Bell Work: Use Unit Circle to Identify the Following:

- $\cos \frac{3\pi}{2}$
- $\sin -\frac{\pi}{6}$
- $\tan \frac{\pi}{3}$
- $\tan \frac{\pi}{2}$

PRE-CALC TRIG

Day 33

4.2 Trigonometric Functions: The Unit Circle

Objective: Evaluate trig functions using the unit circle and a calculator

Definitions of Trigonometric Functions

Let t be a real number and let (x, y) be the point on the unit circle corresponding to t.

Cosine, sine, tangent, secant, cosecant, cotangent

$$\cos t = x \qquad \sec t = \frac{1}{x},\ x \neq 0$$

$$\sin t = y \qquad \csc t = \frac{1}{y},\ y \neq 0$$

$$\tan t = \frac{y}{x},\ x \neq 0 \qquad \cot t = \frac{x}{y},\ y \neq 0$$

Example:
Evaluate the 6 trigonometric functions for $\frac{\pi}{6}$

$\mathbf{cos}\,\frac{\pi}{6} =$

$\mathbf{sin}\,\frac{\pi}{6} =$

$\mathbf{tan}\,\frac{\pi}{6} =$

$\mathbf{sec}\,\frac{\pi}{6} =$

$\mathbf{csc}\,\frac{\pi}{6} =$

$\mathbf{cot}\,\frac{\pi}{6} =$

Additional Examples

- Evaluate the following:

$$\cos\frac{7\pi}{4} =$$

$$\sin\frac{3\pi}{2} =$$

$$\tan\ \frac{5\pi}{4} =$$

$$\sec\frac{5\pi}{6} =$$

Additional Examples

- Evaluate the following:

$$\cos\frac{7\pi}{4} = \frac{\sqrt{2}}{2}$$

$$\sin\frac{3\pi}{2} = -1$$

$$\tan\frac{5\pi}{4} = 1$$

$$\sec\frac{5\pi}{6} = \frac{1}{\frac{-\sqrt{3}}{2}} = \frac{-2\sqrt{3}}{3}$$

Review: Domain and Range

- The domain of sine and cosine functions is all real numbers (you can go around the circle in either direction for as long as you'd like). To determine the range, consider the unit circle: [cos t = x and sin t = y]
- Because (x, y) is on the unit circle, you know they have to be between -1 and 1. Therefore → $-1 \leq \cos t \leq 1$ $-1 \leq \sin t \leq 1$
- Adding 2π completes a second revolution and the values of cos(t + 2π) and sin(t + 2π) correspond to cos(t) and sin(t). This is true no matter how many revolutions, n (positive or negative) are made.
- cos(t + $2\pi n) = \cos(t)$
- sin(t + $2\pi n) = \sin(t)$

Repetitive functions like this are called periodic.

Even and Odd Trig Functions

Even

Cosine and Secant functions are even.

cos(-t) = cos t　　　　sec(-t) = sect t

Odd

Sine, cosecant, tangent, and cotangent functions are odd.

sin(-t) = - sin t　　　　csc(-t) = - csc t

tan(-t) = - tan t　　　　cot(-t) = - cot t

4.3 Right Triangle Trigonometry

Objective: Evaluate trig functions of acute angles and use fundamental trig identities while solving real-life problems

Right Triangle Definitions of Trigonometric Functions

Let θ be an acute angle of a right triangle. The six trig functions of angle θ are defined:

Cosine, sine, tangent, secant, cosecant, cotangent

$$\cos\theta = \frac{adj}{hyp} \qquad \sec\theta = \frac{hyp}{adj}$$

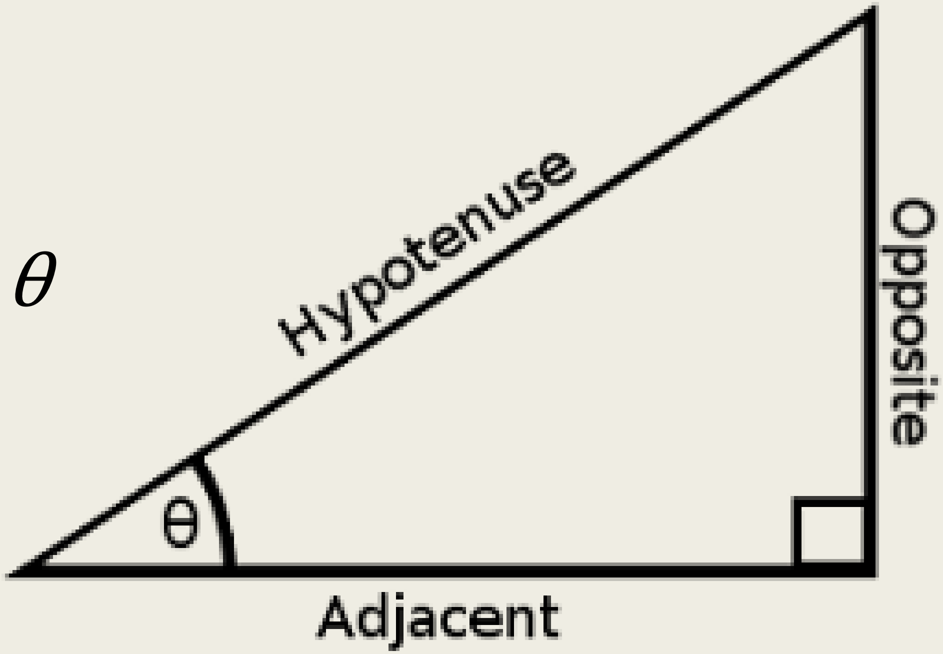
$$\sin\theta = \frac{opp}{hyp} \qquad \csc\theta = \frac{hyp}{opp}$$

$$\tan\theta = \frac{opp}{adj} \qquad \cot\theta = \frac{adj}{opp}$$

----SOH----CAH----TOA----

The abbreviations in the previous slide: opp, adj, hyp represent the lengths of the sides of the right triangle (as seen in the picture)

opp = length of side opposite given θ
adj = length of side adjacent to given θ
hyp = length for of hypotenuse



Example:
Evaluate the 6 trigonometric functions for θ for a triangle with a hypotenuse of 5 and side opposite of θ equal to 3.

$\boldsymbol{\cos\theta =}$

$\boldsymbol{\sin\theta =}$

$\boldsymbol{\tan\theta =}$

$\boldsymbol{\sec\theta =}$

$\boldsymbol{\csc\theta =}$

$\boldsymbol{\cot\theta =}$

Fundamental Trigonometric Identities

- These identities will be vital as we move through this chapter and into second semester

Reciprocal Identities

$$\cos\theta = \frac{1}{sec\ \theta}$$

$$\sec\theta = \frac{1}{cos\ \theta}$$

$$\sin\theta = \frac{1}{csc\ \theta}$$

$$\csc\theta = \frac{1}{sin\ \theta}$$

$$\tan\theta = \frac{1}{cot\ \theta}$$

$$\cot\theta = \frac{1}{tan\ \theta}$$

Quotient Identities

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\cot\theta = \frac{\cos\theta}{\sin\theta}$$

Pythagorean Identities

$$\boldsymbol{sin^2\theta + cos^2\theta = 1}$$

Therefore,

$$\boldsymbol{1 + tan^2\theta = sec^2\theta}$$

But how?

$$\boldsymbol{1 + cot^2\theta = csc^2\theta}$$

Example

Let θ be an acute angle such that $\sin\theta = 0.6$. Find the value of:

1.) $\cos\theta$

2.) $\tan\theta$

Solution:

Let θ be an acute angle such that $\sin\theta = 0.6$. Find the value of:

1.) $\cos\theta$

$$(0.6)^2 + cos^2\theta = 1 \Rightarrow cos^2\theta = 1 - .36$$

$$cos^2\theta = 0.64$$

$$cos\ \theta = \sqrt{0.64}$$

$$\boldsymbol{cos\ \theta = 0.8}$$

2.) $\tan\theta$

$$\tan\theta = \frac{0.6}{0.8}$$

$$\mathbf{tan}\ \boldsymbol{\theta} = \mathbf{0.75}$$

Example: Applications to Real Life

You're standing just down the street from the 478 foot tall Woodmen Building in downtown Omaha. The measure of the angle of elevation from where you are standing to the top of the Woodmen building is 68.4°. How far away from the building are you standing?

Solution:

You’re standing just down the street from the 478 foot tall Woodmen Building in downtown Omaha. The measure of the angle of elevation from where you are standing to the top of the Woodmen building is 68.4°. How far away from the building are you standing?

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 68.4° = \frac{478}{x} => x\tan 68.4° = 478$$

$$x = \frac{478}{\tan 68.4°}$$

x = 189.25 feet

Example: Second Application

A historic lighthouse is 200 yards directly south from a bike path along a lake. A walk way to the lighthouse is 400 yards long. Find the acute angle θ between the bike path and the walkway.

Example: Second Application

A historic lighthouse is 200 yards directly south from a bike path along a lake. A walk way to the lighthouse is 400 yards long. Find the acute angle θ between the bike path and the walkway.

Note: path is adjacent, walk way is hypotenuse, and 200 yards is opposite

$$\sin\theta = \frac{opp}{hyp} = \frac{200}{400} = \frac{1}{2} => \sin\theta = \frac{1}{2}$$

This is true at 30°

For Next Time

Pg 297 #5, 9-19 (odd), 27, 34,
49-53 (odd), 59

Get a Head Start…

Pg 306 #1, 5-6, 30-31, 37-40, 45, 47,
57, 63-64, 67, 71