




# PRE-CALC & TRIG

Day 45



# Bell Work

Prove:

$$\frac{1}{\cos x + 1} + \frac{1}{\cos x - 1} = -2 \csc x \cot x$$

# From Last Time

Day 1: Pg 385 #3-8, 9-13 (odd), 17, 19, 23

Day 2: Pg 385 #10-20 (even), 25, 29, 31

## 5.3 Solving Trig Equations

**Objective:** To solve equations involving trigonometry using algebraic techniques such as combining like terms, and factoring to isolate the trig function.

Solve:  $\sin x = 1 - \sin x$

Hint: Combine like terms

Solve:  $2 \sin x = 1$

$$2 \sin x = 1$$

$$\sin x = \frac{1}{2}$$

$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6}$$

$$x = \frac{\pi}{6} + 2n\pi \quad x = \frac{5\pi}{6} + 2n\pi$$

Solve:  $2\sin^2 x - \sin x - 1 = 0$

Hint: Factor

Solve:  $2\sin^2 x - \sin x - 1 = 0$

Hint: Factor

$$(2 \sin x + 1)(\sin x - 1) = 0$$

$$(2 \sin x + 1) = 0$$

$$\sin x = -1/2$$

$$x = \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$(\sin x - 1) = 0$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$\pi$



Solve  $\cot x \cos^2 x = 2 \cot x$

Factor

Solve  $\cot x \cos^2 x = 2 \cot x$   
Factor

$$\cot x \cos^2 x - 2 \cot x = 0$$

$$\cot x (\cos^2 x - 2) = 0$$

$$\cot x = 0$$

$$\begin{aligned}\cos^2 x - 2 &= 0 \\ \cos^2 x &= 2\end{aligned}$$

$$\cos x = \sqrt{2}$$

$$x = \frac{\pi}{2} + n\pi$$

***No solution,  
out of range of cos***

Solve:  $2\sin^2 x + 3 \cos x - 3 = 0$

Hint: Write as a single trig function

$$\text{Solve: } 2\sin^2 x + 3 \cos x - 3 = 0$$

Hint: Write as a single trig function

$$2(1 - \cos^2 x) + 3 \cos x - 3 = 0$$

$$2 - 2\cos^2 x + 3 \cos x - 3 = 0$$

$$-2\cos^2 x + 3 \cos x - 1 = 0$$

$$2\cos^2 x - 3 \cos x + 1 = 0$$

$$(2 \cos x - 1)(\cos x - 1) = 0$$

$$(2 \cos x - 1) = 0$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$(\cos x - 1) = 0$$

$$\cos x = 1$$

$$x = 0$$

Find all solutions of  
 $\cos x + 1 = \sin x$  *in the interval*  $[0, 2\pi)$

Hint: Square and convert to quadratic

$$\cos x + 1 = \sin x \quad \text{in the interval } [0, 2\pi)$$

$$\cos^2 x + 2 \cos x + 1 = \sin^2 x \quad \text{square both sides}$$

$$\cos^2 x + 2 \cos x + 1 = 1 - \cos^2 x \quad \text{Pythagorean id}$$

$$\cos^2 x + \cos^2 x + 2 \cos x + 1 - 1 + \cos^2 x$$

$$2\cos^2 x + 2 \cos x = 0$$

$$2 \cos x (\cos x + 1) = 0$$

$$2 \cos x = 0$$

$$\cos x + 1 = 0$$

$$\cos x = 0$$

$$\cos x = -1$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

and

$$x = \pi$$

But.....

But... since you squared in the beginning  
you need to check for extraneous  
solutions...

$$\cos \frac{\pi}{2} + 1 = \sin \frac{\pi}{2} \rightarrow 0 + 1 = 1 \quad \frac{\pi}{2} \text{ is solution}$$

$$\cos \frac{3\pi}{2} + 1 = \sin \frac{3\pi}{2} \rightarrow 0 + 1 \neq -1 \quad \frac{3\pi}{2} \text{ is extraneous}$$

$$\cos \pi + 1 = \sin \pi \rightarrow -1 + 1 = 0 \quad \pi \text{ is solution}$$

Solve  $2 \cos 3x - 1 = 0$

Hint: Multiple angles



Solve  $2 \cos 3x - 1 = 0$

Hint: Multiple angles

$$2 \cos 3x = 1$$

$$\cos 3x = 1/2$$

$$3x = \frac{\pi}{3} + 2n\pi$$

$$3x = \frac{5\pi}{3} + 2n\pi$$

$$x = \frac{\pi}{9} + \frac{2n\pi}{9}$$

$$x = \frac{5\pi}{9} + \frac{2n\pi}{9}$$

Solve  $\sec^2 x - 2 \tan x = 4$

$$\text{Solve } \sec^2 x - 2 \tan x = 4$$

$$(1 + \tan^2 x) - 2 \tan x - 4 = 0$$

*Pythagorean id*

$$\tan^2 x - 2 \tan x - 3 = 0$$

$$(\tan x - 3)(\tan x + 1) = 0$$

$$\tan x - 3 = 0$$

$$\tan x + 1 = 0$$

$$\tan x = 3$$

$$\tan x = -1$$

$$x = \arctan 3$$

$$x = -\frac{\pi}{4}$$

Recall range of

inverse tangent is  $(-\frac{\pi}{2}, \frac{\pi}{2})$

$$\mathbf{x = \arctan 3 + n\pi}$$

$$\mathbf{x = -\frac{\pi}{4} + n\pi}$$

You can estimate  $\arctan$

# For Next Time

Day 1: Pg 394 #5-7, 11-19 (odd)

Day 2: Pg 394 #9, 27-30, 33-34, 49

Day 3: Pg 394 #21, 39-42, 63, 75