

# PHY3004W Formula Sheet

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## 1 Electromagnetism

$$\begin{aligned}\sin \theta &\equiv \theta \\ \tan \theta &\equiv \sin \theta \\ \cos \theta &\equiv 1\end{aligned}$$

$$\begin{aligned}\nabla \cdot \mathbf{D} &= \rho_f \\ \nabla \cdot \mathbf{B} &= 0 \\ \nabla \times \mathbf{E} &= -\frac{\partial \mathbf{B}}{\partial t} \\ \nabla \times \mathbf{H} &= \mathbf{J}_f + \frac{\partial \mathbf{D}}{\partial t}\end{aligned}$$

$$\begin{aligned}\mathbf{D} &= \epsilon_0 \mathbf{E} + \mathbf{P} \\ \mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M})\end{aligned}$$

$$\begin{aligned}\oint \mathbf{D} \cdot d\mathbf{a} &= Q_{f,enc} \\ \oint \mathbf{H} \cdot d\mathbf{l} &= I_{f,enc}(t) + \frac{d\Phi_{\mathbf{D}}}{dt} \\ \oint \mathbf{B} \cdot d\mathbf{a} &= 0 \\ \oint \mathbf{E} \cdot d\mathbf{l} &= -\frac{d\Phi_{\mathbf{B}}}{dt}\end{aligned}$$

$$\begin{aligned}D_1^\perp - D_2^\perp &= \sigma_f \\ B_1^\perp - B_2^\perp &= 0 \\ \mathbf{E}_1^\parallel - \mathbf{E}_2^\parallel &= 0 \\ \mathbf{H}_1^\parallel - \mathbf{H}_2^\parallel &= \mathbf{K}_f \times \hat{\mathbf{n}}\end{aligned}$$

$$\begin{aligned}\mathbf{S} &= \frac{1}{\mu} \mathbf{E} \times \mathbf{B} \\ u &= \frac{1}{2} \left( \epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \\ I &= \langle S \rangle = \frac{1}{2} v \epsilon E_0^2 \\ P &= \int \mathbf{S} \cdot d\mathbf{a} \\ \nabla \cdot \mathbf{J} &= -\frac{\partial \rho}{\partial t}\end{aligned}$$

$$\frac{\sin \theta_I}{\sin \theta_T} = \frac{k_T}{k_I} = \frac{n_2}{n_1}$$

$$\begin{aligned}R &= \frac{I_R}{I_I} \\ T &= \frac{I_T}{I_I} \\ R + T &= 1\end{aligned}$$

### 1.1 Special Relativity

$$\begin{aligned}\beta &= \frac{v}{c} \\ \gamma &= \frac{1}{\sqrt{1 - \beta^2}}\end{aligned}$$

$$\Lambda = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Lambda_\alpha^\mu g_{\mu\nu} \Lambda_\beta^\nu = g_{\alpha\beta}$$

$$I = s^2 = x_\mu x^\mu = c^2 t^2 - d^2$$

$$d\tau = \frac{dt}{\gamma} \quad (\text{proper time})$$

$$\eta^\mu = \frac{dx^\mu}{d\tau} = \begin{pmatrix} \gamma c \\ \gamma \mathbf{u} \end{pmatrix} \quad (\text{proper velocity})$$

$$p^\mu = m\eta^\mu = \begin{pmatrix} \gamma mc \\ \mathbf{p} \end{pmatrix} \quad (4\text{-momentum})$$

$$\begin{aligned}\eta_\mu \eta^\mu &= c^2 \\ p_\mu p^\mu &= E^2/c^2 - \mathbf{p}^2 = m^2 c^2\end{aligned}$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$K^\mu = q F^{\mu\nu} g_{\nu\alpha} \eta^\alpha$$

$$\mathbf{J}^\mu = \begin{pmatrix} c\rho \\ \mathbf{J} \end{pmatrix}$$

$$\partial_\mu J^\mu = 0$$

$$\mathbf{A}(\mathbf{x}) = \frac{1}{4\pi} \int \frac{\nabla' \times \mathbf{J}(\mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|} d\mathbf{x}'$$

$$\hat{\mathbf{S}} = \frac{\hbar}{2} \boldsymbol{\sigma}$$

$$\begin{aligned} \mathbf{B} &= \nabla \times \mathbf{A} \\ \mathbf{E} &= -\nabla V - \frac{\partial \mathbf{A}}{\partial t} \end{aligned}$$

$$\begin{aligned} \sigma_x &= \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \\ \sigma_y &= \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ \sigma_z &= \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{aligned}$$

$$A_{\text{ret}}^\mu(\mathbf{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{J^\mu(x^0 - |\mathbf{x} - \mathbf{x}'|, \mathbf{x}')}{|\mathbf{x} - \mathbf{x}'|}$$

$$[S^i, S^j] = i\hbar \epsilon^{ijk} S^k$$

## 1.2 Radiation

$$\mathbf{a} \times (\mathbf{b} \times \mathbf{c}) = \mathbf{b}(\mathbf{a} \cdot \mathbf{c}) - \mathbf{c}(\mathbf{a} \cdot \mathbf{b}) \quad (1.1)$$

$$\begin{aligned} \mathbf{E} &= \frac{q}{4\pi\epsilon} \left( \frac{\hat{\mathbf{r}} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^3 |\mathbf{r}|^2} \right. \\ &\quad \left. + \frac{\hat{\mathbf{r}} \times ((\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^3 c |\mathbf{r}|} \right) \quad (\text{for point charge}) \end{aligned}$$

$$\mathbf{E} = \frac{q}{4\pi\epsilon_0} \frac{\hat{\mathbf{r}} - \boldsymbol{\beta}}{\gamma^2(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^3 |\mathbf{r}|^2} \quad (\text{constant velocity})$$

$$\frac{dP}{d\Omega} = \frac{\mu_0 q^2}{16\pi c} \frac{(\hat{\mathbf{r}} \times ((\hat{\mathbf{r}} - \boldsymbol{\beta}) \times \mathbf{a}))^2}{(1 - \boldsymbol{\beta} \cdot \hat{\mathbf{r}})^5}$$

$$\begin{aligned} \frac{dP}{d\Omega} &= \frac{\mu_0 q^2 a^2}{16\pi^2 c} \sin^2 \theta \quad (\text{non-rel. limit}) \\ P &= \frac{\mu_0 q^2 a^2}{6\pi c} \end{aligned}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c} (a^2 - |\boldsymbol{\beta} \times \mathbf{a}|^2) \quad (\text{rel. generalisation})$$

## 1.3 Radiation of dipole

$$\mathbf{p}(t) = e^{-i\omega t} \int \rho(\mathbf{x}') \mathbf{x}' d^3x' \quad (\text{dipole moment})$$

$$P = \frac{\mu_0 p_0^2 \omega^4}{12\pi c} \quad (1.2)$$

## 2 Atomic Physics

$$-i\hbar \frac{d}{dt} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle$$

$$[x^i, p^j] = i\hbar \delta^{ij}$$

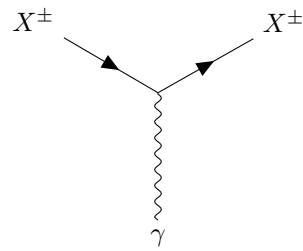
$$[L^i, L^j] = i\hbar \epsilon^{ijk} L^k$$

$$\begin{aligned} [AB, C] &= A[B, C] + [A, B]C \\ [A, BC] &= [A, B]C + B[A, C] \end{aligned}$$

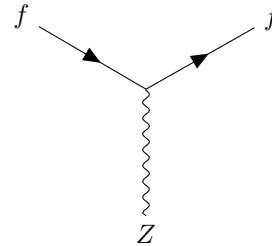
$$\begin{aligned} E_n^{(1)} &= \langle E_n^{(0)} | H_I | E_n^{(0)} \rangle \\ E_n^{(2)} &= \sum_{m \neq n} \langle E_n^{(0)} | H_I | E_m^{(0)} \rangle \frac{\langle E_m^{(0)} | H_I | E_n^{(0)} \rangle}{E_n^{(0)} - E_m^{(0)}} \end{aligned}$$

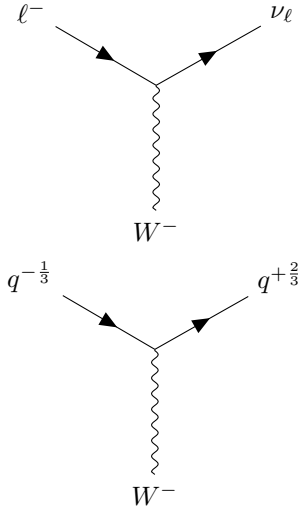
## 3 Particle Physics

$$s = (p_A + p_B)^2 \quad (3.1)$$

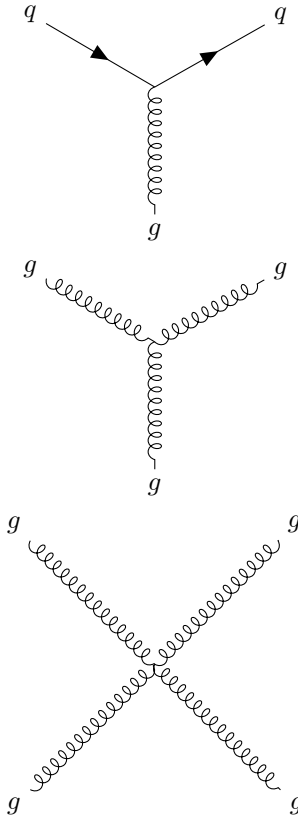


### 3.1 Weak Force





### 3.2 Strong Force



$$\frac{dN_{\text{scat}}}{dt} = \sigma \mathcal{L} \quad (3.2)$$

$$\Gamma = \frac{|\mathbf{p}|}{8\pi m_A^2} |\mathcal{M}|^2 \quad (\text{two-body decay})$$

$$\frac{d\sigma}{d\Omega} = \left( \frac{1}{8\pi} \right)^2 \frac{|\mathcal{M}|^2}{(E_A + E_B)^2} \frac{|\mathbf{p}_f|}{|\mathbf{p}_i|} \quad (2 \rightarrow 2 \text{ scatter})$$

## 4 Nuclear Physics

$$\Delta M(Z, A) = M(Z, A) - [Z(M_p + m_e) + NM_n]$$

$$B(A, Z) = -\Delta M(Z, A)c^2 \quad (\text{Binding energy})$$

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$

$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{dN(t)}{dt} = \lambda N(t)$$

$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{dN(t)}{dt}$$

$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x} \quad (\text{Intensity loss of } \gamma)$$

$$I(x) = I_0 e^{-N\sigma x} \quad (\text{Neutron absorption})$$

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_R W_R D_{\text{tissue},R} \quad (\text{Equivalent Dose})$$

$$N = In\Delta x\sigma$$

$$I = I_0 e^{-n\sigma t} \quad (\text{For a thick target})$$

$$\frac{d\sigma}{d\Sigma}$$

maybe include SEMF?

$$(n\ell_j)^k \quad (\text{Labeling states } [k = 2j + 1])$$

$$S, P, D, F, G, \dots \implies \ell = 0, 1, 2, 3, 4, \dots$$

$$P = (-1)^\ell \quad (\text{for single particle state})$$

## 5 Thermal Physics

$$\begin{aligned} pV &= K_B N T & (\text{Ideal gas law}) \\ pV &= nRT \end{aligned}$$

$$\bar{E}_{\text{kin}} = \frac{1}{2} f N K_B T \quad (\text{Equipartition thm})$$

$$\Delta U = Q + W \quad (\text{1st Law})$$

$$\frac{Q}{\Delta T} = \alpha A \frac{dT}{dx}$$

$$\ln(A!) \approx A \ln(A) - A \quad (\text{Stirling approx.})$$

$$\Omega(N, q) = \frac{(q + N - 1)!}{q!(N - 1)!} \quad (\text{Einstein solid})$$

$$\Omega(N, N_{\uparrow}) = \frac{N!}{N_{\uparrow}!(N - N_{\uparrow})!} \quad (\text{paramagnet})$$

$$S = K_B \ln \Omega \quad (\text{entropy})$$

$$S(U, V, N) = K_B N \left[ \ln \left( \frac{V}{N} \left( \frac{4\pi m U}{3\hbar^2 N} \right)^{\frac{3}{2}} \right) + \frac{5}{2} \right] \quad (\text{Sackur-Tetrode})$$

$$\frac{1}{T} \equiv \left[ \frac{\partial S}{\partial U} \right]_{N, V}$$

$$p \equiv T \left[ \frac{\partial S}{\partial U} \right]_{U, N}$$

$$\eta \equiv \frac{W}{Q_H} \quad (\text{efficiency})$$

$$H \equiv U + pV$$

$$F \equiv U - TS$$

$$G \equiv H - TS$$

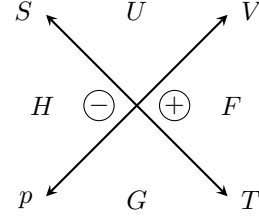
$$dU = TdS - pdV + \mu dN \quad (5.1)$$

$$dF = -SdT - pdV + \mu dN \quad (5.2)$$

$$dG = -SdT + Vdp + \mu dN \quad (5.3)$$

$$dH = TdS + Vdp + \mu dN \quad (5.4)$$

$$\frac{dp}{dT} = \frac{L/T}{\Delta V} \quad (\text{Clausius-Clapeyron relation})$$



$$\left( p + a \frac{N^2}{V^2} \right) (V - bN) = K_B N T \quad (\text{Van der Waals eqn})$$

$$P(S) = \frac{1}{z} \exp \left( -\frac{E(s)}{T} \right)$$

$$z = \sum_s d(s) \exp \left( -\frac{E(s)}{T} \right)$$

$$\langle E \rangle = -\frac{\partial}{\partial \beta} \ln z$$

$$F = -K_B T \ln z$$

$$p(s) = \frac{1}{Z} \sum_s \exp \left( -\frac{E(s) - \mu N(s)}{T} \right)$$

$$\mathcal{Z}(T, V, \mu) = \sum_s \exp \left( -\frac{E(s) - \mu N(s)}{T} \right)$$

## 6 Solid State Physics

$$\mathbf{R} = [n_1, n_2, n_3] = n_1 \mathbf{a}_1 + n_2 \mathbf{a}_2 + n_3 \mathbf{a}_3 \quad (6.1)$$

$$\mathbf{G} = h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3 \quad (6.2)$$

$$d = \frac{2\pi}{|\mathbf{G}_{\min}|} \quad (6.3)$$

$$\mathbf{b}_i \cdot \mathbf{a}_j = 2\pi \delta_{ij} \quad (6.4)$$

$$(hkl) \rightarrow_{\perp} h\mathbf{b}_1 + k\mathbf{b}_2 + l\mathbf{b}_3 \quad (6.5)$$

$$2d \sin(\theta) = n\lambda \quad (\text{Bragg condition})$$

$$f(E, T) = \frac{1}{\exp(\frac{E - \mu}{K_B T}) + 1} \quad (\text{Fermi-Dirac})$$

$$E_f = \frac{\hbar^2}{2m} (3\pi^2 n)^{2/3} \quad (\text{for free electron gas})$$