PRE-CALC & TRIG

Day 46

Bell Work

When $x = 3$ and $y = 5$, by how much does the value of $3x^2 - 2y$ exceed the value of $2x^2 - 3y$?

F. 3

G. 14

H. 17

I. 20

J. 51

From Last Time

Day 1: Pg 394 #5-7, 11-19 (odd)

Day 2: Pg 394 #9, 27-30, 33-34, 49

Day 3: Pg 394 #21, 39-42, 63, 75

5.3 Solving Trig Equations

Objective: To solve equations involving trigonometry using algebraic techniques such as combining like terms, and factoring to isolate the trig function.

Solve: $\sin x = 1 - \sin x$

Hint: Combine like terms

Solve: 2 sin x = 1

$$2\sin x = 1$$

$$\sin x = ½$$

$$x = \frac{\pi}{6} \quad x = \frac{5\pi}{6}$$

$$\boldsymbol{x = \frac{\pi}{6} + 2\mathbf{n}\pi \quad x = \frac{5\pi}{6} + 2\mathbf{n}\pi}$$

Solve: $2sin^2x - \sin x - 1 = 0$
Hint: Factor

Solve: $2sin^2x - \sin x - 1 = 0$
Hint: Factor

$(2\sin x + 1)(\sin x - 1) = 0$

π

$(2\sin x + 1) = 0$

$(\sin x - 1) = 0$

sin x = -1/2

sin x = 1

$\boldsymbol{x = \frac{7\pi}{6}, \frac{11\pi}{6}}$

$\mathbf{x} = \boldsymbol{\frac{\pi}{2}}$

Solve $\cot x \cos^2 x = 2\cot x$

Factor

Solve $\cot x \cos^2 x = 2\cot x$

Factor

$\cot x \cos^2 x - 2\cot x = 0$

$\cot x(\cos^2 x - 2) = 0$

$\cot x = 0$

$\cos^2 x - 2 = 0$

$\cos^2 x = 2$

$\cos x = \sqrt{2}$

x$= \frac{\boldsymbol{\pi}}{\mathbf{2}} + \boldsymbol{n\pi}$

$\boldsymbol{No\ solution}$,

$\boldsymbol{out\ of\ range\ of\ cos}$

Solve: $2sin^2x + 3\cos x - 3 = 0$
Hint: Write as a single trig function

Solve: $2sin^2x + 3\cos x - 3 = 0$

Hint: Write as a single trig function

$2(1 - cos^2x) + 3\cos x - 3 = 0$

$2 - 2cos^2x + 3\cos x - 3 = 0$

$-2cos^2x + 3\cos x - 1 = 0$

$2cos^2x - 3\cos x + 1 = 0$

$(2\cos x - 1)(\cos x - 1) = 0$

$(2\cos x - 1) = 0$ $(\cos x - 1) = 0$

$\cos x = ½$ $\cos x = 1$

$\boldsymbol{x = \frac{\pi}{3}, \frac{5\pi}{3}}$ $\mathbf{x = 0}$

Find all solutions of
$\cos x + 1 = \sin x \quad in\ the\ interval\ [0, 2\pi)$
Hint: Square and covert to quadratic

$$\cos x + 1 = \sin x \quad in\ the\ interval\ [0, 2\pi)$$

$\cos^2 x + 2\cos x + 1 = \sin^2 x$ *square both sides*

$\cos^2 x + 2\cos x + 1 = 1 - \cos^2 x$ *Pythagorean id*

$\cos^2 x + \cos^2 x + 2\cos x + 1 - 1 + \cos^2 x$

$2\cos^2 x + 2\cos x = 0$

$2\cos x(\cos x + 1) = 0$

$2\cos x = 0$ $\cos x + 1 = 0$

$\cos x = 0$ $\cos x = -1$

$\boldsymbol{x = \frac{\pi}{2}\ ,\ \frac{3\pi}{2}}$ and $\boldsymbol{x = \pi}$

But.....

But… since you squared in the beginning you need to check for extraneous solutions…

$$\cos\frac{\boldsymbol{\pi}}{\mathbf{2}} + 1 = \sin\frac{\boldsymbol{\pi}}{\mathbf{2}} \;\rightarrow\; 0 + 1 = 1 \qquad \frac{\boldsymbol{\pi}}{\mathbf{2}}\ \textit{is solution}$$

$$\cos\frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}} + 1 = \sin\frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}} \;\rightarrow 0 + 1 \neq -1 \qquad \frac{\mathbf{3}\boldsymbol{\pi}}{\mathbf{2}}\ \textit{is extraneous}$$

$$\cos\boldsymbol{\pi} + 1 = \sin\boldsymbol{\pi} \rightarrow -1 + 1 = 0 \qquad \boldsymbol{\pi}\ \textit{is solution}$$

Solve $2\cos 3x - 1 = 0$

Hint: Multiple angles

Solve $2\cos 3x - 1 = 0$
Hint: Multiple angles

$2\cos 3x = 1$

$\cos 3x = 1/2$

$3x = \frac{\pi}{3} + 2n\pi$ $\quad$ $3x = \frac{5\pi}{3} + 2n\pi$

$\boldsymbol{x = \frac{\pi}{9} + \frac{2n\pi}{9}}$ $\quad$ $\boldsymbol{x = \frac{5\pi}{9} + \frac{2n\pi}{9}}$

Solve $\sec^2 x - 2\tan x = 4$

Solve $\sec^2 x - 2\tan x = 4$

$(1+\tan^2 x) - 2\tan x - 4 = 0$ *Pythagorean id*

$\tan^2 x - 2\tan x - 3 = 0$

$(\tan x - 3)(\tan x + 1) = 0$

$\tan x - 3 = 0$ $\tan x + 1 = 0$

$\tan x = 3$ $\tan x = -1$

$x = \arctan 3$ $x = -\frac{\pi}{4}$ Recall range of inverse tangent is $(-\frac{\pi}{2}, \frac{\pi}{2})$

$\boldsymbol{x = \arctan 3 + n\pi}$ $\boldsymbol{x = -\frac{\pi}{4} + n\pi}$

You can estimate arctan

5.4 Sum and Difference Formulas

Objective: Use sum and difference formulas to evaluate trig functions, verify identities, and solve trig equations

Sum and Difference Formulas *[page 398]*

$$\sin(u + v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u - v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u + v) = \cos u \cos v - \sin u \sin v$$

$$\cos(u - v) = \cos u \cos v + \sin u \sin v$$

Sum and Difference Formulas *[page 398]*

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

Example 1 (Evaluate a Trig Function)

Find the exact value of $\sin\frac{\pi}{12}$

Find the exact value of $\sin\frac{\pi}{12}$

Since $\frac{\pi}{12}=\frac{\pi}{3}-\frac{\pi}{4}$ then $\sin\frac{\pi}{12}=\sin\left(\frac{\pi}{3}-\frac{\pi}{4}\right)$

$$\sin\frac{\pi}{3}\cos\frac{\pi}{4}-\cos\frac{\pi}{3}\sin\frac{\pi}{4}=\frac{\sqrt{3}}{2}\frac{\sqrt{2}}{2}-\frac{1}{2}\frac{\sqrt{2}}{2}=\frac{\sqrt{6}-\sqrt{2}}{4}$$

Example 2 (Evaluate a Trig Function)

Find the exact value of $\cos 15°$

Find the exact value of $\cos 15°$

Since 15 = 45 – 30 then $\cos 15 = \cos(45 - 30)$

$$\cos 45 \cos 30 + \sin 45 \sin 30 = \frac{\sqrt{2}}{2}\frac{\sqrt{3}}{2} + \frac{\sqrt{2}}{2}\frac{1}{2} = \frac{\sqrt{6}+\sqrt{2}}{4}$$

Example 3 (Simplify the Expression)

Simplify: $\sin(\theta - \frac{\pi}{2})$

Simplify: $\sin(\theta - \frac{\pi}{2})$

$$\sin\theta\cos\frac{\pi}{2} - \cos\theta\sin\frac{\pi}{2} = (\sin\theta)(0) - (\cos\theta)(1) = -(\cos\theta)$$

For Next Time

Pg 402 #7-13 (odd), 37-40