# Bell Work: Simplify

$1.)\ \sin t \csc t - \cos t \sec t$

$2.)\ \csc x \sec x - \tan x$

$3.)\ \dfrac{1 - sin^2 x}{csc^2 x - 1}$

# From Last Time

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57, 63-64, 67, 71

# PRE-CALC TRIG

Day 34

# 4.3 Right Triangle Trigonometry

Objective: Evaluate trig functions of acute angles and use fundamental trig identities while solving real-life problems

# Right Triangle Definitions of Trigonometric Functions

Let $\theta$ be an acute angle of a right triangle. The six trig functions of angle $\theta$ are defined:

*Cosine, sine, tangent, secant, cosecant, cotangent*

$$\mathbf{\cos\theta = \frac{adj}{hyp}} \qquad \mathbf{\sec\theta = \frac{hyp}{adj}}$$

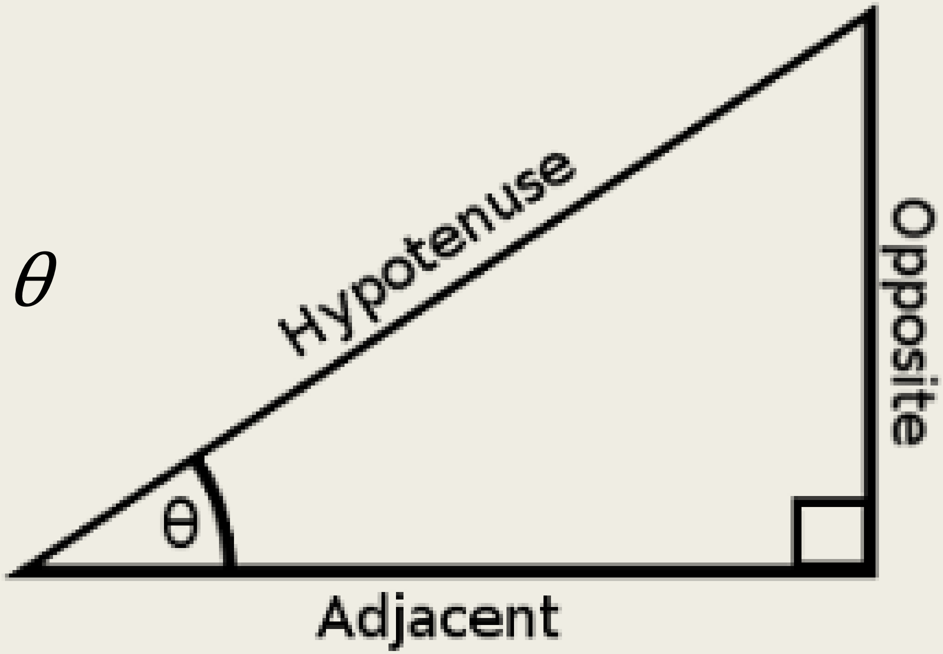
$$\mathbf{\sin\theta = \frac{opp}{hyp}} \qquad \mathbf{\csc\theta = \frac{hyp}{opp}}$$

$$\mathbf{\tan\theta = \frac{opp}{adj}} \qquad \mathbf{\cot\theta = \frac{adj}{opp}}$$

# ----SOH----CAH----TOA----

The abbreviations in the previous slide: opp, adj, hyp represent the lengths of the sides of the right triangle (as seen in the picture)

opp = length of side opposite given $\theta$
adj = length of side adjacent to given $\theta$
hyp = length for of hypotenuse



Example:
Evaluate the 6 trigonometric functions for $\theta$ for a triangle with a hypotenuse of 5 and side opposite of $\theta$ equal to 3.

$\mathbf{\cos\theta} =$

$\mathbf{\sec\theta} =$

$\mathbf{\sin\theta} =$

$\mathbf{\csc\theta} =$

$\mathbf{\tan\theta} =$

$\mathbf{\cot\theta} =$

# Fundamental Trigonometric Identities

- These identities will be vital as we move through this chapter and into second semester

# Reciprocal Identities

$$\cos\theta = \frac{1}{sec\ \theta}$$

$$\sec\theta = \frac{1}{cos\ \theta}$$

$$\sin\theta = \frac{1}{csc\ \theta}$$

$$\csc\theta = \frac{1}{sin\ \theta}$$

$$\tan\theta = \frac{1}{cot\ \theta}$$

$$\cot\theta = \frac{1}{tan\ \theta}$$

# Quotient Identities

$$\tan \theta = \frac{\sin \theta}{\cos \theta}$$

$$\cot \theta = \frac{\cos \theta}{\sin \theta}$$

# *Pythagorean Identities*

$$\boldsymbol{sin^2\theta + cos^2\theta = 1}$$

*Therefore,*

$$\boldsymbol{1 + tan^2\theta = sec^2\theta}$$

But how?

$$\boldsymbol{1 + cot^2\theta = csc^2\theta}$$

# Example

Let $\theta$ be an acute angle such that $\sin\theta = 0.6$. Find the value of:

1.) $\cos\theta$

2.) $\tan\theta$

# Solution:

Let $\theta$ be an acute angle such that $\sin\theta = 0.6$. Find the value of:

1.) $\cos\theta$

$$(0.6)^2 + cos^2\theta = 1 => \ cos^2\theta = 1 - .36$$

$$cos^2\theta = 0.64$$

$$cos\ \theta = \sqrt{0.64}$$

$$\boldsymbol{cos\ \theta = 0.8}$$

2.) $\tan\theta$

$$\tan\theta = \frac{0.6}{0.8}$$

$$\mathbf{tan}\ \boldsymbol{\theta} = \mathbf{0.75}$$

# Example: Applications to Real Life

You’re standing just down the street from the 478 foot tall Woodmen Building in downtown Omaha. The measure of the angle of elevation from where you are standing to the top of the Woodmen building is 68.4°. How far away from the building are you standing?

# Solution:

You're standing just down the street from the 478 foot tall Woodmen Building in downtown Omaha. The measure of the angle of elevation from where you are standing to the top of the Woodmen building is 68.4°.  How far away from the building are you standing?

$$\tan\theta = \frac{opp}{adj}$$

$$\tan 68.4° = \frac{478}{x} => x \tan 68.4° = 478$$

$$x = \frac{478}{\tan 68.4°}$$

**x = 189.25 feet**

# Example: Second Application

A historic lighthouse is 200 yards directly south from a bike path along a lake. A walk way to the lighthouse is 400 yards long. Find the acute angle $\theta$ between the bike path and the walkway.

# Example: Second Application

A historic lighthouse is 200 yards directly south from a bike path along a lake. A walk way to the lighthouse is 400 yards long. Find the acute angle $\theta$ between the bike path and the walkway.

Note: path is adjacent, walk way is hypotenuse, and 200 yards is opposite

$$\cos\theta = \frac{adj}{hyp} = \frac{200}{400} = \frac{1}{2} => \cos\theta = \frac{1}{2}$$

This is true at 60°

# 4.4 Trig Functions of Any Angle

Objective: Evaluate trig functions of any angles

# Definitions of Trigonometric Functions of Any Angle

Let $\theta$ be an angle in standard position with (x, y) a point on the terminal side of $\theta$ and $r = \sqrt{x^2 + y^2} \neq 0$.

$$\boldsymbol{\cos\theta = \frac{x}{r}} \qquad \boldsymbol{\sec\theta = \frac{r}{x},\ \ x \neq 0}$$

$$\boldsymbol{\sin\theta = \frac{y}{r}} \qquad \boldsymbol{\csc\theta = \frac{r}{y},\ \ y \neq 0}$$

$$\boldsymbol{\tan\theta = \frac{y}{x},\ \ x \neq 0} \qquad \boldsymbol{\cot\theta = \frac{x}{y},\ \ y \neq 0}$$

Note: See picture on page 310 for image of what is meant by x, y, and r.

Example: Let (-6, 8) be a point on the terminal side of $\theta$. Find the cosine, sine, and tangent of $\theta$. Also find the secant, cosecant, and cotangent.

Example: Let (-6, 8) be a point on the terminal side of $\theta$. Find the cosine, sine, and tangent of $\theta$. Also find the secant, cosecant, and cotangent.

$$r = \sqrt{(-6)^2 + (8)^2} = \sqrt{36 + 64} = \sqrt{100} = 10...$$

$$\cos\theta = \frac{-6}{10} \qquad \sin\theta = \frac{8}{10} \qquad \tan\theta = \frac{8}{-6}$$

$$\sec\theta = \frac{10}{-6} \qquad \csc\theta = \frac{10}{8} \qquad \cot\theta = \frac{-6}{8}$$

Example: Given that $\tan\theta = -\frac{4}{7}$ and $\cos\theta < 0$, find $\sin\theta$ and $\sec\theta$.

-- Note that $\theta$ is in Quadrant II because that is the only quadrant in which tangent is negative and cos is negative.

Example: Given that $\tan\theta = -\frac{4}{7}$ and $\cos\theta < 0$, find $\sin\theta$ and $\sec\theta$.

$\tan\theta = -\frac{4}{7}$ so you know that $y = 4 \ and \ x = -7$

So, $r = \sqrt{16 + 49} = \sqrt{65}$

$\sin\theta = \frac{y}{r} = \frac{4}{\sqrt{65}} = 0.496$

$\sec\theta = \frac{r}{x} = \frac{\sqrt{65}}{-7} = -1.152$

# Evaluating Trig Functions of Any Angle

To find the value of a trig function of any angle $\theta$

Determine the function value for the associated reference angle $\theta'$ (reference angle)

Depending on the quadrant in which $\theta$ lies, affix the appropriate sign to the function value.

# Reference Angles

*Let $\theta$ be an angle in standard position. The reference angle is the acute angle $\theta'$ formed by the terminal side of $\theta$ and the horizontal axis.*

| Quadrant II | Quadrant III | Quadrant IV |
|---|---|---|
| $\theta' = \pi - \theta(radian)$ | $\theta' = \theta - \pi(radian)$ | $\theta' = 2\pi - \theta(radian)$ |
| $\theta' = 180 - \theta(degree)$ | $\theta' = \theta - 180(degree)$ | $\theta' = 2\pi - 180(degree)$ |

Note: See Figure 4.39 on page 312 for a visual

# Example

Find the reference angle $\theta'$

$1.)\ \theta = 290°$

$2.)\ \theta = 3.4$

# Example

$\theta = 290°$ $\theta' = 360° - 290° = \mathbf{70°}$

$\theta = 3.4$ Note: $\frac{\pi}{2} \approx 1.57$ $\pi \approx 3.14$ $\frac{3\pi}{2} \approx 4.71$

Therefore 3.4 radians is between $\pi\ and\ \frac{3\pi}{2}$

$(in\ otherwords\ the\ 3rd\ quadrant\ )$

$\theta' = 3.4 - \pi \approx \mathbf{0.258}\ \boldsymbol{radians}$

# For Next Time

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Get a Head Start…

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