PHY3004W Formula Sheet

Alex Veltman November 2020

1 Electromagnetism

$$\sin \theta \equiv \theta$$
$$\tan \theta \equiv \sin \theta$$
$$\cos \theta \equiv 1$$

$$\nabla \cdot \boldsymbol{D} = \rho_f$$

$$\nabla \cdot \boldsymbol{B} = 0$$

$$\nabla \times \boldsymbol{E} = \frac{\partial \boldsymbol{B}}{\partial t}$$

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}_f + \frac{\partial \boldsymbol{D}}{\partial t}$$

$$oldsymbol{D} = \epsilon_0 oldsymbol{E} + oldsymbol{P} \ oldsymbol{B} = \mu_0 (oldsymbol{H} + oldsymbol{M})$$

$$\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f,enc}$$

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f,enc}(t) + \frac{d\Phi_{\mathbf{D}}}{dt}$$

$$\oint \mathbf{B} \cdot d\mathbf{a} = 0$$

$$\oint \mathbf{E} \cdot d\mathbf{l} = -\frac{d\Phi_{\mathbf{B}}}{dt}$$

$$egin{aligned} D_1^\perp - D_2^\perp &= \sigma_f \ B_1^\perp - B_2^\perp &= 0 \ oldsymbol{E}_1^\parallel - oldsymbol{E}_2^\parallel &= 0 \ oldsymbol{H}_1^\parallel - oldsymbol{H}_2^\parallel &= oldsymbol{K}_f imes \hat{oldsymbol{n}} \end{aligned}$$

$$\begin{split} \boldsymbol{S} &= \frac{1}{\mu} \boldsymbol{E} \times \boldsymbol{B} \\ \boldsymbol{u} &= \frac{1}{2} \left(\epsilon_0 E^2 + \frac{1}{\mu_0} B^2 \right) \\ \boldsymbol{I} &= \langle S \rangle = \frac{1}{2} v \epsilon E_0^2 \\ \boldsymbol{P} &= \int \boldsymbol{S} \cdot \mathrm{d} \boldsymbol{a} \end{split}$$

1.1 Special Relativity

$$\beta = \frac{v}{c}$$

$$\gamma = \frac{1}{\sqrt{1 - \beta^2}}$$

$$\Lambda = \begin{pmatrix} \gamma & -\beta \gamma & 0 & 0 \\ -\beta \gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$u' = \frac{u - v}{1 - \frac{uv}{c^2}}$$

$$\Lambda^{\mu}_{\alpha} g_{\mu\nu} \Lambda^{\nu}_{\beta} = g_{\alpha\beta}$$

$$I = s^2 = x_{\mu} x^{\mu} = c^2 t^2 - d^2$$

$$d\tau = \frac{dt}{\gamma} \qquad \text{(proper time)}$$

$$\eta^{\mu} = \frac{dx^{\mu}}{d\tau} = \begin{pmatrix} \gamma c \\ \gamma u \end{pmatrix} \qquad \text{(proper velocity)}$$

$$\eta^{\mu} = m \eta^{\mu} = \begin{pmatrix} E/c \\ p \end{pmatrix} \qquad \text{(proper velocity)}$$

$$\eta_{\mu} \eta^{\mu} = c^2$$

$$p_{\mu} p^{\mu} = E^2/c^2 - p^2 = m^2 c^2$$

$$F^{\mu\nu} = \begin{pmatrix} 0 & -E_x/c & -E_y/c & -E_z/c \\ E_x/c & 0 & -B_z & B_y \\ E_y/c & B_z & 0 & -B_x \\ E_z/c & -B_y & B_x & 0 \end{pmatrix}$$

$$K^{\mu} = q F^{\mu\nu} g_{\nu\alpha} \eta^{\alpha}$$

$$J^{\mu} = \begin{pmatrix} c\rho \\ J \end{pmatrix}$$

$$\partial_{\mu} J^{\mu} = 0$$

$$A(x) = \frac{1}{4\pi} \int \frac{\nabla' \times J(x')}{|x - x'|} dx$$

$$B = \nabla \times A$$

$$E = -\nabla V - \frac{\partial A}{\partial \mu}$$

 $A_{\text{ret}}^{\mu}(\boldsymbol{x}) = \frac{\mu_0}{4\pi} \int d^3x' \frac{J'(x^0 - |\boldsymbol{x} - \boldsymbol{x}', \boldsymbol{x}')}{|\boldsymbol{x} - \boldsymbol{x}'|}$

1.2 Radiation

$$\begin{split} \boldsymbol{E} &= \frac{q}{4\pi\epsilon} \Big(\frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2 (1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 |\boldsymbol{r}|^2} \\ &+ \frac{\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \dot{\boldsymbol{\beta}})}{(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 c |\boldsymbol{r}|} \Big) \qquad \text{(for point charge)} \end{split}$$

$$E = \frac{q}{4\pi\epsilon_0} \frac{\hat{\boldsymbol{r}} - \boldsymbol{\beta}}{\gamma^2 (1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^3 |\boldsymbol{r}|^2} \quad \text{(constant velocity)}$$
$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2}{16\pi c} \frac{(\hat{\boldsymbol{r}} \times ((\hat{\boldsymbol{r}} - \boldsymbol{\beta}) \times \boldsymbol{a}))^2}{(1 - \boldsymbol{\beta} \cdot \hat{\boldsymbol{r}})^5}$$

$$\frac{\mathrm{d}P}{\mathrm{d}\Omega} = \frac{\mu_0 q^2 a^2}{16\pi^2 c} \sin^2\theta \qquad \qquad \text{(non-rel. limit)}$$

$$P = \frac{\mu_0 q^2 a^2}{6\pi c}$$

$$P = \frac{\mu_0 q^2 \gamma^2}{6\pi c} (a^2 - |\boldsymbol{\beta} \times \boldsymbol{a}|^2) \quad \text{(rel. generalisation)}$$

todo Maybe do dipole stuff?

1.3 Radiation of dipole

$$p(t) = e^{-i\omega t} \int \rho x' x' d^3 x'$$
 (dipole moment)

2 Nuclear Physics

$$\Delta M(Z,A) = M(Z,A) - [Z(M_p + m_e) + NM_n]$$

$$B(A, Z) = -\Delta M(Z, A)c^2$$
 (Binding energy)

$$Q = [(M_i + M_I) - (M_f + M_F)]c^2$$

$$Q = B(f) + B(F) - B(i) + B(I)$$

$$T_{1/2} = \frac{\ln 2}{\lambda}$$

$$-\frac{\mathrm{d}N(t)}{\mathrm{d}t} = \lambda N(t)$$
$$N(t) = N(0)e^{-\lambda N(t)}$$

$$R(t) = -\frac{\mathrm{d}N(t)}{\mathrm{d}t}$$
$$R(t) = \lambda N(0)e^{-\lambda N(t)}$$

$$I(x) = I_0 e^{-\mu x}$$
 (Intensity loss of γ)

$$I(x) = I_0 e^{-N\sigma x}$$
 (Neutron absorption)

$$D_{\text{tissue}} = \frac{E_{\text{deposited}}}{m_{\text{tissue}}}$$

$$H_{\text{tissue}} = \sum_{R} W_R D_{\text{tissue},R}$$
 (Equivalent Dose)

$$\begin{split} N &= In\Delta x \sigma \\ I &= I_0 e^{-n\sigma t} \end{split} \tag{For a thick target)} \end{split}$$

 $\frac{\mathrm{d}\sigma}{\mathrm{d}\Sigma}$

maybe include SEMF?

$$(n\ell_j)^k$$
 (Labeling states $[k=2j+1]$)

$$S, P, D, F, G, \dots \implies \ell = 0, 1, 2, 3, 4, \dots$$

$$P = (-1)^{\ell}$$
 (for single particle state)

3 Atomic Physics

$$-i\hbar \frac{\mathrm{d}}{\mathrm{d}t} |\Psi(t)\rangle = \hat{H} |\Psi(t)\rangle \tag{3.1}$$

$$[x^i, p^j] = i\hbar \delta^{ij} \tag{3.2}$$

$$[L^i, L^j] = i\hbar \epsilon^{ijk} L^k \tag{3.3}$$

$$\hat{\mathbf{S}} = \frac{\hbar}{2}\boldsymbol{\sigma} \tag{3.4}$$

$$[S^i, S^j] = i\hbar \epsilon^{ijk} S^k \tag{3.5}$$

$$[AB, C] = A[B, C] + [A, B]C$$
 (3.6)

4 Thermal Physics

$$pV = K_B NT$$
 (Ideal gas law)
 $pV = nRT$

$$\bar{E_{\rm kin}} = \frac{1}{2} f N K_B T$$
 (Equipartition thm)
$$\Delta U = Q + W \qquad (1 {\rm st~Law})$$

$$\frac{Q}{\Delta T} = \alpha A \frac{{\rm d}T}{{\rm d}r}$$

$$ln(A!) = A ln(A) - A$$
 (Stirling approx.)

$$S = K_B \ln \Omega$$
 (entropy)

$$S(U, V, N) = K_B N \left[\ln \left(\frac{V}{N} \left(\frac{4\pi mU}{3\hbar^2 N} \right)^{\frac{3}{2}} \right) \right]$$
(Sackur-Tetrode)
$$\frac{1}{T} \equiv \left[\frac{\partial S}{\partial U} \right]_{N, V}$$

$$p \equiv T \left[\frac{\partial S}{\partial U} \right]_{U, N}$$

$$\eta \equiv \frac{W}{Q_H}$$
 (efficiency)

$$H \equiv U + pV$$
$$F \equiv U - TS$$
$$G \equiv H - TS$$

$$\frac{\mathrm{d}p}{\mathrm{d}T}\frac{L/T}{\Delta V} \tag{Clausius-Clapeyron relation}$$

$$\left(p + a\frac{N^2}{V^2}\right)(V - bN) = K_B NT$$
(Van der Waals eqn)

$$P(S) = \frac{1}{z} \exp\left(-\frac{E(s)}{T}\right)$$

$$z = \sum_{s} d(s) \exp\left(-\frac{E(s)}{T}\right)$$

$$\langle E \rangle = -\frac{\partial \beta}{\partial \ln} z$$

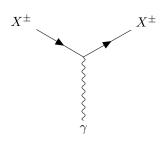
$$F = -T \ln z$$

$$p(s) = \frac{1}{Z} \sum_{s} \exp\left(-\frac{E(s) - \mu N(s)}{T}\right)$$

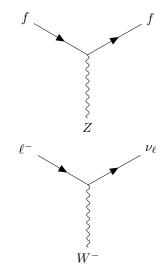
$$\mathcal{Z}(T, V, \mu) = \sum_{s} \exp\left(-\frac{E(s) - \mu N(s)}{T}\right)$$

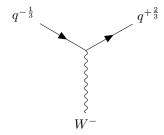
5 Particle Physics

$$s = (p_A + p_B)^2 (5.1)$$

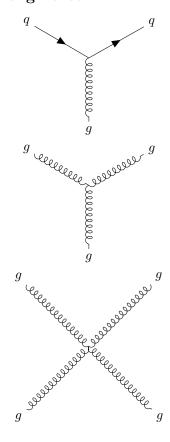


5.1 Weak Force





5.2 Strong Force



$$\frac{\mathrm{d}N_{\mathrm{scat}}}{\mathrm{d}t} = \sigma\mathcal{L} \tag{5.2}$$

$$\Gamma = \frac{|\boldsymbol{p}|}{8\pi m_A^2} |\mathcal{M}|^2 \qquad \qquad \text{(two-body decay)}$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = \left(\frac{1}{8\pi}\right)^2 \frac{|\mathcal{M}|^2}{(E_A + E_B)^2} \frac{|\boldsymbol{p}_f|}{|\boldsymbol{p}_i|} \quad (2 \to 2 \text{ scatter})$$

6 Solid State Physics