ACT Math Facts & Formulas

Numbers, Sequences, Factors

Integers:	..., -3, -2, -1, 0, 1, 2, 3, ...
Rationals:	fractions, that is, anything expressable as a ratio of integers
Reals:	integers plus rationals plus special numbers such as $\sqrt{2}$, $\sqrt{3}$ and π

Order Of Operations: PEMDAS
(Parentheses / Exponents / Multiply / Divide / Add / Subtract)

Arithmetic Sequences: each term is equal to the previous term plus d

Sequence: t_1, $t_1 + d$, $t_1 + 2d$, ...

Example: $d = 4$ and $t_1 = 3$ gives the sequence 3, 7, 11, 15, ...

Geometric Sequences: each term is equal to the previous term times r

Sequence: t_1, $t_1 \cdot r$, $t_1 \cdot r^2$, ...

Example: $r = 2$ and $t_1 = 3$ gives the sequence 3, 6, 12, 24, ...

Factors: the factors of a number divide into that number without a remainder

Example: the factors of 52 are 1, 2, 4, 13, 26, and 52

Multiples: the multiples of a number are divisible by that number without a remainder

Example: the positive multiples of 20 are 20, 40, 60, 80, ...

Percents: use the following formula to find part, whole, or percent

$$\text{part} = \frac{\text{percent}}{100} \times \text{whole}$$

Example: 75% of 300 is what?
Solve $x = (75/100) \times 300$ to get 225

Example: 45 is what percent of 60?
Solve $45 = (x/100) \times 60$ to get 75%

Example: 30 is 20% of what?
Solve $30 = (20/100) \times x$ to get 150

ACT Math Facts & Formulas

Averages, Counting, Statistics, Probability

$$\text{average} = \frac{\text{sum of terms}}{\text{number of terms}}$$

$$\text{average speed} = \frac{\text{total distance}}{\text{total time}}$$

$$\text{sum} = \text{average} \cdot (\text{number of terms})$$

$$\text{mode} = \text{value in the list that appears most often}$$

$$\text{median} = \text{middle value in the list}$$

median of $\{3, 9, 10, 27, 50\} = 10$
median of $\{3, 9, 10, 27\} = (9 + 10)/2 = 9.5$

Fundamental Counting Principle:

If an event can happen in N ways, and another, independent event can happen in M ways, then both events together can happen in $N \times M$ ways. (Extend this for three or more: $N_1 \times N_2 \times N_3 \ldots$)

Probability (Optional):

$$\text{probability} = \frac{\text{number of desired outcomes}}{\text{number of total outcomes}}$$

Example: each ACT math multiple choice question has five possible answers, one of which is the correct answer. If you guess the answer to a question completely at random, your probability of getting it right is $1/5 = 20\%$.

The probability of two different events A and B *both* happening is $P(A \text{ and } B) = P(A) \cdot P(B)$, as long as the events are independent (not mutually exclusive).

Powers, Exponents, Roots

$$x^a \cdot x^b = x^{a+b} \qquad x^a/x^b = x^{a-b} \qquad 1/x^b = x^{-b}$$

$$(x^a)^b = x^{a \cdot b} \qquad (xy)^a = x^a \cdot y^a \qquad (-1)^n = \begin{cases} +1, & \text{if } n \text{ is even;} \\ -1, & \text{if } n \text{ is odd.} \end{cases}$$

$$x^0 = 1 \qquad \sqrt{xy} = \sqrt{x} \cdot \sqrt{y}$$

Factoring, Solving

$$(x+a)(x+b) = x^2 + (b+a)x + ab \qquad \text{“FOIL”}$$

$$a^2 - b^2 = (a+b)(a-b) \qquad \text{“Difference Of Squares”}$$

$$a^2 + 2ab + b^2 = (a+b)(a+b)$$
$$a^2 - 2ab + b^2 = (a-b)(a-b)$$

$$x^2 + (b+a)x + ab = (x+a)(x+b) \qquad \text{“Reverse FOIL”}$$

You can use Reverse FOIL to factor a polynomial by thinking about two numbers a and b which add to the number in front of the x, and which multiply to give the constant. For example, to factor $x^2 + 5x + 6$, the numbers add to 5 and multiply to 6, i.e., $a = 2$ and $b = 3$, so that $x^2 + 5x + 6 = (x + 2)(x + 3)$.

To solve a quadratic such as $x^2+bx+c = 0$, first factor the left side to get $(x+a)(x+b) = 0$, then set each part in parentheses equal to zero. E.g., $x^2 + 4x + 3 = (x + 3)(x + 1) = 0$ so that $x = -3$ or $x = -1$.

To solve two linear equations in x and y: use the first equation to substitute for a variable in the second. E.g., suppose $x + y = 3$ and $4x - y = 2$. The first equation gives $y = 3 - x$, so the second equation becomes $4x - (3 - x) = 2 \Rightarrow 5x - 3 = 2 \Rightarrow x = 1, y = 2$.

Solving two linear equations in x and y is geometrically the same as finding where two lines intersect. In the example above, the lines intersect at the point $(1, 2)$. Two parallel lines will have no solution, and two overlapping lines will have an infinite number of solutions.

Functions

A function is a rule to go from one number (x) to another number (y), usually written

$$y = f(x).$$

The set of possible values of x is called the *domain* of $f()$, and the corresponding set of possible values of y is called the *range* of $f()$. For any given value of x, there can only be one corresponding value y.

Absolute value:

$$|x| = \begin{cases} +x, & \text{if } x \geq 0; \\ -x, & \text{if } x < 0. \end{cases}$$

Logarithms (Optional):

Logarithms are basically the inverse functions of exponentials. The function $\log_b x$ answers the question: b to what power gives x? Here, b is called the logarithmic "base". So, if $y = \log_b x$, then the logarithm function gives the number y such that $b^y = x$. For example, $\log_3 \sqrt{27} = \log_3 \sqrt{3^3} = \log_3 3^{3/2} = 3/2 = 1.5$. Similarly, $\log_b b^n = n$.

A useful rule to know is: $\log_b xy = \log_b x + \log_b y$.

Complex Numbers

A complex number is of the form $a + bi$ where $i^2 = -1$. When multiplying complex numbers, treat i just like any other variable (letter), except remember to replace powers of i with -1 or 1 as follows (the pattern repeats after the first four):

$$\begin{array}{llll} i^0 = 1 & i^1 = i & i^2 = -1 & i^3 = -i \\ i^4 = 1 & i^5 = i & i^6 = -1 & i^7 = -i \end{array}$$

For example, using "FOIL" and $i^2 = -1$: $(1 + 3i)(5 - 2i) = 5 - 2i + 15i - 6i^2 = 11 + 13i$.

Lines (Linear Functions)

Consider the line that goes through points $A(x_1, y_1)$ and $B(x_2, y_2)$.

Distance from A to B: $\sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Mid-point of the segment $\overline{AB}$: $\left(\dfrac{x_1 + x_2}{2}, \dfrac{y_1 + y_2}{2}\right)$

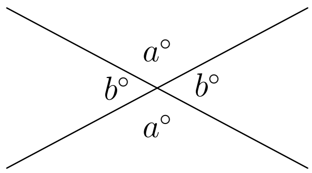
Slope of the line: $\dfrac{y_2 - y_1}{x_2 - x_1} = \dfrac{\text{rise}}{\text{run}}$

Point-slope form: given the slope m and a point (x_1, y_1) on the line, the equation of the line is $(y - y_1) = m(x - x_1)$.

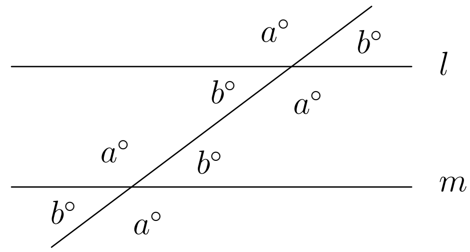
Slope-intercept form: given the slope m and the y-intercept b, then the equation of the line is $y = mx + b$.

To find the equation of the line given two points $A(x_1, y_1)$ and $B(x_2, y_2)$, calculate the slope $m = (y_2 - y_1)/(x_2 - x_1)$ and use the point-slope form.

Parallel lines have equal slopes. Perpendicular lines (i.e., those that make a 90° angle where they intersect) have negative reciprocal slopes: $m_1 \cdot m_2 = -1$.



Intersecting Lines



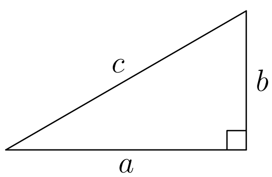
Parallel Lines ($l \parallel m$)

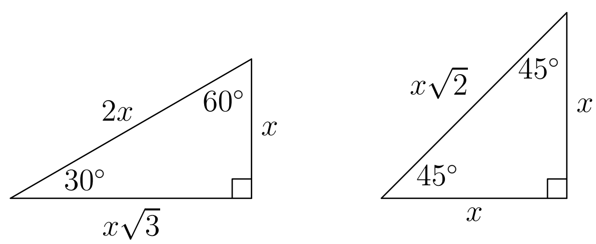
Intersecting lines: opposite angles are equal. Also, each pair of angles along the same line add to 180°. In the figure above, $a + b = 180°$.

Parallel lines: eight angles are formed when a line crosses two parallel lines. The four big angles (a) are equal, and the four small angles (b) are equal.

Triangles

Right triangles:



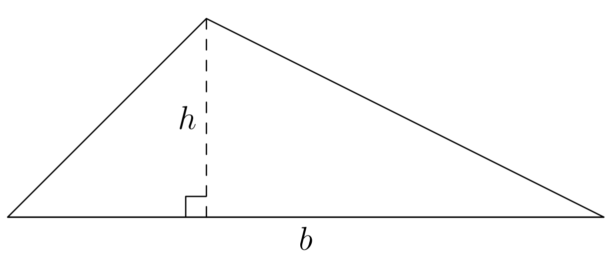


$a^2 + b^2 = c^2$

Special Right Triangles

A good example of a right triangle is one with $a = 3$, $b = 4$, and $c = 5$, also called a 3–4–5 right triangle. Note that multiples of these numbers are also right triangles. For example, if you multiply these numbers by 2, you get $a = 6$, $b = 8$, and $c = 10$ (6–8–10), which is also a right triangle.

All triangles:



$$\text{Area} = \frac{1}{2} \cdot b \cdot h$$

Angles on the inside of any triangle add up to 180°.

The length of one side of any triangle is always *less* than the sum and *more* than the difference of the lengths of the other two sides.

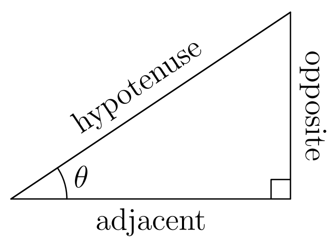
An exterior angle of any triangle is equal to the sum of the two remote interior angles.

Other important triangles:

Equilateral:	These triangles have three equal sides, and all three angles are 60°.
Isosceles:	An isosceles triangle has two equal sides. The "base" angles (the ones opposite the two sides) are equal (see the 45° triangle above).
Similar:	Two or more triangles are similar if they have the same shape. The corresponding angles are equal, and the corresponding sides are in proportion. For example, the 3–4–5 triangle and the 6–8–10 triangle from before are similar since their sides are in a ratio of 2 to 1.

Trigonometry

Referring to the figure below, there are three important functions which are defined for angles in a right triangle:



$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

"SOH" "CAH" "TOA"

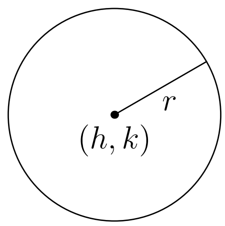
(the last line above shows a mnemonic to remember these functions: "SOH-CAH-TOA")

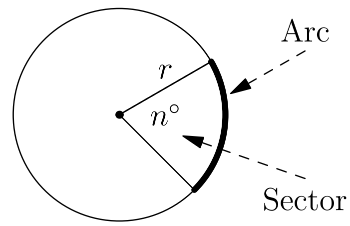
Optional: A useful relationship to remember which works for any angle θ is:

$$\sin^2\theta + \cos^2\theta = 1.$$

For example, if $\theta = 30°$, then (refer to the Special Right Triangles figure) we have $\sin 30° = 1/2$, $\cos 30° = \sqrt{3}/2$, so that $\sin^2 30° + \cos^2 30° = 1/4 + 3/4 = 1$.

Circles





$$\text{Area} = \pi r^2$$
$$\text{Circumference} = 2\pi r$$
$$\text{Full circle} = 360°$$

$$\text{Length Of Arc} = (n°/360°) \cdot 2\pi r$$
$$\text{Area Of Sector} = (n°/360°) \cdot \pi r^2$$

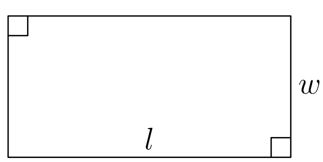
Equation of the circle (above left figure): $(x-h)^2 + (y-k)^2 = r^2.$

Another way to measure angles is with radians. These are defined such that π radians is equal to 180°, so that the number of radians in a circle is 2π (or 360°).

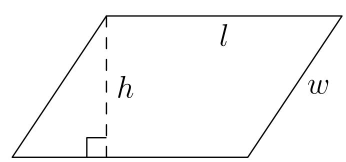
To convert from degrees to radians, just multiply by $\pi/180°$. For example, the number of radians in 45° is 0.785, since $45° \cdot \pi/180° = \pi/4$ rad ≈ 0.785 rad.

Rectangles And Friends

Rectangles and Parallelograms:

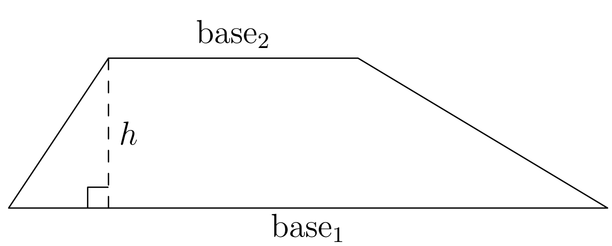


Rectangle
(Square if $l = w$)
Area $= lw$



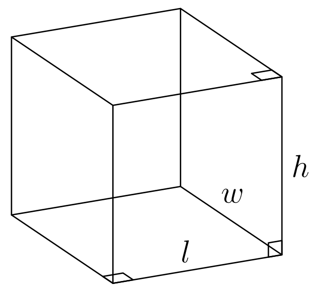
Parallelogram
(Rhombus if $l = w$)
Area $= lh$

Trapezoids (Optional):



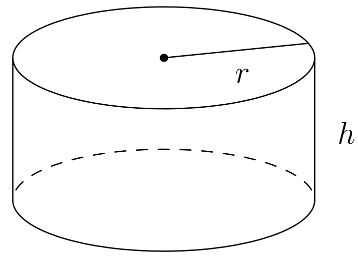
$$\text{Area of trapezoid} = \left(\frac{\text{base}_1 + \text{base}_2}{2}\right) \cdot h$$

Solids (Optional)



Rectangular Solid

Volume $= lwh$



Right Cylinder

Volume $= \pi r^2 h$