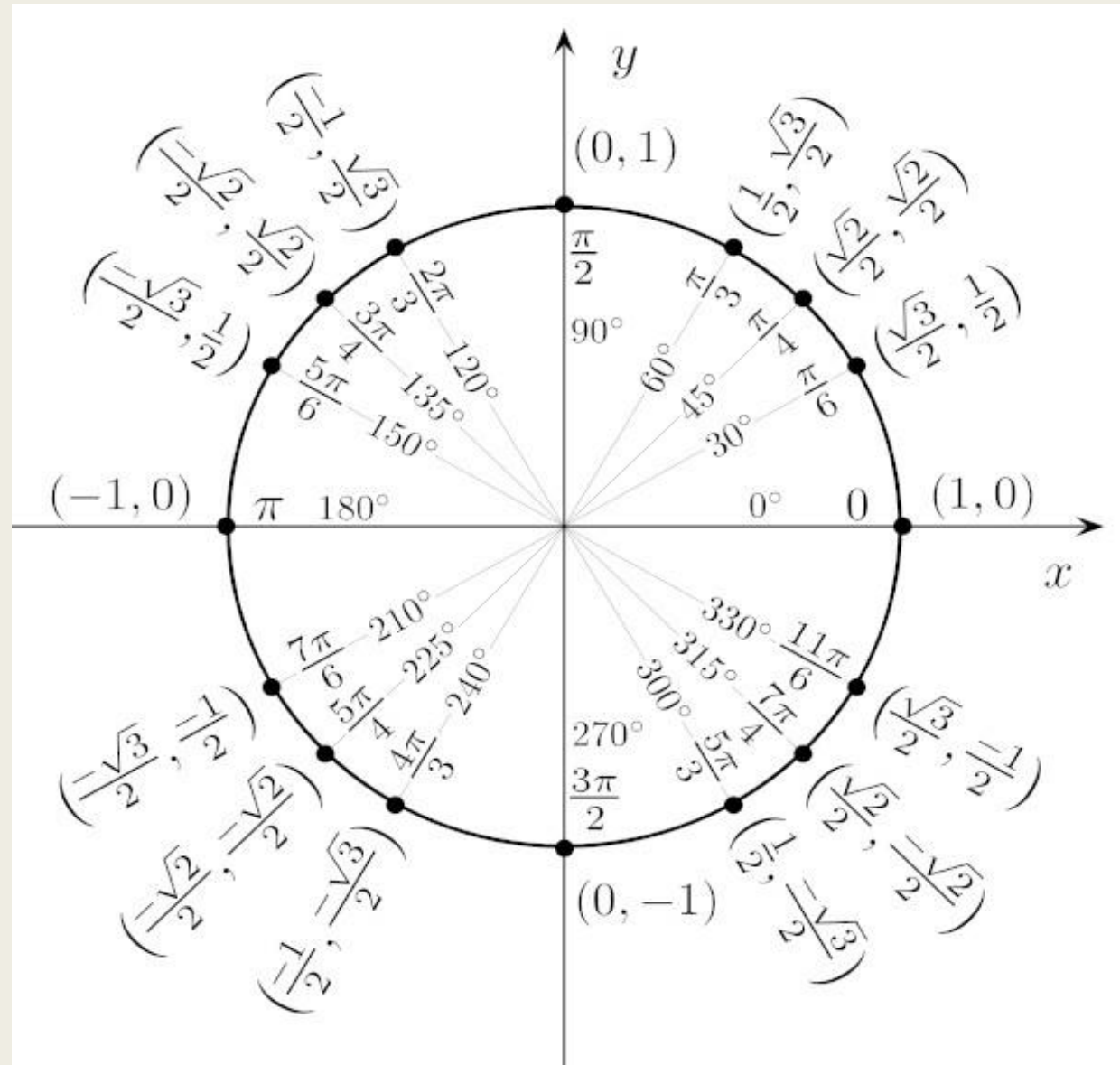


Bell Work: Unit Circle Review

$$\sin \frac{\pi}{2} =$$


$$\cos \frac{3\pi}{2} =$$





PRE-CALC TRIG

Day 29

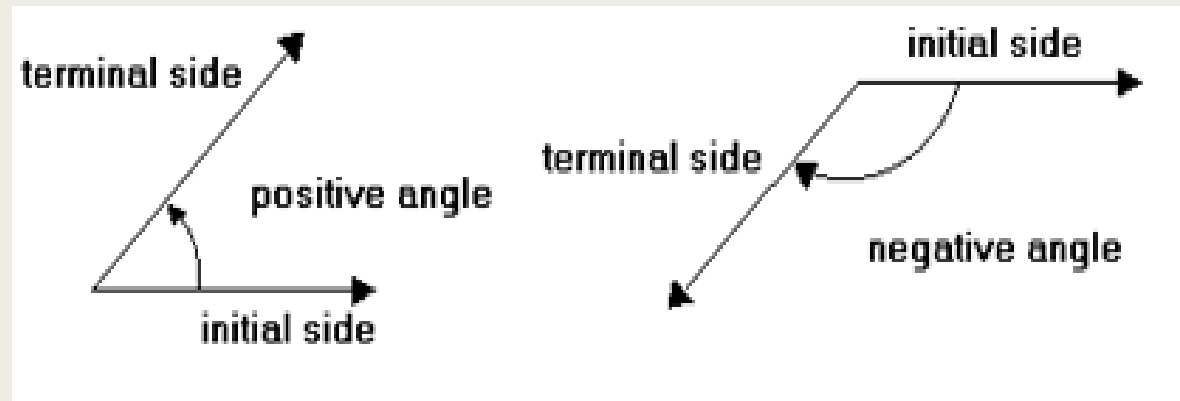


4.1 Radian and Degree Measure

- Objective: Describe angles using radian and degree measure.

Important Vocabulary

- **Trigonometry:** measure of triangles
- **Initial Side:** starting position of the ray
- **Terminal Side:** position after the rotation



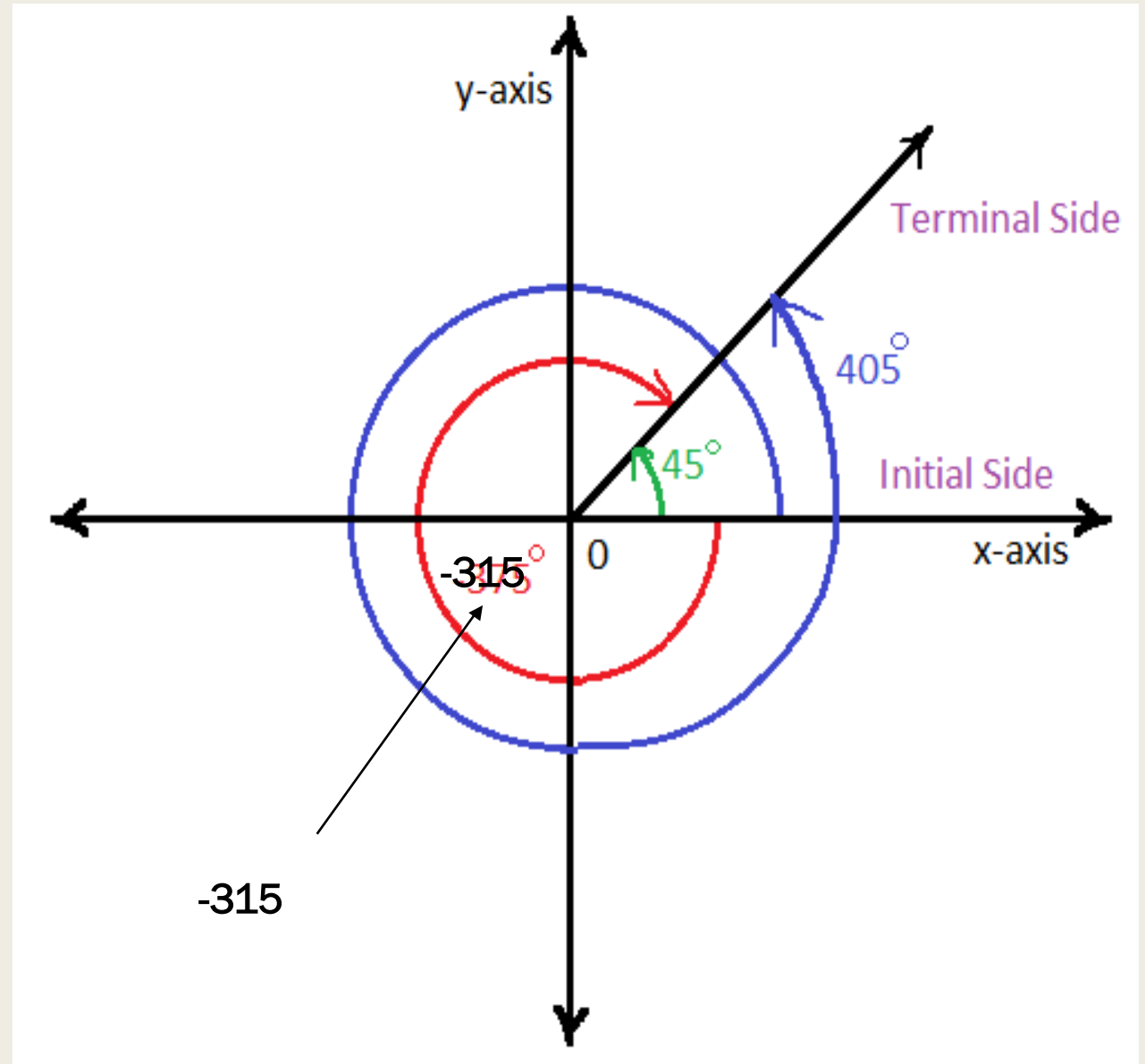
- **Positive Angles** (counter-clockwise) and **Negative Angles** (clockwise)

More Vocabulary

Coterminal: have the same terminal angle

Complementary: sum to $\pi/2$ (90 degrees)

Supplementary: sum to π (180 degrees)



Example of Coterminal Angles

Find coterminal angle of $3\pi/4$

$$3\pi/4 - 2\pi = -\mathbf{5\pi/4}$$

Find coterminal angle of $-3\pi/4$

$$-3\pi/4 + 2\pi = \mathbf{5\pi/4}$$

What the heck is a radian?

One radian is the measure of a central angle θ that intercepts an arc s equal in length to the radius r of a circle. [Watch youtube video] Algebraically, this means that $\theta = \frac{s}{r}$ where θ is measured in radians.

<https://www.youtube.com/watch?v=ifBhTdsTMuE>

Converting Degrees to Radians and vice versa

Degree Measure

$$360^\circ = 2\pi \text{ rad}$$

and

$$180^\circ = \pi \text{ rad}$$

$$1^\circ = \frac{\pi}{180} \text{ rad}$$

and

$$1 \text{ rad} = \frac{180^\circ}{\pi}$$

Therefore:

- To covert degrees to radians, multiply degrees by $\frac{\pi \text{ rad}}{180^\circ}$
- To covert radians to degrees, multiply degrees by $\frac{180^\circ}{\pi \text{ rad}}$

Example: Convert

$$135^\circ =$$

$$\frac{9\pi}{2} \text{ rad} =$$

Example: Worked Out...

$$135^\circ = 135^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \frac{3\pi}{4}$$

$$\frac{9\pi}{2} \text{ rad} = \left(\frac{9\pi}{2} \text{ rad} \right) \left(\frac{180^\circ}{\pi \text{ rad}} \right) = 810^\circ$$

Arc Length

For a circle of radius, r , a central angle θ intercepts an arc of length s given by $s = r \cdot \theta$ (from radian definition above) where θ is measured in radians. Note that if $r = 1$, then $s = \theta$, and the radian measure of θ equals the arc length.

Arc Length: Example

A circle has a radius of 5 inches.

Find the length of the arc, s , intercepted by a central angle of 200 degrees.

Arc Length: Example Worked Out...

Use the formula, $s = r \cdot \theta$ but θ is in radians so covert first.

$$200^\circ = 200^\circ \left(\frac{\pi \text{ rad}}{180^\circ} \right) = \frac{10\pi}{9} \text{ rad}$$

$$s = 5 * \frac{10\pi}{9} \text{ rad} = \frac{50\pi}{9} \text{ rad} \approx \mathbf{17.452 \text{ inches}}$$

Linear and Angular Speed

Consider a particle moving at a constant speed along a circular arc of radius r . If s is the length of the arc traveled in time t , then the linear speed v of the particle is:

$$\text{Linear Speed } v = \frac{\text{arc length}}{\text{time}} = \frac{s}{t}$$

Moreover, if θ is the angle (in radians) corresponding to the arc length s , then the angular speed ω (*lowercase omega*) of the particle is:

$$\text{Angular Speed } \omega = \frac{\text{central angle}}{\text{time}} = \frac{\theta}{t}$$

Area of a Sector of a Circle

For any circle of radius r , the area A of a sector of the circle with central angle θ is given by $A = \frac{1}{2}r^2\theta$ where θ is measured in radians.

Example:

The second hand of a clock is 12.4 cm long. Find the linear speed of the tip of the second hand as it moves around the clock.

Example: Worked Out...

The second hand of a clock is 12.4 cm long. Find the linear speed of the tip of the second hand as it moves around the clock.

- Arc of one revolution = $2\pi r = 2\pi(12.4) = 24.8\pi$
- The time it takes is 60 seconds
- $linear\ speed = \frac{s}{t} = \frac{24.8\pi}{60} = \mathbf{1.299\ cm\ per\ second}$

Example:

The blades of a wind turbine are 250 feet long. The propeller rotates at 12 revolutions per minute.

- 1.) Find the angular speed of the propeller in radians per min
- 2.) Find the linear speed of the tips of the blades.

Example: Worked Out...

The blades of a wind turbine are 250 feet long. The propeller rotates at 12 revolutions per minute.

- 1.) Find the angular speed of the propeller in radians per min
- 2.) Find the linear speed of the tips of the blades.

Because each revolution generates 2π radians, it follows that the propeller turns $(2\pi)(12) = 24\pi$ *radians per minute*.

In other words, $angular\ speed = \frac{\theta}{t} = \frac{24\pi\ rad}{1\ min} = 24\ radians\ per\ min$

The linear speed is:

$$linear\ speed = \frac{s}{t} = \frac{r\theta}{t} = \frac{250 * 24\pi}{1\ minute} = \frac{6000\pi}{1} = 18,849.556\ feet/min$$

Example:

Example: A sprinkler on a golf course sprays water a distance of 50 feet and rotates through an angle of 140° . Find the area of the course watered by the sprinkler.

Example: Worked Out...

Example: A sprinkler on a golf course sprays water a distance of 50 feet and rotates through an angle of 140° . Find the area of the course watered by the sprinkler.

Covert 140° to radians $\Rightarrow \frac{7\pi}{9}$

$$A = \frac{1}{2}r^2\theta = \frac{1}{2}50^2\left(\frac{7\pi}{9}\right) = \frac{8750\pi}{9} \approx$$

3054.326 square feet

For Next Time:

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#17, 23-24, 27, 29, 31 ← Radian

#41, 45-46, 49, 51, 53 ← Degree

#57-58, 61-62, 65-66, 73-74 ← Convert

#89-90, 93-94, 98-99, 109-110,
112, 118-119 ← Applied