PHY3004W Formula Sheet

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1 Electromagnetism

$$\begin{aligned}\sin\theta &\equiv \theta\\ \tan\theta &\equiv \sin\theta\\ \cos\theta &\equiv 1\end{aligned}$$

$$\begin{aligned}\nabla\cdot\boldsymbol{D} &= \rho_f\\ \nabla\cdot\boldsymbol{B} &= 0\\ \nabla\times\boldsymbol{E} &= -\frac{\partial\boldsymbol{B}}{\partial t}\\ \nabla\times\boldsymbol{H} &= \boldsymbol{J}_f + \frac{\partial\boldsymbol{D}}{\partial t}\end{aligned}$$

$$\begin{aligned}\boldsymbol{D} &= \epsilon_0\boldsymbol{E} + \boldsymbol{P}\\ \boldsymbol{B} &= \mu_0(\boldsymbol{H} + \boldsymbol{M})\end{aligned}$$

$$\begin{aligned}\oint \boldsymbol{D}\cdot \mathrm{d}\boldsymbol{a} &= Q_{f,enc}\\ \oint \boldsymbol{H}\cdot \mathrm{d}\boldsymbol{l} &= I_{f,enc}(t) + \frac{\mathrm{d}\Phi_{\boldsymbol{D}}}{\mathrm{d}t}\\ \oint \boldsymbol{B}\cdot \mathrm{d}\boldsymbol{a} &= 0\\ \oint \boldsymbol{E}\cdot \mathrm{d}\boldsymbol{l} &= -\frac{\mathrm{d}\Phi_{\boldsymbol{B}}}{\mathrm{d}t}\end{aligned}$$

$$\begin{aligned}D_1^\perp - D_2^\perp &= \sigma_f\\ B_1^\perp - B_2^\perp &= 0\\ \boldsymbol{E}_1^\parallel - \boldsymbol{E}_2^\parallel &= 0\\ \boldsymbol{H}_1^\parallel - \boldsymbol{H}_2^\parallel &= \boldsymbol{K}_f\times\hat{\boldsymbol{n}}\end{aligned}$$

$$\begin{aligned}\boldsymbol{S} &= \frac{1}{\mu}\boldsymbol{E}\times\boldsymbol{B}\\ u &= \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\\ I &= \langle S\rangle = \frac{1}{2}v\epsilon E_0^2\\ P &= \int \boldsymbol{S}\cdot\mathrm{d}\boldsymbol{a}\end{aligned}$$

$$\nabla\cdot\boldsymbol{J} = -\frac{\partial\rho}{\partial t}$$

$$\frac{\sin\theta_I}{\sin\theta_T} = \frac{k_T}{k_I} = \frac{n_2}{n_1}$$

$$\begin{aligned}R &= \frac{I_R}{I_I}\\ T &= \frac{I_T}{I_I}\\ R + T &= 1\end{aligned}$$

1.1 Special Relativity

$$\begin{aligned}\beta &= \frac{v}{c}\\ \gamma &= \frac{1}{\sqrt{1-\beta^2}}\end{aligned}$$

$$\Lambda = \begin{pmatrix}\gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

$$\Lambda^\mu_\alpha g_{\mu\nu}\Lambda^\nu_\beta = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2t^2 - d^2$$

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{\gamma} \qquad \text{(proper time)}$$

$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \begin{pmatrix}\gamma c\\ \gamma\boldsymbol{u}\end{pmatrix} \qquad \text{(proper velocity)}$$

$$p^\mu = m\eta^\mu = \begin{pmatrix}\gamma mc\\ \boldsymbol{p}\end{pmatrix} \qquad \text{(4-momentum)}$$

$$\begin{aligned}\eta_\mu\eta^\mu &= c^2\\ p_\mu p^\mu &= E^2/c^2 - \boldsymbol{p}^2 = m^2c^2\end{aligned}$$

$$F^{\mu\nu} = \begin{pmatrix}0 & -E_x/c & -E_y/c & -E_z/c\\ E_x/c & 0 & -B_z & B_y\\ E_y/c & B_z & 0 & -B_x\\ E_z/c & -B_y & B_x & 0\end{pmatrix}$$

$$K^\mu = qF^{\mu\nu}g_{\nu\alpha}\eta^\alpha$$

$$J^\mu = \begin{pmatrix}c\rho\\ \boldsymbol{J}\end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi}\int \frac{\nabla' \times \boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}\boldsymbol{x}$$

$$\boldsymbol{B} = \nabla \times A$$
$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$A^\mu_{\text{ret}}(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\int \mathrm{d}^3x' \frac{J^\mu(x^0 - |\boldsymbol{x}-\boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

1.2 Radiation

$$\boldsymbol{a} \times (\boldsymbol{b} \times \boldsymbol{c}) = \boldsymbol{b}(\boldsymbol{a}\cdot\boldsymbol{c}) - \boldsymbol{c}(\boldsymbol{a}\cdot\boldsymbol{b}) \tag{1.1}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon}\Big(\frac{\hat{\boldsymbol{r}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} + \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}}-\boldsymbol{\beta})\times\dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3 c|\boldsymbol{r}|}\Big) \qquad \text{(for point charge)}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}}-\boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} \qquad \text{(constant velocity)}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c}\frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}}-\boldsymbol{\beta})\times\boldsymbol{a}))^2}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\sin^2\theta \qquad \text{(non-rel. limit)}$$
$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c}(a^2 - |\boldsymbol{\beta}\times\boldsymbol{a}|^2) \qquad \text{(rel. generalisation)}$$

1.3 Radiation of dipole

$$\boldsymbol{p}(t) = e^{-i\omega t}\int \rho(\boldsymbol{x}')\boldsymbol{x}'\mathrm{d}^3x' \qquad \text{(dipole moment)}$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \tag{1.2}$$

2 Nuclear Physics

$$\Delta M(Z,A) = M(Z,A) - [Z(M_p + m_e) + NM_n]$$

$$B(A,Z) = -\Delta M(Z,A)c^2 \qquad \text{(Binding energy)}$$

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$
$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lambda N(t)$$
$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{\mathrm{d}N(t)}{\mathrm{d}t}$$
$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x} \qquad \text{(Intensity loss of } \gamma\text{)}$$

$$I(x) = I_0 e^{-N\sigma x} \qquad \text{(Neutron absorption)}$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \qquad \text{(Equivalent Dose)}$$

$$N = In\Delta x\sigma$$
$$I = I_0 e^{-n\sigma t} \qquad \text{(For a thick target)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \qquad \text{(Labeling states } [k = 2j+1])$$

$$S, P, D, F, G, \cdots \implies \ell = 0, 1, 2, 3, 4, \ldots$$

$$P = (-1)^\ell \qquad \text{(for single particle state)}$$

3 Atomic Physics

$$-i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

$$[x^i, p^j] = i\hbar\delta^{ij}$$

$$[L^i, L^j] = i\hbar\epsilon^{ijk}L^k$$

$$\hat{\boldsymbol{S}} = \frac{\hbar}{2}\boldsymbol{\sigma}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$
$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$[S^i, S^j] = i\hbar\epsilon^{ijk}S^k$$

$$[\boldsymbol{AB}, \boldsymbol{C}] = \boldsymbol{A}[\boldsymbol{B}, \boldsymbol{C}] + [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C}$$
$$[\boldsymbol{A}, \boldsymbol{BC}] = [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C} + \boldsymbol{B}[\boldsymbol{A}, \boldsymbol{C}]$$

$$E_n^{(1)} = \langle E_n^{(0)}|H_I|E_n^{(0)}\rangle$$
$$E_n^{(2)} = \sum_{m\neq n}\langle E_n^{(0)}|H_I|E_m^{(0)}\rangle\frac{\langle E_m^{(0)}|H_I|E_n^{(0)}\rangle}{E_n^{(0)} - E_m^{(0)}}$$

4 Thermal Physics

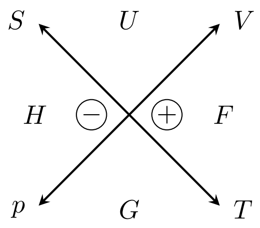
$$pV = K_BNT \qquad \text{(Ideal gas law)}$$
$$pV = nRT$$

$$\bar{E}_{\text{kin}} = \frac{1}{2}fNK_BT \qquad \text{(Equipartition thm)}$$

$$\Delta U = Q + W \qquad \text{(1st Law)}$$

$$\frac{Q}{\Delta T} = \alpha A\frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\ln(A!) = A\ln(A) - A \qquad \text{(Stirling approx.)}$$



$$S = K_B\ln\Omega \qquad \text{(entropy)}$$

$$S(U,V,N) = K_BN\left[\ln\left(\frac{V}{N}\left(\frac{4\pi mU}{3\hbar^2N}\right)^{\frac{3}{2}}\right)\right] \qquad \text{(Sackur-Tetrode)}$$

$$\frac{1}{T} \equiv \left[\frac{\partial S}{\partial U}\right]_{N,V}$$

$$p \equiv T\left[\frac{\partial S}{\partial U}\right]_{U,N}$$

$$\eta \equiv \frac{W}{Q_H} \qquad \text{(efficiency)}$$

$$H \equiv U + pV$$
$$F \equiv U - TS$$
$$G \equiv H - TS$$

$$\frac{\mathrm{d}p}{\mathrm{d}T} = \frac{L/T}{\Delta V} \qquad \text{(Clausius-Clapeyron relation)}$$

$$\left(p + a\frac{N^2}{V^2}\right)(V - bN) = K_BNT \qquad \text{(Van der Waals eqn)}$$

$$P(S) = \frac{1}{z}\exp\left(-\frac{E(s)}{T}\right)$$

$$z = \sum_s d(s)\exp\left(-\frac{E(s)}{T}\right)$$

$$\langle E\rangle = -\frac{\partial\beta}{\partial\ln}z$$

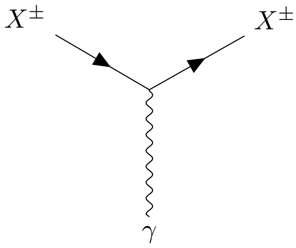
$$F = -T\ln z$$

$$p(s) = \frac{1}{\mathcal{Z}}\sum_s\exp\left(-\frac{E(s) - \mu N(s)}{T}\right)$$

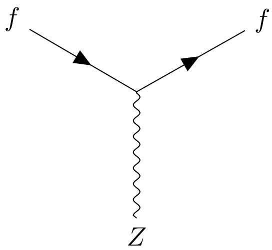
$$\mathcal{Z}(T,V,\mu) = \sum_s\exp\left(-\frac{E(s) - \mu N(s)}{T}\right)$$

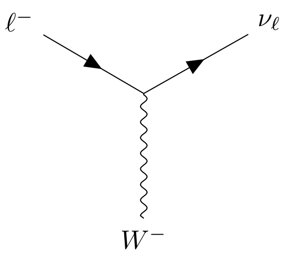
5 Particle Physics

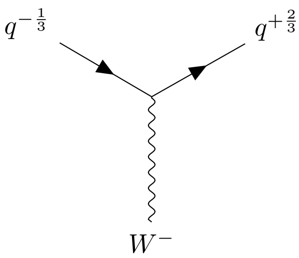
$$s = (p_A + p_B)^2 \tag{5.1}$$



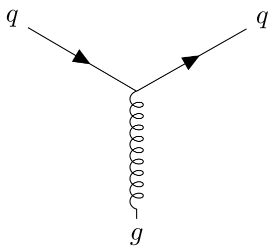
5.1 Weak Force

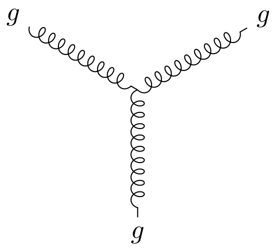


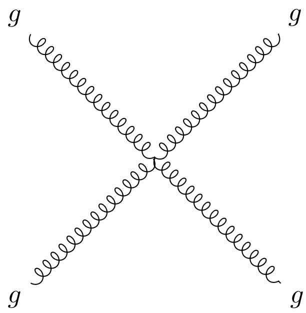




5.2 Strong Force







$$\frac{\mathrm{d}N_{\text{scat}}}{\mathrm{d}t} = \sigma \mathcal{L} \tag{5.2}$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2} |\mathcal{M}|^2 \qquad \text{(two-body decay)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A + E_B)^2} \frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \qquad (2 \to 2 \text{ scatter})$$

6 Solid State Physics