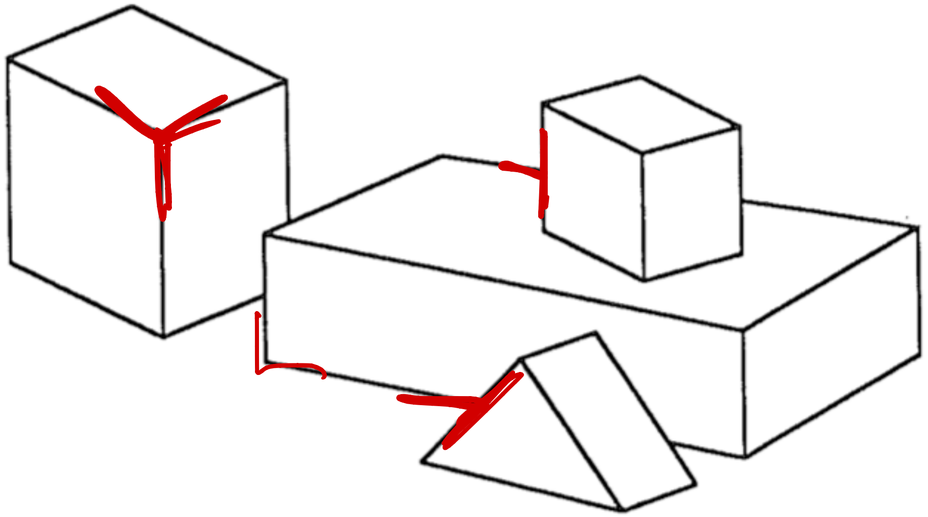
Example: Simple Images

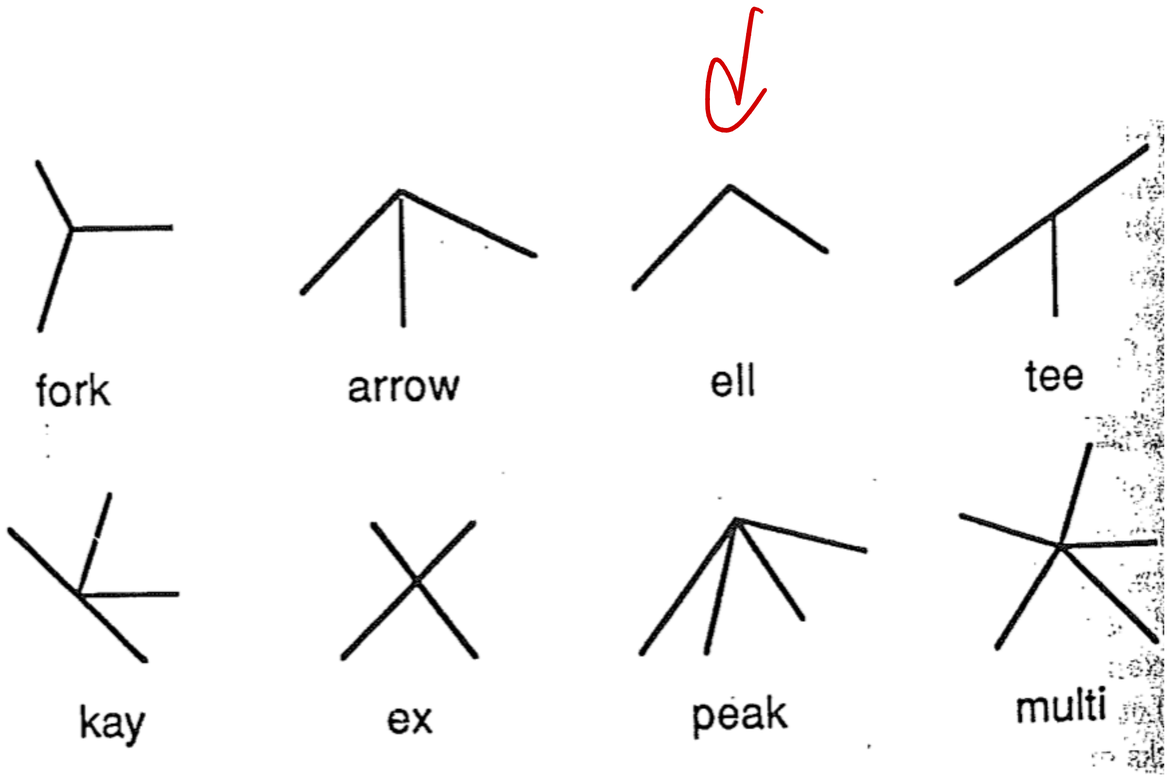
- Objective:
  - Find objects - Prism, pyramid, etc



Knowledge Representation

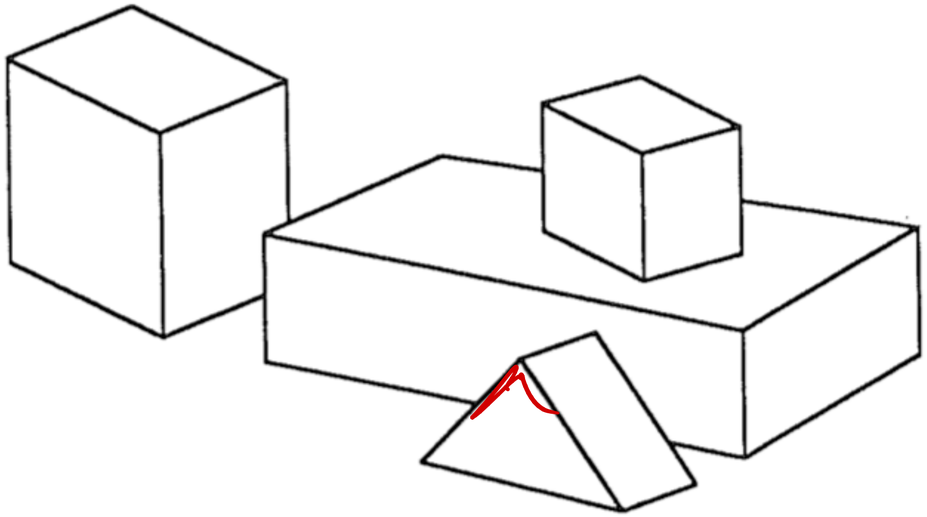
Example: Simple Images

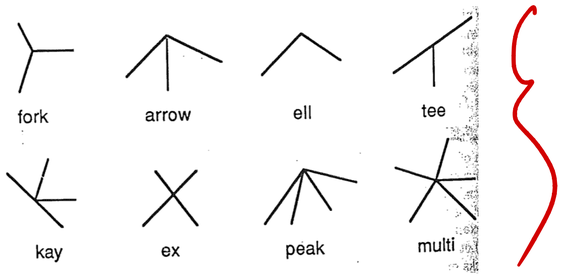


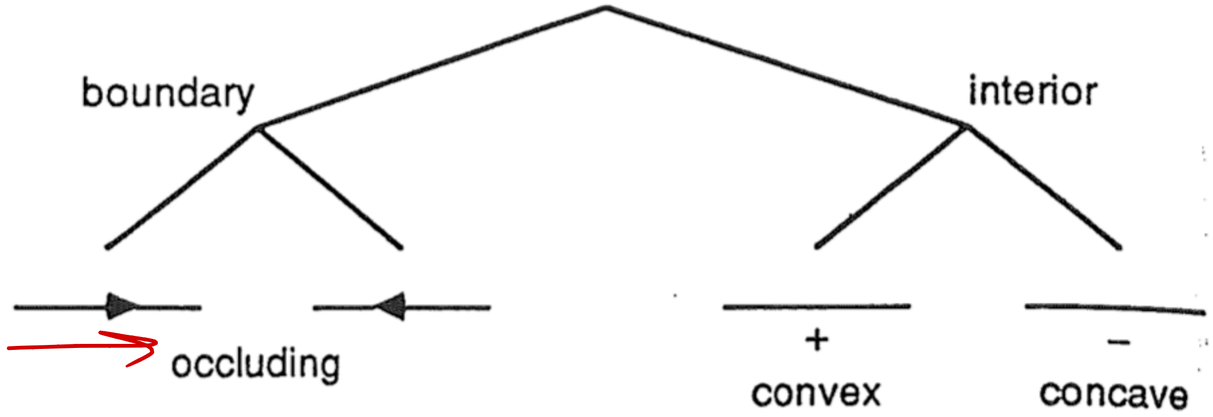


Knowledge Representation

Example: Simple Images

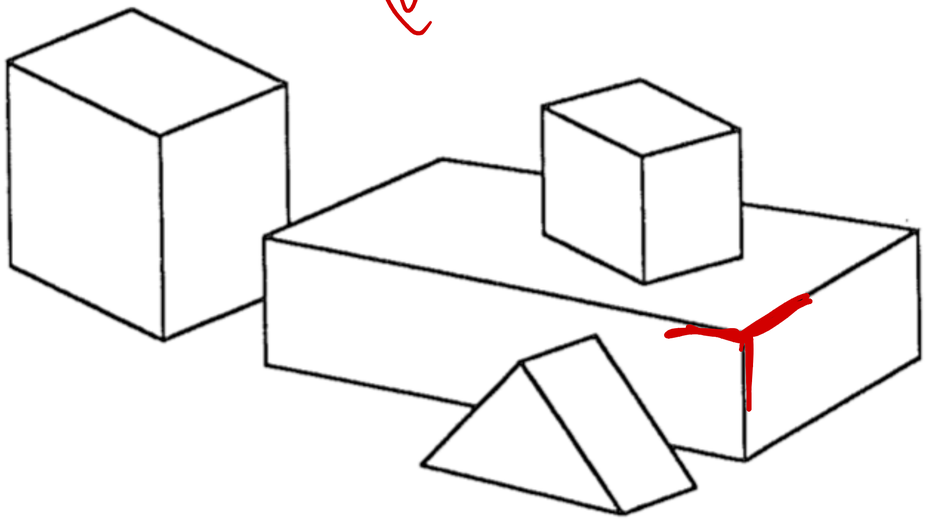


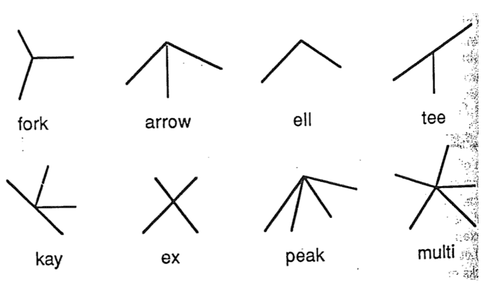


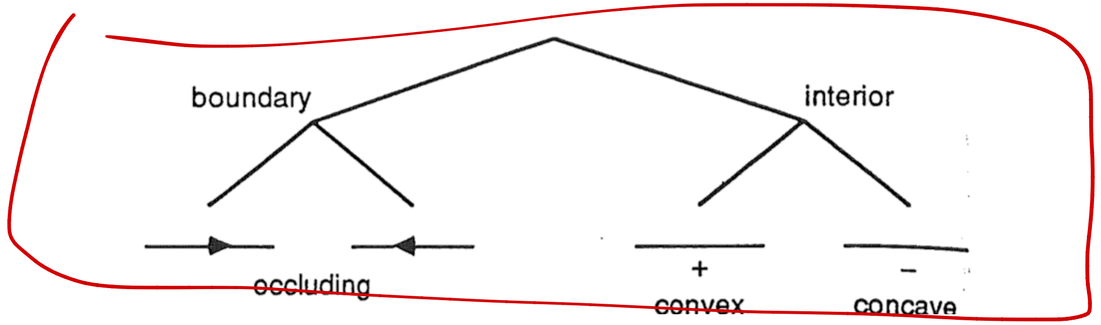


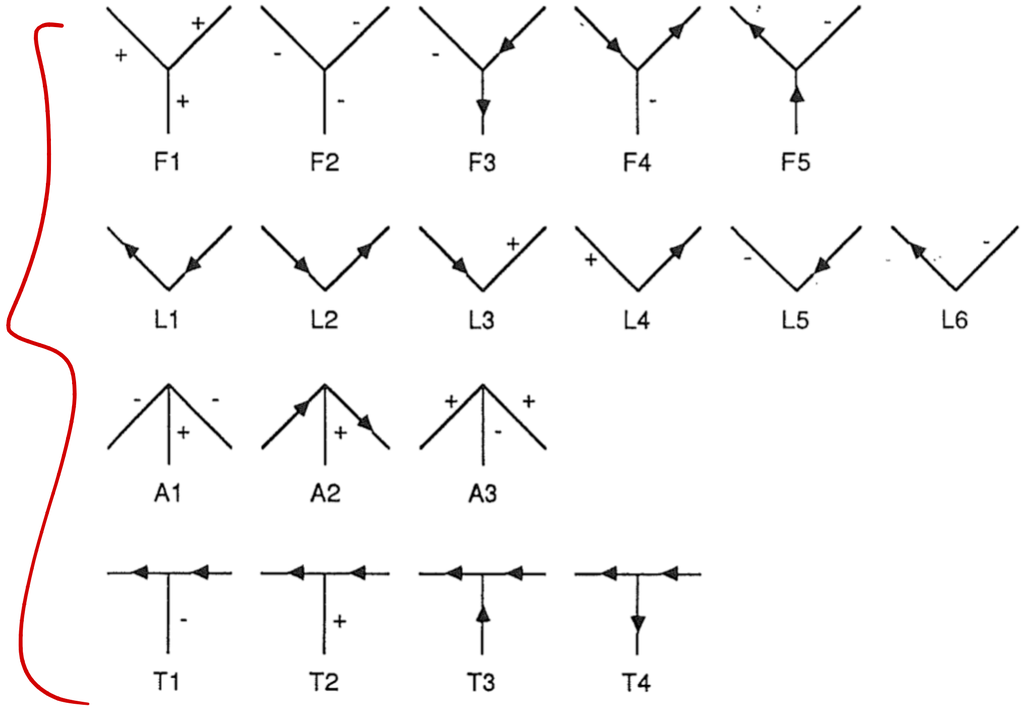
Knowledge Representation

Example: Simple Images



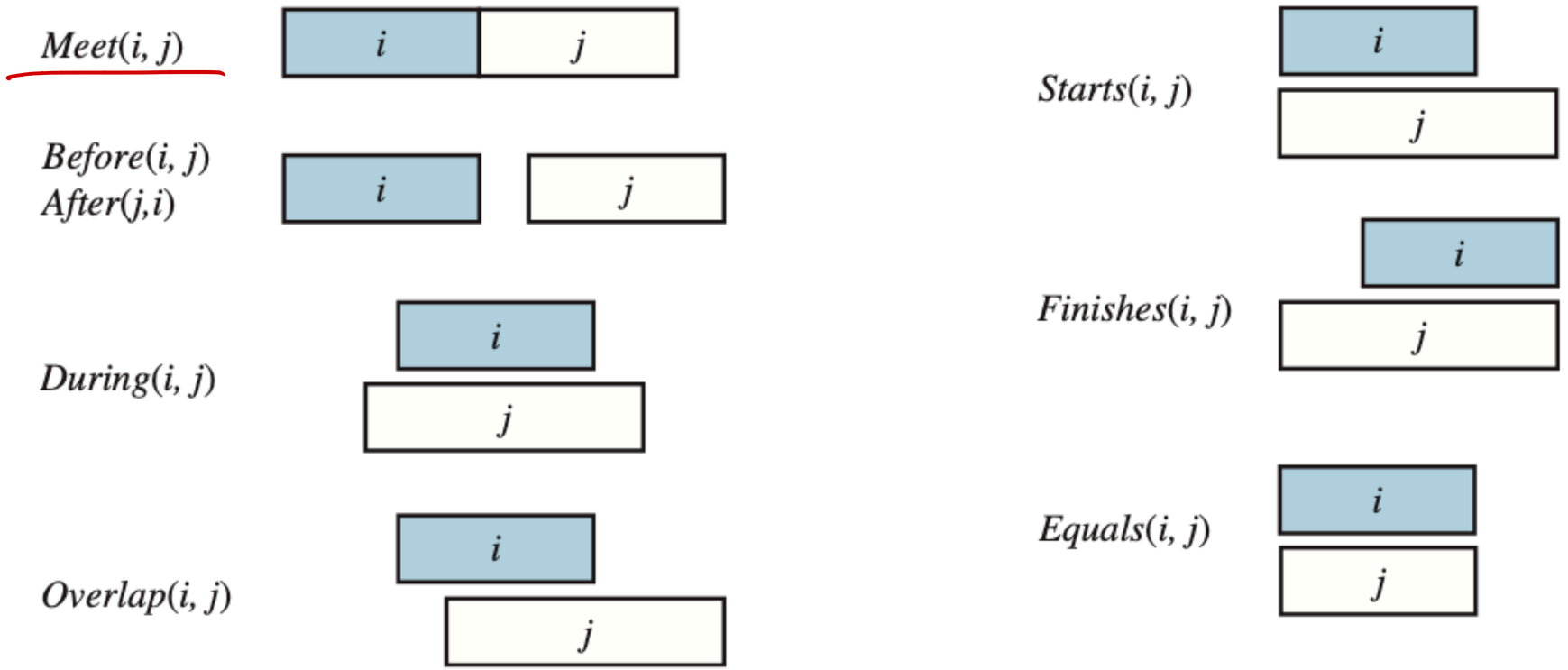






Knowledge Representation

Time - Duration



Languages

Language	Ontological Commitment (What exists in the world)	Epistemological Commitment (What an agent believes about facts)
Propositional logic	facts	true/false/unknown
First-order logic	facts, objects, relations	true/false/unknown
Temporal logic	facts, objects, relations, times	true/false/unknown
Probability theory	facts	degree of belief $\in [0, 1]$
Fuzzy logic	facts with degree of truth $\in [0, 1]$	known interval value