PRE-CALC & TRIG

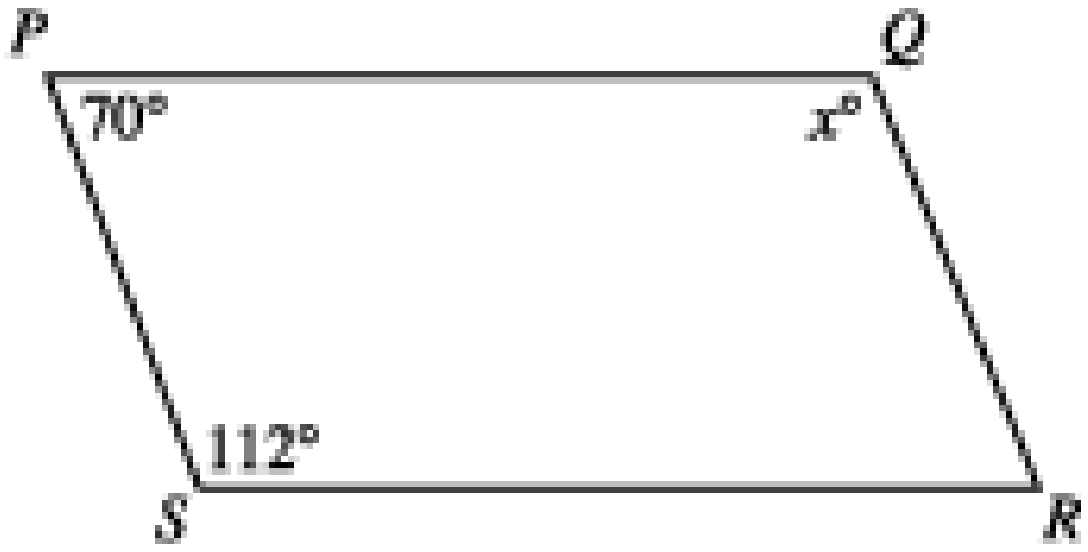
Day 48

Bell Work

In quadrilateral *PQRS* below,
sides *PS* and *QR* are parallel for what value of *x* ?

Individual Question

A.182
B.132
C.112
D.110
E.70



From Last Time

Pg 402 #7-13 (odd), 37-40

5.5 Multiple Angle and Product-To-Sum Formulas

Objective: Use formulas to rewrite and evaluate trig functions

Double-Angle Formulas [page 405]

$$\sin 2u = 2\sin u \cos u$$

$$\cos 2u = \cos^2 u - \sin^2 u \rightarrow 2\cos^2 u - 1 \rightarrow 1 - 2\sin^2 u$$

$$\tan 2u = \frac{2\tan u}{1 - \tan^2 u}$$

Example 1 (Solving Multiple Angle)

Solve $2\cos x + \sin 2x = 0$

Solve $2\cos x + \sin 2x = 0$

$2\cos x + 2\sin x \cos x = 0$ Double-Angle Formula

$2\cos x(1 + \sin x) = 0$ Factor

$2\cos x = 0 \quad and \quad (1 + \sin x) = 0$ Zero-Product… Solve

$x = \frac{\pi}{2}, \frac{3\pi}{2} \qquad x = \frac{3\pi}{2}$ Solutions in $[0, 2\pi)$

So in general the solution is:

$$\boldsymbol{x = \frac{\pi}{2} + 2n\pi \quad and \quad x = \frac{3\pi}{2} + 2n\pi}$$

Power-Reducing Formulas [page 407]

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Example 2 (Power Reducing Formulas)

Rewrite $\sin^4 x$ as a sum of first powers of cosines of multiple angles

$\sin^4 x$

$$\sin^4 x = (\sin^2 x)^2 = (\frac{1-\cos 2u}{2})^2 = (\frac{1-\cos 2u}{2})(\frac{1-\cos 2u}{2})$$ Power reduce, FOIL

$$= \frac{1-2\cos 2x+\cos^2 2x}{4}$$ FOIL, simplify

$$= \frac{1}{4}(1 - 2\cos 2x + \cos^2 2x)$$ Factor out 1/4

$$= \frac{1}{4}\left(1 - 2\cos 2x + \frac{1+\cos 4x}{2}\right)$$ Power Reduce

$$= \left(\frac{1}{4} - \frac{1}{2}\cos 2x + \frac{1}{8} + \frac{\cos 4x}{8}\right)$$ Distribute 1/4

$$= \boldsymbol{\frac{1}{8}(3 - 4\cos 2x + \cos 4x)}$$ Factor 1/8

Half-Angle Formulas [page 408]

$$\sin\frac{u}{2} = \pm\sqrt{\frac{1-\cos \mathrm{u}}{2}}$$

$$\cos\frac{u}{2} = \pm\sqrt{\frac{1+\cos \mathrm{u}}{2}}$$

$$\tan\frac{u}{2} = \frac{1-\cos \mathrm{u}}{\sin u} = \frac{\sin u}{1+\cos \mathrm{u}}$$

Note: The sign of $\sin\frac{u}{2}$ and $\cos\frac{u}{2}$ depends on the quadrant in which $\frac{u}{2}$ lies.

Example 3 (Half-Angle Formulas)

Find the exact value of $\sin 105°$

$Note: 105\ is\ half\ of\ 210$

Find the exact value of $\sin 105°$

$$\sin 105 = \sin\frac{210}{2} = +\sqrt{\frac{1-\cos 210}{2}}$$

Since 105 is in II,

sin 105 is positive

$$\sin 105 = \sqrt{\frac{1-(-\sqrt{3}/2)}{2}}$$

Sub from Unit Circle

$$\sin 105 = \frac{\sqrt{2+\sqrt{3}}}{2}$$

Remove radical from

denominator, simplify

Product-To-Sum Formulas [page 409]

$$\sin \mathrm{u} \sin v = \frac{1}{2}[\cos(u - v) - \cos(u + v)]$$

$$\cos \mathrm{u} \cos v = \frac{1}{2}[\cos(u - v) + \cos(u + v)]$$

$$\sin \mathrm{u} \cos v = \frac{1}{2}[\sin(u + v) + \sin(u - v)]$$

$$\cos \mathrm{u} \sin v = \frac{1}{2}[\sin(u + v) - \sin(u - v)]$$

Example 4 (Product-To-Sum Formulas)

Rewrite the product $\cos 6x \sin 3x$

Rewrite the product $\cos 6x \sin 3x$

$$\cos 6x \sin 3x = \frac{1}{2}[\sin(6x + 3x) - \sin(6x - 3x)]$$

$$= \boldsymbol{\frac{1}{2}\sin(9x) - \frac{1}{2}\sin(3x)}$$

Sum-To-Product Formulas [page 410]

$$\sin \mathrm{u} + \sin v = 2\sin\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\sin \mathrm{u} - \sin v = 2\cos\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

$$\cos \mathrm{u} + \cos v = 2\cos\left(\frac{u+v}{2}\right)\cos\left(\frac{u-v}{2}\right)$$

$$\cos \mathrm{u} - \cos v = -2\sin\left(\frac{u+v}{2}\right)\sin\left(\frac{u-v}{2}\right)$$

Example 5 (Sum-To-Product Formulas)

Find the exact value of $\cos 195° + \cos 105°$

Find the exact value of $\cos 195° + \cos 105°$

$$\cos 195° + \cos 105° = 2\cos\left(\frac{195° + 105°}{2}\right)\cos\left(\frac{195° - 105°}{2}\right)$$

$$2\cos(150°)\cos(45°) = 2\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) = -\frac{\boldsymbol{\sqrt{6}}}{\mathbf{2}}$$

For Next Time

Pg 413 #11-21, 29-31, 43-49, 53-55, 59-61, 81-83, 91-93, 99, 107-109, 111-113

(All Odd)