Bell Work: Evaluate, if you can.

$$\mathbf{1.)\ arccos}\left(\frac{\sqrt{2}}{2}\right)$$

$$\mathbf{2.)}\ \boldsymbol{cos^{-1}}(\mathbf{-1})$$

Bell Work: Simplify

$\mathbf{1.)\ arccos}\left(\frac{\sqrt{2}}{2}\right)$ since $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (for $0 \le y \le \pi$)

it follows that $\mathbf{arccos}\left(\frac{\sqrt{2}}{2}\right) = \frac{\boldsymbol{\pi}}{\mathbf{4}}$

$\mathbf{2.)}\ \boldsymbol{cos^{-1}(-1)}$ since $\cos(\pi) = -1$ (for $0 \le y \le \pi$)

it follows that $\boldsymbol{cos^{-1}(-1)} = \pi$

From last time…

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Get a Head Start…

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PRE-CALC TRIG

Day 36

4.7 Inverse Trig Functions

Objective: Evaluate and graph inverse functions

Think about… In order for a function to have an inverse, it must be one-to-one (must pass Horizontal Line Test). $y = \sin x$ doesn't pass, but if we focus on domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ it will pass.

Inverse of Trig Functions
(explained with sine)

On this interval $y = \sin x$ is increasing.

On this interval $y = \sin x$ takes on full range of values,

$$-1 \leq y = \sin x \leq 1$$

On this interval $y = \sin x$ is one-to-one

Therefore, on restricted domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$, $y = \sin x$ has an inverse.

Denoted: $y = \arcsin x$ or $y = sin^{-1}x$

Definitions of Inverse Sine Function

The inverse sine function is defined by

$y = \arcsin x \qquad if\ and\ only\ if \qquad \sin y = x$

$$where -1 \leq x \leq 1\ and -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

$The\ domain\ of\ \ y = \arcsin x$

$is\ [-1,1] and\ the\ range\ is\ [-\frac{\pi}{2}, \frac{\pi}{2}]$

Examples: If possible, find the exact value.

$\mathbf{1.)\ arcsin}\left(\frac{\sqrt{3}}{2}\right)$

$\mathbf{2.)}\ \boldsymbol{sin^{-1}}\left(\frac{-1}{2}\right)$

$\mathbf{3.)}\ \boldsymbol{sin^{-1}}(\mathbf{5})$

Examples: If possible, find the exact value.

$\mathbf{1.)\ arcsin}\left(\frac{\sqrt{3}}{2}\right)$ since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ (for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$)

it follows that $\mathbf{arcsin}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$

$\mathbf{2.)}\ \boldsymbol{sin^{-1}}\left(\frac{-1}{2}\right)$ since $\sin\left(-\frac{\pi}{6}\right) = \frac{-1}{2}$ (for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$)

it follows that $\boldsymbol{sin^{-1}}\left(\frac{-1}{2}\right) = -\frac{\pi}{6}$

$\mathbf{3.)}\ \boldsymbol{sin^{-1}}(\mathbf{5})$ **Not possible because 5 is not in the**

range for $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Definitions of the Inverse Trig Functions

Functions	*Domain*	*Range*
$y = \arcsin x \quad iff \quad \sin y = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x \quad iff \quad \cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x \quad iff \quad \tan y = x$	$-\infty \leq x \leq \infty$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Graphs on page 343 for a visual

Examples: If possible, find the exact value.

$$\mathbf{1.)\ \arccos\left(\frac{\sqrt{2}}{2}\right)}$$

$$\boldsymbol{2.)\ cos^{-1}(-1)}$$

$$\boldsymbol{3.)\ tan^{-1}(0)}$$

Examples: If possible, find the exact value.

$\mathbf{1.)\ arccos}\left(\frac{\sqrt{2}}{2}\right)$ since $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (for $0 \le y \le \pi$)

it follows that $\mathbf{arccos}\left(\frac{\sqrt{2}}{2}\right) = \frac{\boldsymbol{\pi}}{\mathbf{4}}$

$\mathbf{2.)}\ \boldsymbol{cos^{-1}(-1)}$ since $\cos(\pi) = -1$ (for $0 \le y \le \pi$)

it follows that $\boldsymbol{cos^{-1}(-1)} = \pi$

$\mathbf{3.)}\ \boldsymbol{tan^{-1}(0)}$ since $\tan(0) = 0 (for\ -\frac{\pi}{2} \le y \le \frac{\pi}{2})$

it follows that $\boldsymbol{tan^{-1}(0) = 0}$

Inverse Properties of Trig Functions

$If\ -1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then

sin(arcsin x) = x and arcsin(sin y) = y

$If\ -1 \le x \le 1$ and $0 \le y \le \pi$, then

cos(arccos x) = x and arccos(cos y) = y

$If\ x\ is\ a\ real\ number$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$, then

tan(arctan x) = x and arctan(tan y) = y

Examples: If possible, find the exact value.

1.) tan(arctan 7)

Since 7 is in the domain and is a real number, the inverse property applies

tan(arctan 7) = 7

2.) $\arcsin\left(sin\frac{5\pi}{3}\right)$ $\frac{5\pi}{3}$ does not lie within range of arcsine, but its coterminal angle of $\frac{-\pi}{3}$ does…

$$\boldsymbol{arcsin\left(sin\frac{5\pi}{3}\right)=\frac{-\pi}{3}}$$

4.) $cos(cos^{-1}\pi)$

$cos^{-1}\pi$ is not defined because the domain is [-1,1]

Calculus Preview Examples:

Find the exact value.

$$1.)\ \tan\left(arccos\frac{2}{3}\right)$$

Solution

If you let $u = arccos\frac{2}{3}$ [note tan(arccos(u)) now]

Then $\cos u = \frac{2}{3}$ since it is positive it is in Quad I)

And we know that $x = 2, r = 3$

and therefore $y = \sqrt{5}$ (Draw triangle if needed)

$\tan u = \frac{\sqrt{5}}{2}$ Therefore, $\mathbf{tan}\left(\boldsymbol{arccos}\frac{\mathbf{2}}{\mathbf{3}}\right) = \frac{\sqrt{\mathbf{5}}}{\mathbf{2}}$

Calculus Preview Examples:

Find the exact value.

$$2.)\ \cos\left(\arcsin\left(\frac{-3}{5}\right)\right)$$

Solution

If you let $u = arcsin\frac{-3}{5}$ [note cos(arcsin(u)) now]

Then $\sin u = \frac{-3}{5}$ (since sin is neg.. it is in Quad IV)

And we know that $y = -3$,

$r = 5$ and therefore $x = 4$ (Draw triangle if needed)

$\cos u = \frac{4}{5}$ Therefore, $\boldsymbol{\cos\left(arcsin\frac{-3}{5}\right) = \frac{4}{5}}$

4.8 Applications and Models

Objective: To Solve Real Life Problems Involving Right Triangles and Harmonic Motion

Example.

You are standing about 600 feet from the state capital. The angle of elevation to the top of the dome (base of the podium of the sower) is 33.55°. How tall is the sower (with podium included) if the angle of elevation to the top of the sower's head is 35.63°?

Solution

$$h_{capital} = 600 \cdot tan\, 33.55 = 397.88\ feet$$

$$h_{capital+sower} = 600 \cdot tan\, 35.63 = 430.03\ feet$$

$$\boldsymbol{h_{sower} = 430.03 - 397.88 = 32.15\ feet}$$

Additional Example

A safety regulation states that the max angle of elevation for a rescue ladder is 72°. A fire department's longest ladder is 110 feet. There is a cat that needs to be rescued from a tree, and the cat is 105 feet off the ground. Are they able to successfully rescue the cat or do they need to find another ladder?

Solution

A safety regulation states that the max angle of elevation for a rescue ladder is 72°. A fire department's longest ladder is 110 feet. There is a cat that needs to be rescued from a tree, and the cat is 105 feet off the ground. Are they able to successfully rescue the cat or do they need to find another ladder?

$$x = 110 \cdot sin\,72$$

$x = 104.6$ ***So the max height it can reach is not high enough to get to the cat, so unless maybe a tall firefighter can reach the cat from the top of the ladder they'll need to find a slightly longer ladder.***

Definition of Simple Harmonic Motion

A point that moves on a coordinate line is said to be simple harmonic motion if its distance, d, from the origin at the time, t, is given by either:

$d = \mathrm{a}\sin \omega t \qquad or \qquad d = \mathrm{a}\cos \omega t$

Where a and ω are real numbers such that $\omega > 0$. The motion has amplitude of |a|, period $\frac{2\pi}{\omega}$, and frequency $\frac{\omega}{2\pi}$

Amplitude = maximum displacement from equilibrium

Period = time for one complete cycle

Frequency = number of cycles per second [page 354 for visual]

Example

A ball is bouncing up and down on a spring. Suppose that 12 inches is the max distance the ball moves vertically (up or down) from its equilibrium (rest) position. Suppose the time it takes for the ball to move from its max displacement (above zero) to its min displacement (below zero) is 6 seconds.

Assuming perfect elasticity, no friction, and no air resistance, the ball would continue to move in a uniform motion. Find the amplitude, period, and frequency.

Solution

Amplitude = |12| = **12**

Period = 6 seconds → period $= \frac{2\pi}{\omega} = 6 \rightarrow \omega = \boldsymbol{\frac{\pi}{3}}$

Frequency = $\frac{\omega}{2\pi} = \frac{\pi/3}{2\pi} = \boldsymbol{\frac{1}{6}}$ cycles per second

$d = 12 sin\frac{\pi}{3}t \qquad \rightarrow graph\ and\ analyze$

Example

Given the equation for simple harmonic motion:

$$d = 4sin\frac{3\pi}{2}t$$

Find the maximum displacement, frequency, the value of d when t = 9,

and the least positive value of t for which d = 0

Solution

<u>Maximum Displacement</u>

Given by the amplitude which is 4 -or- Graph: find the max (from y = 0 equilibrium)

<u>Frequency</u>

$\frac{\omega}{2\pi} = \frac{3\pi/2}{2\pi} = \frac{3}{4}$ -or- Graph: time to complete one cycle

<u>Value of d when t = 9</u>

$d = 4sin\frac{3\pi}{2}(9) \rightarrow\ 4sin\frac{27\pi}{2} = 4(1) = 4$ -or Trace to t = 9

<u>Least positive value of t for which d = 0</u>

$$0 = 4sin\frac{3\pi}{2}t \rightarrow\ 0 = sin\frac{3\pi}{2}t$$

sin is 0 at 0, $\pi, 2\pi, \ldots$ so $\frac{3\pi}{2}t$ =0, $\pi, 2\pi, \ldots$ solve for t… t =0, $\frac{2}{3}, \frac{4}{3}$, …

Therefore the least is 0. -or- Use the Root button

For Next Time

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Get a Head Start…

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