PHY3004W Formula Sheet

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1 Electromagnetism

$$\begin{aligned}\nabla\cdot\boldsymbol{D} =& \rho_f\\ \nabla\cdot\boldsymbol{B} =& 0\\ \nabla\times\boldsymbol{E} =& \frac{\partial\boldsymbol{B}}{\partial t}\\ \nabla\times\boldsymbol{H} =& \boldsymbol{J}_f + \frac{\partial\boldsymbol{D}}{\partial t}\end{aligned}$$

$$\begin{aligned}\boldsymbol{D} =& \epsilon_0\boldsymbol{E} + \boldsymbol{P}\\ \boldsymbol{B} =& \mu_0(\boldsymbol{H} + \boldsymbol{M})\end{aligned}$$

$$\begin{aligned}\oint\boldsymbol{D}\cdot\mathrm{d}\boldsymbol{a} =& Q_{f,enc}\\ \oint\boldsymbol{H}\cdot\mathrm{d}\boldsymbol{l} =& I_{f,enc}(t) + \frac{\mathrm{d}\Phi_{\boldsymbol{D}}}{\mathrm{d}t}\\ \oint\boldsymbol{B}\cdot\mathrm{d}\boldsymbol{a} =& 0\\ \oint\boldsymbol{E}\cdot\mathrm{d}\boldsymbol{l} =& -\frac{\mathrm{d}\Phi_{\boldsymbol{B}}}{\mathrm{d}t}\end{aligned}$$

$$\begin{aligned}D_1^{\perp} - D_2^{\perp} &= \sigma_f\\ B_1^{\perp} - B_2^{\perp} &= 0\\ \boldsymbol{E}_1^{\parallel} - \boldsymbol{E}_2^{\parallel} &= 0\\ \boldsymbol{H}_1^{\parallel} - \boldsymbol{H}_2^{\parallel} &= \boldsymbol{K}_f\times\hat{\boldsymbol{n}}\end{aligned}$$

$$\begin{aligned}\boldsymbol{S} =& \frac{1}{\mu}\boldsymbol{E}\times\boldsymbol{B}\\ u =& \frac{1}{2}\left(\epsilon_0 E^2 + \frac{1}{\mu_0}B^2\right)\\ I =& \langle S\rangle = \frac{1}{2}v\epsilon E_0^2\\ P =& \int\boldsymbol{S}\cdot\mathrm{d}\boldsymbol{a}\end{aligned}$$

1.1 Special Relativity

$$\Lambda = \begin{pmatrix}\gamma & -\beta\gamma & 0 & 0\\ -\beta\gamma & \gamma & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$u' = \frac{u-v}{1-\frac{uv}{c^2}}$$

$$\Lambda^{\mu}_{\alpha} g_{\mu\nu}\Lambda^{\nu}_{\beta} = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2t^2 - d^2$$

$$\begin{aligned}\mathrm{d}\tau &= \frac{\mathrm{d}t}{\gamma} && \text{(proper time)}\\ \eta^\mu &= \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \begin{pmatrix}\gamma c\\ \gamma\boldsymbol{u}\end{pmatrix} && \text{(proper velocity)}\\ \eta^\mu &= m\eta^\mu = \begin{pmatrix}E/c\\ \boldsymbol{p}\end{pmatrix} && \text{(proper velocity)}\end{aligned}$$

$$\begin{aligned}&\eta_\mu\eta^\mu = c^2\\ &p_\mu p^\mu = E^2/c^2 - \boldsymbol{p}^2 = m^2c^2\end{aligned}$$

$$F^{\mu\nu} = \begin{pmatrix}0 & -E_x/c & -E_y/c & -E_z/c\\ E_x/c & 0 & -B_z & B_y\\ E_y/c & B_z & 0 & -B_x\\ E_z/c & -B_y & B_x & 0\end{pmatrix}$$

$$K^\mu = qF^{\mu\nu}g_{\nu\alpha}\eta^\alpha$$

$$J^\mu = \begin{pmatrix}c\rho\\ \boldsymbol{J}\end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi}\int\frac{\nabla'\times\boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}\mathrm{d}\boldsymbol{x}$$

$$\begin{aligned}\boldsymbol{B} &= \nabla\times A\\ \boldsymbol{E} &= -\nabla V - \frac{\partial\boldsymbol{A}}{\partial t}\end{aligned}$$

$$A^\mu_{\mathrm{ret}}(\boldsymbol{x}) = \frac{\mu_0}{4\pi}\int\mathrm{d}^3x'\frac{J'(x^0 - |\boldsymbol{x}-\boldsymbol{x}', \boldsymbol{x}')}{|\boldsymbol{x}-\boldsymbol{x}'|}$$

1.2 Radiation

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon}\Big(\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} + \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3 c|\boldsymbol{r}|}\Big) \qquad \text{(for point charge)}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0}\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^3|\boldsymbol{r}|^2} \qquad \text{(constant velocity)}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c}\frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \boldsymbol{a}))^2}{(1-\boldsymbol{\beta}\cdot\hat{\boldsymbol{r}})^5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c}\sin^2\theta \qquad \text{(non-rel. limit)}$$
$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c}(a^2 - |\boldsymbol{\beta} \times \boldsymbol{a}|^2) \qquad \text{(rel. generalisation)}$$

todo Maybe do dipole stuff?

1.3 Radiation of dipole

$$\boldsymbol{p}(t) = e^{-i\omega t}\int \rho \boldsymbol{x}'\boldsymbol{x}'\mathrm{d}^3x' \qquad \text{(dipole moment)}$$

2 Nuclear Physics

$$\Delta M(Z, A) = M(Z, A) - [Z(M_p + m_e) + NM_n]$$

$$B(A, Z) = -\Delta M(Z, A)c^2 \qquad \text{(Binding energy)}$$

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$
$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lambda N(t)$$
$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{\mathrm{d}N(t)}{\mathrm{d}t}$$
$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x} \qquad \text{(Intensity loss of } \gamma\text{)}$$

$$I(x) = I_0 e^{-N\sigma x} \qquad \text{(Neutron absorption)}$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \qquad \text{(Equivalent Dose)}$$

$$N = In\Delta x\sigma$$
$$I = I_0 e^{-n\sigma t} \qquad \text{(For a thick target)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \qquad \text{(Labeling states } [k = 2j+1]\text{)}$$

$$S, P, D, F, G, \cdots \implies \ell = 0, 1, 2, 3, 4, \ldots$$

$$P = (-1)^\ell \qquad \text{(for single particle state)}$$

3 Atomic Physics

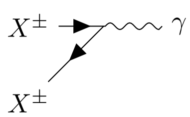
$$[x^i, p^j] = i\hbar\delta^{ij} \tag{3.1}$$

$$[L^i, L^j] = i\hbar\epsilon^{ijk}L^k \tag{3.2}$$

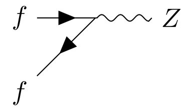
4 Thermal Physics

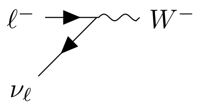
5 Particle Physics

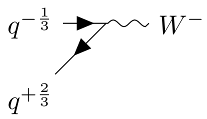
$$s = (p_A + p_B)^2 \tag{5.1}$$



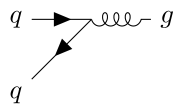
5.1 Weak Force

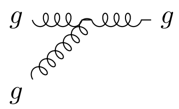


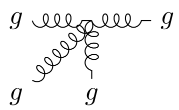




5.2 Strong Force







$$\frac{\mathrm{d}N_{\mathrm{scat}}}{\mathrm{d}t} = \sigma\mathcal{L} \tag{5.2}$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2}|\mathcal{M}|^2 \qquad \text{(two-body decay)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A+E_B)^2}\frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \qquad (2 \to 2 scatter)$$

6 Solid State Physics