ZHANG'S CAMERA CALIBRATION:STEP BY STEP

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1. Introduction

In this text we present an implementation of the algorithm for camera calibration proposed by Zhengyou Zhang in "A Flexible New Technique for Camera Calibration". Our implementation seeks to be as close as possible to the algorithm described by Zhang. The technique proposed by Zhang only requires the camera to observe a planar pattern shown at a few (at least two) different orientations. In the camera's model used we consider the intrinsic parameters, the extrinsic parameters and the radial lens distortion. This method proposes an analytic solution to solve these parameters and then create the best approximation using a minimization method.

2. Description of the implantation

Inputs:

- Three groups of pair points, in each group contain at least 8 pair points corresponding image(m) to the model(M)
- size of image - h and w

Steps:

(1) Normalize all image points.
  - build the normalization of matrix N

$$N = \begin{bmatrix} \frac{2.0}{w} & 0 & -1 \\ 0 & \frac{2.0}{h} & -1 \\ 0 & 0 & 1 \end{bmatrix} \quad matrix's\ normalization$$

  - For all groups
    - For all image points m do $\tilde{m}' = N\tilde{m}$

$$\tilde{m} = \begin{bmatrix} u \\ v \\ 1 \end{bmatrix} \Leftrightarrow m = \begin{bmatrix} u \\ v \end{bmatrix} \quad image\ point$$

(2) Computes the homography (H') for each group - see details on appendix 1
(3) Estimate the value of B:
  - Build matrix V

$$B = \begin{bmatrix} B_{11} & B_{12} & B_{13} \\ B_{12} & B_{22} & B_{23} \\ B_{13} & B_{23} & B_{33} \end{bmatrix}, symetric.$$

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$$b = \begin{bmatrix} B_{11} & B_{12} & B_{22} & B_{13} & B_{23} & B_{33} \end{bmatrix}$$

$$G_i = \begin{bmatrix} P_{i_0}P_{i_1} & (P_{i_0}P_{i_1} + P_{i_3}P_{i_1}) & P_{i_3}P_{i_4} & (P_{i_0}P_{i_7} + P_{i_6}P_{i_1}) & (P_{i_3}P_{i_7} + P_{i_6}P_{i_7}) & P_{i_6}P_{i_7} \\ P_{i_0}^2 - P_{i_1}^2 & 2(P_{i_0}P_{i_1} - P_{i_3}P_{i_1}) & P_{i_3}^2 - P_{i_4}^2 & 2(P_{i_0}P_{i_7} - P_{i_6}P_{i_1}) & 2(P_{i_3}P_{i_7} - P_{i_6}P_{i_7}) & P_{i_6}^2 - P_{i_7}^2 \end{bmatrix}$$

$$V = \begin{bmatrix} G_1 \\ G_2 \\ \vdots \\ G_k \end{bmatrix}$$

- Solve the system Vb=0, where b is the variable
- From solution b build B

(4) From B estimate parameter of A’
- Solve these calculations and mount A’

$$A' = \begin{bmatrix} \alpha' & \gamma' & u_0' \\ 0 & \beta' & v_0' \\ 0 & 0 & 1 \end{bmatrix} \quad \textit{normalized homography}$$

$$v_0' = \frac{B_{12}B_{13} - B_{11}B_{23}}{B_{13}B_{22} - B_{12}^2} \tag{1}$$

$$\lambda' = B_{33} - \frac{B_{12}^2 + v_0'(B_{12}B_{13} - B_{11}B_{23})}{B_{11}} \tag{2}$$

$$\alpha' = \sqrt{\frac{\lambda'}{B_{11}}} \tag{3}$$

$$\beta' = \sqrt{\frac{\lambda' B_{11}}{B_{11}B_{22} - B_{12}^2}} \tag{4}$$

$$\gamma' = \frac{-B_{12}\alpha^2\beta}{\lambda'} \tag{5}$$

$$u_0' = \frac{\gamma' v_0}{\beta'} - \frac{B_{13}\alpha'^2}{\lambda'} \tag{6}$$

$$u_0' = \frac{\gamma' v_0}{\beta'} - \frac{B_{13}\alpha'^2}{\lambda'} \tag{7}$$

- For obtain A : Solve

$$A = N^{-1}A'$$

$$A = \begin{bmatrix} \alpha & \gamma & u_0 \\ 0 & \beta & v_0 \\ 0 & 0 & 1 \end{bmatrix}$$

(5) Estimate the matrix K:
input: -matrix A
-One of the homographies of the matrix K will be relative to the coordinate system

- Undo normalization homography if necessary

$$H = N^{-1}H'$$

$$H = \begin{bmatrix} | & | & | \\ h_1 & h_2 & h_3 \\ | & | & | \end{bmatrix}$$

- Computes the parameter of K

$$K = [R\ t]$$

$$R = \begin{bmatrix} | & | & | \\ r_1 & r_2 & r_3 \\ | & | & | \end{bmatrix}, t = \begin{bmatrix} t_1 \\ t_2 \\ t_3 \end{bmatrix}$$

$$\lambda_{r_1} = \frac{1}{\|A^{-1}h_1\|} \tag{8}$$

$$\lambda_{r_2} = \frac{1}{\|A^{-1}h_2\|} \tag{9}$$

$$\lambda_{r_3} = \frac{\lambda_{r_1} + \lambda_{r_2}}{2} \tag{10}$$

$$r_1 = \lambda_{r_1} A^{-1} h_1 \tag{11}$$

$$r_2 = \lambda_{r_2} A^{-1} h_2 \tag{12}$$

$$r_3 = r_1 \times r_2 \tag{13}$$

$$t = \lambda_{r_3} A^{-1} h_3 \tag{14}$$

- minimize error using Levenberg-Marquardt

(6) Calculate the radial distortion

$$D_i = \begin{bmatrix} (u_i - u_0)(x_i^2 + y_i^2) & (u_i - u_0)(x_i^2 + y_i^2) \\ (v_i - v_0)(x_i^2 + y_i^2) & (v_i - v_0)(x_i^2 + y_i^2) \end{bmatrix}$$

$$d_i = \begin{bmatrix} (uu_i - u_i) \\ (uv_i - v_i) \end{bmatrix}$$

$$\mathbf{D} = \begin{bmatrix} D_1 \\ D_2 \\ \vdots \\ D_n \end{bmatrix}$$

$$\mathbf{k} = \begin{bmatrix} k_1 \\ k_2 \end{bmatrix}$$

$$\mathbf{d} = \begin{bmatrix} d_1 \\ d_2 \\ \vdots \\ d_n \end{bmatrix}$$

- Mount matrix D and d
- solve the system Dk = d, where k is the variable.
- minimize error using Levenberg-Marquardt

3. Appendix 1

Computes the homography (H)
input: list of pairs of points, each pair is composed for a point M of the model and a point m that is the projection of point M in the image

$$H = \begin{bmatrix} P_0 & P_1 & P_2 \\ P_3 & P_4 & P_5 \\ P_6 & P_7 & P_8 \end{bmatrix} = \begin{bmatrix} -- & \bar{h}_1 & -- \\ -- & \bar{h}_2 & -- \\ -- & \bar{h}_3 & -- \end{bmatrix} \textit{ homography}$$

$$L = \begin{bmatrix} \tilde{M}_1^T & 0^T & -u_1\tilde{M}_1^T \\ 0^T & \tilde{M}_1^T & -v_1\tilde{M}_1^T \\ \tilde{M}_2^T & 0^T & -u_2\tilde{M}_2^T \\ 0^T & \tilde{M}_2^T & -v_2\tilde{M}_2^T \\ & \vdots & \\ \tilde{M}_n^T & 0^T & -u_n\tilde{M}_n^T \\ 0^T & \tilde{M}_n^T & -v_n\tilde{M}_n^T \end{bmatrix} = \begin{bmatrix} X_1 & Y_1 & W_1 & 0 & 0 & 0 & -u_1X_1 & -u_1Y_1 & -u_1W_1 \\ 0 & 0 & 0 & X_1 & Y_1 & W_1 & -v_1X_1 & -v_1Y_1 & -v_1W_1 \\ X_2 & Y_2 & W_2 & 0 & 0 & 0 & -u_2X_2 & -u_2Y_2 & -u_2W_2 \\ 0 & 0 & 0 & X_2 & Y_2 & W_2 & -v_2X_2 & -v_2Y_2 & -v_2W_2 \\ & & & & \vdots & & & & \\ X_n & Y_n & W_n & 0 & 0 & 0 & -u_nX_n & -u_nY_n & -u_nW_n \\ 0 & 0 & 0 & X_n & Y_n & W_n & -v_nX_n & -v_nY_n & -v_nW_n \end{bmatrix}$$

$$\tilde{M}_i = \begin{bmatrix} X_i \\ Y_i \\ W_i \end{bmatrix} \Leftrightarrow \begin{bmatrix} X_i \\ Y_i \\ 0 \\ W_i \end{bmatrix}$$

$$x = \begin{bmatrix} \bar{h}_1^T \\ \bar{h}_2^T \\ \bar{h}_3^T \end{bmatrix} = \begin{bmatrix} P_0 \\ P_1 \\ P_2 \\ P_3 \\ P_4 \\ P_5 \\ P_6 \\ P_7 \\ P_8 \end{bmatrix}$$

- mount the matrix L
- solve the system Lx = 0, where x is the variable.
- minimize error using Levenberg-Marquardt.

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