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Advance Search

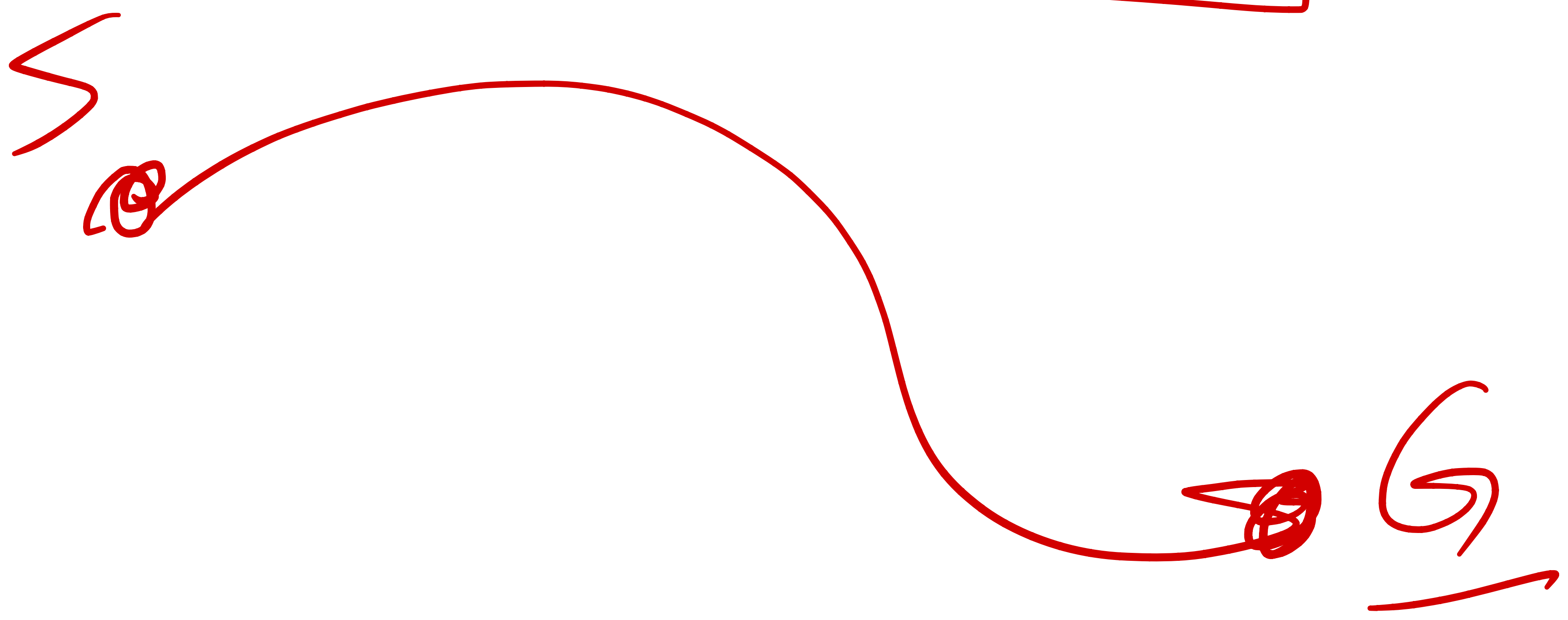
Local Search

Edgar Granados

Problem Solving by Searching

(Recap)

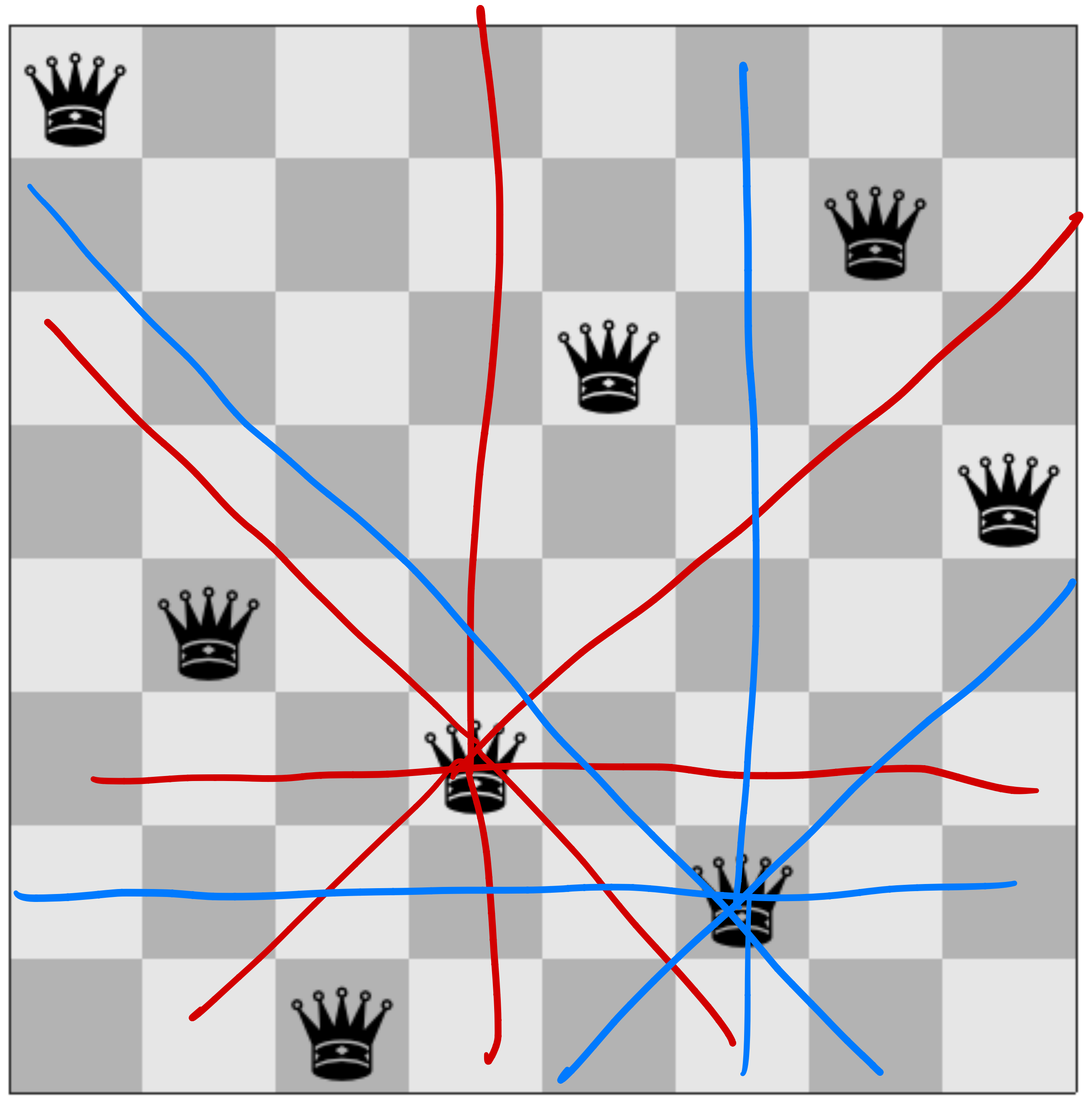
- Find the best sequence of actions - **PATHS**

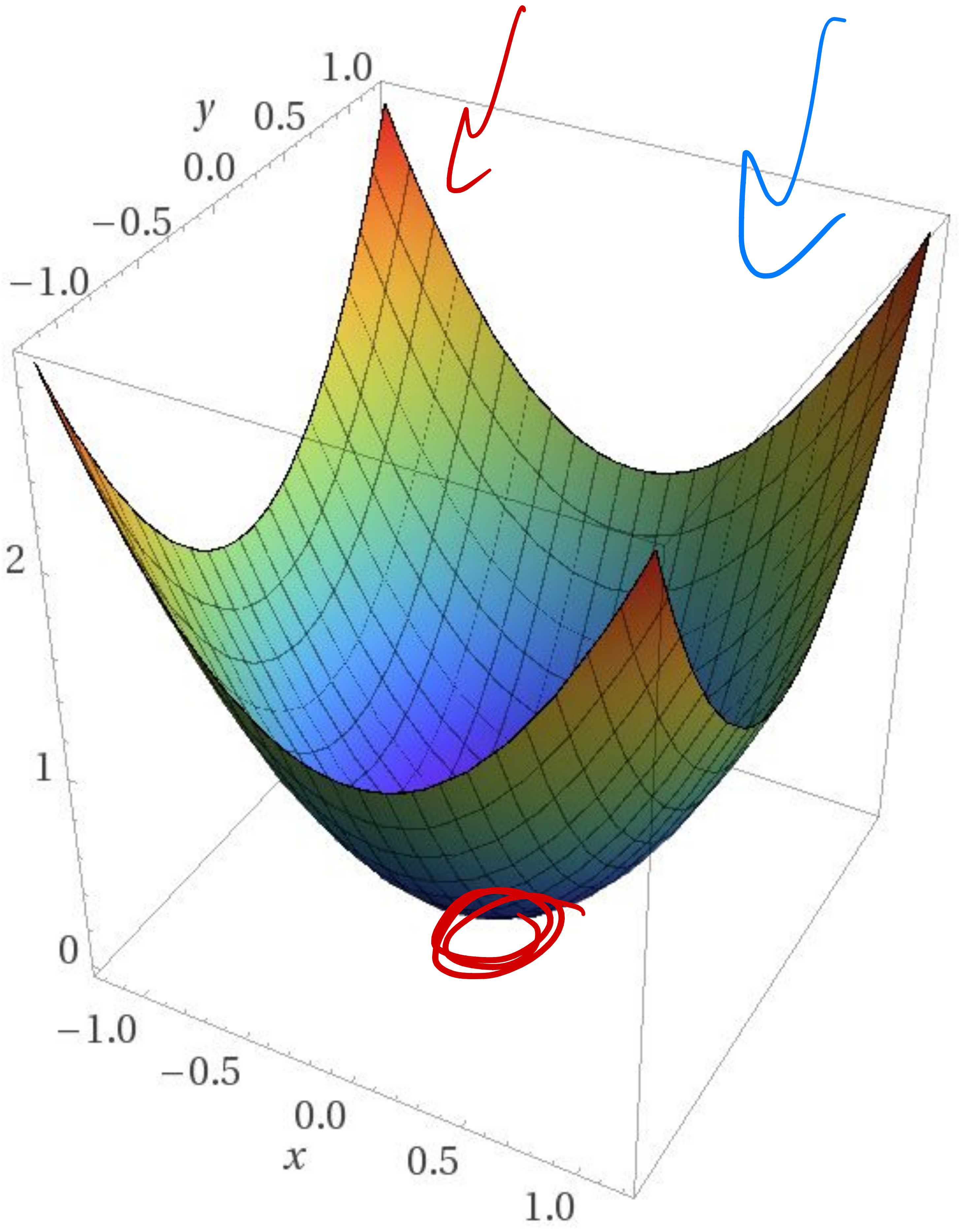


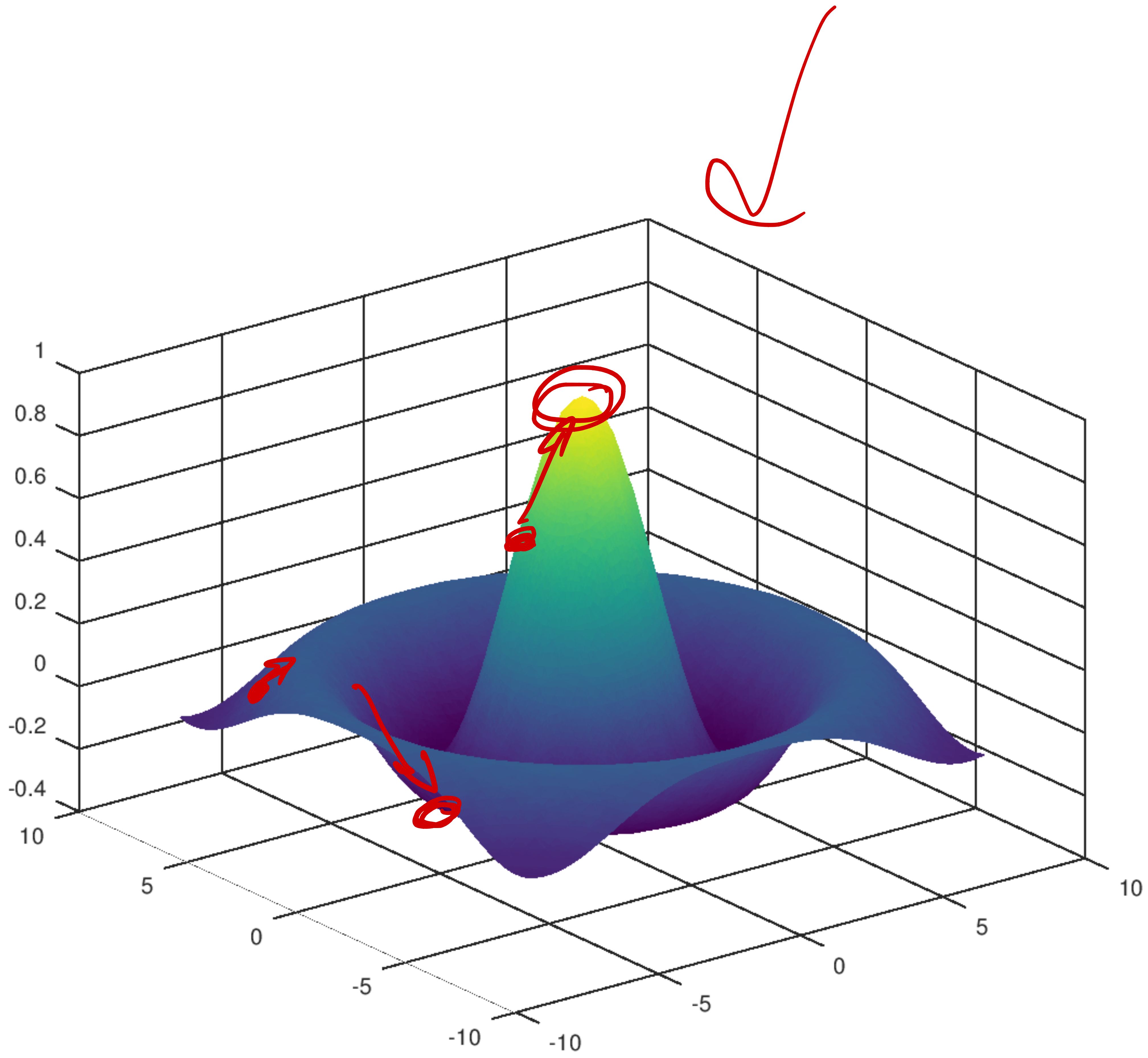
Problem Solving by Searching

(Recap)

N
8 Queen

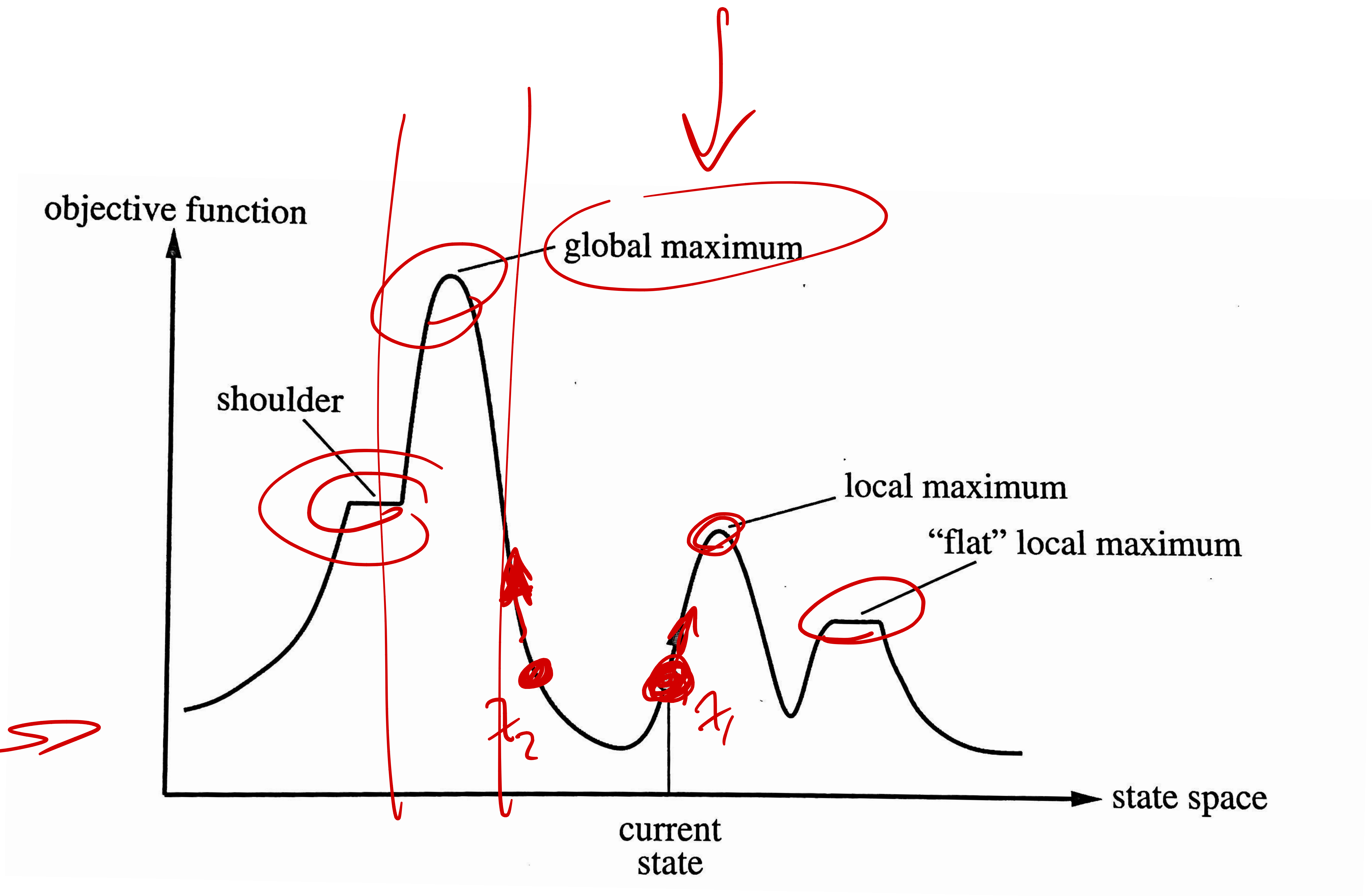






Local Search

- Path not important
  - Only finding the goal
- Move locally
  - Less memory
- Optimization Problems



Local Search

Optimizing

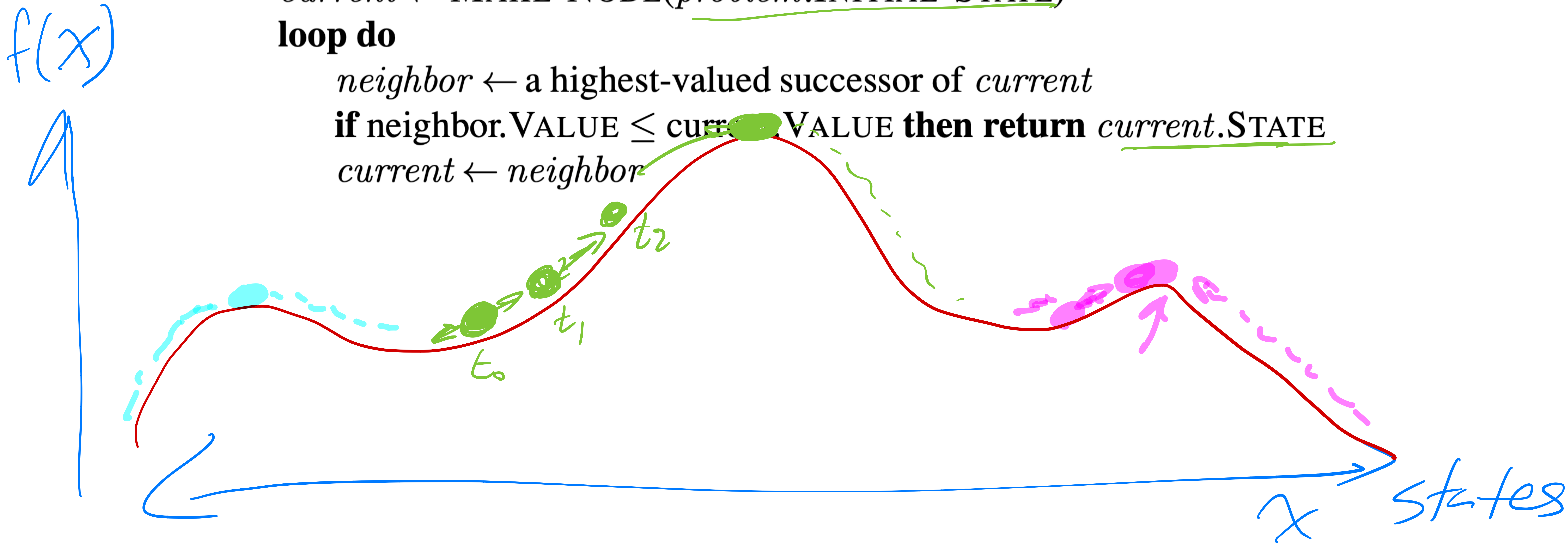
- Optimum
  - Maximize? Reward
  - Minimize? → Penalty
- Global Optimum vs Local Optimum

Hill Climbing

Greedy Local Search

function HILL-CLIMBING(*problem*) **returns** a state that is a local maximum

    current ← MAKE-NODE(problem.INITIAL-STATE)
    loop do
        neighbor ← a highest-valued successor of current
        if neighbor.VALUE ≤ current.VALUE then return current.STATE
        current ← neighbor



Hill Climbing

Example

17

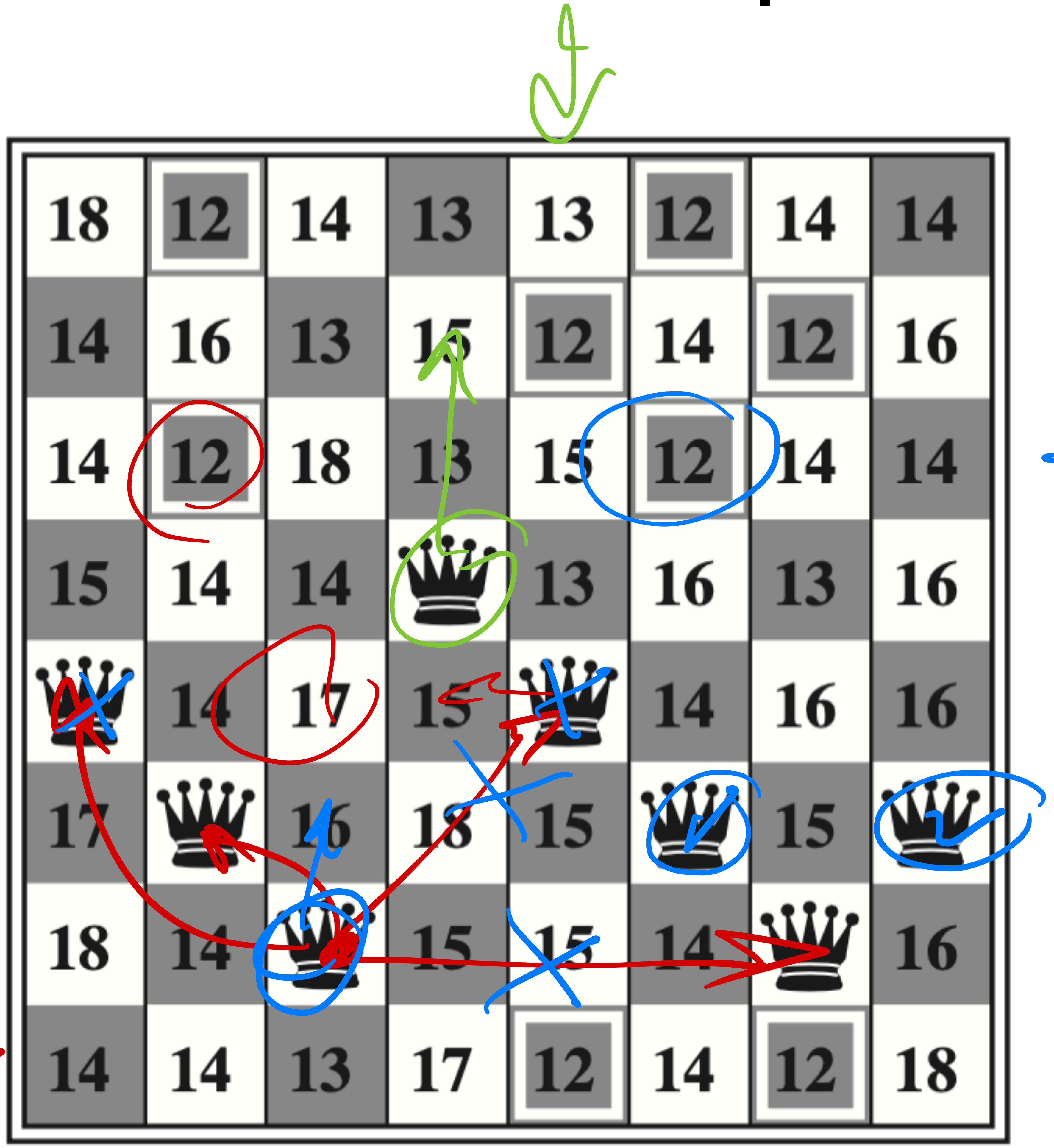
- f(x) = Number of pairs of attacking Queens
- Move queens vertically

3+4+2 = 9

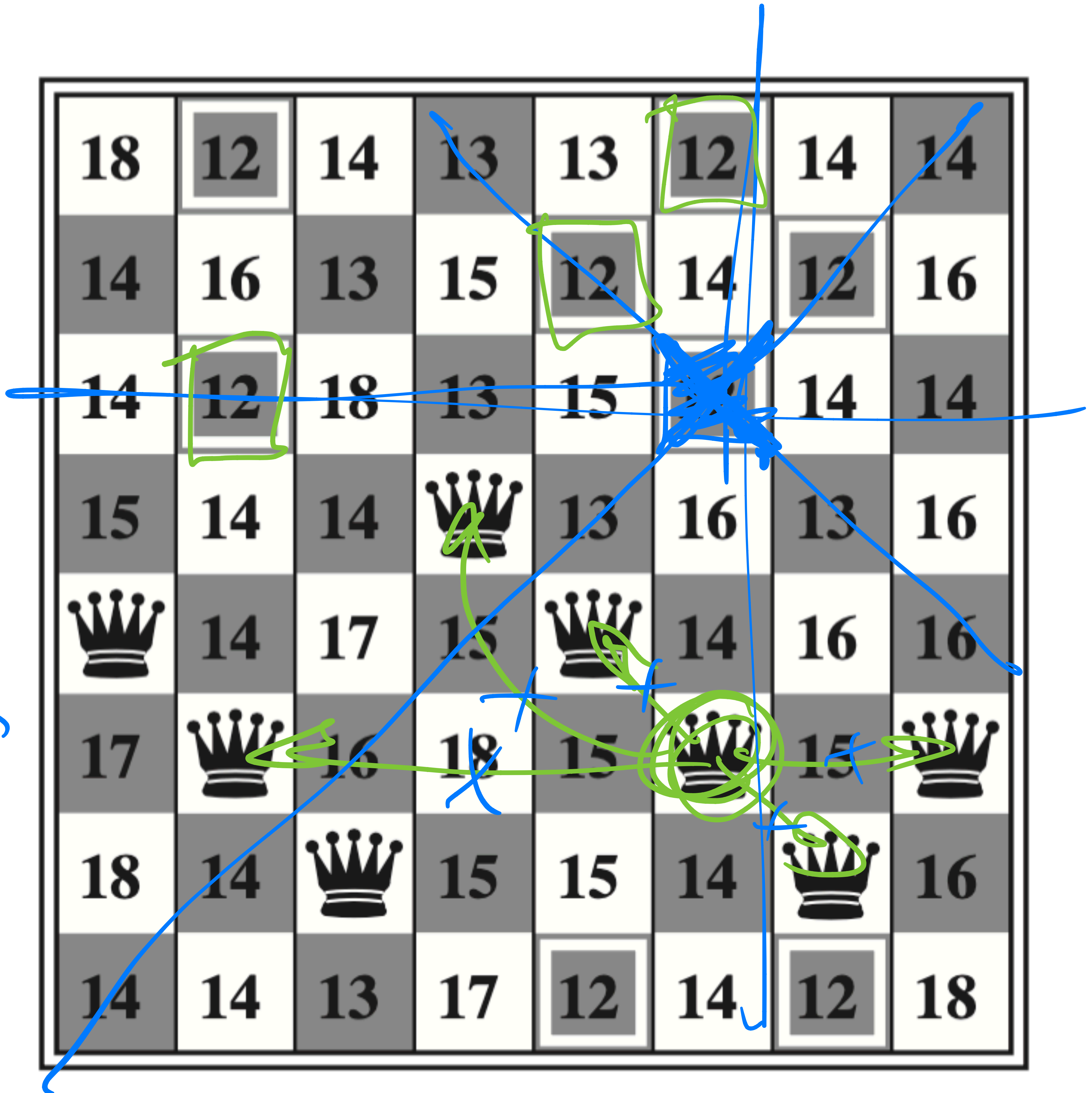
3+2+2 = 7

1 1

17



17 - 3 + 2 = 16

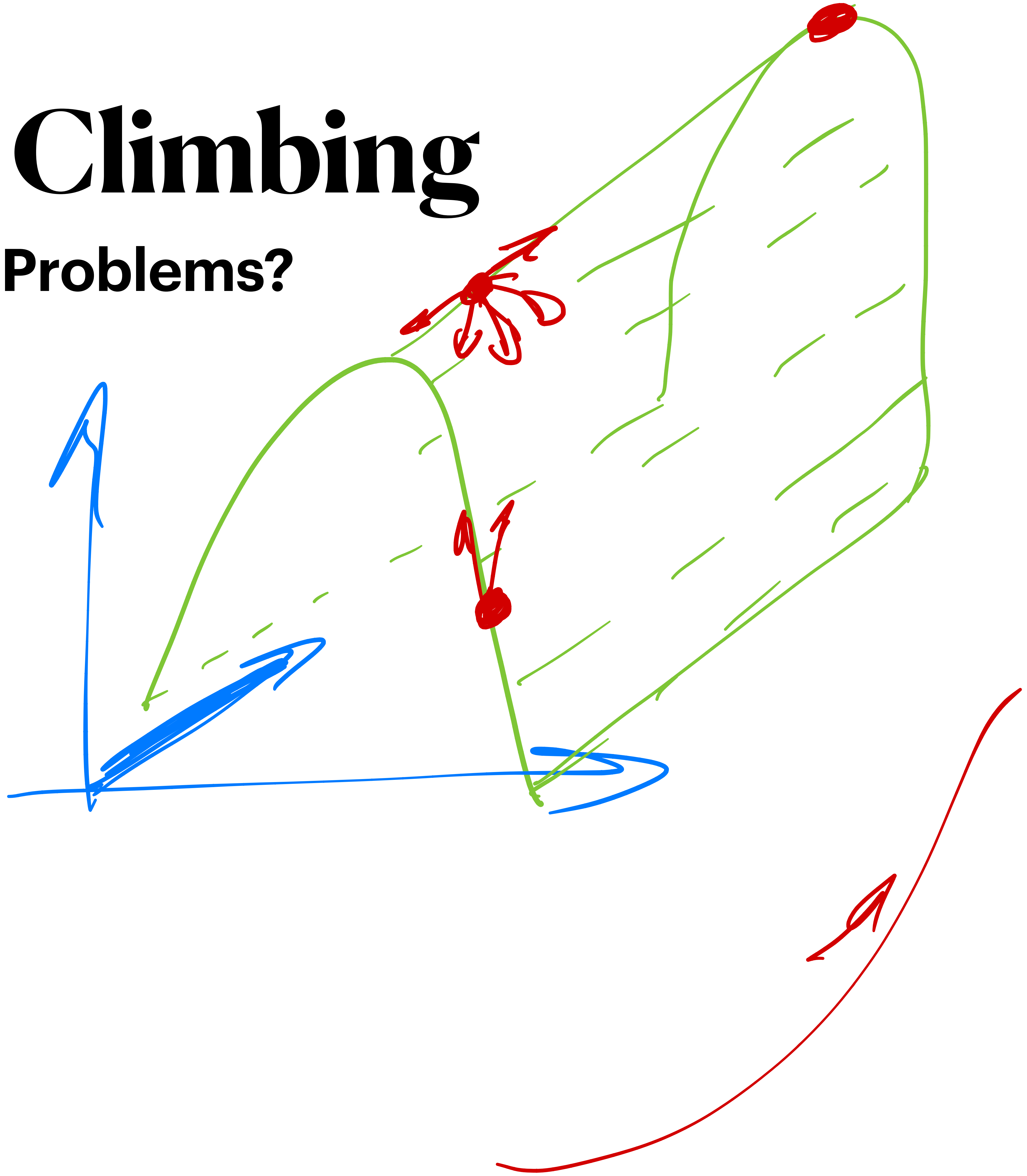


17 - 5 = 12

Hill Climbing

Problems?

1. local optimum
2. Ridges
3. Plateaus



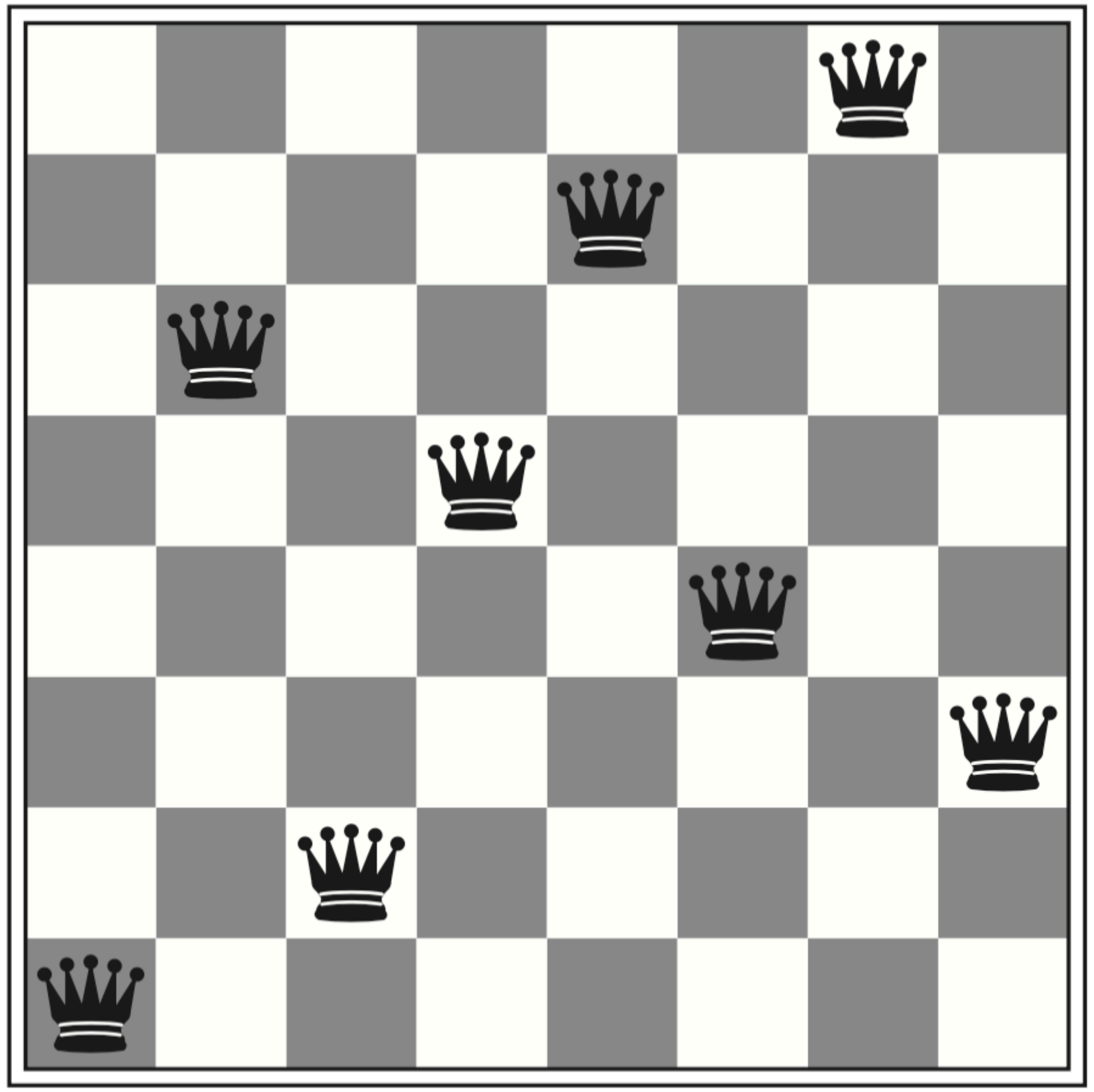
Hill Climbing

8 Queens

min $f(x) = 0$

- Stuck at local maxima 86% of the time
- Finds local maxima 14%

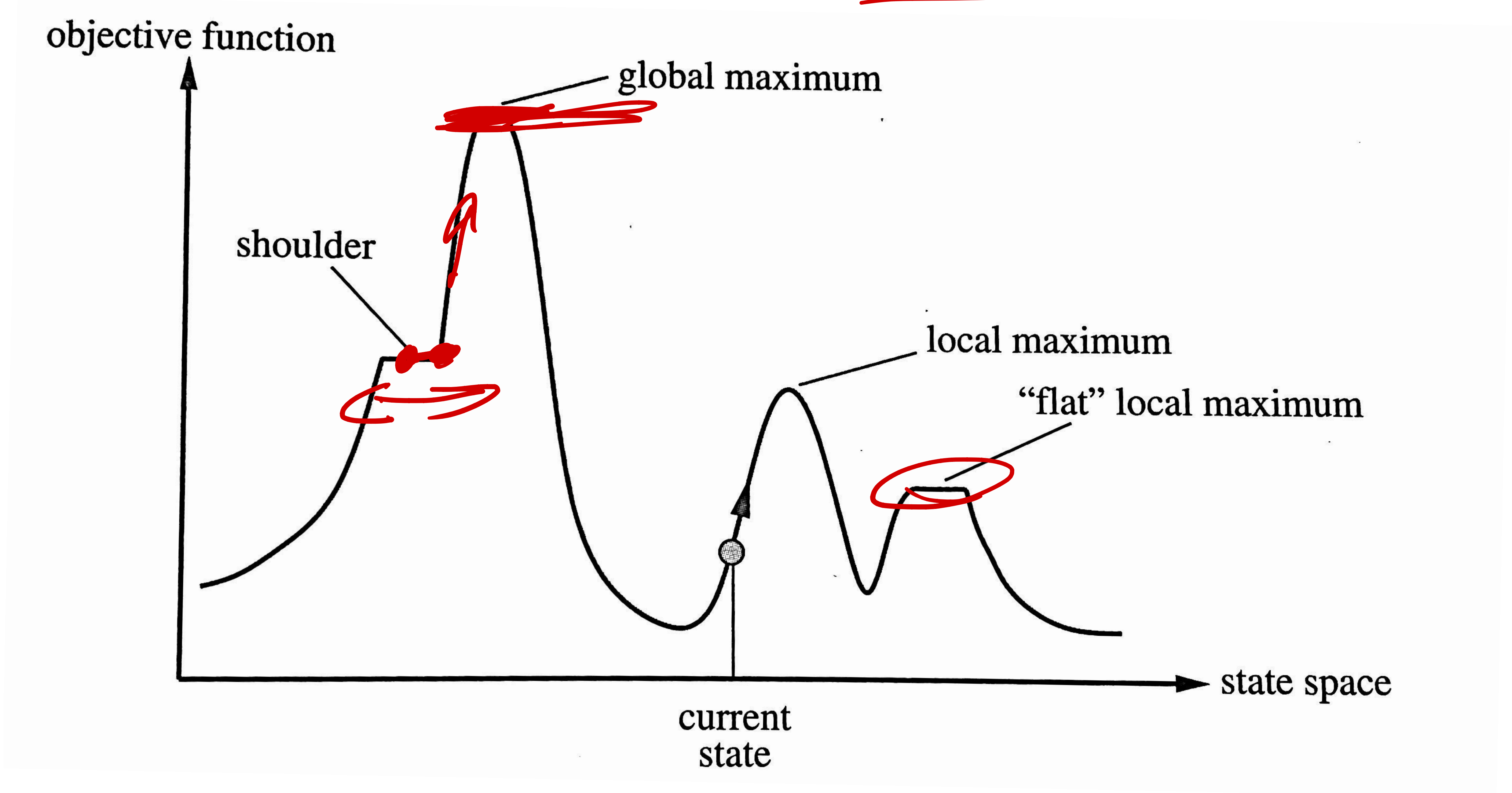
- Solutions?

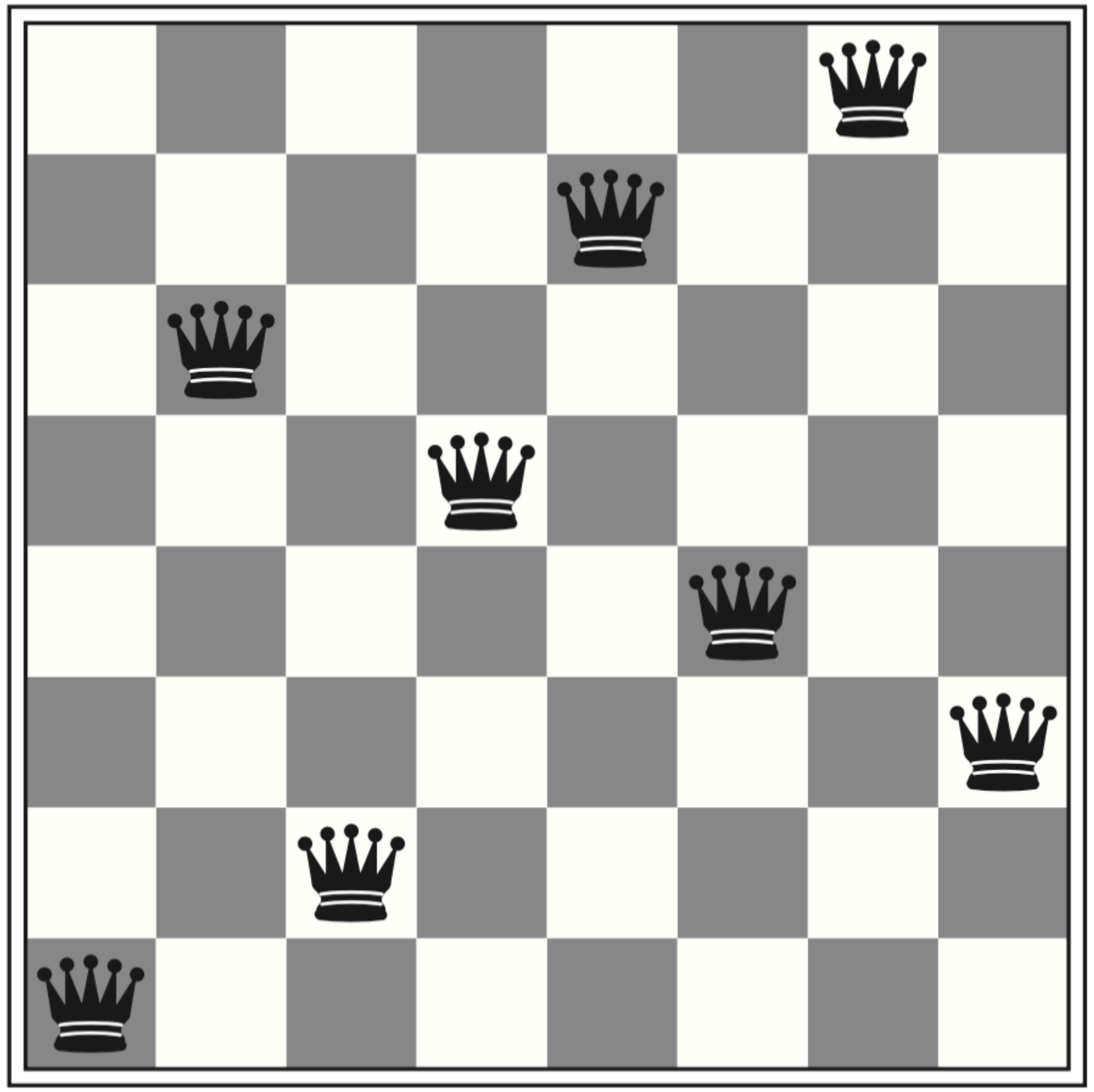


Hill Climbing

Move Sideways

- Move Sideways!
  - Avoid being stuck at shoulders
- 8 Queen - Finds global maxima 94%



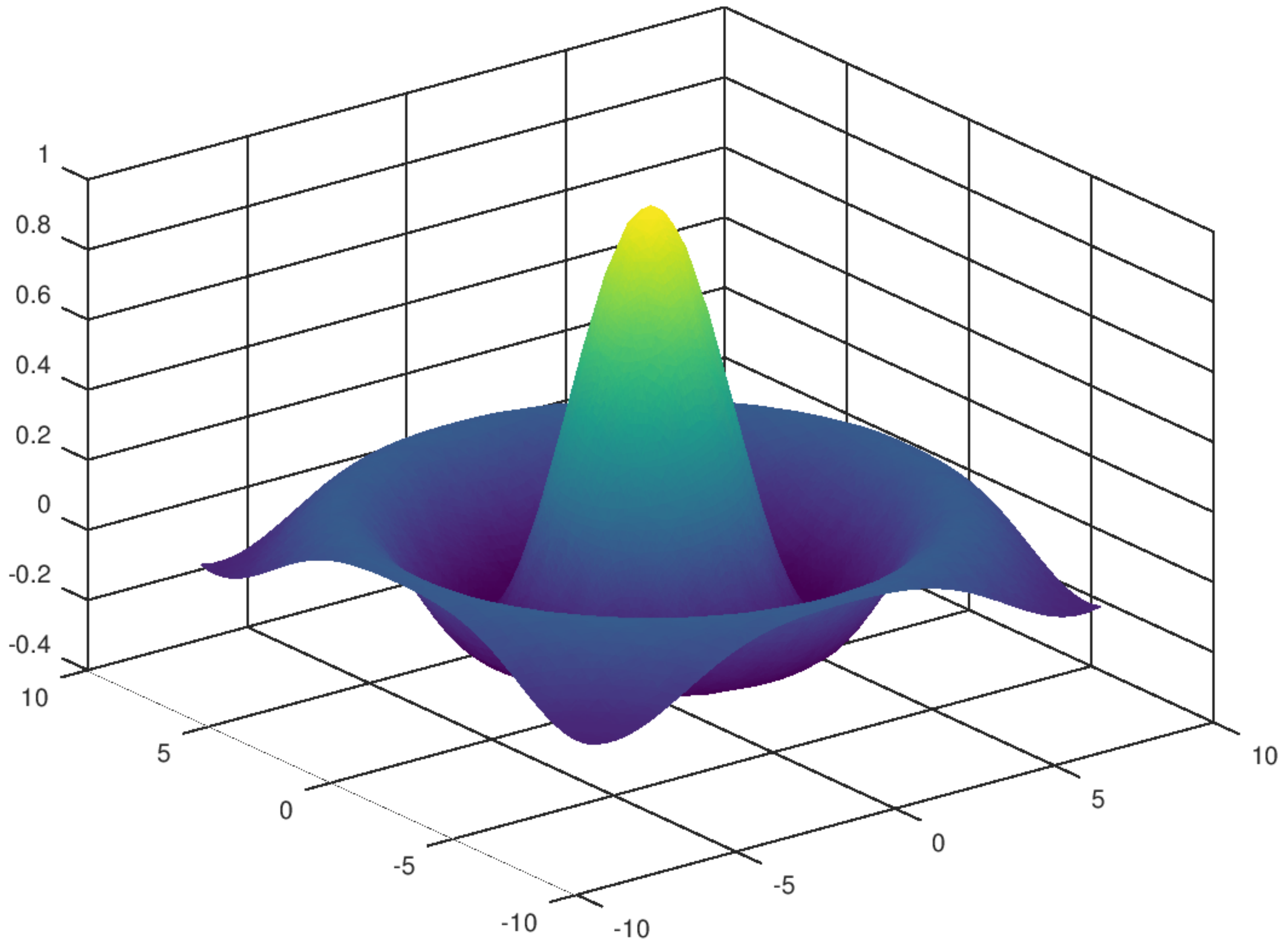


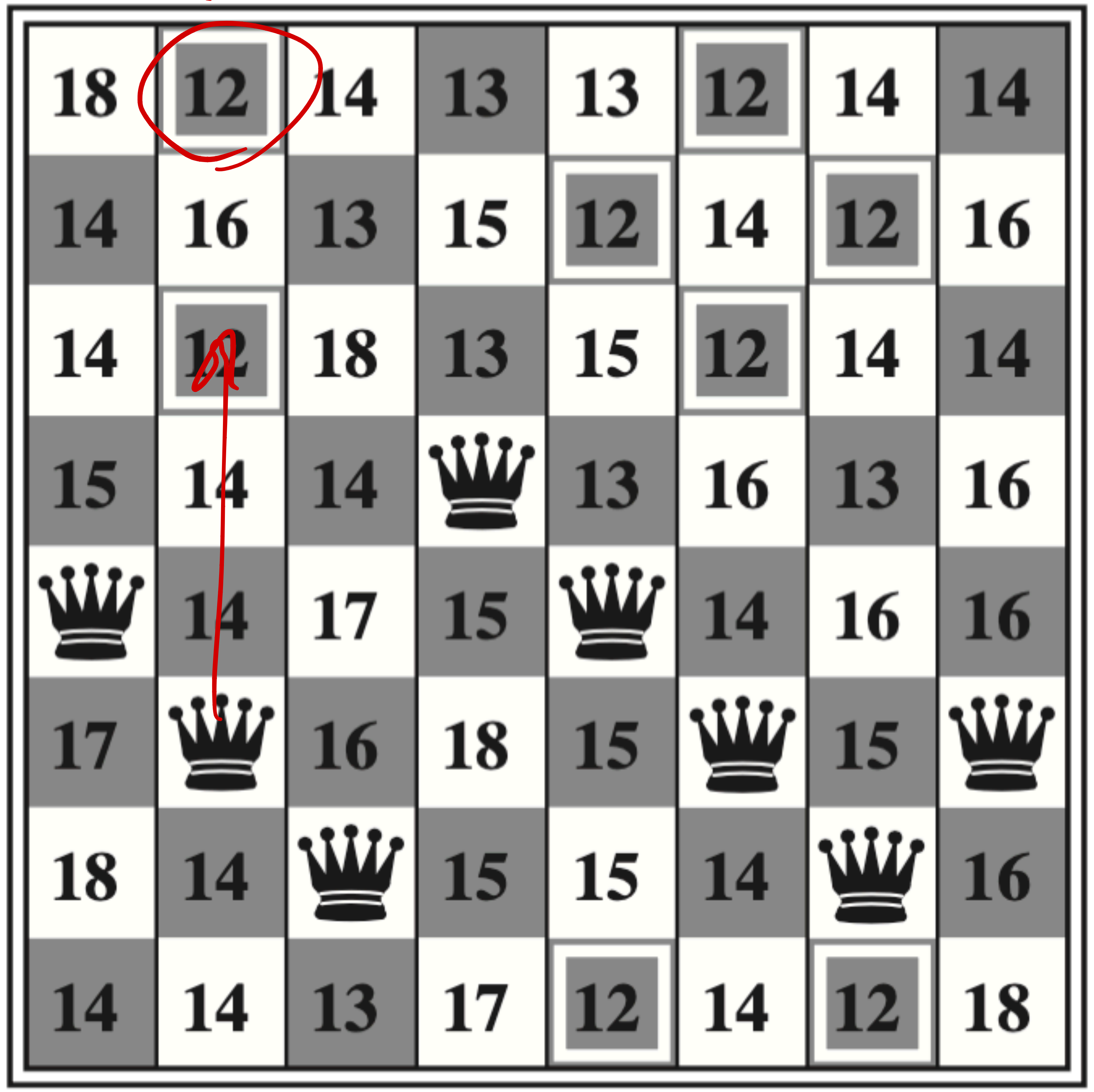
Hill Climbing

Stochastic

$P_{12} > P_{13} > P_{14} > \ldots > P_{17}$

- Choose randomly among uphill moves
  - Probabilities vary with the steepness of the moves

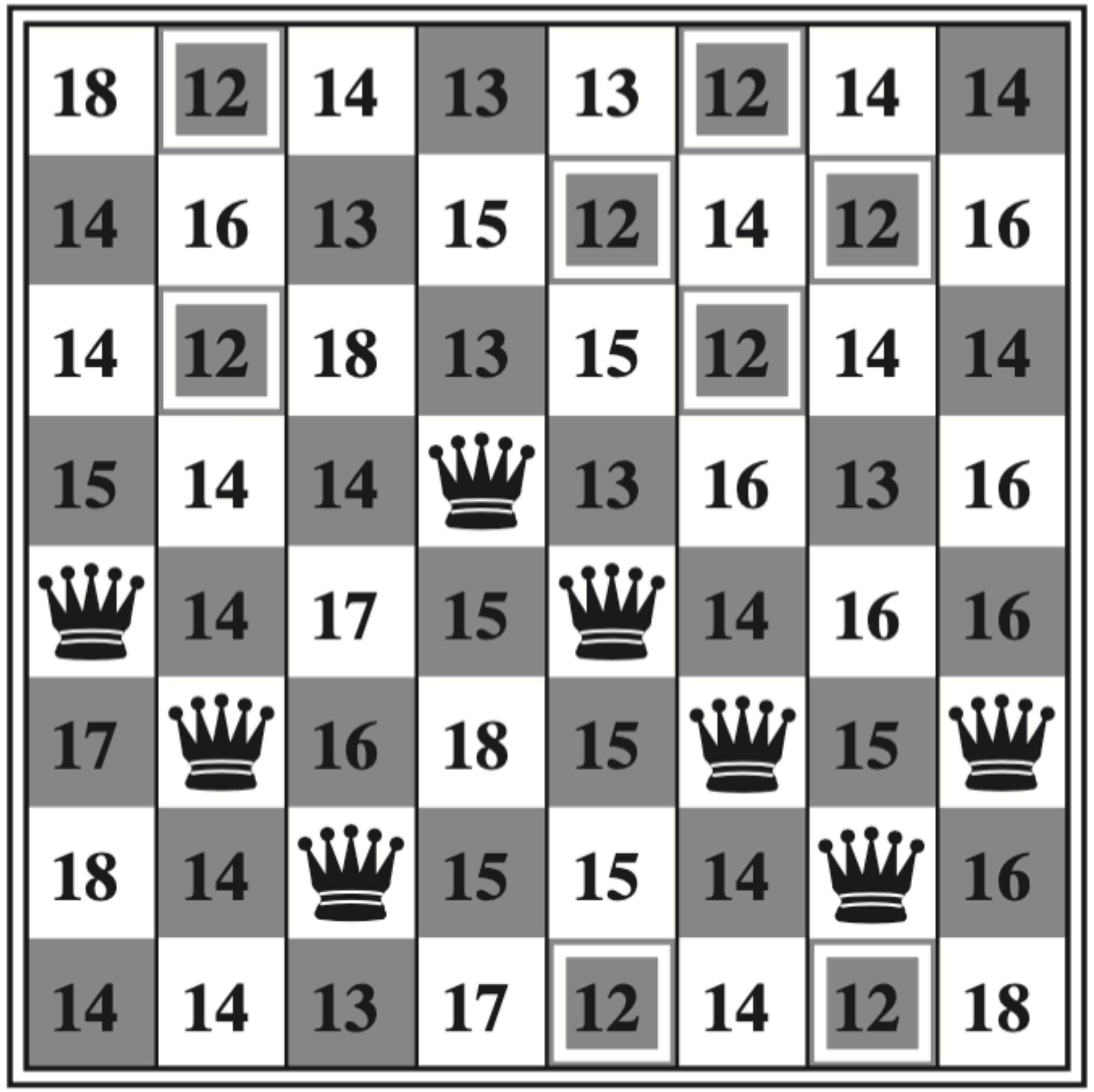




Hill Climbing

First-choice

- Version of Stochastic Hill Climbing
  - Generate random successors



Hill Climbing

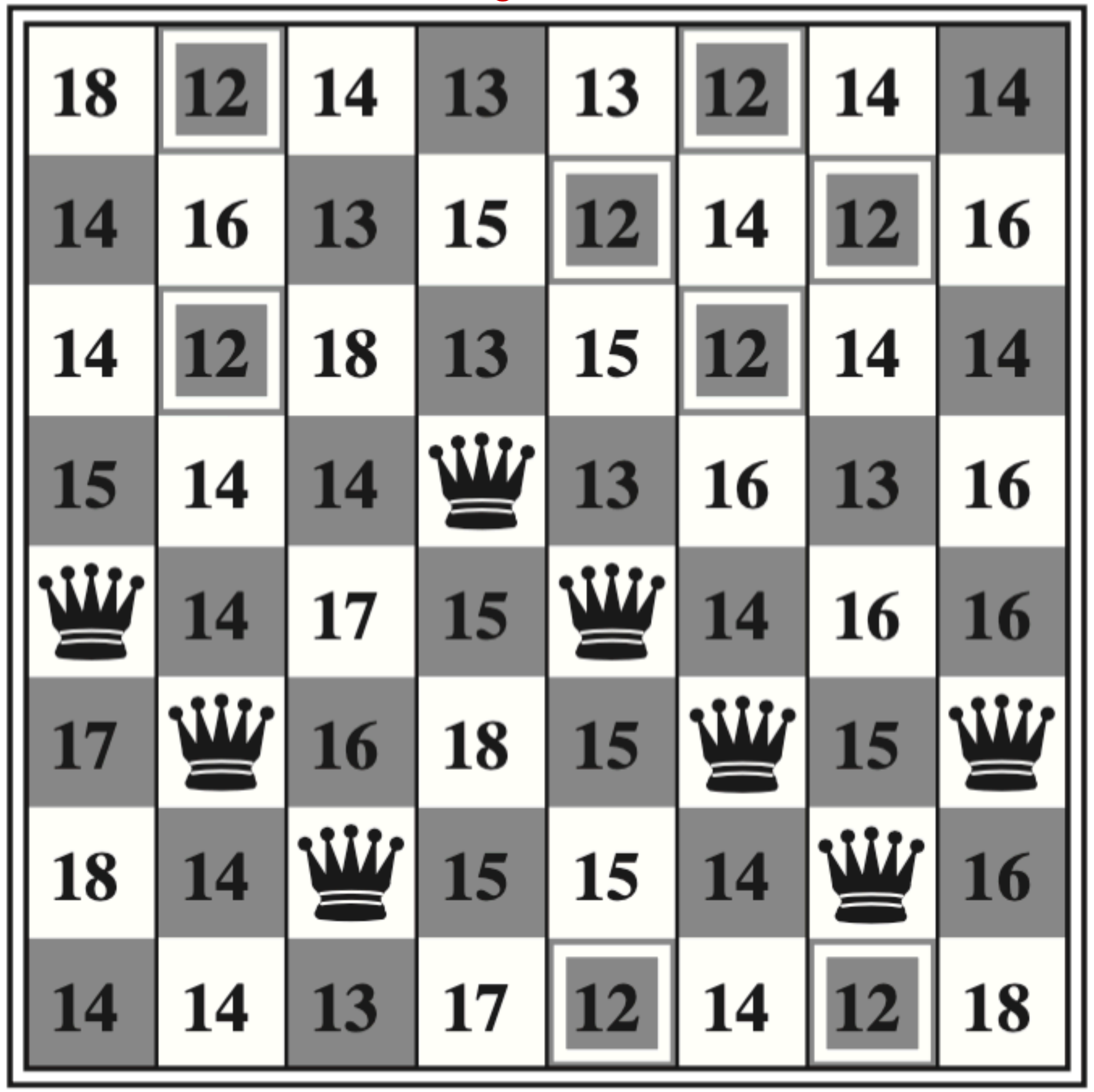
Random-Restart

- Repeat HC from random initial states
- Guaranteed to find the optimal solution if state space is finite

14%

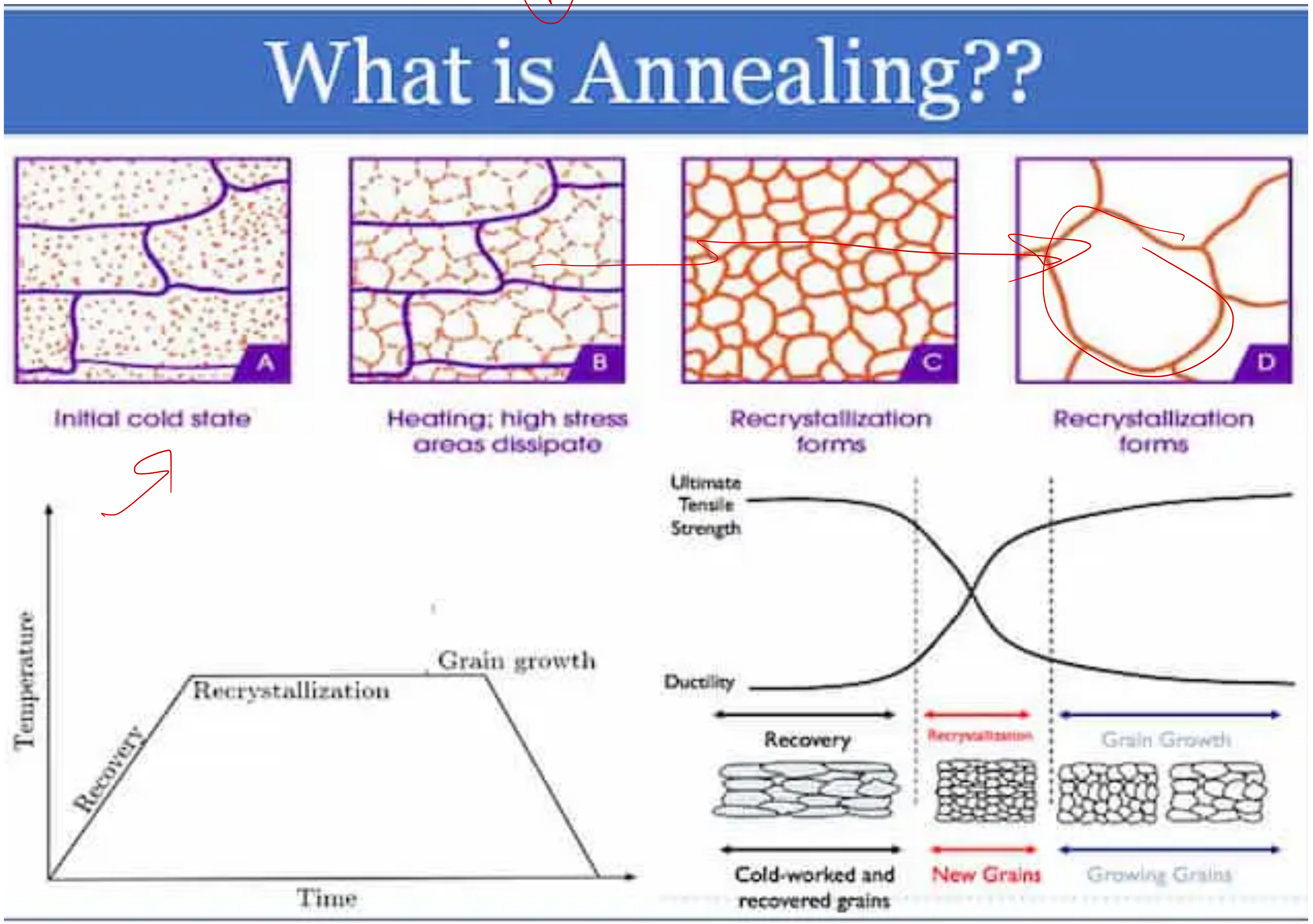
Hill Climbing

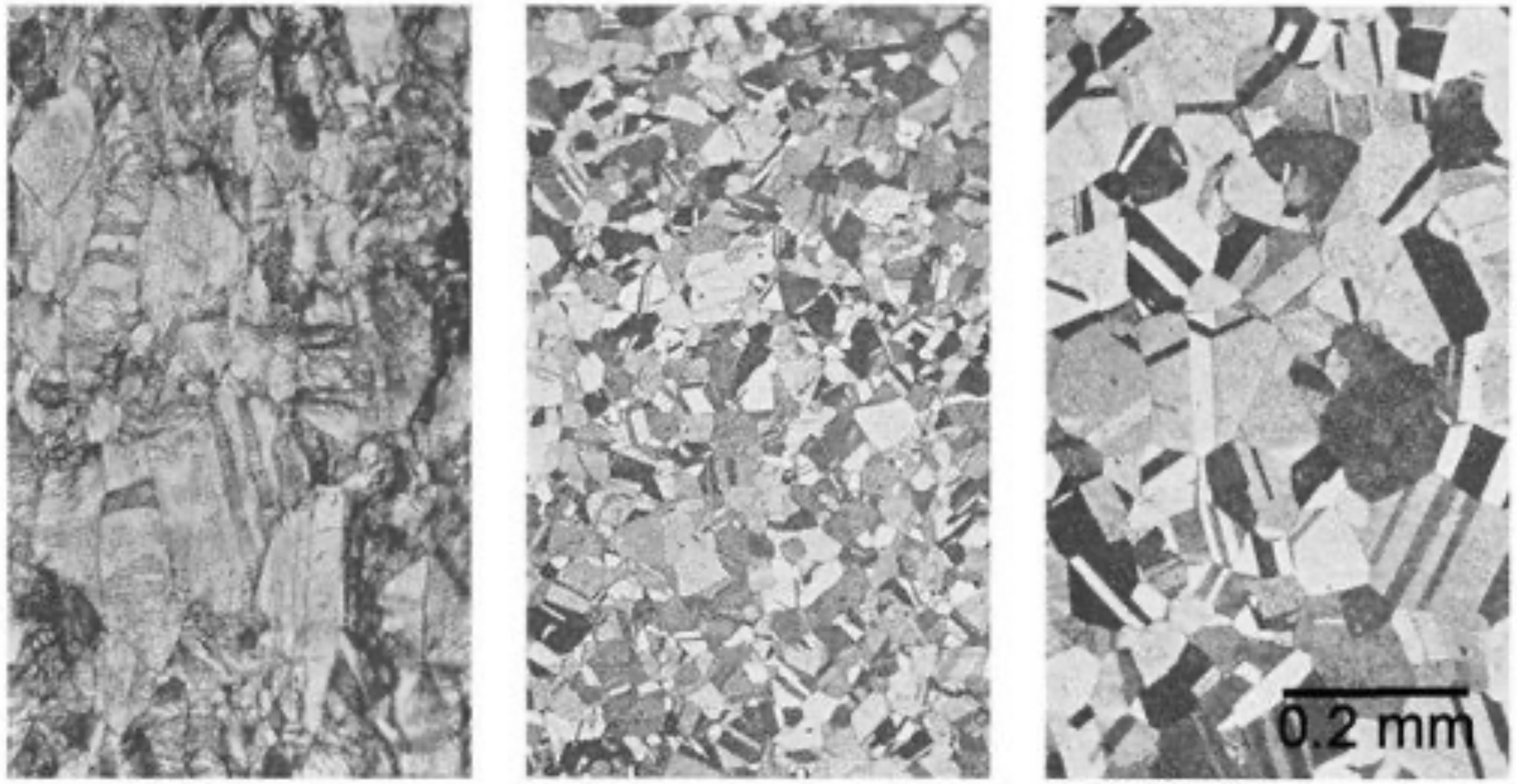
Advance Search



Simulated Annealing

(Inspired by metallurgy)





Cold-rolled

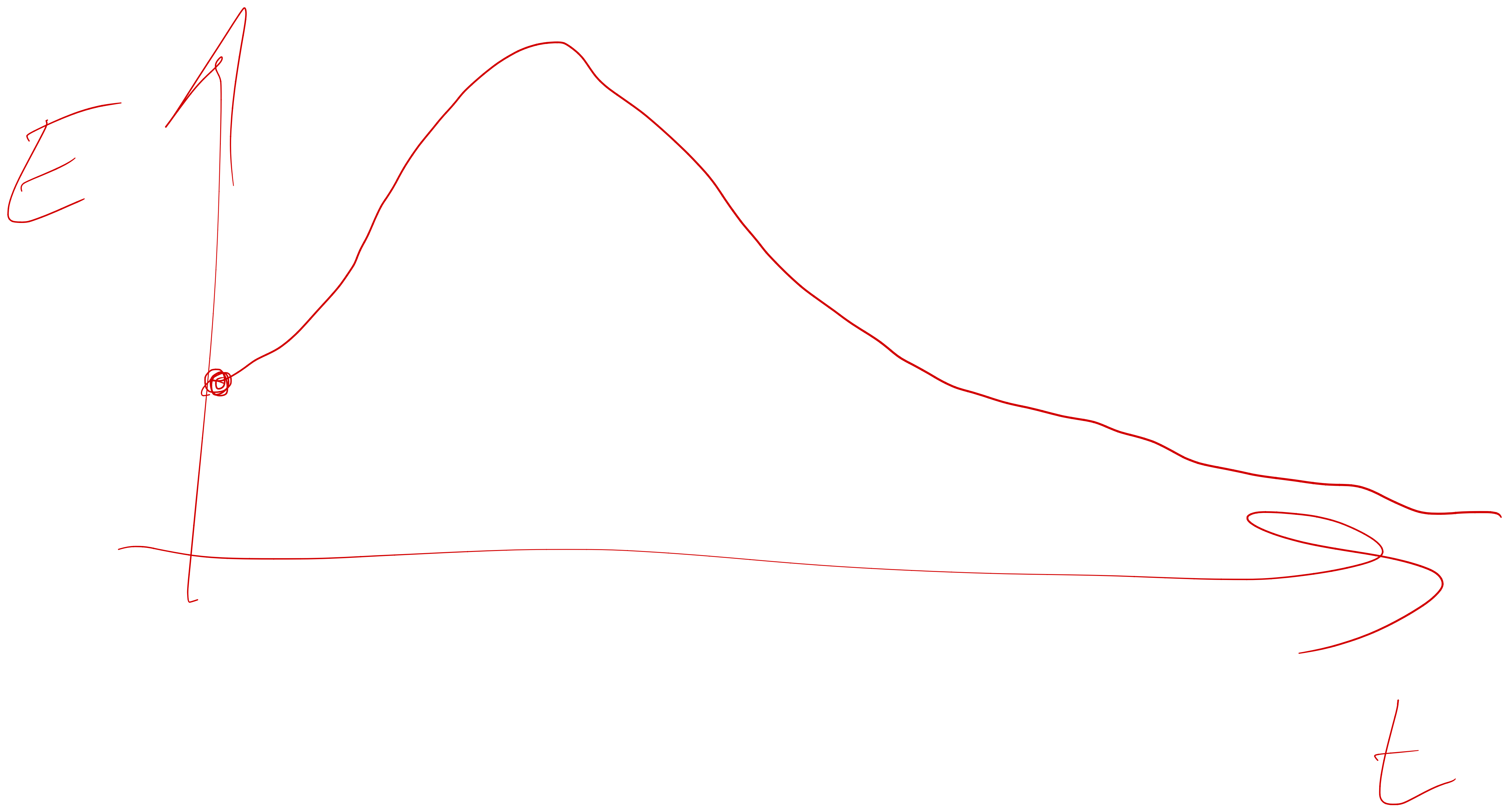
550 °C anneal for 1 hour

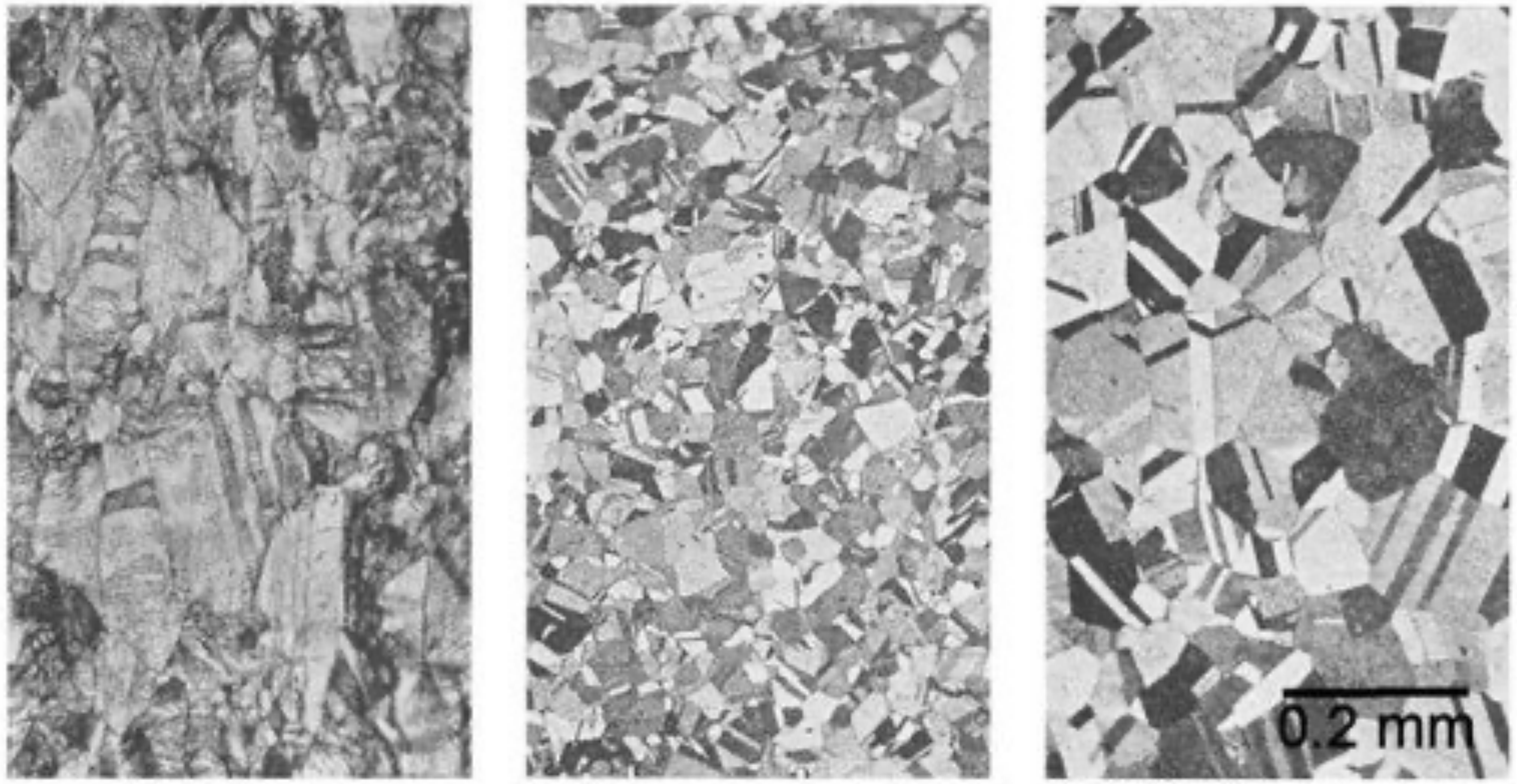
650 °C anneal for 1 hour

Simulated Annealing

(Inspired by metallurgy)

- Increase temperature (energy)
- Let it cool-down (slowly)





Cold-rolled | 550 °C anneal for 1 hour | 650 °C anneal for 1 hour

P = NP?

NP

P

Simulated Annealing

- Popular for solving **combinatorial optimization problems**
  - Optimize an objective function whose domain is a discrete but with a large configuration space
- Examples:
  - Traveling Salesman
  - Job-Scheduling
  - SAT - problem

NP

F(x)

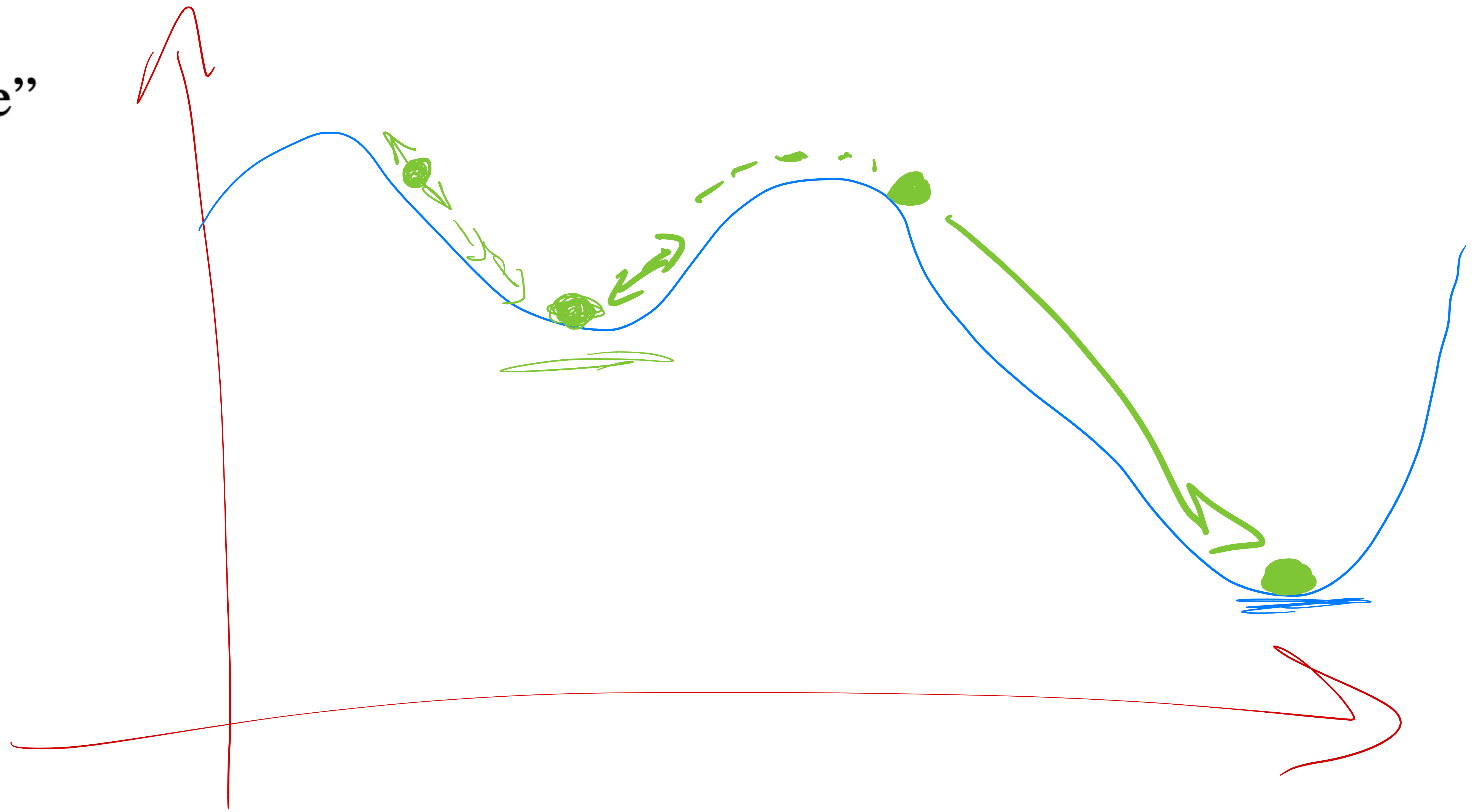
$(x_1 \lor x_2 \lor x_3) \land (x_4 \lor x_5 \lor x_6)$

Simulated Annealing

Steps

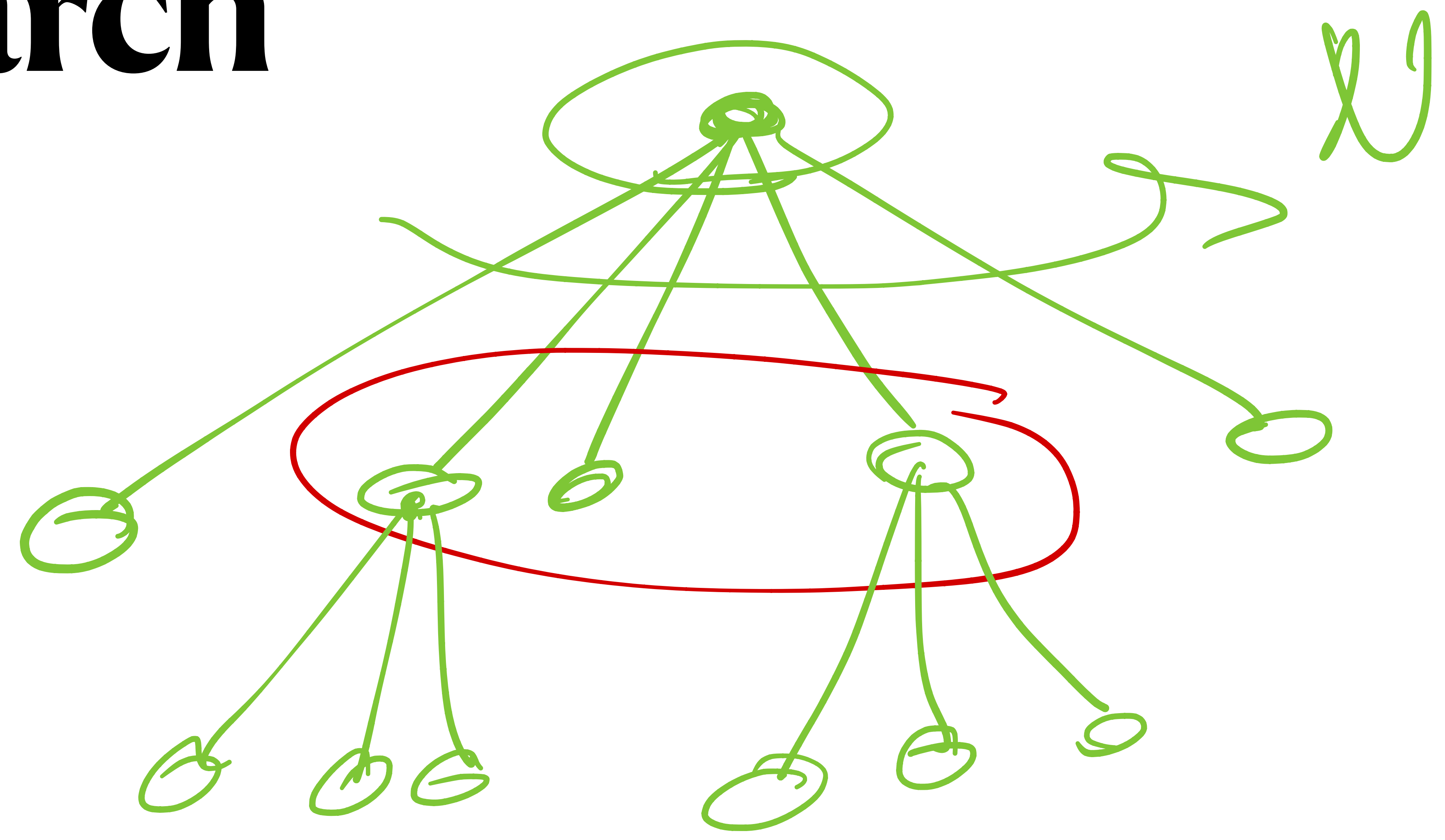
function SIMULATED-ANNEALING(*problem*, *schedule*) **returns** a solution state
  inputs: *problem*, a problem
    schedule, a mapping from time to "temperature"

  current ← MAKE-NODE(*problem*.INITIAL-STATE)
  for $t = 1$ **to** ∞ **do**
    $T \leftarrow schedule(t)$
    if $T = 0$ **then return** *current*
    next ← a randomly selected successor of *current*
    $\Delta E \leftarrow$ *next*.VALUE – *current*.VALUE
    if $\Delta E > 0$ **then** *current* ← *next*
    else *current* ← *next* only with probability $e^{\Delta E/T}$



Example Schedule(t): $T = \dfrac{100}{t^2}$

Beam Search



1. Generate *k* initial (random) states
2. Generate **all** successors
    1. Retain *k* best ones
3. Stop if GOAL-FUNCTION returns true
    1. Or if no improvement is possible

Beam Search

- Similar (not equal) to HC
- Problems with Beam Search?

- Stochastic Beam Search

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Advance Search

Genetic Algorithms

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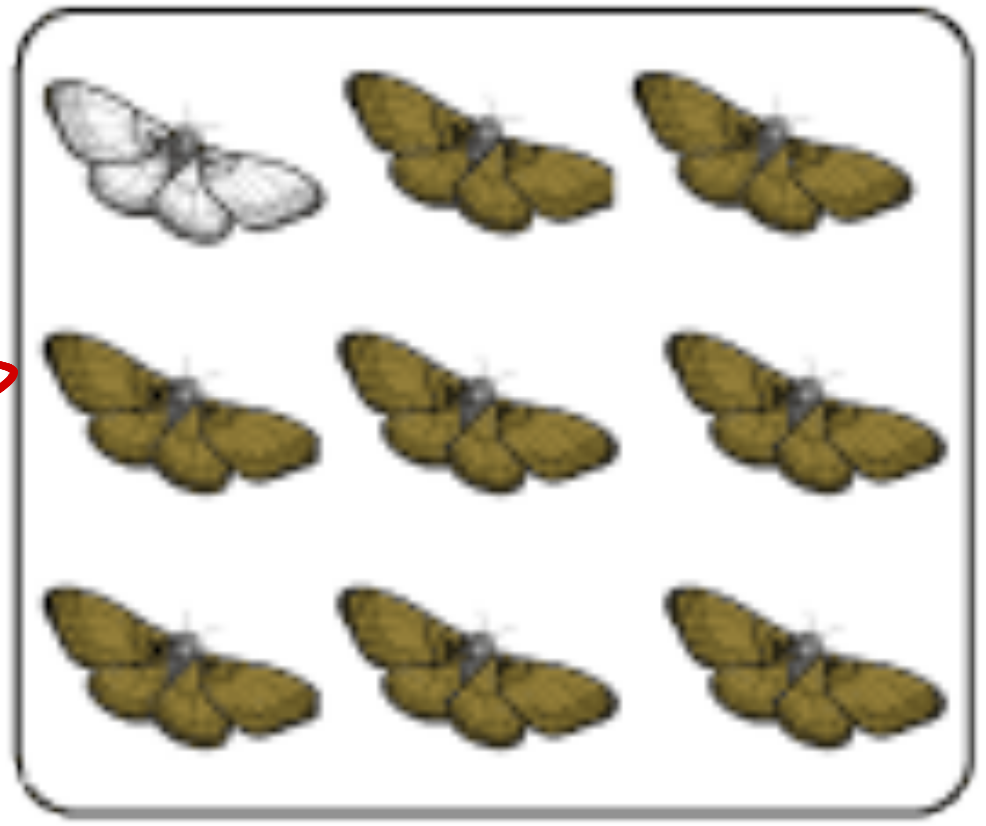
Evolution

(Darwin's theory of natural selection.)



Genetic Algorithms

$\boxed{N}$ k

1. Define
   1. Functions
   2. Individuals
2. Selection
3. Crossover
4. Mutation
5. Evaluate

$\to N$

p_m

Fitness $\simeq$ God

Crossover

Mutation

10110101

a_0 | a_1 | a_2 | . . . . | a_n

(x,y) | (x,y) | (x,y)

1 | 0 | 0 | 1

1 | 1 | 1 | 0

Genetic Algorithms

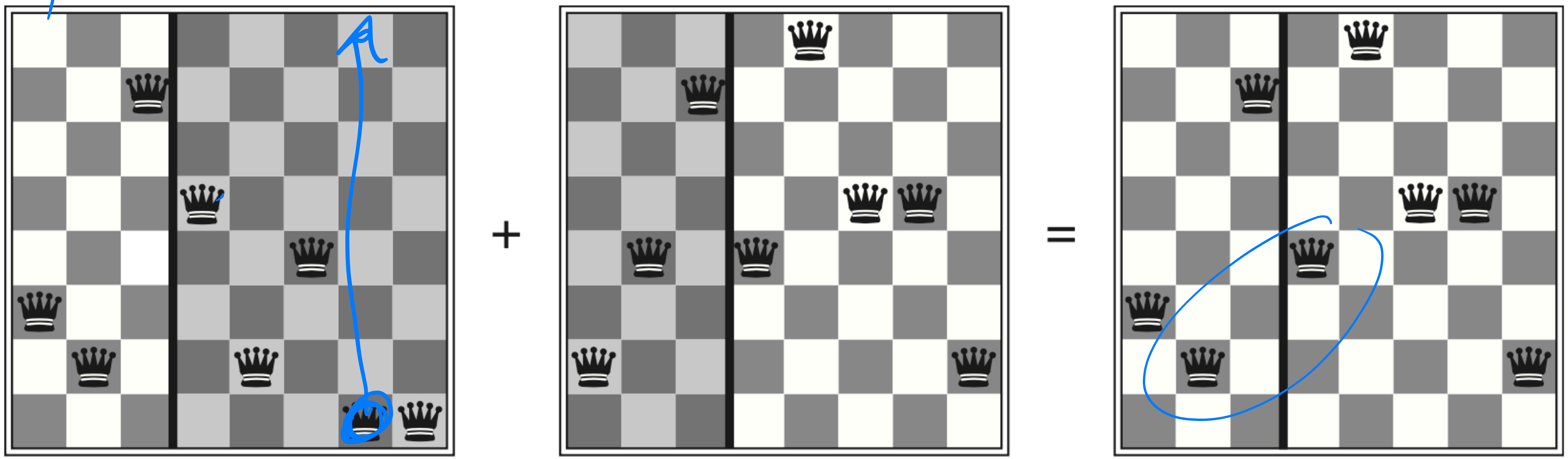
Example: 8 Queens

C_1

$\rightarrow (6, 7, 2, 4$

P_M

C_1



I_1 I_2

Genetic Algorithms

- Deal with complex problems
- Parallelism
- Able to handle different types of objective function - fitness function
- Heuristics

- https://rednuht.org/genetic_cars_2/

Genetic Algorithms

Theorem 4.1.1. *There is a Turing-complete genetic algorithm: There is a genetic algorithm that is able to simulate every step of the computation of* (M, w) *for every Turing machine* M *and input* w.

Loscos Barroso, Daniel. "Generalization and Completeness of Evolutionary Computation." (2018).

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Advance Search

Continuous Spaces

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Linear Programming

$F(x) =$

$$\begin{array}{ll} \text{maximize} & \boxed{c^T x} \\ \text{subject to} & \boxed{Ax \le b,} \ 0 \le x \end{array}$$

$$\begin{array}{ll} \text{minimize} & b^T y \\ \text{subject to} & A^T y \ge c, \ 0 \le y. \end{array}$$

Reward

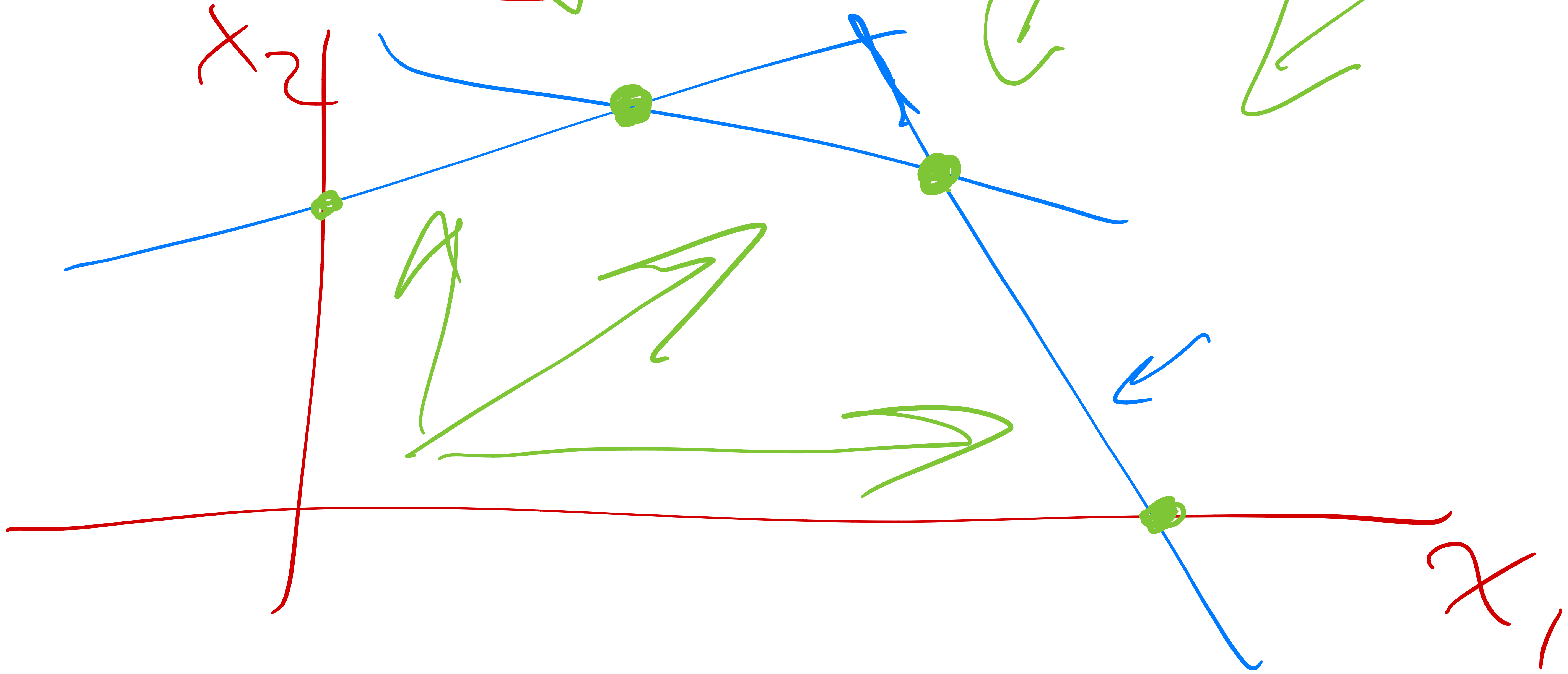
cost

time

Linear Programming

Duality Theorem

$$
\begin{array}{ll} \text{minimize} & b^T y \\ \text{subject to} & A^T y \geq c, \\ & 0 \leq y \end{array}
\quad = \quad
\begin{array}{ll} -\text{maximize} & (-b)^T y \\ \text{subject to} & (-A^T) y \leq (-c), \\ & 0 \leq y. \end{array}
$$



Linear Programming

Example

- Manufacturing is organized into four departments: sheet metal stamping, engine assembly, automobile assembly, and truck as- sembly. The capacity of each department is limited. The following table provides the percentages of each department's monthly capacity that would be consumed by constructing a thousand cars or a thousand trucks

Department	Automobile	Truck
metal stamping	4%	2.86%
engine assembly	3%	6%
automobile assembly	4.44%	0%
truck assembly	0%	6.67%

- The marketing department estimates a profit of \$3000 per car produced and \$2500 per truck produced. If the company decides only to produce cars, it could produce 22,500 of them, generating a total profit of \$67.5 million. On the other hand, if it only produces trucks, it can produce 15, 000 of them, with a total profit of \$37.5 million

Linear Programming

Example

$x_1 = 1000 = 1K$

$x_1 = 20.4$
$x_2 = 6.5$

- The marketing department estimates a profit of \$3000 per car produced and \$2500 per truck produced. If the company decides only to produce cars, it could produce 22,500 of them, generating a total profit of \$67.5 million. On the other hand, if it only produces trucks, it can produce 15, 000 of them, with a total profit of \$37.5 million

Department	Automobile	Truck
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$$\text{Max } f(x) = 3x_1 + 2.5x_2$$

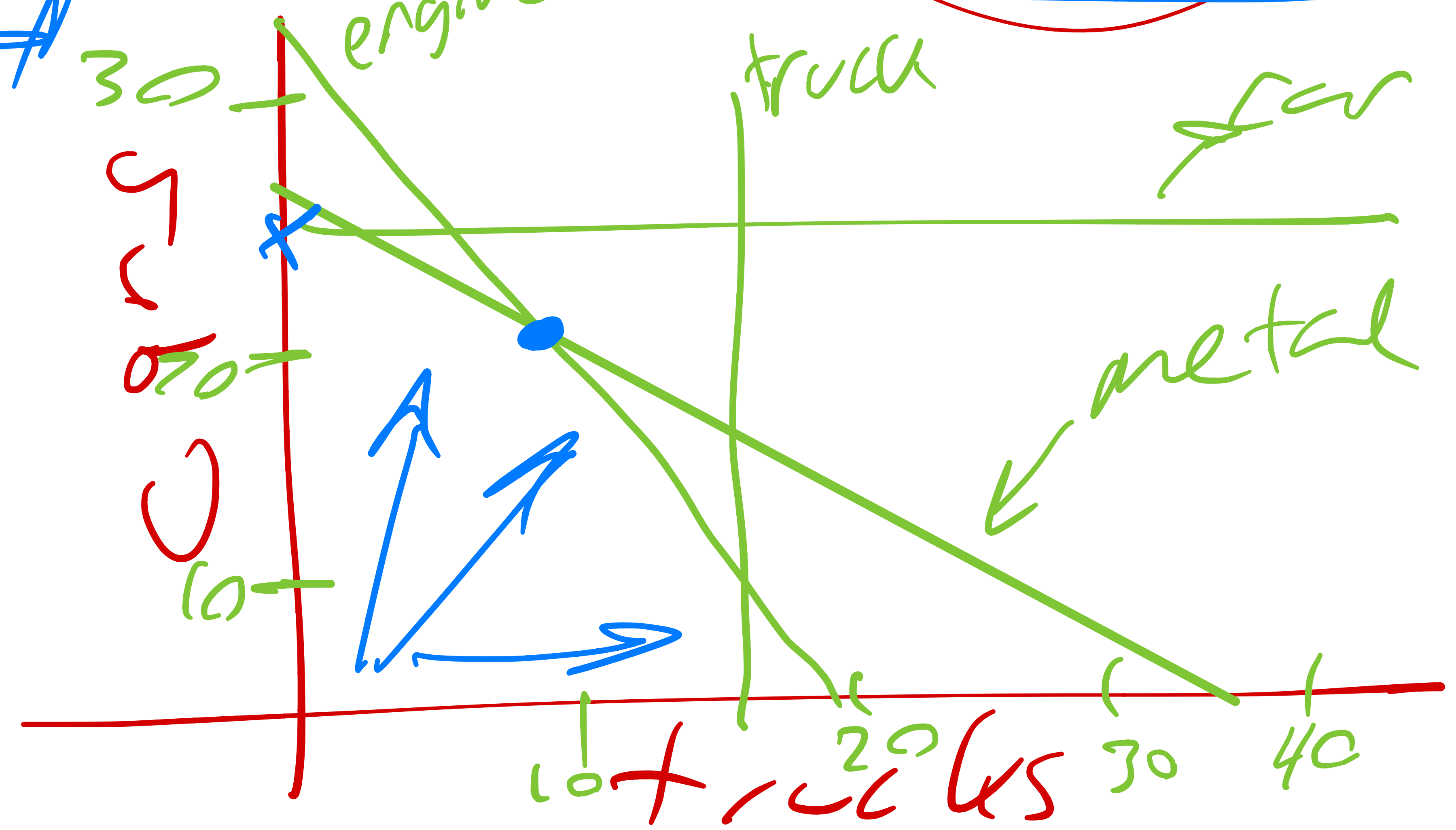
$$3 \cdot 20.4 + (2.5)6.5 = 61.2$$

16.25

$\boxed{77.45}$

S.T.

$$4.44x_1 \le 100$$
$$6.67x_2 \le 100$$
$$4x_1 + 2.86x_2 \le 100$$
$$3x_1 + 6x_2 \le 100$$
$$x_1 > 0;\ x_2 > 0$$



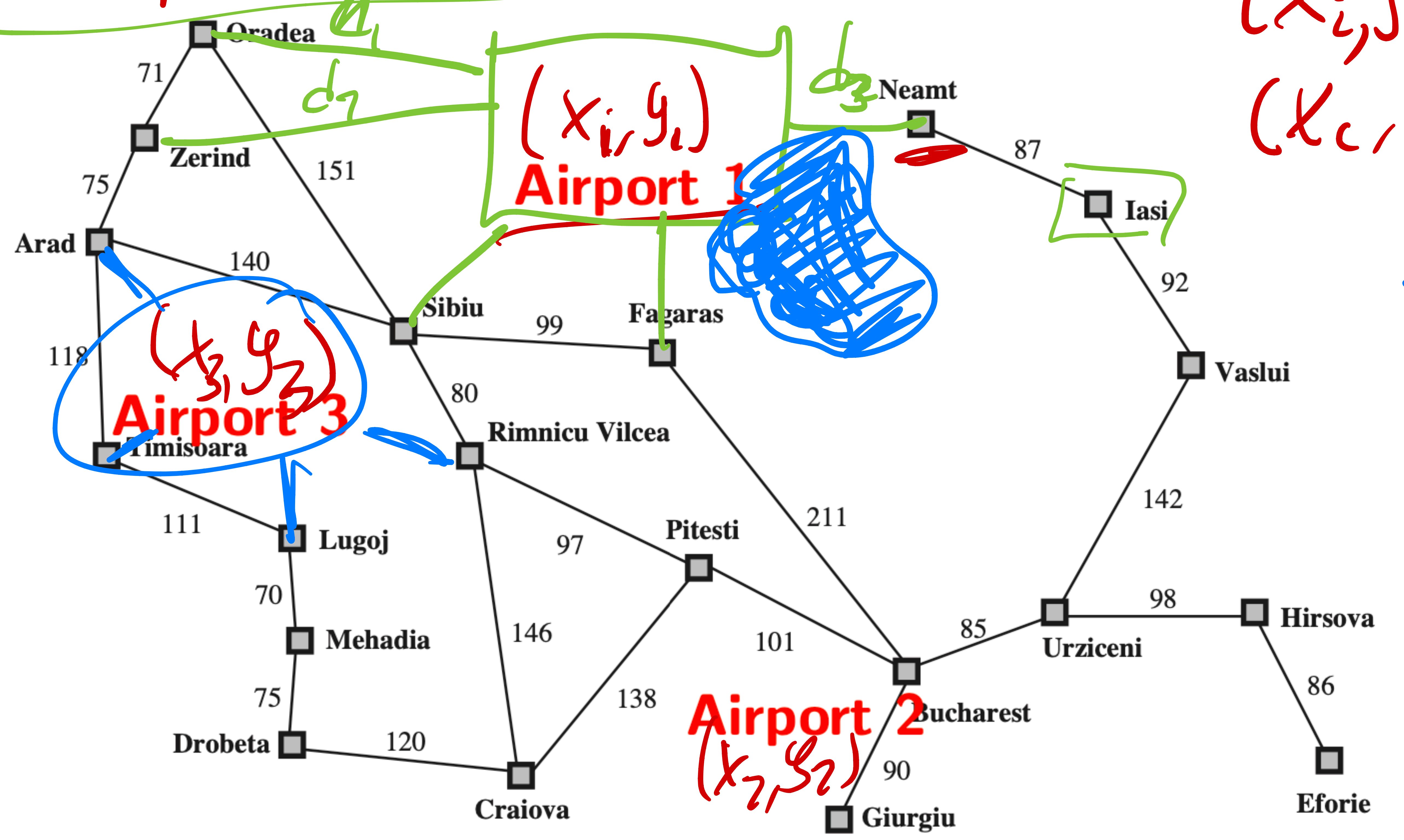
Linear Programming

- Optimization with constraints
- Only linear functions
  - Quadratic programming...

$$\begin{array}{ll} \text{minimize} & \boxed{x^T P x} \\ \text{subject to} & Ax \preceq b \end{array}$$

Location of Airports?

Obj: Min distance between each city & the nearest airport



$C_i \sim$ City that has the airport i as its nearest

$(x_i, y_i) \sim$ Coordinates of Airport i

$(x_c, y_c) \sim$ Coordinates of city c

$$f(x_1, y_1, x_2, y_2, x_3, y_3) = \sum_{i=1}^{3} \sum_{c \in C_i} (x_i - x_c)^2 + (y_i - y_c)^2$$

Gradient Descent

- Compute:

$$\nabla f = \left(\frac{\partial f}{\partial x_1}, \frac{\partial f}{\partial y_1}, \frac{\partial f}{\partial x_2}, \frac{\partial f}{\partial y_2}, \frac{\partial f}{\partial x_3}, \frac{\partial f}{\partial y_3}\right),$$

$$\frac{\partial f}{\partial x_i} = 2\sum_{c \in C_i} (x_i - x_c)$$

- Apply: $x \leftarrow x - \alpha \nabla f.$

THE END

Newton's Method

In 1669, Newton proposed a method for finding a zero of a given function g that consists in iteratively applying the update rule

$$x \leftarrow x \boxed{-} \boxed{\frac{g(x)}{g'(x)}}$$

In optimization, we search for a point where the derivative ∇f is zero. Using Newton's method, we can write the update rule as

$$x \leftarrow x - \frac{f'(x)}{\boxed{f''(x)}}.$$

If f is multivariate, then we can use H_f, the Hessian matrix of second derivatives

$$x \leftarrow x - \boxed{H_f^{-1}(x)}\nabla f(x).$$

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Advance Search

Non-Determinism

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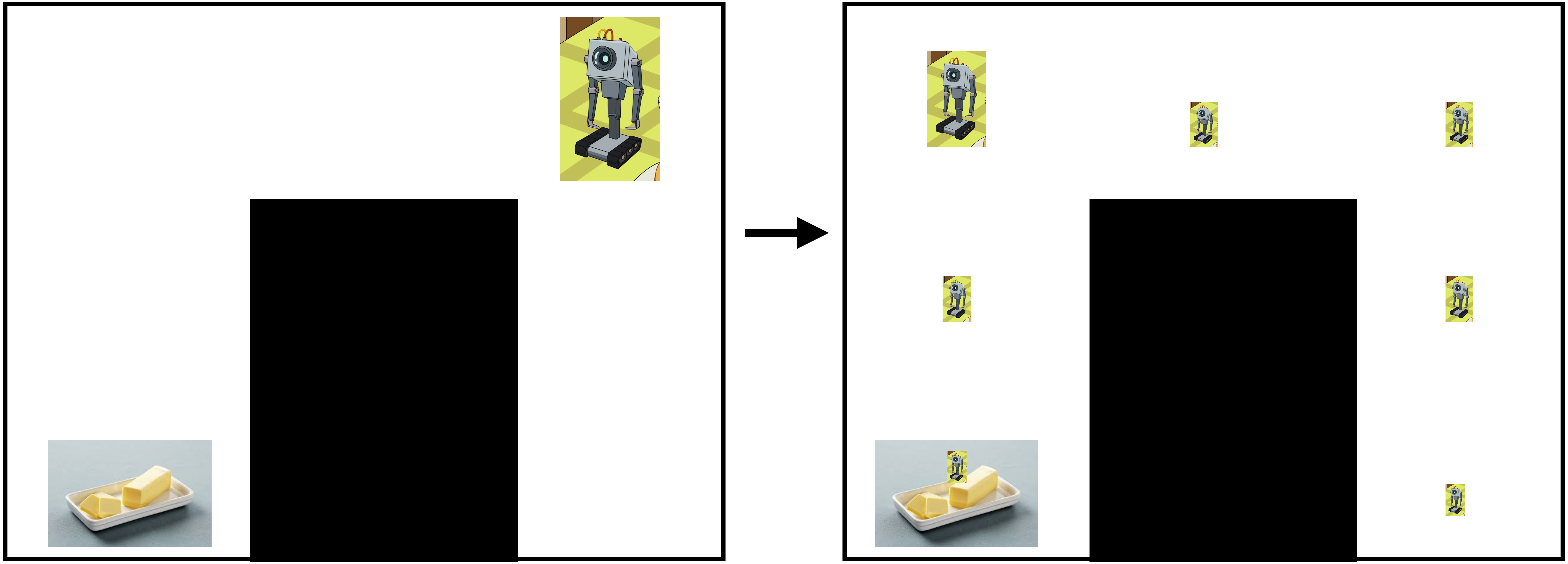
- So far, we have assumed that the agent knows exactly the state of its environment.
- In reality, an agent receives partial, possibly noised, observations. Therefore, the state can only be estimated.
- In this case, the agent needs to remember all its history of actions and observations in order to track the state.
- Replace *states* with *observations*

Partial Observations

Belief State



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Advance Search

Partial Observability

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Advance Search

Contraint Satisfaction Problems

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