CSL703: Logic for Computer Science Oct 16, 2016

Assignment 2

The first assignment implements tautology checker, with the following proposition structure.

```
datatype Prop =
ATOM of string |
NOT of Prop |
AND of Prop * Prop |
OR  of Prop * Prop |
IMP  of Prop * Prop
```

In this assignment, you will implement a generic proof system, and verify its soundness. The proof tree and proof rules may be modeled using the following datatypes:

```
type Sequent = Prop list * Prop
```

```
datatype Proof =
Axiom of Sequent |
UnaryInf of Proof * Sequent |
BinaryInf of Proof * Proof * Sequent
```

```
datatype ProofRule =
AxiomR of Sequent |
UnaryInfR of Sequent * Sequent |
BinaryInfR of Sequent * Sequent * Sequent
```

where, the last sequent in the proof rules will correspond to the conclusion. You will be given a list of proofrules as the input. You need to-

1. Verify that the proof system is sound, i.e., every derived statement in the system is true. Ideally, soundness is proved by showing that we cannot derive contradiction. But it can also be seen in terms of preserving truth. If each proof rule preserves truth, i.e., every model that satisfies all the hypothesis must also satisfy the conclusion, then by structural induction, each proof tree also preserves truth.

   Therefore, you need to verify that each proof rule is sound. For example, the following rule is sound iff the formula $((p \to q) \wedge p) \to q)$ is valid. For this, you may use the tautology checker from assignment 1.

$$\frac{\vdash p \to q \qquad \vdash p}{\vdash q}$$

   In general, the formula corresponding to proof rules is given by the following table

Proof Rule	Corresponding Formula
$\dfrac{}{\Gamma \vdash \psi}$	$\bigwedge \Gamma \to \psi$
$\dfrac{\Gamma' \vdash \psi'}{\Gamma \vdash \psi}$	$(\bigwedge \Gamma' \to \psi') \to (\bigwedge \Gamma \to \psi)$
$\dfrac{\Gamma' \vdash \psi' \qquad \Gamma'' \vdash \psi''}{\Gamma \vdash \psi}$	$(\bigwedge \Gamma' \to \psi') \wedge (\bigwedge \Gamma'' \to \psi'') \to (\bigwedge \Gamma \to \psi)$

Note that if Γ is empty then $\bigwedge \Gamma = \top$, and $\top \to \psi$ is logically equivalent to just ψ (giving us the formula for modus ponens rule above).

2. Given any proof tree as input, verify that it corresponds to the derivation using the given proof rules. For example,

$$\dfrac{\dfrac{}{\vdash (p \to p) \to (q \to (p \to p))} \qquad \dfrac{}{\vdash p \to p}}{\vdash q \to (p \to p)}$$

Above proof tree can be derived using the following rules,

$$\dfrac{}{\vdash X \to (Y \to X)} \qquad\qquad \dfrac{}{\vdash X \to X} \qquad\qquad \dfrac{\vdash X \to Y \qquad \vdash X}{\vdash Y}$$

This involves unifying proof rules with every node of the proof tree, but only the atoms in proof rules must be treated as variable. For example, to unifying the K-rule with its corresponding node in the tree above, we will obtain the substitution $X/p \to p$ and Y/q.