PRE-CALC & TRIG

Day 43

Bell Work

Prove:

$$(\cos x \tan x)^2 + (\sin x \cot x)^2 = 1$$

From Last Time

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5.1 Fundamental Trig Identities

- Objective: **Use fundamental trig identities to evaluate, simplify, and rewrite trig expressions**

Reciprocal Identities

$$\cos\theta = \frac{1}{sec\,\theta} \qquad \sec\theta = \frac{1}{cos\,\theta}$$

$$\sin\theta = \frac{1}{csc\,\theta} \qquad \csc\theta = \frac{1}{sin\,\theta}$$

$$\tan\theta = \frac{1}{cot\,\theta} \qquad \cot\theta = \frac{1}{tan\,\theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta} \qquad \cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\boldsymbol{sin^2\theta + cos^2\theta = 1}$$

Therefore,

$$\boldsymbol{1 + tan^2\theta = sec^2\theta}$$

But how?

$$\boldsymbol{1 + cot^2\theta = csc^2\theta}$$

Example 1: Use the values of $\sec u = -\frac{3}{2}$ and $\tan u > 0$ to find the values of all six trigonometric functions.

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$$\cos u = \frac{1}{\sec u} = \frac{1}{-3/2} = -\frac{\mathbf{2}}{\mathbf{3}} \qquad \sec u = -\frac{\mathbf{3}}{\mathbf{2}}$$

Note: since secant is < 0 and tangent is > 0 we know the value is in Quadrant III. Therefore sine is also negative.

$$\sin^2 u = 1 - cos^2 u = 1 - (-\frac{2}{3})^2$$

$$\sin^2 u = 1 - \frac{4}{9} = \frac{5}{9}$$

$$\boldsymbol{\sin u = -\frac{\sqrt{5}}{3}} \qquad \boldsymbol{csc\, u = \frac{1}{\sqrt{5}/3} = -\frac{3\sqrt{5}}{5}}$$

$$\boldsymbol{\tan u = \frac{\sin u}{\cos u} = \frac{-\frac{\sqrt{5}}{3}}{-\frac{2}{3}} = \frac{\sqrt{5}}{2}} \qquad \boldsymbol{cot\, u = \frac{1}{\sqrt{5}/2} = \frac{2\sqrt{5}}{5}}$$

Solution

$$\cos u = -\frac{2}{3} \qquad \sec u = -\frac{3}{2}$$

$$\sin u = -\frac{\sqrt{5}}{3} \qquad \csc u = -\frac{3\sqrt{5}}{3}$$

$$\tan u = \frac{\sqrt{5}}{2} \qquad \cot u = \frac{2\sqrt{5}}{5}$$

Prove:

$$\sin x \ \cos^2 x - \sin x = -\sin^3 x$$

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$\sin x\,(\cos^2 x - 1)$ ← Factor out sin x

$-\sin x\,(1 - \cos^2 x)$ ← Factor out -1 (combine with 1st step)

$\sin x\,(\sin^2 x)$ ← Pythagorean Identity

$\boldsymbol{-sin^3 x}$ ← Simplify

Simplify: $\sin w + \cot w \cos w = \csc w$

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$$\sin w + \cot w \cos w = \sin w + \frac{cos^2 w}{\sin w} \cos w$$ ← Quotient Property

$$\frac{sin^2 w + cos^2 w}{\sin w}$$ ← Common Denom. & Add

$$\frac{1}{\sin w}$$ ← Pythagorean Identity

$$\boldsymbol{csc\, x}$$ ← Reciprocal Identity

Perform the addition and simplify:

$$\frac{\sin t}{1+\cos t}+\frac{\cos t}{\sin t}$$

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Common Denominator

$$\sin t\,\frac{\sin t}{1+\cos t}+\frac{\cos t}{\sin t}(1+\cos t)$$

Distribute and Combine

$$\frac{\sin^2 t+\cos^2 t+\cos t}{(1+\cos t)(\sin t)}$$

Pythagorean Identity

$$\frac{(1+\cos t)}{(1+\cos t)(\sin t)}$$

Divide by Common Factor

$$\frac{1}{\sin t}$$

Reciprocal Identity

csc t

For Next Time

Day 1: Pg 377 #5, 6, 13, 14, 25–31 (odd), 79, 123

Day 2: Pg 377 #41–49 (odd), 125