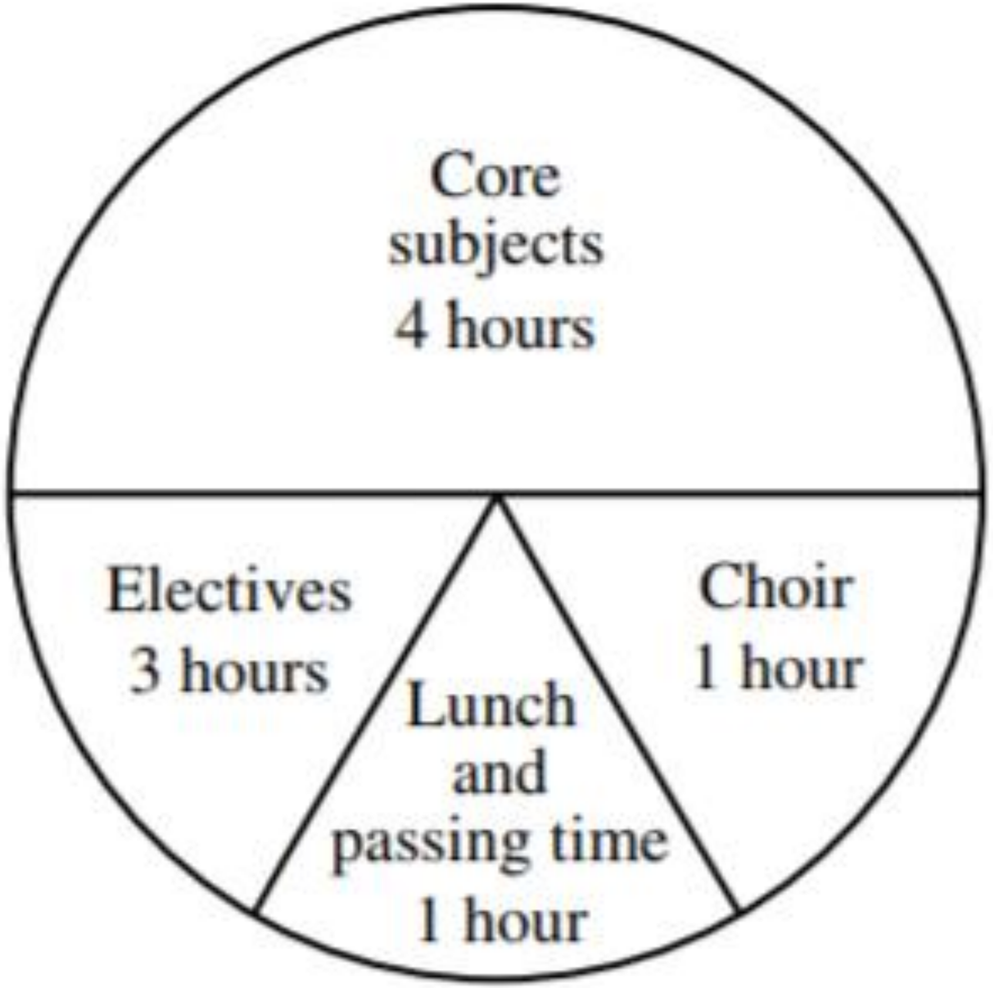
PRE-CALC & TRIG

Day 51

Bell Work

Antwan drew the circle graph below describing his time spent at school in 1 day. His teacher said that the numbers of hours listed were correct, but that the central angle measures for the sectors were not correct. What should be the central angle measure for the Core subjects sector?



F. 72°
G. 80°
H. 160°
J. 200°
K. 288°

<u>6.1 Law of Sines</u>

Objective: Use law of sines to solve and find the area of oblique triangles (AAS, ASA, SSA) while applying it to real life situations

Oblique Triangle: triangles that have no right angles

It is standard practice to put side a across from angle A, side b across from angle B, and side c across from angle C.

To Solve: you need to know at least one side and two other measures (either two sides, two angles, or one side and one angle):

1. Two angles & any side (AAS or ASA)
2. Two sides & an angle opposite one of them (SSA)
3. Three sides (SSS)
4. Two sides & their included angle (SAS)

Note: First two cases can be solved using ***Law of Sines,*** whereas the last two cases require the *Law of Cosines* (next section).

Law of Sines

If ABC is a triangle with sides a, b, and c then

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Note: reciprocal form is also true
(see proof on page 487)

Example 1: Two angles and One side – AAS

For the triangle with C = 108 degrees, B = 24 degrees and b = 25. Find the remaining sides and angles.

Example 2: Two angles and One side – ASA

A pole tilts toward the sun at an 8 degree angle from the vertical, and it casts an 22 foot shadow. The angle of elevation from the tip of the shadow to the top of the pole is 43 degrees. How tall is the pole?

Ambiguous Case (SSA)

If given two sides and one angle, there are three possible situations:

1. No such triangle exists
2. One such triangle exists
3. Two distinct triangles exist

Lets go to page 430 so you can see what we're talking about

Example 3: Single Solution – SSA

For a triangle, $a = 20$ inches, $b = 10$ inches, and $A = 38$ degrees. Find the remaining sides and angles.

Solution: For a triangle, a = 20 inches, b = 10 inches, and A = 38 degrees. Find the remaining sides and angles.

$$\frac{20}{\sin 38} = \frac{10}{\sin B} \rightarrow B = \mathbf{17.93}\ \textbf{degree}$$

C = 180-17.93-38 = **124.07 degree**

$$\frac{20}{\sin 38} = \frac{c}{\sin 124.07} \rightarrow c = \mathbf{26.91}$$

Example 4: Two Solution – SSA

Find two triangles for which a = 12, b = 31 and A = 20.5 degrees

Area of an Oblique Triangle

Note/remember the height of a triangle can be found:
h = b sin A

$$A = \frac{1}{2}(base)height \rightarrow = \frac{1}{2}(c)(bsinA) \rightarrow = \boldsymbol{\frac{1}{2}bc\sin A}$$

$$\boldsymbol{\frac{1}{2}ab\sin C = \frac{1}{2}bc\sin A = \frac{1}{2}ac\sin B}$$

Example 5: Finding the Area of a Triangular Lot

Find the area of a triangular lot having two sides of lengths 90 and 52 and an included angle of 102 degrees.

6.2 Law of Cosines

Objective: Use law of cosines to solve (using SSS, SAS, SSA) and find area (Heron's Area Formula) of oblique triangles while applying it to real life situations

Why can't you use Law of Sines in these cases?!

Law of Cosines (proof on page 488)

Standard Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = a^2 + b^2 - 2ab\cos C$$

Alternative Form

$$\cos A = \frac{b^2+c^2-a^2}{2bc}$$

$$\cos B = \frac{a^2+c^2-b^2}{2ac}$$

$$\cos C = \frac{a^2+b^2-c^2}{2ab}$$

Example 1: Three sides of a triangle – SSS

For the triangle with a = 8 feet, b = 19 feet and c = 14 feet. Find the remaining sides and angles.

[Good idea to find angle opposite largest side 1^{st}

If biggest angle is obtuse, others acute… if acute all acute]

Solution:

$$\cos B = \frac{(8)^2+(14)^2-(19)^2}{2(8)(14)}$$

$\cos B = -0.45089$ $\mathbf{B = 116.80°}$

[Can use Law of Cosine to keep going, but perhaps easier to use Law of Sines now.]

$\mathbf{A = 22.08°}$ $\mathbf{C = 41.12°}$

Example 2: Two Sides and the Included Angle – SAS

For the triangle with b = 9 feet, c = 12 feet and $A = 25°$. Find the remaining sides and angles.

Solution:

$a^2 = (9)^2 + (12)^2 - 2(9)(12)\cos 25$ $\boldsymbol{a = 5.4072}$

[Can use Law of Cosine to keep going, but perhaps easier to use Law of Sines now.]

$$\frac{\sin B}{b} = \frac{\sin A}{a} \rightarrow \sin B = b\left(\frac{\sin A}{a}\right) \rightarrow 9\left(\frac{\sin 25}{5.4072}\right) \rightarrow 0.7034$$

$B_1 = 44.7°$ $C_1 = 110.3°$

$B_2 = 180 - 44.7 = 135.3°$ $C_2 = 19.7°$

But, because side c is the longest side the C has to be the largest angle.

Therefore, $\boldsymbol{B = 44.7°}$ $\boldsymbol{C = 110.3°}$

Example 3: Application of Law of Cosines

The pitcher's mound in softball field is 43 feet from home plate and the distance from home plate and the distance between the bases is 60 feet. (Note pitcher's mound is not halfway between home and 2nd base). How far is the pitcher's mound from first base?

Solution:

$$h^2 = f^2 + p^2 - 2fp\cos H$$

$$h^2 = 60^2 + 43^2 - 2(60)(43)\cos 45$$

[Diagonal of a square, thus 45 degrees]

$$\boldsymbol{h = 42.43\ feet}$$

Heron’s Area Formula

Given any triangle with sides a, b, and c, the area of the triangle is:

$$\boldsymbol{A = \sqrt{s(s-a)(s-b)(s-c)}}$$

$$where\ s = \frac{(a+b+c)}{2}$$

Example 4: Using Heron's Formula

Find the area of a triangle with $a = 43$, $b = 53$, and $c = 72$

Solution:

$$s = \frac{(43 + 53 + 72)}{2} = 84$$

$$A = \sqrt{84(84-43)(84-53)(84-72)} = \mathbf{1131.89}$$

For Next Time

Law of Sines

Pg 434 #1, 4, 5, 7, 9, 25, 27, 39, 40, 45, 46, 49

Law of Cosines

Pg 441 #5, 7, 9, 13, 27, 29, 33, 43, 53