RUTGERS

THE STATE UNIVERSITY
OF NEW JERSEY

Machine Learning

Reinforcement Learning

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Reinforcement Learning

Definition

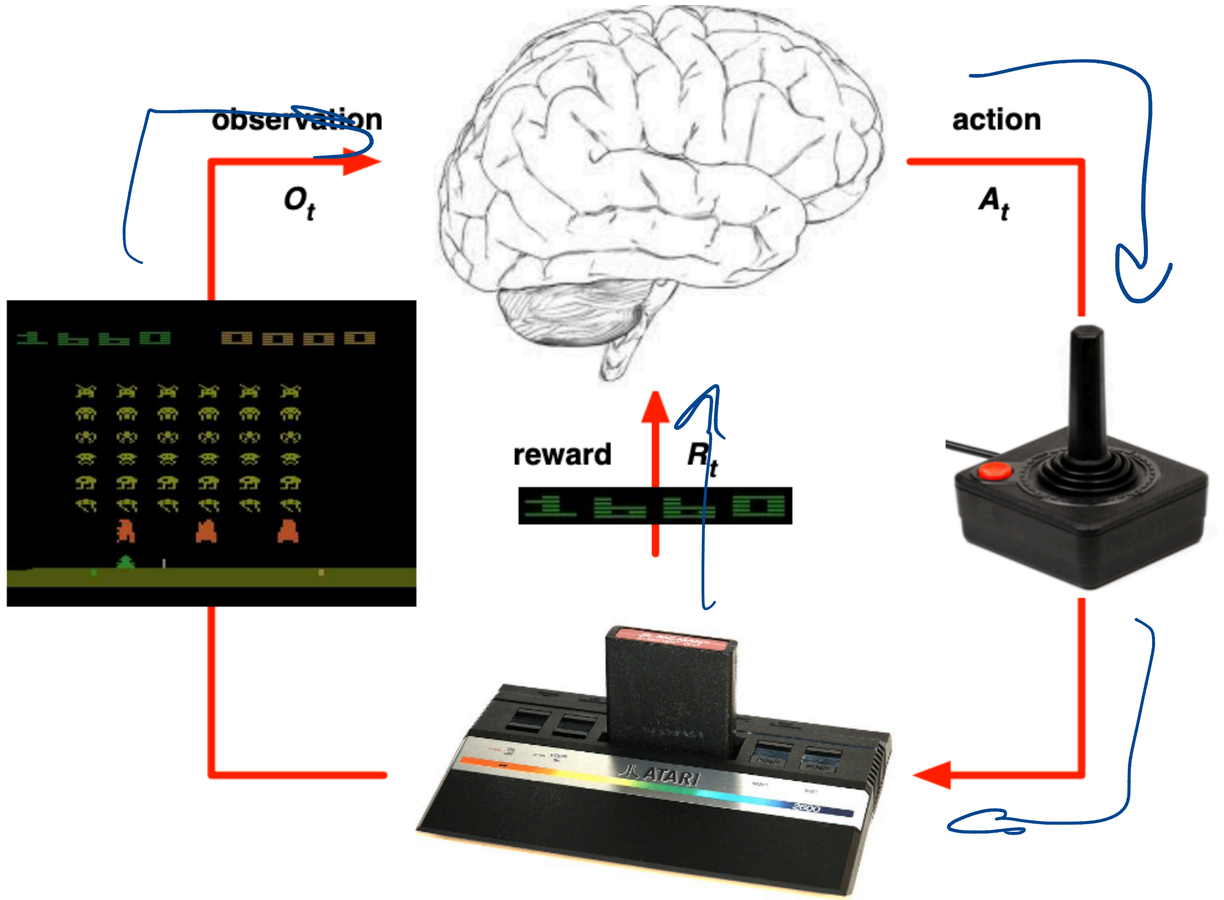
"A way of programming agents by reward and punishment without needing to specify how the task is to be achieved." [L. Kaelbling, M. Littman and A. Moore, 1996]

Reinforcement Learning

Videogames

→ Rules unknown

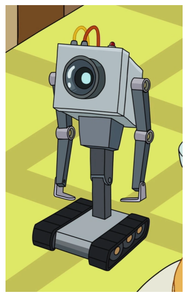
→ Learning from outcomes

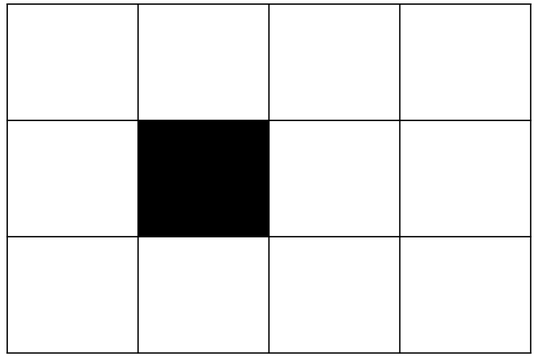


Reinforcement Learning

Behavioral Psychology

Reinforcement Learning





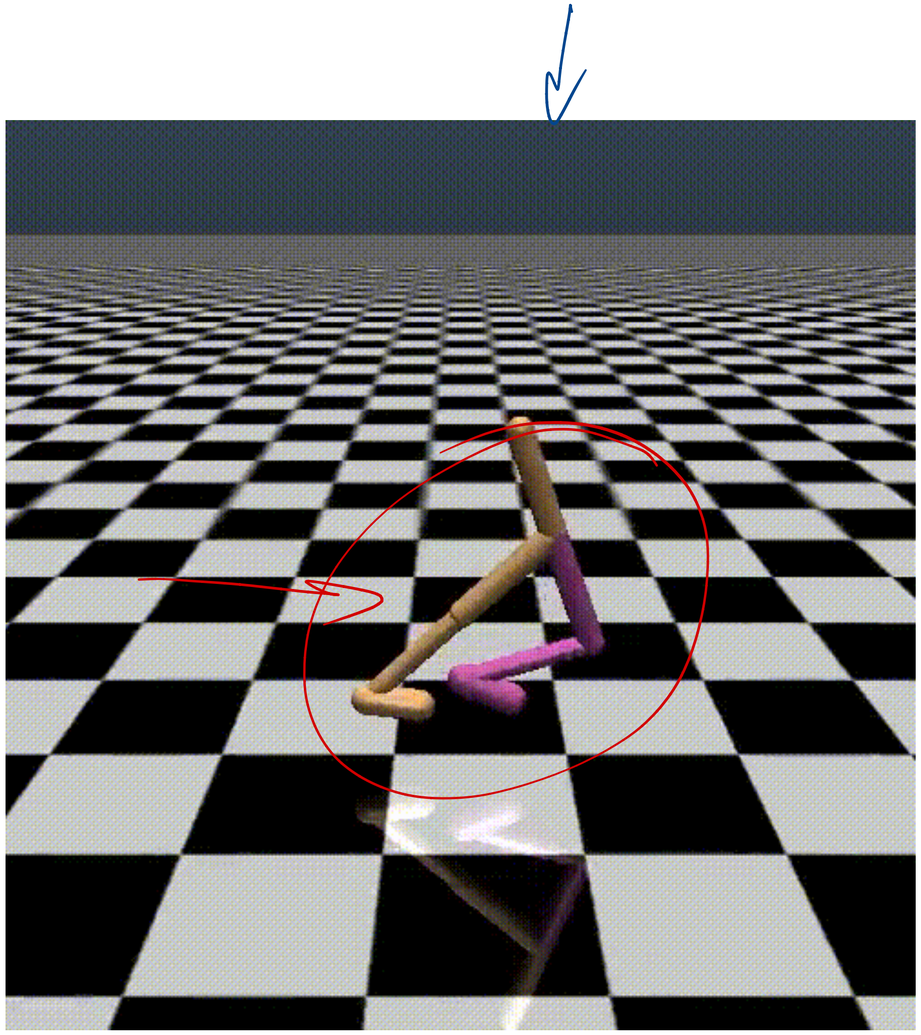
Action

Reward

Observation

Defining Rewards

- Helicopter Maneuvers
- Playing games
- Investment Portfolio
- Power station
- Humanoid walk



Horizons

- Given a reward function, the goal of the agent is to maximize the expected cumulated reward over some number H of steps, called the horizon.

$(s_t, a_t, r_t), (s_{t+1}, a_{t+1}, r_{t+1}) \ldots (s_{t+H-1}, a_{t+H-1}, r_{t+H-1})$

Horizon

Horizons

Finite or infinite?

- If the horizon is infinite, the optimal actions depend only on the state. In our case, the optimal action at any step is to move toward the goal.

$0 < \gamma < 1$

$$r_t + \gamma r_{t+1} + \gamma^2 r_{t+2} + (\ldots)$$

Learning with Markov Decision Processes

- Optimal policy without transition function?

- RL:
  - Finding the optimal policy for an MDP with unknown transition function
  - Learn from experience: Act and then observe the receiving rewards

trial and Error

Model-based Approach

- Collect Data → $\{(s_t, a, s'), (\ldots)\}$
- Estimate T:
  - $T(s, a, s') = P(s'|s,a) \approx \dfrac{\#(s,a,s') \in \text{Data}}{\#(s,a,\text{Anything}) \in \text{Data}}$

Get some estimate of T

Do MDP { Policy iter / Value iter

Model-free Approach

- Learn policy directly from rewards
  - No need to learn the transition function

Req More data

Q-value

- A Q-value is the expected sum of rewards that an agent will receive if it executes action a in state s then follows a policy π for the remaining steps

$$Q^{\pi}(s,a) = R(s,a) + \gamma \sum_{s' \in S} T(s,a,s') V^{\pi}(s')$$

a not necessarily $\pi(s)$

- Value function: (MDP)

$$V^{\pi}(s) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, \pi\right]$$

How good is a state?

- Q-value function

$$Q^{\pi}(s, a) = \mathbb{E}\left[\sum_{t \geq 0} \gamma^t r_t | s_0 = s, a_0 = a, \pi\right]$$

the state-action pair

The Q-learning algorithm

- Q-value updates?

$$Q_{k+1}(s,a) \leftarrow \sum_{s'} T(s,a,s') \left[R(s,a,s') + \gamma \max_{a'} Q_k(s',a') \right]$$

The Q-learning algorithm

- Compute averages as we go:
  - Sample transition (s, a, r, s')
  - $Q(s, a) \leftarrow (1 - \alpha) Q(s, a) + \alpha [r + \alpha \max_{a'} Q(s', a')]$
  - $\pi(s_t) = \arg\max_{a \in \mathcal{A}} Q^t(s_t, a);$

$Q(s,a) \approx r + \gamma \max_{a'} Q(s',a')$

$0 < \alpha < 1$, decreases over time

Alternative Formulation of the Update Equation

$$Q^{t+1}(s_t, a_t) \leftarrow Q^t(s_t, a_t) + \alpha_t[R(s_t, a_t) + \alpha \max_{a' \in A} Q(s_{t+1}, a') - Q^t(s_t, a_t)]$$

Observed Value

Predicted Value

New Value = Old Value + (learning Rate)(Observed Value − Predicted Value)

Temporal Difference TD

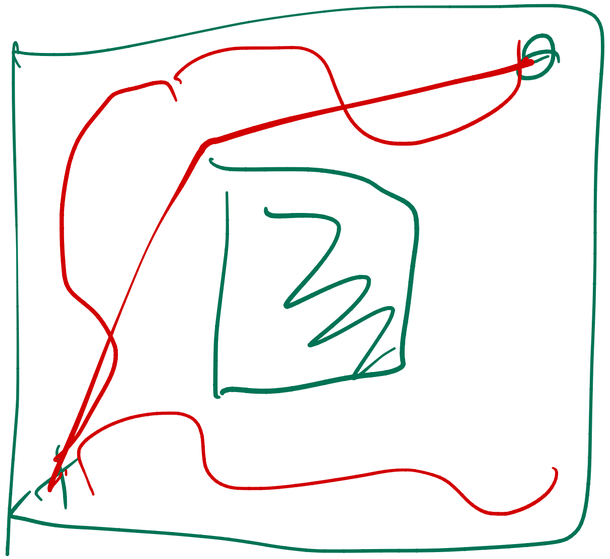
Convergence conditions of tabular Q-learning

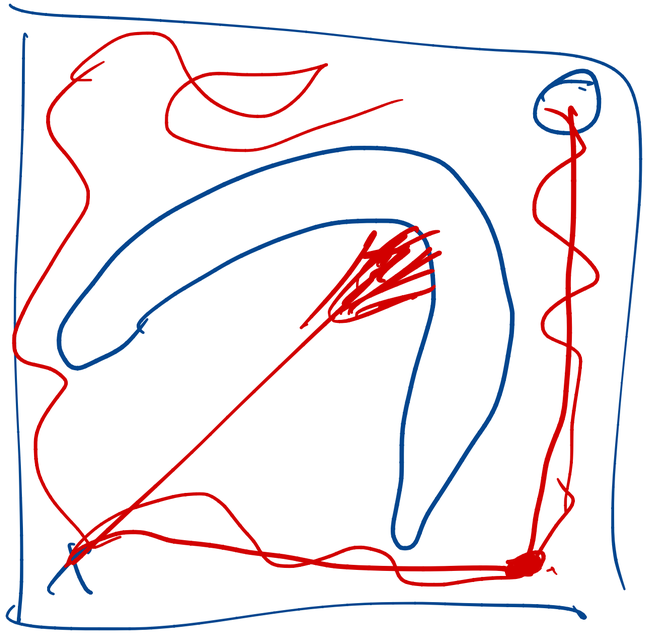
- Robbins-Monro conditions:
  - $\sum_{t=0}^{\infty} \alpha_t^2 < \infty$ → learning rate decreases over time
  - $\sum_{t=0}^{\infty} \alpha_t = \infty$ → but slowly

$\alpha_t = \frac{1}{t}$ $\epsilon_t = \frac{1}{\sqrt{t}}$

Exploration vs Exploitation

- Estimate the structure (size/topology) of the state space
- “Finding the right information” —> Difficult





Exploration

- Sometimes take random actions

ε-greedy

$P_{explore} \rightarrow$ Explore

$1 - P_{explore} \rightarrow$ Act on the Policy

Exploration Function

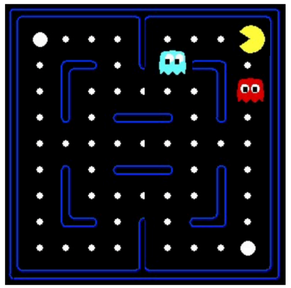
- Takes a value estimate u and a visit count n, and returns an optimistic utility

  - Q-update
  - Modified Q-Update

Basic Q-learning

- Keep table of all q-values

- Generalize



Vector of features

- Features are functions from states to real numbers (often 0/1) that capture important properties of the state

Q-learning with linear Q-functions

$$Q(s,a) \leftarrow Q(s,a) + \alpha(TD)$$

$$w_i \leftarrow w_i + \alpha(TD) f_i(s,a)$$

Deep Q Learning

DQN, Mnih et al., 2013