$$\Pi\,|\mathbf{r}\rangle = |-\mathbf{r}\rangle$$

1 Angular Momentum

$$\mathbf{L} = -i\hbar(\mathbf{r}\times\boldsymbol{\nabla})$$

$$\mathbf{J} = \mathbf{J} + \mathbf{S}$$

$$[J_i, J_j] = i\hbar\epsilon_{ijk}J_j$$

$$\left[J_i, J^2\right] = 0$$

$$L^2\,|l\,m\rangle = \hbar^2 l(l+1)\,|l\,m\rangle$$

$$L_z\,|l\,m\rangle = \hbar^2 m\,|l\,m\rangle$$

Clebsch-Gordan coefficients:

$$|s\,m\rangle = \sum_m C^{s_1 s_2 s}_{m_1 m_2 m}\,|s_1\,m_1\rangle\,|s_2\,m_2\rangle$$

Ritz Method:

$$\bar{H}[\tilde{0}](\lambda_i) = \frac{\langle\tilde{0}|H|\tilde{0}\rangle}{\langle\tilde{0}|\tilde{0}\rangle} \geq 0$$
$$\frac{\partial\bar{H}}{\partial\lambda_i} = 0$$

Hydrogen atom:

$$\psi_{n\ell m}(r,\theta,\varphi) = \sqrt{\left(\frac{2}{na_0^*}\right)^3\frac{(n-\ell-1)!}{2n(n+\ell)!}}e^{-\rho/2}\rho^\ell L^{2\ell+1}_{n-\ell-1}(\rho)Y^m_\ell(\theta,\varphi)$$
$$\rho = \frac{2r}{na_0}$$
$$\psi_{100}(r) = \frac{1}{\sqrt{\pi}a_0^{3/2}}e^{-r/a_0}$$
$$\psi_{2,0,0} = \frac{1}{4\sqrt{2\pi}a_0^{3/2}}\left(2-\frac{r}{a_0}\right)e^{-r/2a_0}$$
$$\psi_{2,1,0} = \frac{1}{4\sqrt{2\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\cos\theta$$
$$\psi_{2,1,\pm1} = \mp\frac{1}{8\sqrt{\pi}a_0^{3/2}}\frac{r}{a_0}e^{-r/2a_0}\sin\theta\;e^{\pm i\varphi}$$

2 Perturbation Theory

$$H = H^{(0)} + H^{(1)}$$

2.1 Time Independent

2.1.1 Non-degenerate

$$E^{(1)}_n = \left\langle\psi^{(0)}_n\middle|H^{(1)}\middle|\psi^{(0)}_n\right\rangle$$

$$\left|\psi^{(1)}_n\right\rangle = \sum_{m\neq n}\frac{\left\langle\psi^{(0)}_m\middle|H^{(1)}\middle|\psi^{(0)}_m\right\rangle}{E^{(0)}_n - E^{(0)}_m}\left|\psi^{(0)}_m\right\rangle$$

$$E^{(2)}_n = \sum_{m\neq n}\frac{\left|\left\langle\psi^{(0)}_m\middle|H^{(1)}\middle|\psi^{(0)}_n\right\rangle\right|^2}{E^{(0)}_n - E^{(0)}_m}$$

2.1.2 Degenerate

$$\hat{H}_{p\alpha\beta} = \left\langle\psi^{(0)}_{n\,\alpha}\middle|H^{(1)}\middle|\psi^{(0)}_{n\,\beta}\right\rangle$$

$$\det(\hat{H}_p - E^{(1)}\mathbb{I}) = 0$$

$$E_{n_a} = E^{(0)}_n + E^{(1)}_{n_\alpha} \qquad (\alpha = 1,2,\ldots,f)$$
$$|psi\rangle$$

3 Interaction Picture

$$|\alpha(t)\rangle_I = e^{\frac{i}{\hbar}H_0 t}\,|\alpha(t)\rangle_S$$

$$A_I = e^{\frac{i}{\hbar}H_0 t}A_S e^{-\frac{i}{\hbar}H_0 t}$$

3.1 Time Dependent

$$P_{f\leftarrow i}(t) = |\mathcal{A}_{f\leftarrow i}|^2 = |\langle f|U_I|i\rangle|^2 = \left|c^{(1)}_n(t) + c^{(2)}_n(t) + \ldots\right|^2$$

$$c^{(0)}_n = \delta_{ni}$$
$$c^{(1)}_n = \frac{-i}{\hbar}\int_{t_0}^{t} dt'\,\langle n|V_I(t')|i\rangle$$
$$= \frac{-i}{\hbar}\int_{t_0}^{t} dt'\,e^{i\omega_{ni}t'}V_{ni}(t')$$

Fermi Golden Rule for constant potential:

$$R_{f\to i} = \frac{2\pi}{\hbar}|\langle f|V|i\rangle|^2\delta(E_f - E_i)$$

Fermi Golden Rule for Harmonic Pertubation:

$$V(t) = ve^{i\omega t} + v^\dagger e^{-\omega t}$$

$$R_{f\to i} = \frac{2\pi}{\hbar}|\langle f|V|i\rangle|^2\delta(E_f - E_i + \hbar\omega) + \frac{2\pi}{\hbar}|\langle f|V|i\rangle|^2\delta(E_f - E_i - \hbar\omega)$$

4 Scattering Theory

Lippemann-Schwinger equation:

$$\left|\psi^\pm\right\rangle = |\phi\rangle + \frac{1}{E - H_0 \pm i\epsilon}\left|\psi^\pm\right\rangle$$

$$\psi^\pm(x) = \phi(x) + \int dx'\,\langle x|\frac{1}{E - H_0 \pm i\epsilon}|x'\rangle\,\left\langle x'\middle|V\middle|\psi^\pm\right\rangle$$

$$\psi^\pm(x) = \phi(x) - \frac{2m}{\hbar^2}\int d^3x'\,\frac{e^{\pm ik|\mathbf{x}-\mathbf{x}'|}}{4\pi|\mathbf{x}-\mathbf{x}'|}\left\langle x'\middle|V\middle|\psi^\pm\right\rangle$$

for local potential V.

$$\psi^+(\mathbf{x}) = \frac{1}{(2\pi)^{3/2}}\left(e^{i\mathbf{k}\cdot\mathbf{x}} + \frac{e^{i\mathbf{k}\cdot\mathbf{x}}}{\mathbf{r}}f(\mathbf{k}',\mathbf{k})\right)$$

$$f(\mathbf{k}',\mathbf{k}) = -\frac{1}{4\pi}(2\pi)^3\frac{2m}{\hbar^2}\langle\mathbf{k}'|V|\psi^{(+)}\rangle$$

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\Omega} = |f(\mathbf{k}',\mathbf{k})|^2$$

Born Approximation:

$$f^{(1)}(\mathbf{k}',\mathbf{k}) = -\frac{1}{4\pi}\frac{2m}{\hbar^2}\int d^3x' e^{i(\mathbf{k}-\mathbf{k}')\cdot\mathbf{x}'}V(\mathbf{x}')$$

For Spherical Potential $V(\mathbf{r})$,

$$f^{(1)}(\theta) = -\frac{2m}{\hbar^2}\frac{1}{q}\int_0^\infty dr\, rV(r)\sin(qr)$$

where

$$q = |\mathbf{k} - \mathbf{k}'| = 2k\sin(\theta/2)$$

Optical Theorem:

$$\sigma_{\text{total}} = \frac{4\pi}{k}\,\text{Im}\, f(\theta = 0)$$

where $f(\theta = 0) = f(\mathbf{k}', \mathbf{k})$.
Rayleigh Formula:

$$e^{ikz} = \sum_{l=0}^{\infty} i^l (2l+1) j_l(kr) P_l(\cos\theta)$$

where $j_l(kr)$ is a spherical bessel function.
Partial Waves:

$$\phi(r,\theta) = A\left(e^{ikz} + k\sum_{l=0}^{\infty} i^{l+1}(2l+1)a_l h_l^{(1)}(kr) P_l(cos\theta)\right)$$