PHY3004W Formula Sheet

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1 Electromagnetism

$$\sin\theta \equiv \theta$$
$$\tan\theta \equiv \sin\theta$$
$$\cos\theta \equiv 1$$

$$\nabla \cdot \boldsymbol{D} = \rho_f$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$

$$\oint \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{a} = Q_{f,enc}$$
$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{f,enc}(t) + \frac{\mathrm{d}\Phi_{\boldsymbol{D}}}{\mathrm{d}t}$$
$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a} = 0$$
$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi_{\boldsymbol{B}}}{\mathrm{d}t}$$

$$D_1^{\perp} - D_2^{\perp} = \sigma_f$$
$$B_1^{\perp} - B_2^{\perp} = 0$$
$$\boldsymbol{E}_1^{\parallel} - \boldsymbol{E}_2^{\parallel} = 0$$
$$\boldsymbol{H}_1^{\parallel} - \boldsymbol{H}_2^{\parallel} = \boldsymbol{K}_f \times \hat{\boldsymbol{n}}$$

$$\boldsymbol{S} = \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B}$$
$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$
$$I = \langle S \rangle = \frac{1}{2} v \epsilon E_0^2$$
$$P = \int \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a}$$

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

$$\frac{\sin\theta_I}{\sin\theta_T} = \frac{k_T}{k_I} = \frac{n_2}{n_1}$$

$$R = \frac{I_R}{I_I}$$
$$T = \frac{I_T}{I_I}$$
$$R + T = 1$$

1.1 Special Relativity

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Lambda^{\mu}_{\alpha} g_{\mu\nu} \Lambda^{\nu}_{\beta} = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2 t^2 - d^2$$

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{\gamma} \qquad \text{(proper time)}$$
$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \begin{pmatrix} \gamma c \\ \gamma \boldsymbol{u} \end{pmatrix} \qquad \text{(proper velocity)}$$
$$p^\mu = m\eta^\mu = \begin{pmatrix} \gamma m c \\ \boldsymbol{p} \end{pmatrix} \qquad \text{(4-momentum)}$$

$$\eta_\mu \eta^\mu = c^2$$
$$p_\mu p^\mu = E^2/c^2 - \boldsymbol{p}^2 = m^2 c^2$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$K^\mu = q F^{\mu\nu} g_{\nu\alpha} \eta^\alpha$$

$$J^\mu = \begin{pmatrix} c\rho \\ \boldsymbol{J} \end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi}\int \frac{\nabla' \times \boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}\boldsymbol{x}$$

$$\begin{aligned}\boldsymbol{B} &= \nabla \times A \\ \boldsymbol{E} &= -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}\end{aligned}$$

$$A^\mu_{\mathrm{ret}}(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\int \mathrm{d}^3x' \, \frac{J^\mu(x^0 - |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

1.2 Radiation

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a}\cdot\boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a}\cdot\boldsymbol{b}) \tag{1.1}$$

$$\begin{aligned}\boldsymbol{E} = \frac{q}{4\pi\epsilon}\Big(&\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} \\ &+ \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3 c|\boldsymbol{r}|}\Big) \qquad \text{(for point charge)}\end{aligned}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} \qquad \text{(constant velocity)}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c}\frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \boldsymbol{a}))^2}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^5}$$

$$\begin{aligned}\frac{\mathrm{d}P}{\mathrm{d}\Omega} &= \frac{\mu_0 q^2 a^2}{16\pi^2 c}\sin^2\theta \qquad \text{(non-rel. limit)} \\ P &= \frac{\mu_0 q^2 a^2}{6\pi c}\end{aligned}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c}(a^2 - |\boldsymbol{\beta} \times \boldsymbol{a}|^2) \qquad \text{(rel. generalisation)}$$

1.3 Radiation of dipole

$$\boldsymbol{p}(t) = e^{-i\omega t}\int \rho(\boldsymbol{x}')\boldsymbol{x}'\mathrm{d}^3x' \qquad \text{(dipole moment)}$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \tag{1.2}$$

2 Atomic Physics

$$-i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

$$[x^i, p^j] = i\hbar\delta^{ij}$$

$$[L^i, L^j] = i\hbar\epsilon^{ijk}L^k$$

$$\hat{\boldsymbol{S}} = \frac{\hbar}{2}\boldsymbol{\sigma}$$

$$\begin{aligned}\sigma_x &= \begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix} \\ \sigma_y &= \begin{pmatrix}0 & -i\\ i & 0\end{pmatrix} \\ \sigma_z &= \begin{pmatrix}1 & 0\\ 0 & -1\end{pmatrix}\end{aligned}$$

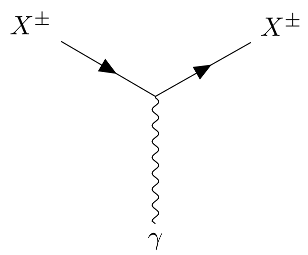
$$[S^i, S^j] = i\hbar\epsilon^{ijk}S^k$$

$$\begin{aligned}[\boldsymbol{AB}, \boldsymbol{C}] &= \boldsymbol{A}[\boldsymbol{B}, \boldsymbol{C}] + [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C} \\ [\boldsymbol{A}, \boldsymbol{BC}] &= [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C} + \boldsymbol{B}[\boldsymbol{A}, \boldsymbol{C}]\end{aligned}$$

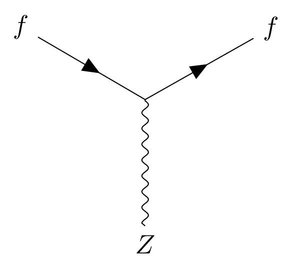
$$\begin{aligned}E_n^{(1)} &= \langle E_n^{(0)}|H_I|E_n^{(0)}\rangle \\ E_n^{(2)} &= \sum_{m\neq n}\langle E_n^{(0)}|H_I|E_m^{(0)}\rangle\frac{\langle E_m^{(0)}|H_I|E_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}\end{aligned}$$

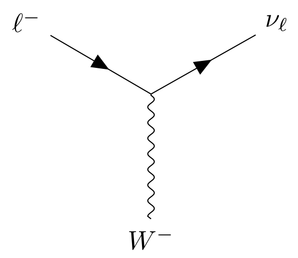
3 Particle Physics

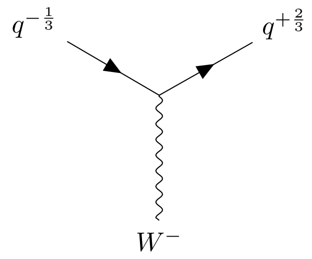
$$s = (p_A + p_B)^2 \tag{3.1}$$



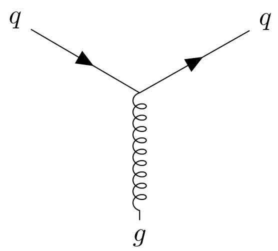
3.1 Weak Force

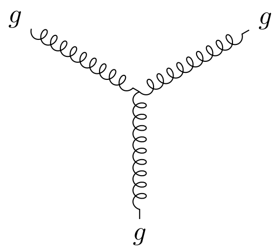


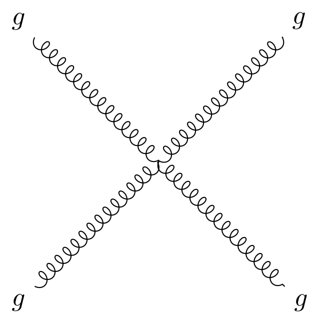




3.2 Strong Force







$$\frac{\mathrm{d}N_{\text{scat}}}{\mathrm{d}t} = \sigma\mathcal{L} \tag{3.2}$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2}|\mathcal{M}|^2 \qquad \text{(two-body decay)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A+E_B)^2}\frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \qquad (2 \to 2 \text{ scatter})$$

4 Nuclear Physics

$$\Delta M(Z,A) = M(Z,A) - [Z(M_p + m_e) + NM_n]$$

$$B(A,Z) = -\Delta M(Z,A)c^2 \qquad \text{(Binding energy)}$$

$$\begin{aligned} Q &= [(M_i + M_I) - (M_f + M_F)]c^2 \\ Q &= B(f) + B(F) - B(i) + B(I) \end{aligned}$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$\begin{aligned} -\frac{\mathrm{d}N(t)}{\mathrm{d}t} &= \lambda N(t) \\ N(t) &= N(0)e^{-\lambda t} \end{aligned}$$

$$\begin{aligned} R(t) &= -\frac{\mathrm{d}N(t)}{\mathrm{d}t} \\ R(t) &= \lambda N(0)e^{-\lambda t} \end{aligned}$$

$$\mu_m = \frac{\mu}{\rho} \qquad \text{(Mass attenuation)}$$

$$I(x) = I_0 e^{-\mu x} \qquad \text{(Intensity loss of } \gamma)$$

$$I(x) = I_0 e^{-N\sigma x} \qquad \text{(Neutron absorption)}$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \qquad \text{(Equivalent Dose)}$$

$$\begin{aligned} N &= In\Delta x\sigma \\ I &= I_0 e^{-n\sigma t} \end{aligned} \qquad \text{(For a thick target)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \qquad \text{(Labeling states } [k = 2j+1])$$

$$S, P, D, F, G, \cdots \implies \ell = 0, 1, 2, 3, 4, \ldots$$

$$P = (-1)^\ell \qquad \text{(for single particle state)}$$

5 Thermal Physics

$$pV = K_B NT \qquad \text{(Ideal gas law)}$$
$$pV = nRT$$

$$\bar{E}_{\text{kin}} = \frac{1}{2} f N K_B T \qquad \text{(Equipartition thm)}$$

$$\Delta U = Q + W \qquad \text{(1st Law)}$$

$$\frac{Q}{\Delta T} = \alpha A \frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\ln(A!) \approx A \ln(A) - A \qquad \text{(Stirling approx.)}$$

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N-1)!} \qquad \text{(Einstein solid)}$$

$$\Omega(N, N_\uparrow) = \frac{N!}{N_\uparrow (N - N_\uparrow)!} \qquad \text{(paramagnet)}$$

$$S = K_B \ln \Omega \qquad \text{(entropy)}$$

$$S(U, V, N) = K_B N \left[\ln \left(\frac{V}{N} \left(\frac{4\pi m U}{3\hbar^2 N} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right]$$
(Sackur-Tetrode)

$$\frac{1}{T} \equiv \left[\frac{\partial S}{\partial U} \right]_{N,V}$$

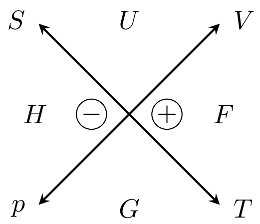
$$p \equiv T \left[\frac{\partial S}{\partial U} \right]_{U,N}$$

$$\eta \equiv \frac{W}{Q_H} \qquad \text{(efficiency)}$$

$$H \equiv U + pV$$
$$F \equiv U - TS$$
$$G \equiv H - TS$$

$$\mathrm{d}U = T\mathrm{d}S - p\mathrm{d}V + \mu \mathrm{d}N \tag{5.1}$$
$$\mathrm{d}F = -S\mathrm{d}T - p\mathrm{d}V + \mu \mathrm{d}N \tag{5.2}$$
$$\mathrm{d}G = -S\mathrm{d}T + V\mathrm{d}p + \mu \mathrm{d}N \tag{5.3}$$
$$\mathrm{d}H = T\mathrm{d}S + V\mathrm{d}p + \mu \mathrm{d}N \tag{5.4}$$

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L/T}{\Delta V} \qquad \text{(Clausius-Clapeyron relation)}$$



$$\left(p + a\frac{N^2}{V^2} \right)(V - bN) = K_B NT$$
(Van der Waals eqn)

$$P(S) = \frac{1}{z} \exp\left(-\frac{E(s)}{T} \right)$$

$$z = \sum_s d(s) \exp\left(-\frac{E(s)}{T} \right)$$

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln z$$

$$F = -K_B T \ln z$$

$$P(s) = \frac{1}{\mathcal{Z}} \sum_s \exp\left(-\frac{E(s) - \mu N(s)}{K_B T} \right)$$

$$\mathcal{Z}(T, V, \mu) = \frac{1}{\mathcal{Z}} \exp\left(-\frac{E(s) - \mu N(s)}{K_B T} \right)$$

$$\epsilon_F = \frac{\hbar^2}{8m} \left(\frac{3}{\pi} \frac{N}{V} \right)^{2/3}$$

$$N = \int_0^\infty \mathrm{d}\epsilon g(\epsilon) n_F(\epsilon, T, \mu)$$

$$U = \int_0^\infty \mathrm{d}\epsilon g(\epsilon) n_F(\epsilon, T, \mu) \epsilon$$

6 Solid State Physics

$$\boldsymbol{R} = [n_1, n_2, n_3] = n_1 \boldsymbol{a}_1 + n_2 \boldsymbol{a}_2 + n_3 \boldsymbol{a}_3 \tag{6.1}$$

$$\boldsymbol{G} = h\boldsymbol{b}_1 + k\boldsymbol{b}_2 + l\boldsymbol{b}_3 \tag{6.2}$$

$$d = \frac{2\pi}{|\boldsymbol{G}_{\min}|} \tag{6.3}$$

$$\boldsymbol{b}_i \cdot \boldsymbol{a}_j = 2\pi \delta_{ij} \tag{6.4}$$

$$(hkl) \to_{\perp} h\boldsymbol{b}_1 + k\boldsymbol{b}_2 + l\boldsymbol{b}_3 \tag{6.5}$$

$$2d\sin(\theta) = n\lambda \qquad \text{(Bragg condition)}$$

$$f(E,T) = \frac{1}{\exp(\frac{E-\mu}{K_B T}) + 1} \qquad \text{(Fermi-Dirac)}$$

$$E_f = \frac{\hbar^2}{2m}(3\pi^2 n)^{2/3} \qquad \text{(for free electron gas)}$$