INF251 Computer Graphics
Autumn 2016
Exercise 05
Due to 23.11.2016

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Problem 1. Splines
Assume you are given six pairwise different control points $p_i, 0 \leq i \leq 5$. We construct three different spline curves from these points. Fill in YES or NO in the following table, according to the relationship of the entries.
(12 points)

the corresponding ...	interpolates both p_0 and p_5	interpolates all p_i	changing p_1 has a global influence on the whole curve	degree of smoothness in all curve points is (C_0, G_1, C_1, C_2, more)	The shape of the curve is dependent on the coordinate system
...5th-order Bezier curve					
...natural cubic spline					
... uniform (not open) quadratic B-spline					

Problem 2. Splines

- $p_{k-1}(t), p_k(t)$ and $p_{k+1}(t)$ are three 2D spline curves that are part of a natural cubic spline interpolation of 2D points and two of them are given as follows:

$$p_{k-1}(t) = \begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot t^3 + \begin{pmatrix} 4 \\ 3 \end{pmatrix} \cdot t^2 + \begin{pmatrix} 1 \\ -4 \end{pmatrix} \cdot t + \begin{pmatrix} 1 \\ 2 \end{pmatrix} \tag{1}$$

$$p_{k+1}(t) = \begin{pmatrix} 1 \\ -1 \end{pmatrix} \cdot t^3 + \begin{pmatrix} -2 \\ 0 \end{pmatrix} \cdot t^2 + \begin{pmatrix} -1 \\ 5 \end{pmatrix} \cdot t + \begin{pmatrix} 5 \\ 5 \end{pmatrix} \tag{2}$$

  1. At which 2D point do $p_k(t)$ and $p_{k+1}(t)$ meet?
  2. What is the tangent vector of the spline curve in that point?

  (6 points)

- Find the one 2D point $\tilde{t}$ of $p_{k-1}(t)$ which has the smallest y-coordinate of all points $p_{k-1}(t)$ (6 points)

- what is the x-coordinate of $p_k \frac{1}{2}$?
  Note 1: being part of this natural cubic spline interpolation, $p_k(t)$ is fully determined (implicitly) by the definitions of $p_{k-1}(t)$ and $p_{k+1}(t)$.
  Note 2: You can save 50% of the computation by only searching for x.
  (16 points)

(28 points)

Problem 3. Splines
Compute the natural cube spline interpolant of the given points:

x	0	1	2	3
f(x)	0	2	1	0

(12 points)

Problem 4. Splines
In the below grids ...

- ... place six control points of a closed, fifth-degree Bezier curve which is min. C^2-continuous in all points (also in the closing point) in the left grid, and label them with p_0, p_1, p_2, p_3, p_4 and p_5. Snap the points to the grid.
  Just mark the locations of these points, with a x and label these locations - you don't have to draw the curve.
- ... place five control points in the right grid in a way that they will form the letter S. Label them accordingly!

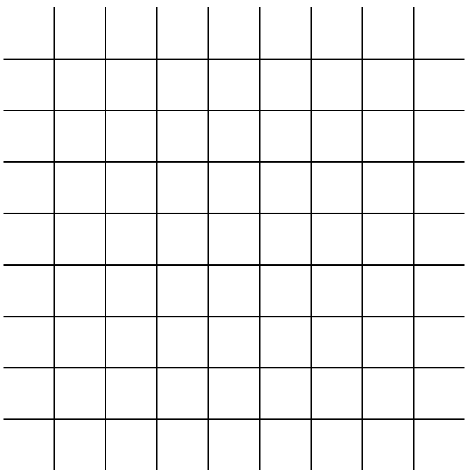

Figure 1: Left Grid

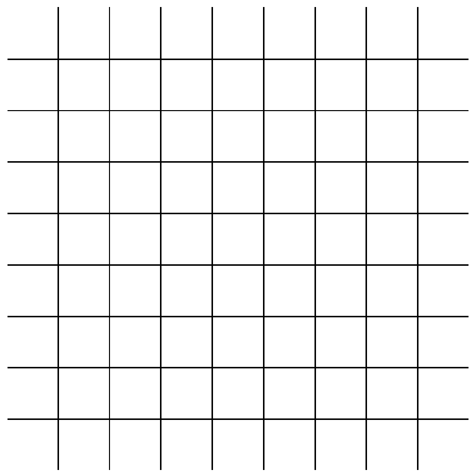

Figure 2: Right Grid

(10 points)