Q) % error in radius of sphere = 2%. Find % error in area and volume.

→

$$A = 4\pi r^2$$

$$\frac{\Delta A}{A} = 2\left(\frac{\Delta r}{r}\right) = 2\times 2 = 4\%$$

$$V = \frac{4}{3}\pi r^3$$

$$\frac{\Delta V}{V} = 3\left(\frac{\Delta r}{r}\right) = 3\times 2 = 6\%$$

* **Order of magnitude :-**

→ Order of magnitude of a physical quantity is that of 10 which is closest to its magnitude.

Number (given) $= a \times 10^x$

⇒ When, $a \leq 5$, $a \approx 1$, and, when $5 < a \leq 10$, $a \approx 10$

$\therefore x =$ order of magnitude

	Scientific notatn	Order of magnitude
8	≈ 10	1
49	$4.9 \times 10^1 \approx 1 \times 10^1$	1
555	$5.55 \times 10^2 \approx 10 \times 10^2$	3
999	$9.99 \times 10^2 \approx 10 \times 10^2$	3
1001	$1.001 \times 10^3 \approx 1 \times 10^3$	3
753000	$7.53000 \times 10^5 \approx 10 \times 10^5$	6
0.135	$1.35 \times 10^{-1} \approx 1 \times 10^{-1}$	−1
0.05	$5 \times 10^{-2} \approx 1 \times 10^{-2}$	−2
0.99	$9.9 \times 10^{-1} \approx 10 \times 10^{-1}$	0

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Q) What is order of magnitude of mass of electron and mass of earth.

⇒ mass of ~~earth~~ electron $= 9.11 \times 10^{-31}$ kg

$\approx 10 \times 10^{-31}$

$\approx 10^{-30}$

⇒ mass of earth, $= 6 \times 10^{24}$

$\approx 10 \times 10^{24}$

$\approx 10^{25}$

⇒ order of magnitude of, electron $= \cancel{10^{-3}}\ \cancel{-30}\ 10^{-30}$ kg

earth $= \cancel{25}\ 10^{25}$ kg

⇒ Mass of proton $= 1.67 \times 10^{-27}$ kg

$\approx 1 \times 10^{-27}$

⇒ Order of magnitude of proton $= \cancel{-27}\ 10^{-27}$ kg

Q) What is order of speed of light.

⇒ order of speed of light $= 3 \times 10^{8}$

$\approx 1 \times 10^{8}$

$\approx 10^{8}$ m/s

* Significant figures :- (sig. fig)

⇒ No. of ~~sure~~certain digit + 1 uncertain digit

Ex:- $97\cancel{6}.0008 \pm 0.0001$ ⇒ sig fig $= 7$

⇒ Rules :-

1) All non-zero digits are sig. fig.
2) Leading zeroes are not sig. fig.

3) Trapped zeroes are sig. fig.

4) Trailing zeroes

- Not sig. fig.
  - ex:- 3200 has 2
  - 35300 has 3
- Are sig. fig. (after decimal)
  - 3.00 has 3
  - 1.0 has 2
- Are sig. fig. (for quantities with unit)
  - 100 m $\rightarrow$ 3
  - 3000 sec $\rightarrow$ 4

$\Rightarrow$ Greater the sig. fig. ; greater preciseness.

$$\boxed{\begin{aligned} &3 \times 10^8 \Rightarrow (3 \pm 1) \times 10^8 \, m/s \\ &2.99 \times 10^8 \Rightarrow (2.99 \pm 0.01) \times 10^8 \, m/s \\ &2.9987 \times 10^8 \Rightarrow (2.9987 \pm 0.0001) \times 10^8 \, m/s \end{aligned}}$$

$\Rightarrow$ No. of sig. fig. don't change by changing system of unit.

$\Rightarrow$ Ex:- Distance = 3578 m $\Rightarrow$ sig fig = 4

$\Rightarrow$ In CGS system of unit.

Distance = 357800 cm $\Rightarrow$ Sig. fig $\neq$ 6

$\Rightarrow$ Sig. fig = 4

$\because$ ~~35~~ $3.578 \times 10^5 \Rightarrow$ sig. fig = 4

$\Rightarrow$ No. of sig. fig. in 2000

$\Rightarrow$ For ~~∞~~ numbers sig. fig is ∞.

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Page

0.007 $\longrightarrow$	~~3~~ 1
2.67×10^{24} $\longrightarrow$	3
0.2730 $\longrightarrow$	4
6.320 $\longrightarrow$	4
6320 $\longrightarrow$	3
6320m $\longrightarrow$	4
0.0006032 $\longrightarrow$	4
2.000 m $\longrightarrow$	4
5100 kg $\longrightarrow$	4
0.050 m $\longrightarrow$	2

⇒ **Rules for round off:-**

⇒ If digit to be droped is:- ~~smal~~

1) Small than 5, then preceding digit is left unchanged
2) Grater than 5, then preceding ~~di~~ digit is increa by one.
3) 5 followed by non-zero digits, then ~~d~~ preceding digit is increased by one.
4) 5 then preceding digit is left unchanged
5) 5 then preceding digit is ~~left~~ odd, then it is increased by one.

* Sum or difference of sig. fig.

→ The final result should be reported to the same number of decimal places as that of the no. with minimum no. of decimal place.

→ Ex:- 436.32 + 227.2 + 0.301 = 663.821 ⇒ 663.8g

* Product or division of sig. fig.:-

→ Final result should be reported to the same number of sig. fig as that of number with minimum no of sig. fig.

→ Ex:- 1) $l = 1.567 cm$
$b = 10.4 cm$

⇒ Area $= l \times b = (1.567 \times 10.4) = 16.2968 = 16.3 cm^3$

(2) $m = 8.254 g$
$V = 2.68 cm^3$

$\rho = \frac{m}{V} = \frac{8.254}{2.68} = 3.079850 = 3.08 gcm^{-3}$

Q) Volume of one cube is $3.58 cm^3$. Find volume of such 500 cubes.
→ $3.58 \times 50 = 179$

Q) $r = 2.57 cm$ Find surface area.
→ $A = 4\pi r^2 = 4 \times 3.142 \times (2.57)^2$
$= 83.0103$
$= 83.0 cm^2$

Q) Length and breadth of thin rectangular plate is $l = 16.2 cm$ and $b = 10.1 cm$. Find area.
→ $l = (16.2 \pm 0.1) cm$
$b = (10.1 \pm 0.1) cm$

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$\Rightarrow A = l \times b$

$= 16.2 \times 10.1$

$= 163.62\ cm^2$

$\Rightarrow A = lb$

$\Rightarrow \frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$

$= \frac{0.1}{16.2} + \frac{0.1}{10.1}$

$\frac{\Delta A}{A} = 0.1\left(\frac{16.2 + 10.1}{16.2 \times 10.1}\right)$

$\frac{\Delta A}{A} = \cancel{0.1}\ 0.01607$

$\Rightarrow \frac{\Delta A}{A} \times 100 = \cancel{0.0}\ 1.6\ \%$

$\Delta A = 0.016 \times 163.62 = 2.61\ cm^2$

$A \pm \Delta A = 163.62 \pm 2.61\ cm^2$

$\boxed{A \pm \Delta A = 164 \pm 2.61}$

27-1-24

Ch-3

Motion In A Straight Line

* Motion :- Motion is changes in position with respect to its surrounding with passage of time.

→ Motion is combined property of object and ~~observer~~ objective

* Particle :- Point like object having mass.

* Mechanics :- The branch of physics which studies and deals with motion of object is called Mechanics.

Mechanics

Kinematics	Dynamics
→ The branch which deals the motion without its causes.	→ The branch which deals the motion with its causes.

* Reference frame :- The situation and position from where observer takes its reference ~~frame~~.

* Types of frame of referances :-

(i) Inertial frame of referance

→ (ii) Non-identical frame of reference.

Q.) Define and explain one dimensional ~~points~~ motion, two dimensional motion and three dimensional motion.

→ For ~~di~~ discription of given motion if we require ~~th~~ one coordinate then it is called one dimensional motion.

→ e.g., free falling body, motion of vehical on straight line.

→ For complete discription of given motion if we require two coordinates then it is called two dimensional motion.

→ e.g., motion of kicked football, motion of carrom stricker on carrom board.

→ For ~~discripti~~ discription of given motion if we require three coordinates then it is called three dimensional motion.

→ e.g., kite flying, flying of ~~birds~~ beez.

* Speed :-

(i) Uniform speed :- object travel equal distance in equal interval of time.

or

→ If speed of object is not changing with time.

→ Here object may move on straight path, circular path, curved path.

(ii) non-uniform speed :-

Object ~~trval~~ travel unequal distance in equal interval of time, or, speed of object changes with time.

(iii) Average speed :-

→ In reality object always moving with variable speed so, to get overall idea (how rapidly object moves from one point to another point), we define Avg speed $\langle v \rangle$ or $\bar{v}$

(iv) Instantaneous speed

Q) Car covers first half of distance b/w 2 places at a speed of 40 km/h and second half at 60 km/h. What is avg speed.

→ $\quad d \quad\quad\quad d$

$v_1 = 40$ km/h $\quad\quad v_2 = 60$ km/h

→ Since speed are different car will take diff. time

→ Let distance $= 2d$

→ For first half, time $t_1 = \frac{d}{v_1}$

→ For second half time, $t_2 = \frac{d}{v_2}$

⇒ Total time, $t = \frac{d}{v_1} + \frac{d}{v_2} = \frac{d}{40} + \frac{d}{60}$

⇒ Avg speed, $\langle v \rangle = \frac{\text{total distance}}{\text{total time}}$

$$= \frac{2d}{\frac{d}{40} + \frac{d}{60}}$$

$$= \frac{2}{\frac{1}{40} + \frac{1}{60}}$$

$$= 48 \text{ km/h}$$

Q) Car travels along a straight line for 1^{st} half time with speed 50 km/h and 2^{nd} half time with speed 60 km/h. Find avg speed of car.

→ Supposed total time $= 2t$

→ So, $\frac{1}{2}$ time interval, distance, $d_1 = v_1 \times t = 50 \times t = 50t$

→ 2^{nd} half time interval, distance travelled, $d_2 = v_2 \times t$
$= 60t$

⇒ Total distance $= 50t + 60t = 110t$

$\therefore$ Avg speed, $\langle v \rangle = \frac{\text{total distance}}{\text{total time}} = \frac{110t}{2t} = 55 \frac{km}{h}$

Q⇒ Given object travells diff. distance, in different time intervals with different speeds. Find avg speed

→ suppose object travels distances, $d_1, d_2, \ldots d_n$ in time intervals $t_1, t_2, t_3, \ldots t_n$ with speeds $v_1, v_2, v \ldots, v_n$

$$\boxed{\langle v \rangle = \frac{\text{total distance}}{\text{total time}} = \frac{d_1 + d_2 + d_3 + \cdots + d_n}{t_1 + t_2 + t_3 + \cdots + t_n}} \text{ — (1)}$$

→ $t_1 = \frac{d_1}{v_1}$, $t_2 = \frac{d_2}{v_2}$, $t_n = \frac{d_n}{v_n}$

⇒ From eq (1),

$$\langle v \rangle = \frac{d_1 + d_2 + \cdots + d_n}{\frac{d_1}{v_1} + \frac{d_2}{v_2} + \cdots + \frac{d_n}{v_n}}$$

⇒ Suppose $d_1 = d_2 = d_3 = d_n$ (case 1)

$$\langle v \rangle = \frac{nd}{d\left(\frac{1}{v_1} + \frac{1}{v_2} + \dots \frac{1}{v_n}\right)}$$

$$\Rightarrow \boxed{\frac{n}{\langle v \rangle} = \frac{1}{v_1} + \frac{1}{v_2} + \frac{1}{v_3} + \dots \frac{1}{v_n}} \leftarrow MCQ$$

$\Rightarrow$ Suppose $n = 2$

$$\frac{2}{\langle v \rangle} = \frac{1}{v_1} + \frac{1}{v_2} = \frac{v_1 + v_2}{v_1 v_2}$$

$$\boxed{\langle v \rangle = \frac{2 v_1 v_2}{v_1 + v_2}} \leftarrow MCQ$$

$\Rightarrow$ Case-2,

$$d_1 = v_1 t_1,\ d_2 = v_2 t_2,\ \dots,\ d_n = v_n t_n$$

$$\langle v \rangle = \frac{v_1 t_1 + v_2 t_2 + \dots + v_n t_n}{t_1 + t_2 + \dots + t_n}$$

$\Rightarrow$ Suppose $t_1 = t_2 = \dots = t_n = t$

$$\langle v \rangle = \frac{t(v_1 + v_2 + \dots + v_n)}{nt}$$

$$\langle v \rangle = \frac{v_1 + v_2 + v_3 + \dots v_n}{n}$$

For $n=2$, $\boxed{\langle v \rangle = \frac{v_1 + v_2}{2}} \leftarrow MCQ$

Q) Car travels 1 half distance b/w 2 places with a speed of 60 Km/h, what should be its speed rest half of distance, so, that, it avg speed 90 Km/h.

→ We know that, for equal time intervals, avg spee

$$\langle v \rangle = \frac{v_1 + v_2 + \dots + v_n}{n}$$ Wrong eq

$$90 = \frac{60 + v_2}{2} \quad [\because \langle v \rangle = 90, v_1 = 60, n = 2 \text{ (Given)}]$$

$$180 = 60 + v_2$$

$$\boxed{v_2 = 120 \text{ Km/h}}$$

Q) A train moves with speed of 30 Km/h in the 1st 15 min, with another speed of 40 Km/h the ~~nex~~ next 15 min and with speed of 60 Km/h in the last 30 min. Calculate the avg speed of train.

→ Train moves with $v_1 = 30$ Km/h for 15 min, then, $v_2 = 40$ Km/h for 15 min and 60 Km/h for 30 min.

$$\therefore d_1 = v_1 t_1 = 30 \times \frac{1}{4} = \frac{30}{4}, \quad d_2 = 40 \times \frac{1}{4} = 10$$

$$\Rightarrow \text{Avg speed}, \langle v_3 \rangle = \frac{\frac{30}{4} + 10}{\frac{1}{4} + \frac{1}{4}} = 35 \text{ Km/h}$$

$$\Rightarrow d_3 = v_3 t_3 = 35 \times \frac{1}{2} = \frac{35}{2}, \quad d_4 = v_4 t_4 = 60 \times \frac{1}{2} = 30$$

$$\Rightarrow \text{Avg speed}, \langle v \rangle = \frac{\frac{35}{2} + 30}{\frac{1}{2} + \frac{1}{2}} = \frac{\frac{35 + 60}{2}}{1} = \frac{95}{2} = 47.5 \text{ Km/h}$$

$\therefore$ Avg speed of train $= 47.5$ Km/h

Scanned with CamScanner

Q) A body travelling along a straight line traversed $1/3^{rd}$ of total distance with velocity V_0, the remain part of distance was covered with velocity V_1 for half time and velocity V_2 for other half of time. Find avg velocity over the whole time of motion.

→

Let total distance $= 3d$

$\therefore$ Time to travel d is, $t_0 = \frac{d}{V_0}$

$\Rightarrow$ Total time of rest distance $= 2t$

$\Rightarrow 2d = V_1 t + V_2 t$

$t = \frac{2d}{V_1+V_2}$

$2t = \frac{4d}{V_1+V_2}$

$$\Rightarrow \langle V \rangle = \frac{3d}{t_0 + 2t} = \frac{3d}{\frac{d}{V_0} + \frac{4d}{V_1+V_2}} = \frac{3V_0(V_1+V_2)}{4V_0+V_1+V_2}$$

Q) A person moves on a semi-circular track of radius 42m during a morning walk. He starts at one end of the track and reaches other end. Find distance covered by and displacement of person.

→ Distance $= \frac{2\pi r}{2} = \frac{22}{7} \times 42 = 134\text{m}$

Displacement $= 84\text{m}$

2) A car covers first half of distance between two places at a speed of $40 Kmh^{-1}$ and the second half at $60 Kmh^{-1}$. What is avg speed of car.

→ Avg speed $\langle v \rangle = \frac{2 v_1 v_2}{v_1 + v_2}$ [∵ Distance are equal]

$$= \frac{2 \times 40 \times 60}{40 + 60}$$

$$= \frac{4800}{100}$$

$$= 48 Kmh^{-1}$$

3) A train moves with a speed of $30 Kmh^{-1}$ in the first 15 min, with $40 Kmh^{-1}$ for 15min and 60Kmh for 30 min. Find avg speed.

→ See questions done before.

4) A body travels first half of total distance with velocity v_1 and second half with v_2. Calculate avg velocity.

→ $d_1 = d_2$

$\Rightarrow d_1 = v_1 t_1$, $d_2 = v_2 t_2$

$t_1 = \frac{d_1}{v_1}$ $t_2 = \frac{d_2}{v_2}$

$$\Rightarrow \langle v \rangle = \frac{d_1 + d_2}{t_1 + t_2} = \frac{d_1 + d_2}{\frac{d_1}{v_1} + \frac{d_2}{v_2}} = \frac{2d_1}{\frac{v_1 d_1 + v_2 d_1}{v_1 v_2}} = \frac{2 v_1 v_2}{v_1 + v_2}$$

5) A body travells distance S_1 with velocity V_1 and distance S_2 with V_2 in same direction. Calculate avg velocity.

→ Total distance $= S_1 + S_2$

$$\Rightarrow t_1 = \frac{S_1}{V_1}, \quad t_2 = \frac{S_2}{V_2}$$

$$\therefore \langle V \rangle = \frac{S_1 + S_2}{\frac{S_1}{V_1} + \frac{S_2}{V_2}} = \frac{(S_1 + S_2)}{\frac{S_1V_2 + S_2V_2}{V_1V_2}}$$

$$\boxed{\langle V \rangle = \frac{(S_1 + S_2)(V_1V_2)}{S_1V_2 + S_2V_1}}$$

6) A car covers $\frac{1}{4}$ of total distance with velocity of $20ms^{-1}$, next $\frac{1}{4}$ part with $40ms^{-1}$, next $\frac{1}{4}$ part with $20ms^{-1}$ and last $\frac{1}{4}$ part with $40ms^{-1}$. Find avg velocity.

→ $\cancel{\langle V \rangle =}$

$$\frac{4}{\langle v \rangle} = \frac{1}{V_1} + \frac{1}{V_2} + \frac{1}{V_3} + \frac{1}{V_4}$$

$$\frac{4}{\langle v \rangle} = \frac{1}{20} + \frac{1}{40} + \frac{1}{20} + \frac{1}{40}$$

$$\frac{4}{\langle v \rangle} = \frac{2}{20} + \frac{2}{40} = \frac{20 + 10}{200}$$

$$\cancel{\langle v \rangle = \frac{4 \times 200}{30}}$$

$$\langle v \rangle = \frac{800}{30} = 26.67 ms^{-1}$$

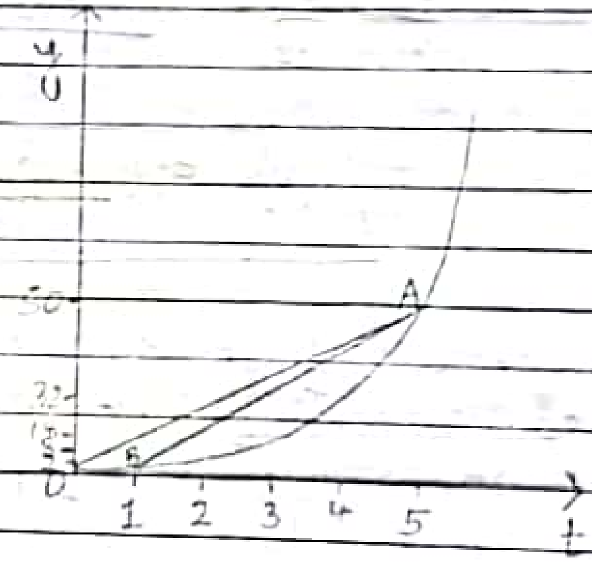
7) A car travels along a straight line of first half with 50 km/h the second half time with 60 Kmh^{-1}. Find avg speed of car.

→ $\langle v \rangle = \frac{50+60}{2} = 55\ Kmh^{-1}$

* Angular velocity :- Angle travelled in unit time

1) $y(t) = 2t^2$

t	y
0	0
1	2
2	8
3	18
4	32
5	50



1) Find avg speed between 0 and 5 sec

→ $\langle v \rangle = \frac{\Delta x}{\Delta y} = \frac{y(5)-y(0)}{5} = \frac{50}{5} = 10\ m/s$

$\langle v \rangle$ = slope of line OA

2) avg speed between 1 and 5 sec

$\langle v \rangle = \frac{\Delta x}{\Delta y} = \frac{y(5)-y(1)}{5-1} = \frac{50-2}{4} = \frac{48}{4} = 12\ m/s$

$\langle v \rangle$ = slope of AB.

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Page

* **Drawback of avg. speed:-**

1) From avg. speed we can't obtain value of speed at any instant of time on the path at any point.

* **Instantaneous speed:-**

Q) $y = 2t^2$, find instantaneous speed at ~~t=~~ 3^{rd} second.

→ avg. speed between 3 and 5 sec

$$\langle v \rangle = \frac{y(5) - y(3)}{5-3} = \frac{50-18}{2} = 16 \, m/s$$

avg. speed between 3 and 4 sec,

$$\langle v \rangle = \frac{y(4) - y(3)}{4-3} = \frac{32-18}{1} = 14 \, m/s$$

avg. speed between 3 and 3.1 sec,

$$\langle v \rangle = \frac{y(3.1) - y(3)}{3.1-3} = \frac{\overset{19.22}{\cancel{20.52}} - 18}{0.1} = 12.2 \, m/s$$

avg. speed btw 3 and 3.01 sec,

$$\langle v \rangle = \frac{y(3.01) - y(3)}{3.01-3} = \frac{18.12-18}{0.01} = 12.02 \, m/s$$

avg. speed btw 3 and 3.001 sec,

$$\langle v \rangle = \frac{y(3.001) - y(3)}{3.001-3} = \frac{18.012-18}{0.001} = 12.002 \, m/s$$

$$\boxed{\frac{dy}{dt} = \frac{d}{dt}(2t^2) = 4t}$$

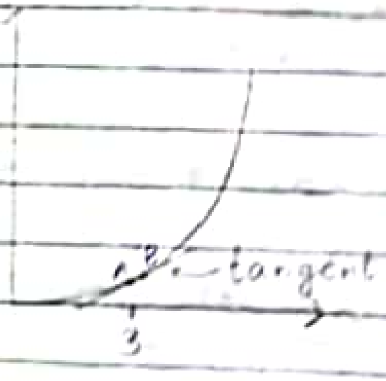
⇒ Instantaneous speed = $\frac{dy}{dt} = 4 \times 2 = 12$ m/s

⇒ As we give less and less time interval to particle to change its ~~speed~~ position by decreases and ratio $\left(\frac{\Delta y}{\Delta t}\right)$ also decreases.

⇒ When Δt is very small mathematically $\Delta t \to 0$, the ratio $\frac{\Delta y}{\Delta t}$ approaches to a specific constant value.

⇒ By decreasing further time interval, these ratio will not decrease, this ratio is called instantaneous speed.

⇒ Thus, instant. speed is avg. speed between time (t) and $(t + \Delta t)$, here $\Delta t \to 0$.



As shown in graph, as we ↓ time interval line AB slowly lining towards time axis, as $\Delta t \to 0$ (slope) this line becomes tangential to point A.

⇒ Now slope of tangent gives instantaneous speed at $t=3$.

⇒ Thus slope of tangent drawn at any point on distance verses time graph, gives instant speed at that instant.

⇒ $\cancel{y=2}$ $y(t)=2t$, $\frac{dy}{dt}=2$, speed constant

$y(t)=2t^2$, $\frac{dy}{dt}=4t$, speed variable

∴ When, $y \propto t$, speed is constant

$y \propto t^n$, speed is variable

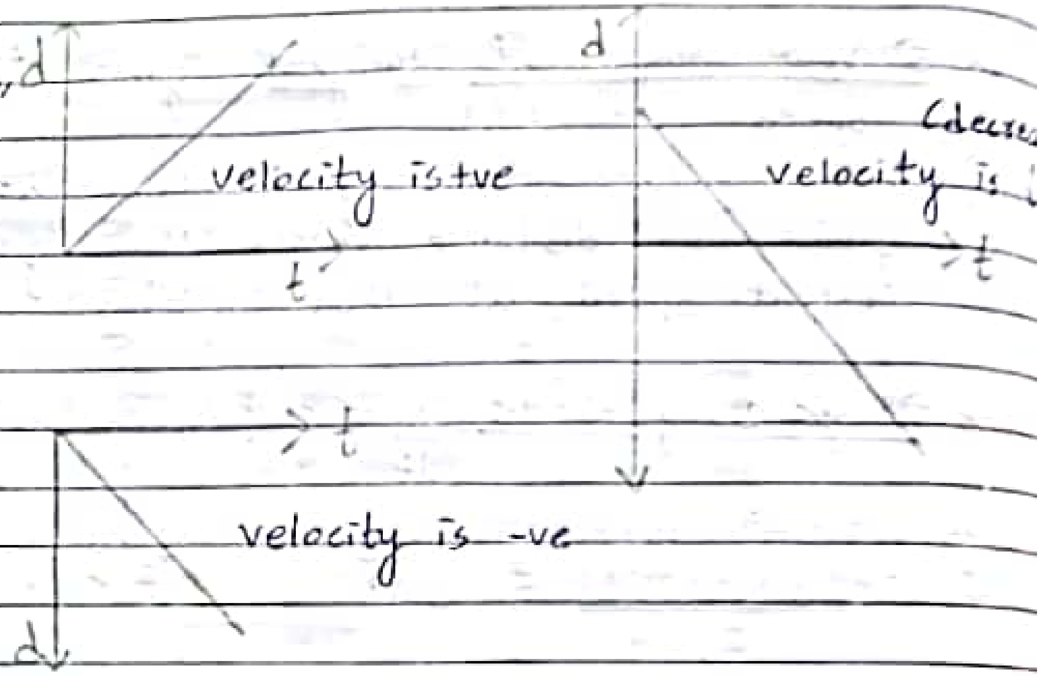
⇒ 1) <u>Constant Velocity</u>:- Object travel equal displ. in equal time ~~in~~ interval.

→ When ~~obj~~ object will ~~move~~ move with constant velocity, it ~~will~~ must be travelling on ~~strait~~ straight line.

→ When object will move will constant speed, it <u>MAY</u> move on a straight line.

→ When object is moving with constant velocity it will have constant speed but reverse may not be possible.

⇒ e.g.,

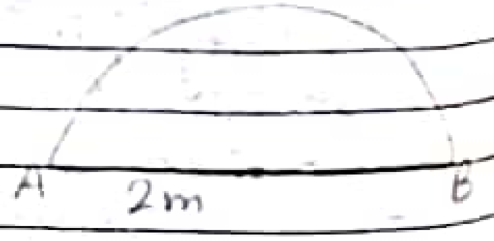


⇒ ⟨2⟩ <u>Variabe velocity</u>:-

→ Velocity of object changes with time.

⇒ ⟨3⟩ <u>Avg. Velocity</u>, $\langle v \rangle = \dfrac{\text{displacement}}{\text{time}}$

Q) ~~Obt~~ Object is moving on a semi circular path in time interval 4s. Find avg. velocity and avg. speed.



→ Avg. speed $= \dfrac{\pi r}{24}$

$= \dfrac{\pi}{2}$ m/s

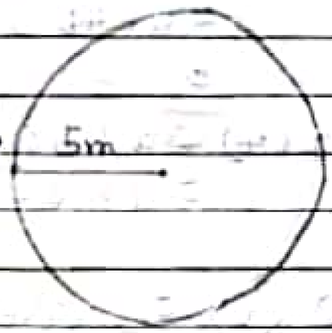
Avg velocity $= \dfrac{2r}{4} = \dfrac{4}{4} = 1\ ms^{-1}$

Q) Object is moving on a circular path of radius 5m. It completes 1 revolution in 10sec. Find avg. speed and avg. velocity.

→ Average speed $= \frac{2\pi r}{10}$

$= \frac{2\times 3.14\times 5}{10}$

$= 3.14\ m\,s^{-1}$



⇒ Avg. velocity $= \frac{0}{10}$

$= 0\ m\,s^{-1}$

⇒ $\boxed{\text{Avg. speed} \geq \text{Avg. Velocity}}$ ⇐

⇒ ⟨4⟩ Instantanuos velocity :-

$$v = \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t} = \frac{dy}{dt}$$

Q) page no. 113, (3.2)

→ $x = a + bt^2$

$a = 8.5\ m$

$b = 2.5\ m\,s^{-2}$

⇒ $v = \frac{dx}{dt} = \frac{d}{dt}(a+bt^2) = 2bt$

$= 5t$

⇒ $\boxed{V_0 = 5\times 0 = 0}$ Initial velocity

$\Rightarrow v = 5 \times 2 = 10\, m s^{-1}$

$\Rightarrow x(2) = a + b(2)^2$
$= a + 4b$

$\Rightarrow x(4) = a + b(4)^2$
$= a + 16b$

$\Rightarrow \langle v \rangle = \frac{x(4) - x(2)}{4-2} = \frac{a + 16b - a - 4b}{2}$
$= \frac{12b}{2}$
$= 6b$
$= 6 \times 2.5$
$= 15\, m s^{-1}$

$\Rightarrow$ (1) <u>Constant acceleration:-</u>

$\rightarrow$ When rate of change of velocity is constant.

$\Rightarrow$ (2) <u>Varied acceleration:-</u>

$\rightarrow$ When accelerat^n changes with time.
$\rightarrow$ When rate of change of velocity change with time.

$\Rightarrow$ (3) <u>Avg. acceleration:-</u> $\langle a \rangle = \frac{v_2 - v_1}{\Delta t}$

$\Rightarrow$ ⟨4⟩ Instantaneous acceleration:-

$$a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

Since, $v = \frac{dx}{dt}$

$$\therefore a = \frac{d}{dt}\left(\frac{dx}{dt}\right)$$

$$\Rightarrow \boxed{a = \frac{d^2(x)}{(dt)^2}} \Leftrightarrow \text{second derivative } (\ddot{x})$$

Q) Eq^n of position for particle moving on straight line path is given by $x(t) = 2 - 5t + t^3$. At $t = 2$ sec, find velocity and accelerat^n of particle.

$\rightarrow$ $\frac{dx}{dt} = v = 0 - 5 + 3t^2$

$\Rightarrow v = 3(2)^2 - 5 = 12 - 5 = 7\ m/s$

$v = -5 + 3t^2$

$\cancel{\frac{dx}{}}\ \frac{dv}{dt} = a = 0 + 6t$

$\Rightarrow$ at $t = 2$ sec, $a = 12\ m/s^2$

Q) Displacement of particle is given by $y(t) = a + bt + \cancel{}ct^2 - dt^4$. Find out initial velocity and initial acceleration.

$\rightarrow$ $v = \frac{dy}{dt} = 0 + b + 2ct - 4dt^3$

$\Rightarrow$ Initial velocity, $t = 0$,

$$\therefore v_0 = b + 2c(0) - 4d(0)^3$$
$$= b \, m/s$$

$$\Rightarrow a = \frac{dv}{dt} = 0 + 2c - 12dt^2$$

$\Rightarrow$ Intial acc., $t = 0$,

$$\therefore a_0 = 2c - 12d(0)^2$$
$$= 2c \, m/s^2$$

$$y \propto t$$

$$y = 2t$$

$$v = \frac{dy}{dt} = 2$$

$$a = \frac{dv}{dt} = 0$$

$\Rightarrow$ When displacement is directly proport^n to time, velocity is constant and acceleraat^n is 0.

$$y \propto t^2$$

$$y = 2t^2$$

$$v = \frac{dy}{dt} = 4t$$

$$a = \frac{dv}{dt} = 4$$

⇒ When ~~displace~~ max. power of time ~~is~~ in displacement's ~~ex~~ eq^n is 2, then velocity is variable and acceleration is constant.

$$y \propto t^{n} \quad n > 2$$

$$y = 3t^3$$

$$\Rightarrow \quad v = \frac{dy}{dt} = 9t^2$$

$$a = \frac{dv}{dt} = 18t$$

⇒ When displacement is directly proportional to t^n, where $n > 2$, then ~~va~~ velocity and accelert^n, both are variable.

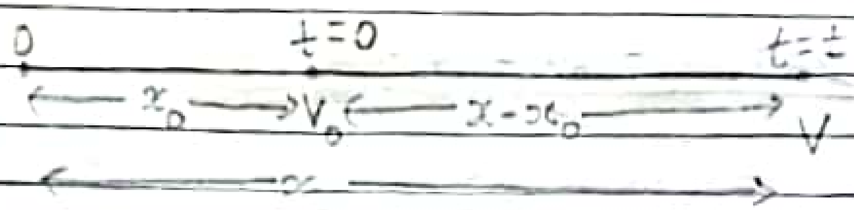
* Derive eq^n of constant accelerat^n :-

→ Suppose, as shown in figure at $t = 0$ particle is at x_0.

→ It's V_0

→ Particle is moving on a straight line with constant accelerat^n a.

→ Its position at time $t = t$ is x and finall velocity is v, displacement is $x - x_0$.



⇒ As we know, acceleration,

$$a = \frac{\text{change in velocity}}{\text{change in time}}$$

$$a = \frac{v - v_0}{t}$$

$$\boxed{v = v_0 + at} \text{ --- (I)}$$

⇒ Avg. Velocity $\langle v \rangle = \left(\frac{v + v_0}{2}\right)$ [∵ Only applicable when its constant accelera…

⇒ displacement $= x - x_0 = d$

$$d = \langle v \rangle \times t$$

$$\boxed{d = \left(\frac{v + v_0}{2}\right)t} \text{ --- (II)}$$

⇒ From (I) and (II),

$$d = \left(\frac{v_0 + at + v_0}{2}\right)t$$

$$\boxed{d = v_0 t + \frac{1}{2}at^2} \text{ --- (III)}$$

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⇒ We know that,

$$a = \frac{v - v_0}{t}$$

$$t = \frac{v - v_0}{a} \quad \text{— (a)}$$

⇒ From eq (a) and (ii),

$$d = \left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)$$

$$d = \frac{v^2 - v_0^2}{2a}$$

$$\boxed{v^2 = v_0^2 + 2ad} \quad \text{— (iv)}$$

(i) $v = v_0 + at$

(ii) $d = \left(\frac{v + v_0}{2}\right) t$

(iii) $d = v_0 t + \frac{1}{2} a t^2$

(iv) $v^2 = v_0^2 + 2ad$

⇒ In this four equations we need to put v_0, v, a and d with there signs(+ or −)

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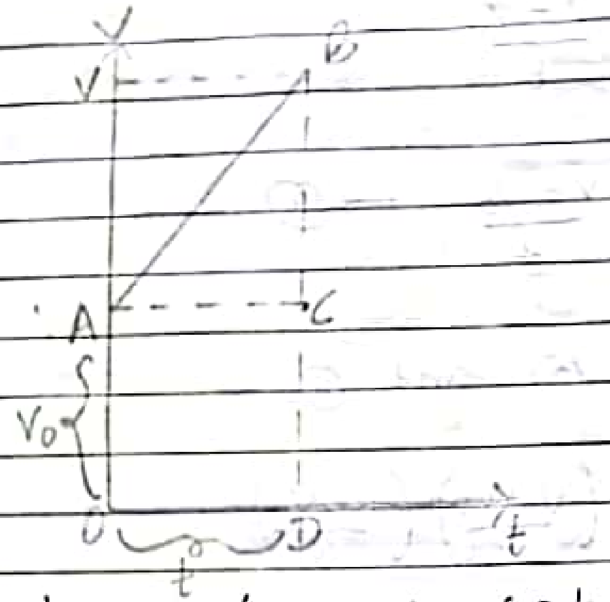
* **Graphical derivatn :-**



slope of graph $v \to t$

$$a = \frac{BC}{AC} = \frac{v - v_0}{t - 0}$$

$$\Rightarrow \boxed{v = v_0 + at} \text{ --- (1)}$$

$\Rightarrow$ Area under the graph of $v \to t$ gives, displacement.

$\therefore$ Area = Area of ΔABC + Area of $\square ODCA$

$$d = \frac{1}{2}(BC)(AC) + (OD)(DC)$$

$$d = \frac{1}{2}(v - v_0)(t) + (t)(v_0)$$

$$d = t\left[\frac{v}{2} - \frac{v_0}{2} + v_0\right]$$

$$d = t\left[\frac{v}{2} + \frac{v_0}{2}\right]$$

$$\boxed{d = \left(\frac{v + v_0}{2}\right)t} \text{ --- (ii)}$$

⇒ From (i) and (ii),

$$\boxed{d = v_0 t + \frac{1}{2} a t^2} \text{ — (iii)}$$

⇒ We know that,

$$\frac{v - v_0}{a} = t \text{ — (a)}$$

⇒ From eq (a) and (ii),

$$d = \left[\left(\frac{v + v_0}{2}\right)\left(\frac{v - v_0}{a}\right)\right]$$

$$d = \frac{v^2 - v_0^2}{2a}$$

$$\boxed{v^2 = v_0^2 + 2ad} \text{ — (iv)}$$

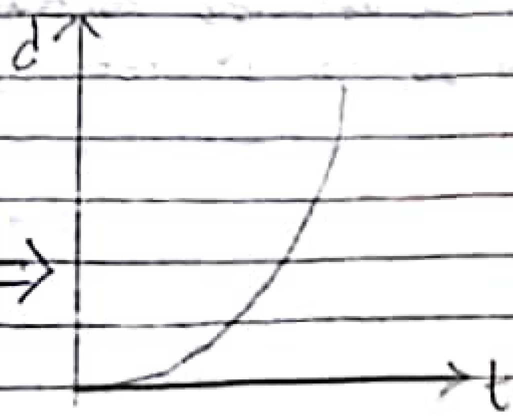
$$d = v_0 t + \frac{1}{2} a t^2$$

⇒ Let, $v_0 = 0$

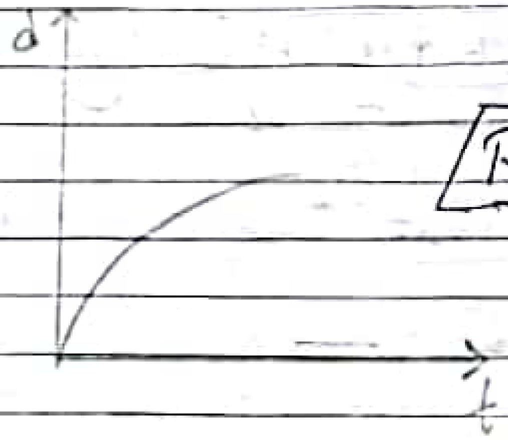
$$d = \frac{1}{2} a t^2$$

$$d \propto t^2$$

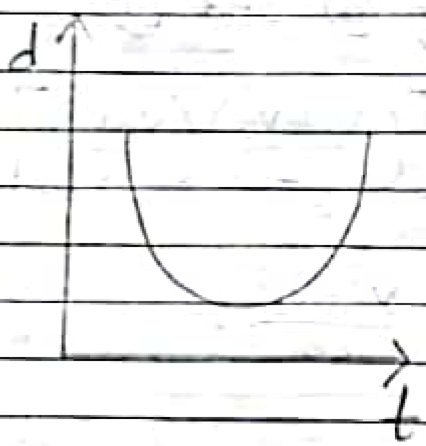
⇒ So graph of $d \to t$, for acceleration is parabolic. ⇒



=> For retardation, graph will be,

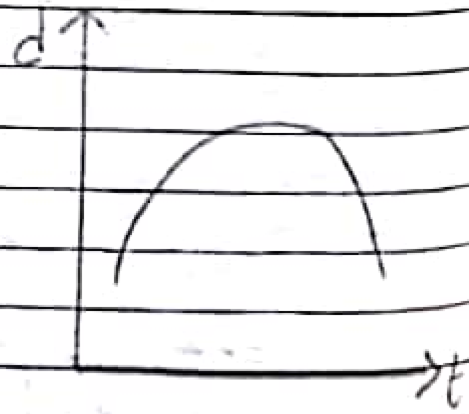


RETARDATION



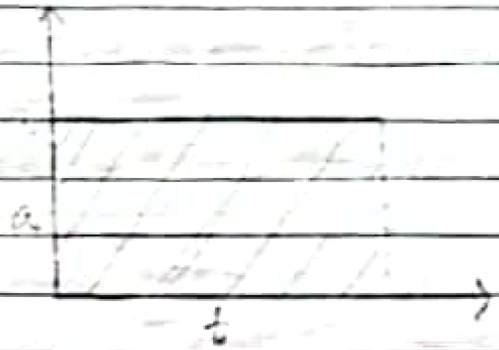
=> Slope continuosly increases and here slope gives instantaneous velocity. So, velocity continuosly increases and hence its acceleration.

=> Here slope continuosly ↓, so, velocity continuosly ↓, so, it is retardation.



$V = V_0 + at$

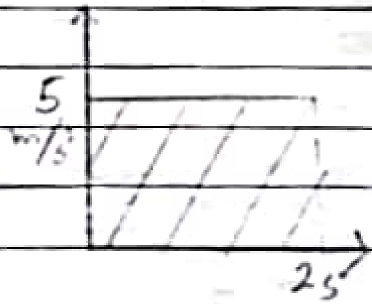
$d = \left(\frac{V^2 - V_0^2}{2a}\right)$



⇒ Area under the graph of a→t gives ~~at~~ change in velocity.

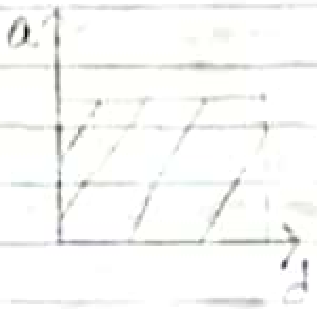
⇒ $\boxed{at = V - V_0 = \text{change in velocity}}$

Q) Initial velocity of given particle is 0 and its acceleration and time are as shown in fig. Find final velocity.



⇒ $at = 5 \times 2 = 10$

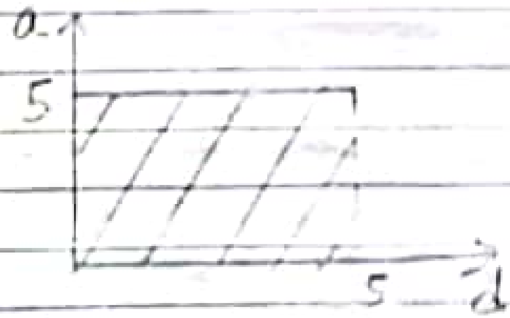
⇒ $V = V_0 + at$
$= 0 + 10$
$= 10\ m/s$



⇒ Area under the graph of a-d gives $\left(\frac{v^2 - v_0^2}{2}\right)$

⇒ $\boxed{ad = \frac{v^2 - v_0^2}{2}}$

Q) For a particle starting from rest ~~area~~ acceleration → displacement is as shown in fig.



⇒ $ad = \frac{1}{2} \times 5 \times 5 = 12.5$

⇒ $12.5 = \frac{v^2 - v_0^2}{2}$

$25 = v^2 - \cancel{v_0^2}\,(0)^2$

$\boxed{v = 5\,m/s}$

* Motion takes place under gravity:-

i) $d = v_0 t \mp \frac{1}{2} g t^2$

(ii) $v = v_0 \mp g t$

(iii) $d = \left(\frac{v + v_0}{2}\right) t$

(iv) $v^2 = v_0^2 \mp 2 g s$

⇒ Vertical upward direction is considered as +ve y-axis

⇒ Vertical downward direction is considered as -ve y-axis

⇒ Gravitatⁿ force is downward.
∴ so, acceleratⁿ g is -ve.

* Freely falling body:-

→ When object is released in gravity, with $v_0 = 0$, $a = -g$, then it is called free fall.

⟨1⟩ $h = -\frac{1}{2} g t^2$

⟨2⟩ $v = -g t$

⟨3⟩ $h = \left(\frac{v_0}{2}\right) t$

⟨4⟩ $h = \frac{+v_0^2}{2g}$

⇒ Velocity -ve means :-

1) Displacement of ~~abject~~ object decreases

2) Object is moving towards origin along selected axis.

Q) Object is thrown ~~is~~ in upward direction with velocity V_0. Find maximum height, time taken and time of flight.

⇒ At maximum height, $v=0$

⇒ Maximum height $= \frac{-V_0^2}{-2g} = \frac{V_0^2}{2g}$

⇒ Time taken, $V = V_0 + at$

$$0 = V_0 - gt$$
$$t = \frac{V_0}{g}$$

⇒ Time of ~~flit~~ flight (time ~~of~~ for which projectile remains in air), $t = 2\frac{V_0}{g}$

⇒ When object is thrown in upward direction, it attains same height twice, with same velocity.

$$h = \frac{V^2 - V_0^2}{2a} \Rightarrow \boxed{V = \pm\sqrt{2ah + V_0^2}}$$

Q) A race car accelerates on road from rest to a speed of 180 kmh^{-1} in 25s. Assuming uniform accelerat^n of car throughout. find distance covered in this time.

→ $V_0 = 0$, $V = 180\ kmh^{-1} = \overset{10}{180} \times \frac{5}{18} = 50\ m/s^{-1}$,

$t = 25\ s$

$\Rightarrow a = \frac{v-u}{t} = \frac{50}{25} = 2\ m/s^2$

$\Rightarrow d = ut + \frac{1}{2}at^2$

$= 0 + \frac{1}{2} \times 2(25)^2$

$= 625\ m$

Q) A car moving along a straight highway with speed of 126 kmh^{-1} is brought to a stop within a distance of 200m. What is retardat^n of car, and how long does it take for car to stop?

→ $V_0 = 126\ kmh^{-1}$, $V = 0$, $s = 200m$, $a = ?$ and $t = ?$

$= 126 \times \frac{5}{18}$

$= 35\ ms^{-1}$

$\therefore V^2 = V_0^2 + 2as$

$0 = (35)^2 + 2a(200)$

$-1225 = 400a$

$a = \frac{-1225}{400} = -3.06\ ms^{-2}$

$\Rightarrow t = \frac{V - V_0}{a}$

$= \frac{-126}{-3.06}$

$= 41.2\ s$

3-Q) An electron travelling with speed of 5×10^3 passes through electric field with an acceleration of $10^2\ m\,s^{-2}$, (i) How long will it take for e^- to double its speed?
(ii) What will be distance covered by electron in this time.

4-Q) A driver takes 0.2 s to apply brakes after he sees a need for it. This is called reation time of driver. If he is driving car at speed of $54\ Km\,h^{-1}$ and the brakes cause a deceleration of $6\ m\,s^{-2}$, find the distance travelled by car after he sees the need to put brakes.

5-Q) A bullet travelling with $16\ m\,s^{-1}$ penetrates trunk and comes to rest in 0.4m. Find time taken during retardation.

4-Q) → $v_0 = 54\ Km\,h^{-1}$
$= 15\ m$

$\Rightarrow d = vt$
$= 15\times0.2$
$= 3\ m$

$\Rightarrow a = -6\ m\,s^{-2}$
$v = 0$

$$\Rightarrow d = \frac{v^2 - v_0^2}{2a} = \frac{-(15)^2}{-2(6)} = \frac{+225}{+12} = 18.75m$$

$\Rightarrow$ Total distance $= 3 + 18.75 = 21.75m$

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5-Q) $V_0 = 16ms^{-1}$, $d = 0.4m$, $V = 0$, $a = ?$

$$\Rightarrow d = \left(\frac{V - V_0}{2}\right) t$$

$$0.4 = \left(\frac{0 - 16}{2}\right) t$$

$$t = 0.05s$$

3-Q) (i) $10 \times 10^3 = 5 \times 10^3 + 10^{12}(t)$

$$5 \times 10^3 = 10^{12}(t)$$

$$t = 5 \times 10^{-9}$$

(ii) $d = \left(\frac{V + V_0}{2}\right) t = \left(\frac{10 \times 10^3 + 5 \times 10^3}{2}\right) 5 \times 10^{-9}$

$$= \frac{15 \times 10^3}{2} \times 5 \times 10^{-9}$$

$$= 375 \times 10^{-8} m = 3.75 \times 10^{-4} m$$

* **Applying equations of Kinematics :-**

1) Make a drawing or graph of present situation.

2) Decide which direct^n are called +ve and -ve.

3) Write down the values with proper + and - signs for any of the 5 kinematics variable.

4) Be alert for phrase like, starts from rest (initial velocity is zero), $V_0 = 0$, free fall $\Rightarrow V_0 = 0$, come to rest $\Rightarrow V = 0$.

5) Identify variable that your are being asked,

to determine.

6) to solve problems, you must have atleast values of 3 of 5 kinematic equatn, and then apply suitable equation.

7) Remember that motion of two objects may be interelated, so they may share common variables.

8) When motion of object is divided into segments final velocity of one segment is initial velocity for next segment.

9) They may be a two possible answers.

10) Freely falling body.

Q) A motor cycle started from rest moves with $a = 2.6\ m s^{-2}$, after travelling 120 m dis, it retards with $a = -1.5 m s^{-2}$ until its velocity becomes $12 m s^{-1}$. Find out total distance travelled by motor cycle.

→

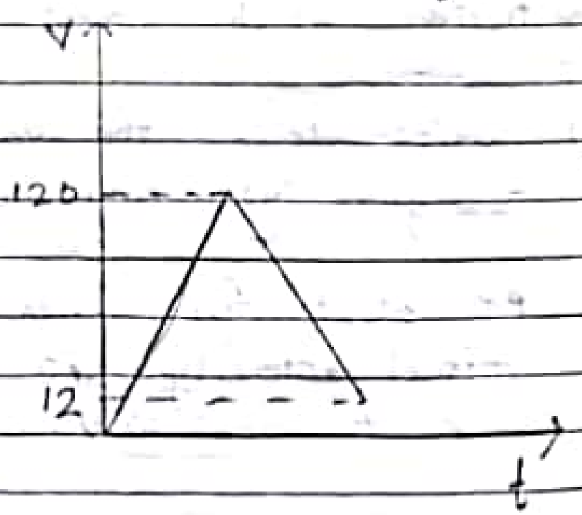
$a = 2.6\ m s^{-2}$

$d = 120 m$

$a_1 = -1.5 m s^{-2}$

$d_1 = ?$

$v_1 = 12 m s^{-1}$



⇒ final velocity,

$$d = \frac{v^2 - v_0^2}{2a}$$

$$v^2 = 2ad$$

$$v^2 = 2 \times 2.6 \times 120$$

$$\boxed{v^2 = 52 \times 12}$$

⇒ This final velocity will be initial velocity for next segment.

$$\Rightarrow d_1 = \frac{v_1^2 - v_0^2}{2a_2}$$

$$d_1 = \frac{(12)^2 - (52 \times 12)}{2 \times (-1.5)}$$

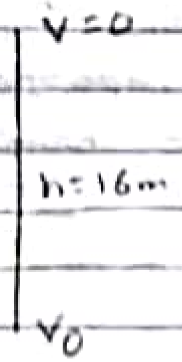
$$= \frac{\cancel{12}^{-4}(12 - 52)}{\cancel{2}\,\cancel{-3}}$$

$$= -4(-40)$$

$$\boxed{d_1 = 160m}$$

⇒ Total distance = 120+160
= 280m

Q) object is projected in vertically upwards direction, attain a maximum height of 16m. Find out height at which its velocity becomes half of its initial velocity.



→ $$h = \frac{v^2 - v_0^2}{2a}$$

$$16 = \frac{0 - v_0^2}{-2g}$$

$$\boxed{16 = \frac{v_0^2}{2g}} \quad — (1)$$

⇒ Let at height h_1, velocity becomes half of initial velocity.

$$\therefore h_1 = \frac{\left(\frac{V_0}{2}\right)^2 - (V_0)^2}{-2g}$$

$$= \frac{-\frac{3V_0^2}{4}}{-2g}$$

$$= \frac{3}{4}\left(\frac{V_0^2}{2g}\right)$$

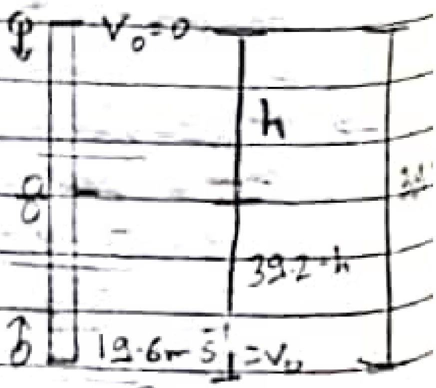
⇒ From eq ①,

$$h_1 = \frac{3}{4} \times 16$$

$$h_1 = 12\,m$$

Q) Height of tower is 39.2m From top of tower one object is released, at the same time another object is thrown in upward direction, from bottom of tower with $v = 19.6\,ms^{-1}$. When and where both will meet.

→ From top of the tower, supposed both will meet at height h and time t.



$\Rightarrow d = v_0 t + \frac{1}{2} a t^2$

$\Rightarrow$ For 1^{st} object,

$$\boxed{+h = 0 + \frac{1}{2} g t^2} \quad \text{--- (1)}$$

$\Rightarrow$ For second ~~oth~~ object,

$$39.2 - h = 19.6t - \frac{1}{2} g t^2$$

$\Rightarrow$ From eq. (1),

$$39.2 - \cancel{h} = 19.6t - \cancel{h}$$

$$\boxed{t = 2 \text{ sec}}$$

$\Rightarrow$ In eq (1),

$$h = \frac{1}{2} g t^2$$

$$h = \frac{1}{\cancel{2}} \times 9.8 \times \cancel{4}^{2}$$

$$\boxed{h = 19.6}$$

Q) Given object is moving with constant acceler-
at the end of 10sec, its velocity is $48ms^{-1}$, and
at the end of 15sec, its velocity is $68ms^{-1}$.
Find out total distance travelled in 15sec.

→ When, $v_1 = 48ms^{-1}$, $t = 10s$, $a = ?$,

$$\therefore V = V_0 + at$$

$$\boxed{48 = V_0 + 10a} \quad \text{— (I)}$$

⇒ When, $v_2 = 68ms^{-1}$, $t = 15s$, $a = ?$,

$$\therefore \boxed{68 = V_0 + 15a} \quad \text{— (II)}$$

⇒ Subtracting eq (I) & (II),

$$\begin{aligned} 48 &= V_0 + 10a \\ -\ 68 &= -V_0 - 15a \\ \hline -20 &= -5a \\ a &= 4ms^{-2} \end{aligned}$$

⇒ In eq (I),

$$48 = V_0 + 10(4)$$

$$\boxed{V_0 = 8ms^{-1}}$$

$$\therefore d = V_0 t + \frac{1}{2}at^2$$

$$= 8(15) + \frac{1}{2}(4)(15)^2$$

$$= 120 + 450$$

$$\boxed{d = 570m}$$

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Q) Distance between 2 stations is 40 Km. A train travells this distance in 1 h, & when train start it motion from 1^{st} station it accelerates for 5 Km then for 20 Km, train moves with constant velocity, and finally train retards and come to rest after travelling 15 Km distance. Find out max velocity and avg. velocity of train.

⇒ $d = 40$ Km

$t = 1$ hr

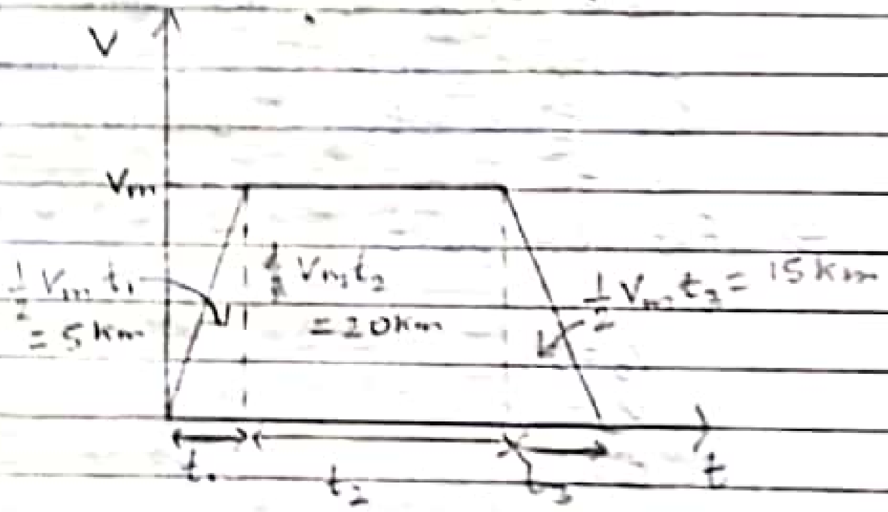
$d_1 = 5$ Km

$d_2 = 20$ Km

$d_3 = 15$ Km

$\langle v \rangle = ?$

$V_m = ?$



$$\Rightarrow 5 = \frac{1}{2} V_m t_1 \Rightarrow t_1 = \frac{10}{V_m}$$

$$20 = V_m t_2 \Rightarrow t_2 = \frac{20}{V_m}$$

$$15 = \frac{1}{2} V_m t_3 \Rightarrow t_3 = \frac{30}{V_m}$$

$$\Rightarrow t = t_1 + t_2 + t_3$$

$$1 = \frac{10}{V_m} + \frac{20}{V_m} + \frac{30}{V_m}$$

$$\boxed{V_m = 60 \text{ Km/h}}$$

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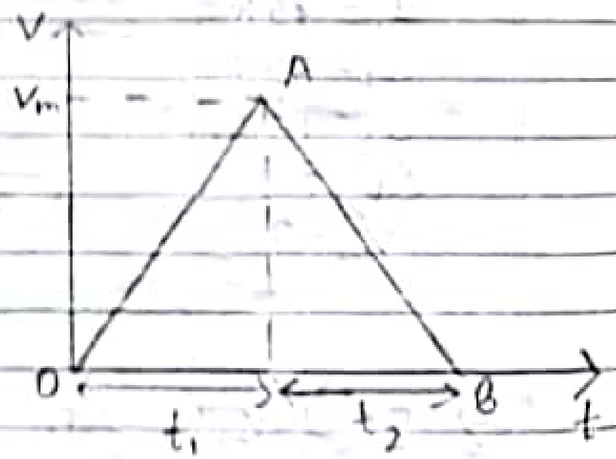
$\Rightarrow \langle v \rangle = \frac{\text{displacement}}{\text{time}}$

$= \frac{40}{1}$

$\langle v \rangle = 40\ km\,h^{-1}$

Q) A body started from rest, moves with constant acceleration α, for time t_1 and acquires constant velocity v_m, then it retards with β for time t_2 and come to rest. Total time is t.

→ For OA part,



$V_0 = 0$
$V = V_m$
$a = \alpha$
$t = t_1$

$\Rightarrow V = V_0 + at$

$\boxed{V_m = 0 + \alpha(t_1)}$ — (i)

$\boxed{t_1 = \frac{V_m}{\alpha}}$ — (ii)

→ For AB part,

$V_0 = V_m$
$V = 0$
$a = \beta$
$t = t_2$

$\Rightarrow 0 = V_m + (-\beta)(t_2)$ $\Rightarrow \boxed{V_m = \beta t_2}$ — (iii)

$\Rightarrow \boxed{t_2 = \frac{V_m}{\beta}}$ — (iv)

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⇒ From eq (i) & (iii),

$$\alpha t_1 = \beta t_2$$

$$\Rightarrow \boxed{\frac{t_1}{t_2} = \frac{\beta}{\alpha}} \text{ — (v)}$$

⇒ From eq (ii) & (iv),

$$t = t_1 + t_2$$

$$= \frac{V_m}{\alpha} + \frac{V_m}{\beta}$$

$$t = V_m\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

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$$\boxed{V_m = \cancel{V_0}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t}$$

⇒ For OA part,

$$d = \frac{v^2 - v_0^2}{2a}$$

$$\boxed{d_1 = \frac{V_m^2}{2\alpha}} \text{ — (vi)}$$

⇒ For AB part,

$$\boxed{d_2 = \frac{V_m^2}{2\beta}} \text{ — (vii)}$$

$\Rightarrow$ ~~Form~~ From eq (v), (vi), (vii),

$$\boxed{\frac{t_1}{t_2} = \frac{d_1}{d_2} = \frac{\beta}{\alpha}}$$

$\Rightarrow$ $d = d_1 + d_2$

$$d = \frac{v_m^2}{2\alpha} + \frac{v_m^2}{2\beta}$$

$$= \frac{v_m^2}{2}\left(\frac{1}{\alpha} + \frac{1}{\beta}\right)$$

$$= \frac{1}{2}\left[\left(\frac{\alpha\beta}{\alpha+\beta}\right)t\right]^2\left(\frac{\alpha+\beta}{\alpha\beta}\right)$$

$$= \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)^2\left(\frac{\alpha+\beta}{\alpha\beta}\right)t^2$$

$$\boxed{d = \frac{1}{2}\left(\frac{\alpha\beta}{\alpha+\beta}\right)t^2}$$

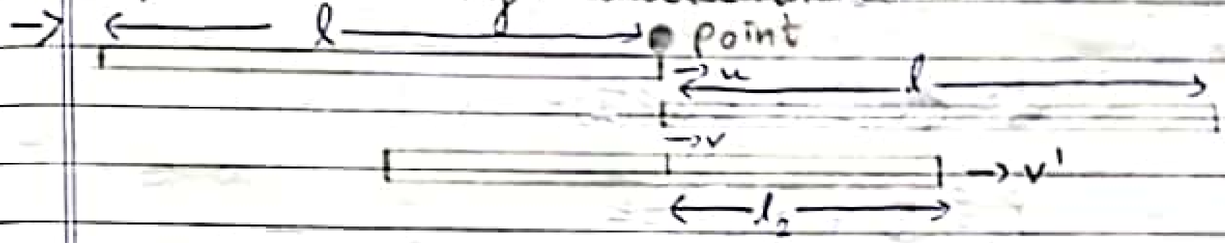
$\Rightarrow$ Area under graph,

$$d = \frac{1}{2}v_m t_1 + \frac{1}{2}v_m t_2$$

$$d = \frac{1}{2}v_m(t_1+t_2)$$

$$\boxed{d = \frac{1}{2}v_m t}$$

Q) Train is moving with ~~ces~~ constant acceleration, when two ends of train are passing a point their respective speeds are u and v. Find out speed of mid-point of train when it passes through ~~there~~ there.



⇒ After travelling distance l ~~tra~~ speed of train becomes v from u

$$\therefore \boxed{a = \frac{v^2-u^2}{2l}} \quad \text{—— (I)}$$

⇒ After travelling distance $\frac{l}{2}$, speed of train becomes v' from u.

$$\therefore \cancel{\frac{l}{2}} = \frac{l}{2} = \frac{(v')^2-u^2}{2a}$$

$$\boxed{a = \frac{(v')^2-u^2}{l}} \quad \text{—— (II)}$$

⇒ From eq (I) & (II),

$$\frac{v^2-u^2}{2l} = \frac{(v')^2-u^2}{l}$$

MIMP $\boxed{v' = \sqrt{\frac{v^2+u^2}{2}}}$

Q) Object is falling freely, find out ratio of the total distances travelled in 1s, 2s, 3s, 4s, ...

→ $V_0 = 0$, $a = -g$.

$$d = V_0 t + \frac{1}{2}at^2$$

$$d_1 = 0(1) + \frac{1}{2} g t(1)^2$$

$$d_2 = 0 + \frac{1}{2} g (2)^2$$

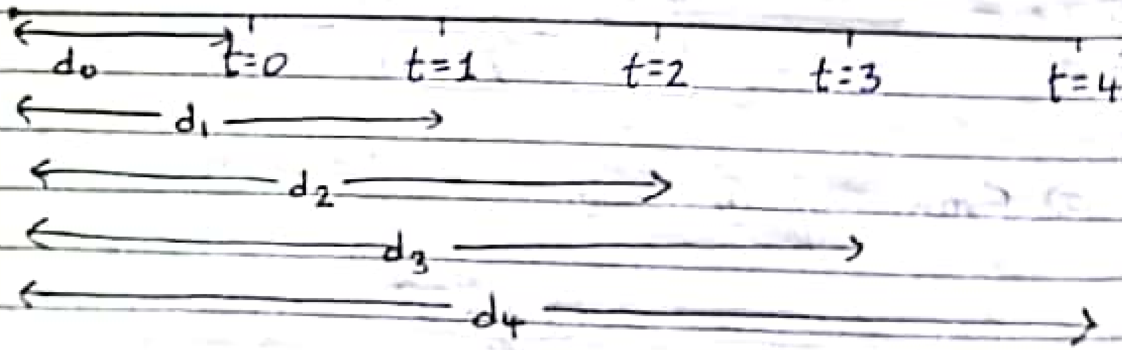
$$d_3 = 0 + \frac{1}{2} g (3)^2$$

$$d_4 = 0 + \frac{1}{2} g (4)^2$$

$\therefore d_1 : d_2 : d_3 : d_4 : .... = 1 : 4 : 9 : 16 : ......$

Q) Object is moving with constant acceleratⁿ a and velocity V_0. Find out distance travelled in n^{th} sec.

→



⇒ Distance travelled in

1^{st} sec, $S_1 = d_1 - d_0 = d_1 - d_{1-1}$
2^{nd} sec, $S_2 = d_2 - d_1 = d_2 - d_{2-1}$
3^{rd} sec, $S_3 = d_3 - d_2 = d_3 - d_{3-1}$
4^{th} sec, $S_4 = d_4 - d_3 = d_4 - d_{4-1}$

∴ Distance travelled in n^{th} second,

$$\boxed{S_n = d_n - d_{n-1}}$$

$$\Rightarrow S_n = \left(V_0 t + \frac{1}{2} a t^2\right) - \left(V_0 t + \frac{1}{2} a t^2\right)$$

$$= \left(V_0 n + \frac{1}{2} a n^2\right) - \left[V_0 (n-1) + \frac{1}{2} a (n-1)^2\right]$$

$$= \cancel{V_0 n} + \cancel{\frac{1}{2} a n^2} - \cancel{V_0 n} + V_0 - \cancel{\frac{1}{2} a n^2} + an - \frac{a}{2}$$

$$= V_0 + an - \frac{a}{2}$$

$$\boxed{S_n = V_0 + a\left(n - \frac{1}{2}\right)}$$

S_n = distance travelled in n^{th} second.

Q) Initial velocity of objects is 0, moves with constant acceleratn. Find out ratio of distances travelled in 3^{rd} and 4^{th} sec.

→ $S_3 = a\left(3 - \frac{1}{2}\right) = a\frac{5}{2}$

$S_4 = a\left(4 - \frac{1}{2}\right) = a\frac{7}{2}$ ∴ $S_3 : S_4 = 5:7$

Q) For object performing free fall find out ratio of the distances travelled in 1^{st}s, 2^{nd}s, 3^{rd}s, 4^{th}s.

→ $v_0 = 0$, $a = g$.

$$S_n = v_0 + a\left(n - \frac{1}{2}\right)$$

$$\Rightarrow S_1 = g\left(1 - \frac{1}{2}\right) = g\frac{1}{2}$$

$$S_2 = g\left(2 - \frac{1}{2}\right) = \frac{3g}{2}$$

$$S_3 = g\left(3 - \frac{1}{2}\right) = \frac{5g}{2}$$

$$S_4 = g\left(4 - \frac{1}{2}\right) = \frac{7g}{2}$$

$$\Rightarrow S_1 : S_2 : S_3 : S_4 \ldots = 1 : 3 : 5 : 7 \ldots$$

Q) Object is falling freely from tower, if it travells $d = \frac{1}{2}$ height of tower, in its last second of jorney find height of tower.

→ Supposed height of tower $= h$

total time taken $= t$

$$\Rightarrow h = v_0 t + \frac{1}{2} a t^2$$

$$\boxed{h = \frac{1}{2} g t^2} \quad \text{— (1)}$$

⇒ Distance travelled in n^{th} sec. $= \frac{h}{2}$

$$S_n = \frac{h}{2} = g\left(t - \frac{1}{2}\right)$$

~~⇒ From eq ①,~~

$$\cancel{S_n = g\left(t - \frac{1}{2}\right)}$$

$$h = 2g\left(t - \frac{1}{2}\right) \quad \text{—(ii)}$$

⇒ From eq ① & ⑪,

$$\frac{1}{2} g t^2 = 2g\left(t - \frac{1}{2}\right)$$

$$t^2 = 4\left(t - \frac{1}{2}\right)$$

$$t^2 - 4t + 2 = 0$$

$$t^2 - 4t + 4 = 2$$

$$(t-2)^2 = 2$$

$$t - 2 = \pm\sqrt{2}$$

$$t = 2 \pm \sqrt{2}$$

$\therefore$ $t = 2 + \sqrt{2},\ 2 - \sqrt{2}$

$\because$ Since t can't be less than 1 sec. $(2 - 1.4 = 0.6)$

$\therefore$ $t = 2 + \sqrt{2}$

⇒ In eq ~~Eq~~ ①,

$$h = \frac{1}{2} g (2 + \sqrt{2})^2$$

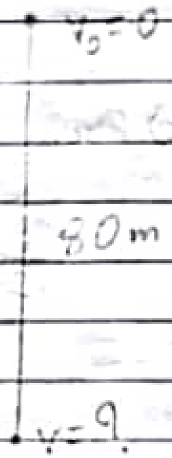
$$= \frac{1}{2} g (4 + 4\sqrt{2} + 2) = 2g + 2\sqrt{2}g + g = g(2 + 2\sqrt{2} + 1)$$

$$= 58.28\,\text{m}$$

* Motion under gravity :-

Q) A particle is dropped from top of a tower of height 80m. Find the time of journey and speed ~~of~~ with which it strikes the ground.

→



$\cancel{v = v_0 + at}$

$\cancel{= 0 + (9.8)t}$

$\Rightarrow v = \sqrt{v_0^2 + 2as}$

$= \sqrt{0 + 2(9.8)(80)}$

$= \cancel{\sqrt{1568}}$

$\Rightarrow d = v_0 t + \frac{1}{2} a t^2$

$80 = 0 + \frac{1}{2}(10)t^2$

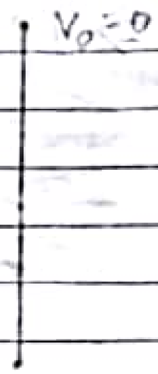
$t = 4 \text{ sec}$

$\Rightarrow v = v_0 + at$

$= 0 + 10(4)$

$= 40 \, m s^{-1}$

Q) A particle is dropped from top of tower. Its displacement in first three seconds, and in the last second is the same. Find height of tower.



→ distance in first 3 seconds,

$d = v_0 t + \frac{1}{2} a t^2$

$= \frac{1}{2} g (3)^2$ — ①

→ Distance travelled in last second,

$$S_n = V_0 + a(n - \tfrac{1}{2})$$

$$S_n = 0 + g(n - \tfrac{1}{2}) \quad \text{— (ii)}$$

⇒ From equatⁿ (i) & (ii),

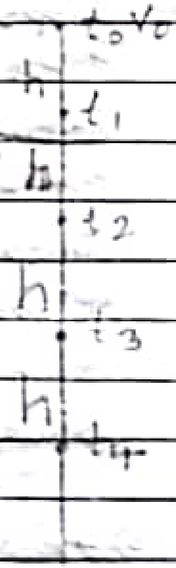
$$d = S_n$$

$$\frac{1}{2} g (3)^2 = g(n - \tfrac{1}{2})$$

$$9 = 2n - 1$$

$$\Rightarrow \boxed{n = 5 \text{ sec}}$$

Q) Particle is dropped from ~~height~~ top of tower, find ratio time in falling successive distance h.



→ $h = V_0 t + \frac{1}{2} a t^2$

$$h = \frac{1}{2} g t^2$$

$$\Rightarrow t_1 = \sqrt{\frac{2h}{g}}$$

$$t_2 = \sqrt{\frac{2(2h)}{g}}$$

$$\Rightarrow t_3 = \sqrt{\frac{3(2h)}{g}}$$

$$t_4 = \sqrt{\frac{4(2h)}{g}}$$

⇒ Time for successive h,

$$t_1 - 0 = \sqrt{\frac{2h}{g}} - 0 = \sqrt{\frac{2h}{g}}\,(1-0)$$

$$t_2 - t_1 = \sqrt{\frac{2(2h)}{g}} - \sqrt{\frac{2h}{g}} = \sqrt{\frac{2h}{g}}\,(\sqrt{2}-1)$$

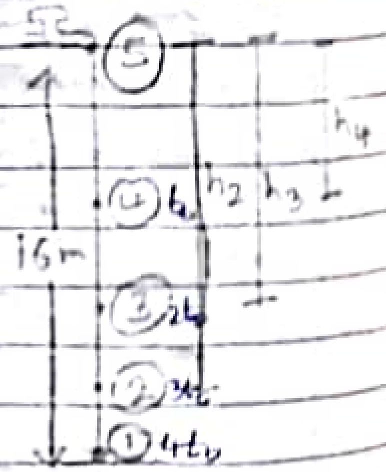
$$t_3 - t_2 = \sqrt{\frac{3(2h)}{g}} - \sqrt{\frac{2(2h)}{g}} = \sqrt{\frac{2h}{g}}\,(\sqrt{3}-\sqrt{2})$$

$$t_4 - t_3 = \sqrt{\frac{4(2h)}{g}} - \sqrt{\frac{3(2h)}{g}} = \sqrt{\frac{2h}{g}}\,(\sqrt{4}-\sqrt{3})$$

⇒ Ratio of time $= 1-0 : \sqrt{2}-1 : \sqrt{3}-\sqrt{2} : \sqrt{4}-\sqrt{3} : \sqrt{5}-$

Q) From top of a building, 16m high water drops are falling at equal intervals of time such that when the first drop reaches ~~group~~ ground, the fifth drop just starts. Find distance between successive drops at that instant.

→ Suppose regular time interval is ~~to~~ t_0.



Total time $4t_0$.

⇒ For ~~4th~~ 1st droplet,

$$h_1 = \frac{1}{2} g (4t_0)^2$$

$$16 = \frac{1}{2} g (16 t_0^2)$$

$$\frac{1}{2} g t_0^2 = 1\text{m}$$

⇒ For 2^{nd} droplet,

$$h_2 = \frac{1}{2} g (3t_0)^2$$
$$= \frac{1}{2} g t_0^2 (9)$$
$$= 9 \, m$$

⇒ For 3^{rd} droplet,

$$h_3 = \frac{1}{2} g (2t_0)^2 = \frac{1}{2} g t_0^2 (4) = 4m$$

⇒ For 4^{th} droplet,

$$h_4 = \frac{1}{2} g (t_0)^2 = \frac{1}{2} g t_0^2 = 1 \, m$$

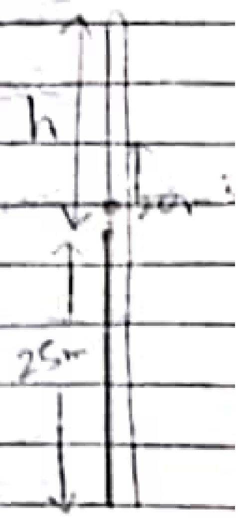
⇒ Distance between successive droplets,

$$= h_1 - h_2, \; h_2 - h_3, \; h_3 - h_4,$$
$$= 16-9, \; 9-4, \; \cancel{9} \, 4-1$$
$$= 7, 5, 3$$

4) ⟨3.4⟩ $h = \frac{v^2 - v_0^2}{2a}$

$$h = \frac{0-(20)^2}{-2\times 10}$$
$$= 20m$$

$$\Rightarrow h = v_0 t + \frac{1}{2} a t^2$$
$$\Rightarrow \cancel{-25} \; -25 = 20t - \frac{1}{2}(10)t^2$$
$$t^2 - 4t - 5 = 0$$
$$\therefore (t-5)(t+5) = 0$$



$\therefore t = 5$ sec

or

~~h =~~

$\Rightarrow v = v_0 + at$

$0 = 20 - 10(t)$

$t = 2$

$\Rightarrow 45 = v_0 t + \frac{1}{2} a t^2$

$45 = \frac{1}{2}(10) t^2$

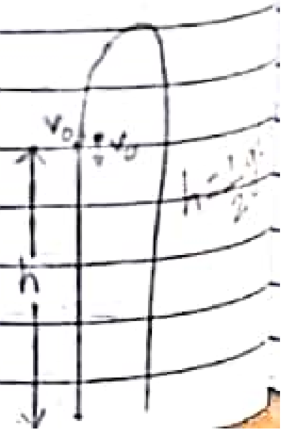
$t^2 = 9$

$t = 3$ sec

$\therefore$ Total time $= 2 + 3 = 5$ sec

Q) From a tower, two objects are projected with same velocity, first ~~with~~ in upward direction which takes time t_1 to reach bottom of tower second takes time t_2 to reach bottom of the tower. Then find height of the tower. Third objects falls freely from tower it takes time t_3 to reach bottom of ~~t~~tower. Find relation betwee t_1, t_2, t_3.

$\rightarrow$ Suppose speed $= v_0$



$\rightarrow$ 1^{st} object (thrown),

$-h = v_0 t_1 - \frac{1}{2} g t_1^2$

$h = -v_0 t_1 + \frac{1}{2} g t_1^2$ —— (1)

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→ 2^{nd} object, (thrown)

$$-h = -V_0 t_2 - \frac{1}{2} g t_2^2$$

$$h = V_0 t_2 + \frac{1}{2} g t_2^2 \quad \text{— (II)}$$

⇒ Multiplay eq (I) by t_2 & (II) by t_1 and add them,

$$ht_2 = -\cancel{V_0 t_1 t_2} + \frac{1}{2} g t_1^2 t_2$$
$$ht_1 = \cancel{V_0 t_1 t_2} + \frac{1}{2} g t_2^2 t_1$$

$$h(\cancel{t_1 + t_2}) = \frac{1}{2} g t_1 t_2 (\cancel{t_1 + t_2})$$

$$\boxed{h = \frac{1}{2} g t_1 t_2} \quad \text{— (III)}$$

⇒ 3^{rd} object (freely falling),

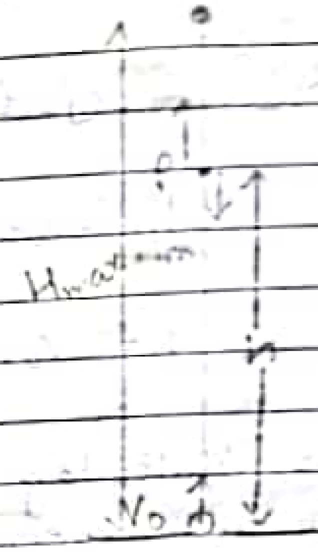
$$h = \frac{1}{2} g t^2 \quad \text{— (IV)}$$

⇒ From eq (III) & (IV),

$$\cancel{\frac{1}{2} g}\, t_1 t_2 = \cancel{\frac{1}{2} g}\, t^2$$

$$\therefore \boxed{t = \sqrt{t_1 t_2}}$$

Q) An object is thrown upwards with velocity V_0, it acquireds maximum height H_{max}, it passes throught point p twice, once in upwards journey and in downward journey. Find V_0, h and H_{max}?



→ Upward journey t_1 and downward journey t_2

$\therefore\ v_0 = ?,\ h = ?,\ H_{max} = ?$

⇒ Upward

$$h = v_0 t_1 - \frac{1}{2} g t_1^2 \quad \text{— (i)}$$

⇒ downward,

$$h = v_0 t_2 - \frac{1}{2} g t_2^2 \quad \text{— (ii)}$$

From (i) & (ii),

$$v_0 t_1 - \frac{1}{2} g t_1^2 = v_0 t_2 - \frac{1}{2} g t_2^2$$

$$v_0(t_1 - t_2) = \frac{1}{2} g (t_1^2 - t_2^2)$$

$$v_0 \cancel{(t_1 - t_2)} = \frac{1}{2} g \cancel{(t_1 - t_2)} (t_1 + t_2)$$

$$\boxed{v_0 = \frac{1}{2} g (t_1 + t_2)} \quad \text{— (iii)}$$

⇒ From eq (i) and (iii),

$$h = \left[\frac{1}{2} g (t_1 + t_2)\right] t_1 - \frac{1}{2} g t_1^2$$

$$h = \cancel{\frac{1}{2} g t_1^2} + \frac{1}{2} g t_2 t_1 - \cancel{\frac{1}{2} g t_1^2}$$

$$\boxed{h = \frac{1}{2} g t_1 t_2}$$

$$\Rightarrow H_{max} = \frac{v - V_0^2}{-2g}$$

$$= \frac{V_0^2}{2g}$$

$$= \frac{\frac{1}{4}g^2(t_1+t_2)^2}{2g}$$

$$\boxed{H_{max} = \frac{g}{8}(t_1+t_2)^2}$$

or

Let, object reach point P at time t,

$$h = V_0 t - \frac{1}{2}gt^2$$

$$\frac{1}{2}gt^2 - V_0 t + h = 0$$

$$\Rightarrow t = \frac{-(-V_0) \pm \sqrt{(-V_0)^2 - 4(\frac{1}{2}g)(h)}}{2(\frac{1}{2}g)}$$

$$= \frac{V_0 \pm \sqrt{V_0^2 - 2gh}}{g}$$

$\Rightarrow$ In upwards jorney, $t = \frac{V_0}{g} - \frac{\sqrt{V_0^2 - 2gh}}{g}$ — (i)

$\Rightarrow$ In downwards jorney, $t = \frac{V_0}{g} + \frac{\sqrt{V_0^2 - 2gh}}{g}$ — (ii)

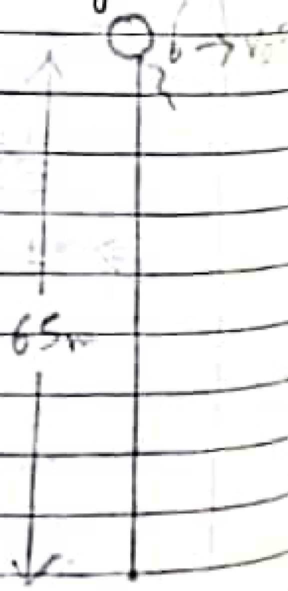
$\Rightarrow$ Total time of flight, add eq (i) & (ii),

$$\boxed{t_1+t_2 = \cancel{2\frac{\sqrt{v_0^2-2gh}}{g}} \quad \frac{2v_0}{g}}$$

$\Rightarrow$ subtract t_2-t_1,

$$\Rightarrow \boxed{t_2-t_1 = \frac{2\sqrt{v_0^2-2gh}}{g}}$$

Q) A balloon carrying a stone is moving vertically upwards with velocity 12m/s. When balloon is at height 65m, stone is dropped. After how much time and with what velocity will it strike the ground?



$\rightarrow$ $h = -65m$

$v_0 = +12ms^{-1}$

$a = -10ms^{-2}$

$t = ?$

$\Rightarrow h = v_0t + \frac{1}{2}(10)t^2$

$-65 = 12t - \frac{1}{2}(10)t^2$

$-65 = 12t - 5t^2$

$5t^2 - 12t - 65 = 0$

$\cancel{t^2}$

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$$\Rightarrow t = \frac{-(-12) \pm \sqrt{(-12)^2 - 4(5)(-65)}}{2(5)}$$

$$= \frac{+12 \pm \sqrt{144 + 1300}}{10}$$

$$= \frac{12 \pm 38}{10}$$

$$\Rightarrow t = \frac{12+38}{10}, \frac{12-38}{10}$$

$$= 5\,sec, -2.6\,sec$$

$\Rightarrow t \neq -2.6\,sec$ (Neglected)

$\therefore t = 5\,sec$

$\therefore \quad V = V_0 + at$

$V = 12 - (10 \times 5)$

$\boxed{V = -38\,m\,s^{-1}}$

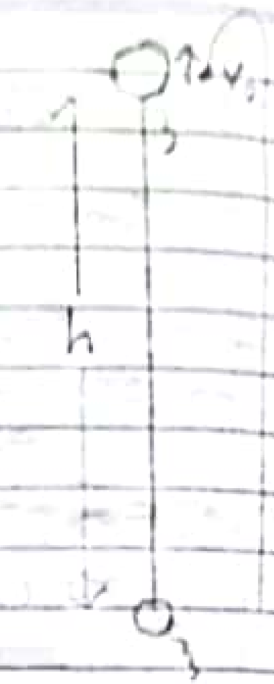
$\Rightarrow$ Distance travelled by balloon.

$h = 12 \times 5 = 60\,m$

$\therefore$ Total $= 60 + 65 = 125\,m$

Q) A balloon starts to fly in upward direction with constant acceleration $1.25\,m\,s^{-2}$, after 8sec stone is ~~xd~~ a released from it. Find out time taken by stone to reach ground.

→ Distance travelled by balloon,



$h = v_0 t + \frac{1}{2} a t^2$

$d = \frac{1}{2} \times 1.25 \times 64$

$\boxed{d = 40\,m}$

⇒ Velocity of balloon,

$v = v_0 + at$

$= 0 + 1.25 \times 8$

$\boxed{v = 10\,ms^{-1}}$

⇒ Stone performs motion under gravity,

$a = -10\,ms^{-2}$

$v = +10\,ms^{-1}$

$h = -40\,m$

$\Rightarrow h = v_0 t + \frac{1}{2} a t^2$

$h = 10t - \frac{1}{2}(10)t^2$

$-40 = 10t - 5t^2$

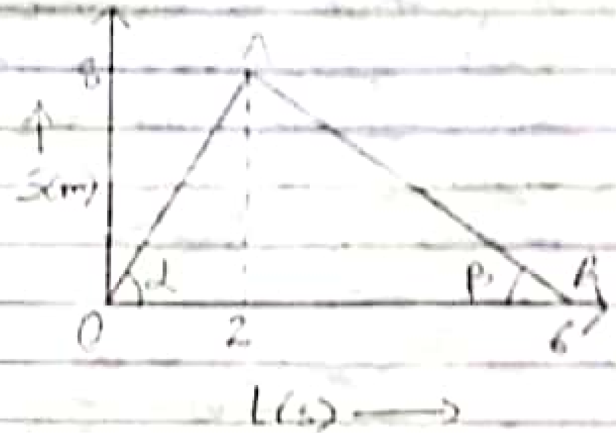
$5t^2 - 10t - 40 = 0$

$t^2 - 2t - 8 = 0$

$(t-4)(t+2) = 0$

$\therefore t = 4\,sec$

Q) For a particle moving in a straight line, the position-time graph is as shown in figure. Find the velocity of particle at time t=1s and t=4s. Also sketch the v->t graph.



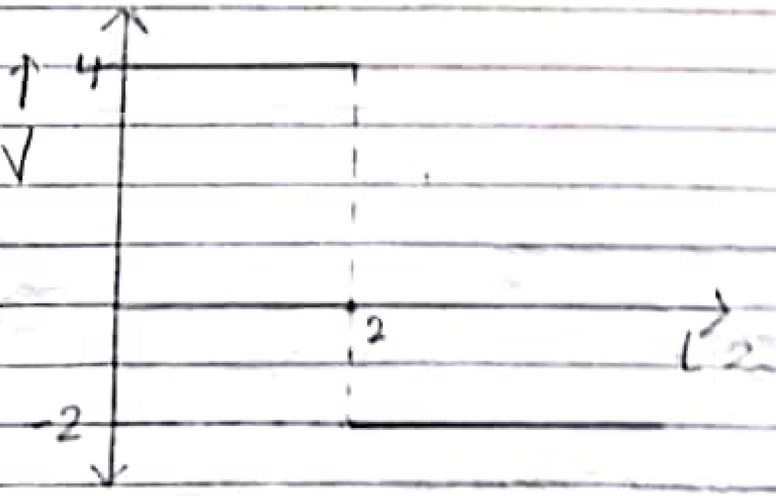
-> Slope of graph OA,

$v = \frac{8}{2} = 4\ ms^{-1}$

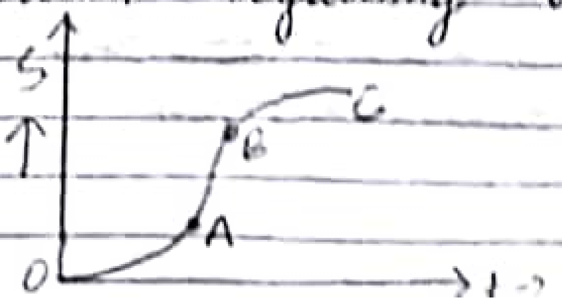
=> Slope of AB,

$v = -\frac{8}{4} = -2\ ms^{-1}$

=> Slope of v->t graph,

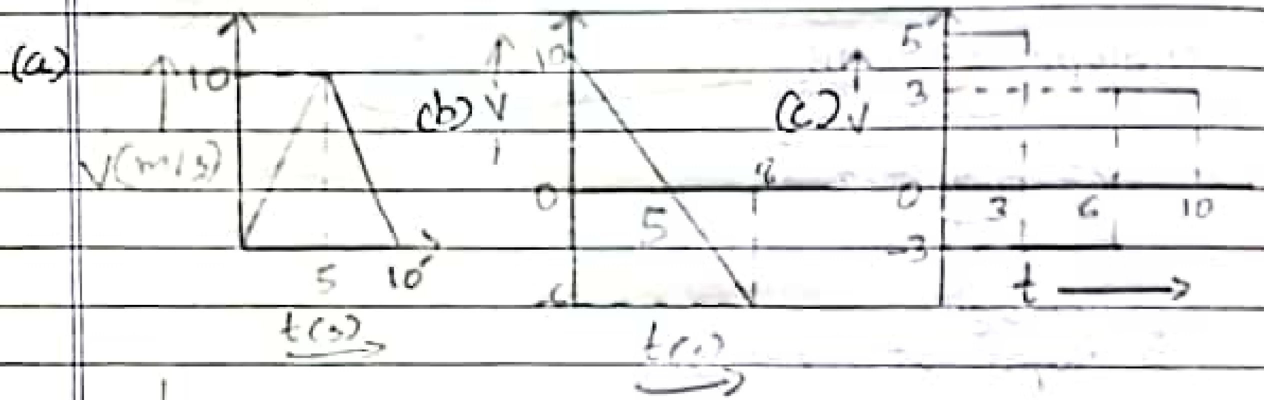


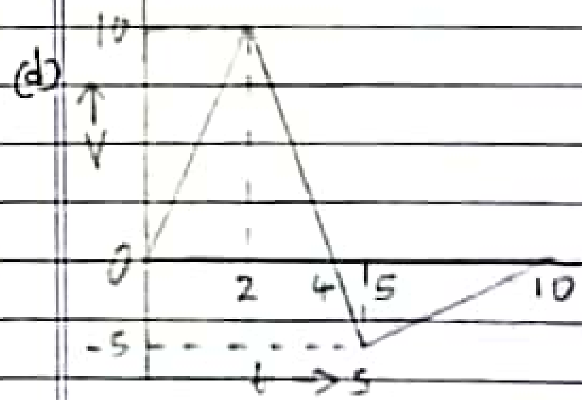
Q) Consider following s->t graph. What conclusion can be drawn regarding velocity of the particle.



→ $O \to A$, velocity increases (accelerated)
$A \to B$, velocity constant
$B \to C$, velocity decreases (deceleration)

Q) A particle is moving in a straight line. Its v→t graph in different cases are as shown below. Find the displacement and distance in time interval $t=0$ to $t=10s$ for following cases:-





→ (a) $d = \frac{1}{2} \times 10 \times 10 = 50m$

(b) Area $= \frac{1}{2} \times 5 \times 10 = 25m$, $\frac{1}{2} \times 3 \times 6 = -9m$

distance $= 25 + 9 =$ ~~36m~~ $34m$
displacement $= 25 - 9 = 16m$

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