

$$\begin{aligned} \textcircled{1} \quad S &\rightarrow A \mid S\#S \mid S@S \\ A &\rightarrow C \mid CA \\ C &\rightarrow a \mid b \mid c \end{aligned}$$

i) aa###bb

Cannot be derived, thus not in language.

Reason: Cannot achieve "##" i.e. 2 #s side by side.

attempt:

$$\begin{aligned} S &\rightarrow S\#S \\ &\rightarrow A\#S \\ &\rightarrow CA\#S \\ &\rightarrow aA\#S \\ &\rightarrow aC\#S \\ &\rightarrow aa\#S \\ &\rightarrow aa\#A \\ &\rightarrow aa\#CA \\ &\rightarrow aa\#bA \\ &\rightarrow aa\#bC \\ &\rightarrow aa\#bb. \text{ (Couldn't get 2 \#s anyhow).} \end{aligned}$$

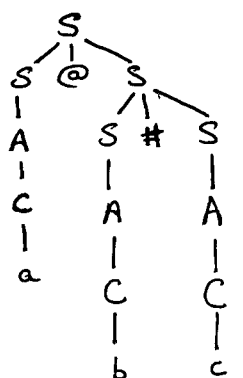
(ii) a@b#c

It is a language in  $G$ .

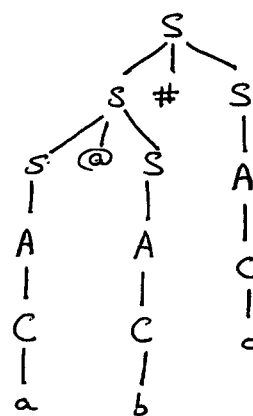
proof:

$$\begin{aligned} S &\Rightarrow S@S \\ &\rightarrow S@S\#S \\ &\rightarrow A@S\#S \\ &\rightarrow C@S\#S \\ &\rightarrow a@S\#S \\ &\rightarrow a@A\#S \\ &\rightarrow a@C\#S \\ &\rightarrow a@b\#S \\ &\rightarrow a@b\#A \\ &\rightarrow a@b\#C \\ &\rightarrow \boxed{a@b\#c} \end{aligned}$$

Parse tree 1



Parse tree 2

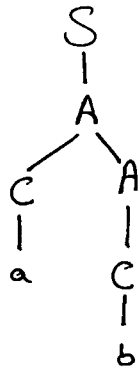


BUT, it is ambiguous, since it can be derived using multiple alternative ways.

(iii)

ab :It is a language in  $G$  and is not ambiguousProof:Parse tree:

$S \rightarrow A$   
 $\rightarrow CA$   
 $\rightarrow aA$   
 $\rightarrow aC$   
 $\rightarrow ab$

① Rewrite grammar  $G$ :

- no string is ambiguous
- '#' higher precedence than '@'
- '#' and '@' both are left associative

 $S \rightarrow S @ A / A$  $A \rightarrow A \# C / A C / C$  $C \rightarrow a / b / c$ 

$\Rightarrow$  lower in the tree means higher precedence  
 (Thus '#' has higher precedence than '@')

$\Rightarrow$  Expansions from the left.  
 (Left associativity)

$\Rightarrow$  For all strings above, there is only 1 derivation & parse tree. Thus unambiguous.

② For given grammar H:

For each production given:

- determine if it is in the language H
- explain why or why not.

i)  $a = 0, 6;$

It is in language H:

Proof: (Bottom-up approach)

$\langle var \rangle = \langle Number \rangle [, \langle var \rangle];$

$\langle var \rangle = \langle value \rangle [, \langle value \rangle];$

$\langle Assignment \rangle$

$\langle Statement \rangle \Rightarrow \text{Language}$

(Top-down approach)

$\hookrightarrow$  Exact reverse of bottom-up.

ii)  $a = b, c, 1;$

It's not in language H

Proof:

$\langle Statement \rangle \rightarrow \langle Assignment \rangle$

$\langle Assignment \rangle \rightarrow \langle var \rangle = \langle value \rangle [, \langle value \rangle];$

$a = b, c, \dots ??$

We can have a maximum of 2 non-terminals ~~=~~ / variables on the right of " $=$ ".

(iii)  $\text{while}(a) \{ b = 0; \text{while}(b) \{ \} \}$  :

It is in language H. ~~to~~

Proof: (Bottom-up approach)

$\text{while}(\langle var \rangle) \{ \langle var \rangle = \langle Number \rangle; \text{while}(\langle var \rangle) \{ \} \}$

$\text{while}(\langle value \rangle) \{ \langle var \rangle = \langle value \rangle; \text{while}(\langle value \rangle) \{ \langle Statement \rangle \} \}$

$\text{while}(\langle value \rangle) \{ \langle var \rangle = \langle value \rangle; \langle While \rangle \}$

$\text{while}(\langle value \rangle) \{ \text{Assignment} \& \text{Statement} \}$

$\text{while}(\langle value \rangle) \{ \langle Statement \rangle \}$

$\bullet \langle while \rangle$

$\langle Statement \rangle$

(iv)  $a=1; \text{while}(a) \{ a=1; \text{while}(a=0) \}$

It is not in language:

Proof:

$a=1; \text{while}(a) \{ a=1; \text{while}(a=0); \}$

$\langle \text{var} \rangle = \langle \text{number} \rangle; \text{while}(\langle \text{var} \rangle) \{ \langle \text{var} \rangle = \langle \text{number} \rangle \text{ while} \dots \}$

$\langle \text{var} \rangle = \langle \text{value} \rangle; \text{while}(\langle \text{var} \rangle) \{ \langle \text{Assignment} \rangle \langle \text{While} \rangle \}$

$\langle \text{var} \rangle = \langle \text{value} \rangle; \text{while}(\langle \text{var} \rangle) \{ \langle \text{Statement} \rangle \}$

$\langle \text{var} \rangle = \langle \text{value} \rangle; \langle \text{while} \rangle$

$\langle \text{Assignment} \rangle \langle \text{Statement} \rangle$

$\langle \text{Statement} \rangle \langle \text{Statement} \rangle$

There can't be <sup>an</sup> ~~a~~ extension to Statement  
or in other words

There can't be 2 or more Statements <sup>side by side</sup>, according to the language unless  
inside ' { } ':

(3) Write grammar for BNF in BNF:

$\langle \text{BNF} \rangle ::= \langle \text{rule} \rangle \mid \langle \text{rule} \rangle \langle \text{BNF} \rangle$

$\langle \text{rule} \rangle ::= \langle " \rangle \langle \text{rulename} \rangle \langle " \rangle ::= \langle \text{expr} \rangle$

$\langle \text{expr} \rangle ::= \langle \text{list} \rangle \mid \langle \text{list} \rangle \mid \langle \text{expr} \rangle$

$\langle \text{list} \rangle ::= \langle \text{term} \rangle \mid \langle \text{term} \rangle \langle \text{list} \rangle$

$\langle \text{term} \rangle ::= \langle \text{text} \rangle \mid \langle " \rangle \langle \text{rulename} \rangle \langle " \rangle$

$\langle \text{rulename} \rangle ::= \langle \text{text} \rangle$

$\langle \text{text} \rangle ::= \langle \text{char} \rangle \langle \text{text} \rangle \mid \langle \text{char} \rangle$

$\langle \text{char} \rangle ::= \langle \text{letter} \rangle \mid \langle \text{digit} \rangle \mid \langle \text{symbol} \rangle$

$\langle \text{letter} \rangle ::= \langle "A" \rangle \mid \langle "B" \rangle \mid \langle "C" \rangle \mid \dots \mid \langle "a" \rangle \mid \langle "b" \rangle \mid \langle "c" \rangle \mid \dots$

$\langle \text{digit} \rangle ::= \langle "0" \rangle \mid \langle "1" \rangle \mid \langle "2" \rangle \mid \dots$

$\langle \text{symbol} \rangle ::= \langle " \rangle \mid \langle " \rangle \mid \langle " \rangle \mid \dots$

There can be other small bifurcations as well, but the logic to be followed is the same.