PHY3004W Formula Sheet

Alex Veltman November 2020

1 Electromagnetism

$$\sin\theta \equiv \theta$$
$$\tan\theta \equiv \sin\theta$$
$$\cos\theta \equiv 1$$

$$\nabla \cdot \boldsymbol{D} = \rho_f$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{B} = \mu_0(\boldsymbol{H} + \boldsymbol{M})$$

$$\oint \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{a} = Q_{f,enc}$$
$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{f,enc}(t) + \frac{\mathrm{d}\Phi_{\boldsymbol{D}}}{\mathrm{d}t}$$
$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a} = 0$$
$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi_{\boldsymbol{B}}}{\mathrm{d}t}$$

$$D_1^{\perp} - D_2^{\perp} = \sigma_f$$
$$B_1^{\perp} - B_2^{\perp} = 0$$
$$\boldsymbol{E}_1^{\parallel} - \boldsymbol{E}_2^{\parallel} = 0$$
$$\boldsymbol{H}_1^{\parallel} - \boldsymbol{H}_2^{\parallel} = \boldsymbol{K}_f \times \hat{\boldsymbol{n}}$$

$$\boldsymbol{S} = \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B}$$
$$u = \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2\right)$$
$$I = \langle S \rangle = \frac{1}{2} v \epsilon E_0^2$$
$$P = \int \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a}$$

$$\nabla \cdot \boldsymbol{J} = -\frac{\partial \rho}{\partial t}$$

1.1 Special Relativity

$$\beta = \frac{v}{c}$$
$$\gamma = \frac{1}{\sqrt{1-\beta^2}}$$
$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$
$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Lambda^{\mu}_{\alpha} g_{\mu\nu} \Lambda^{\nu}_{\beta} = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2 t^2 - d^2$$

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{\gamma} \qquad \text{(proper time)}$$
$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \begin{pmatrix} \gamma c \\ \gamma \boldsymbol{u} \end{pmatrix} \qquad \text{(proper velocity)}$$
$$\eta^\mu = m\eta^\mu = \begin{pmatrix} E/c \\ \boldsymbol{p} \end{pmatrix} \qquad \text{(proper velocity)}$$

$$\eta_\mu \eta^\mu = c^2$$
$$p_\mu p^\mu = E^2/c^2 - \boldsymbol{p}^2 = m^2 c^2$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$K^\mu = q F^{\mu\nu} g_{\nu\alpha} \eta^\alpha$$

$$J^\mu = \begin{pmatrix} c\rho \\ \boldsymbol{J} \end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi} \int \frac{\nabla' \times \boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} \mathrm{d}\boldsymbol{x}$$

$$\boldsymbol{B} = \nabla \times A$$
$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$A^\mu_{\mathrm{ret}}(\boldsymbol{x}) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3 x' \, \frac{J^\mu(x^0 - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}$$

1.2 Radiation

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon}\Big(\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} + \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3 c|\boldsymbol{r}|}\Big) \quad \text{(for point charge)}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} \quad \text{(constant velocity)}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c}\frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \boldsymbol{a}))^2}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\sin^2\theta \quad \text{(non-rel. limit)}$$
$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c}(a^2 - |\boldsymbol{\beta}\times\boldsymbol{a}|^2) \quad \text{(rel. generalisation)}$$

1.3 Radiation of dipole

$$\boldsymbol{p}(t) = e^{-i\omega t}\int \rho(\boldsymbol{x}')\boldsymbol{x}'\mathrm{d}^3x' \quad \text{(dipole moment)}$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \quad (1.1)$$

2 Nuclear Physics

$$\Delta M(Z,A) = M(Z,A) - [Z(M_p + m_e) + NM_n]$$

$$B(A,Z) = -\Delta M(Z,A)c^2 \quad \text{(Binding energy)}$$

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$
$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lambda N(t)$$
$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{\mathrm{d}N(t)}{\mathrm{d}t}$$
$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x} \quad \text{(Intensity loss of } \gamma\text{)}$$

$$I(x) = I_0 e^{-N\sigma x} \quad \text{(Neutron absorption)}$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \quad \text{(Equivalent Dose)}$$

$$N = In\Delta x\sigma$$
$$I = I_0 e^{-n\sigma t} \quad \text{(For a thick target)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \quad \text{(Labeling states } [k = 2j+1]\text{)}$$

$$S, P, D, F, G, \cdots \implies \ell = 0, 1, 2, 3, 4, \ldots$$

$$P = (-1)^\ell \quad \text{(for single particle state)}$$

3 Atomic Physics

$$-i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle$$

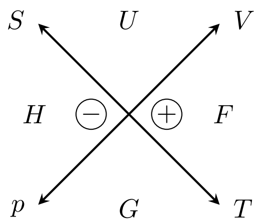
$$[x^i, p^j] = i\hbar\delta^{ij}$$

$$[L^i, L^j] = i\hbar\epsilon^{ijk}L^k$$

$$\hat{\boldsymbol{S}} = \frac{\hbar}{2}\boldsymbol{\sigma}$$

$$[S^i, S^j] = i\hbar\epsilon^{ijk}S^k$$

$$[\boldsymbol{AB}, \boldsymbol{C}] = \boldsymbol{A}[\boldsymbol{B}, \boldsymbol{C}] + [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C}$$



4 Thermal Physics

$$pV = K_B NT \qquad \text{(Ideal gas law)}$$
$$pV = nRT$$

$$\bar{E}_{\text{kin}} = \frac{1}{2} f N K_B T \qquad \text{(Equipartition thm)}$$

$$\Delta U = Q + W \qquad \text{(1st Law)}$$

$$\frac{Q}{\Delta T} = \alpha A \frac{\mathrm{d}T}{\mathrm{d}x}$$

$$\ln(A!) = A\ln(A) - A \qquad \text{(Stirling approx.)}$$

$$S = K_B \ln \Omega \qquad \text{(entropy)}$$

$$S(U,V,N) = K_B N \left[\ln\left(\frac{V}{N} \left(\frac{4\pi m U}{3\hbar^2 N} \right)^{\frac{3}{2}} \right) \right] \qquad \text{(Sackur-Tetrode)}$$

$$\frac{1}{T} \equiv \left[\frac{\partial S}{\partial U} \right]_{N,V}$$

$$p \equiv T \left[\frac{\partial S}{\partial U} \right]_{U,N}$$

$$\eta \equiv \frac{W}{Q_H} \qquad \text{(efficiency)}$$

$$H \equiv U + pV$$
$$F \equiv U - TS$$
$$G \equiv H - TS$$

$$\frac{\mathrm{d}p}{\mathrm{d}T} \frac{L/T}{\Delta V} \qquad \text{(Clausius-Clapeyron relation)}$$

$$\left(p + a\frac{N^2}{V^2} \right)(V - bN) = K_B NT \qquad \text{(Van der Waals eqn)}$$

$$P(S) = \frac{1}{z} \exp\left(-\frac{E(s)}{T} \right)$$

$$z = \sum_s d(s) \exp\left(-\frac{E(s)}{T} \right)$$

$$\langle E \rangle = -\frac{\partial \beta}{\partial \ln} z$$

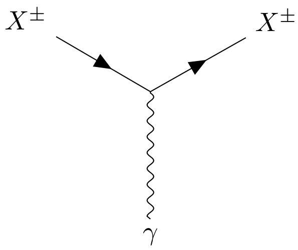
$$F = -T \ln z$$

$$p(s) = \frac{1}{\mathcal{Z}} \sum_s \exp\left(-\frac{E(s) - \mu N(s)}{T} \right)$$

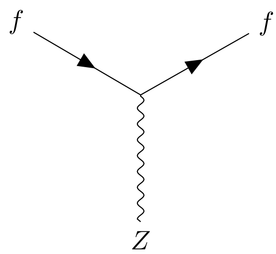
$$\mathcal{Z}(T,V,\mu) = \sum_s \exp\left(-\frac{E(s) - \mu N(s)}{T} \right)$$

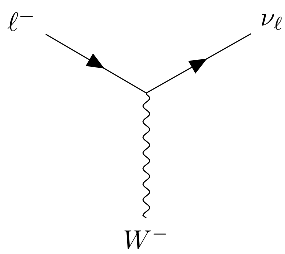
5 Particle Physics

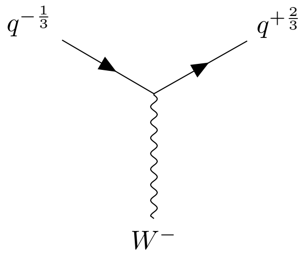
$$s = (p_A + p_B)^2 \tag{5.1}$$



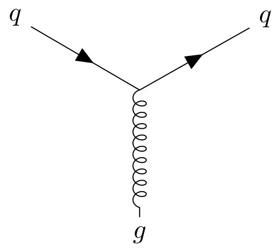
5.1 Weak Force

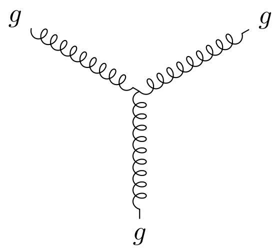


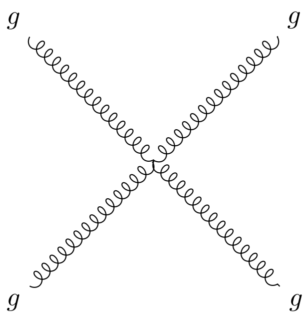




5.2 Strong Force







$$\frac{\mathrm{d}N_{\text{scat}}}{\mathrm{d}t} = \sigma\mathcal{L} \tag{5.2}$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2}|\mathcal{M}|^2 \qquad \text{(two-body decay)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A + E_B)^2}\frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \qquad (2 \to 2 \text{ scatter})$$

6 Solid State Physics