PHY3004W Formula Sheet

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1   Electromagnetism

$$\nabla \cdot \boldsymbol{D} = \rho_f$$
$$\nabla \cdot \boldsymbol{B} = 0$$
$$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}$$
$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$\boldsymbol{D} = \epsilon_0 \boldsymbol{E} + \boldsymbol{P}$$
$$\boldsymbol{B} = \mu_0 (\boldsymbol{H} + \boldsymbol{M})$$

$$\oint \boldsymbol{D} \cdot \mathrm{d}\boldsymbol{a} = Q_{f,enc}$$

$$\oint \boldsymbol{H} \cdot \mathrm{d}\boldsymbol{l} = I_{f,enc}(t) + \frac{\mathrm{d}\Phi_D}{\mathrm{d}t}$$

$$\oint \boldsymbol{B} \cdot \mathrm{d}\boldsymbol{a} = 0$$

$$\oint \boldsymbol{E} \cdot \mathrm{d}\boldsymbol{l} = -\frac{\mathrm{d}\Phi_B}{\mathrm{d}t}$$

$$D_1^\perp - D_2^\perp = \sigma_f$$
$$B_1^\perp - B_2^\perp = 0$$
$$\boldsymbol{E}_1^\parallel - \boldsymbol{E}_2^\parallel = 0$$
$$\boldsymbol{H}_1^\parallel - \boldsymbol{H}_2^\parallel = \boldsymbol{K}_f \times \hat{\boldsymbol{n}}$$

$$\boldsymbol{S} = \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B}$$
$$u = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right)$$
$$I = \langle S \rangle = \frac{1}{2} v\epsilon E_0^2$$
$$P = \int \boldsymbol{S} \cdot \mathrm{d}\boldsymbol{a}$$

1.1   Special Relativity

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Lambda_\alpha^\mu g_{\mu\nu} \Lambda_\beta^\nu = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2 t^2 - d^2$$

$$\mathrm{d}\tau = \frac{\mathrm{d}t}{\gamma} \qquad \text{(proper time)}$$
$$\eta^\mu = \frac{\mathrm{d}x^\mu}{\mathrm{d}\tau} = \begin{pmatrix} \gamma c \\ \gamma \boldsymbol{u} \end{pmatrix} \qquad \text{(proper velocity)}$$
$$\eta^\mu = m\eta^\mu = \begin{pmatrix} E/c \\ \boldsymbol{p} \end{pmatrix} \qquad \text{(proper velocity)}$$

$$\eta_\mu \eta^\mu = c^2$$
$$p_\mu p^\mu = E^2/c^2 - \boldsymbol{p}^2 = m^2 c^2$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$K^\mu = q F^{\mu\nu} g_{\nu\alpha} \eta^\alpha$$

$$J^\mu = \begin{pmatrix} c\rho \\ \boldsymbol{J} \end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\boldsymbol{A}(\boldsymbol{x}) = \frac{1}{4\pi} \int \frac{\nabla' \times \boldsymbol{J}(\boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|} \mathrm{d}\boldsymbol{x}$$

$$\boldsymbol{B} = \nabla \times A$$
$$\boldsymbol{E} = -\nabla V - \frac{\partial \boldsymbol{A}}{\partial t}$$

$$A_{\text{ret}}^\mu(\boldsymbol{x}) = \frac{\mu_0}{4\pi} \int \mathrm{d}^3 x' \frac{J'(x^0 - |\boldsymbol{x} - \boldsymbol{x}'|, \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}$$

1.2 Radiation

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon} \Big(\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 |\boldsymbol{r}|^2}$$
$$+ \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 c |\boldsymbol{r}|} \Big) \qquad \text{(for point charge)}$$

$$\boldsymbol{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 |\boldsymbol{r}|^2} \qquad \text{(constant velocity)}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c} \frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \boldsymbol{a}))^2}{(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \sin^2\theta \qquad \text{(non-rel. limit)}$$
$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c}(a^2 - |\boldsymbol{\beta} \times \boldsymbol{a}|^2) \qquad \text{(rel. generalisation)}$$

todo Maybe do dipole stuff?

1.3 Radiation of dipole

$$\boldsymbol{p}(t) = e^{-i\omega t} \int \rho \boldsymbol{x}' \boldsymbol{x}' \mathrm{d}^3 x' \qquad \text{(dipole moment)}$$

2 Nuclear Physics

$$\Delta M(Z, A) = M(Z, A) - [Z(M_p + m_e) + NM_n]$$

$$B(A, Z) = -\Delta M(Z, A)c^2 \qquad \text{(Binding energy)}$$

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$
$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lambda N(t)$$
$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{\mathrm{d}N(t)}{\mathrm{d}t}$$
$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x} \qquad \text{(Intensity loss of } \gamma\text{)}$$

$$I(x) = I_0 e^{-N\sigma x} \qquad \text{(Neutron absorption)}$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \qquad \text{(Equivalent Dose)}$$

$$N = In\Delta x\sigma$$
$$I = I_0 e^{-n\sigma t} \qquad \text{(For a thick target)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \qquad \text{(Labeling states } [k = 2j + 1]\text{)}$$

$$S, P, D, F, G, \cdots \implies \ell = 0, 1, 2, 3, 4, \ldots$$

$$P = (-1)^\ell \qquad \text{(for single particle state)}$$

3 Atomic Physics

$$-i\hbar\frac{\mathrm{d}}{\mathrm{d}t}|\Psi(t)\rangle = \hat{H}|\Psi(t)\rangle \qquad (3.1)$$

$$[x^i, p^j] = i\hbar\delta^{ij} \qquad (3.2)$$

$$[L^i, L^j] = i\hbar\epsilon^{ijk}L^k \qquad (3.3)$$

$$\hat{\boldsymbol{S}} = \frac{\hbar}{2}\boldsymbol{\sigma} \qquad (3.4)$$
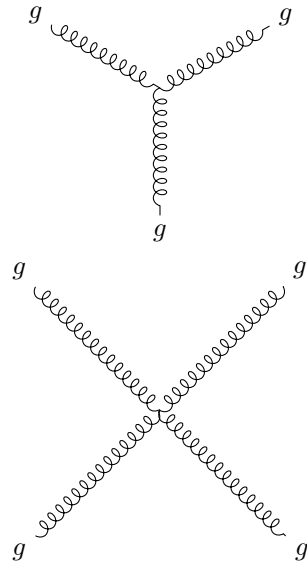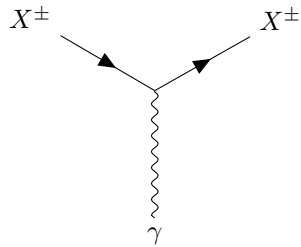
$$[S^i, S^j] = i\hbar\epsilon^{ijk}S^k \qquad (3.5)$$

$$[\boldsymbol{AB}, \boldsymbol{C}] = \boldsymbol{A}[\boldsymbol{B}, \boldsymbol{C}] + [\boldsymbol{A}, \boldsymbol{B}]\boldsymbol{C} \qquad (3.6)$$

4 Thermal Physics

5 Particle Physics

$$s = (p_A + p_B)^2 \qquad (5.1)$$

5.1 Weak Force
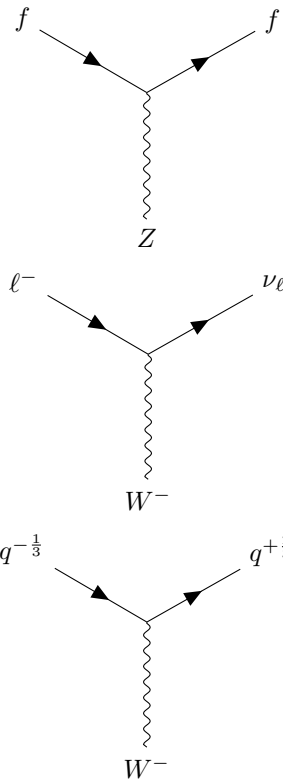
5.2 Strong Force

$$\frac{\mathrm{d}N_{\mathrm{scat}}}{\mathrm{d}t} = \sigma \mathcal{L} \qquad (5.2)$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2} |\mathcal{M}|^2 \qquad (\text{two-body decay})$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A + E_B)^2} \frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \qquad (2 \to 2\,scatter)$$

6 Solid State Physics