Bell Work: Simplify

$$If\ the \tan\theta = -\frac{4}{3}\ and \sin\theta > 0.$$

What quadrant is the θ located in and what are the other 5 trig functions?

For Next Time

Page 306 #1, 5-6, 30-31, 37-40, 45, 47, 57, 63-64, 67, 71

Get a Head Start…

Page 316 #9, 11, 13, 19-24, 37-44, 45-51 (odd), 69, 71, 97

PRE-CALC TRIG

Day 35

4.4 Trig Functions of Any Angle

Objective: Evaluate trig functions of any angles

Definitions of Trigonometric Functions of Any Angle

Let θ be an angle in standard position with (x, y) a point on the terminal side of θ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\boldsymbol{\cos\theta = \frac{x}{r}} \qquad \boldsymbol{\sec\theta = \frac{r}{x},\ x \neq 0}$$

$$\boldsymbol{\sin\theta = \frac{y}{r}} \qquad \boldsymbol{\csc\theta = \frac{r}{y},\ y \neq 0}$$

$$\boldsymbol{\tan\theta = \frac{y}{x},\ \ x \neq 0} \qquad \boldsymbol{\cot\theta = \frac{x}{y},\ y \neq 0}$$

Note: See picture on page 310 for image of what is meant by x, y, and r.

Example: Let (-6, 8) be a point on the terminal side of θ. Find the cosine, sine, and tangent of θ. Also find the secant, cosecant, and cotangent.

Example: Let (-6, 8) be a point on the terminal side of θ. Find the cosine, sine, and tangent of θ. Also find the secant, cosecant, and cotangent.

$$r = \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10...$$

$$\cos\theta = \frac{-6}{10} \qquad \sin\theta = \frac{8}{10} \qquad \tan\theta = \frac{8}{-6}$$

$$\sec\theta = \frac{10}{-6} \qquad \csc\theta = \frac{10}{8} \qquad \cot\theta = \frac{-6}{8}$$

Example: Given that $\tan\theta = -\frac{4}{7}$ and $\cos\theta < 0$, find $\sin\theta$ and $\sec\theta$.

-- Note that θ is in Quadrant II because that is the only quadrant in which tangent is negative and cos is negative.

Example: Given that $\tan\theta = -\frac{4}{7}$ and $\cos\theta < 0$, find $\sin\theta$ and $\sec\theta$.

$\tan\theta = -\frac{4}{7}$ so you know that $y = 4\ and\ x = -7$

So, $r = \sqrt{16+49} = \sqrt{65}$

$\sin\theta = \frac{y}{r} = \frac{4}{\sqrt{65}} = 0.496$ $\sec\theta = \frac{r}{x} = \frac{\sqrt{65}}{-7} = -1.152$

Evaluating Trig Functions of Any Angle

To find the value of a trig function of any angle θ

Determine the function value for the associated reference angle θ' (reference angle)

Depending on the quadrant in which θ lies, affix the appropriate sign to the function value.

Reference Angles

Let θ be an angle in standard position. The reference angle is the acute angle θ' formed by the terminal side of θ and the horizontal axis.

Quadrant II	Quadrant III	Quadrant IV
$\theta' = \pi - \theta(radian)$	$\theta' = \theta - \pi(radian)$	$\theta' = 2\pi - \theta(radian)$
$\theta' = 180 - \theta(degree)$	$\theta' = \theta - 180(degree)$	$\theta' = 2\pi - 180(degree)$

Note: See Figure 4.39 on page 312 for a visual

Example

Find the reference angle θ'

$1.)\ \theta = 290°$

$2.)\ \theta = 3.4$

Example

$\theta = 290°$ $\qquad\qquad$ $\theta' = 360° - 290° = \mathbf{70}°$

$\theta = 3.4$ $\qquad$ Note: $\frac{\pi}{2} \approx 1.57 \quad \pi \approx 3.14 \quad \frac{3\pi}{2} \approx 4.71$

Therefore 3.4 radians is between $\pi\ and\ \frac{3\pi}{2}$
$(in\ otherwords\ the\ 3rd\ quadrant\ )$

$\theta' = 3.4 - \pi \approx \boldsymbol{0.258\ radians}$

<u>4.7 Inverse Trig Functions</u>

Objective: Evaluate and graph inverse functions

Think about… In order for a function to have an inverse, it must be one-to-one (must pass Horizontal Line Test). $y = \sin x$ doesn't pass, but if we focus on domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}$ it will pass.

Inverse of Trig Functions
(explained with sine)

On this interval $y = \sin x$ is increasing.

On this interval $y = \sin x$ takes on full range of values,

$$-1 \leq y = \sin x \leq 1$$

On this interval $y = \sin x$ is one-to-one

Therefore, on restricted domain $-\frac{\pi}{2} \leq x \leq \frac{\pi}{2}, y = \sin x$ has an inverse.

Denoted: $y = \arcsin x$ or $y = sin^{-1}x$

Definitions of Inverse Sine Function

The inverse sine function is defined by

$y = \arcsin x \qquad if\ and\ only\ if \qquad \sin y = x$

$$where -1 \leq x \leq 1\ and -\frac{\pi}{2} \leq y \leq \frac{\pi}{2}.$$

$The\ domain\ of\ \ y = \arcsin x$

$is\ [-1,1] and\ the\ range\ is\ [-\frac{\pi}{2}, \frac{\pi}{2}]$

Examples: If possible, find the exact value.

$\mathbf{1.})\ \mathbf{arcsin}\left(\frac{\sqrt{3}}{2}\right)$

$\mathbf{2.})\ \boldsymbol{sin^{-1}}\left(\frac{-1}{2}\right)$

$\mathbf{3.})\ \boldsymbol{sin^{-1}}(\mathbf{5})$

Examples: If possible, find the exact value.

$\mathbf{1.)\ arcsin}\left(\frac{\sqrt{3}}{2}\right)$ since $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$ (for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$)

it follows that $\mathbf{arcsin}\left(\frac{\sqrt{3}}{2}\right) = \frac{\boldsymbol{\pi}}{\mathbf{3}}$

$\mathbf{2.)}\ \boldsymbol{sin^{-1}}\left(\frac{\mathbf{-1}}{\mathbf{2}}\right)$ since $\sin\left(-\frac{\pi}{6}\right) = \frac{-1}{2}$ (for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$)

it follows that $\boldsymbol{sin^{-1}}\left(\frac{\mathbf{-1}}{\mathbf{2}}\right) = -\frac{\boldsymbol{\pi}}{\mathbf{6}}$

$\mathbf{3.)}\ \boldsymbol{sin^{-1}}(\mathbf{5})$ Not possible because 5 is not in the

range for $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Definitions of the Inverse Trig Functions

Functions	*Domain*	*Range*
$y = \arcsin x \quad iff \quad \sin y = x$	$-1 \leq x \leq 1$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$
$y = \arccos x \quad iff \quad \cos y = x$	$-1 \leq x \leq 1$	$0 \leq y \leq \pi$
$y = \arctan x \quad iff \quad \tan y = x$	$-\infty \leq x \leq \infty$	$-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$

Graphs on page 343 for a visual

Examples: If possible, find the exact value.

$\mathbf{1.)\ arccos\left(\frac{\sqrt{2}}{2}\right)}$

$\mathbf{2.)}\ \boldsymbol{cos^{-1}(-1)}$

$\mathbf{3.)}\ \boldsymbol{tan^{-1}(0)}$

Examples: If possible, find the exact value.

$\mathbf{1.)}\ \mathbf{arccos}\left(\frac{\sqrt{2}}{2}\right)$ since $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$ (for $0 \leq y \leq \pi$)

it follows that $\mathbf{arccos}\left(\frac{\sqrt{2}}{2}\right) = \frac{\boldsymbol{\pi}}{\mathbf{4}}$

$\mathbf{2.)}\ \boldsymbol{cos^{-1}}(\mathbf{-1})$ since $\cos(\pi) = -1$ (for $0 \leq y \leq \pi$)

it follows that $\boldsymbol{cos^{-1}}(\mathbf{-1}) = \pi$

$\mathbf{3.)}\ \boldsymbol{tan^{-1}}(\mathbf{0})$ since $\tan(0) = 0 (for\ -\frac{\pi}{2} \leq y \leq \frac{\pi}{2})$

it follows that $\boldsymbol{tan^{-1}}(\mathbf{0}) = \mathbf{0}$

Inverse Properties of Trig Functions

$If\ -1 \leq x \leq 1$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then

sin(arcsin x) = x and arcsin(sin y) = y

$If\ -1 \leq x \leq 1$ and $0 \leq y \leq \pi$, then

cos(arccos x) = x and arccos(cos y) = y

$If\ x\ is\ a\ real\ number$ and $-\frac{\pi}{2} \leq y \leq \frac{\pi}{2}$, then

tan(arctan x) = x and arctan(tan y) = y

Examples: If possible, find the exact value.

1.) tan(arctan 7)

Since 7 is in the domain and is a real number, the inverse property applies

tan(arctan 7) = 7

2.) $\arcsin\left(sin\frac{5\pi}{3}\right)$

$\frac{5\pi}{3}$ does not lie within range of arcsine, but its coterminal angle of $\frac{-\pi}{3}$ does…

$$\boldsymbol{arcsin\left(sin\frac{5\pi}{3}\right)=\frac{-\pi}{3}}$$

4.) $cos(cos^{-1}\pi)$

$cos^{-1}\pi$ is not defined because the domain is [-1,1]

Calculus Preview Examples:

Find the exact value.

$$1.)\ \tan\left(arccos\,\frac{2}{3}\right)$$

Solution

If you let $u = arccos\frac{2}{3}$ [note tan(arccos(u)) now]

Then $\cos u = \frac{2}{3}$ since it is positive it is in Quad I)

And we know that $x = 2, r = 3$

and therefore $y = \sqrt{5}$ (Draw triangle if needed)

$\tan u = \frac{\sqrt{5}}{2}$ Therefore, $\mathbf{tan}\left(\boldsymbol{arccos}\frac{\mathbf{2}}{\mathbf{3}}\right) = \frac{\sqrt{\mathbf{5}}}{\mathbf{2}}$

Calculus Preview Examples:

Find the exact value.

$$2.)\ \cos\left(\arcsin\left(\frac{-3}{5}\right)\right)$$

Solution

If you let $u = arcsin\frac{-3}{5}$ [note cos(arcsin(u)) now]

Then $\sin u = \frac{-3}{5}$ (since sin is neg.. it is in Quad IV)

And we know that $y = -3$,

$r = 5$ and therefore $x = 4$ (Draw triangle if needed)

$\cos u = \frac{4}{5}$ Therefore, $\boldsymbol{\cos\left(arcsin\frac{-3}{5}\right) = \frac{4}{5}}$

For Next Time

Page 316 #9, 11, 13, 19-24, 37-44,
45-51 (odd), 69, 71, 97

Get a Head Start…

Page 347 #1-4, 5-9 (odd), 23-27 (odd),
43, 49-51, 55, 105, 111