

Bell Work

Tell me anything that you know about $P(x)$.

$$P(x) = x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 = 0$$

From Last Time...

- Pg 157 #55, 59, 61, 67, 69, 75
- Pg 146 #48, 51

More so for discussion next time...

- Pg 164 #21, 31, 51, 69, 97



PRE-CALC TRIG

Day 16



2.5 Zeros of Polynomial Functions

- **Objective:** Use Fundamental Theorem of Algebra to determine number of solutions and then find the zeros.

The Fundamental Theorem of Algebra

- If $f(x)$ is a polynomial of degree n where $n \geq 0$, then the equation $f(x) = 0$ has exactly n roots, including multiple and complex roots.
- In other words: Any n^{th} degree polynomial function has exactly n zeros

Bell Work Explained:

- What are the roots for the following equation?

$$P(x) = x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 = 0$$

There are 5 zeros (solutions) because the degree is 5.

Graph to find the real ones: 1 , 2, -2 (so 2 must be imaginary...)

How to find the imaginary:

- Step2: Since $P(1)=0$ we know it is a root (Remainder Thm) and therefore $x-1$ is a factor. Use synthetic division to factor out $x-1$.

$$\begin{array}{r}
 x^5 - x^4 - 3x^3 + 3x^2 - 4x + 4 \\
 \hline
 1 \quad 1 \quad 0 \quad -3 \quad 0 \quad -4 \\
 \hline
 1 \quad 0 \quad -3 \quad 0 \quad -4 \quad 0 \\
 \hline
 2 \quad 2 \quad 4 \quad 2 \quad 4 \\
 \hline
 1 \quad 2 \quad 1 \quad 2 \quad 0 \\
 -2 \quad -2 \quad 0 \quad -2 \\
 \hline
 1 \quad 0 \quad 1 \quad 0
 \end{array}$$

Therefore, $(x-1)(x-2)(x+2)(x^2 + 1) = 0 \Rightarrow (x - 1)(x + 2)(x - 2)(x + i)(x - i) = 0$

The roots are 1, -2, 2, -i and i

Find all Zeros

- $x^5 - x^4 - 7x^3 + 7x^2 - 18x + 18 = 0$

- $x^4 + 2x^3 - 4x^2 - 7x - 2 = 0$

For next time...

- Pg 176 #9, 19, 27, 31, 44, 71, 77, 113, 118

Additional Information

The Rational Zero Theorem;

If $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_1x + a_0$ has integer coefficients, then every rational zero of $f(x)$ has the following form:

$$\frac{p}{q} = \frac{\text{factor of constant term } a_0}{\text{factor of leading coefficient } a_n}$$

Conjugate Pairs

Let $f(x)$ be a polynomial function that has real coefficients. If $a+bi$, where b isn't 0, is a zero of the function, the conjugate $a-bi$ is also a zero of the function.