Bell Work

- Let’s jump right into homework from last time, and your *new* material today. We have two ‘review’ sections to get through.

From Last Time…

- Page 146 #25, 29, 43, 47, 57, 97

PRE-CALC TRIG

Day 15

2.3 Polynomial and Synthetic Division

Objective: To understand (but potentially never use) synthetic division to find zeros and remainders

2.4 Complex Numbers

Objective: To add, subtract, multiply and divide complex numbers as well as find complex solutions (non-real zeros)

Why do we need to know Synthetic Division exists?

- Division of Polynomials: Useful for factoring and finding zeros when dividing by linear expressions, also most helpful when needing to find imaginary roots

- Long Division: divide by non-linear expressions (won't use!!)

- Synthetic Division: simplifies long-division by dividing by a linear expression $x - a$.

Synthetic Division Review

- When dividing a polynomial by an expression of the form **x – a**, you can use synthetic substitution as a form of synthetic division.
- The final constants equal the coefficients of the quotient
- Exponents for the quotient = exponent from original – 1

  (for each column)

*if the last column equals 0 then x – a is a factor

Steps to Synthetic Division

- Write the equation in standard form

  (put 0's in for exponents not represented)
- Multiply leading coefficient by the value of the variable
- Sum the next coefficient with the answer from Step 2
- Multiply the answer from Step 3 by the value of the variable
- Sum the next coefficient with the answer from Step 4
- Continue until each coefficient has been used

Synthetic Example

Example 1: Divide Using Synthetic Division.

f(x) = $2x^4 - 8x^2 + 5x - 7$ divided by $x - 3$

$2x^4 + 0x^3 - 8x^2 + 5x - 7$

	2	0	-8	5	-7
3		6	18	30	105
	2	6	10	35	98

$2x^4 - 8x^2 + 5x - 7 = \mathbf{(2x^3 + 6x^2 + 10x + 35)(x-3) + 98}$

Side Note:

Remainder Theorem;

If a polynomial f(x) is divided by x – k, then the remainder is

r = f(k)

i.e. in this problem, the remainder is 98 and f(3) = 98

What are complex numbers?

<u>Point of Emphasis:</u>

*Complex answers come in **pairs**, don't touch x-axis when graphed, and you can use synthetic division with real solutions until you reach a quadratic in which you can use the Quadratic Formula to find complex roots when needed*

Quick Review of imaginary numbers…

Imaginary Unit: i, defined as

$i = \sqrt{-1}$, or, **i^2 =** -1

Simplify using imaginary, i

Example 1:

$\sqrt{-31} = \pm i\sqrt{31}$

Example 2:

$\sqrt{-25} = \pm 5i$

Example 3:

$\sqrt{-18} = i3\sqrt{2} = \pm 3i\sqrt{2}$

For next time…

- Pg 157 #55, 59, 61, 67, 69, 75
- Pg 146 #48, 51

More so for discussion next time…

- Pg 164 #21, 31, 51, 69, 97

Additional Info on Complex Numbers

Imaginary Number: $a + bi$; when $b \neq 0$
Complex Number: Imaginary and real numbers together

Standard Form: **$a + bi$**
Imaginary Number: $a + bi$; when $b \neq 0$
Pure Imaginary Number: $a + bi$; when $a = 0$ and $b \neq 0$, or, **bi**

Complex Number Plane: the point (a, b) represents the complex number $a+bi$

x-axis is the real axis
y-axis is the imaginary axix

Additional Info on Complex Numbers

Find the absolute value of a complex number: (distance formula)

$$|a + bi| = \sqrt{a^2 + b^2}$$

<u>Sum of Complex Numbers</u>:

$(a + bi) + (c + di) = (a + c) + (b + d)i$

<u>Difference of Complex Numbers</u>:

$(a + bi) – (c + di) = (a – c) + (b – d)i$

- Add or Subtract Real Numbers
- Add or Subtract Imaginary Numbers

Additional Info on Complex Numbers

Multiplying Complex Numbers;

1) Use distributive or FOIL methods
2) Simplify (combine like terms and use $i^2 = -1$)
3) Put into Standard Form $(a + bi)$

Examples:

$5i(-2 + i)$

$(7 - 4i)(-1 + 2i)$

$-10i + i^2$
$-10i + (-1)$
$\mathbf{-1 - 10}\boldsymbol{i}$

$7 + 14i + 4i - 8i^2$
$7 + 18i - 8(-1)$
$7 + 18i + 8$
$\mathbf{15 + 18}\boldsymbol{i}$

Additional Info on Complex Numbers

Complex Conjugates: a + bi, and, a – bi

- Product is always a real number
- *Used to eliminate complex numbers from denominators*

<u>The Square Root of a Negative Number</u>;

- If r is a positive real number, then $\sqrt{-r} = i\sqrt{r}$
- $(i\sqrt{r})^2 = -r$

<u>Dividing Complex Numbers</u>;

1) Multiple numerator and denominator by complex conjugate of denominator (use FOIL)
2) Simplify (combine like terms and use $i^2 = -1$)
3) Put into Standard Form (a + bi)

Additional Info on Complex Numbers

Solving Quadratics Equations with Complex Solutions

1) Isolate the squared term on one side of the equation
2) Square root both sides
3) Substitute $i = \sqrt{-1}$ into equation and simplify radical
4) Write solution in Standard Form $(a + bi)$

Examples:

$x^2 = -9$

$2x^2 + 3 = -13$