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2nd order ordinary linear differential equation:-

A differential equation of the form

$$y'' + P(x)y' + Q(x)y = r(x) \quad \text{--- (i)}$$

is a linear 2nd order non-homogenous differential equation

Homogenous differential eq^n of 2nd order if all the terms have a dependent variable present

$$y'' + P(x)y' + Q(x)y = r(x)$$

The eq^n (i) becomes homogenous if $r(x) = 0$.

$$y'' + P(x)y' + Q(x)y = 0 \quad \text{--- (ii)}$$

$\{v_1, v_2, v_3 \ldots\ldots v_n\}$ is LD

A set of vectors are Linearly Dependent if $\alpha_1 v_1 + \alpha_2 v_2 + \alpha_3 v_3 + \ldots + \alpha_n v_n = 0$

with atleast one $\alpha_i \neq 0, i = 1, 2, \ldots n$

if $\alpha_1 \neq 0$,

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$$\alpha_1 = \alpha_2 = \alpha_3 = \cdots \alpha_n = 0.$$

Ex:-

$$y'' - y = 0.$$

$$\cancel{\frac{d^2y}{dx^2} = y}$$

$y_1 = e^{x}$ is a solution

$y_2 = e^{-x}$ is a solution

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Reduction of order method :-

Let y_1 be one of the solutions of a linear homogenous 2nd order differential equation of type

$$y'' + p(x)y' + q(x)y = 0 \quad \text{——— (i)}$$

Find out $y_2 = ?$

Ans:- Use reduction of order method

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Putting this value in eq^n (i) we get,

$$v''y_1 + 2v'y_1' + vy_1'' + P\{v'y_1 + vy_1'\} + Q\{vy_1\} = 0.$$

$$v''y_1 + 2v'y_1' + vy_1'' + Pv'y_1 + Pvy_1' + Qvy_1 = 0.$$

$$\Rightarrow v''y_1 + 2v'y_1' + vy_1'' + P(v'y_1 + vy_1') + Q(vy_1) = 0$$

The simplification of the above equation can be made by collecting the term v'', v', v

$$v''y_1 + v'(2y_1' + Py_1) + v(\cancel{y_1'' + Py_1' + Qy_1}^{\,0}) = 0.$$

As y_1 is a solution of eq^n-1 therefore the co-efficient of v in the above eq^n vanishes.

$$v''y_1 + v'(2y_1' + Py_1) = 0.$$

$$\Rightarrow v'' + \frac{v'(2y_1' + Py_1)}{y_1} = 0. \quad \text{——— (3)}.$$

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let $V = v'$

$$V' + V\left(2 \times \frac{y_1'}{y_1} + P\right) = 0,$$

$$\int \frac{dV}{V} = -\int \left(\frac{2y_1'}{y_1} + P\right) dx$$

$$\ln|V| = -2\int \frac{y_1'}{y_1}\, dx - \int P\, dx$$

$$\ln|V| = -2\ln y_1 - \int P\, dx$$

$$\ln V + \ln y_1^2 =$$

$$\ln\left(V \cdot y_1^2\right) = -\int P\, dx$$

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Que:- Find the 2nd solution y_2 of the following D.E $= x^2y'' - xy' + y = 0$
If the 1st sol^n $y_1 = x$ by reduction of order.

Answer let $y_2 = Vy_1$

$\Rightarrow y_2 = Vx$

$\Rightarrow y_2' = V + x \times \frac{dv}{dx}$

$\Rightarrow V = \frac{1}{y_1^2} e^{-\int P dx}$

$V = \frac{1}{x^2} \times e^{\int x dx}$

$= \frac{1}{x^2} \times e^{x^2/2}$

$= \frac{e^{x^2/2}}{x^2}$

$V = \frac{e^{x^2/2}}{x^2}$

$v = \int V dx$

$= \int \frac{e^{x^2/2}}{x^2} dx$

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$$x^2y'' - xy' + y = 0$$

$$\Rightarrow \quad y'' - \frac{y'}{x} + \frac{y}{x^2} = 0$$

$$V = \frac{1}{y_1^2} \times e^{-\int P\,dx}$$

$$= \frac{1}{x^2} \times e^{\int 1/x\,dx}$$

$$V = \frac{1}{x^2} \times e^{\ln x}$$

$$V = \frac{1}{x^2} \times x$$

$$V = \frac{1}{x}$$

$$V = \int \frac{1}{x}\,dx$$

$$V = \ln|x|$$

$$V = \ln|x|$$

$$y_2 = y_1 \times V$$
$$= x \ln x$$

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$x^2y'' - 5xy' + 9y = 0$. $y_1 = x^3$

$y'' - \frac{5y'}{x} + \frac{9y}{x^2} = 0$.

$V = \frac{1}{y_1^2} \times e^{-\int P\,dx}$

$= \frac{1}{x^6} \times e^{5 \times \ln x}$

$= \frac{1}{x^6} \times x^5$

$V = \ln|x|$

$y_2 = x^3 \ln|x|$

$xy'' + 2y' + xy = 0$. $y_1 = \frac{\sin x}{x}$,

$y'' + \frac{2}{x}y' + y = 0$.

$V = \frac{x^2}{\sin^2 x} \times e^{-\ln x^2}$ $V = \frac{x^2}{\sin^2 x} \times \frac{1}{x^2}$

$= \frac{x^4}{\sin^2 x}$ $V = \frac{x^4}{\sin^2 x}$ $\operatorname{cosec}^2 x$

$V =$

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$$\cancel{V = \int \frac{x^4}{\sin^2 x}\, dx}$$

$$\cancel{V = \int x^4 \operatorname{cosec}^2 x\, dx}$$

$\operatorname{cosec}^2 x - \cot^2 x = 1$

$$V = \int \operatorname{cosec}^2 x\, dx$$

$$= \cancel{\int 1 + \cot^2 x\, dx}$$

$$= \int \operatorname{cosec}^2 x\, dx$$

$$= -\cot x$$

$$y_2 = -\cot x \times \frac{\sin x}{x}$$

$$= -\frac{\cos x}{\cancel{\sin x}} \times \frac{\cancel{\sin x}}{x}$$

$$= -\frac{\cos x}{x}$$

solution of linear ~~order~~ homogenous 2nd order differential equation with constant co-efficients:-

standard form:- $y'' + ay' + by = 0$ — (i)

[Where a & b are real numbers]

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Find the Auxulary equation :

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0.$$

$$\lambda^2 + a\lambda + b = 0.$$

$$\lambda = \frac{-a \pm \sqrt{a^2-4b}}{2}$$

$$= \frac{-a \pm \sqrt{a^2-4b}}{2}$$

Case 1: If $a^2 > 4b$ then

λ_1, λ_2 are two real and distinct roots.

So, the solⁿ is

$$\boxed{y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}}$$

Case 2:

If $a^2 = 4b$

then, $\lambda_1 = \lambda_2 = \lambda$

Real and repeated

$$y_h = c_1 e^{\lambda x} + c_2 x e^{\lambda x}$$

$$= e^{\lambda x}(c_1 + c_2 x)$$

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$y'' - y = 0$

$\lambda^2 - 1 = 0$

$(\lambda + 1)(\lambda - 1) = 0$

$\lambda = -1$ and $\lambda = 1$

$$\boxed{y_h = c_1 e^{x} + c_2 e^{-x}}$$

$y'' - y' - 2y = 0$.

$y(0) = 4$
$y'(0) = -5$

$\lambda^2 - \lambda - 2 = 0$

$\lambda^2 - 2\lambda + \lambda - 2 = 0$

$\lambda(\lambda - 2) + 1(\lambda - 2) = 0$

$\lambda = 2$ & $\lambda = -1$

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At $x = 0$ $y = 4$

$c_1 + c_2 = 4$.

$y'_h = c_1 e^{2x} \cdot 2 - c_2 e^{-x}$

$-5 = 2c_1 - c_2$

$\Rightarrow$ $2c_1 - c_2 = -5$

$c_1 + c_2 = 4$

$3c_1 = -1$

$c_1 = -1/3$

$2 \times \left(\frac{-1}{3}\right) - c_2 = -5$

$c_2 = \frac{-2}{3} + 5$

$= \frac{-2+15}{3} = 13/3$.

$y_h = \frac{-1}{3} e^{2x} + \frac{13}{3} e^{-x}$

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Que: $y'' + 8y' + 16y = 0$

$\lambda^2 + 8\lambda + 16 = 0$

$\Rightarrow \lambda^2 + 4\lambda + 4\lambda + 16 = 0$

$\Rightarrow \lambda(\lambda+4) + 4(\lambda+4) = 0$

$\lambda = -4, -4$

So, $\lambda_1 = \lambda_2 = \lambda$.

$y_h = e^{\lambda x}(c_1 + c_2 x)$.

$\Rightarrow e^{-4x}[c_1 + c_2 x]$

$= e^{-4x}[c_1 + c_2 x]$

$y'' - 4y' + 4y = 0$. $\quad y(0) = 3$, $y'(0) = 1$

$\lambda^2 - 4\lambda + 4 = 0$.

$\lambda^2 - 2\lambda - 2\lambda + 4 = 0$

$\lambda(\lambda - 2) - 2(\lambda - 2) = 0$

$\lambda = 2, 2$

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$y_h = e^{\lambda x}[c_1 + c_2 x]$

$y_h = e^{2x}[c_1 + c_2 x]$.

$3 = c_1$.

$y'_h = e^{2x}[c_2] + (c_1 + c_2 x)e^{2x} \cdot 2$

$1 = 2c_1 + c_2$

$2c_1 = 1$ $\quad c_2 = -5$

$c_1 = 1/2$

$y_h = e^{2x}[3 - 5x]$

$y_h = e^{2x}[3 - 5x]$

$8y'' - 2y' - y = 0$

$y'' - \frac{y'}{4} - \frac{1}{8}y = 0$.

$\lambda^2 - \frac{\lambda}{4} - \frac{1}{8} = 0$.

$8\lambda^2 - 2\lambda - 1 = 0$.

$8\lambda^2 - 4\lambda + 2\lambda - 1 = 0$.

$\lambda = \frac{1}{2}$

$\lambda = \frac{-1}{4}$

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λ_1 and λ_2 are $\alpha + i\beta$ and $\alpha - i\beta$

Then the homogenous solⁿ

$$\boxed{y_h = e^{\alpha x}(A\cos\beta x + B\sin\beta x)}$$

$$\boxed{y_h = e^{\alpha x}(A\cos\beta x + B\sin\beta x)}$$

Ques:- $y'' + y = 0$

$\lambda^2 + 1 = 0$

$\lambda^2 = -1$

$\lambda = 0 \pm i$

$\lambda = 0 + i$

$$y_h = e^{0x}(A\cos x + B\sin x)$$

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$\lambda = -1/3$

$y_h = e^{\lambda x}(c_1 + c_2 x)$

$= e^{-1/3\, x}(c_1 + c_2 x)$

$4 = c_1 + c_2 \times 0$

$c_1 = 4$

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$c_1 = 4 \qquad c_2 = -3$.

$$y_h = e^{-\frac{1}{3}x}[4 - 3x].$$

$$y'' + 2y' + 2y = 0. \qquad y(0) = 1$$

$$y(\pi/2) = 0.$$

$$\lambda^2 + 2\lambda + 2 = 0.$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-2 \pm \sqrt{4 - 4\times 2}}{2}$$

$$= \frac{-2 \pm \sqrt{4 - 8}}{2}$$

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$$y_h = e^{-x}(A\cos\beta x + B\sin\beta x)$$

$$y_h = e^{-x}(A\cos x + B\sin x)$$

$$1 = (A\cos 0 + B\sin 0)$$

$$\Rightarrow 1 = A$$

$$0 = e^{\pi/2}(A\cos 90 + B\sin 90)$$

$$0 = e^{-\pi/2}(0 + B)$$

$$B = 0$$

$$y_h = e^{-x}(\cos x)$$

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$$3C_2 = -23.21$$

$$C_2 = -23/3$$

$$C_2 = \frac{-21}{3}$$

$$= -7$$

$$C_1 = \frac{13}{2}$$

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$y_h = -7e^{-x} + 3e^{2x}$.

Que:- $4y'' + 16y' + 17y = 0$ $\quad y(0) = -0.5$

$y'(0) = 1$

(2) $3y'' - 8y' - 3y = 0$ $\quad y(-3) = 1$

$y(3) = \frac{1}{e^2}$

(3) $y'' - 2y' + (4\pi^2 + 1)y = 0$

$y(0) = -2$

$y'(0) = 6\pi - 2$

$$\begin{array}{r|l} 2 & 68 \\ \hline 2 & 34 \\ \hline & 17 \end{array}$$

$$\begin{array}{r} 17 \\ 4 \\ \hline 68 \end{array}$$

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$$(D^2 - 4D)y = 0.$$

$$(64D^2 + 16D + 1)y = 0$$

$$(9D^2 + 6D + 1)y = 0.$$

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$$9\lambda^2 + 4\lambda + 1 = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \times 9 \times 1}}{18}$$

$$= \frac{-4 \pm \sqrt{16 - 36}}{18}$$

$$= \frac{-4 \pm \sqrt{-20}}{18}$$

$$= \frac{-4 \pm 2\sqrt{5}}{18}$$

$$= \frac{-2 \pm \sqrt{5}}{9}$$

$$= \frac{-2}{9} \pm \frac{\sqrt{5}}{9}$$

$$\alpha = \frac{-2}{9} \quad \& \quad \beta = \frac{\sqrt{5}}{9}$$

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1) $4y'' + 16y' + 17y = 0$ $\qquad y(0) = -1/2$

$y'(0) = 1$

$\Rightarrow y'' + 4y' + \frac{17}{4}y = 0$

$\Rightarrow \lambda^2 + 4\lambda + \frac{17}{4} = 0$

$\Rightarrow 4\lambda^2 + 4\lambda + 17 = 0$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-4 \pm \sqrt{16 - 4 \times 4 \times 17}}{8}$$

$$= \frac{-4 \pm \sqrt{16 - 272}}{8}$$

$$= \frac{-4 \pm \sqrt{-256}}{8}$$

$$= \frac{-4 \pm 16\sqrt{-1}}{8}$$

$$= \frac{-\cancel{4}(1 \pm 4\sqrt{-1})}{\cancel{8}\,2}$$

$$= \frac{-1}{2} \pm 2i$$

$$\begin{array}{r} 256 \\ 16 \\ \hline 272 \\ 16 \\ \hline 256 \end{array}$$

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$\alpha = \frac{1}{2}$ $\beta = 2$.

So, $y_h = e^{-\frac{1}{2}x}(A\cos 2x + B\sin 2x)$.

$\frac{-1}{2} = (A\cos 0 + B\sin 0)$.

$\Rightarrow \frac{-1}{2} = A$.

$y'_h = e^{-\frac{1}{2}x} \cdot \left(\frac{-1}{2}\right)(A\cos 2x + B\sin 2x)$

$+ e^{-\frac{1}{2}x}\left[A(-\sin 2x)\cdot 2 + B\cos 2x \cdot 2\right]$.

$\Rightarrow 1 = \left(-\frac{1}{2}\right)(A) + 2B$

$= 2B - \frac{A}{2}$

$= 2B - \left(\frac{-1}{2} \div 2\right)$.

$1 = 2B + \frac{1}{4}$

$\Rightarrow 1 - \frac{1}{4} = 2B$

$\Rightarrow \frac{4-1}{4} = 2B$

$\Rightarrow B = 3/8$

$y_h = e^{-\frac{1}{2}x}\left(\frac{-1}{2}A\cos 2x + \frac{3}{8}\sin 2x\right)$

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$$y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$$

$$y_h = c_1 e^{3x} + c_2 e^{-1/3\, x}$$

$$1 = c_1 x e^{-1} + c_2 x e^{\cancel{1/3} x (\cancel{-3})}$$

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$$1 = \frac{c_1}{e^9} + c_2 e$$

$$\frac{1}{e^2} = c_1 e^9 + \frac{c_2}{e}$$

$$\Rightarrow \cancel{e^9}$$

$$\Rightarrow \frac{1}{e^{\cancel{2}}} = \frac{e c_1 e^9 + c_2}{\cancel{e}}$$

$$\Rightarrow \frac{1}{e} = c_1 e^{10} + c_2$$

$$\Rightarrow 1 = c_1 e^{11} + c_2 e$$

$$\Rightarrow 1 = \frac{c_1}{e^9} + c_2 e$$

$$1 = c_1 e^{11} + c_2 e$$

$(-)$

$$\Rightarrow \frac{1}{e^2} = c_1 e^9 + \frac{c_2}{e}$$

$$\Rightarrow \frac{1}{e^{\cancel{2}}} = \frac{c_1 e^{10} + c_2}{\cancel{e}}$$

$$\Rightarrow 1 = c_1 e^{11} + c_2 e \quad \text{—— (11)}$$

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$$1 = 0 + c_2 e$$

$$c_2 = \frac{1}{e}$$

So, $c_1 = 0$ and $c_2 = \frac{1}{e}$

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$$\lambda^2 - 2\lambda + 4\pi^2 + 1 = 0$$

$$\lambda^2 - 2\lambda + (4\pi^2 + 1) = 0$$

$$\lambda = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{2 \pm \sqrt{4 - 4 \times 1 \times (4\pi^2 + 1)}}{2}$$

$$= \frac{2 \pm \sqrt{4 - (16\pi^2 + 4)}}{2}$$

$$= \frac{2 \pm \sqrt{\cancel{4} - \cancel{4} - 16\pi^2}}{2}$$

$$= \frac{2 \pm \sqrt{16\pi^2(-1)}}{2}$$

$$= \frac{2 \pm 4\pi\sqrt{-1}}{2}$$

$$= \cancel{2}\left(\frac{1 \pm 2\pi i}{\cancel{2}}\right)$$

$$= 1 \pm 2\pi i$$

$\alpha = 1 \qquad \beta = 2\pi$

$$y_h = e^{\alpha x}(A\cos\beta x + B\sin\beta x)$$

$$= e^{x}(A\cos 2\pi x + B\sin 2\pi x)$$

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$$y_h = e^{x}(A\cos 2\pi x + B\sin 2\pi x)$$

$$-2 = e^{0}(A\cos 2\pi\times 0 + B\sin 2\pi\times 0)$$

$$-2 = A$$

$$y'_h = e^{x}[A(-\sin 2\pi x)\times 2\pi + B\cos 2\pi x\cdot 2\pi]$$
$$+(A\cos 2\pi x + B\sin 2\pi x)e^{x}$$

$$6\pi - 2 = 0$$

$$6\pi - 2 = 2\pi B + A$$

$$\cancel{6}^{3}\cancel{\pi} - \cancel{2} = \cancel{2}\cancel{\pi}B - \cancel{2}$$

$$B = 3$$

$$64D^2 + 16D + 1 = 0$$

$$D^2 + \frac{D}{4} + \frac{1}{64} = 0$$

$$64\lambda^2 + 16\lambda + 1 = 0$$

$$\lambda = \frac{-16 \pm \sqrt{256 - 4\times 64}}{128}$$

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$\frac{64}{8}$, $x^2 - 4 = 0$. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

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Page No. $= \frac{-0 \pm \sqrt{0 - 4 \times 1(1-4)}}{2}$

$\lambda = \frac{-1}{8}$ $= \pm$

$y_h = e^{\lambda x} (c_1 + c_2 x)$

$= e^{-1/8 x} (c_1 + c_2 x)$

(4) $(D^2 - 4D) y = 0$

$\Rightarrow \lambda^2 - 4 = 0$

$\Rightarrow (\lambda + 2)(\lambda - 2) = 0$

$\Rightarrow \lambda = 2$ and $\lambda = -2$.

$y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

$= c_1 e^{2x} + c_2 e^{-2x}$

$= c_1 e^{2x} + c_2 e^{-2x}$

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$\ln x \overset{4}{(-)} + \ln x^4 \overset{12}{(-)} = 0$ [L.D]

(e^x, e^{-x})

$c_1 \cdot e^x + c_2 \cdot e^{-x} = 0$

$c_1 e^x + \frac{c_2}{e^x} = 0$ (L.I)

<u>The mathematical formulla for L.I or L.D</u>

<u>Wronskian (W)</u> :-

let y_1 and y_2 be the two basis of solutions of a given 2nd order differential equation

(1) If $W(y_1, y_2) = 0$

then $\{y_1, y_2\} = L.D$

(2) If $W(y_1, y_2) \neq 0$

then $\{(y_1, y_2)\} = L.I$

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$$W(y_1, y_2) = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}.$$

If $y_1 y_2' - y_1' y_2 = 0$.
then L.D.
else ..
~~to it is L.D~~.

Find the solution of-

$y'' - 2y' + y = 0$

$\Rightarrow \lambda^2 - 2\lambda + 1 = 0$

$\Rightarrow \lambda^2 - \lambda - \lambda + 1 = 0$

$\Rightarrow \lambda(\lambda - 1) - 1(\lambda - 1) = 0$

$\lambda = 1$ and $\lambda = 1$

$y_h = c_1 e^{x} + c_2 e^{x} x$.

$\begin{vmatrix} -1 & 1 \\ 0 & 0 \end{vmatrix} = 0 - 0$
$= 0$ (L.D).

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$\cancel{\lambda_1 = 3}$

$$y_h = c_1 e^{3x} + c_2 x e^{3x}$$

$\cancel{2y'' - 6y' + 3 = 0}$

$$(\lambda - 3)^2 = 0$$

$$\lambda^2 - 6\lambda + 9 = 0$$

$$y'' - 6y' + 9y = 0$$

$x - 3e^{x} + e^{3x}$

$$\begin{vmatrix} e^{3x} & xe^{3x} \\ 3e^{3x} & x - 3e^{3x} + e^{3x} \end{vmatrix}$$

$$e^{3x}(3xe^{3x} + e^{3x}) - 3xe^{9x} \neq 0$$

$$3x\cancel{e}^{6x} + e^{6x} - 3x\cancel{e}^{6x}$$

$$e^{9} e^{6x} \neq 0 \quad [L.I]$$

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Euler-Cauchy equation:-

The differential eqn of the form

$$x^2y'' + axy' + by = 0 \quad \text{--- (i)}$$

$a, b \in R$

is called Euler Cauchy equation.

$y = x^m$

$y' = m \cdot x^{m-1}$ $\quad y'' = m(m-1)x^{m-2}$

$$x^2\left[m(m-1)x^{m-2}\right] + axmx^{m-1} + bx^m = 0$$

$$\Rightarrow x^m\left\{m(m-1) + am + b\right\} = 0.$$

So, the auxillary eqn

$$x^m\left(m^2 - m + am + b\right) = 0.$$

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m_1 and m_2 then the solution

$$\boxed{y_h = c_1 x^{m_1} + c_2 x^{m_2}}$$

<u>Case 2</u>: If the roots of the auxillary eq^n are real and repeated then the sol^n

$$\boxed{y_h = (c_1 + c_2 \ln x) x^m}$$

<u>Case 3</u>: If the roots of the auxillary eq^n are complex conjugate to each other

$\alpha \pm i\beta$, then the sol^n

$$\boxed{y_h = x^{\alpha} (A \cos \beta \ln x + B \sin \beta \ln x)}$$

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$$x^m\left[m(m-1) - 2.5m - 2\right] = 0.$$

$$m^2 - m - 2.5m - 2 = 0$$

$$m^2 - 3.5m - 2 = 0$$

$$10m^2 - 35m - 20 = 0.$$

$$\Rightarrow 10m^2 - 40m + 5m - 20 = 0$$

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Ans:- $x^2y'' - 3xy' + 4y = 0.$

$\Rightarrow \quad y = x^m$

$\Rightarrow \quad y' = m \cdot x^{m-1}$

$y'' = m(m-1) \cdot x^{m-2}$

$\Rightarrow \quad x^2[m(m-1) \cdot x^{m-2}] - 3x[m \cdot x^{m-1}] + 4x^m = 0$

$\Rightarrow \quad x^m[m(m-1) - 3m + 4] = 0.$

$\Rightarrow \quad x^m[m^2 - m - 3m + 4] = 0$

$m^2 - 4m + 4 = 0$

$m^2 - 2m - 2m + 4 = 0$

$m(m-2) - 2(m-2) = 0$

$\boxed{m = 2, 2}$

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$m = 2$ and $m = 3$.

$$\boxed{c_1 x^2 + c_2 x^3 = y_h}$$

Qu:- $x^2 y'' + 7xy' + 13y = 0$.

$m^2 + m(a-1) + b = 0$.

i) $m^2 + m(6) + 13 = 0$

i) $m^2 + 6m + 13 = 0$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-6 \pm \sqrt{36 - 4 \times 1 \times 13}}{2}$$

$$= \frac{-6 \pm \sqrt{36 - 52}}{2}$$

$$= \frac{-6 \pm \sqrt{-16}}{2}$$

$$= \frac{-6 \pm 4\sqrt{-1}}{2}$$

$$= -3 \pm 2i.$$

$\Rightarrow \alpha = -3$ & $\beta = 2$

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$m^2 + m(a-1) + b = 0.$

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$y_h = x^{-3}\left[A\cos(2\ln x) + B\sin(2\ln x)\right].$

Onl:- $(x^2D^2 - 3xD + 4)y = 0.$ $\quad y(1) = 0.$ $\quad y'(1) = 3.$

$m^2 + m(a-1) + b = 0.$

$\Rightarrow m^2 + m(-3-1) + 4 = 0$

$\Rightarrow m^2 - 4m + 4 = 0.$

$m^2 - 2m - 2m + 4 = 0.$

$\Rightarrow m(m-2) - 2(m-2) = 0$

$m = 2.$

So, $(c_1 + c_2\ln x)x^2 = y_h$

$\Rightarrow c_1 = 0.$

$y'_h = 2x(c_1 + c_2\ln x) + x^2\left[0 + \frac{1}{x}\times c_2\right].$

$= 2(0+0) + x^2 \times \frac{c_2}{x}$

$3 = c_2 \times x.$

$c_2 = 3.$

~~y_h~~ $y_h = (3\ln x)x^2.$

$y_h = 3x^2\ln x$

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Answers

1. $4x^2y'' + 24xy' + 25y = 0$ $\quad y(1) = 2$, $y'(1) = -4$.

$y = x^m$

$\Rightarrow y' = m \cdot x^{m-1}$

$\Rightarrow y'' = m(m-1)\, x^{m-2}$

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$$\Rightarrow m = \frac{-20 \pm \sqrt{400 - 4\times4\times25}}{8}$$

$$= \frac{-20 \pm \sqrt{400 - 400}}{8}$$

$$= \frac{-\cancel{20}\,\cancel{10}\,5}{\cancel{8}\,\cancel{4}\,2} = -5/2.$$

$$x^m [c_1 + c_2 \ln x] = y_h$$

$$\Rightarrow x^{-5/2} [c_1 + c_2 \ln x] = y_h.$$

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2nd Order Non-homogenous D.E with const. co-efficients :-

$$y'' + ay' + by = r(x) \quad \text{——— (i)}$$

Find y_h :-

Put $r(x) = 0$

Find y_p :- (Particular solution)

The General solⁿ :-

$$\boxed{y = y_h + y_p}$$

y_p :-

CASE 1 :

If $r(x) = ke^{\alpha x}$ and α is not a root of the auxillary eqⁿ then trial

$$y_p = ce^{\alpha x}$$

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$\lambda^2 - 3\lambda + 2 = 0$

$\Rightarrow \lambda^2 - 2\lambda - \lambda + 2 = 0$

$\lambda = 1$ and 2

$y_h = e^{x} c_1 + e^{2x} c_2$.

$y_p = c e^{-x}$

$y' = c \times (-1) \cdot e^{-x}$

$y'' = c \cdot e^{-x}$

$c \cdot e^{-x} + 3c e^{-x} + 2c e^{-x} = e^{-x}$

$\Rightarrow \quad 6c \cancel{e^{-x}} = \cancel{e^{-x}}$

$c = \frac{1}{6}$.

$y_p = \frac{1}{6} e^{-x}$ — (2)

$y = y_h + y_p$

$y = e^{x} c_1 + e^{2x} c_2 + \frac{1}{6} e^{-x}$

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Case 2: (for y_p).

In case 1, if α is the root of the auxillary eqn and appearing once, and appearing once then the trial

$y_p = cxe^{\alpha x}$

If α is appearing twice then,

$y_p = cx^2e^{\alpha x}$

$y'' - 3y' + 2y = e^x$

$e^x = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda = 1$ and $\lambda = 2$.

$e^x = 0$.

$y_h = e^x c_1 + c_2 e^{2x}$

$y_p = cxe^x$

$y' = c[x \cdot e^x + e^x]$ — (1)

$y'' = c[x \cdot e^x + e^x + e^x]$

$= c[xe^x + 2e^x]$ — (2)

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$2c_2 = 4 \quad c = -1$

$c_2 = 2.$

$y_p = -1 x e^{x}$

$y = -x e^{x} + c_1 e^{x} + c_2 e^{2x}.$

$\cancel{= e^{x}(2x + c_1 + c_2)}$

$= c_1 e^{x} + c_2 e^{2x} - x e^{x}$

$= e^{x}[c_1 + c_2 - x].$

$y'' + 2y' + y = e^{-x} \qquad y(0) = -1$

$y'(0) = +1$

$\lambda^2 + 2\lambda + 1 = 0.$

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$y_h = e^{\lambda x}[c_1 + c_2 x]$

$= e^{-x}[c_1 + c_2 x]$.

$e^{-x} = 0$.

$y_p = c e^{-x}$ $\quad c x^2 e^{-x}$

$y' = c \cdot (-1) e^{-x} = -c e^{-x}$.

$y'' = c \cdot (-1)(-1) e^{-x} = c e^{-x}$

$c e^{-x} - 2c e^{-x} + c e^{-x} = e^{-x}$

$y' = c\left[2x \cdot e^{-x} - x^2 e^{-x}\right]$.

$y'' = c\left[2\{x \cdot e^{-x}(-1) + e^{-x}\} + x^2 e^{-x} + e^{-x} \cdot 2x\right]$

$= c\left[\cancel{-2x e^{-x}} + 2e^{-x} + x^2 e^{-x} + \cancel{2x e^{-x}}\right]$

$= c\left[2e^{-x} + x^2 e^{-x}\right]$.

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$\lambda = \pm 2i$

$y_h = e^{0x}(4\cos 2x + 3\sin 2x)$ — (i)

$y_p = \cancel{8x^2 + 0x + 0} \; a_0 + a_1 x + a_2 x^2$

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$\Rightarrow 2a_2 + 4a_0 = 0$.

$4a_1 = 0$.

$\boxed{a_1 = 0}$.

$4a_2 = 8$

$\boxed{a_2 = 2}$

$\boxed{a_0 = -1}$

$y_p = -1 + 2x^2$
$= 2x^2 - 1$ ——— (ii)

$y = (A\cos 2x + B\sin 2x) + 2x^2 - 1$

Ans:- $y'' + 2y' + 10y = 25x^2 + 3$.

$y'' + 2y' + 10y = 0$.

$\lambda^2 + 2\lambda + 10 = 0$.

$\Rightarrow \lambda = \dfrac{-2 \pm \sqrt{4 - 4\times 10\times 1}}{2}$

$= \dfrac{-2 \pm \sqrt{-36}}{2}$

$= \dfrac{-2 \pm 6i}{2} = \boxed{-1 \pm 3i}$.

$\alpha = -1$
$\beta = 3$,

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$y_h = e^{-x}(A\cos 3x + B\sin 3x)$

$y_p = 25x^2 + 3$

$= a_0 + a_1 x + a_2 x^2$

$y' = a_1 + 2a_2 x$

$y'' = 2a_2$

$2a_2 + 2(a_1 + 2a_2 x) + 10(a_0 + a_1 x + a_2 x^2)$

$= 25x^2 + 3$

$\Rightarrow \boxed{2a_2 + 2a_1} + 4a_2 x + 10a_0 + 10a_1 x + 10a_2 x^2$

$= 25x^2 + 3$

$\Rightarrow 2a_2 + 2a_1 + 10a_0 + 4a_2 x + 10a_1 x + 10a_2 x^2$

$= 25x^2 + 0x + 3$

$\Rightarrow 2a_2 + 2a_1 + 10a_0 = 3$

$10a_2 = 25 \quad \Rightarrow \boxed{a_2 = \frac{25}{10} = \frac{5}{2}}$

$4a_2 + 10a_1 = 0$

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$$\cancel{4}^{2} \times \frac{5}{\cancel{2}} + 10a_1 = 0$$

$$10a_1 = -10$$

$$\boxed{a_1 = -1}$$

$$\cancel{2} \times \frac{5}{\cancel{2}} + 2 \times (-1) + 10a_0 = 3$$

$$\cancel{5 - 2} + 10a_0 = \cancel{3}$$

$$a_0 = 0$$

$$y_p = 0 + (-1)x + \frac{5}{2}x^2$$

$$= 2.5x^2 - x$$

$$\boxed{y = e^{-x}(A\cos 3x + B\sin 3x) + 2.5x^2 - x}$$

<u>Case 4</u>: If $r(x) = k\cos\omega x$ or $k\sin\omega x$ then,

Trial $y_p = M\cos\omega x + N\sin\omega x$

<u>Ex</u>:- $y'' + 4y = \sin 3x$

$$\lambda^2 + 4 = 0$$

$$\lambda^2 = -4$$

$$\lambda = \pm 2i$$

$$y_h = e^{0x}(A\cos 2x + B\sin 2x)$$

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Ans:-

$y_p = M\cos 3x + N\sin 3x$

$y' = -3M\sin 3x + 3N\cos 3x$

$y'' = -9M\cos 3x - 9N\sin 3x$

$y'' + 4y = \sin 3x$

$-9M\cos 3x - 9N\sin 3x + 4(M\cos 3x + N\sin 3x)$
$= \sin 3x$

$\Rightarrow -9M\cos 3x - 9N\sin 3x + 4M\cos 3x + 4N\sin 3x$
$= \sin 3x$

$\Rightarrow -5M\cos 3x - 5N\sin 3x = \sin 3x + 0\cdot\cos 3x$

$5M = 0$

$-5N = 1$

$N = -1/5$

$M = 0$

$y_p = \frac{-1}{5}\sin 3x + 0$

$y_h = (A\cos 2x + B\sin 2x) - \frac{1}{5}\sin 3x$

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Que:- $y'' + 2y' + 5y = e^{0.5x} + 40\cos 4x - 55\sin 4x$

$y'' + 2y' + 5y = 0$

$A^2 + 2A + 5 = 0$

$A = \frac{-2 \pm \sqrt{4-20}}{2}$

$= -1 \pm 2i$

$y_h = e^{-x}(A\cos 2x + B\sin 2x)$

$y_p = e^{0.5x} + 40\cos 4x - 55\sin 4x$

$y_p = Ce^{0.5x} + M\cos 4x + N\sin 4x$

$y'_p = Ce^{0.5x} \cdot 0.5 + 4N\cos 4x - 4M\sin 4x$

$y''_p = 0.25Ce^{0.5x} - 16N\sin 4x - 16M\cos 4x$

$\cancel{+16N\sin 4x - 16M\cos 4x}$

$0.25Ce^{0.5x} + 2 \times C \times 0.5 \times e^{0.5x} + 8N\cos 4x - 8M\sin 4x$

$+ 5e^{0.5x}C + 200\cos 4x - 275\sin 4x$

$= e^{0.5x} + 40\cos 4x - 55\sin 4x$

$\Rightarrow e^{0.5x}C(5 + 0.25 + 1) - 8N\cos 4x - 8M\sin 4x$

$+ 200\cos 4x - 275\sin 4x$

$= e^{0.5x} - 8N\cos 4x$

$- 8M\sin 4x$

$\cancel{+ 200\cos 4x - 275\sin 4x}$

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$6.25\,C\,\cancel{e^{0.5x}} = \cancel{e^{0.5x}}$

$$\boxed{C = 1/6.25}$$

$$\boxed{M = 0 \qquad N = 5}$$

Case 5:- If $r(x) = ke^{\alpha x}\cos\omega x$
or
$ke^{\alpha x}\sin\omega x$

Then,
the trial $y_p = e^{\alpha x}(M_1 \sin\omega x + N_1 \cos\omega x)$

[where α is not a root of auxillary eqn].

$$\boxed{\begin{aligned} M_1 &= M \cdot C \\ N_1 &= N \cdot C \end{aligned}}$$

$y_p = x e^{\alpha x}(M_1 \sin\omega x + N_1 \cos\omega x)$

if α is appearing once in auxillary eqn

else,

$y_p = x^2 e^{\alpha x}(M_1 \sin\omega x + N_1 \cos\omega x)$

[if α is appearing twice]

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Ans. $y'' + 6y' + 9y = 50e^{-x}\cos x$.

$r(\lambda) = 0$.

$\lambda^2 + 6\lambda + 9 = 0$.

$\lambda^2 + 3\lambda + 3\lambda + 9 = 0$

$\lambda(\lambda+3) + 3(\lambda+3) = 0$.

$\lambda = -3, -3$.

$y_p = x^2 e^{-3x}(M_1 \sin x + N_1 \cos x)$

$y' = x^2 e^{-3x}[M\cos x - N\sin x]$.

$+ [x^2 \cdot e^{-3x} \cdot (-3) + e^{-3x} \cdot 2x](M\cos x - N\sin x)$

$y'' = x^2 e^{-3x}[-M\sin x - N\cos x] +$

$(M\cos x - N\sin x)[x^2 \cdot e^{-3x} \cdot (-3) + e^{-3x} \cdot 2x]$.

$y_p = e^{-3x}(M_1 \sin x + N_1 \cos x)$.

$y' = e^{-3x} \cdot [M\sin x + N\cos x]$.

$+ M\sin x \, e^{-3x}(M\cos x - N\sin x)$.

ke^{-x}

ce^{-x}

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$+ 6(-3) \cdot e^{-3x} (M\cos x - N\sin x)$

$+ e^{-3x} (-M\sin x - N\cos x) -$

$3M\cos x + 3N\sin x$

$= 9e^{-3x} M\sin x + 9e^{-3x} N\cos x$

$- 3e^{-3x} M\cos x + 3e^{-3x} N\sin x$

$- e^{-3x} M\sin x - e^{-3x} N\cos x -$

$3M\cos x + 3N\sin x$

$= e^{-3x}[9M\sin x + 9N\cos x - 3M\cos x$
$+ 3N\sin x - M\sin x - N\cos x$
$- 3M\cos x + 3N\sin x]$

$= e^{-3x}[8M\sin x + 8N\cos x - 6M\cos x$
$+ 6N\sin x]$

$e^{-3x}[8M\sin x + 8N\cos x - 6M\cos x + 6N\sin x]$

$+ 6\{[-3e^{-3x} M\sin x - 3e^{-3x} N\cos x]$
$+ e^{-3x} M\cos x - e^{-3x} N\sin x\}$

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$$y'' + 8y' + 16y = 64 \cosh 4x$$

$$= \cancel{64}^{32} \left(\frac{e^{4x} + e^{-4x}}{\cancel{2}} \right)$$

$$= 32 e^{4x} + 32 e^{-4x}$$

$$r(x) = 32 e^{4x} + 32 e^{-4x}$$

$$= c e^{4x} + c_1 x e^{-4x}$$

$$y'' - 4y = e^{-2x} - 3x$$

$$r(x) = e^{-2x} - 3x$$

$$\lambda^2 - 4 = 0$$

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Ans:- $3y'' + 10y' + 3y = 9x$.

$\Rightarrow 3y'' + 10y' + 3y = 0$.

$\Rightarrow y'' + \frac{10}{3}y' + y = 0$.

$\Rightarrow \lambda^2 + \frac{10}{3}\lambda + 1 = 0$.

$\Rightarrow 3\lambda^2 + 10\lambda + 3 = 0$.

$\Rightarrow 3\lambda^2 + 9\lambda + 1\lambda + 3 = 0$

$\Rightarrow 3\lambda(\lambda+3) + 1(\lambda+3) = 0$.

$(\lambda+3)(3\lambda+1) = 0$.

$\lambda = -3$ & $\lambda = -1/3$.

$y_h = c_1 e^{\lambda_1 x} + c_2 e^{\lambda_2 x}$

$= c_1 e^{-3x} + c_2 e^{-1/3 x}$

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$0 + 10a_1 + 3a_0 + 3a_1 x = 3x$

$\Rightarrow \quad 10a_1 + 3a_0 = 0.$

$3a_1 x = 3x$

$a_1 = 1$

$10 + 3a_0 = 0.$

$3a_0 = -10$

$a_0 = -10/3$

$y_p = -10/3 + x.$

$$\boxed{y = c_1 e^{-3x} + c_2 e^{-1/3 x} - \frac{10}{3} + x}$$

<u>Ques:-</u> $y'' + y' = x^2 + 2x + 2$ $\qquad y(0) = 8$

$\Rightarrow \quad y'' + y' + 0y = 0.$ $\qquad y'(0) = -1.$

$\Rightarrow \quad \lambda^2 + \lambda = 0.$

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$r(x) = x^2 + 2x + 2$

$y = a_0 + a_1 x + a_2 x^2$

$y' = 0 + a_1 + 2a_2 x$

$y'' = 0 + 0 + 2a_2$
$= 2a_2$.

$2a_2 + a_1 + 2a_2 x = x^2 + 2x + 2$.

$\Rightarrow 2a_2 + a_1 = 2$.

$2a_2 x = 2x$
$a_2 = 1$

$\cancel{2} + a_1 = 0 + \cancel{2}$
$a_1 = 0$.

$y_p = x^2 + 0$
$= x^2 + a_0$.

$y = c_1 + c_2 e^{-x} + x^2 + a_0$.

At $x = 0$ $y = 8$.

$\Rightarrow 8 = c_1 + c_2 + 0 + a_0$.

$\Rightarrow 8 = c_1 + c_2 + a_0$.

$y' = 0 + c_2 \cdot e^{-x} \cdot (-1) + 2x$
$-1 = -c_2 \cdot e^{-x} + 2x$
$-1 = -c_2$. $\Rightarrow c_2 = 1$

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$c_2 = 1$

$c_1 + c_2 = 8$

$c_1 + 1 = 8$

$\boxed{c_1 = 7 \quad 7 - a_0}$

So, $\boxed{y = 7 + e^{-x} + x^2 - a_0}$

$y'' - y = 2e^{x} + 6e^{2x}$

$\lambda^2 - 1 = 0$

$\lambda = \pm 1$

$y_h = c_1 e^{x} + c_2 e^{-x}$

$y_p = 2e^{x} + 6e^{2x}$

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$\Rightarrow 5ce^{2x} = 2e^{x} + 6e^{2x}$

$\Rightarrow 5ce^{2x} + 0 \cdot ce^{x} = 2e^{x} + 6e^{2x}$

$y'' = c[x \cdot e^{x} + 2e^{x}] + 4c_1 e^{2x}$

$y_p = cx\cancel{e^{x}} + 2ce^{x} + 4c_1 e^{2x} = cx\cancel{e^{x}} - c_1 e^{2x}$

$\Rightarrow 2ce^{x} + 3c_1 e^{2x} = 2e^{x} + 6e^{2x}$

$2c\cancel{e^{x}} = 2\cancel{e^{x}}$

$c = 1$

$3c_1 \cancel{e^{2x}} = 6\cancel{e^{2x}}$

$c_1 = 2$

$c = 1$

$c_1 = 2$

$y = y_h + y_p$

$= c_1 e^{x} + c_2 e^{-x} + xe^{x} + 2e^{2x}$

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Qu: (i) $y'' + 2y' - 35y = 12e^{5x} + 37\sin 5x$

(ii) $y'' - 2.8y' + 1.96y = 2e^{1.4x}$

$y(0) = 0$

$y'(0) = 0$

(i) $y'' + 2y' - 35y = 0$

$\Rightarrow \lambda^2 + 2\lambda - 35 = 0$

$\Rightarrow \lambda^2 + 7\lambda - 5\lambda - 35 = 0$

$\Rightarrow \lambda(\lambda + 7) - 5(\lambda + 7) = 0$

$\lambda = 5 \;\&\; \lambda = -7$

$y_h = c_1 e^{5x} + c_2 e^{-7x}$

$r(x) = 12e^{5x} + 37\sin 5x$

$y_p = cxe^{5x} + (M\cos 5x + N\sin 5x)$

$\Rightarrow y' = c[x \cdot e^{5x} \cdot 5 + e^{5x} \cdot x] + (N\cos 5x \cdot 5 - M\sin 5x \cdot 5)$

$\Rightarrow y'' = c[5x \cdot e^{5x} \cdot 5 + 5x \cdot e^{5x} \cdot 5 + 5e^{5x}]$

$+ (-5N\sin 5x \cdot 5 - 5M\cos 5x \cdot 5)$

$= c[25xe^{5x} + 10e^{5x}]$

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$\Rightarrow 25\cancel{Cxe^{5x}} + 10Ce^{5x} - 25N\sin 5x - 25M\cos 5x$

$+ \cancel{10Cxe^{5x}} + Ce^{5x} + 5N\cos 5x - 5M\sin 5x$

$- \cancel{35Cxe^{5x}} - 35M\cos 5x - 35N\sin 5x$

$= 12e^{5x} + 37\sin 5x$

$\Rightarrow 11Ce^{5x} - 60N\sin 5x - 5M\sin 5x$

$- 60M\cos 5x + 5N\cos 5x$

$\cancel{\Rightarrow 12e^{5x} - \sin 5x(60N + 5M)}$

$\cancel{- \cos 5x(60M - 5N)}$.

$\Rightarrow 11Ce^{5x} - \sin 5x(60N + 5M) - \cos 5x(60M - 5N)$

$= 12Ce^{5x} + 37\sin 5x + 0.\cos 5x$

$\Rightarrow 11C = 12$

$C = \frac{12}{11}$

$\cancel{\Rightarrow 25M = -37}$

$\cancel{M = \frac{-37}{25}}$

$-60N - 5M = 37$

$60M - 5N = 0$

$\Rightarrow 60N + 5M = -37$

$60M - 5N = 0$

$\Rightarrow 60N + 5M = -37$

$\cancel{-600}$

$N - 720M = 0$

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Variation of Parameters :-

$$y'' + P(x)y' + Q(x)y = r(x) \quad \text{--- (1)}$$

The particular sol^n y_p is obtained using variation of parameter method as follows.

$$y_p = -y_1 \int \frac{y_2\, r(x)}{W}\, dx + y_2 \int \frac{y_1\, r(x)}{W}\, dx \quad \text{--- (2)}$$

Where y_1 and y_2 are basis of sol^n of homogenous solution

and W is $\begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$

Om:-

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$$y_p = -\cos x \int \sin x \sec x \, dx + \sin x \int 1 \, dx$$

$$= -\cos x \ln(\sec x) + x \sin x.$$

$$y = y_h + y_p$$

$$= A\cos x + B\sin x - \cos x \ln \sec x + x \sin x$$

Qu:- $x^2 y'' - 4xy' + 6y = 21x^{-4}$

$\Rightarrow$ $y = x^m$

$y' = m \cdot x^{m-1}$

$y'' = m(m-1) \cdot x^{m-2}$

$\Rightarrow$ $x^m \cdot m(m-1) - 4(m \cdot x^m) + 6x^m = 0$

$\Rightarrow$ $x^m [m(m-1) - 4m + 6] = 0$

$\Rightarrow$ $m^2 - m - 4m + 6 \rightsquigarrow$

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$= \frac{1}{2} \times x^{-4}$ (Answer).

Qn:- $xy'' - y' = (3+x)$

$\Rightarrow x^2y'' - xy' = 3x + x^2$.

$\Rightarrow y = x^m$

$y' = m \cdot x^{m-1}$

$y'' = m(m-1) \cdot x^{m-2}$

$\Rightarrow m(m-1) - m = 0$

$\Rightarrow m^2 - m - m = 0$

$m^2 - 2m = 0$

$m(m-2) = 0$

$m = 0$ & $m = 2$

$c_1 x^0 + c_2 x^2$

$c_1 + c_2 x^2 = y_h$

$y_1 = 1$ $\quad y_2 = x^2$

$$W = \begin{vmatrix} 1 & x^2 \\ 0 & 2x \end{vmatrix} = 2x.$$

Qn:

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$r(x) = 3x^2$

$y'' - \frac{1}{x} y' = \frac{3}{x} + 1$

$\Rightarrow y_p = \frac{3}{x} + 1.$

$-y_1 \int \frac{y_2 \, r(x)}{W} dx + y_2 \int \frac{y_1 \, r(x)}{W} dx$

$\Rightarrow -1 \int \frac{x^2 \times \frac{3}{x} + 1}{2x} + 2x \int \frac{1 + \frac{3}{x}}{2x} dx$

$\downarrow$ solve further.

<u>Qn 1:-</u> $(x^2 D^2 - 2x D + 2) y = x^3 + x.$

$\Rightarrow x^2 y'' - 2xy' + 2y = x^3 + x^3$

$\Rightarrow m(m-1) - 2m + 2 = 0.$

$\Rightarrow m^2 - 3m + 2 = 0$

$\Rightarrow m^2 - m - 2m + 2 = 0$

$m(m-1) - 2(m-1) = 0$

$m = 1 \;\&\; 2$

$y_h = c_1 x + c_2 x^2$

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$$y_p = \frac{x^3+x}{x^2}$$

$$= x + \frac{1}{x}$$

$$y_p = -y_1 \int \frac{y_2\, r(x)}{W}\, dx + y_2 \int \frac{y_1\, r(x)}{W}\, dx$$

$$W = \begin{vmatrix} x & x^2 \\ 1 & 2x \end{vmatrix}$$

$$= 2x^2 - x^2 = x^2$$

$$= -x \int \frac{\cancel{x^2} \times \left(x + \frac{1}{x}\right)}{\cancel{x^2}} + x^2 \int \frac{\cancel{x}\left(x + \frac{1}{x}\right)}{x^{\cancel{2}}}$$

$$= -x \int x + \frac{1}{x}\, dx + x^2 \int 1 + \frac{1}{x^2}\, dx$$

$$r(x) = x \sin x$$

$$-x \int \frac{\cancel{x^2} \times x \sin x}{\cancel{x^2}} + x^2 \int \frac{\cancel{x} \times x \sin x}{\cancel{x^2}}$$

$$= -x \int x \sin x$$

$$\downarrow$$

$$I$$

Ans:

(2)

$I = \int x \cos x$

$= x \int \cos x - \int \frac{d}{dx}(x) \int \cos x$

$= x \sin x - \int \sin x$

$= \cancel{\sin x (x-1)}, \cancel{x \sin x + \cos x}$

$y_p = \cancel{-x \sin x (x-1) + x^2 \sin x}$

$= \cancel{-x^2 \sin x + x \sin x + x^2 \sin x}$

$= \cancel{x \sin x}$

$= x \sin x - \sin x$

$y_p = \cancel{-x \sin x - \sin x}$

Que:- $(x^2 D^2 + x D - 9) y = 48 x^5$

(2) $4x^2 y'' + 8xy' - 3y = 7x^2 - 15x^3$

$= \int x \cos x$

$= x \int \cos x - \int \frac{d}{dx}(x) \cdot \int \cos x$

$= x \sin x - \int \sin x$

$= x \sin x + \cos x$

$y_p = \cancel{-x^2 \sin x} - x\cos x + \cancel{x^2 \sin x}$

$= -x\cos x$

Applications of electric circuits :-

(1) $E_R = RI$

$E_L = L\dfrac{dI}{dt}$

$E_C = \dfrac{1}{C}\int I\,dt$

$RI + LI' + \dfrac{1}{C}\int I\,dt = E_0 \sin\omega t$

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the D.E for RLC circuit is ~~LCR~~

$$LI' + RI + \frac{1}{C}\int I\,dt = E_0 \sin\omega t$$

we change this eqn into a 2nd order eqn
let us differentiate both sides
w.r.t t, so the above eqn

$$LI'' + RI' + \frac{1}{C} \times I = E_0 \cdot \omega \cos\omega t$$, this is

the 2nd order D.E for RLC circuit.
Here the homogeneous eqn is $LI'' + RI' + \frac{1}{C} \times I = 0$

Solving these homogenous eqn we get our complimentary function, solution which is called the steady state current for the RLC circuit,

But the total soln of the non-homogenous equation is called the transient solution.

Qu:- Find the current in RLC circuit where
$R = 2\Omega$, $L = 1H$, $C = 0.5\,F$ and
$E = 50 \sin t$
$E_0 = 50$, $\omega = 1$

$$LI'' + RI' + \frac{1}{C} I = E_0 \omega \cos\omega t$$

$$\Rightarrow I'' + 2I' + 2I = 50 \cos t$$

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Hence

$\Rightarrow \quad r'' + 2r' + 2r = 120\cos t$

$\Rightarrow \quad r'' + 2r' + 2r = 0$

$\Rightarrow \quad \cancel{r'' + r' + 2r + 2r}$

$\Rightarrow \quad \lambda^2 + 2\lambda + 2 = 0$

$\Rightarrow \quad \lambda^2 = \dfrac{-2 \pm \sqrt{4 - 4 \times 1 \times 2}}{2}$

$= \dfrac{-2 \pm \sqrt{-4}}{2}$

$= \dfrac{-2 \pm 2i}{2}$

$= -1 \pm i$

$\alpha = -1 \quad , \beta = -i$

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$\Rightarrow -N\sin t - M\cos t + 3N\cos t - 2M\sin t$

$+ 2M\cos t + 2N\sin t = y_p$

$\Rightarrow N\sin t + M\cos t - 2M\sin t + 2N\cos t$

$\Rightarrow \sin t(N - 2M) + \cos t(M + 2N) = y_p$.

$M + 2N = 10$.

$N - 2M = 0$.

$M + 2N = 10$.

$2M - N = 0$.

$2M + 4N = 100$.

$2M - N = 0$.

$(-) \quad (+)$

$5N = 100$

$N = 20$.

$\boxed{\begin{array}{l} N = 20 \\ M = 10 \end{array}}$

$2M - 20 = 20$

$2M = 20$

$M = 10$

$y_p = 10\cos t + 20\sin t$

$y_h = e^{-t}(A\cos t + B\sin t) + 10\cos t + 20\sin t$.

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$5\Gamma'' + 20\Gamma' + 100\Gamma = 4125 \sin 4t + 1700 \cos 4t$

$\Gamma'' + 4\Gamma' + 20\Gamma = 825 \sin 4t + 340 \cos 4t$

$\lambda^2 + 4\lambda + 20 = 0$

$\lambda = \dfrac{-4 \pm \sqrt{16 - 4 \times 20}}{2}$

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$\begin{array}{r} 85 \\ 140\overline{)425} \\ 25 \end{array}$ $\quad$ $\begin{array}{r} 7.10 \\ 80 \\ 16 \\ \hline 64 \end{array}$

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$y' = 4N\cos 4t - 4M\sin 4t$

$y'' = -16N\sin 4t - 16M\cos 4t$

$\Rightarrow -16N\sin 4t - 16M\cos 4t + 16N\cos 4t - 16M\sin 4t$

$+ 20M\cos 4t + 20N\sin 4t = y_p$

$\Rightarrow$ ~~$4N\sin 4t + 4M\cos 4t - 16M\cos 4t + 16N\cos 4t$~~

$\Rightarrow$ ~~$\cos 4t(4M - 16M)$~~

$\therefore$ ~~$4N\sin 4t + 4M\cos 4t + 16N\cos 4t$~~ $= y_p$.

$\Rightarrow$ ~~$4N\sin 4t + \cos 4t(4M + 16N)$~~.

$N = 0$.

$4M + 16N = 20$.

$4M = 340$

$M = \frac{340}{4}$ 85

$\Rightarrow 4N\sin 4t + 4M\cos 4t + 16N\cos 4t - 16M\sin 4t$

$\Rightarrow \cos 4t(4M + 16N) +$

$\sin 4t(4N - 16M)$.

$4M + 16N = 340$.

$16M - 4N = 0$.

$\Rightarrow 4M + 16N = 340$

$4M - N = 0$.

$(-) \quad (+)$

$17N = 340$

$N = 20$.

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$16M - 80 = 0$

$16M = 80$

$M = 5$

$\therefore M = 5$ and $N = 20$

$y = e^{-2x}(A\cos 4t + B\sin 4t) + 5\cos 4t + 20\sin 4t$

Find the transient current in RLC circuit where,

$R = 80\,\Omega$, $L = 10\,H$, $C = 0.004\,F$

$E = 240 \cdot 5 \sin 10t$, $I(0) = 0$, $I'(0) = 0$

$$L I'' + R I' + \frac{1}{C} I = 2405 \cos 10t$$

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$\frac{4}{1000}$ 4

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$y_h = e^{-4x}(A\cos 3x + B\sin 3x)$.

$y_p = M\cos 10t + N\sin 10t$.

$y' = 10N\cos 10t - 10M\sin 10t$

$y'' = -100N\sin t - 100M\sin 10t$.

$-100N\sin 10t - 100M\sin 10t + 80N\cos 10t - 80M\sin 10t$

$+ 25M\sin 10t + 25N\cos$

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Que:- Find the current in LC circuit where $L = 2H$, $C = 0.005\ F$ and $E = 220 \sin 4t$.

$$L I'' + \frac{I}{C} = 220 \times 4 \cos 4t$$

$$\Rightarrow 2I'' + 200I = 880 \cos 4t$$

$$\Rightarrow I'' + 100I = 440 \cos 4t$$

$$\lambda^2 + 100 = 0$$

$$\Rightarrow \lambda = \pm 10i$$

$$y_h = e^{0x} (A \cos 10x + B \sin 10x)$$

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Properties :-

1. $A+B = B+A$
2. $(A+B)+C = A+(B+C)$
3. $A+O = O+A = A$
4. $A+(-A) = -A+A = O$
5. $(\alpha+\beta)A = \alpha A + \beta A$
6. $\alpha(A+B) = \alpha A + \alpha B$

Upper Ar Matrix :-

A square matrix is an upper triangular matrix if all the elements present below the main diagonal must zero

Diagonal :-

A square matrix is a diagonal matrix if all the elements present above or below the main diagonal must be zero

7. $(A+B)^T = A^T + B^T = B^T + A^T$

$(cA)^T = cA^T$ where $c \in R$.

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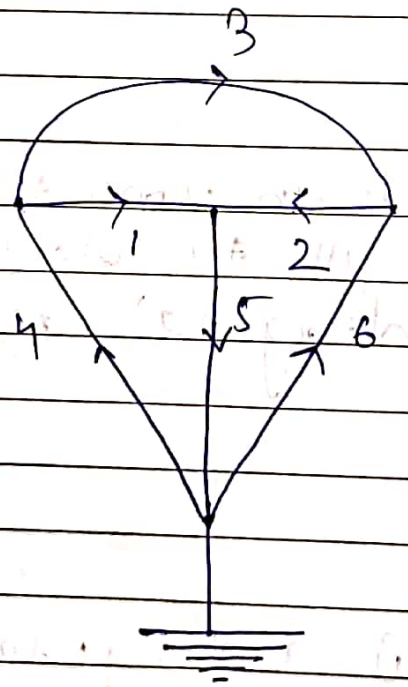
Nodal, incidence, Matrix :-

Every circuit in electrical system can be represented in the form of a matrix called nodal incidence matrix.

The matrix is represented as a_{jk} =

$$\begin{cases} +1 & \text{if branch } k \text{ leaves node } j \\ -1 & \text{if branch } k \text{ leaves } j \text{ entering node } j \\ 0 & \text{if branch } k \text{ is not touching node } j \end{cases}$$

Ons:-



	1	2	3	4	5	6
1	1	0	1	-1	0	0

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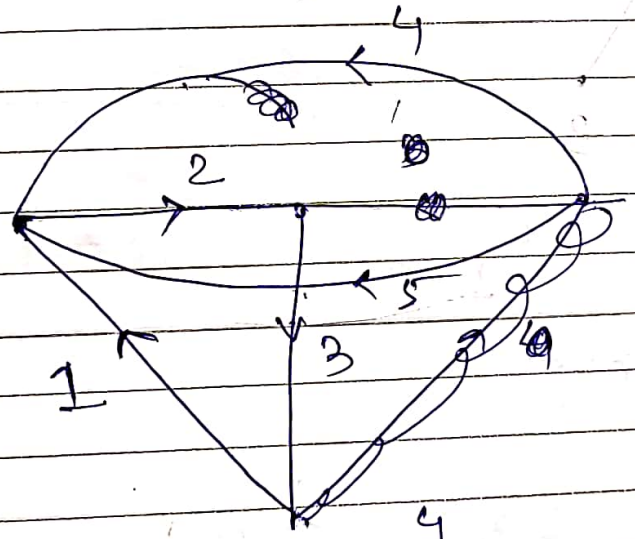
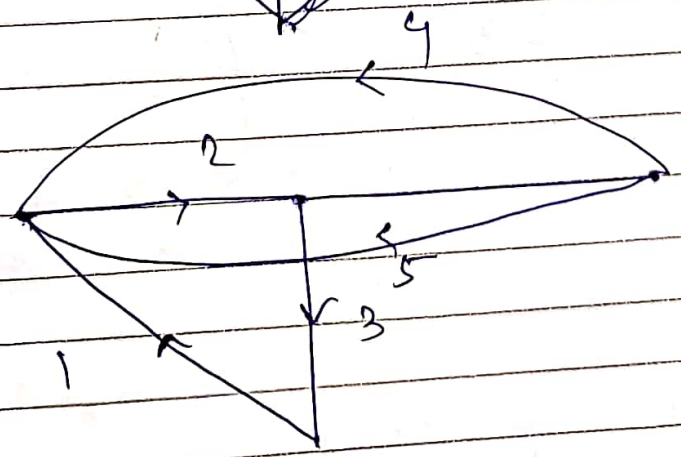
$$\begin{array}{c} \begin{bmatrix} 1 & 0 & 1 & -1 & 0 & 0 \\ -1 & -1 & 0 & 0 & 1 & 0 \\ 0 & 1 & -1 & 0 & 0 & -1 \end{bmatrix} \\ \begin{array}{cccccc} 1 & 2 & 3 & 4 & 5 & \end{array} \end{array} \quad 3\times 6.$$

Ans:-

$$\begin{array}{c} 1 \\ 2 \\ 3 \end{array} \begin{bmatrix} -1 & 1 & 0 & -1 & -1 \\ 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 1 & 0 \end{bmatrix}$$





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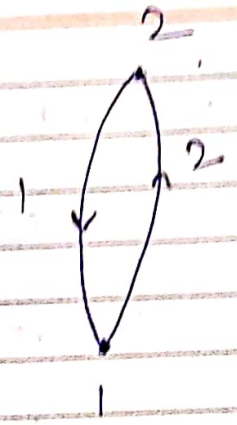
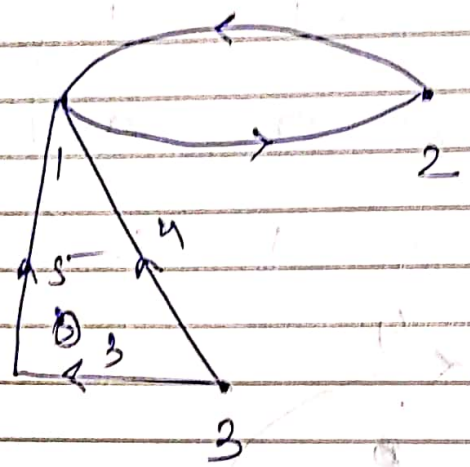
Ans:





$(A)_{m \times n}$ $B_{n \times p}$

$(AB)_{m \times p}$

Ans: $B(A) = \begin{pmatrix} 1 & 2 & 3 \end{pmatrix} \begin{pmatrix} 1 \end{pmatrix}$

$BA = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} \begin{pmatrix} 1 & 2 & 3 \end{pmatrix}$

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 4 & 6 \\ 3 & 6 & 9 \end{bmatrix}_{3 \times 3}$$

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$m \times n \quad n \times m \quad m \times p \; AB \neq BA$

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Properties under multiplication:-

(1) $A \times B \neq B \times A$ in general.

(2) $AB = 0 \Rightarrow$ Doesn't necessary imply that $A = 0$ or $B = 0$ or both.

(3) $AC = CD$
$\Rightarrow A \neq D$

$$A = \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} \qquad C = \begin{bmatrix} 2 & 1 \\ 2 & 2 \end{bmatrix}$$

$$D = \begin{bmatrix} 3 & 0 \\ 1 & 3 \end{bmatrix}$$

$$AC = \begin{bmatrix} 2+2 & 1+2 \\ 4+4 & 2+4 \end{bmatrix} = \begin{bmatrix} 4 & 3 \\ 8 & 6 \end{bmatrix}$$

$$AD = \begin{bmatrix} 3+1 & 0+3 \\ 6+2 & 0+6 \end{bmatrix}$$

$$= \begin{bmatrix} 4 & 3 \\ 8 & 6 \end{bmatrix}$$

$AC = AD$

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$K \in R$ for any two matrices A, B

→ $K(AB) = A(KB) = (KA)B$

→ $(AB)C = A(BC)$

$A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ $B = \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}$ $C = \begin{pmatrix} 2 & 2 \\ 2 & 2 \end{pmatrix}$

→ $(A+B)C = AC + BC$

Scalar Matrix :-

A diagonal matrix is called a scalar matrix if all the elements of the main diagonal are equal

Solution of system of linear equations :-

$AX = B$

$A =$ co-efficient matrix.

Augmented

$$A = \begin{pmatrix} a_{11} & \cdots & a_{1n} & b_1 \\ a_{m1} & \cdots & a_{mn} & b_n \end{pmatrix}$$

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$$= \frac{a_{11}}{a_{21}} \neq \frac{a_{12}}{a_{22}}$$

Unique sol^n

$x + y = 1$
$x - y = 0$.

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} = \frac{b_1}{b_2}$$

Infinite sol^n.

(2)

$$\frac{a_{11}}{a_{21}} = \frac{a_{12}}{a_{22}} \neq \frac{b_1}{b_2}$$

No sol^n

$x + y = 1$
$2x + 2y = 5$

$2(y+1) - 2y = 2$
$2y + 2 - 2y = 2$

Gauss elimination method :-

Row elementary operation :-

(1) We are allowed to interchange any two rows.

(2) A non-zero constant can be multiplied to any row

(3) A non-zero constant can multiplied to a row or eq^n and added to another row

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$x_1 - x_2 + 2x_3 = 1$

$2x_1 + 3x_2 + x_3 = 5$

$-x_1 + 2x_2 - 4x_3 = 2$

$$\begin{array}{ccc} 1 & -1 & 2 \\ 2 & 3 & 1 \\ -1 & 2 & -4 \end{array}$$

$(2) + (-2) \times 1 = 2$, $(3) = (3) + 1$

$x_1 - x_2 + 2x_3 = 1$ —— (1)

$\cancel{0 + 5x_2 + 3x_3 = 60}$

$0 + 5x_2 - 3x_3 = 3$ —— (2')

$0 + x_2 - 2x_3 = 3$ —— (3')

$(3') = 3' + (-1/5)\, 2'$

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$$\frac{15-3}{5} = 12/5$$

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$$-7/5\, x_3 = 12/5$$

$$x_3 = -12/7$$

$$5x_2 - 3\times\left(\frac{-12}{7}\right) = 3$$

$$5x_2 + \frac{36}{7} = 3$$

$$5x_2 = 3 - \frac{36}{7}$$

$$= \frac{21-36}{7}$$

$$\cancel{5}\, x_2 = \frac{-\cancel{15}\,3}{7}$$

$$x_2 = -3/7$$

$$x_1 + \frac{3}{7} - \frac{24}{7} = 1$$

$$x_1 = 1 + \frac{24}{7} - \frac{3}{7}$$

$$x_1 = 1+3$$
$$= 4$$

$$x_1 = 4,\ x_2 = -3/7,\ x_3 = -12/7$$

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$x_2 \rightarrow$ [illegible] $-3+4$ 2)

$3+8+24$

$2x_1 - x_2 + x_3 = 7$

$3x_1 + 2x_2 + 4x_3 = 12$

$4x_1 - 2x_2 + 2x_3 = 14$.

$$\begin{bmatrix} 2 & -1 & 1 \\ 3 & 2 & 4 \\ 4 & -2 & 2 \end{bmatrix} \begin{matrix} = 7 \\ = 12 \\ = 14 \end{matrix}$$

$$\begin{matrix} 2 & -1 & 1 & = 7 \\ 0 & 1 & 11 & = 25 \\ 4 & -2 & 2 & = 14 \end{matrix}$$

$1 = 1 \times 3$.

$2 = 2 \times (-2)$

$$\begin{matrix} 2 & -1 & 1 & = 7 \\ 6 & -3 & 3 & = 21 \\ -6 & -4 & -8 & = -24 \\ 4 & -2 & 2 & = 14 \end{matrix}$$

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$$= \begin{vmatrix} 6 & -3 & 3 \\ 0 & -7 & -5 \\ 4 & -2 & 2 \end{vmatrix} \begin{matrix} = 21 \\ = -3 \\ = 14 \end{matrix}$$

$$= \begin{vmatrix} 12 & -6 & 6 \\ 0 & -7 & -5 \\ -12 & 6 & -6 \end{vmatrix} \begin{matrix} = 42 \\ = -3 \\ = -42 \end{matrix}$$

$$\begin{vmatrix} x_1 & x_2 & x_3 \\ 12 & -6 & 6 \\ 0 & -7x_2 & -5x_3 \\ 0 & 0 & 0x_3 \end{vmatrix} \begin{matrix} \\ = 42 \\ = -3 \\ = 0 \end{matrix}$$

$\cancel{x_3 = 22}$

$-7x_2 + 5x_3 = -3$

$-7x_2 = -3$

$x_2 = 3/7$

$12x_1 - 6x_2 + 6x_3 = 42$

$12x_1 - 6 \times \frac{3}{7} + 0 = 42$

$\Rightarrow 12x_1 - \frac{18}{7} = 42$

$\Rightarrow 12x_1 = 42 + \frac{18}{7}$

$\Rightarrow x_1 = \frac{26}{7}$

It is a case of Infinitely many sol^n.

Ques:- $3x_1 + 2x_2 + x_3 = 3$

$2x_1 + x_2 + x_3 = 0$.

$6x_1 + 2x_2 + 4x_3 = 6$;

$$\begin{bmatrix} 3 & 2 & 1 \\ 2 & 1 & 1 \\ 6 & 2 & 4 \end{bmatrix} = \begin{matrix} 3 \\ 0 \\ 6 \end{matrix}.$$

$R_1 \to 2R_1$; $R_2 \to (-3) R_2$.

$$\begin{bmatrix} 6 & 4 & 2 \\ -6 & -3 & -3 \\ 6 & 2 & 4 \end{bmatrix} \begin{matrix} = 6. \\ = 0 \\ = 6. \end{matrix}$$

$R_2 \to R_1 + R_2$

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$$\begin{bmatrix} 6 & 4 & 2 \\ 0 & 1 & -1 \\ 6 & 2 & 4 \end{bmatrix} \begin{matrix} = 6 \\ = 6 \\ = 6 \end{matrix}$$

$R_3 \longrightarrow (-1) R_3$

$$\begin{bmatrix} 6 & 4 & 2 \\ 0 & 1 & -1 \\ -6 & -2 & -4 \end{bmatrix} \begin{matrix} = 6 \\ 6 \\ -6 \end{matrix}$$

$R_3 \longrightarrow R_1 + R_3$

$$\begin{bmatrix} 6 & 4 & 2 \\ 0 & 1 & -1 \\ 0 & 2 & -2 \end{bmatrix} \begin{matrix} 6 \\ = 6 \\ = 0 \end{matrix}$$

$2x_2 - 2x_3 = 0$. It has ~~many~~ NO. solution.

$x_2 - x_3 = 6$.

$x_2 = 6 + x_3$

$2(6 + x_3) - 2x_3 = 0$

$12 + 2x_3 - 2x_3 = 0$.

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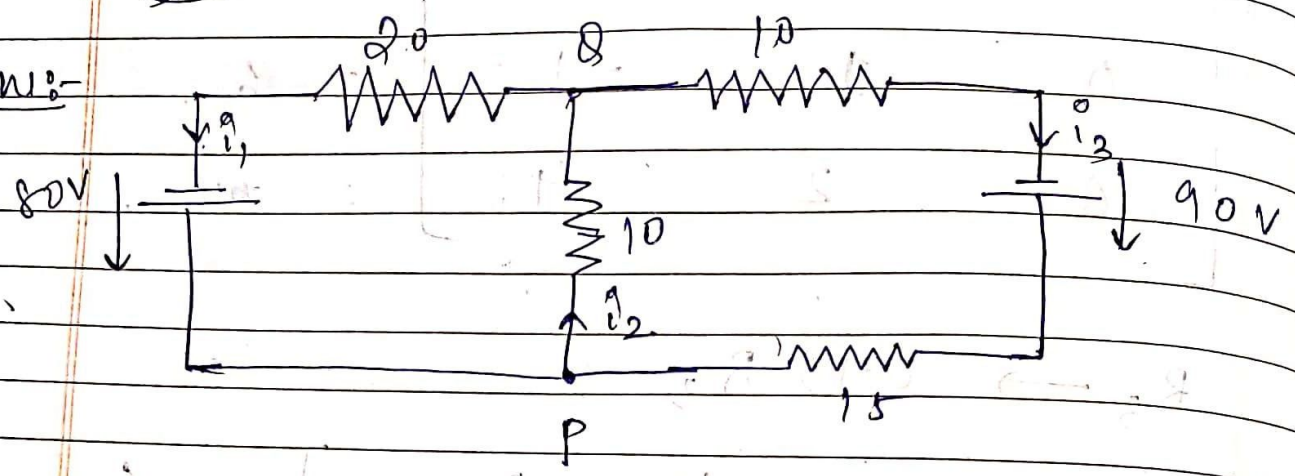
RC3 :

Ques:-



Node P

$-i_1 + i_2 - i_3 = 0$.

Node Q

$i_1 - i_2 + i_3 = 0$.

For the right loop :-

$10 i_2 + 25 i_3 = 90$.

for left loop :-

$10 i_2 + 20 i_1 = 80$.

$$\begin{bmatrix} 1 & -1 & 1 \\ 0 & 10 & 25 \\ 20 & 10 & 0 \end{bmatrix} = \begin{matrix} 0 \\ 90 \\ 80 \end{matrix}$$

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$R_3 \to R_1 \to (-20) R_1$.

$-20 \quad 20 \quad -20 \quad 0$.

$0 \quad 10 \quad 25 \quad = 90$.

$20 \quad 10 \quad 0 \quad = 80$.

$R_3 \to R_1 + R_3$

$-20 \quad 20 \quad -20 = 0$.

$0 \quad 10 \quad 25 = 90$.

$0 \quad 30 \quad -20 = 80$.

$R_2 \to -3 R_2$

$-20 \quad 20 \quad -20 = 0$.

$0 \quad -30 \quad -75 = -270$.

$0 \quad 30 \quad -20 = 80$.

$R_3 \to R_2 + R_3$

$-20 \quad 20 \quad -20 = 0$.

$0 \quad -30 \quad -75 = -270$

$0 \quad 0 \quad -95 = -190$.

$-95 x_3 = -190 \quad x_3 = 2$

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$-30x_2 - 75x_3 = -270$

$30x_2 + 180 = 270$

$30x_2 = 120$

$x_2 = \frac{120}{30} = 4$

$-20x_1 + 80 - 80 = 0$

$x_1 = 0$

So, $x_1 = 0$, $x_2 = 4$, $x_3 = 2$

~~Rank~~ Rank :-

The rank of a matrix A is the maximum of number of linearly independent rows present in A

Determinant method :-

The rank of a matrix is the order of the biggest non-zero minor obtained from that matrix

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$$A = \begin{bmatrix} 3 & 0 & 2 & 2 \\ -6 & 42 & 24 & 54 \\ 21 & -21 & 0 & -15 \end{bmatrix}$$

The rank of a matrix is defined as the order of the biggest non-zero minor obtained from that matrix.

$R_2 = R_2 + 2R_1$

$R_3 = R_3 + (-7)R_1$

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & -21 & -14 & -29 \end{bmatrix}$$

$R_3 = R_3 + (1/2) R_2$

$$\begin{bmatrix} 3 & 0 & 2 & 2 \\ 0 & 42 & 28 & 58 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$ (exists a two biggest 2x2 matrix).

So, the rank is 2.

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$$\begin{array}{r} 14 \\ 4 \\ \hline 56 \end{array}$$

$$\begin{array}{r} -56 \\ 3 \\ \hline 168 \end{array} \qquad \begin{array}{r} 50 \\ 60 \\ 25 \\ \hline 312 \end{array}$$

Ans:-

$$\begin{array}{ccc} C_1 & C_2 & C_3 \end{array}$$

$$= \begin{bmatrix} 3 & -1 & 5 \\ 2 & -4 & 6 \\ 10 & 0 & 14 \end{bmatrix}$$

$$3(-56) + 1(28 - 60) + 5(40)$$

$$\cancel{+240} \quad -168 - 32 + 200$$

$$= \cancel{-210 + 200} \quad -200 + 200$$

$$= \cancel{-10} \quad = 0.$$

$$\begin{array}{r} -16 \\ 42 \\ \hline 26 \end{array}$$

$$\begin{array}{r} 2 \\ 1 \\ 1 \\ \hline 2 \end{array} \quad \begin{array}{r} 1 \\ 6 \\ 4 \\ \hline 1 \end{array} \quad \begin{array}{r} 0 \\ 8 \\ 2 \\ \hline 0 \end{array}$$

The Rank is 2.

Ans:-

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ -3 & 4 & 4 \\ 1 & 2 & 4 \end{bmatrix}$$

$R_3 \to R_1 + R_3$.

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & 5 & 8 \\ 0 & 5 & 8 \\ 1 & 2 & 4 \end{bmatrix}$$

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$R_2 \to (-1) R_2$.

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & -5 & -8 \\ 0 & 5 & 8 \\ 1 & 2 & 4 \end{bmatrix}$$

$R_3 \to R_3 + R_2$.

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & -5 & -8 \\ 0 & 0 & 0 \\ 1 & 2 & 4 \end{bmatrix}$$

$R_3 \to (-1) R_3$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & -5 & -8 \\ 0 & 0 & 0 \\ -1 & -2 & -4 \end{bmatrix}$$

$R_4 \to R_4 + R_1$

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$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & -5 & -8 \\ 0 & 0 & 0 \\ 2 & -1 & 0 \end{bmatrix}$$

$$\begin{bmatrix} 3 & 1 & 4 \\ 0 & -5 & -8 \\ 2 & -1 & 0 \end{bmatrix}$$

$$3(10) - 1(-8) + 4(-5)$$

$$= 30 + 8 - 20$$

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Ans:

$$\begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

$\therefore R_2 = -3R_2$

$$\begin{bmatrix} 9 & 3 & 1 & 0 \\ -9 & 0 & (-3) & 18 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

$$\therefore \begin{bmatrix} 9 & 3 & 1 & 0 \\ 0 & 3 & -2 & 18 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

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