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1 Algorithms (183)

1.0.1 Cormen Edition 3 Exercise 9.1 Question 2 (Page No. 215) top

Prove the lower bound of $\lceil 3n/2 \rceil - 2$ comparisons in the worst case to find both the maximum and minimum of n numbers. (Hint: Consider how many numbers are potentially either the maximum or minimum and investigate how a comparison affects these counts.)

cormen algorithms descriptive

Answer key

1.0.2 Cormen Edition 3 Exercise 9.1 Question 1 (Page No. 215) top

Show that the second smallest of n elements can be found with $n + \lceil lg\, n \rceil - 2$ comparisons in the worst case. (Hint: Also find the smallest element.)

cormen algorithms descriptive

Answer key

1.0.3 Cormen Edition 3 Exercise 1.1 Question 4 (Page No. 11) top

How are the shortest-path and traveling-salesman problems given above similar? How are they different?

cormen algorithms descriptive

1.0.4 Cormen Edition 3 Exercise 1.1 Question 2 (Page No. 11) top

Other than speed, what other measures of efficiency might one use in a real-world setting?

cormen algorithms descriptive

1.0.5 Cormen Edition 3 Exercise 1.1 Question 3 (Page No. 11) top

Select a data structure that you have seen previously, and discuss its strengths and limitations.

cormen algorithms descriptive

1.0.6 Cormen Edition 3 Exercise 1.1 Question 5 (Page No. 11) top

Come up with a real-world problem in which only the best solution will do. Then come up with one in which a solution that is “approximately” the best is good enough.

cormen algorithms descriptive

1.0.7 Cormen Edition 3 Exercise 1.2 Question 1 (Page No. 14) top

Give an example of an application that requires algorithmic content at the application level, and discusses the function of the algorithms involved.

cormen algorithms descriptive

1.0.8 Cormen Edition 3 Exercise 1.1 Question 1 (Page No. 11) top

Give a real-world example that requires sorting or a real-world example that requires computing a convex hull.

cormen algorithms

1.0.9 Cormen Edition 3 Exercise 1.2 Question 3 (Page No. 14) top

What is the smallest value of n such that an algorithm whose running time is $100n^2$ runs faster than an algorithm whose running time is 2^n on the same machine?

cormen algorithms descriptive

1.0.10 Cormen Edition 3 Exercise 2.1 Question 1 (Page No. 22) top

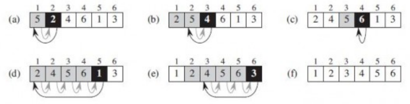


Figure 2.2 The operation of INSERTION-SORT on the array $A = \langle 5, 2, 4, 6, 1, 3 \rangle$. Array indices appear above the rectangles, and values stored in the array positions appear within the rectangles. (a)–(e) The iterations of the **for** loop of lines 1–8. In each iteration, the black rectangle holds the key taken from $A[j]$, which is compared with the values in shaded rectangles to its left in the test of line 5. Shaded arrows show array values moved one position to the right in line 6, and black arrows indicate where the key moves to in line 8. (f) The final sorted array.

Using Figure 2.2 as a model, illustrate the operation of INSERTION-SORT on the array $A = (31, 41, 59, 26, 41, 58)$

cormen algorithms descriptive

Answer key

1.0.11 Cormen Edition 3 Exercise 2.1 Question 4 (Page No. 22-23) top

Consider the problem of adding two n-bit binary integers, stored in two n-element arrays A and B.The sum of the two integers should be stored in binary form in an $(n+1)$-element array C. State the problem formally and write pseudocode for adding the two integers.

cormen algorithms descriptive

Answer key

1.0.12 Cormen Edition 3 Exercise 1.2 Question 2 (Page No. 14) top

Suppose we are comparing implementations of insertion sort and merge sort on the same machine. For inputs of size n, insertion sort runs in $8n^2$ steps, while merge sort runs in $64\ n \lg n$ steps. For which values of n does insertion sort beat merge sort?

cormen algorithms descriptive

1.1 Algorithm Design Techniques (1) top

1.1.1 Algorithm Design Techniques: Cormen Edition 3 Exercise 2.3 Question 7 (Page No. 39) top

Describe a $\Theta(n\ lg\ n)$ time algorithm that, given a set S of n integers and another integer x, determines whether or not there exist two elements in S whose sum is exactly x.

cormen algorithms algorithm-design-techniques descriptive difficult

Answer key

1.2 Asymptotic Notations (15) top

1.2.1 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 1 (Page No. 52) top

Let $f(n)$ and $g(n)$ be asymptotically nonnegative functions. Using the basic definition of Θ notation, prove that $max(f(n), g(n)) = \Theta(f(n) + g(n))$.

cormen algorithms asymptotic-notations descriptive

1.2.2 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 2 (Page No. 52) top

Show that for any real constants a and b, where $b > 0$,
$(n+a)^b = \Theta(n^b)$

cormen algorithms asymptotic-notations descriptive

Answer key

1.2.3 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 3 (Page No. 53) top

Explain why the statement, “The running time of algorithm A is at least $O(n^2)$,” is meaningless.

cormen algorithms asymptotic-notations descriptive

1.2.4 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 4 (Page No. 53) top

Is $2^{n+1} = O(2^n)$? $2^{2n} = O(2^n)$?

cormen algorithms asymptotic-notations descriptive

Answer key

1.2.5 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 6 (Page No. 53) top

Prove that the running time of an algorithm is $\Theta(g(n))$ if and only if its worst-case running time is $O(g(n))$ and its best-case running time is $\Omega(g(n))$.

cormen algorithms asymptotic-notations descriptive

1.2.6 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 7 (Page No. 53) top

Prove that $o(g(n)) \cap \omega(g(n))$ is the empty set.

cormen algorithms asymptotic-notations descriptive

Answer key

1.2.7 Asymptotic Notations: Cormen Edition 3 Exercise 3.1 Question 8 (Page No. 53) top

We can extend our notation to the case of two parameters n and m that can go to infinity independently at different rates.For a given function $g(n, m)$, we denote by $O(g(n, m))$ the set of functions

$O(g(n, m)) = \{f(n, m)$: there exist positive constants c, n_0, and m_0 such that $0 \leq f(n, m) \leq cg(n, m)$ for all $n \geq n_0$ or $m \geq m_0$ $\}$

Give corresponding definitions for $\Omega(g(n, m))$ and $\Theta(g(n, m))$.

cormen algorithms asymptotic-notations descriptive

1.2.8 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 1 (Page No. 60) top

Show that if $f(n)$ and $g(n)$ are monotonically increasing functions, then so are the functions $f(n) + g(n)$ and $f(g(n))$, and if $f(n)$ and $g(n)$ are in addition nonnegative, then $f(n). g(n)$ is monotonically increasing.

cormen algorithms asymptotic-notations descriptive

1.2.9 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 3 (Page No. 60) top

Prove that $n! = \omega(2^n)$ and $n! = o(n^n)$.

cormen algorithms asymptotic-notations descriptive

Answer key

1.2.10 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 4 (Page No. 60) top

Is the function $\lceil \, lg\ n \, \rceil!$ polynomially bounded ? Is the function $\lceil \, lg\ lg\ n \, \rceil!$ polynomially bounded ?

cormen algorithms asymptotic-notations descriptive difficult

1.2.11 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 5 (Page No. 60) top

Which is asymptotically larger: $lg(lg^*n)$ and $lg^*(lgn)$?

cormen algorithms asymptotic-notations descriptive difficult

Answer key

1.2.12 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 6 (Page No. 60) top

Show that the golden ratio ϕ and its conjugate $\hat{\phi}$ both satisfy the equation $x^2 = x + 1$.

cormen algorithms asymptotic-notations descriptive

1.2.13 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 7 (Page No. 60) top

Prove by induction that the i^{th} Fibonacci number satisfies the equality

$F_i = \frac{\phi^i - \hat{\phi}^i}{\sqrt{5}}$

where ϕ is the golden ratio and $\hat{\phi}$ is its conjugate.

cormen algorithms asymptotic-notations

1.2.14 Asymptotic Notations: Cormen Edition 3 Exercise 3.2 Question 8 (Page No. 60) top

Show that $K\ ln\ K = \Theta(n)$ implies $k = \Theta(n/ln\ n)$.

cormen algorithms asymptotic-notations descriptive

1.2.15 Asymptotic Notations: Cormen Edition 3 Exercise 8.1 Question 2 (Page No. 194) top

Obtain asymptotically tight bounds on $lg\ (n!)$ without using Stirling's approximation. Instead, evaluate the summation $\sum_{k=1}^{n} lg\ k$.

cormen algorithms asymptotic-notations descriptive

Answer key

1.3 Binary Heap (1) top

1.3.1 Binary Heap: Cormen Edition 3 Exercise 6.1 Question 5 (Page No. 154) top

Is an array that is in sorted order a min-heap ?

data-structures binary-heap cormen descriptive

Answer key

1.4 Breadth First Search (3) top

1.4.1 Breadth First Search: Cormen Edition 3 Exercise 22.2 Question 6 (Page No. 539) top

Give an example of a directed graph $G = (V, E)$, a source vertex $s \epsilon V$, and a set of tree edges $E_\Pi \subseteq E$ such that for each vertex $v \epsilon V$, the unique simple path in the graph (V, E_Π) from s to v is a shortest path in G, yet the set of edges E_Π cannot be produced by running BFS on G, no matter how the vertices are ordered in each adjacency list.

cormen breadth-first-search graph-algorithms descriptive

Answer key

1.4.2 Breadth First Search: Cormen Edition 3 Exercise 22.2 Question 7 (Page No. 539) top

There are two types of professional wrestlers: "babyfaces" ("good guys") and "heels" ("bad guys"). Between any pair of professional wrestlers, there may or may not be a rivalry. Suppose we have n professional wrestlers and we have a list of r pairs of wrestlers for which there are rivalries. Give an $O(n+r)$ time algorithm that determines whether it is possible to designate some of the wrestlers as babyfaces and the remainder as heels such that each rivalry is between a babyface and a heel. If it is possible to perform such a designation, your algorithm should produce it.

cormen graph-algorithms breadth-first-search descriptive

Answer key

1.4.3 Breadth First Search: Cormen Edition 3 Exercise 22.2 Question 8 (Page No. 539) top

The diameter of a tree $T = (V, E)$ is defined as $max_{u,v\,\epsilon\,V}\ \delta(u, v)$, that is, the largest of all shortest-path distances in the tree. Give an efficient algorithm to compute the diameter of a tree, and analyze the running time of your algorithm.

cormen graph-algorithms breadth-first-search descriptive

Answer key

1.5 Bucket Sort (5) top

1.5.1 Bucket Sort: Cormen Edition 3 Exercise 8.4 Question 1 (Page No. 204) top

```
BUCKET-SORT(A)
let B[0...n-1] be a new array
n = A.length
for i - 0 to n - 1
make B[i] an empty list
for i = 1 to n
insert A[i] into list B[nA[i]]
for i = 0 to n - 1
sort list B[i] with insertion sort
concatenate the lists B[0] , B[1] , ….,B[n-1] together in order
```

illustrate the operation of BUCKET-SORT on the array $A = \langle .79, .13, .16, .64, .39, .20, .89, .53, .71, .42 \rangle$

cormen algorithms sorting bucket-sort descriptive

Answer key

1.5.2 Bucket Sort: Cormen Edition 3 Exercise 8.4 Question 2 (Page No. 204) top

Explain why the worst-case running time for bucket sort is $\Theta(n^2)$. What simple change to the algorithm preserves its linear average-case running time and makes its worst-case running time $O(n\ lg\ n)$?

cormen algorithms sorting bucket-sort descriptive

Answer key

1.5.3 Bucket Sort: Cormen Edition 3 Exercise 8.4 Question 3 (Page No. 204) top

Let X be a random variable that is equal to the number of heads in two flips of a fair coin. What is $E[X^2]$? What is $E^2[X]$?

cormen algorithms sorting bucket-sort expectation descriptive

Answer key

1.5.4 Bucket Sort: Cormen Edition 3 Exercise 8.4 Question 4 (Page No. 204) top

We are given n points in the unit circle, $P_i = (x_i, y_i)$, such that $0 < x_i^2 + y_i^2 < 1$ for $i = 1, 2, \dots, n$.Suppose that the points are uniformly distributed; that is, the probability of finding a point in any region of the circle is proportional to the area of that region. Design an algorithm with an average-case running time of $\Theta(n)$ to sort the n points by their distances $d_i = \sqrt{x_i^2 + y_i^2}$ from the origin. (Hint: Design the bucket sizes in BUCKET-SORT to reflect the uniform distribution of the points in the unit circle.)

cormen algorithms sorting bucket-sort descriptive difficult

1.5.5 Bucket Sort: Cormen Edition 3 Exercise 8.4 Question 5 (Page No. 204) top

A probability distribution function $P(x)$ for a random variable X is defined by $P(x) = Pr\{X \leq x\}$.Suppose that we draw a list of n random variables $X_1, X_2, \dots, X_n$ from a continuous probability distribution function P that is computable in $O(1)$ time. Give an algorithm that sorts these numbers in linear averagecase time.

cormen algorithms sorting bucket-sort descriptive difficult

1.6 Countingsort (4) top

1.6.1 Countingsort: Cormen Edition 3 Exercise 8.2 Question 1 (Page No. 196) top

```
COUNTING-SORT(A, B, k)
```

```
1      let C[0,…,k] be a new array
2      for i = 0 to k
3      C[i] = 0
4      for j = 1 to A.length
5      C[A[j]] = C[A[j]] + 1
6      // C[i] now contains the number of elements equal to i .
7      for i =1 to k
8      C[i] = C[i] + C[i-1]
9      // C[i] now contains the number of elements less than or equal to i .
10     for j = A.length downto 1
11     B[C[A[j]]] = A[j]
12     C[A[j]] = C[A[j]] - 1
```

illustrate the operation of COUNTING-SORT on the array $A = \langle 6, 0, 2, 0, 1, 3, 4, 6, 1, 3, 2 \rangle$

cormen algorithms sorting countingsort descriptive

1.6.2 Countingsort: Cormen Edition 3 Exercise 8.2 Question 2 (Page No. 196) top

Prove that COUNTING-SORT is stable.

cormen algorithms sorting countingsort descriptive

Answer key

1.6.3 Countingsort: Cormen Edition 3 Exercise 8.2 Question 3 (Page No. 196) top

Suppose that we were to rewrite the for loop header in line 10 of the COUNTINGSORT as

```
10   for j = 1 to A.length
```

Show that the algorithm still works properly. Is the modified algorithm stable?

cormen algorithms sorting countingsort descriptive

1.6.4 Countingsort: Cormen Edition 3 Exercise 8.2 Question 4 (Page No. 197) top

Describe an algorithm that, given n integers in the range 0 to k preprocesses its input and then answers any query about how many of the n integers fall into the range $[a..b]$ in $O(1)$ time.Your algorithm should use $\Theta(n+k)$ preprocessing time.

cormen algorithms sorting countingsort descriptive

Answer key

1.7 Divide And Conquer (4) top

1.7.1 Divide And Conquer: Cormen Edition 3 Exercise 4.1 Question 2 (Page No. 74) top

Write pseudo code for the brute-force method of solving the $maximum-subarray$ problem. Your procedure should run in $\Theta(n^2)$ time.

cormen algorithms divide-and-conquer descriptive

1.7.2 Divide And Conquer: Cormen Edition 3 Exercise 4.1 Question 3 (Page No. 74) top

Implement both the brute-force and recursive algorithms for the $maximumsubarray$ problem on your own computer. What problem size n_0 gives the crossover point at which the recursive algorithm beats the brute-force algorithm? Then, change the base case of the recursive algorithm to use the brute-force algorithm whenever the problem size is less than n_0. Does that change the crossover point?

cormen algorithms divide-and-conquer descriptive

1.7.3 Divide And Conquer: Cormen Edition 3 Exercise 4.1 Question 4 (Page No. 74) top

Suppose we change the definition of the $maximum-subarrayproblem$ to allow the result to be an empty subarray, where the sum of the values of an empty subarray is 0. How would you change any of the algorithms that do not allow empty subarrays to permit an empty subarray to be the result?

cormen algorithms divide-and-conquer descriptive

1.7.4 Divide And Conquer: Cormen Edition 3 Exercise 4.1 Question 5 (Page No. 74) top

Use the following ideas to develop a nonrecursive, linear-time algorithm for the maximum-subarray problem. Start at the left end of the array, and progress toward the right, keeping track of the maximum subarray seen so far. Knowing a maximum subarray of $A[1 \ldots j]$, extend the answer to find a maximum subarray ending at index $j+1$ by using the following observation: a maximum subarray of $A[1...j+1]$ is either a maximum subarray of $A[1...j]$ or a subarray $A[1...j+1]$, for some $1 \leq i \leq j+1$. Determine a maximum subarray of the form $A[i \ldots j+1]$ inconstant time based on knowing a maximum subarray ending at index j.

cormen algorithms divide-and-conquer descriptive

1.8 Graph Algorithms (7) top

1.8.1 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 1 (Page No. 592) top

Given an adjacency-list representation of a directed graph, how long does it take to compute the out-degree of every vertex ? How long does it take to compute the in-degrees ?

cormen algorithms graph-algorithms descriptive

Answer key

1.8.2 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 2 (Page No. 592) top

Give an adjacency-list representation for a complete binary tree on 7 vertices. Give an equivalent adjacency-matrix representation. Assume that vertices are numbered from $1\ to\ 7$ as in a binary heap.

cormen algorithms graph-algorithms descriptive

Answer key

1.8.3 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 3 (Page No. 592) top

The transpose of a directed graph $G=(V,E)$ is the graph $G^T=(V,E^T)$, where $E^T=\{(v,u) \in V*V:(u,v) \in E\ \}$ .Thus ,G^T is G with all its edges reversed . Describe efficient algorithms for computing G^T from G,for both the adjacency list and adjacency matrix representations of G. Analyze the running times of your algorithms.

cormen algorithms graph-algorithms descriptive

Answer key

1.8.4 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 4 (Page No. 593) top

Given an adjacency-list representation of a multi graph $G=(V,E)$, describe an $O(V+E)$ time algorithm to compute the adjacency-list representation of the “equivalent” undirected graph $G'=(V,E')$, where E' is consists of the edges in E with all multiple edges between two vertices replaced by a single edge and with all self-loops removed.

cormen algorithms graph-algorithms descriptive

Answer key

1.8.5 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 5 (Page No. 593) top

The square of a directed graph $G=(V,E)$ is the graph $G^2=(V,E^2)$ such that $(u,v) \in E^2$ if and only G contains a path with at most two edges between u and v .Describe efficient algorithms for computing G^2 and G for both the adjacency list and adjacency-matrix representations of G. Analyze the running times of your algorithms.

cormen algorithms graph-algorithms descriptive

1.8.6 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 6 (Page No. 593) top

Most graph algorithms that take an adjacency-matrix representation as input require time $\Omega(V^2)$,but there are some exceptions. Show how to determine whether a directed graph G contains a **universal link** $-$ a vertex with in-degree $|V-1|$ and out-degree 0 in time $O(V)$, given an adjacency matrix for G.

cormen algorithms graph-algorithms descriptive

1.8.7 Graph Algorithms: Cormen Edition 3 Exercise 22.1 Question 8 (Page No. 593) top

Suppose that instead of a linked list, each array entry $adj[u]$ is a hash table containing the vertices v for which $(u, v) \in E$. If all edge lookups are equally likely, what is the expected time to determine whether an edge is in the graph? What disadvantages does this scheme have ? Suggest an alternate data structure for each edge list that solves these problems. Does your alternative have disadvantages compared to the hash table ?

cormen algorithms graph-algorithms descriptive

1.9 Hashing (21) top

1.9.1 Hashing: Cormen Edition 3 Exercise 11.1 Question 1 (Page No. 255) top

Suppose that a dynamic set S is represented by a direct-address table T of length m. Describe a procedure that finds the maximum element of S. What is the worst-case performance of your procedure ?

cormen algorithms hashing

Answer key

1.9.2 Hashing: Cormen Edition 3 Exercise 11.1 Question 2 (Page No. 255) top

A **bit** vector is simply an array of bits ($0s$ and $1s$). A **bit** vector of length m takes much less space than an array of m pointers. Describe how to use a **bit** vector to represent a dynamic set of distinct elements with no satellite data. Dictionary operations should run in $\mathcal{O}(1)$ time.

cormen algorithms hashing descriptive

1.9.3 Hashing: Cormen Edition 3 Exercise 11.1 Question 3 (Page No. 255) top

Suggest how to implement a direct-address table in which the keys of stored elements do not need to be distinct and the elements can have satellite data. All three dictionary operations ($INSERT$,$DELETE$, and $SEARCH$) should run in $\mathcal{O}(1)$ time. (Don't forget that $DELETE$ takes as an argument a pointer to an object to be deleted, not a key.)

cormen algorithms hashing descriptive

1.9.4 Hashing: Cormen Edition 3 Exercise 11.1 Question 4 (Page No. 255) top

We wish to implement a dictionary by using direct addressing on a huge array. At the start, the array entries may contain garbage, and initializing the entire array is impractical because of its size. Describe a scheme for implementing a direct address dictionary on a huge array. Each stored object should use $\mathcal{O}(1)$ space; the operations $SEARCH$, $INSERT$, and $DELETE$ should take $\mathcal{O}(1)$ time each; and initializing the data structure should take $\mathcal{O}(1)$ time. (Hint: Use an additional array, treated somewhat like a stack whose size is the number of keys actually stored in the dictionary, to help determine whether a given entry in the huge array is valid or not.)

cormen algorithms hashing descriptive

1.9.5 Hashing: Cormen Edition 3 Exercise 11.2 Question 1 (Page No. 261) top

Suppose we use a hash function h to hash n distinct keys into an array T of length m. Assuming simple uniform hashing, what is the expected number of collisions ? More precisely, what is the expected cardinality of $\{\{k, l\} : k \neq landh(k) = h(l)\}$?

cormen algorithms hashing descriptive

1.9.6 Hashing: Cormen Edition 3 Exercise 11.2 Question 2 (Page No. 261) top

Demonstrate what happens when we insert the keys $5, 28, 19, 15, 20, 33, 12, 17, 10$ into a hash table with collisions resolved by chaining. Let the table have 9 slots, and let the hash function be $h(k) = k \; mod \; 9$.

cormen algorithms hashing

1.9.7 Hashing: Cormen Edition 3 Exercise 11.2 Question 3 (Page No. 261) top

Professor Marley hypothesizes that he can obtain substantial performance gains by modifying the chaining scheme to keep each list in sorted order. How does the professor's modification affect the running time for successful searches, unsuccessful searches, insertions, and deletions?

cormen algorithms hashing descriptive

1.9.8 Hashing: Cormen Edition 3 Exercise 11.2 Question 4 (Page No. 261) top

Suggest how to allocate and deallocate storage for elements within the hash table itself by linking all unused slots into a free list. Assume that one slot can store a flag and either one element plus a pointer or two pointers. All dictionary and free-list operations should run in $\mathcal{O}(1)$expected time. Does the free list need to be doubly linked, or does a singly linked free list suffice?

cormen algorithms hashing descriptive

1.9.9 Hashing: Cormen Edition 3 Exercise 11.2 Question 5 (Page No. 261) top

Suppose that we are storing a set of n keys into a hash table of size m. Show that if the keys are drawn from a universe U with $|U| > nm$, then U has a subset of size n consisting of keys that all hash to the same slot, so that the worst-case searching time for hashing with chaining is $\Theta(n)$.

cormen algorithms hashing descriptive

1.9.10 Hashing: Cormen Edition 3 Exercise 11.2 Question 6 (Page No. 261) top

Suppose we have stored n keys in a hash table of size m, with collisions resolved by chaining, and that we know the length of each chain, including the length L of the longest chain. Describe a procedure that selects a key uniformly at random from among the keys in the hash table and returns it in expected time $\mathcal{O}(L * (1 + \frac{1}{\alpha}))$.

cormen algorithms hashing descriptive

1.9.11 Hashing: Cormen Edition 3 Exercise 11.3 Question 1 (Page No. 268) top

Suppose we wish to search a linked list of length n, where each element contains a key k along with a hash value $h(k)$. Each key is a long character string. How might we take advantage of the hash values when searching the list for an element with a given key ?

cormen algorithms hashing descriptive

1.9.12 Hashing: Cormen Edition 3 Exercise 11.3 Question 2 (Page No. 268) top

Suppose that we hash a string of r characters into m slots by treating it as a $radix - 128$ number and then using the division method. We can easily represent the number m as a $32 - bit$ computer word, but the string of r characters, treated as a $radix - 128$ number, takes many words. How can we apply the division method to compute the hash value of the character string without using more than a constant number of words of storage outside the string itself ?

cormen algorithms hashing descriptive

1.9.13 Hashing: Cormen Edition 3 Exercise 11.3 Question 3 (Page No. 269) top

Consider a version of the division method in which $h(k) = k \; mod \, m$, where $m = 2^p - 1$ and k is a character string interpreted in $radix \; 2^p$. Show that if we can derive string x from string y by permuting its characters, then x and y hash to the same value. Give an example of an application in which this property would be undesirable in a hash function.

cormen algorithms hashing descriptive

1.9.14 Hashing: Cormen Edition 3 Exercise 11.3 Question 4 (Page No. 269) top

Consider a hash table of size $m = 1000$ and a corresponding hash function $h(k) = \lfloor \; m(kA \; mod \, 1) \rfloor$ for $A = \frac{(\sqrt{5}-1)}{2}$ .Compute the locations to which the keys $61, 62, 63, 64$, and 65 are mapped.

cormen algorithms hashing

Answer key

1.9.15 Hashing: Cormen Edition 3 Exercise 11.3 Question 5 (Page No. 269) top

Define a family $\mathscr{H}$ of hash functions from a finite set U to a finite set B to be universal if for all pairs of distinct elements k and l in U,

$$Pr\{h(k) = h(l)\} \leq \epsilon$$

where the probability is over the choice of the hash function h drawn at random from the family $\mathscr{H}$. Show that an $\epsilon-$universal family of hash functions must have

$\epsilon \geq \frac{1}{|B|} - \frac{1}{|U|}$

cormen algorithms hashing descriptive

1.9.16 Hashing: Cormen Edition 3 Exercise 11.3 Question 6 (Page No. 269) top

Let U be a set of $n - tuples$ of values drawn from $\mathbb{Z}_\text{p}$, let $B = \mathbb{Z}_\text{p}$, where p is prime. Define the hash function $h_b : U \rightarrow B$ for $b \in \mathbb{Z}_\text{p}$ on an input $n-$tuple $\langle a_0, a_1, \ldots, a_{n-1} \rangle$ from U as

$$h(\langle a_0, a_1, \ldots a_{n-1} \rangle) = \left(\sum_{j=0}^{n-1} a_j b^j \right) mod\, p$$

let $\mathscr{H} = \{h_b : b \in \mathbb{Z}_\text{p}\}$ Argue that $\mathscr{H}$ is $n(n-1)/p$- universal according to the definition of the ϵ universal.

cormen algorithms hashing descriptive

1.9.17 Hashing: Cormen Edition 3 Exercise 11.4 Question 1 (Page No. 277) top

Consider inserting the keys $10, 22, 31, 4, 15, 28, 17, 88, 59$ into a hash table of length $m = 11$ using open addressing with the auxiliary hash function $h'(k) = k$. Illustrate the result of inserting these keys using linear probing, using quadratic probing with $c_1 = 1$ and $c_2 = 3$ and using double hashing with $h_1(k) = k$ and $h_2(k) = 1 + (k\, mod\, (m-1))$.

cormen algorithms hashing descriptive

1.9.18 Hashing: Cormen Edition 3 Exercise 11.4 Question 2 (Page No. 277) top

Write pseudo code for $HASH - DELETE$ as outlined in the text, and modify $HASHINSERT$ to handle the special value $DELETED$.

cormen algorithms hashing descriptive

1.9.19 Hashing: Cormen Edition 3 Exercise 11.4 Question 3 (Page No. 277) top

Consider an open-address hash table with uniform hashing. Give upper bounds on the expected number of probes in an unsuccessful search and on the expected number of probes in a successful search when the load factor is $3/4$ and when it is $7/8$.

cormen algorithms hashing descriptive

1.9.20 Hashing: Cormen Edition 3 Exercise 11.4 Question 4 (Page No. 277) top

Suppose that we use double hashing to resolve collisions--that is, we use the hash function $h(k, i) = (h_1(k) + ih_2(k))\, mod\, m$.Show that if m and $h_2(k)$ have greatest common divisor $d \geq 1$ for some key k,then an unsuccessful search for key k examines $(1/d)th$ of the hash table before returning to slot $h_1(k)$.Thus, when $d = 1$, so that m and $h_2(k)$ are relatively prime, the search may examine the entire hash table.

cormen algorithms hashing descriptive

1.9.21 Hashing: Cormen Edition 3 Exercise 11.4 Question 5 (Page No. 277) top

Consider an open-address hash table with a load factor α Find the nonzero value α for which the expected number of probes in an unsuccessful search equals twice the expected number of probes in a successful search.

cormen algorithms hashing descriptive

1.10 Heap (23) top

1.10.1 Heap: Cormen Edition 3 Exercise 6.1 Question 1 (Page No. 153) top

What are the minimum and maximum numbers of elements in a heap of height h?

cormen algorithms heap descriptive

Answer key

1.10.2 Heap: Cormen Edition 3 Exercise 6.1 Question 2 (Page No. 153) top

Show that an n-element heap has height $\lfloor lg\, n \rfloor$.

cormen algorithms heap descriptive

1.10.3 Heap: Cormen Edition 3 Exercise 6.1 Question 3 (Page No. 153) top

Show that in any subtree of a max-heap, the root of the subtree contains the largest value occurring anywhere in that subtree.

cormen algorithms heap descriptive

1.10.4 Heap: Cormen Edition 3 Exercise 6.1 Question 4 (Page No. 154) top

Where in a max-heap might the smallest element reside, assuming that all elements are distinct ?

cormen algorithms sorting heap descriptive

Answer key

1.10.5 Heap: Cormen Edition 3 Exercise 6.1 Question 6 (Page No. 154) top

Is the array with values $23, 17, 14; 6, 13, 10, 1, 5, 7, 12$ a max-heap ?

cormen algorithms heap descriptive

Answer key

1.10.6 Heap: Cormen Edition 3 Exercise 6.1 Question 7 (Page No. 154) top

Show that, with the array representation for storing an n-element heap, the leaves are the nodes indexed by $\lfloor n/2 \rfloor + 1$, $\lfloor n/2 \rfloor + 2, \ldots, n$

cormen algorithms heap descriptive

1.10.7 Heap: Cormen Edition 3 Exercise 6.2 Question 1 (Page No. 156) top

```
MAX-HEAPIFY(A,i)

l=Left(i)
r=Right(i)
if l <= A.heapsize and A[l] > A[i]
largest=l
else largest = i
if r <= A.heapsize and A[r] > A[largest]
largest=r
if largest!=i
exchange A[i] with A[largest]
MAX-HEAPIFY(A,largest)
```

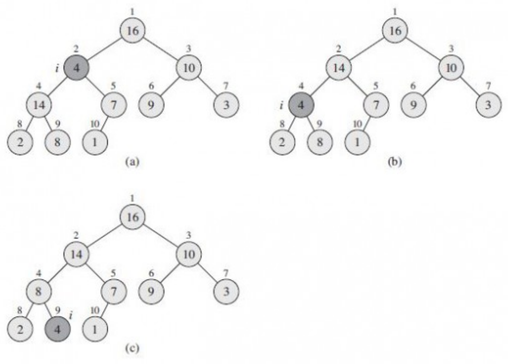


Figure 6.2 The action of MAX-HEAPIFY$(A, 2)$, where $A.heap\text{-}size = 10$. **(a)** The initial configuration, with $A[2]$ at node $i = 2$ violating the max-heap property since it is not larger than both children. The max-heap property is restored for node 2 in **(b)** by exchanging $A[2]$ with $A[4]$, which destroys the max-heap property for node 4. The recursive call MAX-HEAPIFY$(A, 4)$ now has $i = 4$. After swapping $A[4]$ with $A[9]$, as shown in (c), node 4 is fixed up, and the recursive call MAX-HEAPIFY$(A, 9)$ yields no further change to the data structure.

Using Figure 6.2 as a model, illustrate the operation of MAX-HEAPIFY(A,3) on the array $A = \langle 27, 17, 3, 16, 13, 10, 1, 5, 7, 12, 4, 8, 9, 0 \rangle$

cormen algorithms heap descriptive

1.10.8 Heap: Cormen Edition 3 Exercise 6.2 Question 2 (Page No. 156) top

Starting with the procedure MAX-HEAPIFY, write pseudocode for the procedure MIN-HEAPIFY (A, i), which performs the corresponding manipulation on a minheap. How does the running time of MIN-HEAPIFY compare to that of MAXHEAPIFY?

cormen algorithms heap descriptive

1.10.9 Heap: Cormen Edition 3 Exercise 6.2 Question 3 (Page No. 156) top

What is the effect of calling MAX-HEAPIFY(A, i) when the element $A[i]$ is larger than its children?

cormen algorithms heap descriptive

1.10.10 Heap: Cormen Edition 3 Exercise 6.2 Question 4 (Page No. 156) top

What is the effect of calling MAX-HEAPIFY(A, i) for $i > A.heapsize/2$.

cormen algorithms heap descriptive

1.10.11 Heap: Cormen Edition 3 Exercise 6.2 Question 5 (Page No. 156) top

The code for MAX-HEAPIFY is quite efficient in terms of constant factors, except possibly for the recursive call in line 10, which might cause some compilers to produce inefficient code. Write an efficient MAX-HEAPIFY that uses an iterative control construct (a loop) instead of recursion.

cormen algorithms heap descriptive

1.10.12 Heap: Cormen Edition 3 Exercise 6.2 Question 6 (Page No. 156) top

Show that the worst-case running time of MAX-HEAPIFY on a heap of size n is $\Omega(lg\, n)$.(Hint: For a heap with n nodes, give node values that cause MAXHEAPIFY to be called recursively at every node on a simple path from the root down to a leaf).

cormen algorithms heap descriptive

1.10.13 Heap: Cormen Edition 3 Exercise 6.3 Question 1 (Page No. 159) top

```
BUILD-MAX-HEAP(A)
     A.heapsize=A.length
     for i=A.length/2 downto 1
```

```
3     MAX-HEAPIFY(A,i)
```

Using Figure 6.3 as a model, illustrate the operation of BUILD-MAX-HEAP on the array $A = \langle 5, 3, 17, 10, 84, 19, 6, 22, 9 \rangle$

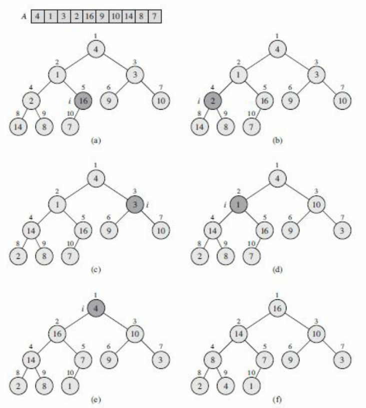


Figure 6.3 The operation of BUILD-MAX-HEAP, showing the data structure before the call to MAX-HEAPIFY in line 3 of BUILD-MAX-HEAP. **(a)** A 10-element input array A and the binary tree it represents. The figure shows that the loop index i refers to node 5 before the call MAX-HEAPIFY(A, i). **(b)** The data structure that results. The loop index i for the next iteration refers to node 4. **(c)–(e)** Subsequent iterations of the **for** loop in BUILD-MAX-HEAP. Observe that whenever MAX-HEAPIFY is called on a node, the two subtrees of that node are both max-heaps. **(f)** The max-heap after BUILD-MAX-HEAP finishes.

cormen algorithms heap descriptive

1.10.14 Heap: Cormen Edition 3 Exercise 6.3 Question 2 (Page No. 159) top

Why do we want the loop index i in line 2 of BUILD-MAX-HEAP to decrease from $\lfloor A.length/2 \rfloor$ to 1 rather than increase from 1 to $\lfloor A.length/2 \rfloor$?

cormen algorithms heap descriptive

1.10.15 Heap: Cormen Edition 3 Exercise 6.3 Question 3 (Page No. 159) top

Show that there are at most $\lceil n/2^{h+1} \rceil$ nodes of height h in any $n-$element heap.

cormen algorithms heap descriptive

1.10.16 Heap: Cormen Edition 3 Exercise 6.5 Question 1 (Page No. 164) top

```
HEAP-EXTRACT-MAX(A)

if A.heap-size < 1
error "heap underflow"
max=A[1]
A[1]=A[A.heapsize]
A.heapsize=A.heapsize-1
MAX-HEAPIFY(A,1)
return max
```

Illustrate the operation of HEAP-EXTRACT-MAX on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

cormen algorithms heap descriptive

1.10.17 Heap: Cormen Edition 3 Exercise 6.5 Question 2 (Page No. 165) top

```
HEAP-INCREASE-KEY(A,i,key)
if key < A[i]
error "new key is smaller than current key"
A[i] = key
while i > 1 and A[parent(i)] < A[i]
exchange A[i] with A[parent(i)]
```

```
6    i=parent(i)

MAX-HEAP-INSERT(A,key)
1       A.heapsize = A.heapsize + 1
2       A[A.heapsize] = -infinity{largest negative number that can be represent in our p
3       HEAP-INCREASE-KEY(A,A.heapsize,key)
```

Illustrate the operation of MAX-HEAP-INSERT$(A, 10)$ on the heap $A = \langle 15, 13, 9, 5, 12, 8, 7, 4, 0, 6, 2, 1 \rangle$.

cormen algorithms heap descriptive

1.10.18 Heap: Cormen Edition 3 Exercise 6.5 Question 3 (Page No. 165) top

Write pseudo code for the procedures HEAP-MINIMUM, HEAP-EXTRACT-MIN, HEAP-DECREASE-KEY, and MIN-HEAP-INSERT that implement a min-priority queue with a min-heap.

cormen algorithms heap descriptive

1.10.19 Heap: Cormen Edition 3 Exercise 6.5 Question 4 (Page No. 165) top

Why do we bother setting the key of the inserted node to $-\infty$ in line 2 of MAX-HEAP-INSERT when the next thing we do is increase its key to the desired value?

cormen algorithms heap descriptive

1.10.20 Heap: Cormen Edition 3 Exercise 6.5 Question 5 (Page No. 166) top

Argue the correctness of HEAP-INCREASE-KEY using the following loop invariant:

At the start of each iteration of the while loop of lines $4-6$, the subarray $A[1..A.heapsize]$ satisfies the max-heap property, except that there may be one violation:: $A[i]$ may be larger than $A[parent(i)]$.

You may assume that the subarray $A[a..heapsize]$ satisfies the max-heap property at the time HEAP-INCREASE-KEY is called.

cormen algorithms heap descriptive

1.10.21 Heap: Cormen Edition 3 Exercise 6.5 Question 6 (Page No. 166) top

Each exchange operation on line 5 of HEAP-INCREASE-KEY typically requires three assignments. Show how to use the idea of the inner loop of INSERTION-SORT to reduce the three assignments down to just one assignment.

cormen algorithms heap descriptive

1.10.22 Heap: Cormen Edition 3 Exercise 6.5 Question 8 (Page No. 166) top

The operation HEAP-DELETE(A, i) deletes the item in node i from heap A. Give an implementation of HEAP-DELETE that runs in $O(lg\ n)$ time for an $n-$element max-heap.

cormen algorithms heap descriptive

Answer key

1.10.23 Heap: Cormen Edition 3 Exercise 6.5 Question 9 (Page No. 166) top

Give an $O(n \lg\ k)$- time algorithm to merge k sorted lists into one sorted list, where n is the total number of elements in all the input lists. (Hint: Use a minheap for k-way merging.)

cormen algorithms heap descriptive

1.11 Heap Sort (5) top

1.11.1 Heap Sort: Cormen Edition 3 Exercise 6.4 Question 1 (Page No. 160) top

```
HEAPSORT(A)

BUILD-MAX-HEAP(A)
for i = A.length down to 2
```

```
3    exchange A[1] with A[i]
4    A.heapsize=A.heapsize - 1
5    MAX-HEAPIFY(A,1)
```

illustrate the operation of HEAPSORT on the array $A = \langle 5, 13, 2, 25, 7, 17, 20, 8, 4 \rangle$

cormen algorithms heap heap-sort descriptive

1.11.2 Heap Sort: Cormen Edition 3 Exercise 6.4 Question 2 (Page No. 160) top

Argue the correctness of HEAPSORT using the following loop invariant:

At the start of each iteration of the for loop of lines $2-5$,the subarray $A[1..i]$ is a max-heap containing the i smallest elements of $A[1..n]$, and the subarray $A[i+1..n]$ contains the $n-i$ largest elements of $A[1..n]$, sorted.

cormen algorithms heap heap-sort descriptive

1.11.3 Heap Sort: Cormen Edition 3 Exercise 6.4 Question 3 (Page No. 160) top

What is the running time of HEAPSORT on an array A of length n that is already sorted in increasing order? What about decreasing order?

cormen algorithms heap heap-sort descriptive

1.11.4 Heap Sort: Cormen Edition 3 Exercise 6.4 Question 4 (Page No. 160) top

Show that the worst-case running time of HEAPSORT is $\Omega(n \lg\ n)$.

cormen algorithms heap heap-sort descriptive

1.11.5 Heap Sort: Cormen Edition 3 Exercise 6.4 Question 5 (Page No. 161) top

Show that when all elements are distinct, the best-case running time of HEAPSORT is $\Omega(n \lg\ n)$.

cormen algorithms heap heap-sort descriptive difficult

1.12 Inversion (4) top

1.12.1 Inversion: Cormen Edition 3 Exercise 2.4 Question 1 (Page No. 41) top

List the five inversions of the array $\langle 2, 3, 8, 6, 1 \rangle$

cormen algorithms inversion descriptive

Answer key

1.12.2 Inversion: Cormen Edition 3 Exercise 2.4 Question 2 (Page No. 42) top

What array with elements from the set $\{1, 2, \ldots n\}$ has the most inversions? How many does it have?

cormen algorithms inversion descriptive

Answer key

1.12.3 Inversion: Cormen Edition 3 Exercise 2.4 Question 3 (Page No. 42) top

What is the relationship between the running time of insertion sort and the number of inversions in the input array? Justify your answer.

cormen algorithms inversion descriptive

1.12.4 Inversion: Cormen Edition 3 Exercise 2.4 Question 4 (Page No. 42) top

Give an algorithm that determines the number of inversions in any permutation on n elements in $\Theta(n\ lg\ n)$ worst-case time. (Hint: Modify merge sort.)

cormen algorithms algorithm-design-techniques inversion descriptive

1.13 Kruskals Algorithm (1) top

1.13.1 Kruskals Algorithm: Cormen Edition 3 Exercise 23.2 Question 6 (Page No. 637) top

Suppose that edge weights are uniformly distributed over half open interval $[0, 1)$. Which algorithm kruskal's or prim's can make you run faster?

algorithms descriptive cormen minimum-spanning-tree kruskals-algorithm prims-algorithm

Answer key

1.14 Linked List (8) top

1.14.1 Linked List: Cormen Edition 3 Exercise 10.2 Question 1 (Page No. 240) top

Can you implement the dynamic-set operation $INSERT$ on a singly linked list in $O(1)$ time? How about $DELETE$?

cormen data-structures linked-list descriptive

Answer key

1.14.2 Linked List: Cormen Edition 3 Exercise 10.2 Question 2 (Page No. 240) top

Implement a stack using a singly linked list L. The operations $PUSH$ and POP should still take $O(1)$ time.

cormen data-structures linked-list descriptive

Answer key

1.14.3 Linked List: Cormen Edition 3 Exercise 10.2 Question 3 (Page No. 240) top

Implement a queue by a singly linked list L. The operations of $ENQUEUE$ and $DEQUEUE$ should still take $O(1)$ time.

cormen data-structures linked-list descriptive

Answer key

1.14.4 Linked List: Cormen Edition 3 Exercise 10.2 Question 4 (Page No. 240) top

```
LIST-SEARCH'(L, k)
    x = L.nil.next
    while x != L.nil and x.key != k
    x = x.next
    return x
```

As written, each loop iteration in the LIST-SEARCH' procedure requires two tests: one for $x \neq L.nil$ and one for $x.key \neq k$. Show how to eliminate the test for $x \neq L.nil$ in each iteration.

cormen data-structures linked-list descriptive

1.14.5 Linked List: Cormen Edition 3 Exercise 10.2 Question 5 (Page No. 240) top

Implement the dictionary operations $INSERT$, $DELETE$, and $SEARCH$ using singly linked, circular lists. What are the running times of your procedures?

cormen data-structures linked-list descriptive

Answer key

1.14.6 Linked List: Cormen Edition 3 Exercise 10.2 Question 6 (Page No. 241) top

The dynamic-set operation $UNION$ takes two disjoint sets S_1 and S_2 as input, and it returns a set $S = S_1 \cup S_2$ consisting of all the elements of S_1 and S_2.The sets S_1 and S_2 are usually destroyed by the operation. Show how to support $UNION$ in $O(1)$ time using a suitable list data structure.

cormen data-structures linked-list descriptive

Answer key

1.14.7 Linked List: Cormen Edition 3 Exercise 10.2 Question 7 (Page No. 241) top

Give a $\Theta(n)$ time nonrecursive procedure that reverses a singly linked list of n elements. The procedure should use no more than constant storage beyond that needed for the list itself.

cormen data-structures linked-list descriptive

Answer key

1.14.8 Linked List: Cormen Edition 3 Exercise 10.2 Question 8 (Page No. 241) top

Explain how to implement doubly linked lists using only one pointer value $x.np$ per item instead of the usual two (next and prev). Assume that all pointer values can be interpreted as k-bit integers, and define $x.np$ to be $x.np = x.next$ $XOR\ x.prev$, the k-bit "exclusive-or" of $x.next$ and $x.prev$.(The value NIL is represented by 0.)Be sure to describe what information you need to access the head of the list. Show how to implement the $SEARCH$, $INSERT$, and $DELETE$ operations on such a list. Also, show how to reverse such a list in $O(1)$ time.

cormen data-structures linked-list descriptive difficult

1.15 Master Theorem (7) top

1.15.1 Master Theorem: Cormen Edition 3 Exercise 4.5 Question 1 (Page No. 96) top

Use the master method to give tight asymptotic bounds for the following recurrences.

a. $T(n) = 2T(n/4) + 1$
b. $T(n) = 2T(n/4) + \sqrt{n}$
c. $T(n) = 2T(n/4) + n$
d. $T(n) = 2T(n/4) + n^2$

cormen algorithms recurrence-relation master-theorem

1.15.2 Master Theorem: Cormen Edition 3 Exercise 4.5 Question 2 (Page No. 97) top

Professor Caesar wishes to develop a matrix-multiplication algorithm that is asymptotically faster than Strassen's algorithm. His algorithm will use the divide and conquer method, dividing each matrix into pieces of size $n/4 * n/4$,and the divide and combine steps together will take $\Theta(n^2)$ time. He needs to determine how many subproblems his algorithm has to create in order to beat Strassen's algorithm. If his algorithm creates a subproblems, then the recurrence for the running time $T(n)$ becomes $T(n) = aT(n/4) + \Theta(n^2)$.What is the largest integer value of a for which Professor Caesar's algorithm would be asymptotically faster than Strassen's algorithm?

cormen algorithms recurrence-relation master-theorem descriptive

Answer key

1.15.3 Master Theorem: Cormen Edition 3 Exercise 4.5 Question 3 (Page No. 97) top

Use the master method to show that the solution to the binary-search recurrence $T(n) = T(n/2) + \Theta(1)$ is $T(n) = \Theta(lg\, n)$.

cormen algorithms recurrence-relation master-theorem descriptive

1.15.4 Master Theorem: Cormen Edition 3 Exercise 4.5 Question 4 (Page No. 97) top

Can the master method be applied to the recurrence $T(n) = 4T(n/2) + n^2\ lg\, n$?Why or why not? Give an asymptotic upper bound for this recurrence.

cormen algorithms recurrence-relation master-theorem descriptive

1.15.5 Master Theorem: Cormen Edition 3 Exercise 4.5 Question 5 (Page No. 97) top

Consider the regularity condition $af(n/b) \leq cf(n)$ for some constant $c < 1$,which is part of case 3 of the master theorem. Give an example of constants $a \geq 1$ and $b > 1$ and a function $f(n)$ that satisfies all the conditions in case 3 of the master theorem except the regularity condition.

cormen algorithms recurrence-relation master-theorem descriptive difficult

1.15.6 Master Theorem: Cormen Edition 3 Exercise 4.6 Question 2 (Page No. 106) top

Show that if $f(n) = \Theta(n^{log_b a} \lg^k n)$, where $k \geq 0$ then the master recurrence has solution $T(n) = \Theta(n^{log_b a} \lg^{k+1} n)$.For simplicity, confine your analysis to exact powers of b.

cormen algorithms recurrence-relation master-theorem descriptive difficult

1.15.7 Master Theorem: Cormen Edition 3 Exercise 4.6 Question 3 (Page No. 106) top

Show that case 3 of the master theorem is overstated, in the sense that the regularity condition $af(n/b) \geq cf(n)$ for some constant $c < 1$ implies that there exists a constant $\epsilon > 0$ such that $f(n) = \Omega(n^{log_b a+\epsilon})$.

cormen algorithms recurrence-relation master-theorem descriptive difficult

1.16 Merge Sort (2) top

1.16.1 Merge Sort: Cormen Edition 3 Exercise 2.3 Question 1 (Page No. 37) top

Using Figure 2.4 as a model, illustrate the operation of merge sort on the array $A = \langle 3, 41, 52, 26, 38, 57, 9, 49 \rangle$

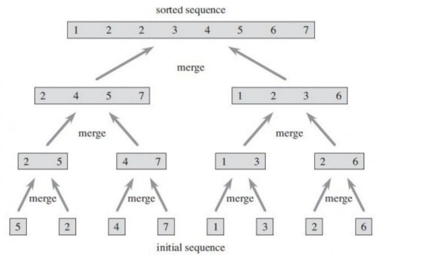


Figure 2.4 The operation of merge sort on the array $A = \langle 5, 2, 4, 7, 1, 3, 2, 6 \rangle$. The lengths of the sorted sequences being merged increase as the algorithm progresses from bottom to top.

cormen algorithms sorting merge-sort descriptive

Answer key

1.16.2 Merge Sort: Cormen Edition 3 Exercise 2.3 Question 2 (Page No. 37) top

Rewrite the MERGE procedure so that it does not use sentinels, instead of stopping once either array L or R has had all its elements copied back to A and then copying the remainder of the other array back into A.

cormen algorithms sorting merge-sort descriptive

1.17 Priority Queue (1) top

1.17.1 Priority Queue: Cormen Edition 3 Exercise 6.5 Question 7 (Page No. 166) top

Show how to implement a first-in, first-out queue with a priority queue. Show how to implement a stack with a priority queue.

cormen algorithms heap priority-queue descriptive

1.18 Queue (4) top

1.18.1 Queue: Cormen Edition 3 Exercise 10.1 Question 3 (Page No. 235) top

```
ENQUEUE(Q, x)
      Q[Q.tail] = x
      if Q.tail == Q.length
      Q.tail = 1
      else Q.tail = Q.tail + 1


DEQUEUE(Q)
       x = Q[Q.head]
       if Q.head == Q.length
       Q.head = 1
       else Q.head = Q.head + 1
       return x
```

illustrate the result of each operation in the sequence ENQUEUE(Q,4),ENQUEUE(Q,1),ENQUEUE(Q,3),DEQUEUE(Q),ENQUEUE(Q,8),DEQUEUE(Q) on an initially empty queue Q stored in array $Q[1...6]$.

cornen data-structures queue descriptive

Answer key

1.18.2 Queue: Cormen Edition 3 Exercise 10.1 Question 4 (Page No. 235) top

Rewrite ENQUEUE and DEQUEUE to detect underflow and overflow of a queue.

cormen data-structures queue descriptive

Answer key

1.18.3 Queue: Cormen Edition 3 Exercise 10.1 Question 5 (Page No. 236) top

Whereas a stack allows insertion and deletion of elements at only one end, and a queue allows insertion at one end and deletion at the other end, a deque (double ended queue) allows insertion and deletion at both ends. Write four $O(1)$ time procedures to insert elements into and delete elements from both ends of a deque implemented by an array.

cormen algorithms data-structures queue descriptive

Answer key

1.18.4 Queue: Cormen Edition 3 Exercise 10.1 Question 6 (Page No. 236) top

Show how to implement a queue using two stacks. Analyze the running time of the queue operations.

cormen data-structures queue descriptive

Answer key

1.19 Quick Sort (16) top

1.19.1 Quick Sort: Cormen Edition 3 Exercise 7.1 Question 1 (Page No. 173) top

```
PARTITION(A,p,r)

x = A[r]
i = p - 1
for j = p to r - 1
if A[j] <= x
i = i + 1
exchange A[i] with A[j]
exchange A[i+1] with A[r]
return i + 1
```

illustrate the operation of PARTITION on the array $A = \langle 13, 19, 9, 5, 12, 8, 7, 4, 21, 2, 6, 11 \rangle$

cormen algorithms sorting quick-sort descriptive

Answer key

1.19.2 Quick Sort: Cormen Edition 3 Exercise 7.1 Question 2 (Page No. 174) top

What value of q does PARTITION return when all elements in the array $A[p..r]$ have the same value? Modify PARTITION so that $q = \lfloor (p+r)/2 \rfloor$ when all elements in the array $A[p..r]$ have the same value.

cormen algorithms sorting quick-sort descriptive

Answer key

1.19.3 Quick Sort: Cormen Edition 3 Exercise 7.1 Question 3 (Page No. 174) top

Give a brief argument that the running time of PARTITION on a subarray of size n is $\Theta(n)$.

cormen algorithms quick-sort descriptive

Answer key

1.19.4 Quick Sort: Cormen Edition 3 Exercise 7.1 Question 4 (Page No. 174) top

```
QUICKSORT(A,p,r)
    if p < r
    q = PARTITION(A,p,r)
    QUICKSORT(A, p , q-1)
    QUICKSORT(A, q + 1, r)
```

How would you modify QUICKSORT to sort into nonincreasing order?

cormen algorithms quick-sort descriptive

Answer key

1.19.5 Quick Sort: Cormen Edition 3 Exercise 7.2 Question 2 (Page No. 178) top

What is the running time of QUICKSORT when all elements of the array A have the same value?

cormen algorithms quick-sort time-complexity descriptive

Answer key

1.19.6 Quick Sort: Cormen Edition 3 Exercise 7.2 Question 3 (Page No. 178) top

Show that the running time of QUICKSORT is $\Theta(n^2)$ when the array A contains distinct elements and is sorted in decreasing order.

cormen algorithms quick-sort time-complexity descriptive

Answer key

1.19.7 Quick Sort: Cormen Edition 3 Exercise 7.2 Question 4 (Page No. 178) top

Banks often record transactions on an account in order of the times of the transactions, but many people like to receive their bank statements with checks listed in order by check number. People usually write checks in order by check number, and merchants usually cash them with reasonable dispatch. The problem of converting time-of-transaction ordering to check-number ordering is, therefore, the problem of sorting almost-sorted input. Argue that the procedure INSERTION-SORT would tend to beat the procedure QUICKSORT on this problem.

cormen algorithms sorting quick-sort descriptive

1.19.8 Quick Sort: Cormen Edition 3 Exercise 7.2 Question 5 (Page No. 178) top

Suppose that the splits at every level of quicksort are in the proportion $1-\alpha$ to α, where $0 < \alpha \leq 1/2$ is a constant. Show that the minimum depth of a leaf in the recursion tree is approximately $-lg\; n/lg\; \alpha$ and the maximum depth is approximately $-lg\; n/lg\; (1-\alpha)$.(Don't worry about integer round-off.)

cormen algorithms quick-sort descriptive

1.19.9 Quick Sort: Cormen Edition 3 Exercise 7.2 Question 6 (Page No. 179) top

Argue that for any constant $0 < \alpha \leq 1/2$, the probability is approximately $1-2\alpha$ that on a random input array, PARTITION produces a split more balanced than $1-\alpha$ to α.

cormen algorithms quick-sort descriptive difficult

Answer key

1.19.10 Quick Sort: Cormen Edition 3 Exercise 7.3 Question 1 (Page No. 180) top

Why do we analyze the expected running time of a randomized algorithm and not its worst-case running time?

cormen algorithms quick-sort descriptive

Answer key

1.19.11 Quick Sort: Cormen Edition 3 Exercise 7.3 Question 2 (Page No. 180) top

```
RANDOMIZED-QUICKSORT(A, p, r)
     if p < r
     q = RANDOMIZED-PARTITION(A, p, r)
     RANDOMIZED-QUICKSORT(A, p, q - 1)
     RANDOMIZED-QUICKSORT(A, q + 1, r)
```

```
RANDOMIZED-PARTITION(A, p, r)
     i = RANDOM(p, r)
     exchange A[r] with A[i]
     return PARTITION(A, p, r)
```

When RANDOMIZED-QUICKSORT runs, how many calls are made to the random number generator RANDOM in the worst case? How about in the best case? Give your answer in terms of Θ notation.

cormen algorithms quick-sort descriptive

Answer key

1.19.12 Quick Sort: Cormen Edition 3 Exercise 7.4 Question 2 (Page No. 184) top

Show that quicksort's best-case running time is $\Omega(n\ lg\ n)$.

cormen algorithms quick-sort time-complexity descriptive

Answer key

1.19.13 Quick Sort: Cormen Edition 3 Exercise 7.4 Question 3 (Page No. 184) top

Show that the expression $q^2 + (n - q - 1)^2$ achieves a maximum over $q = 0, 1, \dots, n - 1$ when $q = 0$ or $q = n - 1$.

cormen algorithms quick-sort descriptive

Answer key

1.19.14 Quick Sort: Cormen Edition 3 Exercise 7.4 Question 4 (Page No. 184) top

Show that RANDOMIZED-QUICKSORT's expected running time is $\Omega(n\ lg\ n)$.

cormen algorithms quick-sort time-complexity descriptive

Answer key

1.19.15 Quick Sort: Cormen Edition 3 Exercise 7.4 Question 5 (Page No. 185) top

We can improve the running time of quicksort in practice by taking advantage of the fast running time of insertion sort when its input is "nearly" sorted. Upon calling quicksort on a subarray with fewer than k elements, let it simply return without sorting the subarray. After the top-level call to quicksort returns, run insertion sort on the entire array to finish the sorting process. Argue that this sorting algorithm runs in $O(nk + n\ lg\ (n/k))$ expected time. How should we pick k, both in theory and in practice?

cormen algorithms quick-sort descriptive

Answer key

1.19.16 Quick Sort: Cormen Edition 3 Exercise 7.4 Question 6 (Page No. 185) top

Consider modifying the PARTITION procedure by randomly picking three elements from the array A and partitioning about their median (the middle value of the three elements). Approximate the probability of getting at worst a α-to-$(1 - \alpha)$ split, as a function of α in the range $0 < \alpha < 1$.

cormen algorithms quick-sort descriptive difficult

Answer key

1.20 Radix Sort (3) top

1.20.1 Radix Sort: Cormen Edition 3 Exercise 8.3 Question 1 (Page No. 199) top

```
RADIX-SORT(A, d)
1     for i = 1 to d
2     use a stable sort to sort array A on digit i
```

illustrate the operation of RADIX-SORT on the following list of English words: COW, DOG, SEA, RUG, ROW, MOB, BOX, TAB, BAR, EAR, TAR, DIG, BIG, TEA, NOW, FOX.

cormen algorithms sorting radix-sort descriptive

Answer key

1.20.2 Radix Sort: Cormen Edition 3 Exercise 8.3 Question 3 (Page No. 200) top

Use induction to prove that radix sort works. Where does your proof need the assumption that the intermediate sort is stable?

cormen algorithms sorting radix-sort descriptive

1.20.3 Radix Sort: Cormen Edition 3 Exercise 8.3 Question 4 (Page No. 200) top

Show how to sort n integers in the range 0 to $n^3 - 1$ in $O(n)$ time.

cormen algorithms sorting radix-sort descriptive

Answer key

1.21 Recurrence Relation (19) top

1.21.1 Recurrence Relation: Cormen Edition 3 Exercise 2.3 Question 3 (Page No. 39) top

Use mathematical induction to show that when n is an exact power of 2, the solution of the recurrence

$$T(n) = \begin{cases} 2, & \text{if n=2,} \\ 2T(n & / & 2) + n, \quad \text{if n=}2^k\text{,for k >1} \end{cases}$$

is $T(n) = n\ lg\ n$.

cormen algorithms recurrence-relation time-complexity descriptive

Answer key

1.21.2 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 1 (Page No. 87) top

Show that the solution of $T(n) = T(n-1) + n$ is $O(n^2)$

cormen algorithms recurrence-relation descriptive

1.21.3 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 2 (Page No. 87) top

Show that the solution of $T(n) = T(\lceil n/2 \rceil) + 1$ is $O(lg\ n)$.

cormen algorithms recurrence-relation descriptive

1.21.4 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 3 (Page No. 87) top

We saw that the solution of $T(n) = T(\lceil n/2 \rceil) + n$ is $O(lg\ n)$. Show that the solution of this recurrence is also $\Omega(n\ lg\ n)$. Conclude that the solution is $\Theta(n \log\ n)$.

cormen algorithms recurrence-relation descriptive

1.21.5 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 6 (Page No. 87) top

Show that the solution to $T(n) = T(\lfloor n/2 \rfloor + 17) + n$ is $O(n \ log \ n)$

cormen algorithms recurrence-relation descriptive

1.21.6 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 7 (Page No. 87) top

Using the master method, you can show that the solution to the recurrence $T(n) = 4T(n/3) + n$ is $T(n) = O(n^{log_3 4})$.Show that a substitution proof with the assumption $T(n) \leq cn^{log_3 4}$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

cormen algorithms recurrence-relation descriptive

1.21.7 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 8 (Page No. 87) top

Using the master method, you can show that the solution to the recurrence $T(n) = 4T(n/2) + n^2$ is $T(n) = \Theta(n^2)$.Show that a substitution proof with the assumption $T(n) \leq cn^2$ fails. Then show how to subtract off a lower-order term to make a substitution proof work.

cormen algorithms recurrence-relation descriptive

1.21.8 Recurrence Relation: Cormen Edition 3 Exercise 4.3 Question 9 (Page No. 88) top

Solve the recurrence $T(n) = 3T(\sqrt{n}) + log \ n$ by making a change of variables.Your solution should be asymptotically tight. Do not worry about whether values are integral.

cormen algorithms recurrence-relation descriptive

Answer key

1.21.9 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 1 (Page No. 92) top

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 3T(\lfloor n/2 \rfloor) + n$. Use the substitution method to verify your answer.

cormen algorithms recurrence-relation descriptive

1.21.10 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 2 (Page No. 92) top

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T(n/2) + n^2$.Use the substitution method to verify your answer

cormen algorithms recurrence-relation descriptive

Answer key

1.21.11 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 3 (Page No. 93) top

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 4T(n/2 + 2) + n$.Use the substitution method to verify your answer.

cormen algorithms recurrence-relation descriptive

1.21.12 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 4 (Page No. 93) top

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = 2T(n-1) + 1$.Use the substitution method to verify your answer.

cormen algorithms recurrence-relation descriptive

1.21.13 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 5 (Page No. 93) top

Use a recursion tree to determine a good asymptotic upper bound on the recurrence $T(n) = T(n-1) + T(n/2) + n$.Use the substitution method to verify your answer.

cormen algorithms recurrence-relation descriptive

1.21.14 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 6 (Page No. 93) top

Argue that the solution to the recurrence $T(n) = T(n/3) + T(2n/3) + cn$,where c is a constant, is $\Omega(n \lg n)$ by appealing to a recursion tree.

cormen algorithms recurrence-relation descriptive

1.21.15 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 7 (Page No. 93) top

Draw the recursion tree for $T(n) = 4T(\lfloor n/2 \rfloor) + cn$, where c is a constant,and provide a tight asymptotic bound on its solution. Verify your bound by the substitution method.

cormen algorithms recurrence-relation descriptive

1.21.16 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 8 (Page No. 93) top

Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(n-a) + T(a) + cn$, where $a \geq 1$ and $c > 0$ are constants.

cormen algorithms recurrence-relation descriptive

1.21.17 Recurrence Relation: Cormen Edition 3 Exercise 4.4 Question 9 (Page No. 93) top

Use a recursion tree to give an asymptotically tight solution to the recurrence $T(n) = T(\alpha n) + T((1-\alpha)n) + cn$,where α is a constant in the range $0 < \alpha < 1$ and $c > 0$ is also constant.

cormen algorithms recurrence-relation descriptive

1.21.18 Recurrence Relation: Cormen Edition 3 Exercise 7.2 Question 1 (Page No. 178) top

Use the substitution method to prove that the recurrence $T(n) = T(n-1) + \Theta(n)$ has the solution $T(n) = \Theta(n^2)$.

cormen algorithms recurrence-relation descriptive

Answer key

1.21.19 Recurrence Relation: Cormen Edition 3 Exercise 7.4 Question 1 (Page No. 184) top

Show that in the recurrence

$T(n) = \max_{0<q\leq n-1}(T(q) + T(n-q-1)) + \Theta(n)$

$T(n) = \Omega(n^2)$

cormen algorithms recurrence-relation descriptive

1.22 Searching (4) top

1.22.1 Searching: Cormen Edition 3 Exercise 2.1 Question 3 (Page No. 22) top

Consider the searching problem:
Input: A sequence of n numbers $A = \langle a_1, a_2, \dots a_n \rangle$ and a value v
Output: An index i such that $v = A[i]$ or the special value NIL if v does not appear in A.
Write pseudocode for **linear search**, which scans through the sequence, looking for v. Using a loop invariant, prove that your algorithm is correct. Make sure that your loop invariant fulfills the three necessary properties.

cormen algorithms searching descriptive

1.22.2 Searching: Cormen Edition 3 Exercise 2.2 Question 3 (Page No. 29) top

Consider the linear search again (see Exercise 2.1-3). How many elements of the input sequence need to be checked on the average, assuming that the element being searched for is equally likely to be any element in the array? How about in the worst case? What are the average-case and worst-case running times of linear search in ,-notation? Justify your answers.

cormen algorithms searching descriptive

Answer key

1.22.3 Searching: Cormen Edition 3 Exercise 2.3 Question 5 (Page No. 39) top

Referring back to the searching problem (see Exercise 2.1-3), observe that if the sequence A is sorted, we can check the midpoint of the sequence against v and eliminate half of the sequence from further consideration. The **binary search** algorithm repeats this procedure, halving the size of the remaining portion of the sequence each time. Write pseudocode, either iterative or recursive, for binary search. Argue that the worst-case running time of binary search is $\Theta(lg\, n)$.

cormen algorithms searching descriptive

1.22.4 Searching: Cormen Edition 3 Exercise 2.3 Question 6 (Page No. 39) top

Observe that the while loop of the INSERTION-SORT procedure uses a linear search to scan (backward) through the sorted subarray $A[i \dots j-1]$ Can we use a binary search (see Exercise 2.3-5) instead to improve the overall worst-case running time of insertion sort to $\Theta(n\ lg\, n)$?

algorithms cormen searching descriptive

Answer key

1.23 Sorting (5) top

1.23.1 Sorting: Cormen Edition 3 Exercise 2.1 Question 2 (Page No. 22) top

Rewrite the INSERTION-SORT procedure to sort into non-increasing instead of non-decreasing order.

cormen algorithms sorting descriptive

Answer key

1.23.2 Sorting: Cormen Edition 3 Exercise 2.3 Question 4 (Page No. 38) top

We can express the insertion sort as a recursive procedure as follows.In order to sort $A[1 \dots n]$, we recursively sort $A[1 \dots n-1]$ and then insert $A[n]$ into the sorted array $A[1 \dots n-1]$. Write a recurrence for the running time of this recursive version of insertion sort.

cormen algorithms sorting time-complexity descriptive

Answer key

1.23.3 Sorting: Cormen Edition 3 Exercise 8.1 Question 1 (Page No. 193) top

What is the smallest possible depth of a leaf in a decision tree for a comparison sort?

cormen algorithms sorting descriptive

Answer key

1.23.4 Sorting: Cormen Edition 3 Exercise 8.1 Question 3 (Page No. 194) top

Show that there is no comparison sort whose running time is linear for at least half of the $n!$ inputs of length n.What about a fraction of $1/n$ inputs of length n? What about a fraction $1/2^n$?

cormen algorithms sorting descriptive

1.23.5 Sorting: Cormen Edition 3 Exercise 8.1 Question 4 (Page No. 194) top

Suppose that you are given a sequence of n elements to sort.The input sequence consists of n/k subsequences, each containing k elements.The elements in a given subsequence are all smaller than the elements in the succeeding subsequence and larger than the elements in the preceding subsequence. Thus, all that is needed to sort the whole sequence of length n is to sort the k elements in each of the n/k sequences Show an $\Omega(n\ lg\, k)$ lower bound on the number of comparisons needed to solve this variant of the sorting problem. (Hint: It is not rigorous to simply combine the lower bounds for the individual subsequences.)

cormen algorithms sorting descriptive

1.24 Stablesort (1) top

1.24.1 Stablesort: Cormen Edition 3 Exercise 8.3 Question 2 (Page No. 200) top

Which of the following sorting algorithms are stable: insertion sort, merge sort, heapsort, and quicksort? Give a simple scheme that makes any sorting algorithm stable. How much additional time and space does your scheme entail?

cormen algorithms sorting stablesort descriptive

Answer key

1.25 Stack (3) top

1.25.1 Stack: Cormen Edition 3 Exercise 10.1 Question 1 (Page No. 235) top

```
STACK-EMPTY(S)
    if S.top == 0
    return TRUE
    else return FALSE

PUSH(S , x)
    S.top = S.top + 1
    S[S.top] = x

POP(S)
    if STACK-EMPTY(S)
    error "underflow"
    else S.top = S.top - 1
    return S[S.top + 1]
```

illustrate the result of each operation in the sequence $PUSH(S, 4), PUSH(S, 1), PUSH(S, 3), POP(S), PUSH(S, 8), POP(S)$ on an initially empty stack S stored in array $S[1...6]$

cormen data-structures stack descriptive

Answer key

1.25.2 Stack: Cormen Edition 3 Exercise 10.1 Question 2 (Page No. 235) top

Explain how to implement two stacks in one array $A[1...n]$ in such a way that neither stack overflows unless the total number of elements in both stacks together is n.The $PUSH$ and POP operations should run in $O(1)$ time.

cormen data-structures stack descriptive

Answer key

1.25.3 Stack: Cormen Edition 3 Exercise 10.1 Question 7 (Page No. 236) top

Show how to implement a stack using two queues. Analyze the running time of the stack operations.

cormen data-structures stack descriptive

Answer key

1.26 Time Complexity (3) top

1.26.1 Time Complexity: Cormen Edition 3 Exercise 2.2 Question 1 (Page No. 29) top

Express the function $n^3/1000 - 100n^2 - 100n + 3$ in terms of Θ notation.

cormen algorithms time-complexity descriptive

Answer key

1.26.2 Time Complexity: Cormen Edition 3 Exercise 2.2 Question 2 (Page No. 29) top

Consider sorting n numbers stored in an array A by first finding the smallest element of A and exchanging it with the element in $A[1]$. Then find the second smallest element of A, and exchange it with $A[2]$. Continue in this manner for the first $n - 1$ elements of A.

A. Write pseudocode for this algorithm, which is known as selection sort.
B. What loop invariant does this algorithm maintain?
C. Why does it need to run for only the first $n-1$ elements, rather than for all n elements?
D. Give the best-case and worst-case running times of selection sort in Θ-notation

algorithms cormen time-complexity descriptive

Answer key

1.26.3 Time Complexity: Cormen Edition 3 Exercise 2.2 Question 4 (Page No. 29) top

How can we modify almost any algorithm to have a good best-case running time?

cormen algorithms time-complexity descriptive

Answer key

1.27 Tree (1) top

1.27.1 Tree: Cormen Edition 3 Exercise 12.1 Question 5 (Page No. 289) top

Argue that since sorting n elements takes $\Omega(n \ lgn)$ time in the worst case in the comparison model, any comparison-based algorithm for constructing a BST from an arbitrary list of n elements takes $\Omega(n \ lgn)$ time in the worst case.

cormen algorithms descriptive binary-search-tree binary-tree tree

Answer key

Answer Keys

1.0.1	N/A	1.0.2	N/A	1.0.3	N/A	1.0.4	N/A	1.0.5	N/A
1.0.6	N/A	1.0.7	N/A	1.0.8	Q-Q	1.0.9	N/A	1.0.10	N/A
1.0.11	N/A	1.0.12	N/A	1.1.1	N/A	1.2.1	N/A	1.2.2	N/A
1.2.3	N/A	1.2.4	N/A	1.2.5	N/A	1.2.6	N/A	1.2.7	N/A
1.2.8	N/A	1.2.9	N/A	1.2.10	N/A	1.2.11	N/A	1.2.12	N/A
1.2.13	Q-Q	1.2.14	N/A	1.2.15	N/A	1.3.1	N/A	1.4.1	N/A
1.4.2	N/A	1.4.3	N/A	1.5.1	N/A	1.5.2	N/A	1.5.3	N/A
1.5.4	N/A	1.5.5	N/A	1.6.1	N/A	1.6.2	N/A	1.6.3	N/A
1.6.4	N/A	1.7.1	N/A	1.7.2	N/A	1.7.3	N/A	1.7.4	N/A
1.8.1	N/A	1.8.2	N/A	1.8.3	N/A	1.8.4	N/A	1.8.5	N/A
1.8.6	N/A	1.8.7	N/A	1.9.1	Q-Q	1.9.2	N/A	1.9.3	N/A
1.9.4	N/A	1.9.5	N/A	1.9.6	Q-Q	1.9.7	N/A	1.9.8	N/A
1.9.9	N/A	1.9.10	N/A	1.9.11	N/A	1.9.12	N/A	1.9.13	N/A
1.9.14	Q-Q	1.9.15	N/A	1.9.16	N/A	1.9.17	N/A	1.9.18	N/A
1.9.19	N/A	1.9.20	N/A	1.9.21	N/A	1.10.1	N/A	1.10.2	N/A
1.10.3	N/A	1.10.4	N/A	1.10.5	N/A	1.10.6	N/A	1.10.7	N/A
1.10.8	N/A	1.10.9	N/A	1.10.10	N/A	1.10.11	N/A	1.10.12	N/A
1.10.13	N/A	1.10.14	N/A	1.10.15	N/A	1.10.16	N/A	1.10.17	N/A
1.10.18	N/A	1.10.19	N/A	1.10.20	N/A	1.10.21	N/A	1.10.22	N/A
1.10.23	N/A	1.11.1	N/A	1.11.2	N/A	1.11.3	N/A	1.11.4	N/A
1.11.5	N/A	1.12.1	N/A	1.12.2	N/A	1.12.3	N/A	1.12.4	N/A
1.13.1	N/A	1.14.1	N/A	1.14.2	N/A	1.14.3	N/A	1.14.4	N/A

1.14.5	N/A	1.14.6	N/A	1.14.7	N/A	1.14.8	N/A	1.15.1	Q-Q
1.15.2	N/A	1.15.3	N/A	1.15.4	N/A	1.15.5	N/A	1.15.6	N/A
1.15.7	N/A	1.16.1	N/A	1.16.2	N/A	1.17.1	N/A	1.18.1	N/A
1.18.2	N/A	1.18.3	N/A	1.18.4	N/A	1.19.1	N/A	1.19.2	N/A
1.19.3	N/A	1.19.4	N/A	1.19.5	N/A	1.19.6	N/A	1.19.7	N/A
1.19.8	N/A	1.19.9	N/A	1.19.10	N/A	1.19.11	N/A	1.19.12	N/A
1.19.13	N/A	1.19.14	N/A	1.19.15	N/A	1.19.16	N/A	1.20.1	N/A
1.20.2	N/A	1.20.3	N/A	1.21.1	N/A	1.21.2	N/A	1.21.3	N/A
1.21.4	N/A	1.21.5	N/A	1.21.6	N/A	1.21.7	N/A	1.21.8	N/A
1.21.9	N/A	1.21.10	N/A	1.21.11	N/A	1.21.12	N/A	1.21.13	N/A
1.21.14	N/A	1.21.15	N/A	1.21.16	N/A	1.21.17	N/A	1.21.18	N/A
1.21.19	N/A	1.22.1	N/A	1.22.2	N/A	1.22.3	N/A	1.22.4	N/A
1.23.1	N/A	1.23.2	N/A	1.23.3	N/A	1.23.4	N/A	1.23.5	N/A
1.24.1	N/A	1.25.1	N/A	1.25.2	N/A	1.25.3	N/A	1.26.1	N/A
1.26.2	N/A	1.26.3	N/A	1.27.1	N/A				

2 CO and Architecture (2)

2.1 Input Output (1) top

2.1.1 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 11 (Page No. 430) top

A computer has a three-stage pipeline as shown in Fig. 1-7(a). On each clock cycle, one new instruction is fetched from memory at the address pointed to by the PC and put into the pipeline and the PC advanced. Each instruction occupies exactly one memory word. The instructions already in the pipeline are each advanced one stage. When an interrupt occurs, the current PC is pushed onto the stack, and the PC is set to the address of the interrupt handler. Then the pipeline is shifted right one stage and the first instruction of the interrupt handler is fetched into the pipeline. Does this machine have precise interrupts? Defend your answer.

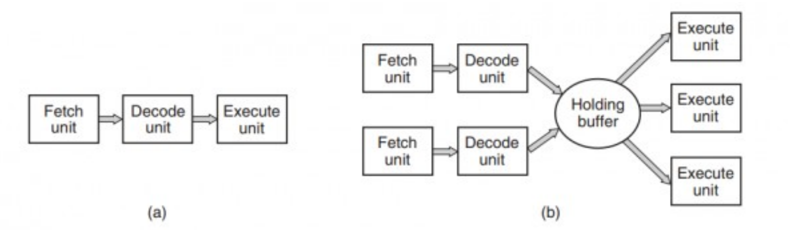


Figure 1-7. (a) A three-stage pipeline. (b) A superscalar CPU.

tanenbaum operating-system input-output pipelining interrupts descriptive

2.2 Machine Instructions (1) top

2.2.1 Machine Instructions: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 14 (Page No. 82) top

A computer has a pipeline with four stages. Each stage takes the same time to do its work, namely, 1 nsec. How many instructions per second can this machine execute?

tanenbaum operating-system machine-instructions pipelining descriptive

Answer key

Answer Keys

2.1.1	N/A	2.2.1	N/A

3.0.1 Ullman (Compiler Design) Edition 2 Exercise 1.1 Question 1 (Page No. 3) top

What is the difference between a compiler and an interpreter?

ullman compiler-design

3.0.2 Ullman (Compiler Design) Edition 2 Exercise 1.6 Question 4 (Page No. 36) top

What is printed by the following C code?

```
#define a (x+1)
int x = 2;
void b() {x = a; printf("%d\n",x);}
void c() {int x = 1; printf("%d\n"),a;}
void main() {b(); c();}
```

ullman compiler-design

Answer key

3.0.3 Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 4 (Page No. 207 - 208) top

There is an extended grammar notation in common use. In this notation, square and curly braces in production bodies are metasymbols (like $\rightarrow$ or $|$) with the following meanings:

i. Square braces around a grammar symbol or symbols denotes that these constructs are optional. Thus, production $A \rightarrow X[Y]Z$ has the same effect as the two productions $A \rightarrow XYZ$ and $A \rightarrow XZ$.
ii. Curly braces around a grammar symbol or symbols says that these symbols may be repeated any number of times, including zero times. Thus, $A \rightarrow X\{YZ\}$ has the same effect as the infinite sequence of productions $A \rightarrow X, A \rightarrow XYZ, A \rightarrow XYZYZ$,and so on.

Show that these two extensions do not add power to grammars; that is, any language that can be generated by a grammar with these extensions can be generated by a grammar without the extensions.

ullman compiler-design descriptive

3.0.4 Ullman (Compiler Design) Edition 2 Exercise 1.1 Question 3 (Page No. 3) top

What advantages are there to a language processing system in which the compiler produces assembly language rather than machine language?

ullman compiler-design

3.0.5 Ullman (Compiler Design) Edition 2 Exercise 1.1 Question 4 (Page No. 3) top

A compiler that translates a high-level language into another high-level language is called a source-to-source translator. What advantages are there to using C as a target language for a compiler?

ullman compiler-design

3.0.6 Ullman (Compiler Design) Edition 2 Exercise 1.1 Question 5 (Page No. 3) top

Describe some of the tasks that an assembler needs to perform.

ullman compiler-design descriptive

3.0.7 Ullman (Compiler Design) Edition 2 Exercise 1.3 Question 1 (Page No. 14 - 15) top

Indicate which of the following terms:

a. imperative
b. declarative
c. von Neumann
d. object-oriented
e. functional
f. third-generation
g. fourth-generation
h. scripting

apply to which of the following languages:

1. C
2. C++
3. Cobol
4. Fortran
5. Java
6. Lisp
7. ML
8. Perl
9. Python
10. VB.

ullman compiler-design

3.0.8 Ullman (Compiler Design) Edition 2 Exercise 1.6 Question 1 (Page No. 35 - 36) top

For the block-structured C code, indicate the values assigned to w, x, y, and z.

```
int w,x,y,z;
int i = 4; int j = 5;
{
   int j = 7;
   i = 6;
   w = i + j;
}
x = i + j;
{
   int i = 8;
   y = i + j;
}
z = i + j;
```

ullman compiler-design

3.0.9 Ullman (Compiler Design) Edition 2 Exercise 1.6 Question 2 (Page No. 35 - 36) top

For the block-structured C code, indicate the values assigned to w, x, y and z.

```
int w,x,y,z;
int i = 3; int j = 4;
{
   int i = 5;
   w = i + j;
}
x = i + j;
{
   int j = 6;
   i = 7;
   y = i + j;
}
z = i + j;
```

ullman compiler-design

3.0.10 Ullman (Compiler Design) Edition 2 Exercise 1.6 Question 3 (Page No. 35 - 36) top

For the block-structured code, assuming the usual static scoping of declarations, give the scope for each of the twelve declarations.

```
{
   int w,x,y,z;          /* Block B1 */
   {
      int x,z;          /* Block B2 */
      {
         int w,x;       /* Block B3 */
      }
   }

   {
       int w,x;        /* Block B4 */
       {
          int y,z;    /* Block B5 */
       }
    }
}
```

ullman compiler-design

3.0.11 Ullman (Compiler Design) Edition 2 Exercise 1.1 Question 2 (Page No. 3) top

What are the advantages of

a. a compiler over an interpreter?
b. an interpreter over a compiler?

ullman compiler-design

3.1 Ambiguous Grammar (1) top

3.1.1 Ambiguous Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 9 (Page No. 259) top

The following is an ambiguous grammar:

- $S \rightarrow AS \mid b$
- $A \rightarrow SA \mid a$

Construct for this grammar its collection of sets of $LR(0)$ items. If we try to build an LR-parsing table for the grammar, there are certain conflicting actions. What are they? Suppose we tried to use the parsing table by nondeterministically choosing a possible action whenever there is a conflict. Show all the possible sequences of actions on input $abab$.

ullman compiler-design ambiguous-grammar lr-parser descriptive

3.2 Annotated Parse Trees (1) top

3.2.1 Annotated Parse Trees: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 2 (Page No. 317) top

For the SDD of Fig. 5.8, give annotated parse trees for the following expressions:

a. int a,b,c.
b. float w,x,y,z.

	PRODUCTION	SEMANTIC RULES
1)	$D \rightarrow T\ L$	$L.inh = T.type$
2)	$T \rightarrow \mathbf{int}$	$T.type = \text{integer}$
3)	$T \rightarrow \mathbf{float}$	$T.type = \text{float}$
4)	$L \rightarrow L_1\ ,\ \mathbf{id}$	$L_1.inh = L.inh$ $addType(\mathbf{id}.entry, L.inh)$
5)	$L \rightarrow \mathbf{id}$	$addType(\mathbf{id}.entry, L.inh)$

Figure 5.8: Syntax-directed definition for simple type declarations

ullman compiler-design syntax-directed-translation parsing annotated-parse-trees

3.3 Bottom Up Parses (1) top

3.3.1 Bottom Up Parses: Ullman (Compiler Design) Edition 2 Exercise 4.5 Question 3 (Page No. 241) top

Give bottom-up parses for the following input strings and grammars:

a. The input 000111 according to the grammar of $S \rightarrow 0\,S\,1 \mid 0\,1$.
b. The input $aaa * a + +$according to the grammar of $S \rightarrow SS+ \mid SS* \mid a$.

ullman compiler-design bottom-up-parses grammar descriptive

3.4 Compiler Tokenization (2) top

3.4.1 Compiler Tokenization: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 1 (Page No. 125) top

Consult the language reference manuals to determine

i. the sets of characters that form the input alphabet (excluding those that may only appear in character strings or comments),
ii. the lexical form of numerical constants, and
iii. the lexical form of identifiers, for each of the following languages:

a. C
b. C++
c. C#
d. Fortran
e. Java
f. Lisp
g. SQL

ullman compiler-design lexical-analysis compiler-tokenization descriptive

3.4.2 Compiler Tokenization: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 4 (Page No. 125) top

Most languages are case sensitive, so keywords can be written only one way, and the regular expressions describing their lexeme is very simple. However, some languages, like SQL, are case insensitive, so a keyword can be written either in lowercase or in uppercase, or in any mixture of cases. Thus, the SQL keyword SELECT can also be written select, Select, or sElEcT, for instance. Show how to write a regular expression for a keyword in a case-insensitive language. Illustrate the idea by writing the expression for "select" in SQL.

ullman compiler-design regular-expression compiler-tokenization descriptive

3.5 Conjunctive Normal Form (1) top

3.5.1 Conjunctive Normal Form: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 8 (Page No. 232) top

A grammar is said to be in Chomsky Normal Form (CNF) if every production is either of the form $A \rightarrow BC$ or of the form
$A \rightarrow a$, where A, B, and C are nonterminals, and a is a terminal. Show how to convert any grammar into a CNF grammar for the same language (with the possible exception of the empty string - no CNF grammar can generate ϵ).

ullman compiler-design conjunctive-normal-form grammar descriptive

3.6 Context Free Grammar (9) top

3.6.1 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 1 (Page No. 51) top

Consider the context-free grammar

$S \rightarrow SS+ \mid SS^* \mid a$

a. Show how the string $aa+a^*$ can be generated by this grammar.
b. Construct a parse tree for this string.
c. What language does this grammar generate? Justify your answer.

ullman compiler-design context-free-grammar

3.6.2 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 2 (Page No. 51) top

What language is generated by the following grammars? In each case justify your answer.

a. $S \rightarrow 0S1 \mid 01$
b. $S \rightarrow +SS \mid -SS \mid a$
c. $S \rightarrow S(S)S \mid \epsilon$
d. $S \rightarrow aSbS \mid bSaS \mid \epsilon$
e. $S \rightarrow a \mid S+S \mid SS \mid S^* \mid (S)$

ullman compiler-design context-free-grammar

3.6.3 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 3 (Page No. 51) top

Which of the grammars are ambiguous?

a. $S \rightarrow 0S1 \mid 01$
b. $S \rightarrow +SS \mid -SS \mid a$
c. $S \rightarrow S(S)S \mid \epsilon$
d. $S \rightarrow aSbS \mid bSaS \mid \epsilon$
e. $S \rightarrow a \mid S+S \mid SS \mid S^* \mid (S)$

ullman compiler-design context-free-grammar ambiguous

Answer key

3.6.4 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 4 (Page No. 51 - 52) top

Construct unambiguous context-free grammars for each of the following languages. In each case show that your grammar is correct.

a. Arithmetic expressions in postfix notation.
b. Left-associative lists of identifiers separated by commas.
c. Right-associative lists of identifiers separated by commas.
d. Arithmetic expressions of integers and identifiers with the four binary operators $+, -, *, /$.
e. Add unary plus and minus to the arithmetic operators of (d).

ullman compiler-design context-free-grammar

3.6.5 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 6 (Page No. 52) top

Construct a context-free grammar for roman numerals.

ullman compiler-design context-free-grammar

3.6.6 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 1 (Page No. 206) top

Consider the context-free grammar:

$S \rightarrow SS+ \mid SS*$

and the string $aa + a*$.

a. Give a leftmost derivation for the string.
b. Give a rightmost derivation for the string.
c. Give a parse tree for the string.
d. Is the grammar ambiguous or unambiguous? Justify your answer.
e. Describe the language generated by this grammar.

ullman compiler-design context-free-grammar parsing ambiguous descriptive

Answer key

3.6.7 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 2 (Page No. 206 - 207) top

Repeat Question $4.2.1$ for each of the following grammars and strings:

a. $S \rightarrow 0S1 \mid 01$ with string 000111.
b. $S \rightarrow +SS \mid *SS \mid a$ with string $+*aaa$.
c. $S \rightarrow S(S)S \mid \epsilon$ with string $(()())$.
d. $S \rightarrow S+S \mid SS \mid (S) \mid S* \mid a$ with string $(a+a)*a$.
e. $S \rightarrow (L) \mid a$ and $L \rightarrow L, S \mid S$ with string $((a,a),a,(a))$.
f. $S \rightarrow aSbS \mid bSaS \mid \epsilon$ with string $aabbab$.
g. The following grammar for boolean expressions:

- $bexpr \ \rightarrow \ bexpr\ or\ bterm \mid bterm$
- $bterm \ \rightarrow \ bterm\ and\ bfactor \mid bfactor$
- $bfactor \ \rightarrow \ not\ bfactor \mid (bexpr) \mid true \mid false$

ullman compiler-design context-free-grammar parsing ambiguous descriptive

3.6.8 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 3 (Page No. 207) top

Design grammars for the following languages:

a. The set of all strings of $0's$ and $1's$ such that every 0 is immediately followed by at least one 1.

b. The set of all strings of $0's$ and $1's$ that are palindromes; that is, the string reads the same backward as forward.
c. The set of all strings of $0's$ and $1's$ with an equal number of $0's$ and $1's$.
d. The set of all strings of $0's$ and $1's$ with an unequal number of $0's$ and $1's$.
e. The set of all strings of $0's$ and $1's$ in which 011 does not appear as a substring.
f. The set of all strings of $0's$ and $1's$ of the form xy, where $x \neq y$ and x and y are of the same length.

ullman compiler-design context-free-grammar descriptive

3.6.9 Context Free Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 10 (Page No. 233) top

Show how, having filled in the table as in Question 4.4.9, we can in $O(n)$ time recover a parse tree for $a_1a_2 \cdots a_n$. Hint: modify the table so it records, for each nonterminal A in each table entry T_{ij}, some pair of nonterminals in other table entries that justified putting A in T_{ij}.

ullman compiler-design context-free-grammar descriptive

3.7 Cyk Algorithm (1) top

3.7.1 Cyk Algorithm: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 9 (Page No. 232) top

Every language that has a context-free grammar can be recognized in at most $O(n^3)$ time for strings of length n. A simple way to do so,called the Cocke- Younger-Kasami (or CYK) algorithm is based on dynamic programming. That is, given a string $a_1a_2 \cdots a_n$, we construct an n-by-n table T such that T_{ij} is the set of nonterminals that generate the substring $a_ia_{i+1} \cdots a_j$. If the underlying grammar is in CNF (see question 4.4.8), then one table entry can be filled in in $O(n)$ time, provided we fill the entries in the proper order: lowest value of $j-i$ first. Write an algorithm that correctly fills in the entries of the table, and show that your algorithm takes $O(n_3)$ time. Having filled in the table, how do you determine whether $a_la_2 \cdots a_n$ is in the language?

ullman compiler-design context-free-grammar cyk-algorithm descriptive

3.8 Dependency Graph (1) top

3.8.1 Dependency Graph: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 1 (Page No. 317) top

What are all the topological sorts for the dependency graph of Fig. 5.7?

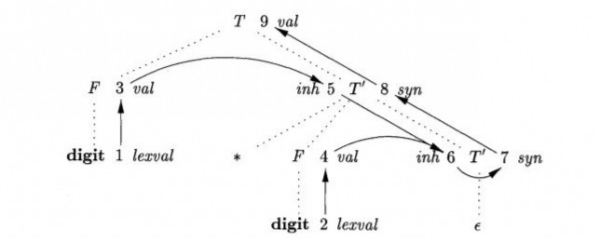


Figure 5.7: Dependency graph for the annotated parse tree of Fig. 5.5

ullman compiler-design syntax-directed-translation dependency-graph topological-sort

3.9 Directed Acyclic Graph (2) top

3.9.1 Directed Acyclic Graph: Ullman (Compiler Design) Edition 2 Exercise 6.1 Question 1 (Page No. 362) top

Construct the DAG for the expression
$((x+y)-((x+y)*(x-y)))+((x+y)*(x-y))$

ullman compiler-design three-address-code directed-acyclic-graph descriptive

Answer key

3.9.2 Directed Acyclic Graph: Ullman (Compiler Design) Edition 2 Exercise 6.1 Question 2 (Page No. 363) top

Construct the DAG and identify the value numbers for the subexpressions of the following expressions, assuming $+$ associates from the left.

a. $a+b+(a+b)$

b. $a + b + a + b$
c. $a + a + ((a + a + a + (a + a + a + a))$

ullman compiler-design three-address-code directed-acyclic-graph descriptive

Answer key

3.10 First And Follow (2) top

3.10.1 First And Follow: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 3 (Page No. 231) top

Compute FIRST and FOLLOW for the grammar of $S \rightarrow SS+ \mid SS* \mid a$

ullman compiler-design grammar first-and-follow descriptive

Answer key

3.10.2 First And Follow: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 4 (Page No. 231) top

Compute FIRST and FOLLOW for each of the grammars of

a. $S \rightarrow 0S1 \mid 01$
b. $S \rightarrow +SS \mid *SS \mid a$
c. $S \rightarrow S(S)S \mid \epsilon$
d. $S \rightarrow S + S \mid SS \mid (S) \mid S* \mid a$
e. $S \rightarrow (L) \mid a$ and $L \rightarrow L, S \mid S$
f. $S \rightarrow aSbS \mid bSaS \mid \epsilon$
g. The following grammar for boolean expressions:

- $bexpr \ \rightarrow \ bexpr \ or \ bterm \mid bterm$
- $bterm \ \rightarrow \ bterm \ and \ bfactor \mid bfactor$
- $bfactor \ \rightarrow \ not \ bfactor \mid (bexpr) \mid true \mid false$

ullman compiler-design grammar first-and-follow descriptive

3.11 Grammar (15) top

3.11.1 Grammar: Ullman (Compiler Design) Edition 2 Exercise 2.2 Question 5 (Page No. 52) top

Show that all binary strings generated by the following grammar have

a. values divisible by 3. Hint. Use induction on the number of nodes in a parse tree.

$$num \rightarrow 11 \mid 1001 \mid num \ 0 \mid num \ num$$

b. Does the grammar generate all binary strings with values divisible by 3?

ullman compiler-design grammar

3.11.2 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 5 (Page No. 208) top

Use the braces described in Question $4.2.4$ to simplify the following grammar for statement blocks and conditional statements:

$$\begin{aligned} stmt &\rightarrow if \ expr \ then \ stmt \ else \ stmt \\ &\mid if \ stmt \ then \ stmt \\ &\mid begin \ stmtList \ end \\ stmtList &\rightarrow stmt; stmtList \mid stmt \end{aligned}$$

ullman compiler-design grammar descriptive

3.11.3 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 6 (Page No. 208) top

Extend the idea of Question $4.2.4$ to allow any regular expression of grammar symbols in the body of a production. Show that this extension does not allow grammars to define any new languages.

ullman compiler-design grammar descriptive

3.11.4 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 7 (Page No. 208) top

A grammar symbol X (terminal or nonterminal) is useless if there is no derivation of the form $S \overset{*}{\Rightarrow} wXy \overset{*}{\Rightarrow} wxy$. That is, X can never appear in the derivation of any sentence.

a. Give an algorithm to eliminate from a grammar all productions containing useless symbols.
b. Apply your algorithm to the grammar:

- $S \rightarrow 0 \mid A$
- $A \rightarrow AB$
- $B \rightarrow 1$

ullman compiler-design grammar descriptive

3.11.5 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.2 Question 8 (Page No. 208 - 209) top

The grammar in Fig. 4.7 generates declarations for a single numerical identifier; these declarations involve four different, independent properties of numbers.

stmt	$\rightarrow$	**declare id** *optionList*
optionList	$\rightarrow$	*optionList option* | ϵ
option	$\rightarrow$	*mode* | *scale* | *precision* | *base*
mode	$\rightarrow$	**real** | **complex**
scale	$\rightarrow$	**fixed** | **floating**
precision	$\rightarrow$	**single** | **double**
base	$\rightarrow$	**binary** | **decimal**

Figure 4.7: A grammar for multi-attribute declarations

a. Generalize the grammar of Fig. 4.7 by allowing n options A_i, for some fixed n and for $i = 1, 2 \cdots, n$, where A_i can be either a_i or b_i. Your grammar should use only $O(n)$ grammar symbols and have a total length of productions that is $O(n)$.
b. The grammar of Fig. 4.7 and its generalization in part (a) allow declarations that are contradictory and/or redundant, such as: *declare foo real fixed real floating.* We could insist that the syntax of the language forbid such declarations; that is, every declaration generated by the grammar has exactly one value for each of the n options. If we do, then for any fixed n there is only a finite number of legal declarations. The language of legal declarations thus has a grammar (and also a regular expression), as any finite language does. The obvious grammar, in which the start symbol has a production for every legal declaration has $n!$ productions and a total production length of $O(n \times n!)$. You must do better: a total production length that is $(n2^n)$.
c. Show that any grammar for part (b) must have a total production length of at least 2^n.
d. What does part (c) say about the feasibility of enforcing nonredundancy and noncontradiction among options in declarations via the syntax of the programming language?

ullman compiler-design grammar descriptive

3.11.6 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.3 Question 3 (Page No. 217) top

The following grammar is proposed to remove the "danglingelse ambiguity" discussed in Section $4.3.2$:

- $stmt \rightarrow if\ expr\ then\ stmt \mid matchedstmt$
- $matchedstmt \rightarrow if\ expr\ then\ matchedstmt\ else\ stmt \mid other$

Show that this grammar is still ambiguous.

ullman compiler-design grammar ambiguous descriptive

3.11.7 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 11 (Page No. 233) top

Modify your algorithm of Question $4.4.9$ so that it will find, for any string, the smallest number of insert, delete, and mutate errors (each error a single character) needed to turn the string into a string in the language of the underlying grammar.

$$
\begin{array}{lcl}
\mathit{stmt} & \rightarrow & \textbf{if } e \textbf{ then } \mathit{stmt}\ \mathit{stmtTail} \\
 & | & \textbf{while } e \textbf{ do } \mathit{stmt} \\
 & | & \textbf{begin } \mathit{list} \textbf{ end} \\
 & | & s \\
\mathit{stmtTail} & \rightarrow & \textbf{else } \mathit{stmt} \\
 & | & \epsilon \\
\mathit{list} & \rightarrow & \mathit{stmt}\ \mathit{listTail} \\
\mathit{listTail} & \rightarrow & ;\ \mathit{list} \\
 & \rightarrow & \epsilon
\end{array}
$$

Figure 4.24: A grammar for certain kinds of statements

ullman compiler-design grammar descriptive

3.11.8 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 6 (Page No. 232) top

A grammar is ϵ-free if no production body is ϵ (called an ϵ-production).

a. Give an algorithm to convert any grammar into an ϵ-free grammar that generates the same language (with the possible exception of the empty string - no ϵ-free grammar can generate ϵ).
b. Apply your algorithm to the grammar $S \rightarrow aSbS \mid bSaS \mid \epsilon$. Hint: First find all the nonterminals that are nullable, meaning that they generate ϵ, perhaps by a long derivation.

ullman compiler-design grammar descriptive

3.11.9 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 7 (Page No. 232) top

A single production is a production whose body is a single nonterminal, i.e., a production of the form $A \rightarrow A$.

a. Give an algorithm to convert any grammar into an ϵ-free grammar, with no single productions, that generates the same language (with the possible exception of the empty string) Hint: First eliminate ϵ-productions, and then find for which pairs of nonterminals A and B does $A \overset{*}{\Rightarrow} B$ by a sequence of single productions.
b. Apply your algorithm to the grammar

$$E \rightarrow E + T \mid T$$

$$T \rightarrow T * F \mid F$$

$$F \rightarrow (E) \mid id$$

c. Show that, as a consequence of part (a), we can convert a grammar into an equivalent grammar that has no cycles (derivations of one or more steps in which $A \overset{*}{\Rightarrow} A$ for some nonterminal A).

ullman compiler-design grammar descriptive

3.11.10 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.5 Question 1 (Page No. 240) top

For the grammar $S \rightarrow 0\,S\,1 \mid 0\,1$ of Question $4.2.2(a)$,indicate the handle in each of the following right-sentential forms:

a. 000111
b. $00S11$

ullman compiler-design grammar descriptive

3.11.11 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.7 Question 2 (Page No. 278) top

Repeat Exercise $4.7.1$ for each of the (augmented) grammars of Exercise $4.2.2(a) - (g)$.

ullman compiler-design grammar parsing descriptive

3.11.12 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.7 Question 3 (Page No. 278) top

For the grammar of Exercise $4.7.1$, use Algorithm 4.63 to compute the collection of LALR sets of items from the kernels of the $LR(0)$ sets of items.

ullman compiler-design grammar parsing descriptive

3.11.13 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.7 Question 4 (Page No. 278) top

Show that the following grammar

- $S \rightarrow Aa \mid bAc \mid dc \mid bda$
- $A \rightarrow d$

is LALR(1) but not SLR(1).

ullman compiler-design grammar parsing descriptive

Answer key

3.11.14 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.7 Question 5 (Page No. 278) top

Show that the following grammar

- $S \rightarrow Aa \mid bAc \mid Bc \mid bBa$
- $A \rightarrow d$
- $B \rightarrow d$

is LR(1) but not LALR(1).

ullman compiler-design grammar parsing descriptive

Answer key

3.11.15 Grammar: Ullman (Compiler Design) Edition 2 Exercise 4.8 Question 1 (Page No. 285 - 286) top

The following is an ambiguous grammar for expressions with n binary, infix operators, at n different levels of precedence:

- $E \rightarrow E\theta_1 E \mid E\theta_2 E \mid \cdots E\theta_n E \mid (E) \mid id$

a. As a function of n, what are the SLR sets of items?
b. How would you resolve the conflicts in the SLR items so that all operators are left associative, and θ_1 takes precedence over θ_2, which takes precedence over θ_3, and so on?
c. Show the SLR parsing table that results from your decisions in part (b).

STACK	SYMBOLS	INPUT	ACTION
0		**id** +) \$	
0 3	**id**	+) \$	
0 1	E	+) \$	
0 1 4	$E+$	) \$	"unbalanced right parenthesis" e2 removes right parenthesis
0 1 4	$E+$	\$	"missing operand" e1 pushes state 3 onto stack
0 1 4 3	$E+$**id**	\$	
0 1 4 7	$E+$	\$	
0 1	$E+$	\$	

Figure 4.54: Parsing and error recovery moves made by an LR parser

d. Repeat parts (a) and (c) for the unambiguous grammar, which defines the same set of expressions, shown in Fig. 4.55.
e. How do the counts of the number of sets of items and the sizes of the tables for the two (ambiguous and unambiguous) grammars compare? What does that comparison tell you about the use of ambiguous expression grammars?

$$
\begin{array}{lcl}
E_1 & \rightarrow & E_1 \ \theta \ E_2 \mid E_2 \\
E_2 & \rightarrow & E_2 \ \theta \ E_3 \mid E_3 \\
 & \cdots & \\
E_n & \rightarrow & E_n \ \theta \ E_{n+1} \mid E_{n+1} \\
E_{n+1} & \rightarrow & (\ E_1\ ) \mid \textbf{id}
\end{array}
$$

Figure 4.55: Unambiguous grammar for n operators

ullman compiler-design grammar ambiguous parsing

3.12 Infix Expressions (1) top

3.12.1 Infix Expressions: Ullman (Compiler Design) Edition 2 Exercise 5.3 Question 2 (Page No. 323) top

Give an SDD to translate infix expressions with $+$ and $*$ into equivalent expressions without redundant parentheses. For example, since both operators associate from the left, and $*$ takes precedence over $+, ((a*(b+c))*(d))$ translates into $a*(b+c)*d$.

ullman compiler-design syntax-directed-translation infix-expressions

3.13 Left Recursion (4) top

3.13.1 Left Recursion: Ullman (Compiler Design) Edition 2 Exercise 4.3 Question 1 (Page No. 216) top

The following is a grammar for regular expressions over symbols a and b only, using $+$ in place of $|$ for union, to avoid conflict with the use of vertical bar as a metasymbol in grammars:

- $rexpr \rightarrow rexpr + rterm \mid rterm$
- $rterm \rightarrow rterm\, rfactor \mid rfactor$
- $rfactor \rightarrow rfactor * \mid rprimary$
- $rprimary \rightarrow a \mid b$

a. Left factor this grammar.
b. Does left factoring make the grammar suitable for top-down parsing?
c. In addition to left factoring, eliminate left recursion from the original grammar.
d. Is the resulting grammar suitable for top-down parsing?

ullman compiler-design regular-expression left-recursion descriptive

3.13.2 Left Recursion: Ullman (Compiler Design) Edition 2 Exercise 4.3 Question 2 (Page No. 216 - 217) top

Repeat Exercise 4.3.1 on the following grammars:

a. $S \rightarrow SS+ \mid SS* \mid a$
b. $S \rightarrow 0S1 \mid 01$
c. $S \rightarrow S(S)S \mid \epsilon$
d. $S \rightarrow (L) \mid a$ and $L \rightarrow L, S \mid S$
e. The following grammar for boolean expressions

- bex
- bte
- $bfactor \rightarrow not\, bfactor \mid (bexpr) \mid true \mid false$

ullman compiler-design regular-expression left-recursion descriptive

3.13.3 Left Recursion: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 1 (Page No. 231) top

For each of the following grammars, devise predictive parsers and show the parsing tables. You may left-factor and/or eliminate left-recursion from your grammars first.

a. $S \rightarrow 0S1 \mid 01$
b. $S \rightarrow +SS \mid *SS \mid a$
c. $S \rightarrow S(S)S \mid \epsilon$
d. $S \rightarrow S+S \mid SS \mid (S) \mid S* \mid a$ $.
e. $S \rightarrow (L) \mid a$ and $L \rightarrow L, S \mid S$
f. The following grammar for boolean expressions:

- $bexpr \;\rightarrow\; bexpr\; or\; bterm \mid bterm$
- $bterm \;\rightarrow\; bterm\; and\; bfactor \mid bfactor$
- $bfactor \;\rightarrow\; not\; bfactor \mid (bexpr) \mid true \mid false$

ullman compiler-design grammar left-recursion descriptive

3.13.4 Left Recursion: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 3 (Page No. 337) top

The following SDT computes the value of a string of $0's$ and $1's$ interpreted as a positive, binary integer.

- $B \to B_1 0 \{B.val = 2 \times B_1.val\} \mid B_1 1 \{B.val = 2 \times B_1.val + 1\} \mid 1 \{B.val = 1\}$

Rewrite this SDT so the underlying grammar is not left recursive, and yet the same value of $B.val$ is computed for the entire input string.

ullman compiler-design syntax-directed-translation grammar left-recursion descriptive

Answer key

3.14 Lexemes (2) top

3.14.1 Lexemes: Ullman (Compiler Design) Edition 2 Exercise 3.1 Question 1 (Page No. 114) top

Divide the following C + + program:

```
float limitedSquare(x) float x {
/* returns x-squared, but never more than 100 */
return (x<=-10.0 || x>=10.0)?100:x*x;
}
```

into appropriate lexemes, using the discussion of Section 3.1.2 as a guide. Which lexemes should get associated lexical values? What should those values be?

ullman compiler-design lexical-analysis lexemes

3.14.2 Lexemes: Ullman (Compiler Design) Edition 2 Exercise 3.1 Question 2 (Page No. 114 - 115) top

Tagged languages like HTML or XML are different from conventional programming languages in that the punctuation (tags) are either very numerous (as in HTML) or a user-definable set (as in XML). Further, tags can often have parameters. Suggest how to divide the following HTML document:

```
Here is a photo of <B>my house</B>:
<P><IMG SRC = "house. gif"><BR>
See <A HREF = "morePix. html1">More Pictures</A> if you
liked that one. <P>
```

into appropriate lexemes. Which lexemes should get associated lexical values,and what should those values be?

ullman compiler-design lexical-analysis lexemes

Answer key

3.15 Lexical Analyzer (3) top

3.15.1 Lexical Analyzer: Ullman (Compiler Design) Edition 2 Exercise 2.6 Question 1 (Page No. 84 - 85) top

Extend the lexical analyzer in Section 2.6.5 to remove comments, defined as follows:

a. A comment begins with $//$ and includes all characters until the end of that line.
b. A comment begins with $/*$ and includes all characters through the next occurrence of the character sequence $*/$.

ullman compiler-design lexical-analyzer

3.15.2 Lexical Analyzer: Ullman (Compiler Design) Edition 2 Exercise 2.6 Question 2 (Page No. 85) top

Extend the lexical analyzer in Section 2.6.5 to recognize the relational operators $<, <=, ==, != , >=, >$.

ullman compiler-design lexical-analyzer

3.15.3 Lexical Analyzer: Ullman (Compiler Design) Edition 2 Exercise 2.6 Question 3 (Page No. 85) top

Extend the lexical analyzer in Section 2.6.5 to recognize floating point numbers such as $2., 3.14$, and $.5$.

ullman compiler-design lexical-analyzer

3.16 Ll Parser (2) top

3.16.1 Ll Parser: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 5 (Page No. 258) top

Show that the following grammar:

- $S \to AaAb \mid BbBa$
- $A \to \epsilon$
- $A \to \epsilon$

is LL(1) but not SLR(1).

ullman compiler-design grammar parsing ll-parser descriptive

3.16.2 Ll Parser: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 6 (Page No. 258) top

Show that the following grammar:

- $S \to SA \mid A$
- $A \to a$

is SLR(1) but not LL(1).

ullman compiler-design grammar parsing ll-parser descriptive

3.17 Lr Parser (5) top

3.17.1 Lr Parser: Ullman (Compiler Design) Edition 2 Exercise 4.5 Question 2 (Page No. 240 - 241) top

Repeat Question 4.5.1 for the grammar $S \to S\,S+ \mid S\,S* \mid a$ of Exercise 4.2.1 and the following right-sentential forms:

a. $SSS + a * +$.
b. $SS + a * a+$.
c. $aaa * a + +$.

ullman compiler-design grammar lr-parser descriptive

3.17.2 Lr Parser: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 7 (Page No. 258) top

Consider the family of grammars G_n, defined by:

- $S \to A_i b_i$ for $1 \leq i \leq n$
- $A_i \to a_j A_i \mid a_j$ for $1 \leq i, j \leq n$ and $i \neq j$

Show that:

a. G_n, has $2n^2 - n$ productions.
b. G_n, has $2^n + n^2 + n$ sets of $LR(0)$ items.
c. G_n is $SLR(1)$.

What does this analysis say about how large LR parsers can get?

ullman compiler-design grammar parsing lr-parser descriptive

3.17.3 Lr Parser: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 8 (Page No. 259) top

We suggested that individual items could be regarded as states of a nondeterministic finite automaton, while sets of valid items are the states of a deterministic finite automaton (see the box on "Items as States of an NFA" in Section 4.6.5). For the grammar $S \to SS+ \mid SS* \mid a$ of Question 4.2.1:

a. Draw the transition diagram (NFA) for the valid items of this grammar according to the rule given in the box cited above.
b. Apply the subset construction (Algorithm 3.20) to your NFA from part (a). How does the resulting DFA compare to the set of $LR(0)$ items for the grammar?
c. Show that in all cases, the subset construction applied to the NFA that comes from the valid items for a grammar produces the $LR(0)$ sets of items.

ullman compiler-design finite-automata grammar lr-parser descriptive

3.17.4 Lr Parser: Ullman (Compiler Design) Edition 2 Exercise 4.7 Question 1 (Page No. 278) top

Construct the

a. canonical LR, and
b. LALR

sets of items for the grammar $S \rightarrow SS+ \mid SS* \mid a$ of Question $4.2.1$.

ullman compiler-design grammar lr-parser descriptive

3.17.5 Lr Parser: Ullman (Compiler Design) Edition 2 Exercise 4.8 Question 2 (Page No. 286 - 287) top

In Fig. 4.56 is a grammar for certain statements, similar to that discussed in Question $4.4.12$. Again, e and s are terminals standing for conditional expressions and "other statements," respectively.

a. Build an LR parsing table for this grammar, resolving conflicts in the usual way for the dangling-else problem.
b. Implement error correction by filling in the blank entries in the parsing table with extra reduce-actions or suitable error-recovery routines.
c. Show the behavior of your parser on the following inputs:

i. if e then s ; if e then s end
ii. while e do begin s ; if e then s ; end

4.9. *PARSER GENERATORS* 287

$$
\begin{array}{lcl}
stmt & \rightarrow & \textbf{if } e \textbf{ then } stmt \\
 & \mid & \textbf{if } e \textbf{ then } stmt \textbf{ else } stmt \\
 & \mid & \textbf{while } e \textbf{ do } stmt \\
 & \mid & \textbf{begin } list \textbf{ end} \\
 & \mid & s \\
list & \rightarrow & list \ ; \ stmt \\
 & \mid & stmt
\end{array}
$$

Figure 4.56: A grammar for certain kinds of statements

ullman compiler-design grammar parsing lr-parser descriptive

3.18 Parsing (1) top

3.18.1 Parsing: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 3 (Page No. 258) top

Show the actions of your parsing table from Question $4.6.2$ on the input $aa * a+$.

ullman compiler-design parsing descriptive

3.19 Postfix Notation (1) top

3.19.1 Postfix Notation: Ullman (Compiler Design) Edition 2 Exercise 5.3 Question 1 (Page No. 323) top

Below is a grammar for expressions involving operator $+$ and integer or floating-point operands. Floating-point numbers are distinguished by having a decimal point.

- $E \rightarrow E + T \mid T$
- $T \rightarrow num.num \mid num$

a. Give an SDD to determine the type of each term T and expression E.
b. Extend your SDD of (a) to translate expressions into postfix notation.Use the unary operator **intToFloat** to turn an integer into an equivalent float.

ullman compiler-design syntax-directed-translation grammar postfix-notation

3.20 Predictive Parser (2) top

3.20.1 Predictive Parser: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 12 (Page No. 233) top

In Fig. 4.24 is a grammar for certain statements. You may take e and s to be terminals standing for conditional expressions and "other statements," respectively. If we resolve the conflict regarding expansion of the optional "else" (nonterminal stmtTail) by preferring to consume an else from the input whenever we see one, we can build a predictive parser for this grammar. Using the idea of synchronizing symbols described in Section 4.4.5:

a. Build an error-correcting predictive parsing table for the grammar.
b. Show the behavior of your parser on the following inputs:

i. if e then s ; if e then s end
ii. while e do begin s ; if e then s ; end

$$\begin{array}{lcl}
\mathit{stmt} & \rightarrow & \textbf{if } e \textbf{ then } \mathit{stmt}\ \mathit{stmtTail} \\
 & \mid & \textbf{while } e \textbf{ do } \mathit{stmt} \\
 & \mid & \textbf{begin } \mathit{list} \textbf{ end} \\
 & \mid & s \\
\mathit{stmtTail} & \rightarrow & \textbf{else } \mathit{stmt} \\
 & \mid & \epsilon \\
\mathit{list} & \rightarrow & \mathit{stmt}\ \mathit{listTail} \\
\mathit{listTail} & \rightarrow & ;\ \mathit{list} \\
 & \rightarrow & \epsilon
\end{array}$$

Figure 4.24: A grammar for certain kinds of statements

ullman compiler-design grammar predictive-parser descriptive

3.20.2 Predictive Parser: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 2 (Page No. 231) top

Is it possible, by modifying the grammar in any way, to construct a predictive parser for the language of $S \rightarrow SS+ \mid SS* \mid a$ (postfix expressions with operand a)?

ullman compiler-design predictive-parser descriptive

3.21 Recursive Descent Parser (2) top

3.21.1 Recursive Descent Parser: Ullman (Compiler Design) Edition 2 Exercise 2.4 Question 1 (Page No. 68) top

Construct recursive-descent parsers, starting with the following grammars:

a. $S \rightarrow +SS \mid -SS \mid a$
b. $S \rightarrow S(S)S \mid \epsilon$
c. $S \rightarrow 0S1 \mid 01$

ullman compiler-design recursive-descent-parser

Answer key

3.21.2 Recursive Descent Parser: Ullman (Compiler Design) Edition 2 Exercise 4.4 Question 5 (Page No. 231 - 232) top

The grammar $S \rightarrow a\ S\ a \mid\ a\ a$ generates all even-length strings of $a's$. We can devise a recursive-descent parser with backtrack for this grammar. If we choose to expand by production $S \rightarrow a\ a$ first, then we shall only recognize the string aa. Thus, any reasonable recursive-descent parser will try $S \rightarrow a\ S\ a$ first.

a. Show that this recursive-descent parser recognizes inputs aa, $aaaa$, and $aaaaaaaa$, but not $aaaaaa$.
b. What language does this recursive-descent parser recognize?

The following exercises are useful steps in the construction of a "Chomsky Normal Form" grammar from arbitrary grammars, as defined in Question 4.4.8.

ullman compiler-design conjunctive-normal-form recursive-descent-parser descriptive

3.22 Regular Expression (9) top

3.22.1 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 10 (Page No. 127) top

The operator ^ matches the left end of a line, and $ matches the right end of a line. The operator ^ is also used to introduce complemented character classes, but the context always makes it clear which meaning is intended. For example, ^ [^aeiou]* $ matches any complete line that does not contain a lowercase vowel.

a. How do you tell which meaning of ^ is intended?
b. Can you always replace a regular expression using the ^ and $ operators by an equivalent expression that does not use either of these operators?

ullman compiler-design regular-expression descriptive

3.22.2 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 11 (Page No. 127 - 128) top

The UNIX shell command sh uses the operators in Fig. 3.9 in filename expressions to describe sets of file names. For example, the filename expression *.o matches all filenames ending in. o; sort 1. ? matches all filenames of the form sort. c, where c is any character. Show how sh filename expressions can be replaced by equivalent regular expressions using only the basic union, concatenation, and closure operators.

EXPRESSION	MATCHES	EXAMPLE
$'s'$	string s literally	`'\'`
$\backslash c$	character c literally	`\'`
*	any string	`*.o`
?	any character	`sort1.?`
$[s]$	any character in s	`sort1.[cso]`

Figure 3.9: Filename expressions used by the shell command **sh**

ullman compiler-design regular-expression descriptive

3.22.3 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 12 (Page No. 128) top

SQL allows a rudimentary form of patterns in which two characters have special meaning: underscore (_) stands for any one character and percent-sign (%) stands for any string of 0 or more characters. In addition, the programmer may define any character, say e, to be the escape character, so e preceding an e preceding -, %, or another e gives the character that follows its literal meaning. Show how to express any SQL pattern as a regular expression, given that we know which character is the escape character.

ullman compiler-design regular-expression descriptive

3.22.4 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 2 (Page No. 125) top

Describe the languages denoted by the following regular expressions:

a. $a(a \mid b)^*a$.
b. $((\epsilon \mid a)b^*)^*$.
c. $(a \mid b)^*a(a \mid b)(a \mid b)$.
d. $a^*ba^*ba^*ba^*$.
e. $(aa \mid bb)^*((ab \mid ba)(aa \mid bb)^*(ab \mid ba)(aa \mid bb)^*)^*$.

ullman compiler-design regular-expression descriptive

Answer key

3.22.5 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 5 (Page No. 125 - 126) top

Write regular definitions for the following languages:

a. All strings of lowercase letters that contain the five vowels in order.
b. All strings of lowercase letters in which the letters are in ascending lexicographic order.
c. Comments, consisting of a string surrounded by /* and */, without an intervening */, unless it is inside double-quotes (").
d. All strings of digits with no repeated digits. Hint: Try this problem first with a few digits, such as $\{0, 1, 2\}$.

e. All strings of digits with at most one repeated digit.
f. All strings of a's and b's with an even number of a's and an odd number of b's.
g. The set of Chess moves, in the informal notation, such as $p - k_4$ or $kbp \times qn$.
h. All strings of a's and b's that do not contain the substring abb.
i. All strings of a's and b's that do not contain the subsequence abb.

ullman compiler-design descriptive regular-expression

3.22.6 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 6 (Page No. 126) top

Write character classes for the following sets of characters:

a. The first ten letters (up to "j") in either upper or lower case.
b. The lowercase consonants.
c. The "digits" in a hexadecimal number (choose either upper or lower case for the "digits" above 9).
d. The characters that can appear at the end of a legitimate English sentence (e.g., exclamation point).

The following exercises, up to and including Exercise $3.3.10$, discuss the extended regular-expression notation from Lex (the lexical-analyzer generator that we shall discuss extensively in Section 3.5). The extended notation is listed in Fig. 3.8.

EXPRESSION	MATCHES	EXAMPLE
c	the one non-operator character c	a
\\c	character c literally	*
"s"	string s literally	"**"
.	any character but newline	a.*b
^	beginning of a line	^abc
$	end of a line	abc$
[s]	any one of the characters in string s	[abc]
[^s]	any one character not in string s	[^abc]
$r*$	zero or more strings matching r	a*
$r+$	one or more strings matching r	a+
$r?$	zero or one r	a?
$r\{m,n\}$	between m and n occurrences of r	a[1,5]
r_1r_2	an r_1 followed by an r_2	ab
$r_1 \mid r_2$	an r_1 or an r_2	a|b
(r)	same as r	(a|b)
r_1/r_2	r_1 when followed by r_2	abc/123

Figure 3.8: Lex regular expressions

ullman compiler-design regular-expression descriptive

3.22.7 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 7 (Page No. 126) top

Note that these regular expressions give all of the following
symbols (operator characters) a special meaning:
\ " . ^ $ [] * + ? {} | /
Their special meaning must be turned off if they are needed to represent themselves in a character string. We can do so by quoting the character within a string of length one or more; e.g., the regular expression "**" matches the string **. We can also get the literal meaning of an operator character by preceding it by a backslash. Thus, the regular expression ** also matches the string **. Write a regular expression that matches the string "\.

ullman compiler-design regular-expression descriptive

3.22.8 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 8 (Page No. 126 - 127) top

In Lex, a complemented character class represents any character except the ones listed in the character class. We denote a complemented class by using ^ as the first character; this symbol (caret) is not itself part of the class being complemented, unless it is listed within the class itself. Thus, [^ A-Za-z] matches any character that is not an uppercase or lowercase letter, and [^\^] represents any character but the caret (or newline, since newline cannot be in any character class). Show that for every regular expression with complemented character classes, there is an equivalent regular expression without complemented character classes.

EXPRESSION	MATCHES	EXAMPLE
c	the one non-operator character c	a
$\backslash c$	character c literally	*
"s"	string s literally	"**"
.	any character but newline	a.*b
^	beginning of a line	^abc
$	end of a line	abc$
[s]	any one of the characters in string s	[abc]
[^s]	any one character not in string s	[^abc]
$r*$	zero or more strings matching r	a*
$r+$	one or more strings matching r	a+
$r?$	zero or one r	a?
$r\{m,n\}$	between m and n occurrences of r	a[1,5]
r_1r_2	an r_1 followed by an r_2	ab
$r_1 \mid r_2$	an r_1 or an r_2	a|b
(r)	same as r	(a|b)
r_1/r_2	r_1 when followed by r_2	abc/123

Figure 3.8: Lex regular expressions

ullman compiler-design regular-expression descriptive

3.22.9 Regular Expression: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 9 (Page No. 127) top

The regular expression $r\{m, n\}$ matches from m to n occurrences of the pattern r. For example, $a[1, 5]$ matches a string of one to five a's. Show that for every regular expression containing repetition operators of this form, there is an equivalent regular expression without repetition operators.

ullman compiler-design regular-expression descriptive

3.23 Slr Item (2) top

3.23.1 Slr Item: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 2 (Page No. 258) top

Construct the SLR sets of items for the (augmented) grammar of Question $4.2.1$. Compute the GOTO function for these sets of items. Show the parsing table for this grammar. Is the grammar SLR?

ullman compiler-design slr-item goto-function descriptive

3.23.2 Slr Item: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 4 (Page No. 258) top

For each of the (augmented) grammars of Question $4.2.2(a)-(g)$:

a. Construct the SLR sets of items and their GOTO function.
b. Indicate any action conflicts in your sets of items.
c. Construct the SLR-parsing table, if one exists.

ullman compiler-design parsing slr-item goto-function descriptive

3.24 Strings (1) top

3.24.1 Strings: Ullman (Compiler Design) Edition 2 Exercise 3.3 Question 3 (Page No. 125) top

In a string of length n, how many of the following are there?

a. Prefixes.
b. Suffixes.
c. Proper prefixes.
d. Substrings.
e. Subsequences.

ullman compiler-design strings descriptive

Answer key

3.25 Syntax Directed Translation (19) top

3.25.1 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 2.3 Question 1 (Page No. 60) top

Construct a syntax-directed translation scheme that translates arithmetic expressions from infix notation into prefix notation in which an operator appears before its operands; e.g., $-xy$ is the prefix notation for $x-y$. Give annotated parse trees for the inputs $9 - 5 + 2$ and $9 - 5 * 2$.

ullman compiler-design syntax-directed-translation parsing

Answer key

3.25.2 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 2.3 Question 2 (Page No. 60) top

Construct a syntax-directed translation scheme that translates arithmetic expressions from postfix notation into infix notation. Give annotated parse trees for the inputs $95 - 2*$ and $952 * -$.

ullman compiler-design syntax-directed-translation parsing

3.25.3 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 2.3 Question 3 (Page No. 60) top

Construct a syntax-directed translation scheme that translates integers into roman numerals.

ullman compiler-design syntax-directed-translation

3.25.4 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 2.3 Question 4 (Page No. 60) top

Construct a syntax-directed translation scheme that translates roman numerals into integers.

ullman compiler-design syntax-directed-translation

3.25.5 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 2.3 Question 5 (Page No. 60) top

Construct a syntax-directed translation scheme that translates postfix arithmetic expressions into equivalent infix arithmetic expressions.

ullman compiler-design syntax-directed-translation

3.25.6 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.1 Question 1 (Page No. 309 - 310) top

	PRODUCTION	SEMANTIC RULES
1)	$L \to E$ **n**	$L.val = E.val$
2)	$E \to E_1 + T$	$E.val = E_1.val + T.val$
3)	$E \to T$	$E.val = T.val$
4)	$T \to T_1 * F$	$T.val = T_1.val \times F.val$
5)	$T \to F$	$T.val = F.val$
6)	$F \to (E)$	$F.val = E.val$
7)	$F \to$ **digit**	$F.val =$ **digit**.lexval

Figure 5.1: Syntax-directed definition of a simple desk calculator

For the SDD(SYNTAX-DIRECTED DEFINITIONS) of Fig. 5.1, give annotated parse trees for the following expressions:

a. $(3 + 4) * (5 + 6)n$.
b. $1 * 2 * 3 * (4 + 5)n$.
c. $(9 + 8 * (7 + 6) + 5) * 4n$.

ullman compiler-design syntax-directed-translation parsing

Answer key

3.25.7 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.1 Question 2 (Page No. 310) top

Extend the SDD of Fig. 5.4 to handle expressions as in Fig. 5.1.

	Production	Semantic Rules
1)	$L \to E\ \mathbf{n}$	$L.val = E.val$
2)	$E \to E_1 + T$	$E.val = E_1.val + T.val$
3)	$E \to T$	$E.val = T.val$
4)	$T \to T_1 * F$	$T.val = T_1.val \times F.val$
5)	$T \to F$	$T.val = F.val$
6)	$F \to (E)$	$F.val = E.val$
7)	$F \to \mathbf{digit}$	$F.val = \mathbf{digit}.\text{lexval}$

Figure 5.1: Syntax-directed definition of a simple desk calculator

	Production	Semantic Rules
1)	$T \to F\,T'$	$T'.inh = F.val$ $T.val = T'.syn$
2)	$T' \to * F\,T'_1$	$T'_1.inh = T'.inh \times F.val$ $T'.syn = T'_1.syn$
3)	$T' \to \epsilon$	$T'.syn = T'.inh$
4)	$F \to \mathbf{digit}$	$F.val = \mathbf{digit}.lexval$

Figure 5.4: An SDD based on a grammar suitable for top-down parsing

ullman compiler-design syntax-directed-translation parsing

3.25.8 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.1 Question 3 (Page No. 310) top

	Production	Semantic Rules
1)	$T \to F\,T'$	$T'.inh = F.val$ $T.val = T'.syn$
2)	$T' \to * F\,T'_1$	$T'_1.inh = T'.inh \times F.val$ $T'.syn = T'_1.syn$
3)	$T' \to \epsilon$	$T'.syn = T'.inh$
4)	$F \to \mathbf{digit}$	$F.val = \mathbf{digit}.lexval$

Figure 5.4: An SDD based on a grammar suitable for top-down parsing

For the SDD(SYNTAX-DIRECTED DEFINITIONS) of Fig. 5.4, give annotated parse trees for the following expressions:

1. $(3+4)*(5+6)n.$
2. $1*2*3*(4+5)n.$
3. $(9+8*(7+6)+5)*4n.$

ullman compiler-design parsing syntax-directed-translation

3.25.9 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 3 (Page No. 317) top

Suppose that we have a production $A \to BCD$. Each of the four nonterminals A, B, C, and D have two attributes: s is a synthesized attribute, and i is an inherited attribute. For each of the sets of rules below, tell whether

i. the rules are consistent with an S-attributed definition
ii. the rules are consistent with an L-attributed definition, and
iii. whether the rules are consistent with any evaluation order at all?

a. $A.s = B.i + C.s.$
b. $A.s = B.i + C.s$ and $D.i = A.i + B.s.$
c. $A.s = B.s + D.s$
d. $A.s = D.i, B.i = A.s + C.s, C.i = B.s,$ and $D.i = B.i + C.i.$

ullman compiler-design syntax-directed-translation

Answer key

3.25.10 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 4 (Page No. 317) top

This grammar generates binary numbers with a "decimal" point:

- $S \rightarrow L.L \mid L$
- $L \rightarrow LB \mid B$
- $B \rightarrow 0 \mid 1$

Design an L-attributed SDD to compute $S.val$, the decimal-number value of an input string. For example, the translation of string 101.101 should be the decimal number 5.625. Hint: use an inherited attribute $L.side$ that tells which side of the decimal point a bit is on.

ullman compiler-design syntax-directed-translation grammar parsing

3.25.11 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 5 (Page No. 317) top

This grammar generates binary numbers with a "decimal" point:

- $S \rightarrow L.L \mid L$
- $L \rightarrow LB \mid B$
- $B \rightarrow 0 \mid 1$

Design an S-attributed SDD to compute $S.val$, the decimal-number value of an input string. For example, the translation of string 101.101 should be the decimal number 5.625.

ullman compiler-design syntax-directed-translation grammar parsing

Answer key

3.25.12 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.2 Question 6 (Page No. 317) top

Implement Algorithm 3.23, which converts a regular expression into a nondeterministic finite automaton, by an L-attributed SDD on a top-down parsable grammar. Assume that there is a token char representing any character, and that char.$lexval$ is the character it represents. You may also assume the existence of a function $new()$ that returns a new state, that is, a state never before returned by this function. Use any convenient notation to specify the transitions of the NFA.

ullman compiler-design syntax-directed-translation regular-expression finite-automata parsing

3.25.13 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.3 Question 3 (Page No. 323) top

Give an SDD to differentiate expressions such as $x * (3 * x + x * x)$ involving the operators $+$ and $*$, the variable x, and constants. Assume that no simplification occurs, so that, for example, $3 * x$ will be translated into $3 * 1 + 0 * x$.

ullman compiler-design syntax-directed-translation

3.25.14 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 1 (Page No. 336) top

We mentioned in Section $5.4.2$ that it is possible to deduce, from the LR state on the parsing stack, what grammar symbol is represented by the state. How would we discover this information?

PRODUCTION	ACTIONS
$L \rightarrow E\ \mathbf{n}$	{ print(*stack*[*top* − 1].*val*); *top* = *top* − 1; }
$E \rightarrow E_1 + T$	{ *stack*[*top* − 2].*val* = *stack*[*top* − 2].*val* + *stack*[*top*].*val*; *top* = *top* − 2; }
$E \rightarrow T$	
$T \rightarrow T_1 * F$	{ *stack*[*top* − 2].*val* = *stack*[*top* − 2].*val* × *stack*[*top*].*val*; *top* = *top* − 2; }
$T \rightarrow F$	
$F \rightarrow (\ E\ )$	{ *stack*[*top* − 2].*val* = *stack*[*top* − 1].*val*; *top* = *top* − 2; }
$F \rightarrow \mathbf{digit}$	

Figure 5.20: Implementing the desk calculator on a bottom-up parsing stack

ullman compiler-design syntax-directed-translation grammar descriptive

3.25.15 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 2 (Page No. 336 - 337) top

Rewrite the following SDT:

- $A \to A\{a\}B \mid AB\{b\} \mid 0$
- $B \to B\{c\}A \mid BA\{d\} \mid 1$

so that the underlying grammar becomes non-left-recursive. Here, a, b, c, and d are actions, and 0 and 1 are terminals.

ullman compiler-design syntax-directed-translation grammar descriptive

Answer key

3.25.16 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 4 (Page No. 337) top

Write L-attributed SDD's analogous to that of Example 5.19 for the following productions, each of which represents a familiar flow-of-control construct, as in the programming language C. You may need to generate a three address statement to jump to a particular label L, in which case you should generate **goto** L.

a. $S \to f(C)S_1 \; else \; S_2$
b. $S \to do \; S_1 \; while \; (C)$
c. $S \to' \{' L'\}'; \; L \to LS \mid \epsilon$

Note that any statement in the list can have a jump from its middle to the next statement, so it is not sufficient simply to generate code for each statement in order.

ullman compiler-design syntax-directed-translation grammar descriptive

3.25.17 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 5 (Page No. 337) top

Write L-attributed SDT's analogous to that of Example 5.19 for the following productions, each of which represents a familiar flow-of-control construct, as in the programming language C. You may need to generate a three address statement to jump to a particular label L, in which case you should generate **goto** L.

1. $S \to f(C)S_1 \; else \; S_2$
2. $S \to do \; S_1 \; while \; (C)$
3. $S \to' \{' L'\}'; \; L \to LS \mid \epsilon$

Note that any statement in the list can have a jump from its middle to the next statement, so it is not sufficient simply to generate code for each statement in order.

ullman compiler-design syntax-directed-translation grammar descriptive

3.25.18 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 6 (Page No. 337) top

Modify the SDD of Fig. 5.25 to include a synthesized attribute $B.le$, the length of a box. The length of the concatenation of two boxes is the sum of the lengths of each. Then add your new rules to the proper positions in the SDT of Fig. 5.26.

	PRODUCTION	SEMANTIC RULES
1)	$S \to B$	$B.ps = 10$
2)	$B \to B_1 \; B_2$	$B_1.ps = B.ps$ $B_2.ps = B.ps$ $B.ht = \max(B_1.ht, B_2.ht)$ $B.dp = \max(B_1.dp, B_2.dp)$
3)	$B \to B_1 \; \textbf{sub} \; B_2$	$B_1.ps = B.ps$ $B_2.ps = 0.7 \times B.ps$ $B.ht = \max(B_1.ht, B_2.ht - 0.25 \times B.ps)$ $B.dp = \max(B_1.dp, B_2.dp + 0.25 \times B.ps)$
4)	$B \to (\; B_1 \;)$	$B_1.ps = B.ps$ $B.ht = B_1.ht$ $B.dp = B_1.dp$
5)	$B \to \textbf{text}$	$B.ht = getHt(B.ps, \textbf{text}.lexval)$ $B.dp = getDp(B.ps, \textbf{text}.lexval)$

Figure 5.25: SDD for typesetting boxes

Production			Actions
1)	$S \rightarrow$		$\{\ B.ps = 10;\ \}$
		B	
2)	$B \rightarrow$		$\{ B_1.ps = B.ps;\ \}$
		B_1	$\{ B_2.ps = B.ps;\ \}$
		B_2	$\{\ B.ht = \max(B_1.ht, B_2.ht);$ $B.dp = \max(B_1.dp, B_2.dp);\ \}$
3)	$B \rightarrow$		$\{ B_1.ps = B.ps;\ \}$
		B_1 **sub**	$\{ B_2.ps = 0.7 \times B.ps;\ \}$
		B_2	$\{\ B.ht = \max(B_1.ht, B_2.ht - 0.25 \times B.ps);$ $B.dp = \max(B_1.dp, B_2.dp + 0.25 \times B.ps);\ \}$
4)	$B \rightarrow$	(	$\{ B_1.ps = B.ps;\ \}$
		B_1)	$\{\ B.ht = B_1.ht;$ $B.dp = B_1.dp;\ \}$
5)	$B \rightarrow$	**text**	$\{\ B.ht = getHt(B.ps, \textbf{text}.lexval);$ $B.dp = getDp(B.ps, \textbf{text}.lexval);\ \}$

Figure 5.26: SDT for typesetting boxes

ullman compiler-design syntax-directed-translation grammar descriptive

3.25.19 Syntax Directed Translation: Ullman (Compiler Design) Edition 2 Exercise 5.4 Question 7 (Page No. 337) top

Modify the SDD of Fig. 5.25 to include superscripts denoted by operator **sup** between boxes. If box B_2 is a superscript of box B_1, then position the baseline of B_2 0.6 times the point size of B_1 above the baseline of B_1. Add the new production and rules to the SDT of Fig. 5.26.

	Production	Semantic Rules
1)	$S \rightarrow B$	$B.ps = 10$
2)	$B \rightarrow B_1\ B_2$	$B_1.ps = B.ps$ $B_2.ps = B.ps$ $B.ht = \max(B_1.ht, B_2.ht)$ $B.dp = \max(B_1.dp, B_2.dp)$
3)	$B \rightarrow B_1$ **sub** B_2	$B_1.ps = B.ps$ $B_2.ps = 0.7 \times B.ps$ $B.ht = \max(B_1.ht, B_2.ht - 0.25 \times B.ps)$ $B.dp = \max(B_1.dp, B_2.dp + 0.25 \times B.ps)$
4)	$B \rightarrow (\ B_1\ )$	$B_1.ps = B.ps$ $B.ht = B_1.ht$ $B.dp = B_1.dp$
5)	$B \rightarrow$ **text**	$B.ht = getHt(B.ps, \textbf{text}.lexval)$ $B.dp = getDp(B.ps, \textbf{text}.lexval)$

Figure 5.25: SDD for typesetting boxes

Production			Actions
1)	$S \rightarrow$		$\{\ B.ps = 10;\ \}$
		B	
2)	$B \rightarrow$		$\{ B_1.ps = B.ps;\ \}$
		B_1	$\{ B_2.ps = B.ps;\ \}$
		B_2	$\{\ B.ht = \max(B_1.ht, B_2.ht);$ $B.dp = \max(B_1.dp, B_2.dp);\ \}$
3)	$B \rightarrow$		$\{ B_1.ps = B.ps;\ \}$
		B_1 **sub**	$\{ B_2.ps = 0.7 \times B.ps;\ \}$
		B_2	$\{\ B.ht = \max(B_1.ht, B_2.ht - 0.25 \times B.ps);$ $B.dp = \max(B_1.dp, B_2.dp + 0.25 \times B.ps);\ \}$
4)	$B \rightarrow$	(	$\{ B_1.ps = B.ps;\ \}$
		B_1)	$\{\ B.ht = B_1.ht;$ $B.dp = B_1.dp;\ \}$
5)	$B \rightarrow$	**text**	$\{\ B.ht = getHt(B.ps, \textbf{text}.lexval);$ $B.dp = getDp(B.ps, \textbf{text}.lexval);\ \}$

Figure 5.26: SDT for typesetting boxes

ullman compiler-design syntax-directed-translation grammar descriptive

3.26 Three Address Code (18) top

3.26.1 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 2.8 Question 1 (Page No. 105) top

For-statements in C and Java have the form:
for $(expr_l; expr_2; expr_3) stmt$
The first expression is executed before the loop; it is typically used for initializing the loop index. The second expression is a test made before each iteration of the loop; the loop is exited if the expression becomes 0. The loop itself can be thought of as the statement $\{strmt\ expr_3;\}$. The third expression is executed at the end of each iteration; it is typically used to increment the loop index. The meaning of the for-statement is similar to $expr_1; while(expr2)\ \{stmt\ expr_3;\}$
Define a class For for for-statements, similar to class If in Fig. 2.43

```
class If extends Stmt {
        Expr E; Stmt S;
        public If(Expr x, Stmt y) { E = x; S = y; after = newlabel(); }
        public void gen() {
                Expr n = E.rvalue();
                emit( "ifFalse " + n.toString() + " goto " + after);
                S.gen();
                emit(after + ":");
        }
}
```

Figure 2.43: Function *gen* in class *If* generates three-address code

.

ullman compiler-design three-address-code

3.26.2 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 2.8 Question 2 (Page No. 105) top

The programming language C does not have a boolean type. Show how a C compiler might translate an if-statement into three-address code.

ullman compiler-design three-address-code

Answer key

3.26.3 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.2 Question 1 (Page No. 370) top

Translate the arithmetic expression $a + -(b + c)$ into:

a. A syntax tree. b. Quadruples. c. Triples. d. Indirect triples

ullman compiler-design three-address-code intermediate-code descriptive

Answer key

3.26.4 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.2 Question 2 (Page No. 370) top

Translate the following arithmetic expression into:

i. $a = b[i] + c[j]$
ii. $a[i] = b * c - b * d$
iii. $x = f(y + 1) + 2$
iv. $x = *p + \&y$

a. A Syntax tree b. Quadruples c. Triples d. Indirect triples

ullman compiler-design three-address-code intermediate-code descriptive

Answer key

3.26.5 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.2 Question 3 (Page No. 370) top

Show how to transform a three-address code sequence into one in which each defined variable gets a unique variable name.

ullman compiler-design three-address-code intermediate-code descriptive

Answer key

3.26.6 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.3 Question 1 (Page No. 378) top

Determine the types and relative addresses for the identifiers in the following sequence of declarations:

```
float x;
record { float x; float y; } p;
record { int tag; float x; float y; } q;
```

ullman compiler-design three-address-code intermediate-code descriptive

3.26.7 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.3 Question 2 (Page No. 378) top

Extend the handling of field names in Fig. 6.18 to classes and single-inheritance class hierarchies.

a. Give an implementation of class Env that allows linked symbol tables, so that a subclass can either redefine a field name or refer directly to a field name in a superclass.
b. Give a translation scheme that allocates a contiguous data area for the fields in a class, including inherited fields. Inherited fields must maintain the relative addresses they were assigned in the layout for the superclass.

$T \rightarrow$ **record** '{' $\{$ $Env.push(top)$; $top =$ **new** $Env()$;
$\quad$ $Stack.push(offset)$; $offset = 0$; $\}$
$\quad D$ '}' $\{$ $T.type = record(top)$; $T.width = offset$;
$\quad$ $top = Env.pop()$; $offset = Stack.pop()$; $\}$

Figure 6.18: Handling of field names in records

ullman compiler-design three-address-code intermediate-code descriptive

3.26.8 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 1 (Page No. 384) top

Add to the translation of Fig. 6.19 rules for the following productions:

- $E \rightarrow E_1 * E_2$
- $E \rightarrow +E_1$ (unary plus)

PRODUCTION	SEMANTIC RULES
$S \rightarrow$ **id** = E ;	$S.code = E.code \;\|\|$ $gen(top.get(\textbf{id}.lexeme)\; '=' \; E.addr)$
$E \rightarrow E_1 + E_2$	$E.addr =$ **new** $Temp()$ $E.code = E_1.code \;\|\|\; E_2.code \;\|\|$ $gen(E.addr\; '=' \; E_1.addr\; '+' \; E_2.addr)$
$\mid\; - E_1$	$E.addr =$ **new** $Temp()$ $E.code = E_1.code \;\|\|$ $gen(E.addr\; '=' \; '\textbf{minus}' \; E_1.addr)$
$\mid\; (E_1)$	$E.addr = E_1.addr$ $E.code = E_1.code$
$\mid$ **id**	$E.addr = top.get(\textbf{id}.lexeme)$ $E.code = ''$

Figure 6.19: Three-address code for expressions

ullman compiler-design intermediate-code three-address-code descriptive

3.26.9 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 2 (Page No. 384) top

Add to the translation of Fig. 6.19 rules for the following productions:

- $E \rightarrow E_1 * E_2$
- $E \rightarrow +E_1$ (unary plus)

6.4. TRANSLATION OF EXPRESSIONS 381

```
S → id = E ;     { gen( top.get(id.lexeme) '=' E.addr); }

E → E1 + E2      { E.addr = new Temp();
                   gen(E.addr '=' E1.addr '+' E2.addr); }

  | - E1         { E.addr = new Temp();
                   gen(E.addr '=' 'minus' E1.addr); }

  | ( E1 )       { E.addr = E1.addr; }

  | id           { E.addr = top.get(id.lexeme); }
```

Figure 6.20: Generating three-address code for expressions incrementally

ullman compiler-design intermediate-code three-address-code descriptive

3.26.10 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 3 (Page No. 385) top

Use the translation of Fig. 6.22 to translate the following assignments:

a. $x = a[i] + b[j]$
b. $x = a[i][j] + b[i][j]$
c. $x = a[b[i][j]][c[[k]]$

```
S → id = E ;     { gen( top.get(id.lexeme) '=' E.addr); }

  | L = E ;      { gen(L.addr.base '[' L.addr ']' '=' E.addr); }

E → E1 + E2      { E.addr = new Temp();
                   gen(E.addr '=' E1.addr '+' E2.addr); }

  | id           { E.addr = top.get(id.lexeme); }

  | L            { E.addr = new Temp();
                   gen(E.addr '=' L.array.base '[' L.addr ']'); }

L → id [ E ]     { L.array = top.get(id.lexeme);
                   L.type = L.array.type.elem;
                   L.addr = new Temp();
                   gen(L.addr '=' E.addr '*' L.type.width); }

  | L1 [ E ]     { L.array = L1.array;
                   L.type = L1.type.elem;
                   t = new Temp();
                   L.addr = new Temp();
                   gen(t '=' E.addr '*' L.type.width); }
                   gen(L.addr '=' L1.addr '+' t); }
```

Figure 6.22: Semantic actions for array references

ullman compiler-design intermediate-code three-address-code descriptive

3.26.11 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 4 (Page No. 385) top

Revise the translation of Fig. 6.22 for array references of the Fortran style, that is, $id[E_1, E_2, \cdots, E_n]$ for an $n-$dimensional array.


```
S → id = E ;     { gen( top.get(id.lexeme) '=' E.addr); }

  | L = E ;      { gen(L.addr.base '[' L.addr ']' '=' E.addr); }

E → E1 + E2      { E.addr = new Temp();
                   gen(E.addr '=' E1.addr '+' E2.addr); }

  | id           { E.addr = top.get(id.lexeme); }

  | L            { E.addr = new Temp();
                   gen(E.addr '=' L.array.base '[' L.addr ']'); }

L → id [ E ]     { L.array = top.get(id.lexeme);
                   L.type = L.array.type.elem;
                   L.addr = new Temp();
                   gen(L.addr '=' E.addr '*' L.type.width); }

  | L1 [ E ]     { L.array = L1.array;
                   L.type = L1.type.elem;
                   t = new Temp();
                   L.addr = new Temp();
                   gen(t '=' E.addr '*' L.type.width); }
                   gen(L.addr '=' L1.addr '+' t); }
```

Figure 6.22: Semantic actions for array references

ullman compiler-design intermediate-code three-address-code descriptive

3.26.12 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 5 (Page No. 385 - 386) top

Generalize formula (6.7) to multidimensional arrays, and indicate what values can be stored in the symbol table and used to compute offsets. Consider the following cases:

a. An array A of two dimensions, in row-major form. The first dimension has indexes running from l_1 to h_l, and the second dimension has indexes from l_2 to h_2. The width of a single array element is w.
b. The same as (a), but with the array stored in column-major form.
c. An array A of k dimensions, stored in row-major form, with elements of
d. size w. The j^{th} dimension has indexes running from l_j to h_j.The same as (c) but with the array stored in column-major form.

ullman compiler-design intermediate-code three-address-code descriptive

3.26.13 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 6 (Page No. 386) top

An integer array $A[i, j]$ has index i ranging from 1 to 10 and index j ranging from 1 to 20. Integers take 4 bytes each. Suppose array A is stored starting at byte 0. Find the location of:

a. $A[4, 5]$
b. $A[10, 8]$
c. $A[3, 17]$

ullman compiler-design intermediate-code three-address-code descriptive

Answer key

3.26.14 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 7 (Page No. 386) top

An integer array $A[i, j]$ has index i ranging from 1 to 10 and index j ranging from 1 to 20. Integers take 4 bytes each. Suppose array A is stored starting at byte 0. Find the location of:

a. $A[4, 5]$
b. $A[10, 8]$
c. $A[3, 17]$

if A is stored in column-major order.

ullman compiler-design intermediate-code three-address-code descriptive

3.26.15 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 8 (Page No. 386) top

A real array $A[i, j, k]$ has index i ranging from 1 to 4, index j ranging from 0 to 4, and index k ranging from 5 to 10. Reals take 8 bytes each. Suppose array A is stored starting at byte 0. Find the location of:

a. $A[3, 4, 5]$
b. $A[1, 2, 7]$
c. $A[4, 3, 9]$

ullman compiler-design intermediate-code three-address-code descriptive

Answer key

3.26.16 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.4 Question 9 (Page No. 386) top

A real array $A[i, j, k]$ has index i ranging from 1 to 4, index j ranging from 0 to 4, and index k ranging from 5 to 10. Reals take 8 bytes each. Suppose array A is stored starting at byte 0. Find the location of:

a. $A[3, 4, 5]$
b. $A[1, 2, 7]$
c. $A[4, 3, 9]$

if A is stored in column-major order.

ullman compiler-design intermediate-code three-address-code descriptive

3.26.17 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.5 Question 1 (Page No. 398) top

Assuming that function $widen$ in Fig. 6.26 can handle any of the types in the hierarchy of Fig. $6.25(a)$, translate the expressions below. Assume that c and d are characters, s and t are short integers, i and j are integers, and x is a float.

a. $x = s + c$
b. $i = s + c$
c. $x = (s + c) * (t + d)$

ullman compiler-design intermediate-code three-address-code descriptive

3.26.18 Three Address Code: Ullman (Compiler Design) Edition 2 Exercise 6.5 Question 2 (Page No. 399) top

As in Ada, suppose that each expression must have a unique type, but that from a subexpression, by itself, all we can deduce is a set of possible types. That is, the application of function E_1 to argument E_2, represented by $E \rightarrow E_l(E_2)$, has the associated rule

$$E.type = \{t \mid \text{for some } s \text{ in } E_2.type, s \rightarrow t \text{ is in } E_1.type\}$$

Describe an SDD that determines a unique type for each subexpression by using an attribute $type$ to synthesize a set of possible types bottom-up, and, once the unique type of the overall expression is determined, proceeds top-down to determine attribute $unique$ for the type of each subexpression.

ullman compiler-design intermediate-code three-address-code descriptive

3.27 Viable Prefix (1) top

3.27.1 Viable Prefix: Ullman (Compiler Design) Edition 2 Exercise 4.6 Question 1 (Page No. 257 - 258) top

Describe all the viable prefixes for the following grammars:

a. The grammar $S \rightarrow 0S1 \mid 01$ of Question $4.2.2(a)$.
b. The grammar $S \rightarrow SS+ \mid SS* \mid a$ of Question $4.2.1$.
c. The grammar $S \rightarrow S(S) \mid \epsilon$ of Question $4.2.2(c)$.

ullman compiler-design grammar viable-prefix descriptive

3.28 Yacc (4) top

3.28.1 Yacc: Ullman (Compiler Design) Edition 2 Exercise 4.9 Question 1 (Page No. 297) top

Write a $Yacc$ program that takes boolean expressions as input [as given by the grammar of Question $4.2.2(g)$] and produces the truth value of the expressions.

ullman compiler-design grammar yacc descriptive

3.28.2 Yacc: Ullman (Compiler Design) Edition 2 Exercise 4.9 Question 2 (Page No. 297) top

Write a $Yacc$ program that takes lists (as defined by the grammar of Question $4.2.2(e)$, but with any single character as an element, not just a) and produces as output a linear representation of the same list; i.e., a single list of the elements, in the same order that they appear in the input.

ullman compiler-design grammar yacc descriptive

3.28.3 Yacc: Ullman (Compiler Design) Edition 2 Exercise 4.9 Question 3 (Page No. 297) top

Write a $Yacc$ program that tells whether its input is a palindrome (sequence of characters that read the same forward and backward).

ullman compiler-design grammar yacc descriptive

3.28.4 Yacc: Ullman (Compiler Design) Edition 2 Exercise 4.9 Question 4 (Page No. 297) top

Write a $Yacc$ program that takes regular expressions (as defined by the grammar of Question $4.2.2(d)$, but with any single character as an argument, not just a) and produces as output a transition table for a nondeterministic finite automaton recognizing the same language.

ullman compiler-design grammar yacc descriptive

Answer Keys

3.0.1	Q-Q	3.0.2	Q-Q	3.0.3	N/A	3.0.4	Q-Q	3.0.5	Q-Q
3.0.6	N/A	3.0.7	Q-Q	3.0.8	Q-Q	3.0.9	Q-Q	3.0.10	Q-Q
3.0.11	Q-Q	3.1.1	N/A	3.2.1	Q-Q	3.3.1	N/A	3.4.1	N/A
3.4.2	N/A	3.5.1	N/A	3.6.1	Q-Q	3.6.2	Q-Q	3.6.3	Q-Q
3.6.4	Q-Q	3.6.5	Q-Q	3.6.6	N/A	3.6.7	N/A	3.6.8	N/A
3.6.9	N/A	3.7.1	N/A	3.8.1	Q-Q	3.9.1	N/A	3.9.2	N/A
3.10.1	N/A	3.10.2	N/A	3.11.1	Q-Q	3.11.2	N/A	3.11.3	N/A
3.11.4	N/A	3.11.5	N/A	3.11.6	N/A	3.11.7	N/A	3.11.8	N/A
3.11.9	N/A	3.11.10	N/A	3.11.11	N/A	3.11.12	N/A	3.11.13	N/A
3.11.14	N/A	3.11.15	Q-Q	3.12.1	Q-Q	3.13.1	N/A	3.13.2	N/A
3.13.3	N/A	3.13.4	N/A	3.14.1	Q-Q	3.14.2	Q-Q	3.15.1	Q-Q
3.15.2	Q-Q	3.15.3	Q-Q	3.16.1	N/A	3.16.2	N/A	3.17.1	N/A
3.17.2	N/A	3.17.3	N/A	3.17.4	N/A	3.17.5	N/A	3.18.1	N/A
3.19.1	Q-Q	3.20.1	N/A	3.20.2	N/A	3.21.1	Q-Q	3.21.2	N/A
3.22.1	N/A	3.22.2	N/A	3.22.3	N/A	3.22.4	N/A	3.22.5	N/A
3.22.6	N/A	3.22.7	N/A	3.22.8	N/A	3.22.9	N/A	3.23.1	N/A
3.23.2	N/A	3.24.1	N/A	3.25.1	Q-Q	3.25.2	Q-Q	3.25.3	Q-Q
3.25.4	Q-Q	3.25.5	Q-Q	3.25.6	Q-Q	3.25.7	Q-Q	3.25.8	Q-Q
3.25.9	Q-Q	3.25.10	Q-Q	3.25.11	Q-Q	3.25.12	Q-Q	3.25.13	Q-Q
3.25.14	N/A	3.25.15	N/A	3.25.16	N/A	3.25.17	N/A	3.25.18	N/A
3.25.19	N/A	3.26.1	Q-Q	3.26.2	Q-Q	3.26.3	N/A	3.26.4	N/A
3.26.5	N/A	3.26.6	N/A	3.26.7	N/A	3.26.8	N/A	3.26.9	N/A
3.26.10	N/A	3.26.11	N/A	3.26.12	N/A	3.26.13	N/A	3.26.14	N/A
3.26.15	N/A	3.26.16	N/A	3.26.17	N/A	3.26.18	N/A	3.27.1	N/A
3.28.1	N/A	3.28.2	N/A	3.28.3	N/A	3.28.4	N/A		

4.0.1 Peterson Davie Computer Networks top

Can a node receive its own copy of LSP(link state packet)?

Answer key

4.0.2 Peterson Davie 2.19a top

Suppose we want to transmit the message 1011001001001011 and protect it from errors using the CRC8 polynomial.

$x^8 + x^2 + x^1 + 1$

(a) Use polynomial long division to determine the message that
should be transmitted.

peterson-davie

Answer key

4.0.3 Peterson Davie 2.23c top

Consider an ARQ algorithm running over a 40-km point-to-point fiber link.

(c) Why might it still be possible for the ARQ algorithm to time out and retransmit a frame, given this timeout value?

peterson-davie

4.0.4 Peterson Davie 2.15 top

Prove that the Internet checksum computation shown in the text is independent of byte order (host order or network order) except that the bytes in the final checksum should be swapped later to be in the correct order. Specifically, show that the sum of 16-bit words can be computed in either byte order. For example, if the one's complement sum (denoted by $+'$) of 16-bit words is represented as follows,

$[A, B] +' [C, D] +' \ldots +' [Y, Z]$

The following swapped sum is the same as the original sum above:

$[B, A] +' [D, C] +' \ldots +' [Z, Y]$

peterson-davie

4.0.5 Peterson Davie 2.16 top

Suppose that one byte in a buffer covered by the Internet checksum algorithm needs to be decremented (e.g., a header hop count field). Give an algorithm to compute the revised checksum without rescanning the entire buffer. Your algorithm should consider whether the byte in question is low order or high order.

peterson-davie descriptive

4.0.6 Peterson Davie 2.17 top

Show that the Internet checksum can be computed by first taking the 32-bit ones complement sum of the buffer in 32-bit units, then taking the 16-bit ones complement sum of the upper and lower half words, and finishing as before by complementing the result. (To take a 32-bit ones complement sum on 32-bit twos complement hardware, you need access to the "overflow" bit.)

peterson-davie difficult

4.0.7 Peterson Davie 2.18a top

Suppose we want to transmit the message 11100011 and protect it from errors using the CRC polynomial $x^3 + 1$.

(a) Use polynomial long division to determine the message that should be transmitted.

peterson-davie

Answer key

4.0.8 Peterson Davie 2.18b top

Suppose we want to transmit the message 11100011 and protect it from errors using the CRC polynomial $x^3 + 1$.

(b) Suppose the leftmost bit of the message is inverted due to noise on the transmission link. What is the result of the receiver's CRC calculation? How does the receiver know that an error has occurred?

peterson-davie

Answer key

4.0.9 Peterson Davie 2.19b top

Suppose we want to transmit the message 1011001001001011 and protect it from errors using the CRC8 polynomial

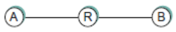
$x^8 + x^2 + x^1 + 1$

(b) Suppose the leftmost bit of the message is inverted due to noise on the transmission link. What is the result of the receiver's CRC calculation? How does the receiver know that an error has occurred?

peterson-davie

Answer key

4.0.10 Peterson Davie 2.36a top



(Figure 2.37)

Suppose A is connected to B via an intermediate router R, as shown in Figure 2.37. The $A - R$ and $R - B$ links each accept and transmit only one packet per second in each direction (so two packets take 2 seconds), and the two directions transmit independently. Assume A sends to B using the sliding window protocol with $SWS = 4$.

(a) For Time = $0, 1, 2, 3, 4, 5$, state what packets arrive at and leave each node, or label them on a timeline.

peterson-davie

Answer key

4.0.11 Peterson Davie 2.24 top

Suppose you are designing a sliding window protocol for a 1-Mbps point-to-point link to the moon, which has a one-way latency of 1.25 seconds. Assuming that each frame carries 1 KB of data, what is the minimum number of bits you need for the sequence number?

peterson-davie

Answer key

4.0.12 Peterson Davie 2.20a top

The CRC algorithm as presented in this chapter requires lots of bit manipulations. It is, however, possible to do polynomial long division taking multiple bits at a time, via a table-driven method, that enables efficient software implementations of CRC.We outline the strategy here for long division 3 bits at a time (see Table 2.5); in practice, we would divide 8 bits at a time, and the table would have 256 entries. Let the divisor polynomial $C = C(x)$ be $x^3 + x^2 + 1$, or 1101. To build the table for C, we take each 3-bit sequence, p, append three trailing 0s, and then find the quotient $q = p \frown 000 \div C$,

p	$q = p^\frown 000 \div C$	$C \times q$
000	000	000 000
001	001	001 101
010	011	010 __
011	0__	011 __
100	111	100 011
101	110	101 110
110	100	110 __
111	__	111 __

Table 2.5 Table-Driven CRC Calculation

Ignoring the remainder. The third column is the product $C \times q$, the first 3 bits of which should equal p.

(a) Verify, for $p = 110$, that the quotients $q = p \frown 000 \div C$ and $q = p \frown 111 \div C$ are the same; that is, it doesn't matter what the trailing bits are.

peterson-davie

4.0.13 Peterson Davie 2.20c top

The CRC algorithm as presented in this chapter requires lots of bit manipulations. It is, however, possible to do polynomial long division taking multiple bits at a time, via a table-driven method, that enables efficient software implementations of CRC.We outline the strategy here for long division 3 bits at a time (see Table 2.5); in practice, we would divide 8 bits at a time, and the table would have 256 entries. Let the divisor polynomial $C = C(x)$ be $x^3 + x^2 + 1$, or 1101. To build the table for C, we take each 3-bit sequence, p, append three trailing 0s, and then find the quotient $q = p \frown 000 \div C$,

p	$q = p^\frown 000 \div C$	$C \times q$
000	000	000 000
001	001	001 101
010	011	010 __
011	0__	011 __
100	111	100 011
101	110	101 110
110	100	110 __
111	__	111 __

Table 2.5 Table-Driven CRC Calculation

Ignoring the remainder. The third column is the product $C \times q$, the first 3 bits of which should equal p

(c) Use the table to divide 101001011001100 by C. Hint: The first 3 bits of the dividend are $p = 101$, so from the table the corresponding first 3 bits of the quotient are $q = 110$.Write the 110 above the second 3 bits of the dividend, and subtract $C \times q = 101110$, again from the table, from the first 6 bits of the dividend. Keep going in groups of 3 bits. There should be no remainder.

peterson-davie

4.0.14 Peterson Davie 2.22 top

Consider an ARQ protocol that uses only negative acknowledgments $(NAKs)$, but no positive acknowledgments $(ACKs)$. Describe what timeouts would have to be scheduled. Explain why an ACK-based protocol is usually preferred to a NAK-based protocol.

peterson-davie descriptive

4.0.15 Peterson Davie 2.23a top

Consider an ARQ algorithm running over a 40-km point-to-point fiber link.

(a) Compute the one-way propagation delay for this link, assuming that the speed of light is 2×108 m/s in the fiber.

peterson-davie

Answer key

4.0.16 Peterson Davie 2.23b top

Consider an ARQ algorithm running over a 40-km point-to-point fiber link.

(b) Suggest a suitable timeout value for the ARQ algorithm to use.

peterson-davie

Answer key

4.0.17 Peterson Davie 2.25a top

Suppose you are designing a sliding window protocol for a 1-Mbps point-to-point link to the stationary satellite revolving around the Earth at an altitude of 3×104 km. Assuming that each frame carries 1 KB of data, what is the minimum number of bits you need for the sequence number in the following cases? Assume the speed of light is 3×108 m/s.

(a) $RWS = 1$

peterson-davie

Answer key

4.0.18 Peterson Davie 2.32a top

Draw a timeline diagram for the sliding window algorithm with $SWS = RWS = 4$ frames in the following two situations. Assume the receiver sends a duplicate acknowledgment if it does not receive the expected frame. For example, it sends DUPACK[2] when it expects to see Frame[2] but receives Frame[3] instead. Also, the receiver sends a cumulative acknowledgment after it receives all the outstanding frames. For example, it sends $ACK[5]$ when it receives the lost frame Frame[2] after it already received Frame[3], Frame[4], and Frame[5]. Use a timeout interval of about $2 \times RTT$.

(a) Frame 2 is lost. Retransmission takes place upon timeout (as usual).

peterson-davie descriptive

4.0.19 Peterson Davie 2.44c top

Suppose the Ethernet transmission algorithm is modified as follows: After each successful transmission attempt, a host waits one or two slot times before attempting to transmit again, and otherwise backs off the usual way.

(c) Suppose the Ethernet transmission algorithm is modified as follows: After each successful transmission attempt, a host waits one or two slot times before attempting to transmit again, and otherwise backs off the usual way.

peterson-davie descriptive

4.0.20 Peterson Davie 2.31a top

Draw a timeline diagram for the sliding window algorithm with $SWS = RWS = 3$ frames, for the following two situations. Use a timeout interval of about $2 \times RTT$.

(a) Frame 4 is lost.

peterson-davie descriptive

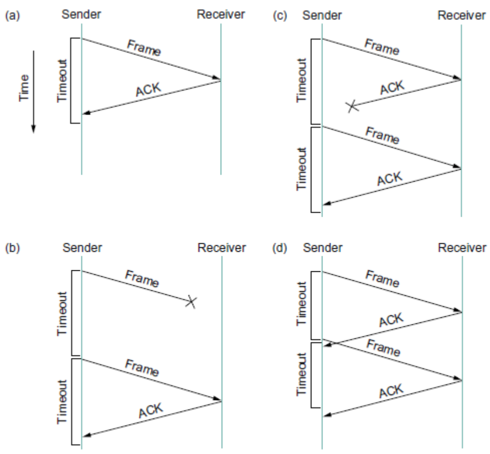
4.0.21 Peterson Davie 2.26 top

The text suggests that the sliding window protocol can be used to implement flow control. We can imagine doing this by having the receiver delay ACKs, that is, not send the ACK until there is free buffer space to hold the next frame. In doing so, each ACK would simultaneously acknowledge the receipt of the last frame and tell the source that there is now free buffer space available to hold the next frame. Explain why implementing flow control in this way is not a good idea.

peterson-davie descriptive

4.0.22 Peterson Davie 2.27 top

Implicit in the stop-and-wait scenarios of Figure 2.17 is the notion that the receiver will retransmit its ACK immediately on receipt of the duplicate data frame. Suppose instead that the receiver keeps its own timer and retransmits its ACK only after the next expected frame has not arrived within the timeout interval. Draw timelines illustrating the scenarios in Figure $2.17(b)$ to (d); assume the receiver's timeout value is twice the sender's. Also redraw (c) assuming the receiver's timeout value is half the sender's.



■ FIGURE 2.17 Timeline showing four different scenarios for the stop-and-wait algorithm. (a) The ACK is received before the timer expires; (b) the original frame is lost; (c) the ACK is lost; (d) the timeout fires too soon.

Figure 2.17 illustrates four different scenarios that result from this basic

peterson-davie descriptive

4.0.23 Peterson Davie 2.28a top

In stop-and-wait transmission, suppose that both sender and receiver retransmit their last frame immediately on receipt of a duplicate ACK or data frame; such a strategy is superficially reasonable because receipt of such a duplicate is most likely to mean the other side has experienced a timeout.

(a) Draw a timeline showing what will happen if the first data frame is somehow duplicated, but no frame is lost. How long will the duplications continue? This situation is known as the Sorcerer's Apprentice bug.

peterson-davie descriptive

4.0.24 Peterson Davie 2.28b top

In stop-and-wait transmission, suppose that both sender and receiver retransmit their last frame immediately on receipt of a duplicate ACK or data frame; such a strategy is superficially reasonable because receipt of such a duplicate is most likely to mean the other side has experienced a timeout.

(b) Suppose that, like data, ACKs are retransmitted if there is no response within the timeout period. Suppose also that both sides use the same timeout interval. Identify a reasonably likely scenario for triggering the Sorcerer's Apprentice bug.

peterson-davie descriptive

4.0.25 Peterson Davie 2.29 top

Give some details of how you might augment the sliding window protocol with flow control by having ACKs carry additional information that reduces the SWS as the receiver runs out of buffer space. Illustrate your protocol with a timeline for a transmission; assume the initial SWS and RWS are 4, the link speed is instantaneous, and the receiver can free buffers at the rate of one per second (i.e., the receiver is the bottleneck). Show what happens at $T = 0, T = 1, \ldots\ldots, T = 4$ seconds.

peterson-davie

4.0.26 Peterson Davie 2.30 top

Describe a protocol combining the sliding window algorithm with selective ACKs. Your protocol should retransmit promptly, but not if a frame simply arrives one or two positions out of order. Your protocol should also make explicit what happens if several consecutive frames are lost.

peterson-davie descriptive

4.0.27 Peterson Davie 2.25b top

Suppose you are designing a sliding window protocol for a 1-Mbps point-to-point link to the stationary satellite revolving around the Earth at an altitude of 3×104 km. Assuming that each frame carries 1 KB of data, what is the minimum number of bits you need for the sequence number in the following cases? Assume the speed of light is 3×108 m/s.

(b) $RWS = SWS$

peterson-davie

Answer key

4.0.28 Peterson Davie 2.35c top

Suppose that we run the sliding window algorithm with $SWS = 5$ and $RWS = 3$, and no out-of-order arrivals.

(c) State a general rule for the minimum MaxSeqNum in terms of SWS and RWS.

peterson-davie descriptive

Answer key

4.0.29 Peterson Davie 2.31b top

Draw a timeline diagram for the sliding window algorithm with $SWS = RWS = 3$ frames, for the following two situations. Use a timeout interval of about $2 \times RTT$

(b) Frames 4 to 6 are lost.

peterson-davie descriptive

4.0.30 Peterson Davie 2.32b top

Draw a timeline diagram for the sliding window algorithm with $SWS = RWS = 4$ frames in the following two situations. Assume the receiver sends a duplicate acknowledgment if it does not receive the expected frame. For example, it sends DUPACK[2] when it expects to see Frame[2] but receives Frame[3] instead. Also, the receiver sends a cumulative acknowledgment after it receives all the outstanding frames. For example, it sends $ACK[5]$ when it receives the lost frame Frame[2] after it already received Frame[3], Frame[4], and Frame[5]. Use a timeout interval of about $2 \times RTT$.

(b) Frame 2 is lost. Retransmission takes place either upon receipt of the first DUPACK or upon timeout. Does this scheme reduce the transaction time? (Note that some end-to-end protocols, such as variants of TCP, use similar schemes for fast retransmission.)

peterson-davie descriptive

4.0.31 Peterson Davie 2.33 top

Suppose that we attempt to run the sliding window algorithm with $SWS = RWS = 3$ and with MaxSeqNum = 5. The Nth packet $DATA[N]$ thus actually contains $N \mod 5$ in its sequence number field. Give an example in which the algorithm becomes confused; that is, a scenario in which the receiver expects $DATA[5]$ and accepts $DATA[0]$ - which has the same transmitted sequence number - in its stead. No packets may arrive out of order. Note that this implies MaxSeqNum ≥ 6 is necessary as well as sufficient.

peterson-davie descriptive

4.0.32 Peterson Davie 2.34 top

Consider the sliding window algorithm with $SWS = RWS = 3$, with no out-of-order arrivals and with infinite-precision sequence numbers

(a) Show that if $DATA[6]$ is in the receive window, then $DATA[0]$ (or in general any older data) cannot arrive at the receiver (and hence that MaxSeqNum = 6 would have sufficed).

These amount to a proof of the formula given in Section $2.5.2$, particularized to the case $SWS = 3$. Note that part (b) implies that the scenario of the previous problem cannot be reversed to involve a failure to distinguish $ACK[0]$ and $ACK[5]$.

peterson-davie descriptive

4.0.33 Peterson Davie 2.34b top

Consider the sliding window algorithm with $SWS = RWS = 3$, with no out-of-order arrivals and with infinite-precision sequence numbers.

(b) Show that if $ACK[6]$ may be sent (or, more literally, that $DATA[5]$ is in the sending window), then $ACK[2]$ (or earlier) cannot be received.

These amount to a proof of the formula given in Section 2.5.2, particularized to the case $SWS = 3$. Note that part (b) implies that the scenario of the previous problem cannot be reversed to involve a failure to distinguish $ACK[0]$ and $ACK[5]$.

peterson-davie descriptive

4.0.34 Peterson Davie 2.35a top

Suppose that we run the sliding window algorithm with $SWS = 5$ and $RWS = 3$, and no out-of-order arrivals.

(a) Find the smallest value for MaxSeqNum. You may assume that it suffices to find the smallest MaxSeqNum such that if DATA[MaxSeqNum] is in the receive window, then $DATA[0]$ can no longer arrive.

peterson-davie descriptive

Answer key

4.0.35 Peterson Davie 2.35b top

Suppose that we run the sliding window algorithm with $SWS = 5$ and $RWS = 3$, and no out-of-order arrivals.

(b) Give an example showing that MaxSeqNum -1 is not sufficient.

peterson-davie descriptive

Answer key

4.0.36 Peterson Davie 2.20b top

The CRC algorithm as presented in this chapter requires lots of bit manipulations. It is, however, possible to do polynomial long division taking multiple bits at a time, via a table-driven method, that enables efficient software implementations of CRC.We outline the strategy here for long division 3 bits at a time (see Table 2.5); in practice, we would divide 8 bits at a time, and the table would have 256 entries. Let the divisor polynomial $C = C(x)$ be $x^3 + x^2 + 1$, or 1101. To build the table for C, we take each 3-bit sequence, p, append three trailing 0s, and then find the quotient $q = p \frown 000 \div C$,

Table 2.5 Table-Driven CRC Calculation

p	$q = p^\frown 000 \div C$	$C \times q$
000	000	000 000
001	001	001 101
010	011	010 ___
011	0__	011 ___
100	111	100 011
101	110	101 110
110	100	110 ___
111	___	111 ___

Ignoring the remainder. The third column is the product $C \times q$, the first 3 bits of which should equal p.

(b) Fill in the missing entries in the table.

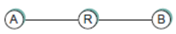
peterson-davie

4.0.37 Peterson Davie 2.14 top

Show that the Internet checksum will never be $0xFFFF$ (that is, the final value of sum will not be $0x0000$) unless every byte in the buffer is 0. (Internet specifications in fact require that a checksum of $0x0000$ be transmitted as $0xFFFF$; the value $0x0000$ is then reserved for an omitted checksum. Note that, in ones complement arithmetic, $0x0000$ and $0xFFFF$ are both representations of the number 0).

peterson-davie descriptive

4.0.38 Peterson Davie 2.36b top



Suppose A is connected to B via an intermediate router R, as shown in Figure 2.37. The $A-R$ and $R-B$ links each accept and transmit only one packet per second in each direction (so two packets take 2 seconds), and the two directions transmit independently. Assume A sends to B using the sliding window protocol with $SWS = 4$.

(b) What happens if the links have a propagation delay of 1.0 second, but accept immediately as many packets as are offered (i.e., latency = 1 second but bandwidth is infinite)?

peterson-davie

4.0.39 Peterson Davie 2.44a top

Suppose the Ethernet transmission algorithm is modified as follows: After each successful transmission attempt, a host waits one or two slot times before attempting to transmit again, and otherwise backs off the usual way.

(a) Explain why the capture effect of the previous exercise is now much less likely.

peterson-davie descriptive

4.0.40 Peterson Davie 2.39 top

What kind of problems can arise when two hosts on the same Ethernet share the same hardware address? Describe what happens and why that behavior is a problem.

peterson-davie descriptive

Answer key

4.0.41 Peterson Davie 2.40 top

The 1982 Ethernet specification allowed between any two stations up to 1500mof coaxial cable, 1000mof other point-to-point link cable, and two repeaters. Each station or repeater connects to the coaxial cable via up to 50mof "drop cable." Typical delays associated with each device are given in Table 2.6 (where c = speed of light in a vacuum = 3×108 m/s). What is the worst-case round-trip propagation delay, measured in bits, due to the sources listed? (This list is not complete; other sources of delay include sense time and signal rise time.)

Table 2.6 Typical Delays Associated with Various Devices (Exercise 40)

Item	Delay
Coaxial cable	Propagation speed .77c
Link/drop cable	Propagation speed .65c
Repeaters	Approximately 0.6 μs each
Transceivers	Approximately 0.2 μs each

peterson-davie

4.0.42 Peterson Davie 2.41 top

Coaxial cable Ethernet was limited to a maximum of 500m between repeaters, which regenerate the signal to 100% of its original amplitude. Along one 500-m segment, the signal could decay to no less than 14% of its original value $(8.5dB)$. Along 1500 m, then, the decay might be $(0.14)^3 = 0.3\%$. Such a signal, even along 2500 m, is still strong enough to be read; why then are repeaters required every 500 m?

peterson-davie difficult

4.0.43 Peterson Davie 2.42a top

Suppose the round-trip propagation delay for Ethernet is $46.4\mu s$. This yields a minimum packet size of 512 bits (464 bits corresponding to propagation delay $+$ 48 bits of jam signal).

(a) What happens to the minimum packet size if the delay time is held constant, and the signalling rate rises to 100 Mbps?

peterson-davie

4.0.44 Peterson Davie 2.42b top

Suppose the round-trip propagation delay for Ethernet is $46.4\mu s$. This yields a minimum packet size of 512 bits (464 bits corresponding to propagation delay $+48$ bits of jam signal).

(b) What are the drawbacks to so large a minimum packet size?

peterson-davie

Answer key

4.0.45 Peterson Davie 2.42c top

Suppose the round-trip propagation delay for Ethernet is $46.4 \mu s$. This yields a minimum packet size of 512 bits (464 bits corresponding to propagation delay $+48$ bits of jam signal).

(c) If compatibility were not an issue, how might the specifications be written so as to permit a smaller minimum packet size?

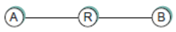
peterson-davie

4.0.46 Peterson Davie 3.28 top

Suppose a switch is built using a computer workstation and that it can forward packets at a rate of $500,000$ packets per second, regardless (within limits) of size. Assume the workstation uses direct memory access (DMA) to move data in and out of its main memory, which has a bandwidth of 2 Gbps, and that the I/O bus has a bandwidth of 1 Gbps. At what packet size would the bus bandwidth become the limiting factor?

peterson-davie

4.0.47 Peterson Davie 2.37 top



Suppose A is connected to B via an intermediate router R, as in the previous problem. The $A-R$ link is instantaneous, but the $R-B$ link transmits only one packet each second, one at a time (so two packets take 2 seconds). Assume A sends to B using the sliding window protocol with $SWS=4$. For Time = $0,1,2,3,4$, state what packets arrive at and are sent from A and B. How large does the queue at R grow?

peterson-davie

Answer key

4.0.48 Peterson Davie 2.44b top

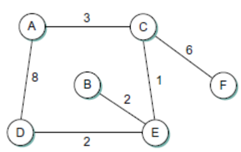
Suppose the Ethernet transmission algorithm is modified as follows: After each successful transmission attempt, a host waits one or two slot times before attempting to transmit again, and otherwise backs off the usual way.

(b) Show how the strategy above can now lead to a pair of hosts capturing the Ethernet, alternating transmissions, and locking out a third.

peterson-davie descriptive

4.0.49 Peterson Davie 3.3 top

For the network given in Figure 3.45, give the datagram forwarding table for each node. The links are labeled with relative costs; your tables should forward each packet via the lowest-cost path to its destination.

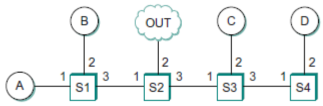


peterson-davie descriptive

Answer key

4.0.50 Peterson Davie 3.4 top

Give forwarding tables for switches $S1$ to $S4$ in Figure 3.46. Each switch should have a default routing entry, chosen to forward packets with unrecognized destination addresses toward OUT. Any specific destination table entries duplicated by the default entry should then be eliminated.



peterson-davie descriptive

Answer key

4.0.51 Peterson Davie 3.23 top

Suppose a bridge has two of its ports on the same network. How might the bridge detect and correct this?

peterson-davie descriptive

Answer key

4.0.52 Peterson Davie 3.24 top

What percentage of an ATM link's total bandwidth is consumed by the ATM cell headers? Ignore padding to fill cells or ATM adaptation layer headers.

peterson-davie descriptive

4.0.53 Peterson Davie 3.25 top

Cell switching methods (like ATM) essentially always use virtual circuit switching rather than datagram forwarding. Give a specific argument why this is so (consider the preceding question).

peterson-davie descriptive

Answer key

4.0.54 Peterson Davie 3.26 top

Suppose a workstation has an I/O bus speed of 800 Mbps and memory bandwidth of 2 Gbps. Assuming direct memory access (DMA) is used to move data in and out of main memory, how many interfaces to 100-Mbps Ethernet links could a switch based on this workstation handle?

peterson-davie descriptive

Answer key

4.0.55 Peterson Davie 3.29a top

Suppose that a switch is designed to have both input and output $FIFO$ buffering. As packets arrive on an input port they are inserted at the tail of the $FIFO$. The switch then tries to forward the packets at the head of each $FIFO$ to the tail of the appropriate output $FIFO$.

(a) Explain under what circumstances such a switch can lose a packet destined for an output port whose $FIFO$ is empty

peterson-davie descriptive

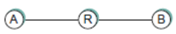
4.0.56 Peterson Davie 3.36 top

Suppose a TCP message that contains 1024 bytes of data and 20 bytes of TCP header is passed to IP for delivery across two networks interconnected by a router (i.e., it travels from the source host to a router to the destination host). The first network has an MTU of 1024 bytes; the second has an MTU of 576 bytes. Each network's MTU gives the size of the largest IP datagram that can be carried in a link layer frame. Give the sizes and offsets of the sequence of fragments delivered to the network layer at the destination host. Assume all IP headers are 20 bytes.

peterson-davie descriptive

Answer key

4.0.57 Peterson Davie 2.38 top



Consider the situation in the previous exercise, except this time assume that the router has a queue size of 1; that is, it can hold one packet in addition to the one it is sending (in each direction). Let $A's$ timeout be 5 seconds, and let SWS again be 4. Show what happens at each second from Time = 0 until all four packets from the first window-full are successfully delivered.

peterson-davie descriptive

4.0.58 Peterson Davie 5th Editon top

Calculate the total time required to transfer a 1.5-MB file, assuming an RTT of 80ms, a packet size of 1 KB data, and an initial 2×RTT of "handshaking" before data is sent. The bandwidth is 10 Mbps, but after we finish sending each data packet we must wait one RTT before sending the next.

Answer key

4.0.59 Peterson Davie 3.29c top

Suppose that a switch is designed to have both input and output $FIFO$ buffering. As packets arrive on an input port they are inserted at the tail of the $FIFO$. The switch then tries to forward the packets at the head of each $FIFO$ to the tail of the appropriate output $FIFO$.

(c) Assume that the $FIFO$ buffering memory can be redistributed freely. Suggest a reshuffling of the buffers that avoids the above problem, and explain why it does so.

peterson-davie descriptive

4.0.60 Peterson Davie 3.29c top

Suppose that a switch is designed to have both input and output $FIFO$ buffering. As packets arrive on an input port they are inserted at the tail of the $FIFO$. The switch then tries to forward the packets at the head of each $FIFO$ to the tail of the appropriate output $FIFO$.

(c) Assume that the $FIFO$ buffering memory can be redistributed freely. Suggest a reshuffling of the buffers that avoids the above problem, and explain why it does so.

peterson-davie descriptive

4.0.61 Peterson Davie 3.30 top

A stage of an $n \times n$ banyan network consists of $(n/2) 2 \times 2$ switching elements. The first stage directs packets to the correct half of the network, the next stage to the correct quarter, and so on, until the packet is routed to the correct output. Derive an expression for the number of 2×2 switching elements needed to make an $n \times n$ banyan network. Verify your answer for $n = 8$.

peterson-davie descriptive difficult

4.0.62 Peterson Davie 3.32a top

Suppose a 10-Mbps Ethernet hub (repeater) is replaced by a 10-Mbps switch, in an environment where all traffic is between a single server and N "clients. "Because all traffic must still traverse the server switch link, nominally there is no improvement in bandwidth.

(a) Would you expect any improvement in bandwidth? If so, why?

peterson-davie descriptive

4.0.63 Peterson Davie 3.32b top

Suppose a 10-Mbps Ethernet hub (repeater) is replaced by a 10-Mbps switch, in an environment where all traffic is between a single server and N "clients." Because all traffic must still traverse the server-switch link, nominally there is no improvement in bandwidth.

(b) What other advantages and drawbacks might a switch offer versus a hub?

peterson-davie descriptive

Answer key

4.0.64 Peterson Davie 3.33 top

What aspect of IP addresses makes it necessary to have one address per network interface, rather than just one per host? In light of your answer, why does IP tolerate point-to-point interfaces that have nonunique addresses or no addresses?

peterson-davie descriptive

Answer key

4.0.65 Peterson Davie 3.34 top

Why does the Offset field in the IP header measure the offset in 8-byte units? (Hint: Recall that the Offset field is 13 bits long.)

peterson-davie descriptive

Answer key

4.0.66 Peterson Davie 3.29b top

Suppose that a switch is designed to have both input and output $FIFO$ buffering. As packets arrive on an input port they are inserted at the tail of the $FIFO$. The switch then tries to forward the packets at the head of each $FIFO$ to the tail of the appropriate output $FIFO$.

(b) What is this behavior called?

peterson-davie descriptive

4.0.67 Peterson Davie 3.35 top

Some signalling errors can cause entire ranges of bits in a packet to be overwritten by all 0s or all 1s. Suppose all the bits in the packet, including the Internet checksum, are overwritten. Could a packet with all 0s or all 1s be a legal $IPv4$ packet? Will the Internet checksum catch that error? Why or why not?

peterson-davie descriptive

Answer key

4.0.68 Peterson Davie 3.37 top

Path MTU is the smallest MTU of any link on the current path (route) between two hosts. Assume we could discover the path MTU of the path used in the previous exercise, and that we use this value as the MTU for all the path segments. Give the sizes and offsets of the sequence of fragments delivered to the network layer at the destination host.

peterson-davie descriptive

Answer key

4.0.69 Peterson Davie 3.38a top

Suppose an IP packet is fragmented into 10 fragments, each with a 1% (independent) probability of loss. To a reasonable approximation, this means there is a 10% chance of losing the whole packet due to loss of a fragment. What is the probability of net loss of the whole packet if the packet is transmitted twice,

(a) Assuming all fragments received must have been part of the same transmission?

peterson-davie descriptive difficult

Answer key

4.0.70 Peterson Davie 3.38b top

Suppose an IP packet is fragmented into 10 fragments, each with a 1% (independent) probability of loss. To a reasonable approximation, this means there is a 10% chance of losing the whole packet due to loss of a fragment. What is the probability of net loss of the whole packet if the packet is transmitted twice,

(b) Assuming any given fragment may have been part of either transmission?

peterson-davie descriptive difficult

Answer key

4.0.71 Peterson Davie 3.38c top

Suppose an IP packet is fragmented into 10 fragments, each with a 1% (independent) probability of loss. To a reasonable approximation, this means there is a 10% chance of losing the whole packet due to loss of a fragment. What is the probability of net loss of the whole packet if the packet is transmitted twice,

(c) Explain how use of the Ident field might be applicable here.

peterson-davie descriptive difficult

Answer key

4.1 Error Correction (3) top

4.1.1 Error Correction: Peterson Davie 2.13 top

Show that two-dimensional parity provides the receiver enough information to correct any 1-bit error (assuming the receiver knows only 1 bit is bad), but not any 2-bit error.

peterson-davie descriptive error-correction

Answer key

4.1.2 Error Correction: Peterson Davie 2.21a top

With 1 parity bit we can detect all 1-bit errors. Show that at least one generalization fails, as follows:

(a) Show that if messages m are 8 bits long, then there is no error detection code $e = e(m)$ of size 2 bits that can detect all 2-bit errors. Hint: Consider the set M of all 8-bit messages with a single 1 bit; note that any message from M can be transmuted into any other with a 2-bit error, and show that some pair of messages m_1 and m_2 in M must have the same error code e.

peterson-davie difficult error-correction

4.1.3 Error Correction: Peterson Davie 2.21b top

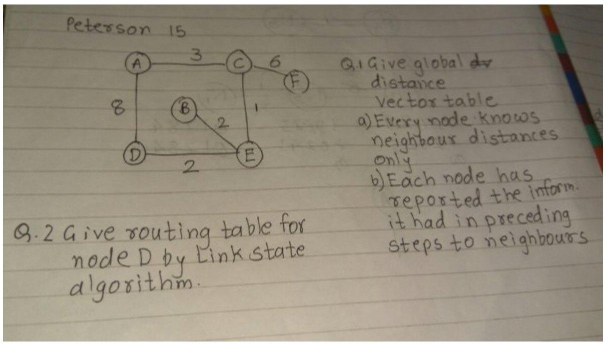
With 1 parity bit we can detect all 1-bit errors. Show that at least one generalization fails, as follows:

(b) Find an N (not necessarily minimal) such that no 32-bit error detection code applied to N-bit blocks can detect all errors altering up to 8 bits.

peterson-davie error-correction difficult

4.2 Link State Routing (1) top

4.2.1 Link State Routing: Peterson Davie Q.15,17 top



Graph links are given as A-C=3,A-D=8,B-E=2,C-F=6,C-E=1,D-E=2 (A)Give global distance vector table (B)Routing table at node D by Link State algorithm (C) Suppose the forwarding tables are all established as in (A) and then the C–E link fails. Give (i) The tables of A, B, D, and F after C and E have reported the news. (ii) The tables of A and D after their next mutual exchange. (iii) The table of C after A exchanges with it.

link-state-routing distance-vector-routing peterson-davie computer-networks

4.3 Routers Bridge Hubs Switches (1) top

4.3.1 Routers Bridge Hubs Switches: Peterson Davie 3.27 top

Suppose a workstation has an I/O bus speed of 1 Gbps and memory bandwidth of 2 Gbps. Assuming DMA is used to move data in and out of main memory, how many interfaces to 100-Mbps Ethernet links could a switch based on this workstation handle?

peterson-davie routers-bridge-hubs-switches numerical-answers

Answer key

Answer Keys

4.0.1	Q-Q	4.0.2	Q-Q	4.0.3	Q-Q	4.0.4	Q-Q	4.0.5	N/A
4.0.6	Q-Q	4.0.7	Q-Q	4.0.8	Q-Q	4.0.9	Q-Q	4.0.10	Q-Q
4.0.11	Q-Q	4.0.12	Q-Q	4.0.13	Q-Q	4.0.14	N/A	4.0.15	Q-Q
4.0.16	Q-Q	4.0.17	Q-Q	4.0.18	N/A	4.0.19	N/A	4.0.20	N/A
4.0.21	N/A	4.0.22	N/A	4.0.23	N/A	4.0.24	N/A	4.0.25	Q-Q
4.0.26	N/A	4.0.27	Q-Q	4.0.28	N/A	4.0.29	N/A	4.0.30	N/A
4.0.31	N/A	4.0.32	N/A	4.0.33	N/A	4.0.34	N/A	4.0.35	N/A
4.0.36	Q-Q	4.0.37	N/A	4.0.38	Q-Q	4.0.39	N/A	4.0.40	N/A
4.0.41	Q-Q	4.0.42	Q-Q	4.0.43	Q-Q	4.0.44	Q-Q	4.0.45	Q-Q
4.0.46	Q-Q	4.0.47	Q-Q	4.0.48	N/A	4.0.49	N/A	4.0.50	N/A
4.0.51	N/A	4.0.52	N/A	4.0.53	N/A	4.0.54	N/A	4.0.55	N/A
4.0.56	N/A	4.0.57	N/A	4.0.58	Q-Q	4.0.59	N/A	4.0.60	N/A
4.0.61	N/A	4.0.62	N/A	4.0.63	N/A	4.0.64	N/A	4.0.65	N/A
4.0.66	N/A	4.0.67	N/A	4.0.68	N/A	4.0.69	N/A	4.0.70	N/A
4.0.71	N/A	4.1.1	N/A	4.1.2	Q-Q	4.1.3	Q-Q	4.2.1	Q-Q
4.3.1	Q-Q								

5 Databases (107)

5.0.1 DBMS Korth Edition 6 Exercise 1 Question 1 (Page No. 33) top

What Are two Disadvantages of Databases?

databases korth-edition6 descriptive

Answer key

5.0.2 DBMS Korth Edition 6 Exercise 1 Question 8 (Page No. 34) top

List four significant differences between a file-processing system and a DBMS.

databases korth-edition6 descriptive

Answer key

5.0.3 DBMS Korth Edition 6 Exercise 1 Question 6 (Page No. 34) top

Keyword queries used in Web search are quite different from database queries. List key differences between the two, in terms of the way the queries are specified, and in terms of what is the result of a query.

databases korth-edition6 descriptive

5.0.4 DBMS Korth Edition 6 Exercise 1 Question 3 (Page No. 34) top

List six major steps that you would take in setting up a database for a particular enterprise.

databases korth-edition6 descriptive

5.0.5 DBMS Korth Edition 6 Exercise 1 Question 9 (Page No. 34) top

Explain the concept of physical data independence, and its importance in database systems.

databases korth-edition6 descriptive

Answer key

5.0.6 DBMS Korth Edition 6 Exercise 1 Question 12 (Page No. 34) top

Describe at least 3 tables that might be used to store information in a social-networking system such as Facebook.

databases korth-edition6 descriptive

5.0.7 DBMS Korth Edition 6 Exercise 1 Question 7 (Page No. 34) top

List four applications you have used that most likely employed a database system to store persistent data

databases korth-edition6 descriptive

5.0.8 DBMS Korth Edition 6 Exercise 1 Question 10 (Page No. 34) top

List five responsibilities of a database-management system. For each responsibility, explain the problems that would arise if the responsibility were not discharged.

databases korth-edition6 descriptive

5.0.9 DBMS Korth Edition 6 Exercise 1 Question 2 (Page No. 33) top

List five ways in which the type declaration system of a language such as Java or C++ differs from the data definition language used in a database.

databases korth-edition6 descriptive

5.0.10 DBMS Korth Edition 6 Exercise 1 Question 11 (Page No. 34) top

List at least two reasons why database systems support data manipulation using a declarative query language such as SQL, instead of just providing a a library of C or C++ functions to carry out data manipulation.

databases korth-edition6 descriptive

5.1 Database Design (25) top

5.1.1 Database Design: DBMS Korth Edition 6 Exercise 7 Question 1 (Page No. 316) top

Construct an E-R diagram for a car insurance company whose customers own one or more cars each. Each car has associated with it zero to any number of recorded accidents. Each insurance policy covers one or more cars and has one or more premium payments associated with it. Each payment is for a particular period of time and has an associated due date, and the date when the payment was received.

databases korth-edition6 database-design er-diagram

5.1.2 Database Design: DBMS Korth Edition 6 Exercise 7 Question 10 (Page No. 317) top

Consider a many-to-one relationship R between entity sets A and B. Suppose the relation created from R is combined with the relation created from A. In SQL, attributes participating in a foreign key constraint can be null. Explain how a constraint on total participation of A in R can be enforced using not null constraints in SQL.

databases korth-edition6 database-design er-diagram descriptive

5.1.3 Database Design: DBMS Korth Edition 6 Exercise 7 Question 11 (Page No. 317) top

In SQL, foreign key constraints can only reference the primary key attributes of the referenced relation, or other attributes declared to be a super key using the unique constraint. As a result, total participation constraints on a many-to-many relationship (or on the "one" side of a one-to-many relationship) cannot be enforced on the relations created from the relationship, using primary key, foreign key and not null constraints on the relations.

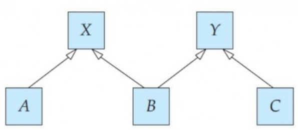
a. Explain why.

b. Explain how to enforce total participation constraints using complex check constraints or assertions (see Section 4.4.7 of korth). (Unfortunately, these features are not supported on any widely used database currently.)

databases korth-edition6 database-design er-diagram descriptive

5.1.4 Database Design: DBMS Korth Edition 6 Exercise 7 Question 12 (Page No. 317-318) top

Figure shows a lattice structure of generalization and specialization (attributes not shown). For entity sets A, B, and C, explain how attributes are inherited from the higher-level entity sets X and Y. Discuss how to handle a case where an attribute of X has the same name as some attribute



of Y.

databases korth-edition6 database-design er-diagram descriptive

5.1.5 Database Design: DBMS Korth Edition 6 Exercise 7 Question 13 (Page No. 318) top

Temporal changes: An E-R diagram usually models the state of an enterprise at a point in time. Suppose we wish to track *temporal changes*, that is,

changes to data over time. For example, Zhang may have been a student between 1 September 2005 31 May 2009, while Shankar may have had instructor Einstein as advisor from 31 May 2008 to 5 December 2008, and again from 1 June 2009 to 5 January 2010. Similarly, attribute values of an entity or relationship, such as *title* and *credits* of course, *salary*, or even *name* of instructor, and tot cred of student, can change over time.
One way to model temporal changes is as follows.We define a new data type called valid_time, which is a time-interval, or a set of time-intervals. We then associate a valid_time attribute with each entity and relationship, recording the time periods during which the entity or relationship is valid. The end-time of an interval can be infinity; for example, if Shankar became a student on 2 September 2008, and is still a student, we can represent the end-time of the valid_time interval as infinity for the Shankar entity. Similarly, we model attributes that can change over time as a set of values, each with its own valid_time.

a. Draw an E-R diagram with the student and instructor entities, and the advisor relationship, with the above extensions to track temporal changes.
b. Convert the above E-R diagram into a set of relations.

databases korth-edition6 database-design er-diagram

Answer key

5.1.6 Database Design: DBMS Korth Edition 6 Exercise 7 Question 14 (Page No. 318) top

Explain the distinctions among the terms primary key, candidate key, and superkey.

databases korth-edition6 database-design er-diagram descriptive

Answer key

5.1.7 Database Design: DBMS Korth Edition 6 Exercise 7 Question 15 (Page No. 319) top

Construct an E-R diagram for a hospital with a set of patients and a set of medical doctors. Associate with each patient a log of the various tests and examinations conducted.

databases korth-edition6 database-design er-diagram

Answer key

5.1.8 Database Design: DBMS Korth Edition 6 Exercise 7 Question 18 (Page No. 319) top

Explain the difference between a weak and a strong entity set.

databases korth-edition6 database-design er-diagram descriptive

Answer key

5.1.9 Database Design: DBMS Korth Edition 6 Exercise 7 Question 19 (Page No. 319) top

We can convert any weak entity set to a strong entity set by simply adding appropriate attributes. Why, then, do we have weak entity sets?

databases korth-edition6 database-design er-diagram descriptive

5.1.10 Database Design: DBMS Korth Edition 6 Exercise 7 Question 2 (Page No. 315) top

Consider a database used to record the marks that students get in different exams of different course offerings (sections).

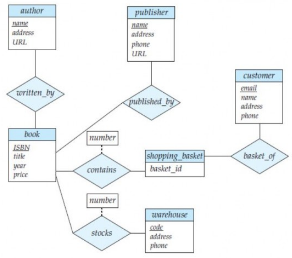
a. Construct an E-R diagram that models exams as entities, and uses a ternary relationship, for the database.

b. Construct an alternative E-R diagram that uses only a binary relationship between students and sections. Make sure that only one relationship exists between a particular student and section pair, yet you can represent the marks that a student gets in different exams.

databases korth-edition6 database-design er-diagram

5.1.11 Database Design: DBMS Korth Edition 6 Exercise 7 Question 20 (Page No. 319-320) top

Consider the E-R diagram in Figure given below, which models an online bookstore.
a. List the entity sets and their primary keys.



b. Suppose the bookstore adds Blu-ray discs and downloadable videos to its collection. The same item may be present in one or both formats, with differing prices. Extend the E-R diagram to model this addition, ignoring the effect on shopping baskets.

c. Now extend the E-R diagram, using generalization, to model the case where a shopping basket may contain any combination of books, Blu-ray discs, or downloadable video.

databases korth-edition6 database-design er-diagram descriptive

5.1.12 Database Design: DBMS Korth Edition 6 Exercise 7 Question 21 (Page No. 319) top

Design a database for an automobile company to provide to its dealers to assist them in maintaining customer records and dealer inventory and to assist sales staff in ordering cars.
Each vehicle is identified by a vehicle identification number (VIN). Each individual vehicle is a particular model of a particular brand offered by the company (e.g., the XF is a model of the car brand Jaguar of Tata Motors).
Each model can be offered with a variety of options, but an individual car may have only some (or none) of the available options. The database needs to store information about models, brands, and options, as well as information about individual dealers, customers, and cars.
Your design should include an E-R diagram, a set of relational schemas, and a list of constraints, including primary-key and foreign-key constraints.

databases korth-edition6 database-design er-diagram

5.1.13 Database Design: DBMS Korth Edition 6 Exercise 7 Question 22 (Page No. 319-320) top

Design a database for a worldwide package delivery company (e.g., DHL or FedEx). The database must be able to keep track of customers (who ship items) and customers (who receive items); some customers may do both. Each package must be identifiable and trackable, so the database must
be able to store the location of the package and its history of locations.
Locations include trucks, planes, airports, and warehouses.
Your design should include an E-R diagram, a set of relational schemas, and a list of constraints, including primary-key and foreign-key constraints.

databases korth-edition6 er-diagram database-design

5.1.14 Database Design: DBMS Korth Edition 6 Exercise 7 Question 23 (Page No. 320) top

Design a database for an airline. The database must keep track of customers and their reservations, flights and their status, seat assignments on individual flights, and the schedule and routing of future flights.
Your design should include an E-R diagram, a set of relational schemas,

and a list of constraints, including primary-key and foreign-key constraints.

databases korth-edition6 database-design er-diagram

5.1.15 Database Design: DBMS Korth Edition 6 Exercise 7 Question 26 (Page No. 321) top

Design a generalization-specialization hierarchy for a motor vehicle sales company. The company sells motorcycles, passenger cars, vans, and buses. Justify your placement of attributes at each level of the hierarchy. Explain why they should not be placed at a higher or lower level.

databases korth-edition6 database-design er-diagram descriptive

5.1.16 Database Design: DBMS Korth Edition 6 Exercise 7 Question 27 (Page No. 321) top

Explain the distinction between condition-defined and user-defined constraints. Which of these constraints can the system check automatically? Explain your answer.

databases korth-edition6 database-design er-diagram descriptive

5.1.17 Database Design: DBMS Korth Edition 6 Exercise 7 Question 28 (Page No. 321) top

Explain the distinction between disjoint and overlapping constraints.

databases korth-edition6 database-design er-diagram descriptive

5.1.18 Database Design: DBMS Korth Edition 6 Exercise 7 Question 29 (Page No. 321) top

Explain the distinction between total and partial constraints.

databases korth-edition6 database-design er-diagram descriptive

Answer key

5.1.19 Database Design: DBMS Korth Edition 6 Exercise 7 Question 3,17 (Page No. 316-319) top

Design an E-R diagram for keeping track of the exploits of your favorite sports team. You should store the matches played, the scores in each match, the players in each match, and individual player statistics for each match. Summary statistics should be modeled as derived attributes.

b)Extend the E-R diagram of above exercise to track the same information for all teams in a league.

databases korth-edition6 database-design er-diagram

5.1.20 Database Design: DBMS Korth Edition 6 Exercise 7 Question 4 (Page No. 316) top

Consider an E-R diagram in which the same entity set appears several times, with its attributes repeated in more than one occurrence. Why is allowing this redundancy a bad practice that one should avoid?

databases korth-edition6 database-design er-diagram

5.1.21 Database Design: DBMS Korth Edition 6 Exercise 7 Question 5 (Page No. 316) top

An E-R diagram can be viewed as a graph. What do the following mean in terms of the structure of an enterprise schema?
a. The graph is disconnected.
b. The graph has a cycle.

databases korth-edition6 database-design er-diagram

5.1.22 Database Design: DBMS Korth Edition 6 Exercise 7 Question 6 (Page No. 316-317) top

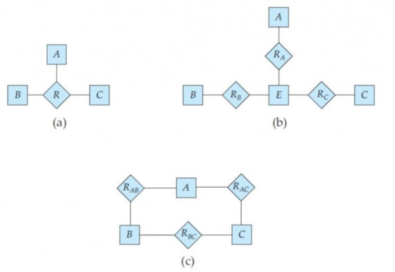
Consider the representation of a ternary relationship using binary relationships as described in this book and illustrated in Figure (b) (attributes not shown)

a. Show a simple instance of E, A, B,C, RA, RB, and RC that cannot correspond to any instance of A, B,C, and R.

b. Modify the E-R diagram of Figure (b) to introduce constraints that will guarantee that any instance of E, A, B, C, RA, RB, and RC that satisfies the constraints will correspond to an instance of A, B, C, and R.

c. Modify the translation above to handle total participation constraints on the ternary relationship.

d. The above representation requires that we create a primary-key attribute for E. Show how to treat E as a weak entity set so that a



primary-key attribute is not required.

databases korth-edition6 database-design er-diagram

5.1.23 Database Design: DBMS Korth Edition 6 Exercise 7 Question 7 (Page No. 317) top

A weak entity set can always be made into a strong entity set by adding to its attributes the primary-key attributes of its identifying entity set. Outline what sort of redundancy will result if we do so.

databases database-design er-diagram descriptive

5.1.24 Database Design: DBMS Korth Edition 6 Exercise 7 Question 8 (Page No. 317) top

Consider a relation such as sec_course, generated from a many-to-one relationship sec_course. Do the primary and foreign key constraints created on the relation enforce the many-to-one cardinality constraint? Explain why.

databases korth-edition6 database-design er-diagram descriptive

5.1.25 Database Design: DBMS Korth Edition 6 Exercise 7 Question 9 (Page No. 317) top

Suppose the advisor relationship was one-to-one. What extra constraints are required on the relation advisor to ensure that the one-to-one cardinality constraint is enforced?

databases korth-edition6 database-design er-diagram descriptive

5.2 Rdbms (1) top

5.2.1 Rdbms: DBMS Korth Edition 6 Exercise 2 Question 4 (Page No. 53) top

In the instance of instructor shown below, no two instructors have the same name. From this, can we conclude that name can be used as a

ID	*name*	*dept_name*	*salary*
10101	Srinivasan	Comp. Sci.	65000
12121	Wu	Finance	90000
15151	Mozart	Music	40000
22222	Einstein	Physics	95000
32343	El Said	History	60000
33456	Gold	Physics	87000
45565	Katz	Comp. Sci.	75000
58583	Califieri	History	62000
76543	Singh	Finance	80000
76766	Crick	Biology	72000
83821	Brandt	Comp. Sci.	92000
98345	Kim	Elec. Eng.	80000

Figure 2.1 The *instructor* relation.

superkey (or primary key) of instructor?

databases korth-edition6 relational-model rdbms

5.3 Relational Calculus (5) top

5.3.1 Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 17 (Page No. 254) top

Let R = (A, B) and S = (A, C), and let r (R) and s(S) be relations.
Write SQL Queries equivalent to the following domain relational-calculus expressions:
a. {< a > | $\exists b(< a, b > \epsilon, r \wedge b = 17)$}
b. {< a, b, c > | < a, b > $\epsilon, r \wedge < a, c > \epsilon,$ s}
c. {< a > |$\exists b(< a, b > \epsilon, r) \vee \forall c(\exists d(< d, c > \epsilon s) \Rightarrow < a, c > \epsilon,$ s)}
d. {< a > | $\exists c(< a, c > \epsilon, s \wedge \exists b1, b2(< a, b1 > \epsilon, r \wedge < c, b2 > \epsilon r \wedge b1 > b2$))}

databases korth-edition6 relational-model relational-calculus sql

5.3.2 Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 18 (Page No. 254) top

Let R = (A, B) and S = (A, C), and let r (R) and s(S) be relations.
Using the special constant null, write tuple-relational-calculus expressions equivalent to each of the following:
a. r ROJ s
b. r FOJ s
c. r LOJ s

databases korth-edition6 relational-model relational-algebra relational-calculus

Answer key

5.3.3 Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 19 (Page No. 254) top

Give a tuple-relational-calculus expression to find the maximum value in relation r (A).

databases korth-edition6 relational-model relational-calculus

5.3.4 Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 6 (Page No. 251) top

Let R = (A, B, C), and let r1 and r2 both be relations on schema R. Give an expression in the domain relational calculus that is equivalent to each of the following:

a. $\prod_A(r1)$
b. $\sigma_{B=17}(r1)$
c. r1 ∪ r2
d. r1 ∩ r2
e. r1 − r2
f. $\prod_{A,B}(r1) \bowtie \prod_{B,C}(r2)$

databases korth-edition6 relational-calculus

5.3.5 Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 7 (Page No. 251) top

Let R = (A, B) and S = (A,C), and let r (R) and s(S) be relations.Write expressions in relational algebra for each of the following queries:
a. {< a > | ∃ b (< a, b > ∈ r ∧ b = 7)}
b. {< a, b, c > | < a, b > ∈ r ∧ < a, c > ∈ s}
c. {< a > | ∃ c (< a, c > ∈ s ∧ ∃ b1, b2 (< a, b1 > ∈ r ∧ < c, b2 > ∈ r ∧ b1 > b2))}

databases korth-edition6 relational-model relational-algebra relational-calculus

5.4 Relational Model (64) top

5.4.1 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 1 (Page No. 53) top

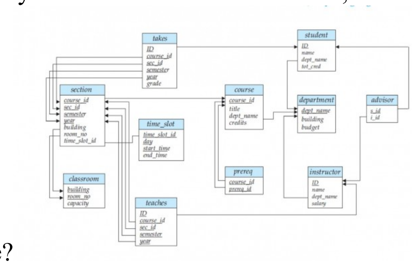
Consider the relational database given below . What are the appropriate primary keys?

employee (person name, street, city)
works (person name, company name, salary)
company (company name, city)

databases korth-edition6 relational-model descriptive

5.4.2 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 10 (Page No. 54) top

Consider the advisor relation shown in Figure , with s id as the primary
key of advisor. Suppose a student can have more than one advisor. Then,
would s id still be a primary key of the advisor relation? If not, what should



the primary key of advisor be?

databases korth-edition6 relational-model relational-algebra

5.4.3 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 11 (Page No. 54) top

Describe the differences in meaning between the terms relation and relation schema.

databases korth-edition6 relational-model descriptive

Answer key

5.4.4 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 12 (Page No. 55) top

Consider the relational database shown below. Give an expression in the
relational algebra to express each of the following queries:
a. Find the names of all employees who work for "First Bank Corporation".
b. Find the names and cities of residence of all employeeswho work for
"First Bank Corporation".
c. Find the names, street address, and cities of residence of all employees
who work for "First Bank Corporation" and earn more than $10,000.

employee (person name, street, city)
works (person name, company name, salary)
company (company name, city)

databases korth-edition6 relational-model relational-algebra

5.4.5 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 13 (Page No. 55) top

Consider the bank database. Give an expression in the relational
algebra for each of the following queries:
a. Find all loan numbers with a loan value greater than $10,000.b. Find the names of all depositors who have an account with a value greater than$6,000.
c. Find the names of all depositors who have an account with a value
greater than $6,000 at the "Uptown" branch.

branch(branch name, branch city, assets)
customer (customer name, customer street, customer city)
loan (loan number, branch name, amount)
borrower (customer name, loan number)
account (account number, branch name, balance)
depositor (customer name, account number)

databases korth-edition6 relational-model relational-algebra

Answer key

5.4.6 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 14 (Page No. 55) top

List two reasons why null values might be introduced into the database.

databases korth-edition6 relational-model descriptive

5.4.7 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 15 (Page No. 55) top

Discuss the relative merits of procedural and nonprocedural languages.

relational-model databases korth-edition6 descriptive

5.4.8 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 2 (Page No. 53) top

Consider the foreign key constraint from the dept name attribute of instructor to the department relation. Give examples of inserts and deletes to these relations, which can cause a violation of the foreign key constraint.

databases korth-edition6 relational-model descriptive

5.4.9 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 3 (Page No. 53) top

Consider the time slot relation. Given that a particular time slot can meet more than once in a week, explain why day and start time are part of the primary key of this relation, while end time is not.

databases korth-edition6 relational-model descriptive

5.4.10 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 5 (Page No. 53) top

What is the result of first performing the cross product of student and advisor, and then performing a selection operation on the result with the predicate s id = ID? (Using the symbolic notation of relational algebra, this query can be written as $\sigma sid = ID(student \times adviser).)$

databases korth-edition6 relational-model descriptive relational-algebra

5.4.11 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 6 (Page No. 54) top

Consider the following expressions, which use the result of a relational algebra operation as the input to another operation. For each expression, explain in words what the expression does.

a. $\sigma_{year \geq 2009}(takes) \bowtie student$
b. $\sigma_{year \geq 2009}(takes \bowtie student)$
c. $\pi_{ID,name,course_id}(student \times takes)$

databases korth-edition6 relational-model relational-algebra

5.4.12 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 7 (Page No. 54) top

Consider the relational database given below. Give an expression in the relational algebra to express each of the following queries:
a. Find the names of all employees who live in city “Miami”.
b. Find the names of all employees whose salary is greater than $100,000.
c. Find the names of all employees who live in “Miami” and whose salary is greater than $100,000.

employee (person name, street, city)
works (person name, company name, salary)
company (company name, city)

databases korth-edition6 relational-model relational-algebra

5.4.13 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 8 (Page No. 54) top

Consider the bank database given below. Give an expression in the relational
algebra for each of the following queries.
a. Find the names of all branches located in “Chicago”.
b. Find the names of all borrowers who have a loan in branch “Downtown”.

branch(branch name, branch city, assets)
customer (customer name, customer street, customer city)
loan (loan number, branch name, amount)
borrower (customer name, loan number)
account (account number, branch name, balance)
depositor (customer name, account number)

databases korth-edition6 relational-model relational-algebra

Answer key

5.4.14 Relational Model: DBMS Korth Edition 6 Exercise 2 Question 9 (Page No. 55) top

Consider the bank database given below.
a. What are the appropriate primary keys?
b. Given your choice of primary keys, identify appropriate foreign keys.

branch(branch name, branch city, assets)
customer (customer name, customer street, customer city)
loan (loan number, branch name, amount)
borrower (customer name, loan number)
account (account number, branch name, balance)
depositor (customer name, account number)

databases korth-edition6 relational-model relational-algebra

5.4.15 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 1 (Page No. 105) top

Write the following queries in SQL, using the university schema. (We suggest
you actually run these queries on a database, using the sample data
that we provide on the Web site of the book, db-book.com. Instructions for
setting up a database, and loading sample data, are provided on the above
Web site.)
a. Find the titles of courses in the Comp. Sci. department that have 3
credits.
b. Find the IDs of all students who were taught by an instructor named
Einstein;make sure there are no duplicates in the result.
c. Find the highest salary of any instructor.
d. Find all instructors earning the highest salary (there may be more
than one with the same salary).
e. Find the enrollment of each section that was offered in Autumn 2009.
f. Find the maximum enrollment, across all sections, in Autumn 2009.
g. Find the sections that had themaximum enrollment in Autumn 2009.

databases korth-edition6 relational-model sql

5.4.16 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 10 (Page No. 108) top

Consider the relational database given below Give an expression in SQL
for each of the following queries.
a. Modify the database so that “Jones” now lives in “Newtown”.
b. Give all managers of “First Bank Corporation” a 10 percent raise
unless the salary becomes greater than $100,000; in such cases, give
only a 3 percent raise.

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.17 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 11 (Page No. 108) top

Write the following queries in SQL, using the university schema.

a. Find the names of all students who have taken at least one Comp. Sci. course; make sure there are no duplicate names in the result.

b. Find the IDs and names of all students who have not taken any course offering before Spring 2009.

c. For each department, find the maximum salary of instructors in that department. You may assume that every department has at least one instructor.

d. Find the lowest, across all departments, of the per-department maximum salary computed by the preceding query.

databases korth-edition6 relational-model sql

5.4.18 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 12 (Page No. 109) top

Write the following queries in SQL, using the university schema.

a. Create a new course “CS-001”, titled “Weekly Seminar”,with 0 credits.
b. Create a section of this course in Autumn 2009, with sec id of 1.
c. Enroll every student in the Comp. Sci. department in the above section.
d. Delete enrollments in the above section where the student’s name is Chavez.
e. Delete the course CS-001. What will happen if you run this delete statement without first deleting offerings (sections) of this course.
f. Delete all takes tuples corresponding to any section of any course with the word “database” as a part of the title; ignore case when matching the word with the title.

databases korth-edition6 relational-model sql

5.4.19 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 13 (Page No. 109) top

Write SQL DDL corresponding to the schema given below. Make any reasonable assumptions about data types, and be sure to declare primary and foreign keys.

person (driver id, name, address)
car (license, model, year)
accident (report number, date, location)
owns (driver id, license)
participated (report number, license, driver id, damage amount)

databases korth-edition6 relational-model sql

5.4.20 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 14 (Page No. 109) top

Consider the insurance database, where the primary keys are underlined. Construct the following SQL queries for this relational database.
a. Find the number of accidents in which the cars belonging to “John Smith” were involved.
b. Update the damage amount for the car with the license number “AABB2000” in the accident with report number “AR2197” to $3000.

here is the database

person (driver id, name, address)
car (license, model, year)

accident (report number, date, location)
owns (driver id, license)
participated (report number, license, driver id, damage amount)

databases korth-edition6 relational-model sql

5.4.21 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 15 (Page No. 109) top

Consider the bank database, where the primary keys are underlined.
Construct the following SQL queries for this relational database.
a. Find all customers who have an account at all the branches located in "Brooklyn".
b. Find out the total sum of all loan amounts in the bank.
c. Find the names of all branches that have assets greater than those of at least one branch located in "Brooklyn".

here is the database

branch(branch name, branch city, assets)
customer (customer name, customer street, customer city)
loan (loan number, branch name, amount)
borrower (customer name, loan number)
account (account number, branch name, balance)
depositor (customer name, account number)

databases korth-edition6 relational-model sql

5.4.22 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 16 (Page No. 109) top

Consider the employee database, where the primary keys are underlined. Give an expression in SQL for each of the following queries.

a. Find the names of all employees who work for "First Bank Corporation".

b. Find all employees in the database who live in the same cities as the companies for which they work.

c. Find all employees in the database who live in the same cities and on the same streets as do theirmanagers.

d. Find all employees who earn more than the average salary of all employees of their company.

e. Find the company that has the smallest payroll.

here is the database

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.23 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 17 (Page No. 110) top

Consider the relational database given below. Give an expression in SQL for each of the following queries.
a. Give all employees of "First Bank Corporation" a 10 percent raise.
b. Give all managers of "First Bank Corporation" a 10 percent raise.
c. Delete all tuples in the works relation for employees of "Small Bank Corporation".

here is the database

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.24 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 18 (Page No. 110)

top

List two reasons why null values might be introduced into the database.

databases korth-edition6 relational-model sql descriptive

Answer key

5.4.25 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 19 (Page No. 110)

top

Show that, in SQL, <> all is identical to not in.

databases korth-edition6 relational-model sql descriptive

5.4.26 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 2 (Page No. 106)

top

Suppose you are given a relation grade points(grade, points), which provides a conversion from letter grades in the takes relation to numeric scores; for example an "A" grade could be specified to correspond to 4 points, an "A−" to 3.7 points, a "B+" to 3.3 points, a "B" to 3 points, and so on. The grade points earned by a student for a course offering (section) is defined as the number of credits for the course multiplied by the numeric points for the grade that the student received.
Given the above relation, and our university schema, write each of the following queries in SQL. You can assume for simplicity that no takes tuple has the null value for grade.
a. Find the total grade-points earned by the student with ID 12345, across all courses taken by the student.
b. Find the grade-point average (GPA) for the above student, that is, the total grade-points divided by the total credits for the associated courses.
c. Find the ID and the grade-point average of every student.

databases korth-edition6 relational-model sql

5.4.27 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 20 (Page No. 110)

top

Give an SQL schema definition for the employee database.
Choose an appropriate domain for each attribute and an appropriate primary key for each relation schema.

here is database

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql descriptive

5.4.28 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 21 (Page No. 110)

top

Consider the library database.Write the following queries in SQL.
a. Print the names of members who have borrowed any book published by "McGraw-Hill".
b. Print the names of members who have borrowed all books published by "McGraw-Hill".
c. For each publisher, print the names of members who have borrowed

more than five books of that publisher.
d. Print the average number of books borrowed per member. Take into account that if an member does not borrow any books, then that member does not appear in the borrowed relation at all.

member(memb no, name, age)
book(isbn, title, authors, publisher)
borrowed(memb no, isbn, date)

databases korth-edition6 relational-model sql

5.4.29 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 22 (Page No. 110) top

Rewrite the where clause

where unique (select title from course)

without using the unique construct.

databases korth-edition6 relational-model sql descriptive

Answer key

5.4.30 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 23 (Page No. 111) top

Consider the query:

select course id, semester, year, sec id, avg (tot cred)
from takes natural join student
where year = 2009
group by course id, semester, year, sec id
having count (ID) >= 2;

Explain why joining section as well in the from clause would not change the result.

databases korth-edition6 relational-model sql descriptive

5.4.31 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 24 (Page No. 111) top

Consider the query:

```
with dept total (dept name, value) as
(select dept name, sum(salary)
from instructor
group by dept name),
dept total avg(value) as
(select avg(value)
from dept total)
select dept name
from dept total, dept total avg
where dept total.value >= dept total avg.value;
```

Rewrite this query without using the with construct.

databases korth-edition6 relational-model sql

Answer key

5.4.32 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 3 (Page No. 106) top

Write the following inserts, deletes or updates in SQL, using the university schema.

a. Increase the salary of each instructor in the Comp. Sci. department by 10%.
b. Delete all courses that have never been offered (that is, do not occur in the section relation).

c. Insert every student whose tot cred attribute is greater than 100 as an instructor in the same department, with a salary of $10,000.

databases relational-model sql

5.4.33 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 4 (Page No. 106) top

Consider the insurance database given below , where the primary keys are underlined. Construct the following SQL queries for this relational database.
a. Find the total number of people who owned cars that were involved in accidents in 2009.
b. Add a new accident to the database; assume any values for required attributes.
c. Delete the Mazda belonging to “John Smith”.

person (driver id, name, address)
car (license, model, year)
accident (report number, date, location)
owns (driver id, license)
participated (report number, license, driver id, damage amount)

databases korth-edition6 relational-model sql

5.4.34 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 5 (Page No. 107) top

Suppose that we have a relation marks(ID, score) and we wish to assign grades to students based on the score as follows: grade F if score < 40, grade C if 40 ≤ score < 60, grade B if 60 ≤ score < 80, and grade A if 80 ≤ score. Write SQL queries to do the following:
a. Display the grade for each student, based on the marks relation.
b. Find the number of students with each grade.

databases korth-edition6 relational-model sql

5.4.35 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 6 (Page No. 107) top

The SQL like operator is case sensitive, but the lower() function on strings can be used to perform case insensitive matching. To show how, write a query that finds departments whose names contain the string “sci” as a substring, regardless of the case.

databases korth-edition6 relational-model sql

5.4.36 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 7 (Page No. 107) top

Consider the SQL query

select distinct p.a1
from p, r1, r2
where p.a1 = r1.a1 or p.a1 = r2.a1

Under what conditions does the preceding query select values of p.a1 that are either in r1 or in r2? Examine carefully the cases where one of r1 or r2 may be empty.

databases korth-edition6 relational-model sql

5.4.37 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 8 (Page No. 107) top

Consider the bank database, where the primary keys are underlined. Construct the following SQL queries for this relational database.
a. Find all customers of the bank who have an account but not a loan.
b. Find the names of all customers who live on the same street and in

the same city as “Smith”.
c. Find the names of all branches with customers who have an account in the bank and who live in “Harrison”.

branch(branch name, branch city, assets)
customer (customer name, customer street, customer city)
loan (loan number, branch name, amount)
borrower (customer name, loan number)
account (account number, branch name, balance)
depositor (customer name, account number)

databases korth-edition6 relational-model sql

5.4.38 Relational Model: DBMS Korth Edition 6 Exercise 3 Question 9 (Page No. 107-108) top

Consider the employee database, where the primary keys are underlined. Give an expression in SQL for each of the following queries.

a. Find the names and cities of residence of all employees who work for “First Bank Corporation”.

b. Find the names, street addresses, and cities of residence of all employees who work for “First Bank Corporation” and earn more than $10,000.

c. Find all employees in the database who do not work for “First Bank Corporation”.

d. Find all employees in the database who earn more than each employee of “Small Bank Corporation”.

e. Assume that the companies may be located in several cities. Find all companies located in every city in which “Small Bank Corporation” is located.

f. Find the company that has the most employees.

g. Find those companies whose employees earn a higher salary, on average, than the average salary at “First Bank Corporation”.

here is the database

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.39 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 1 (Page No. 152) top

Write the following queries in SQL:

a. Display a list of all instructors, showing their ID, name, and the number of sections that they have taught. Make sure to show the number of sections as 0 for instructors who have not taught any section. Your query should use an outer join, and should not use scalar sub queries.

b. Write the same query as above, but using a scalar sub query, without outer join.

c. Display the list of all course sections offered in Spring 2010, along with the names of the instructors teaching the section. If a section has more than one instructor, it should appear as many times in the result as it has instructors. If it does not have any instructor, it should still appear in the result with the instructor name set to “—”.

d. Display the list of all departments, with the total number of instructors in each department, without using scalar sub queries. Make sure to correctly handle departments with no instructors.

databases korth-edition6 relational-model sql

5.4.40 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 10 (Page No. 154) top

SQL provides an n-ary operation called coalesce, which is defined as follows: coalesce(A1, A2, . . . , An) returns the first non null Ai in the list A1, A2, . . . , An, and returns null if all of A1, A2, . . . , An are null.
Let a and b be relations with the schemas A(name, address, title), and B(name, address, salary), respectively. Show how to express a natural full outer join b using the full outer-join operation with an on condition and the coalesce operation. Make sure that the result relation does not contain two copies of the attributes name and address, and that the solution is correct even if some tuples in a and b have null values for attributes name or address.

databases korth-edition6 relational-model sql

5.4.41 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 12 (Page No. 155) top

For the database shown below, write a query to find those employees with no manager. Note that an employee may simply have no manager listed or may have a null manager. Write your query using an outer join and then write it again using no outer join at all.

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.42 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 13 (Page No. 155) top

Under what circumstances would the query

```
select *
from student natural full outer join takes natural full outer join course
```

include tuples with null values for the title attribute?

databases korth-edition6 relational-model sql

5.4.43 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 14 (Page No. 155) top

Show how to define a view tot credits (year, num credits), giving the total number of credits taken by students in each year.

databases korth-edition6 relational-model sql

5.4.44 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 16 (Page No. 155) top

Referential-integrity constraints as defined generally involve exactly two relations. Consider a database that includes the relations shown below

. Suppose that we wish to require that every name that appears in address appears in either salaried worker or hourly worker, but not necessarily in both.
a. Propose a syntax for expressing such constraints.
b. Discuss the actions that the system must take to enforce a constraint of this form.

here is the database

salaried worker (name, office, phone, salary)
hourly worker (name, hourly wage)
address (name, street, city)

databases korth-edition6 relational-model sql

5.4.45 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 17 (Page No. 155) top

Explain why, when a manager, say Satoshi, grants an authorization, the grant should be done by the manager role, rather than by the user Satoshi.

databases korth-edition6 relational-model sql

5.4.46 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 2 (Page No. 152) top

Outer join expressions can be computed in SQL without using the SQL outer join operation. To illustrate this fact, show how to rewrite each of the following SQL queries without using the outer join expression.

a. select* from student natural left outer join takes
b. select* from student natural full outer join takes

databases korth-edition6 relational-model sql

5.4.47 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 3 (Page No. 153) top

Suppose we have three relations r (A, B), s(B, C), and t(B, D), with all attributes declared as not null. Consider the expressions
• r natural left outer join (s natural left outer join t), and
• (r natural left outer join s) natural left outer join t
a. Give instances of relations r , s and t such that in the result of the second expression, attribute C has a null value but attribute D has a non-null value.
b. Is the above pattern, with C null and D not null possible in the result of the first expression? Explain why or why not.

databases korth-edition6 relational-model sql

5.4.48 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 5 (Page No. 153) top

Show how to define the view student_grades (ID, GPA) giving the gradepoint average of each student, based on the query given below 3.2; recall that we used a relation grade_points(grade, points) to get the numeric points associated with a letter grade. Make sure your view definition correctly handles the case of null values for the grade attribute of the takes relation.

3.2 Suppose you are given a relation *grade_points*(*grade*, *points*), which provides a conversion from letter grades in the *takes* relation to numeric scores; for example an "A" grade could be specified to correspond to 4 points, an "A−" to 3.7 points, a "B+" to 3.3 points, a "B" to 3 points, and so on. The grade points earned by a student for a course offering (section) is defined as the number of credits for the course multiplied by the numeric points for the grade that the student received.

Given the above relation, and our university schema, write each of the following queries in SQL. You can assume for simplicity that no *takes* tuple has the *null* value for *grade*.

a. Find the total grade-points earned by the student with ID 12345, across all courses taken by the student.
b. Find the grade-point average (*GPA*) for the above student, that is, the total grade-points divided by the total credits for the associated courses.
c. Find the ID and the grade-point average of every student.

databases korth-edition6 relational-model sql

5.4.49 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 6 (Page No. 153) top

Complete the SQL DDL definition of the university database shown below in figure 4.8

```
create table classroom
   (building      varchar (15),
    room_number   varchar (7),
    capacity      numeric (4,0),
    primary key (building, room_number))

create table department
   (dept_name     varchar (20),
    building      varchar (15),
    budget        numeric (12,2) check (budget > 0),
    primary key (dept_name))

create table course
   (course_id     varchar (8),
    title         varchar (50),
    dept_name     varchar (20),
    credits       numeric (2,0) check (credits > 0),
    primary key (course_id),
    foreign key (dept_name) references department)

create table instructor
   (ID            varchar (5),
    name          varchar (20), not null
    dept_name     varchar (20),
    salary        numeric (8,2), check (salary > 29000),
    primary key (ID),
    foreign key (dept_name) references department)

create table section
   (course_id     varchar (8),
    sec_id        varchar (8),
    semester      varchar (6), check (semester in
                        ('Fall', 'Winter', 'Spring', 'Summer'),
    year          numeric (4,0), check (year > 1759 and year < 2100)
    building      varchar (15),
    room_number   varchar (7),
    time_slot_id  varchar (4),
    primary key (course_id, sec_id, semester, year),
    foreign key (course_id) references course,
    foreign key (building, room_number) references classroom)
```

Figure 4.8 SQL data definition for part of the university database.

to include the relations student, takes, advisor, and prereq.

databases korth-edition6 relational-model sql

5.4.50 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 7 (Page No. 154) top

Consider the relational database shown below. Give an SQL DDL definition
of this database. Identify referential-integrity constraints that should hold,
and include them in the DDL definition.

employee (employee name, street, city)
works (employee name, company name, salary)
company (company name, city)
manages (employee name, manager name)

databases korth-edition6 relational-model sql

5.4.51 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 8 (Page No. 154) top

we expect the constraint “an instructor cannot teach sections in two different classrooms in a semester in the same time slot” to hold.
a. Write an SQL query that returns all (instructor, section) combinations
that violate this constraint.
b. Write an SQL assertion to enforce this constraint.

databases korth-edition6 relational-model sql

5.4.52 Relational Model: DBMS Korth Edition 6 Exercise 4 Question 9 (Page No. 154) top

SQL allows a foreign-key dependency to refer to the same relation, as in the
following example:

create table manager
(employee name varchar(20) not null
manager name varchar(20) not null,
primary key employee name,
foreign key (manager name) references manager
on delete cascade)

Here, employee name is a key to the table manager, meaning that each employee
has atmost one manager. The foreign-key clause requires that every
manager also be an employee. Explain exactly what happens when a tuple
in the relation manager is deleted.

databases korth-edition6 relational-model sql

5.4.53 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 1 (Page No. 249) top

Write the following queries in relational algebra, using the university schema.
a. Find the titles of courses in the Comp. Sci. department that have 3 credits.
b. Find the IDs of all students who were taught by an instructor named Einstein; make sure there are no duplicates in the result.
c. Find the highest salary of any instructor.
d. Find all instructors earning the highest salary (there may be more than one with the same salary).

e. Find the enrollment of each section that was offered in Autumn 2009.
f. Find the maximum enrollment, across all sections, in Autumn 2009.
g. Find the sections that had the maximum enrollment in Autumn 2009.

databases korth-edition6 relational-model relational-algebra

5.4.54 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 10 (Page No. 252) top

Write the following queries in relational algebra, using the university schema.
a. Find the names of all students who have taken at least one Comp. Sci. course.
b. Find the IDs and names of all students who have not taken any course offering before Spring 2009.
c. For each department, find the maximum salary of instructors in that department. You may assume that every department has at least one instructor.
d. Find the lowest, across all departments, of the per-department maximum salary computed by the preceding query.

databases korth-edition6 relational-model relational-algebra

5.4.55 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 11 (Page No. 252) top

Consider the relational database shown here, where the primary keys are underlined. Give an expression in the relational algebra to express each of the following queries:
a. Find the names of all employees who work for “First Bank Corporation”.
b. Find the names and cities of residence of all employees who work for “First Bank Corporation”.
c. Find the names, street addresses, and cities of residence of all employees who work for “First Bank Corporation” and earn more than $10,000.
d. Find the names of all employees in this database who live in the same city as the company for which they work.
e. Assume the companies may be located in several cities. Find all companies located in every city in which “Small Bank Corporation” is located.

here is the database

```
employee (person name, street, city )
works (person name, company name, salary)
company (company name, city)
manages (person name, manager name)
```

databases korth-edition6 relational-model relational-algebra

5.4.56 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 12 (Page No. 253) top

Using the university example, write relational-algebra queries to find the course sections taught by more than one instructor in the following ways:
a. Using an aggregate function.
b. Without using any aggregate functions.

databases korth-edition6 relational-model relational-algebra

5.4.57 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 13 (Page No. 253) top

Consider the relational database shown here. Give a relational-algebra expression for each of the following queries:
a. Find the company with the most employees.
b. Find the company with the smallest payroll.
c. Find those companies whose employees earn a higher salary, on average, than the average salary at First Bank Corporation.

here is the database

```
employee (person name, street, city )
works (person name, company name, salary)
company (company name, city)
manages (person name, manager name)
```

databases korth-edition6 relational-algebra relational-model

5.4.58 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 14 (Page No. 253) top

Consider the following relational schema for a library:

```
member(memb_no, name, dob)
books(isbn, title, authors, publisher)
borrowed(memb_no, isbn, date)
```

Write the following queries in relational algebra.
a. Find the names of members who have borrowed any book published by "McGraw-Hill".
b. Find the name of members who have borrowed all books published by "McGraw-Hill".
c. Find the name and membership number of members who have borrowed more than five different books published by "McGraw-Hill".
d. For each publisher, find the name and membership number of members who have borrowed more than five books of that publisher.
e. Find the average number of books borrowed per member. Take into account that if a member does not borrow any books, then that member does not appear in the *borrowed* relation at all.

databases korth-edition6 relational-model relational-algebra

Answer key

5.4.59 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 2 (Page No. 250) top

Consider the relational database given below, where the primary keys are underlined. Give an expression in the relational algebra to express each of the following queries:
a. Find the names of all employees who live in the same city and on the same street as do their managers.
b. Find the names of all employees in this database who do not work for "First Bank Corporation".
c. Find the names of all employees who earn more than every employee of "Small Bank Corporation".

here is the database

```
employee (person name, street, city )
works (person name, company name, salary)
company (company name, city)
manages (person name, manager name)
```

databases korth-edition6 relational-model relational-algebra

5.4.60 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 3 (Page No. 250) top

The natural outer-join operations extend the natural-join operation so that tuples from the participating relations are not lost in the result of the join. Describe how the theta-join operation can be extended so that tuples from the left, right, or both relations are not lost from the result of a theta join.

databases korth-edition6 relational-algebra relational-model

5.4.61 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 4 (Page No. 250) top

(Division operation): The division operator of relational algebra, "÷", is defined as follows. Let r (R) and s(S) be relations, and let S ⊆ R; that is, every attribute of schema S is also in schema R. Then r ÷ s is a relation on schema R − S (that is, on the schema containing all attributes of schema R that are not in schema S). A tuple t is in r ÷ s if and only if both of two conditions hold:
• t is in $\prod_{r-s}(r)$
• For every tuple t_s in s, there is a tuple t_r in r satisfying both of the following:
a.$t_r[S] = t_s[S]$
b. $t_r[R - S] = t$
Given the above definition:

a. Write a relational algebra expression using the division operator to find the IDs of all students who have taken all Comp. Sci. courses. (Hint: project takes to just ID and course_id, and generate the set of all Comp. Sci. course_ids using a select expression, before doing the division.)

b. Show how to write the above query in relational algebra, without using division. (By doing so, you would have shown how to define the division operation using the other relational algebra operations.)

databases korth-edition6 relational-model relational-algebra

5.4.62 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 5 (Page No. 251) top

Let the following relation schemas be given:
R = (A, B,C)
S = (D, E, F)
Let relations r(R) and s(S) be given. Give an expression in the tuple relational calculus that is equivalent to each of the following:
a. $\prod_A(r)$
b. $\sigma_{B=17}(r)$
c. r × s
d. $\prod_{A,F}(\sigma_{C=D}(r \times s))$

databases korth-edition6 relational-model relational-algebra

5.4.63 Relational Model: DBMS Korth Edition 6 Exercise 6 Question 9 (Page No. 252) top

Describe how to translate join expressions in SQL to relational algebra.

databases korth-edition6 relational-model relational-algebra sql

5.4.64 Relational Model: DBMS Korth Edition 7 Question 25 (Page No. 321) top

Consider the relation schemas are shown below, which were generated from the E-R diagram in Figure given below. For each schema, specify what foreign key constraints, if any, should be created.

```
teaches (ID, course id, sec id, semester, year)
takes (ID, course id, sec id, semester, year, grade)
```

```
prereq (course id, prereq id)
advisor (s ID, i ID)
sec course (course id, sec id, semester, year)
sec time slot (course id, sec id, semester, year, time slot id)
sec class (course id, sec id, semester, year, building, room number)
inst dept (ID, dept name)
stud dept (ID, dept name)
course dept (course id, dept name)
```

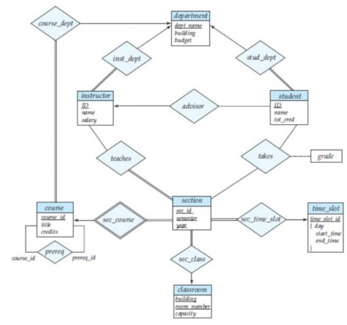


Figure 7.15 E-R diagram for a university enterprise.

databases korth-edition6 relational-model er-diagram descriptive

5.5 Tuple Relational Calculus (2) top

5.5.1 Tuple Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 15 (Page No. 253-254) top

Consider the employee database shown here. Give expressions in tuple relational calculus and domain relational calculus for each of the following queries:
a. Find the names of all employees who work for “First Bank Corporation”.
b. Find the names and cities of residence of all employees who work for “First Bank Corporation”.
c. Find the names, street addresses, and cities of residence of all employees who work for “First Bank Corporation” and earn more than $10,000.
d. Find all employees who live in the same city as that in which the company for which they work is located.
e. Find all employees who live in the same city and on the same street as their managers.
f. Find all employees in the database who do not work for “First Bank Corporation”.
g. Find all employees who earn more than every employee of “Small Bank Corporation”.
h. Assume that the companies may be located in several cities. Find all companies located in every city in which “Small Bank Corporation” is located.

here is the database

```
employee (person name, street, city )
works (person name, company name, salary)
company (company name, city)
manages (person name, manager name)
```

databases korth-edition6 relational-model relational-algebra relational-calculus tuple-relational-calculus

5.5.2 Tuple Relational Calculus: DBMS Korth Edition 6 Exercise 6 Question 8 (Page No. 251) top

Consider the relational database given below where the primary keys are underlined. Give an expression in tuple relational calculus for each of the following queries:

a. Find all employees who work directly for “Jones.”
b. Find all cities of residence of all employees who work directly for “Jones.”
c. Find the name of the manager of the manager of “Jones.”
d. Find those employees who earn more than all employees living in the city “Mumbai.”

here is the database

```
employee (person name, street, city )
works (person name, company name, salary)
company (company name, city)
manages (person name, manager name)
```

databases korth-edition6 relational-model relational-algebra relational-calculus tuple-relational-calculus

Answer key

Answer Keys

5.0.1	N/A	5.0.2	N/A	5.0.3	N/A	5.0.4	N/A	5.0.5	N/A
5.0.6	N/A	5.0.7	N/A	5.0.8	N/A	5.0.9	N/A	5.0.10	N/A
5.1.1	Q-Q	5.1.2	N/A	5.1.3	N/A	5.1.4	N/A	5.1.5	Q-Q
5.1.6	N/A	5.1.7	Q-Q	5.1.8	N/A	5.1.9	N/A	5.1.10	Q-Q
5.1.11	N/A	5.1.12	Q-Q	5.1.13	Q-Q	5.1.14	Q-Q	5.1.15	N/A
5.1.16	N/A	5.1.17	N/A	5.1.18	N/A	5.1.19	Q-Q	5.1.20	Q-Q
5.1.21	Q-Q	5.1.22	Q-Q	5.1.23	N/A	5.1.24	N/A	5.1.25	N/A
5.2.1	Q-Q	5.3.1	Q-Q	5.3.2	Q-Q	5.3.3	Q-Q	5.3.4	Q-Q
5.3.5	Q-Q	5.4.1	N/A	5.4.2	Q-Q	5.4.3	N/A	5.4.4	Q-Q
5.4.5	Q-Q	5.4.6	N/A	5.4.7	N/A	5.4.8	N/A	5.4.9	N/A
5.4.10	N/A	5.4.11	Q-Q	5.4.12	Q-Q	5.4.13	Q-Q	5.4.14	Q-Q
5.4.15	Q-Q	5.4.16	Q-Q	5.4.17	Q-Q	5.4.18	Q-Q	5.4.19	Q-Q
5.4.20	Q-Q	5.4.21	Q-Q	5.4.22	Q-Q	5.4.23	Q-Q	5.4.24	N/A
5.4.25	N/A	5.4.26	Q-Q	5.4.27	N/A	5.4.28	Q-Q	5.4.29	N/A
5.4.30	N/A	5.4.31	Q-Q	5.4.32	Q-Q	5.4.33	Q-Q	5.4.34	Q-Q
5.4.35	Q-Q	5.4.36	Q-Q	5.4.37	Q-Q	5.4.38	Q-Q	5.4.39	Q-Q
5.4.40	Q-Q	5.4.41	Q-Q	5.4.42	Q-Q	5.4.43	Q-Q	5.4.44	Q-Q
5.4.45	Q-Q	5.4.46	Q-Q	5.4.47	Q-Q	5.4.48	Q-Q	5.4.49	Q-Q
5.4.50	Q-Q	5.4.51	Q-Q	5.4.52	Q-Q	5.4.53	Q-Q	5.4.54	Q-Q
5.4.55	Q-Q	5.4.56	Q-Q	5.4.57	Q-Q	5.4.58	Q-Q	5.4.59	Q-Q
5.4.60	Q-Q	5.4.61	Q-Q	5.4.62	Q-Q	5.4.63	Q-Q	5.4.64	N/A
5.5.1	Q-Q	5.5.2	Q-Q						

6 Discrete Mathematics: Combinatory (416)

6.1 Binomial Theorem (39) top

6.1.1 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 1 (Page No. 421) top

Find the expansion of $(x+y)^4$

A. using combinatorial reasoning, as in Example 1.
B. using the binomial theorem.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.2 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 10 (Page No. 421) top

Give a formula for the coefficient of x^k in the expansion of $\left(x+\frac{1}{x}\right)^{100}$, where k is an integer.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.3 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 11 (Page No. 421) top

Give a formula for the coefficient of x^k in the expansion of $\left(x^2-\frac{1}{x}\right)^{100}$, where k is an integer.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.4 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 12 (Page No. 421) top

The row of Pascal's triangle containing the binomial coefficients $\binom{10}{k}$, $0 \leq k \leq 10$, is: $1\ \ 10\ \ 45\ \ 120\ \ 210\ \ 252\ \ 210\ \ 120\ \ 45\ \ 10\ \ 1$ Use Pascal's identity to produce the row immediately following this row in Pascal's triangle.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.5 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 13 (Page No. 421) top

What is the row of Pascal's triangle containing the binomial coefficients $\binom{9}{k}$, $0 \leq k \leq 9$?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.6 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 14 (Page No. 421) top

Show that if n is a positive integer, then $1=\binom{n}{0}<\binom{n}{1}<\cdots<\binom{n}{\lfloor n/2\rfloor}=\binom{n}{\lceil n/2\rceil}>\ldots\binom{n}{n-1}>\binom{n}{n}=1.$

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.7 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 15 (Page No. 421) top

Show that $\binom{n}{k} \leq 2^n$ for all positive integers n and all integers k with $0 \leq k \leq n$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.8 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 16 (Page No. 421) top

A. Use question 14 and Corollary 1 to show that if n is an integer greater than 1, then $\binom{n}{\lfloor n/2\rfloor} \geq \frac{2^n}{2}$.

B. Conclude from part (A) that if n is a positive integer, then $\binom{2n}{n} \geq \frac{4^n}{2n}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.9 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 17 (Page No. 421) top

Show that if n and k are integers with $1 \leq k \leq n$, then $\binom{n}{k} \leq \frac{n^k}{2^{k-1}}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.10 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 18 (Page No. 421) top

Suppose that b is an integer with $b \geq 7$. Use the binomial theorem and the appropriate row of Pascal's triangle to find the base-b expansion of $(11)_b^4$ [that is, the fourth power of the number $(11)_b$ in base-b notation].

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.11 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 19 (Page No. 421) top

Prove Pascal's identity, using the formula for $\binom{n}{r}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.12 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 2 (Page No. 421) top

Find the expansion of $(x+y)^5$

A. using combinatorial reasoning, as in Example 1.
B. using the binomial theorem.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.13 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 20 (Page No. 421) top

Suppose that k and n are integers with $1 \leq k < n$. Prove the hexagon identity $\binom{n-1}{k-1}\binom{n}{k+1}\binom{n+1}{k} = \binom{n-1}{k}\binom{n}{k-1}\binom{n+1}{k+1}$, which relates terms in Pascal's triangle that form a hexagon.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.14 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 21 (Page No. 422) top

Prove that if n and k are integers with $1 \leq k \leq n$, then $k\binom{n}{k} = n\binom{n-1}{k-1}$,

A. using a combinatorial proof. [Hint: Show that the two sides of the identity count the number of ways to select a subset with k elements from a set with n elements and then an element of this subset.]
B. using an algebraic proof based on the formula for $\binom{n}{r}$ given in Theorem 2 in Section 6.3.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.15 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 22 (Page No. 422) top

Prove the identity $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$, whenever n, r, and k are nonnegative integers with $r \leq n$ and $k \leq r$,

A. using a combinatorial argument.
B. using an argument based on the formula for the number of r-combinations of a set with n elements.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.16 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 23 (Page No. 422) top

Show that if n and k are positive integers, then $\binom{n+1}{k} = \dfrac{(n+1)\binom{n}{k-1}}{k}$. Use this identity to construct an inductive definition of the binomial coefficients.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.17 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 24 (Page No. 422) top

Show that if p is a prime and k is an integer such that $1 \leq k \leq p-1$, then p divides $\binom{p}{k}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.18 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 25 (Page No. 422) top

Let n be a positive integer. Show that $\binom{2n}{n+1} + \binom{2n}{n} = \dfrac{\binom{2n+2}{n+1}}{2}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.19 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 26 (Page No. 422) top

Let n and k be integers with $1 \leq k \leq n$. Show that $\displaystyle\sum_{k=1}^{n} \binom{n}{k}\binom{n}{k-1} = \dfrac{\binom{2n+2}{n+1}}{2} - \binom{2n}{n}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.20 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 27 (Page No. 422) top

Prove the hockeystick identity $\displaystyle\sum_{k=0}^{r} \binom{n+k}{k} = \binom{n+r+1}{r}$ whenever n and r are positive integers,

A. using a combinatorial argument.
B. using Pascal's identity.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.21 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 28 (Page No. 422) top

Show that if n is a positive integer, then $\binom{2n}{2} = 2\binom{n}{2} + n^2$

A. using a combinatorial argument.
B. by algebraic manipulation.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.22 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 29 (Page No. 422) top

Give a combinatorial proof that $\displaystyle\sum_{k=1}^{n} k\binom{n}{k} = n2^{n-1}$. [Hint: Count in two ways the number of ways to select a committee and to then select a leader of the committee.]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.23 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 3 (Page No. 421) top

Find the expansion of $(x+y)^6$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.24 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 30 (Page No. 422) top

Give a combinatorial proof that $\sum_{k=1}^{n} k\binom{n}{k}^2 = n\binom{2n-1}{n-1}$. [Hint: Count in two ways the number of ways to select a committee, with n members from a group of n mathematics professors and n computer science professors, such that the chairperson of the committee is a mathematics professor.]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.25 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 31 (Page No. 422) top

Show that a nonempty set has the same number of subsets with an odd number of elements as it does subsets with an even number of elements.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

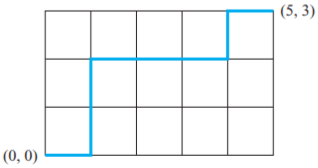
6.1.26 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 32 (Page No. 422) top

Prove the binomial theorem using mathematical induction.

kenneth-rosen discrete-mathematics counting binomial-theorem proof

6.1.27 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 33 (Page No. 422) top

In this exercise we will count the number of paths in the xy plane between the origin $(0,0)$ and point (m,n), where m and n are nonnegative integers, such that each path is made up of a series of steps, where each step is a move one unit to the right or a move one unit upward. (No moves to the left or downward are allowed.) Two such paths from $(0,0)$ to $(5,3)$ are illustrated here.



A. Show that each path of the type described can be represented by a bit string consisting of m $0s$ and n $1s$, where a 0 represents a move one unit to the right and a 1 represents a move one unit upward.
B. Conclude from part (A) that there are $\binom{m+n}{n}$ paths of the desired type.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.28 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 34 (Page No. 422) top

Use question 33 to give an alternative proof of Corollary 2 in Section 6.3, which states that $\binom{n}{k} = \binom{n}{n-k}$ whenever k is an integer with $0 \leq k \leq n.$ [Hint: Consider the number of paths of the type described in question 33 from $(0,0)$ to $(n-k,k)$ and from $(0,0)$ to $(k,n-k).$]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.29 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 35 (Page No. 422) top

Use question 33 to prove Theorem 4. [Hint: Count the number of paths with n steps of the type described in question 33. Every such path must end at one of the points $(n-k,k)$ for $k = 0,1,2,\dots,n.$]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.30 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 36 (Page No. 422) top

Use question 33 to prove Pascal's identity. [Hint: Show that a path of the type described in question 33 from $(0,0)$ to $(n+1-k,k)$ passes through either $(n+1-k,k-1)$ or $(n-k,k)$, but not through both.]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.31 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 37 (Page No. 422) top

Use question 33 to prove the hockeystick identity from question $27.$ [Hint: First, note that the number of paths from $(0,0)$ to $(n+1,r)$ equals $\binom{n+1+r}{r}$. Second, count the number of paths by summing the number of these paths that start by going k units upward for $k=0,1,2,\ldots,r.$]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.32 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 38 (Page No. 422) top

Give a combinatorial proof that if n is a positive integer then $\displaystyle\sum_{k=0}^{n} k^2 \binom{n}{k} = n(n+1)2^{n-2}$. [Hint: Show that both sides count the ways to select a subset of a set of n elements together with two not necessarily distinct elements from this subset. Furthermore, express the right-hand side as $n(n-1)2^{n-2}+n2^{n-1}.$]

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.33 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 39 (Page No. 422 - 423) top

Determine a formula involving binomial coefficients for the nth term of a sequence if its initial terms are those listed. [Hint: Looking at Pascal's triangle will be helpful. Although infinitely many sequences start with a specified set of terms, each of the following lists is the start of a sequence of the type desired.]

A. $1,3,6,10,15,21,28,36,45,55,66,\ldots$
B. $1,4,10,20,35,56,84,120,165,220,\ldots$
C. $1,2,6,20,70,252,924,3432,12870,48620,\ldots$
D. $1,1,2,3,6,10,20,35,70,126,\ldots$
E. $1,1,1,3,1,5,15,35,1,9,\ldots$
F. $1,3,15,84,495,3003,18564,116280,735471,4686825,\ldots$

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

6.1.34 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 4 (Page No. 421) top

Find the coefficient of x^5y^8 in $(x+y)^{13}$.

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.35 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 5 (Page No. 421) top

How many terms are there in the expansion of $(x+y)^{100}$ after like terms are collected?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.36 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 6 (Page No. 421) top

What is the coefficient of x^7 in $(1+x)^{11}$?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.37 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 7 (Page No. 421) top

What is the coefficient of x^9 in $(2-x)^{19}$?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.38 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 8 (Page No. 421) top

What is the coefficient of x^8y^9 in the expansion of $(3x+2y)^{17}$?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.1.39 Binomial Theorem: Kenneth Rosen Edition 7 Exercise 6.4 Question 9 (Page No. 421) top

What is the coefficient of $x^{101}y^{99}$ in the expansion of $(2x-3y)^{200}$?

kenneth-rosen discrete-mathematics counting binomial-theorem descriptive

Answer key

6.2 Counting (329) top

6.2.1 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 1 (Page No. 396) top

There are 18 mathematics majors and 325 computer science majors at a college.

A. In how many ways can two representatives be picked so that one is a mathematics major and the other is a computer science major?
B. In how many ways can one representative be picked who is either a mathematics major or a computer science major?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.2 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 10 (Page No. 396) top

How many bit strings are there of length eight?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.3 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 11 (Page No. 396) top

How many bit strings of length ten both begin and end with a 1?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.4 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 12 (Page No. 396) top

How many bit strings are there of length six or less, not counting the empty string?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.5 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 13 (Page No. 396) top

How many bit strings with length not exceeding n, where n is a positive integer, consist entirely of $1s$, not counting the empty string?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.6 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 14 (Page No. 396) top

. How many bit strings of length n, where n is a positive integer, start and end with $1s$?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.7 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 15 (Page No. 396) top

How many strings are there of lowercase letters of length four or less, not counting the empty string?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.8 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 16 (Page No. 396) top

How many strings are there of four lowercase letters that have the letter x in them?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.9 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 17 (Page No. 396) top

How many strings of five ASCII characters contain the character @ ("at" sign) at least once? [Note: There are 128 different ASCII characters.

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Answer key

6.2.10 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 18 (Page No. 396) top

How many 5-element DNA sequences

A. end with A?
B. start with T and end with G?
C. contain only A and T?
D. do not contain C?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.11 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 19 (Page No. 396) top

How many 6-element RNA sequences

A. do not contain U?
B. end with GU?
C. start with C?
D. contain only A or U?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.12 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 2 (Page No. 396) top

An office building contains 27 floors and has 37 offices on each floor. How many offices are in the building?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.13 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 20 (Page No. 396) top

How many positive integers between 5 and 31

A. are divisible by 3? Which integers are these?
B. are divisible by 4? Which integers are these?
C. are divisible by 3 and by 4? Which integers are these?

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Answer key

6.2.14 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 21 (Page No. 396) top

How many positive integers between 50 and 100

A. are divisible by 7? Which integers are these?
B. are divisible by 11? Which integers are these?
C. are divisible by both 7 and 11? Which integers are these?

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Answer key

6.2.15 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 22 (Page No. 396) top

How many positive integers less than 1000

A. are divisible by 7?
B. are divisible by 7 but not by 11?
C. are divisible by both 7 and 11?
D. are divisible by either 7 or 11?
E. are divisible by exactly one of 7 and 11?
F. are divisible by neither 7 nor 11?
G. have distinct digits?
H. have distinct digits and are even?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.16 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 23 (Page No. 396) top

How many positive integers between 100 and 999 inclusive

A. are divisible by 7?
B. are odd?
C. have the same three decimal digits?
D. are not divisible by 4?
E. are divisible by 3 or 4?
F. are not divisible by either 3 or 4?
G. are divisible by 3 but not by 4?
H. are divisible by 3 and 4?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.17 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 24 (Page No. 396) top

How many positive integers between 1000 and 9999 inclusive

A. are divisible by 9?
B. are even?
C. have distinct digits?
D. are not divisible by 3?
E. are divisible by 5 or 7?
F. fare not divisible by either 5 or 7?
G. are divisible by 5 but not by 7?
H. are divisible by 5 and 7?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.18 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 25 (Page No. 397) top

How many strings of three decimal digits

A. do not contain the same digit three times?
B. begin with an odd digit?
C. have exactly two digits that are $4s$?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.19 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 26 (Page No. 397) top

How many strings of four decimal digits

A. do not contain the same digit twice?
B. end with an even digit?
C. have exactly three digits that are $9s$?

kenneth-rosen discrete-mathematics counting descriptive

6.2.20 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 27 (Page No. 397) top

A committee is formed consisting of one representative from each of the 50 states in the United States, where the representative from a state is either the governor or one of the two senators from that state. How many ways are there to form this committee?

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Answer key

6.2.21 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 28 (Page No. 397) top

How many license plates can be made using either three digits followed by three uppercase English letters or three uppercase English letters followed by three digits?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.22 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 29 (Page No. 397) top

How many license plates can be made using either two uppercase English letters followed by four digits or two digits followed by four uppercase English letters?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.23 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 3 (Page No. 396) top

A multiple-choice test contains 10 questions. There are four possible answers for each question.

A. In how many ways can a student answer the questions on the test if the student answers every question?
B. In how many ways can a student answer the questions on the test if the student can leave answers blank?

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Answer key

6.2.24 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 30 (Page No. 397) top

How many license plates can be made using either three uppercase English letters followed by three digits or four uppercase English letters followed by two digits?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.25 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 31 (Page No. 397) top

How many license plates can be made using either two or three uppercase English letters followed by either two or three digits?

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Answer key

6.2.26 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 32 (Page No. 397) top

How many strings of eight uppercase English letters are there

A. if letters can be repeated?
B. if no letter can be repeated?
C. that start with X, if letters can be repeated?
D. that start with X, if no letter can be repeated?
E. that start and end with X, if letters can be repeated?
F. that start with the letters BO (in that order), if letters can be repeated?
G. that start and end with the letters BO (in that order), if letters can be repeated?
H. that start or end with the letters BO (in that order), if letters can be repeated?

kenneth-rosen discrete-mathematics counting descriptive

6.2.27 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 33 (Page No. 397) top

How many strings of eight English letters are there

A. that contain no vowels, if letters can be repeated?
B. that contain no vowels, if letters cannot be repeated?
C. that start with a vowel, if letters can be repeated?
D. that start with a vowel, if letters cannot be repeated?
E. that contain at least one vowel, if letters can be repeated?
F. that contain exactly one vowel, if letters can be repeated?
G. that start with X and contain at least one vowel, if letters can be repeated?
H. that start and end with X and contain at least one vowel, if letters can be repeated?

kenneth-rosen discrete-mathematics counting descriptive

6.2.28 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 34 (Page No. 397) top

How many different functions are there from a set with 10 elements to sets with the following numbers of elements?

A. 2 B. 3 C. 4 D. 5

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Answer key

6.2.29 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 35 (Page No. 397) top

How many one-to-one functions are there from a set with five elements to sets with the following number of elements?

A. 4 B. 5 C. 6 D. 7

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6.2.30 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 36 (Page No. 397) top

How many functions are there from the set $\{1, 2, \dots, n\}$, where n is a positive integer, to the set $\{0, 1\}$?

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Answer key

6.2.31 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 37 (Page No. 397) top

How many functions are there from the set $\{1, 2, \dots, n\}$, where n is a positive integer, to the set $\{0, 1\}$

A. that are one-to-one?
B. that assign 0 to both 1 and n?
C. that assign 1 to exactly one of the positive integers less than n?

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6.2.32 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 38 (Page No. 397) top

How many partial functions $(\text{see Section } 2.3)$ are there from a set with five elements to sets with each of these number of elements?

A. 1 B. 2 C. 5 D. 9

kenneth-rosen discrete-mathematics counting descriptive

6.2.33 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 39 (Page No. 397) top

How many partial functions $(\text{see Definition } 13 \text{ of Section } 2.3)$ are there from a set with m elements to a set with n elements, where m and n are positive integers?

kenneth-rosen discrete-mathematics counting descriptive

6.2.34 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 4 (Page No. 396) top

A particular brand of shirt comes in 12 colors, has a male version and a female version, and comes in three sizes for each sex. How many different types of this shirt are made?

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Answer key

6.2.35 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 40 (Page No. 397) top

How many subsets of a set with 100 elements have more than one element?

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6.2.36 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 41 (Page No. 397) top

A palindrome is a string whose reversal is identical to the string. How many bit strings of length n are palindromes?

kenneth-rosen discrete-mathematics counting descriptive

6.2.37 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 42 (Page No. 397) top

How many 4-element DNA sequences

A. do not contain the base T?
B. contain the sequence ACG?
C. contain all four bases A, T, C, and G?
D. contain exactly three of the four bases A, T, C, and G?

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6.2.38 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 43 (Page No. 397) top

How many 4-element RNA sequences

A. contain the base U?
B. do not contain the sequence CUG?
C. do not contain all four bases A, U, C, and G?
D. contain exactly two of the four bases A, U, C, and G?

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6.2.39 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 44 (Page No. 397) top

How many ways are there to seat four of a group of ten people around a circular table where two seatings are considered the same when everyone has the same immediate left and immediate right neighbor?

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6.2.40 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 45 (Page No. 397) top

How many ways are there to seat six people around a circular table where two seating are considered the same when everyone has the same two neighbors without regard to whether they are right or left neighbors?

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Answer key

6.2.41 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 46 (Page No. 397) top

In how many ways can a photographer at a wedding arrange 6 people in a row from a group of 10 people, where the bride and the groom are among these 10 people, if

A. the bride must be in the picture?
B. both the bride and groom must be in the picture?
C. exactly one of the bride and the groom is in the picture?

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6.2.42 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 47 (Page No. 397) top

In how many ways can a photographer at a wedding arrange six people in a row, including the bride and groom, if

A. the bride must be next to the groom?
B. the bride is not next to the groom?

C. the bride is positioned somewhere to the left of the groom?

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6.2.43 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 48 (Page No. 398) top

How many bit strings of length seven either begin with two $0s$ or end with three $1s$?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.44 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 49 (Page No. 398) top

How many bit strings of length 10 either begin with three $0s$ or end with two $0s$?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.45 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 5 (Page No. 396) top

Six different airlines fly from New York to Denver and seven fly from Denver to San Francisco. How many different pairs of airlines can you choose on which to book a trip from New York to San Francisco via Denver, when you pick an airline for the flight to Denver and an airline for the continuation flight to San Francisco?

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Answer key

6.2.46 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 50 (Page No. 398) top

How many bit strings of length 10 contain either five consecutive $0s$ or five consecutive $1s$?

kenneth-rosen discrete-mathematics counting descriptive

6.2.47 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 51 (Page No. 398) top

How many bit strings of length eight contain either three consecutive $0s$ or four consecutive $1s$?

kenneth-rosen discrete-mathematics counting descriptive

6.2.48 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 52 (Page No. 398) top

Every student in a discrete mathematics class is either a computer science or a mathematics major or is a joint major in these two subjects. How many students are in the class if there are 38 computer science majors (including joint majors), 23 mathematics majors (including joint majors), and 7 joint majors?

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6.2.49 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 53 (Page No. 398) top

How many positive integers not exceeding 100 are divisible either by 4 or by 6?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.50 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 54 (Page No. 398) top

How many different initials can someone have if a person has at least two, but no more than five, different initials? Assume that each initial is one of the 26 uppercase letters of the English language.

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6.2.51 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 55 (Page No. 398) top

Suppose that a password for a computer system must have at least 8, but no more than 12, characters, where each character in the password is a lowercase English letter, an uppercase English letter, a digit, or one of the six special characters $*, >, <, !, +,$ and $=$.

A. How many different passwords are available for this computer system?
B. How many of these passwords contain at least one occurrence of at least one of the six special characters?
C. Using your answer to part (A), determine how long it takes a hacker to try every possible password, assuming that it takes one nanosecond for a hacker to check each possible password.

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6.2.52 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 56 (Page No. 398) top

The name of a variable in the C programming language is a string that can contain uppercase letters, lowercase letters, digits, or underscores. Further, the first character in the string must be a letter, either uppercase or lowercase, or an underscore. If the name of a variable is determined by its first eight characters, how many different variables can be named in C? (Note that the name of a variable may contain fewer than eight characters.)

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6.2.53 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 57 (Page No. 398) top

The name of a variable in the JAVA programming language is a string of between 1 and $65,535$ characters, inclusive, where each character can be an uppercase or a lowercase letter, a dollar sign, an underscore, or a digit, except that the first character must not be a digit. Determine the number of different variable names in JAVA.

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.54 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 58 (Page No. 398) top

The International Telecommunications Union (ITU) specifies that a telephone number must consist of a country code with between 1 and 3 digits, except that the code 0 is not available for use as a country code, followed by a number with at most 15 digits. How many available possible telephone numbers are there that satisfy these restrictions?

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Answer key

6.2.55 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 59 (Page No. 398) top

Suppose that at some future time every telephone in the world is assigned a number that contains a country code 1 to 3 digits long, that is, of the form X, XX, or XXX, followed by a 10-digit telephone number of the form NXX-NXX-XXXX $(\text{as described in Example } 8)$. How many different telephone numbers would be available worldwide under this numbering plan?

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6.2.56 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 6 (Page No. 396) top

There are four major auto routes from Boston to Detroit and six from Detroit to Los Angeles. How many major auto routes are there from Boston to Los Angeles via Detroit?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.57 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 60 (Page No. 398) top

A key in the Vigenère cryptosystem is a string of English letters, where the case of the letters does not matter. How many different keys for this cryptosystem are there with three, four, five, or six letters?

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6.2.58 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 61 (Page No. 398) top

A wired equivalent privacy (WEP) key for a wireless fidelity (WiFi) network is a string of either $10, 26,$ or 58 hexadecimal digits. How many different WEP keys are there?

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Answer key

6.2.59 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 62 (Page No. 398) top

Suppose that p and q are prime numbers and that $n = pq$. Use the principle of inclusion-exclusion to find the number of positive integers not exceeding n that are relatively prime to n.

kenneth-rosen discrete-mathematics counting descriptive

6.2.60 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 63 (Page No. 398) top

Use the principle of inclusion–exclusion to find the number of positive integers less than $1,000,000$ that are not divisible by either 4 or by 6.

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.61 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 64 (Page No. 398) top

Use a tree diagram to find the number of bit strings of length four with no three consecutive $0s$.

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.62 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 65 (Page No. 398) top

How many ways are there to arrange the letters $a, b, c, \text{ and } d$ such that a is not followed immediately by b?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.63 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 66 (Page No. 398) top

Use a tree diagram to find the number of ways that the World Series can occur, where the first team that wins four games out of seven wins the series.

kenneth-rosen discrete-mathematics counting descriptive

6.2.64 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 67 (Page No. 398) top

Use a tree diagram to determine the number of subsets of $\{3, 7, 9, 11, 24\}$ with the property that the sum of the elements in the subset is less than 28.

kenneth-rosen discrete-mathematics counting descriptive

6.2.65 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 68 (Page No. 398) top

A. Suppose that a store sells six varieties of soft drinks: cola, ginger ale, orange, root beer, lemonade, and cream soda. Use a tree diagram to determine the number of different types of bottles the store must stock to have all varieties available in all size bottles if all varieties are available in 12-ounce bottles, all but lemonade are available in 20-ounce bottles, only cola and ginger ale are available in 32-ounce bottles, and all but lemonade and cream soda are available in 64-ounce bottles?
B. Answer the question in part (A) using counting rules.

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6.2.66 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 69 (Page No. 398) top

A. Suppose that a popular style of running shoe is available for both men and women. The woman's shoe comes in sizes $6, 7, 8, \text{ and } 9$, and the man's shoe comes in sizes $8, 9, 10, 11, \text{ and } 12$. The man's shoe comes in white and black, while the woman's shoe comes in white, red, and black. Use a tree diagram to determine the number of different shoes that a store has to stock to have at least one pair of this type of running shoe for all available sizes and colors for both men and women.
B. Answer the question in part (A) using counting rules.

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6.2.67 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 7 (Page No. 396) top

How many different three-letter initials can people have?

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Answer key

6.2.68 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 70 (Page No. 398) top

Use the product rule to show that there are 2^{2^n} different truth tables for propositions in n variables.

kenneth-rosen discrete-mathematics counting descriptive

6.2.69 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 71 (Page No. 399) top

Use mathematical induction to prove the sum rule for m tasks from the sum rule for two tasks.

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6.2.70 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 72 (Page No. 399) top

Use mathematical induction to prove the product rule for m tasks from the product rule for two tasks.

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6.2.71 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 73 (Page No. 399) top

How many diagonals does a convex polygon with n sides have? (Recall that a polygon is convex if every line segment connecting two points in the interior or boundary of the polygon lies entirely within this set and that a diagonal of a polygon is a line segment connecting two vertices that are not adjacent.)

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6.2.72 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 74 (Page No. 399) top

Data are transmitted over the Internet in datagrams, which are structured blocks of bits. Each datagram contains header information organized into a maximum of 14 different fields (specifying many things, including the source and destination addresses) and a data area that contains the actual data that are transmitted. One of the 14 header fields is the header length field (denoted by HLEN), which is specified by the protocol to be 4 bits long and that specifies the header length in terms of 32-bit blocks of bits. For example, if $\text{HLEN} = 0110$, the header is made up of six 32-bit blocks. Another of the 14 header fields is the 16-bit-long total length field (denoted by TOTAL LENGTH), which specifies the length in bits of the entire datagram, including both the header fields and the data area. The length of the data area is the total length of the datagram minus the length of the header.

A. The largest possible value of TOTAL LENGTH (which is 16 bits long) determines the maximum total length in octets (blocks of 8 bits) of an Internet datagram. What is this value?
B. The largest possible value of HLEN (which is 4 bits long) determines the maximum total header length in 32-bit blocks. What is this value? What is the maximum total header length in octets?
C. The minimum (and most common) header length is 20 octets. What is the maximum total length in octets of the data area of an Internet datagram?
D. How many different strings of octets in the data area can be transmitted if the header length is 20 octets and the total length is as long as possible?

kenneth-rosen discrete-mathematics counting descriptive

6.2.73 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 8 (Page No. 396) top

How many different three-letter initials with none of the letters repeated can people have?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.74 Counting: Kenneth Rosen Edition 7 Exercise 6.1 Question 9 (Page No. 396) top

How many different three-letter initials are there that begin with an A?

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Answer key

6.2.75 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 1 (Page No. 413) top

List all the permutations of $\{a, b, c\}$.

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Answer key

6.2.76 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 10 (Page No. 413) top

There are six different candidates for governor of a state. In how many different orders can the names of the candidates be printed on a ballot?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.77 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 11 (Page No. 413) top

How many bit strings of length 10 contain

A. exactly four $1s$?
B. at most four $1s$?
C. at least four $1s$?
D. an equal number of $0s$ and $1s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.78 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 12 (Page No. 413) top

How many bit strings of length 12 contain

A. exactly three $1s$?
B. at most three $1s$?
C. at least three $1s$?
D. an equal number of $0s$ and $1s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.79 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 13 (Page No. 413) top

A group contains n men and n women. How many ways are there to arrange these people in a row if the men and women alternate?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.80 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 14 (Page No. 413) top

In how many ways can a set of two positive integers less than 100 be chosen?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.81 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 15 (Page No. 413) top

In how many ways can a set of five letters be selected from the English alphabet?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.82 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 16 (Page No. 413) top

How many subsets with an odd number of elements does a set with 10 elements have?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.83 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 17 (Page No. 413) top

How many subsets with more than two elements does a set with 100 elements have?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.84 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 18 (Page No. 413) top

A coin is flipped eight times where each flip comes up either heads or tails. How many possible outcomes

A. are there in total?
B. contain exactly three heads?
C. contain at least three heads?
D. contain the same number of heads and tails?

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Answer key

6.2.85 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 19 (Page No. 413) top

A coin is flipped 10 times where each flip comes up either heads or tails. How many possible outcomes

A. are there in total?
B. contain exactly two heads?
C. contain at most three tails?
D. contain the same number of heads and tails?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.86 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 2 (Page No. 413) top

How many different permutations are there of the set $\{a, b, c, d, e, f, g\}$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.87 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 20 (Page No. 413) top

How many bit strings of length 10 have

A. exactly three $0s$?
B. more $0s$ than $1s$?
C. at least seven $1s$?
D. at least three $1s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.88 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 21 (Page No. 414) top

How many permutations of the letters ABCDEFG contain

A. the string BCD?
B. the string CFGA?
C. the strings BA and GF?
D. the strings ABC and DE?
E. the strings ABC and CDE?
F. the strings CBA and BED?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.89 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 22 (Page No. 414) top

How many permutations of the letters ABCDEFGH contain

A. the string ED?
B. the string CDE?
C. the strings BA and FGH?
D. the strings AB, DE, and GH?
E. the strings CAB and BED?
F. the strings BCA and ABF?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.90 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 23 (Page No. 414) top

How many ways are there for eight men and five women to stand in a line so that no two women stand next to each other? [Hint: First position the men and then consider possible positions for the women.]

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.91 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 24 (Page No. 414) top

How many ways are there for 10 women and six men to stand in a line so that no two men stand next to each other? [Hint: First position the women and then consider possible positions for the men.]

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.92 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 25 (Page No. 414) top

One hundred tickets, numbered $1, 2, 3, \dots, 100,$ are sold to 100 different people for a drawing. Four different prizes are awarded, including a grand prize (a trip to Tahiti). How many ways are there to award the prizes if

A. there are no restrictions?
B. the person holding ticket 47 wins the grand prize?
C. the person holding ticket 47 wins one of the prizes?
D. the person holding ticket 47 does not win a prize?
E. the people holding tickets $19 \text{ and } 47$ both win prizes?
F. the people holding tickets $19, 47, \text{ and } 73$ all win prizes?
G. the people holding tickets $19, 47, 73, \text{ and } 97$ all win prizes?
H. none of the people holding tickets $19, 47, 73, \text{ and } 97$ wins a prize?
I. the grand prize winner is a person holding ticket $19, 47, 73, \text{ or } 97?$
J. the people holding tickets $19 \text{ and } 47$ win prizes, but the people holding tickets $73 \text{ and } 97$ do not win prizes?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.93 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 26 (Page No. 414) top

Thirteen people on a softball team show up for a game.

A. How many ways are there to choose 10 players to take the field?
B. How many ways are there to assign the 10 positions by selecting players from the 13 people who show up?
C. Of the 13 people who show up, three are women. How many ways are there to choose 10 players to take the field if at least one of these players must be a woman?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.94 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 27 (Page No. 414) top

A club has 25 members.

A. How many ways are there to choose four members of the club to serve on an executive committee?
B. How many ways are there to choose a president, vice president, secretary, and treasurer of the club, where no person can hold more than one office?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.95 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 28 (Page No. 414) top

A professor writes 40 discrete mathematics true/false questions. Of the statements in these questions, 17 are true. If the questions can be positioned in any order, how many different answer keys are possible?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.96 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 29 (Page No. 414) top

How many 4-permutations of the positive integers not exceeding 100 contain three consecutive integers $k, k+1, k+2$, in the correct order

A. where these consecutive integers can perhaps be separated by other integers in the permutation?
B. where they are in consecutive positions in the permutation?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.97 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 3 (Page No. 413) top

How many permutations of $\{a, b, c, d, e, f, g\}$ end with a?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.98 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 30 (Page No. 414) top

Seven women and nine men are on the faculty in the mathematics department at a school.

A. How many ways are there to select a committee of five members of the department if at least one woman must be on the committee?
B. How many ways are there to select a committee of five members of the department if at least one woman and at least one man must be on the committee?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.99 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 31 (Page No. 414) top

The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain

A. exactly one vowel?
B. exactly two vowels?
C. at least one vowel?
D. at least two vowels?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.100 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 32 (Page No. 414) top

How many strings of six lowercase letters from the English alphabet contain

A. the letter a?
B. the letters a and b?
C. the letters a and b in consecutive positions with a preceding b, with all the letters distinct?
D. the letters a and b, where a is somewhere to the left of b in the string, with all the letters distinct?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.101 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 33 (Page No. 414) top

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have the same number of men and women?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.102 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 34 (Page No. 414) top

Suppose that a department contains 10 men and 15 women. How many ways are there to form a committee with six members if it must have more women than men?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.103 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 35 (Page No. 414) top

How many bit strings contain exactly eight $0s$ and 10 $1s$ if every 0 must be immediately followed by a 1?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.104 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 36 (Page No. 414) top

How many bit strings contain exactly five $0s$ and 14 $1s$ if every 0 must be immediately followed by two $1s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.105 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 37 (Page No. 414) top

How many bit strings of length 10 contain at least three $1s$ and at least three $0s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.106 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 38 (Page No. 414) top

How many ways are there to select 12 countries in the United Nations to serve on a council if 3 are selected from a block of $45, 4$ are selected from a block of 57, and the others are selected from the remaining 69 countries?

kenneth-rosen discrete-mathematics counting combinatory descriptive

6.2.107 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 39 (Page No. 415) top

How many license plates consisting of three letters followed by three digits contain no letter or digit twice?

kenneth-rosen discrete-mathematics counting combinatory descriptive

6.2.108 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 4 (Page No. 413) top

Let $S = \{1, 2, 3, 4, 5\}$.

A. List all the 3-permutations of S.
B. List all the 3-combinations of S.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.109 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 40 (Page No. 415) top

A circular r-permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.
Find the number of circular 3-permutations of 5 people.

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6.2.110 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 41 (Page No. 415) top

A circular r-permutation of n people is a seating of r of these n people around a circular table, where seatings are considered to be the same if they can be obtained from each other by rotating the table.
Find a formula for the number of circular r-permutations of n people.

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6.2.111 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 42 (Page No. 415) top

Find a formula for the number of ways to seat r of n people around a circular table, where seatings are considered the same if every person has the same two neighbors without regard to which side these neighbors are sitting on.

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6.2.112 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 43 (Page No. 415) top

How many ways are there for a horse race with three horses to finish if ties are possible? [Note: Two or three horses may tie.]

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Answer key

6.2.113 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 44 (Page No. 415) top

How many ways are there for a horse race with four horses to finish if ties are possible? [Note: Any number of the four horses may tie.)

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.114 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 45 (Page No. 415) top

There are six runners in the 100-yard dash. How many ways are there for three medals to be awarded if ties are possible? (The runner or runners who finish with the fastest time receive gold medals, the runner or runners who finish with exactly one runner ahead receive silver medals, and the runner or runners who finish with exactly two runners ahead receive bronze medals.)

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6.2.115 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 46 (Page No. 415) top

This procedure is used to break ties in games in the championship round of the World Cup soccer tournament. Each team selects five players in a prescribed order. Each of these players takes a penalty kick, with a player from the first team followed by a player from the second team and so on, following the order of players specified. If the score is still tied at the end of the 10 penalty kicks, this procedure is repeated. If the score is still tied after 20 penalty kicks, a sudden-death shootout occurs, with the first team scoring an unanswered goal victorious.

A. How many different scoring scenarios are possible if the game is settled in the first round of 10 penalty kicks, where the round ends once it is impossible for a team to equal the number of goals scored by the other team?
B. How many different scoring scenarios for the first and second groups of penalty kicks are possible if the game is settled in the second round of 10 penalty kicks?
C. How many scoring scenarios are possible for the full set of penalty kicks if the game is settled with no more than 10 total additional kicks after the two rounds of five kicks for each team?

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6.2.116 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 5 (Page No. 413) top

Find the value of each of these quantities.

A. $P(6,3)$ B. $P(6,5)$ C. $P(8,1)$ D. $P(8,5)$ E. $P(8,8)$
F. $P(10,9)$

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Answer key

6.2.117 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 6 (Page No. 413) top

Find the value of each of these quantities.

A. $C(5,1)$ B. $C(5,3)$ C. $C(8,4)$ D. $C(8,8)$ E. $C(8,0)$

F. $C(12,6)$

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.118 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 7 (Page No. 413) top

Find the number of 5-permutations of a set with nine elements.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.119 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 8 (Page No. 413) top

In how many different orders can five runners finish a race if no ties are allowed?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.120 Counting: Kenneth Rosen Edition 7 Exercise 6.3 Question 9 (Page No. 413) top

How many possibilities are there for the win, place, and show (first, second, and third) positions in a horse race with 12 horses if all orders of finish are possible?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.121 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 1 (Page No. 432) top

In how many different ways can five elements be selected in order from a set with three elements when repetition is allowed?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.122 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 10 (Page No. 432) top

A croissant shop has plain croissants, cherry croissants, chocolate croissants, almond croissants, apple croissants, and broccoli croissants. How many ways are there to choose

A. a dozen croissants?
B. three dozen croissants?
C. two dozen croissants with at least two of each kind?
D. two dozen croissants with no more than two broccoli croissants?
E. two dozen croissants with at least five chocolate croissants and at least three almond croissants?
F. two dozen croissants with at least one plain croissant, at least two cherry croissants, at least three chocolate croissants, at least one almond croissant, at least two apple croissants, and no more than three broccoli croissants?

kenneth-rosen discrete-mathematics counting combinatory

6.2.123 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 11 (Page No. 432) top

How many ways are there to choose eight coins from a piggy bank containing 100 identical pennies and 80 identical nickels?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.124 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 12 (Page No. 432) top

How many different combinations of pennies, nickels, dimes, quarters, and half dollars can a piggy bank contain if it has 20 coins in it?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.125 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 13 (Page No. 432) top

A book publisher has 3000 copies of a discrete mathematics book. How many ways are there to store these books in their three warehouses if the copies of the book are indistinguishable?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.126 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 14 (Page No. 432) top

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 = 17,$ where $x_1, x_2, x_3,$ and x_4 are nonnegative integers?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.127 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 15 (Page No. 432) top

How many solutions are there to the equation $x1 + x2 + x3 + x4 + x5 = 21,$ where $xi, i = 1, 2, 3, 4, 5,$ is a nonnegative integer such that

A. $x_1 \geq 1$?
B. $x_i \geq 2$ for $i = 1, 2, 3, 4, 5$?
C. $0 \leq x_1 \leq 10$?
D. $0 \leq x_1 \leq 3, 1 \leq x_2 < 4,$ and $x_3 \geq 15$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.128 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 16 (Page No. 432) top

How many solutions are there to the equation $x_1 + x_2 + x_3 + x_4 + x_5 + x_6 = 29,$ where $x_i, i = 1, 2, 3, 4, 5, 6,$ is a nonnegative integer such that

A. $x_i > 1$ for $i = 1, 2, 3, 4, 5, 6$?
B. $x_1 \geq 1, x_2 \geq 2, x_3 \geq 3, x_4 \geq 4, x_5 > 5,$ and $x_6 \geq 6$?
C. $x_1 \leq 5$?
D. $x_1 < 8$ and $x_2 > 8$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.129 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 17 (Page No. 432) top

How many strings of 10 ternary digits $(0, 1,$ or $2)$ are there that contain exactly two $0s$, three $1s$, and five $2s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.130 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 18 (Page No. 432) top

How many strings of 20-decimal digits are there that contain two $0s$, four $1s$, three $2s$, one 3, two $4s$, three $5s$, two $7s$, and three $9s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.131 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 19 (Page No. 432) top

Suppose that a large family has 14 children, including two sets of identical triplets, three sets of identical twins, and two individual children. How many ways are there to seat these children in a row of chairs if the identical triplets or twins cannot be distinguished from one another?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.132 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 2 (Page No. 432) top

In how many different ways can five elements be selected in order from a set with five elements when repetition is allowed?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.133 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 20 (Page No. 432) top

How many solutions are there to the inequality $x_1 + x_2 + x_3 \leq 11,$ where $x_1, x_2,$ and x_3 are nonnegative integers? [Hint: Introduce an auxiliary variable x_4 such that $x_1 + x_2 + x_3 + x_4 = 11.$]

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.134 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 21 (Page No. 432) top

How many ways are there to distribute six indistinguishable balls into nine distinguishable bins?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.135 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 22 (Page No. 432) top

How many ways are there to distribute 12 indistinguishable balls into six distinguishable bins?

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Answer key

6.2.136 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 23 (Page No. 432) top

How many ways are there to distribute 12 distinguishable objects into six distinguishable boxes so that two objects are placed in each box?

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Answer key

6.2.137 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 24 (Page No. 432) top

How many ways are there to distribute 15 distinguishable objects into five distinguishable boxes so that the boxes have one, two, three, four, and five objects in them, respectively.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.138 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 25 (Page No. 433) top

How many positive integers less than $1,000,000$ have the sum of their digits equal to 19?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.139 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 26 (Page No. 433) top

How many positive integers less than $1,000,000$ have exactly one digit equal to 9 and have a sum of digits equal to 13?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.140 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 27 (Page No. 433) top

There are 10 questions on a discrete mathematics final exam. How many ways are there to assign scores to the problems if the sum of the scores is 100 and each question is worth at least 5 points?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.141 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 28 (Page No. 433) top

Show that there are $C(n+r-q_1-q_2-\cdots-q_r-1, n-q_1-q_2-\cdots-q_r)$ different unordered selections of n objects of r different types that include at least q_1 objects of type one, q_2 objects of type two $,\ldots,$ and q_r objects of type $r.$

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6.2.142 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 29 (Page No. 433) top

How many different bit strings can be transmitted if the string must begin with a 1 bit, must include three additional 1 bits (so that a total of four 1 bits is sent), must include a total of 12 0 bits, and must have at least two 0 bits following each 1 bit?

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6.2.143 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 3 (Page No. 432) top

How many strings of six letters are there?

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Answer key

6.2.144 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 30 (Page No. 433) top

How many different strings can be made from the letters in MISSISSIPPI, using all the letters?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.145 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 31 (Page No. 433) top

How many different strings can be made from the letters in $\text{ABRACADABRA},$ using all the letters?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.146 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 32 (Page No. 433) top

How many different strings can be made from the letters in AARDVARK, using all the letters, if all three As must be consecutive?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.147 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 33 (Page No. 433) top

How many different strings can be made from the letters in ORONO, using some or all of the letters?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.148 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 34 (Page No. 433) top

How many strings with five or more characters can be formed from the letters in SEERESS?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.149 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 35 (Page No. 433) top

How many strings with seven or more characters can be formed from the letters in EVERGREEN?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.150 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 36 (Page No. 433) top

How many different bit strings can be formed using six $1s$ and eight $0s$?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.151 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 37 (Page No. 433) top

A student has three mangos, two papayas, and two kiwi fruits. If the student eats one piece of fruit each day, and only the type of fruit matters, in how many different ways can these fruits be consumed?

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Answer key

6.2.152 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 38 (Page No. 433) top

A professor packs her collection of 40 issues of a mathematics journal in four boxes with 10 issues per box. How many ways can she distribute the journals if

A. each box is numbered, so that they are distinguishable?
B. the boxes are identical, so that they cannot be distinguished?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.153 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 39 (Page No. 433) top

How many ways are there to travel in xyz space from the origin $(0,0,0)$ to the point $(4,3,5)$ by taking steps one unit in the positive x direction, one unit in the positive y direction, or one unit in the positive z direction? (Moving in the negative $x, y,$ or z direction is prohibited, so that no backtracking is allowed.)

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Answer key

6.2.154 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 4 (Page No. 432) top

Every day a student randomly chooses a sandwich for lunch from a pile of wrapped sandwiches. If there are six kinds of sandwiches, how many different ways are there for the student to choose sandwiches for the seven days of a week if the order in which the sandwiches are chosen matters?

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Answer key

6.2.155 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 40 (Page No. 433) top

How many ways are there to travel in $xyzw$ space from the origin $(0,0,0,0)$ to the point $(4,3,5,4)$ by taking steps one unit in the positive x, positive y, positive z, or positive w direction?

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6.2.156 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 41 (Page No. 433) top

How many ways are there to deal hands of seven cards to each of five players from a standard deck of 52 cards?

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Answer key

6.2.157 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 42 (Page No. 433) top

In bridge, the 52 cards of a standard deck are dealt to four players. How many different ways are there to deal bridge hands to four players?

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Answer key

6.2.158 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 43 (Page No. 433) top

How many ways are there to deal hands of five cards to each of six players from a deck containing 48 different cards?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.159 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 44 (Page No. 433) top

In how many ways can a dozen books be placed on four distinguishable shelves

A. if the books are indistinguishable copies of the same title?
B. if no two books are the same, and the positions of the books on the shelves matter? [Hint: Break this into 12 tasks, placing each book separately. Start with the sequence $1, 2, 3, 4$ to represent the shelves. Represent the books by $b_i, i = 1, 2, \ldots, 12$. Place b_1 to the right of one of the terms in $1, 2, 3, 4$. Then successively place $b_2, b_3, \ldots,$ and b_{12}.]

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6.2.160 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 45 (Page No. 433) top

How many ways can n books be placed on k distinguishable shelves

A. if the books are indistinguishable copies of the same title?
B. if no two books are the same, and the positions of the books on the shelves matter?

kenneth-rosen discrete-mathematics counting combinatory descriptive

6.2.161 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 46 (Page No. 433) top

A shelf holds 12 books in a row. How many ways are there to choose five books so that no two adjacent books are chosen? [Hint: Represent the books that are chosen by bars and the books not chosen by stars. Count the number of sequences of five bars and seven stars so that no two bars are adjacent.]

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6.2.162 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 47 (Page No. 433) top

Use the product rule to prove Theorem 4, by first placing objects in the first box, then placing objects in the second box, and so on.

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6.2.163 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 48 (Page No. 433) top

Prove Theorem 4 by first setting up a one-to-one correspondence between permutations of n objects with n_i indistinguishable objects of type $i,\ i = 1, 2, 3, \ldots, k$, and the distributions of n objects in k boxes such that n_i objects are placed in box $i,\ i = 1, 2, 3, \ldots, k$ and then applying Theorem 3.

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6.2.164 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 49 (Page No. 433 - 434) top

In this exercise we will prove Theorem 2 by setting up a one-to-one correspondence between the set of r-combinations with repetition allowed of $S = \{1, 2, 3, \ldots, n\}$ and the set of r-combinations of the set $T = \{1, 2, 3, \ldots n + r - 1\}$.

A. Arrange the elements in an r-combination, with repetition allowed, of S into an increasing sequence $x_1 \leq x_2 \leq \cdots \leq x_r$. Show that the sequence formed by adding $k-1$ to the k^{th} term is strictly increasing. Conclude that this sequence is made up of r distinct elements from T.
B. Show that the procedure described in (A) defines a one-to-one correspondence between the set of r-combinations, with repetition allowed, of S and the r-combinations of T. [Hint: Show the correspondence can be reversed by associating to the r-combination $\{x_1, x_2, \ldots, x_r\}$ of T, with $1 \leq x_1 < x_2 < \cdots < x_r \leq n+r-1$, the r-combination with repetition allowed from S, formed by subtracting $k-1$ from the k^{th} element.]
C. Conclude that there are $C(n+r-1, r)$ r-combinations with repetition allowed from a set with n elements.

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6.2.165 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 5 (Page No. 432) top

How many ways are there to assign three jobs to five employees if each employee can be given more than one job?

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Answer key

6.2.166 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 50 (Page No. 434) top

How many ways are there to distribute five distinguishable objects into three indistinguishable boxes?

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6.2.167 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 51 (Page No. 434) top

How many ways are there to distribute six distinguishable objects into four indistinguishable boxes so that each of the boxes contains at least one object?

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6.2.168 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 52 (Page No. 434) top

How many ways are there to put five temporary employees into four identical offices?

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6.2.169 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 53 (Page No. 434) top

How many ways are there to put six temporary employees into four identical offices so that there is at least one temporary employee in each of these four offices?

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6.2.170 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 54 (Page No. 434) top

How many ways are there to distribute five indistinguishable objects into three indistinguishable boxes?

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6.2.171 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 55 (Page No. 434) top

How many ways are there to distribute six indistinguishable objects into four indistinguishable boxes so that each of the boxes contains at least one object?

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6.2.172 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 56 (Page No. 434) top

How many ways are there to pack eight identical DVDs into five indistinguishable boxes so that each box contains at least one DVD?

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6.2.173 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 57 (Page No. 434) top

How many ways are there to pack nine identical DVDs into three indistinguishable boxes so that each box contains at least two DVDs?

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6.2.174 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 58 (Page No. 434) top

How many ways are there to distribute five balls into seven boxes if each box must have at most one ball in it if

A. both the balls and boxes are labeled?
B. the balls are labeled, but the boxes are unlabeled?
C. the balls are unlabeled, but the boxes are labeled?
D. both the balls and boxes are unlabeled?

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6.2.175 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 59 (Page No. 434) top

How many ways are there to distribute five balls into three boxes if each box must have at least one ball in it if

A. both the balls and boxes are labeled?
B. the balls are labeled, but the boxes are unlabeled?
C. the balls are unlabeled, but the boxes are labeled?
D. both the balls and boxes are unlabeled?

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6.2.176 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 6 (Page No. 432) top

How many ways are there to select five unordered elements from a set with three elements when repetition is allowed?

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Answer key

6.2.177 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 60 (Page No. 434) top

Suppose that a basketball league has 32 teams, split into two conferences of 16 teams each. Each conference is split into three divisions. Suppose that the North Central Division has five teams. Each of the teams in the North Central Division plays four games against each of the other teams in this division, three games against each of the 11 remaining teams in the conference, and two games against each of the 16 teams in the other conference. In how many different orders can the games of one of the teams in the North Central Division be scheduled?

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Answer key

6.2.178 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 61 (Page No. 434) top

Suppose that a weapons inspector must inspect each of five different sites twice, visiting one site per day. The inspector is free to select the order in which to visit these sites, but cannot visit site X, the most suspicious site, on two consecutive days. In how many different orders can the inspector visit these sites?

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.179 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 62 (Page No. 434) top

How many different terms are there in the expansion of $(x_1 + x_2 + \cdots + x_m)^n$ after all terms with identical sets of exponents are added?

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6.2.180 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 63 (Page No. 434) top

***63.** Prove the **Multinomial Theorem:** If n is a positive integer, then

$$(x_1 + x_2 + \cdots + x_m)^n = \sum_{n_1+n_2+\cdots+n_m=n} C(n; n_1, n_2, \ldots, n_m) x_1^{n_1} x_2^{n_2} \cdots x_m^{n_m},$$

where

$$C(n; n_1, n_2, \ldots, n_m) = \frac{n!}{n_1!\, n_2! \cdots n_m!}$$

is a **multinomial coefficient**.

Prove the Multinomial Theorem: If n is a positive integer, then $(x_1 + x_2 + \cdots + x_m)^n = \sum\limits_{n_1+n_2+\cdots+n_m=n} C(n : n_1, n_2, \ldots, n_m) x_1^{n_1} x_2^{n_2} \ldots x_m^{n_m}$, where $C(n : n_1, n_2, \ldots, n_m) = \dfrac{n!}{n_1! n_2! \ldots n_m!}$ is a multinomial coefficient.

kenneth-rosen discrete-mathematics counting combinatory proof

6.2.181 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 64 (Page No. 434) top

Find the expansion of $(x + y + z)^4$.

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6.2.182 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 65 (Page No. 434) top

Find the coefficient of $x^3 y^2 z^5$ in $(x + y + z)^{10}$.

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Answer key

6.2.183 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 66 (Page No. 434) top

How many terms are there in the expansion of $(x + y + z)^{100}$?

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Answer key

6.2.184 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 7 (Page No. 432) top

How many ways are there to select three unordered elements from a set with five elements when repetition is allowed?

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Answer key

6.2.185 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 8 (Page No. 432) top

How many different ways are there to choose a dozen donuts from the 21 varieties at a donut shop?

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Answer key

6.2.186 Counting: Kenneth Rosen Edition 7 Exercise 6.5 Question 9 (Page No. 432) top

A bagel shop has onion bagels, poppy seed bagels, egg bagels, salty bagels, pumpernickel bagels, sesame seed bagels, raisin bagels, and plain bagels. How many ways are there to choose

A. six bagels?
B. a dozen bagels?
C. two dozen bagels?
D. a dozen bagels with at least one of each kind?
E. a dozen bagels with at least three egg bagels and no more than two salty bagels?

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Answer key

6.2.187 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 1 (Page No. 438) top

Place these permutations of $\{1,2,3,4,5\}$ in lexicographic order : $43521, 15432, 45321, 23451, 23514, 14532, 21345, 45213, 31452, 31542$.

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Answer key

6.2.188 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 10 (Page No. 438) top

Show that Algorithm 1 produces the next larger permutation in lexicographic order.

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6.2.189 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 11 (Page No. 438) top

Show that Algorithm 3 produces the next larger r-combination in lexicographic order after a given r-combination.

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6.2.190 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 12 (Page No. 438) top

Develop an algorithm for generating the r-permutations of a set of n elements.

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Answer key

6.2.191 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 13 (Page No. 438) top

List all 3-permutations of $\{1,2,3,4,5\}$.

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Answer key

6.2.192 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 14 (Page No. 438) top

The remaining exercises in this section develop another algorithm for generating the permutations of $\{1,2,3,\ldots,n\}$. This algorithm is based on Cantor expansions of integers. Every nonnegative integer less than $n!$ has a unique Cantor expansion $a_{11!}+a_{22!}+\cdots+a_{n-1(n-1)!}$ where ai is a nonnegative integer not exceeding i, for $i=1,2,\ldots n-1$. The integers $a_1,a_2,\ldots,a_{n-1}$ are called the Cantor digits of this integer. Given a permutation of $\{1,2,\ldots,n\}$, let $a_{k-1}, k=2,3,\ldots n$, be the number of integers less than k that follow k in the permutation. For instance, in the permutation $43215, a_1$ is the number of integers less than 2 that follow 2, so $a_1=1$. Similarly, for this example $a_2=2, a_3=3,$ and $a_4=0$. Consider the function from the set of permutations of $\{1,2,3,\ldots,n\}$ to the set of nonnegative integers less than $n!$ that sends a permutation to the integer that has $a_1,a_2,\ldots,a_{n-1}$, defined in this way, as its Cantor digits.

Find the Cantor digits $a_1,a_2,\ldots,a_{n-1}$ that correspond to these permutations.

A. 246531
B. 12345
C. 654321

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6.2.193 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 15 (Page No. 438) top

Show that the correspondence described in the preamble is a bijection between the set of permutations of $\{1,2,3,\ldots,n\}$ and the nonnegative integers less than $n!$.

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6.2.194 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 16 (Page No. 439) top

The remaining exercises in this section develop another algorithm for generating the permutations of $\{1,2,3,\ldots,n\}$. This algorithm is based on Cantor expansions of integers. Every nonnegative integer less than $n!$ has a unique Cantor expansion $a_{11!} + a_{22!} + \cdots + a_{n-1(n-1)!}$ where ai is a nonnegative integer not exceeding i, for $i = 1, 2, \ldots n-1$. The integers $a_1, a_2, \ldots, a_{n-1}$ are called the Cantor digits of this integer. Given a permutation of $\{1, 2, \ldots, n\}$, let $a_{k-1}, k = 2, 3, \ldots n$, be the number of integers less than k that follow k in the permutation. For instance, in the permutation $43215, a_1$ is the number of integers less than 2 that follow 2, so $a_1 = 1$. Similarly, for this example $a_2 = 2, a_3 = 3$, and $a_4 = 0$. Consider the function from the set of permutations of $\{1, 2, 3, \ldots, n\}$ to the set of nonnegative integers less than $n!$ that sends a permutation to the integer that has $a_1, a_2, \ldots, a_{n-1}$, defined in this way, as its Cantor digits.

Find the permutations of $\{1, 2, 3, 4, 5\}$ that correspond to these integers with respect to the correspondence between Cantor expansions and permutations as described in the preamble to question 14.

A. 3
B. 89
C. 111

kenneth-rosen discrete-mathematics counting combinatory descriptive

6.2.195 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 17 (Page No. 438) top

The remaining exercises in this section develop another algorithm for generating the permutations of $\{1, 2, 3, \ldots, n\}$. This algorithm is based on Cantor expansions of integers. Every nonnegative integer less than $n!$ has a unique Cantor expansion $a_{11!} + a_{22!} + \cdots + a_{n-1(n-1)!}$ where ai is a nonnegative integer not exceeding i, for $i = 1, 2, \ldots n-1$. The integers $a_1, a_2, \ldots, a_{n-1}$ are called the Cantor digits of this integer. Given a permutation of $\{1, 2, \ldots, n\}$, let $a_{k-1}, k = 2, 3, \ldots n$, be the number of integers less than k that follow k in the permutation. For instance, in the permutation $43215, a_1$ is the number of integers less than 2 that follow 2, so $a_1 = 1$. Similarly, for this example $a_2 = 2, a_3 = 3$, and $a_4 = 0$. Consider the function from the set of permutations of $\{1, 2, 3, \ldots, n\}$ to the set of nonnegative integers less than $n!$ that sends a permutation to the integer that has $a_1, a_2, \ldots, a_{n-1}$, defined in this way, as its Cantor digits.

Develop an algorithm for producing all permutations of a set of n elements based on the correspondence described in the preamble to question 14.

kenneth-rosen discrete-mathematics counting combinatory descriptive

6.2.196 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 2 (Page No. 438) top

Place these permutations of $\{1, 2, 3, 4, 5, 6\}$ in lexicographic order : $234561, 231456, 165432, 156423, 543216, 541236, 231465, 314562, 432561, 654321, 654312, 435612$.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.197 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 3 (Page No. 438) top

The name of a file in a computer directory consists of three uppercase letters followed by a digit, where each letter is either $A, B,$ or C, and each digit is either 1 or 2. List the name of these files in lexicographic order, where we order letters using the usual alphabetic order of letters.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.198 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 4 (Page No. 438) top

Suppose that the name of a file in a computer directory consists of three digits followed by two lowercase letters and each digit is $0, 1,$ or 2, and each letter is either a or b. List the name of these files in lexicographic order, where we order letters using the usual alphabetic order of letters.

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.199 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 5 (Page No. 438) top

Find the next larger permutation in lexicographic order after each of these permutations.

A. 1432
B. 54123
C. 12453
D. 45231
E. 6714235
F. 31528764

kenneth-rosen discrete-mathematics counting combinatory descriptive

Answer key

6.2.200 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 6 (Page No. 438) top

. Find the next larger permutation in lexicographic order after each of these permutations.

A. 1342
B. 45321
C. 13245
D. 612345
E. 1623547
F. f23587416

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Answer key

6.2.201 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 7 (Page No. 438) top

Use Algorithm 1 to generate the 24 permutations of the first four positive integers in lexicographic order.

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6.2.202 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 8 (Page No. 438) top

Use Algorithm 2 to list all the subsets of the set $\{1, 2, 3, 4\}$.

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6.2.203 Counting: Kenneth Rosen Edition 7 Exercise 6.6 Question 9 (Page No. 438) top

Use Algorithm 3 to list all the 3-combinations of $\{1, 2, 3, 4, 5\}$.

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6.2.204 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 1 (Page No. 510) top

Use mathematical induction to verify the formula derived in Example 2 for the number of moves required to complete the Tower of Hanoi puzzle.

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6.2.205 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 10 (Page No. 511) top

A. Find a recurrence relation for the number of bit strings of length n that contain the string 01.
B. What are the initial conditions?
C. How many bit strings of length seven contain the string 01?

kenneth-rosen discrete-mathematics counting descriptive

Answer key

6.2.206 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 11 (Page No. 511) top

A. Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one stair or two stairs at a time.
B. What are the initial conditions?
C. In how many ways can this person climb a flight of eight stairs?

kenneth-rosen discrete-mathematics counting descriptive

6.2.207 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 12 (Page No. 511) top

A. Find a recurrence relation for the number of ways to climb n stairs if the person climbing the stairs can take one, two, or three stairs at a time.

B. What are the initial conditions?
C. In many ways can this person climb a flight of eight stairs?

kenneth-rosen discrete-mathematics counting descriptive

6.2.208 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 13 (Page No. 511) top

A string that contains only $0s, 1s$, and $2s$ is called a ternary string.

A. Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive $0s.$
B. What are the initial conditions?
C. How many ternary strings of length six do not contain two consecutive $0s$?

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6.2.209 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 14 (Page No. 511) top

A. Find a recurrence relation for the number of ternary strings of length n that contain two consecutive $0s.$
B. What are the initial conditions?
C. How many ternary strings of length six contain two consecutive $0s$?

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6.2.210 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 15 (Page No. 511) top

1. Find a recurrence relation for the number of ternary strings of length n that do not contain two consecutive $0s$ or two consecutive $1s.$
2. What are the initial conditions?
3. How many ternary strings of length six do not contain two consecutive $0s$ or two consecutive $1s$?

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6.2.211 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 16 (Page No. 511) top

A. Find a recurrence relation for the number of ternary strings of length n that contain either two consecutive $0s$ or two consecutive $1s.$
B. What are the initial conditions?
C. How many ternary strings of length six contain two consecutive $0s$ or two consecutive $1s$?

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6.2.212 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 17 (Page No. 511) top

A. Find a recurrence relation for the number of ternary strings of length n that do not contain consecutive symbols that are the same.
B. What are the initial conditions?
C. How many ternary strings of length six do not contain consecutive symbols that are the same?

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6.2.213 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 18 (Page No. 511) top

A. Find a recurrence relation for the number of ternary strings of length n that contain two consecutive symbols that are the same.
B. What are the initial conditions?

C. How many ternary strings of length six contain consecutive symbols that are the same?

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6.2.214 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 19 (Page No. 511) top

Messages are transmitted over a communications channel using two signals. The transmittal of one signal requires 1 microsecond, and the transmittal of the other signal requires 2 microseconds.

A. Find a recurrence relation for the number of different messages consisting of sequences of these two signals, where each signal in the message is immediately followed by the next signal, that can be sent in n microseconds.
B. What are the initial conditions?
C. How many different messages can be sent in 10 microseconds using these two signals?

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6.2.215 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 2 (Page No. 510) top

A. Find a recurrence relation for the number of permutations of a set with n elements.
B. Use this recurrence relation to find the number of permutations of a set with n elements using iteration

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6.2.216 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 20 (Page No. 511) top

A bus driver pays all tolls, using only nickels and dimes, by throwing one coin at a time into the mechanical toll collector.

A. Find a recurrence relation for the number of different ways the bus driver can pay a toll of n cents (where the order in which the coins are used matters).
B. In how many different ways can the driver pay a toll of 45 cents?

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6.2.217 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 21 (Page No. 511) top

A. Find the recurrence relation satisfied by R_n, where R_n is the number of regions that a plane is divided into by n lines, if no two of the lines are parallel and no three of the lines go through the same point.
B. Find R_n using iteration.

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Answer key

6.2.218 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 22 (Page No. 511) top

A. a) Find the recurrence relation satisfied by R_n, where R_n is the number of regions into which the surface of a sphere is divided by n great circles (which are the intersections of the sphere and planes passing through the center of the sphere), if no three of the great circles go through the same point.
B. b) Find R_n using iteration.

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6.2.219 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 23 (Page No. 511) top

A. Find the recurrence relation satisfied by S_n, where S_n is the number of regions into which three-dimensional space is divided by n planes if every three of the planes meet in one point, but no four of the planes go through the same point.

B. Find S_n using iteration.

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6.2.220 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 24 (Page No. 511) top

Find a recurrence relation for the number of bit sequences of length n with an even number of $0s$.

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6.2.221 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 25 (Page No. 511) top

How many bit sequences of length seven contain an even number of $0s$?

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Answer key

6.2.222 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 26 (Page No. 512) top

A. Find a recurrence relation for the number of ways to completely cover a $2 \times n$ checkerboard with 1×2 dominoes. [Hint: Consider separately the coverings where the position in the top right corner of the checkerboard is covered by a domino positioned horizontally and where it is covered by a domino positioned vertically.]
B. What are the initial conditions for the recurrence relation in part (A)?
C. How many ways are there to completely cover a 2×17 checkerboard with 1×2 dominoes?

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6.2.223 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 27 (Page No. 512) top

A. Find a recurrence relation for the number of ways to lay out a walkway with slate tiles if the tiles are red, green, or gray, so that no two red tiles are adjacent and tiles of the same color are considered indistinguishable.
B. What are the initial conditions for the recurrence relation in part (A)?
C. How many ways are there to lay out a path of seven tiles as described in part (A)?

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6.2.224 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 28 (Page No. 512) top

Show that the Fibonacci numbers satisfy the recurrence relation $f_n = 5f_{n-4} + 3f_{n-5}$ for $n = 5, 6, 7, \dots,$ together with the initial conditions $f_0 = 0, f_1 = 1, f_2 = 1, f_3 = 2,$ and $f4 = 3.$ Use this recurrence relation to show that f_{5n} is divisible by 5, for $n = 1, 2, 3, \dots.$

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6.2.225 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 29 (Page No. 512) top

Let $S(m, n)$ denote the number of onto functions from a set with m elements to a set with n elements. Show that $S(m, n)$ satisfies the recurrence relation

$$S(m, n) = n^m - \sum_{k=1}^{n-1} C(n, k) S(m, k)$$

whenever $m \geq n$ and $n > 1$, with the initial condition $S(m, 1) = 1$.

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6.2.226 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 3 (Page No. 510) top

A vending machine dispensing books of stamps accepts only one-dollar coins, $\$1$ bills, and $\$5$ bills.

A. Find a recurrence relation for the number of ways to deposit n dollars in the vending machine, where the order in which the coins and bills are deposited matters.
B. What are the initial conditions?
C. How many ways are there to deposit $\$10$ for a book of stamps?

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6.2.227 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 30 (Page No. 512) top

A. Write out all the ways the product $x_0 \cdot x_1 \cdot x_2 \cdot x_3 \cdot x_4$ can be parenthesized to determine the order of multiplication.
B. Use the recurrence relation developed in Example 5 to calculate C_4, the number of ways to parenthesize the product of five numbers so as to determine the order of multiplication. Verify that you listed the correct number of ways in part (A).
C. Check your result in part (B) by finding C_4, using the closed formula for C_n mentioned in the solution of Example 5.

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6.2.228 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 31 (Page No. 512) top

A. Use the recurrence relation developed in Example 5 to determine C_5, the number of ways to parenthesize the product of six numbers so as to determine the order of multiplication.
B. Check your result with the closed formula for C_5 mentioned in the solution of Example 5.

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6.2.229 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 32 (Page No. 512) top

In the Tower of Hanoi puzzle, suppose our goal is to transfer all n disks from peg 1 to peg 3, but we cannot move a disk directly between pegs 1 and 3. Each move of a disk must be a move involving peg 2. As usual, we cannot place a disk on top of a smaller disk.

A. Find a recurrence relation for the number of moves required to solve the puzzle for n disks with this added restriction.
B. Solve this recurrence relation to find a formula for the number of moves required to solve the puzzle for n disks.
C. How many different arrangements are there of the n disks on three pegs so that no disk is on top of a smaller disk?
D. Show that every allowable arrangement of the n disks occurs in the solution of this variation of the puzzle.

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6.2.230 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 33 (Page No. 512) top

Question $33-37$ deal with a variation of the **Josephus problem** described by Graham, Knuth, and Patashnik in $[G_r K_n P_a 94]$. This problem is based on an account by the historian Flavius Josephus, who was part of a band of 41 Jewish rebels trapped in a cave by the Romans during the Jewish Roman war of the first century. The rebels preferred suicide to capture; they decided to form a circle and to repeatedly count off around the circle, killing every third rebel left alive. However, Josephus and another rebel did not want to be killed this way; they determined the positions where they should stand to be the last two rebels remaining alive. The variation we consider begins with n people, numbered 1 to n, standing around a circle. In each stage, every second person still left alive is eliminated until only one survives. We denote the number of the survivor by $J(n)$.

Determine the value of $J(n)$ for each integer n with $1 \leq n \leq 16$.

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6.2.231 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 34 (Page No. 512) top

Question $33-37$ deal with a variation of the **Josephus problem** described by Graham, Knuth, and Patashnik in $[G_r K_n P_a 94]$. This problem is based on an account by the historian Flavius Josephus, who was part of a band of 41 Jewish rebels trapped in a cave by the Romans during the Jewish Roman war of the first century. The rebels preferred suicide to capture; they decided to form a circle and to repeatedly count off around the circle, killing every third rebel left alive. However, Josephus and another rebel did not want to be killed this way; they determined the positions where they should stand to be the last two rebels remaining alive. The variation we consider begins with n people, numbered 1 to n, standing around a

circle. In each stage, every second person still left alive is eliminated until only one survives. We denote the number of the survivor by $J(n)$.

Use the values you found in question 33 to conjecture a formula for $J(n)$. [Hint: Write $n = 2^m + k$, where m is a nonnegative integer and k is a nonnegative integer less than $2m.$]

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6.2.232 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 35 (Page No. 512) top

Question $33-37$ deal with a variation of the $\textbf{Josephus problem}$ described by Graham, Knuth, and Patashnik in $[G_rK_nP_a94]$. This problem is based on an account by the historian Flavius Josephus, who was part of a band of 41 Jewish rebels trapped in a cave by the Romans during the Jewish Roman war of the first century. The rebels preferred suicide to capture; they decided to form a circle and to repeatedly count off around the circle, killing every third rebel left alive. However, Josephus and another rebel did not want to be killed this way; they determined the positions where they should stand to be the last two rebels remaining alive. The variation we consider begins with n people, numbered 1 to n, standing around a circle. In each stage, every second person still left alive is eliminated until only one survives. We denote the number of the survivor by $J(n)$.

Show that $J(n)$ satisfies the recurrence relation $J(2n) = 2J(n) - 1$ and $J(2n + 1) = 2J(n) + 1,$ for $n \geq 1,$ and $J(1) = 1.$

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6.2.233 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 36 (Page No. 512) top

Question $33-37$ deal with a variation of the $\textbf{Josephus problem}$ described by Graham, Knuth, and Patashnik in $[G_rK_nP_a94]$. This problem is based on an account by the historian Flavius Josephus, who was part of a band of 41 Jewish rebels trapped in a cave by the Romans during the Jewish Roman war of the first century. The rebels preferred suicide to capture; they decided to form a circle and to repeatedly count off around the circle, killing every third rebel left alive. However, Josephus and another rebel did not want to be killed this way; they determined the positions where they should stand to be the last two rebels remaining alive. The variation we consider begins with n people, numbered 1 to n, standing around a circle. In each stage, every second person still left alive is eliminated until only one survives. We denote the number of the survivor by $J(n)$.

Use mathematical induction to prove the formula you conjectured in question 34, making use of the recurrence relation from question 35.

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6.2.234 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 37 (Page No. 512) top

Question $33-37$ deal with a variation of the $\textbf{Josephus problem}$ described by Graham, Knuth, and Patashnik in $[G_rK_nP_a94]$. This problem is based on an account by the historian Flavius Josephus, who was part of a band of 41 Jewish rebels trapped in a cave by the Romans during the Jewish Roman war of the first century. The rebels preferred suicide to capture; they decided to form a circle and to repeatedly count off around the circle, killing every third rebel left alive. However, Josephus and another rebel did not want to be killed this way; they determined the positions where they should stand to be the last two rebels remaining alive. The variation we consider begins with n people, numbered 1 to n, standing around a circle. In each stage, every second person still left alive is eliminated until only one survives. We denote the number of the survivor by $J(n)$.

Determine $J(100), J(1000),$ and $J(10,000)$ from your formula for $J(n)$.

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6.2.235 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 38 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n.$ When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1,$ the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n - k$ smallest disks from peg 1 to peg $2,$ using all four pegs. Next move the stack of the k largest disks from peg 1 to peg $4,$ using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n - k$ smallest disks. Finally, recursively move the smallest $n - k$ disks to peg $4,$ using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k + 1)/2,$ the k^{th} triangular number, that is,

$t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that the Reve's puzzle with three disks can be solved using five, and no fewer, moves.

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6.2.236 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 39 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that the Reve's puzzle with four disks can be solved using nine, and no fewer, moves

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6.2.237 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 4 (Page No. 510) top

A country uses as currency coins with values of 1 peso, 2 pesos, 5 pesos, and 10 pesos and bills with values of 5 pesos, 10 pesos, 20 pesos, 50 pesos, and 100 pesos. Find a recurrence relation for the number of ways to pay a bill of n pesos if the order in which the coins and bills are paid matters.

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6.2.238 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 40 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Describe the moves made by the Frame–Stewart algorithm, with k chosen so that the fewest moves are required, for

A. 5 disks.
B. 6 disks.
C. 7 disks.
D. 8 disks.

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6.2.239 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 41 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that if $R(n)$ is the number of moves used by the Frame–Stewart algorithm to solve the Reve's puzzle with n disks, where k is chosen to be the smallest integer with $n \leq k(k+1)/2$, then $R(n)$ satisfies the recurrence relation $R(n) = 2R(n-k) + 2^k - 1$, with $R(0) = 0$ and $R(1) = 1$.

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6.2.240 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 42 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that if k is as chosen in question 41, then $R(n) - R(n-1) = 2^{k-1}$.

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6.2.241 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 43 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that if k is as chosen in question 41, then $R(n) = \sum\limits_{i=1}^{k} i2^{i-1} - (t_k - n)2^{k-1}$.

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6.2.242 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 44 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks. Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n$. When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n-k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n-k$ smallest disks. Finally, recursively move the smallest $n-k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Use question 43 to give an upper bound on the number of moves required to solve the Reve's puzzle for all integers n with $1 \leq n \leq 25$.

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6.2.243 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 45 (Page No. 512) top

Question $38-45$ involve the Reve's puzzle, the variation of the Tower of Hanoi puzzle with four pegs and n disks.

Before presenting these exercises, we describe the Frame–Stewart algorithm for moving the disks from peg 1 to peg 4 so that no disk is ever on top of a smaller one. This algorithm, given the number of disks n as input, depends on a choice of an integer k with $1 \leq k \leq n.$ When there is only one disk, move it from peg 1 to peg 4 and stop. For $n > 1$, the algorithm proceeds recursively, using these three steps. Recursively move the stack of the $n - k$ smallest disks from peg 1 to peg 2, using all four pegs. Next move the stack of the k largest disks from peg 1 to peg 4, using the three-peg algorithm from the Tower of Hanoi puzzle without using the peg holding the $n - k$ smallest disks. Finally, recursively move the smallest $n - k$ disks to peg 4, using all four pegs. Frame and Stewart showed that to produce the fewest moves using their algorithm, k should be chosen to be the smallest integer such that n does not exceed $t_k = k(k+1)/2$, the k^{th} triangular number, that is, $t_{k-1} < n \leq t_k$. The unsettled conjecture, known as **Frame's conjecture,** is that this algorithm uses the fewest number of moves required to solve the puzzle, no matter how the disks are moved.

Show that $R(n)$ is $O(\sqrt{n}2^{\sqrt{2n}})$.

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6.2.244 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 46 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Find ∇a_n for the sequence $\{a_n\}$, where

A. $a_n = 4.$ B. $a_n = 2n.$ C. $a_n = n^2.$ D. $a_n = 2^n.$

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6.2.245 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 47 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Find $\nabla^2 a_n$ for the sequence $\{a_n\}$, where

A. $a_n = 4.$ B. $a_n = 2n.$ C. $a_n = n^2.$ D. $a_n = 2^n.$

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6.2.246 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 48 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Show that $a_{n-1} = a_n - \nabla a_n$.

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6.2.247 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 49 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Show that $a_{n-2} = a_n - 2\nabla a_n + \nabla^2 a_n$.

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6.2.248 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 5 (Page No. 510) top

How many ways are there to pay a bill of 17 pesos using the currency described in question 4, where the order in which coins and bills are paid matters?

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6.2.249 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 50 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Prove that a_{n-k} can be expressed in terms of $a_n, \nabla a_n, \nabla^2 a_n, \dots, \nabla^k a_n$.

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6.2.250 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 51 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1} a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Express the recurrence relation $a_n = a_{n-1} + a_{n-2}$ in terms of $a_n, \nabla a_n,$ and $\nabla^2 a_n$.

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6.2.251 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 52 (Page No. 512) top

Let $\{a_n\}$ be a sequence of real numbers. The backward differences of this sequence are defined recursively as shown next. The first difference ∇a_n is

$$\nabla a_n = a_n - a_{n-1}.$$

The $(k+1)^{\text{st}}$ difference $\nabla^{k+1} a_n$ is obtained from $\nabla^k a_n$ by

$$\nabla^{k+1}a_n = \nabla^k a_n - \nabla^k a_{n-1}.$$

Show that any recurrence relation for the sequence $\{a_n\}$ can be written in terms of $a_n, \nabla a_n, \nabla^2 a_n, \ldots$ The resulting equation involving the sequences and its differences is called a difference equation.

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6.2.252 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 53 (Page No. 512) top

Construct the algorithm described in the text after Algorithm 1 for determining which talks should be scheduled to maximize the total number of attendees and not just the maximum total number of attendees determined by Algorithm 1.

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6.2.253 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 54 (Page No. 512) top

Use Algorithm 1 to determine the maximum number of total attendees in the talks in Example 6 if w_i, the number of attendees of talk $i, i = 1, 2, \ldots, 7,$ is

A. $20, 10, 50, 30, 15, 25, 40.$
B. $100, 5, 10, 20, 25, 40, 30.$
C. $2, 3, 8, 5, 4, 7, 10.$
D. $10, 8, 7, 25, 20, 30, 5.$

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6.2.254 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 55 (Page No. 512) top

For each part of question 54, use your algorithm from question 53 to find the optimal schedule for talks so that the total number of attendees is maximized.

A. $20, 10, 50, 30, 15, 25, 40.$
B. $100, 5, 10, 20, 25, 40, 30.$
C. $2, 3, 8, 5, 4, 7, 10.$
D. $10, 8, 7, 25, 20, 30, 5.$

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6.2.255 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 56 (Page No. 512) top

In this question, we will develop a dynamic programming algorithm for finding the maximum sum of consecutive terms of a sequence of real numbers. That is, given a sequence of real numbers $a_1, a_2, \ldots, a_n$, the algorithm computes the maximum sum $\displaystyle\sum_{i=j}^{k} a_i$ where $1 \leq j \leq k \leq n.$

A. Show that if all terms of the sequence are nonnegative, this problem is solved by taking the sum of all terms. Then, give an example where the maximum sum of consecutive terms is not the sum of all terms.
B. Let $M(k)$ be the maximum of the sums of consecutive terms of the sequence ending at a_k. That is, $M(k) = \max_{1 \leq j \leq k} \displaystyle\sum_{i=j}^{k} a_i$. Explain why the recurrence relation $M(k) = \max(M(k-1) + a_k, a_k)$ holds for $k = 2, \ldots, n.$
C. Use part (B) to develop a dynamic programming algorithm for solving this problem.
D. Show each step your algorithm from part (C) uses to find the maximum sum of consecutive terms of the sequence $2, -3, 4, 1, -2, 3.$
E. Show that the worst-case complexity in terms of the number of additions and comparisons of your algorithm from part (C) is linear.

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6.2.256 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 57 (Page No. 512) top

Dynamic programming can be used to develop an algorithm for solving the matrix-chain multiplication problem introduced in Section 3.3. This is the problem of determining how the product $A_1A_2 \ldots A_n$ can be computed using the fewest integer multiplications, where $A_1, A_2, \ldots, A_n$ are $m_1 \times m_2, m_2 \times m_3, \ldots, m_n \times m_{n+1}$ matrices, respectively, and each matrix has integer entries. Recall that by the associative law, the product does not depend on the order in which the matrices are multiplied.

A. Show that the brute-force method of determining the minimum number of integer multiplications needed to solve a matrix-chain multiplication problem has exponential worst-case complexity. [Hint: Do this by first showing that the order of multiplication of matrices is specified by parenthesizing the product. Then, use Example 5 and the result of part (A) of question 41 in Section $8.4.$]

B. Denote by A_{ij} the product $A_i A_{i+1} \ldots, A_j,$ and $M(i,j)$ the minimum number of integer multiplications required to find A_{ij}. Show that if the least number of integer multiplications are used to compute A_{ij}, where $i < j$, by splitting the product into the product of A_i through A_k and the product of A_{k+1} through A_j, then the first k terms must be parenthesized so that A_{ik} is computed in the optimal way using $M(i,k)$ integer multiplications and $A_{k+1,j}$ must be parenthesized so that $A_{k+1,j}$ is computed in the optimal way using $M(k+1,j)$ integer multiplications.

C. Explain why part (B) leads to the recurrence relation $M(i,j) = \min_{i \leq k < j}(M(i,k) + M(k+1,j) + m_i m_{k+1} m_{j+1})$ if $1 \leq i \leq j < j \leq n.$

D. Use the recurrence relation in part (C) to construct an efficient algorithm for determining the order the n matrices should be multiplied to use the minimum number of integer multiplications. Store the partial results $M(i,j)$ as you find them so that your algorithm will not have exponential complexity.

E. Show that your algorithm from part (D) has $O(n^3)$ worst-case complexity in terms of multiplications of integers.

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6.2.257 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 6 (Page No. 510) top

A. Find a recurrence relation for the number of strictly increasing sequences of positive integers that have 1 as their first term and n as their last term, where n is a positive integer. That is, sequences $a_1, a_2, \ldots, a_k,$ where $a_1 = 1, a_k = n,$ and $a_j < a_{j+1}$ for $j = 1, 2, \ldots, k-1.$

B. What are the initial conditions?

C. How many sequences of the type described in (A) are there when n is an integer with $n \geq 2$?

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6.2.258 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 7 (Page No. 510 - 511) top

A. Find a recurrence relation for the number of bit strings of length n that contain a pair of consecutive $0s$.

B. What are the initial conditions?

C. How many bit strings of length seven contain two consecutive $0s$?

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6.2.259 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 8 (Page No. 511) top

A. Find a recurrence relation for the number of bit strings of length n that contain three consecutive $0s.$

B. What are the initial conditions?

C. How many bit strings of length seven contain three consecutive $0s$?

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6.2.260 Counting: Kenneth Rosen Edition 7 Exercise 8.1 Question 9 (Page No. 511) top

A. Find a recurrence relation for the number of bit strings of length n that do not contain three consecutive $0s.$

B. What are the initial conditions?

C. How many bit strings of length seven do not contain three consecutive $0s$?

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6.2.261 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 1 (Page No. 524) top

Determine which of these are linear homogeneous recurrence relations with constant coefficients. Also, find the degree of those that are.

A. $a_n = 3a_{n-1} + 4a_{n-2} + 5a_{n-3}$
B. $a_n = 2na_{n-1} + a_{n-2}$
C. $a_n = a_{n-1} + a_{n-4}$
D. $a_n = a_{n-1} + 2$
E. $a_n = a_{n-1}^2 + a_{n-2}$
F. $a_n = a_{n-2}$
G. $a_n = a_{n-1} + n$

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Answer key

6.2.262 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 10 (Page No. 525) top

Prove Theorem 2 : Let c_1 and c_2 be real numbers with $c_2 \neq 0$. Suppose that $r^2 - c_1 r - c_2 = 0$ has only one root r_0. A sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = c_1 a_{n-1} + c_2 a_{n-2}$ if and only if $a_n = \alpha_1 r_0^n + \alpha_2 n r_0^n$, for $n = 0, 1, 2, \ldots,$ where α_1 and α_2 are constants.

kenneth-rosen discrete-mathematics counting recurrence-relation proof

6.2.263 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 11 (Page No. 525) top

The Lucas numbers satisfy the recurrence relation $L_n = L_{n-1} + L_{n-2},$ and the initial conditions $L_0 = 2$ and $L_1 = 1$.

A. Show that $L_n = f_{n-1} + f_{n+1}$ for $n = 2, 3, \ldots,$ where fn is the n^{th} Fibonacci number.
B. Find an explicit formula for the Lucas numbers.

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6.2.264 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 12 (Page No. 525) top

Find the solution to $a_n = 2a_{n-1} + a_{n-2} - 2a_{n-3}$ for $n = 3, 4, 5, \ldots,$ with $a_0 = 3, a_1 = 6,$ and $a_2 = 0.$

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Answer key

6.2.265 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 13 (Page No. 525) top

Find the solution to $a_n = 7a_{n-2} + 6a_{n-3}$ with $a_0 = 9, a_1 = 10,$ and $a_2 = 32.$

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Answer key

6.2.266 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 14 (Page No. 525) top

Find the solution to $a_n = 5a_{n-2} - 4a_{n-4}$ with $a_0 = 3, a_1 = 2, a_2 = 6,$ and $a_3 = 8.$

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Answer key

6.2.267 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 15 (Page No. 525) top

Find the solution to $a_n = 2a_{n-1} + 5a_{n-2} - 6a_{n-3}$ with $a_0 = 7, a_1 = -4,$ and $a_2 = 8.$

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Answer key

6.2.268 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 16 (Page No. 525) top

Prove Theorem 3 :
Let $c_1, c_2, \ldots, c_k$ be real numbers. Suppose that the characteristic equation

$$r^k - c_1 r^{k-1} - \ldots - c_k = 0$$

has k distinct roots $r_1, r_2, \ldots r_k$. Then a sequence $\{a_n\}$ is a solution of the recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

if and only if

$$a_n = \alpha_1 r_1^n + \alpha_2 r_2^n + \cdots + \alpha_k r_k^n$$

for $n = 0, 1, 2, \ldots,$ where $\alpha_1, \alpha_2, \ldots, \alpha_k$ are constants.

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6.2.269 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 17 (Page No. 525) top

Prove this identity relating the Fibonacci numbers and the binomial coefficients: $f_{n+1} = C(n,0) + C(n-1,1) + \cdots + C(n-k,k),$ where n is a positive integer and $k = n/2.$ [Hint: Let $a_n = C(n,0) + C(n-1,1) + \cdots + C(n-k,k)$. Show that the sequence $\{a_n\}$ satisfies the same recurrence relation and initial conditions satisfied by the sequence of Fibonacci numbers.]

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6.2.270 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 18 (Page No. 525) top

Solve the recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3}$ with $a_0 = -5, a_1 = 4,$ and $a_2 = 88.$

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Answer key

6.2.271 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 19 (Page No. 525) top

Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5, a_1 = -9,$ and $a_2 = 15.$

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Answer key

6.2.272 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 2 (Page No. 524) top

Determine which of these are linear homogeneous recurrence relations with constant coefficients. Also, find the degree of those that are.

A. $a_n = 3a_{n-2}$
B. $a_n = 3$
C. $a_n = a_{n-1}^2$
D. $an = a_{n-1} + 2a_{n-3}$
E. $an = a_{n-1}/n$
F. $an = a_{n-1} + a_{n-2} + n + 3$
G. $a_n = 4a_{n-2} + 5a_{n-4} + 9a_{n-7}$

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Answer key

6.2.273 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 20 (Page No. 525) top

Find the general form of the solutions of the recurrence relation $a_n = 8a_{n-2} - 16a_{n-4}$.

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Answer key

6.2.274 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 21 (Page No. 525) top

What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has roots $1, 1, 1, 1, -2, -2, -2, 3, 3, -4$?

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Answer key

6.2.275 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 22 (Page No. 525) top

What is the general form of the solutions of a linear homogeneous recurrence relation if its characteristic equation has the roots $-1, -1, -1, 2, 2, 5, 5, 7$?

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Answer key

6.2.276 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 23 (Page No. 525) top

Consider the nonhomogeneous linear recurrence relation $a_n = 3a_{n-1} + 2^n$.

A. Show that $a_n = -2^{n+1}$ is a solution of this recurrence relation.
B. Use Theorem 5 to find all solutions of this recurrence relation.
C. Find the solution with $a_0 = 1$.

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6.2.277 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 24 (Page No. 525) top

Consider the nonhomogeneous linear recurrence relation $a_n = 2a_{n-1} + 2^n$.

1. Show that $a_n = n2^n$ is a solution of this recurrence relation.
2. Use Theorem 5 to find all solutions of this recurrence relation.
3. Find the solution with $a_0 = 2$.

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6.2.278 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 25 (Page No. 525) top

A. Determine values of the constants A and B such that $a_n = An + B$ is a solution of recurrence relation $a_n = 2a_{n-1} + n + 5$.
B. Use Theorem 5 to find all solutions of this recurrence relation.
C. Find the solution of this recurrence relation with $a_0 = 4$.

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6.2.279 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 26 (Page No. 525) top

What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 6a_{n-1} - 12a_{n-2} + 8a_{n-3} + F(n)$ if

A. $F(n) = n^2$?
B. $F(n) = 2^n$?
C. $F(n) = n2^n$?
D. $F(n) = (-2)^n$?
E. $F(n) = n^2 2^n$?
F. $F(n) = n^3(-2)^n$?
G. $F(n) = 3$?

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6.2.280 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 27 (Page No. 525) top

What is the general form of the particular solution guaranteed to exist by Theorem 6 of the linear nonhomogeneous recurrence relation $a_n = 8a_{n-2} - 16a_{n-4} + F(n)$ if

A. $F(n) = n^3$?
B. $F(n) = (-2)^n$?
C. $F(n) = n2^n$?
D. $F(n) = n^2 4^n$?
E. $F(n) = (n^2 - 2)(-2)^n$?
F. $F(n) = n^4 2^n$?
G. $F(n) = 2$?

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6.2.281 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 28 (Page No. 525) top

A. Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 2n^2$.
B. Find the solution of the recurrence relation in part (A) with initial condition $a_1 = 4$.

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6.2.282 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 29 (Page No. 525) top

A. Find all solutions of the recurrence relation $a_n = 2a_{n-1} + 3n$.
B. Find the solution of the recurrence relation in part (A) with initial condition $a_1 = 5$.

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6.2.283 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 3 (Page No. 524) top

Solve these recurrence relations together with the initial conditions given.

A. $a_n = 2a_{n-1}$ for $n \geq 1, a_0 = 3$
B. $a_n = a_{n-1}$ for $n \geq 1, a_0 = 2$
C. $a_n = 5a_{n-1} - 6a_{n-2}$ for $n \geq 2, a_0 = 1, a_1 = 0$
D. $a_n = 4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 6, a_1 = 8$
E. $a_n = -4a_{n-1} - 4a_{n-2}$ for $n \geq 2, a_0 = 0, a_1 = 1$
F. $a_n = 4a_{n-2}$ for $n \geq 2, a_0 = 0, a_1 = 4$
G. $a_n = a_{n-2}/4$ for $n \geq 2, a_0 = 1, a_1 = 0$

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Answer key

6.2.284 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 30 (Page No. 525) top

A. Find all solutions of the recurrence relation $a_n = -5a_{n-1} - 6a_{n-2} + 42 \cdot 4^n$.
B. Find the solution of this recurrence relation with $a_1 = 56$ and $a_2 = 278$.

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6.2.285 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 31 (Page No. 525) top

Find all solutions of the recurrence relation $a_n = 5a_{n-1} - 6a_{n-2} + 2^n + 3n$. [Hint: Look for a particular solution of the form $qn2^n + p_1n + p_2$, where q, p_1, and p_2 are constants.]

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6.2.286 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 32 (Page No. 525) top

Find the solution of the recurrence relation $a_n = 2a_{n-1} + 3 \cdot 2^n$.

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6.2.287 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 33 (Page No. 525) top

Find all solutions of the recurrence relation $a_n = 4a_{n-1} - 4a_{n-2} + (n+1)2^n$.

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Answer key

6.2.288 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 34 (Page No. 526) top

Find all solutions of the recurrence relation $a_n = 7a_{n-1} - 16a_{n-2} + 12a_{n-3} + n4^n$ with $a_0 = -2, a_1 = 0$, and $a_2 = 5$.

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6.2.289 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 35 (Page No. 526) top

Find the solution of the recurrence relation $a_n = 4a_{n-1} - 3a_{n-2} + 2^n + n + 3$ with $a_0 = 1$ and $a_1 = 4$.

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Answer key

6.2.290 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 36 (Page No. 526) top

Let an be the sum of the first n perfect squares, that is, $a_n = \sum_{k=1}^{n} k^2$. Show that the sequence $\{a_n\}$ satisfies the linear nonhomogeneous recurrence relation $a_n = a_{n-1} + n^2$ and the initial condition $a_1 = 1$. Use Theorem 6 to determine a formula for a_n by solving this recurrence relation.

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6.2.291 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 37 (Page No. 526) top

Let an be the sum of the first n triangular numbers, that is,
$a_n = \sum_{k=1}^{n} t_k$, where $t_k = k(k+1)/2$. Show that $\{an\}$ satisfies the linear nonhomogeneous recurrence relation $a_n = a_{n-1} + n(n+1)/2$ and the initial condition $a_1 = 1$.
Use Theorem 6 to determine a formula for a_n by solving this recurrence relation.

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6.2.292 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 38 (Page No. 526) top

A. Find the characteristic roots of the linear homogeneous recurrence relation $a_n = 2a_{n-1} - 2a_{n-2}$. [Note: These are complex numbers.]
B. Find the solution of the recurrence relation in part (A) with $a_0 = 1$ and $a_1 = 2$.

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Answer key

6.2.293 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 39 (Page No. 526) top

A. a) Find the characteristic roots of the linear homogeneous recurrence relation $a_n = a_{n-4}$. [Note: These include complex numbers.]
B. Find the solution of the recurrence relation in part (A) with $a_0 = 1, a_1 = 0, a_2 = -1,$ and $a_3 = 1$.

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6.2.294 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 4 (Page No. 524) top

Solve these recurrence relations together with the initial conditions given.

A. $a_n = a_{n-1} + 6a_{n-2}$ for $n \geq 2, a_0 = 3, a_1 = 6$
B. $a_n = 7a_{n-1} - 10a_{n-2}$ for $n \geq 2, a_0 = 2, a_1 = 1$
C. $a_n = 6a_{n-1} - 8a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 10$
D. $a_n = 2a_{n-1} - a_{n-2}$ for $n \geq 2, a_0 = 4, a_1 = 1$
E. $a_n = a_{n-2}$ for $n \geq 2, a_0 = 5, a_1 = -1$
F. $a_n = -6a_{n-1} - 9a_{n-2}$ for $n \geq 2, a_0 = 3, a_1 = -3$
G. $a_{n+2} = -4a_{n+1} + 5a_n$ for $n \geq 0, a_0 = 2, a_1 = 8$

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6.2.295 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 40 (Page No. 526) top

Solve the simultaneous recurrence relations

- $a_n = 3a_{n-1} + 2b_{n-1}$
- $b_n = a_{n-1} + 2b_{n-1}$

with $a_0 = 1$ and $b_0 = 2$.

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Answer key

6.2.296 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 41 (Page No. 526) top

A. Use the formula found in Example 4 for f_n, the n^{th} Fibonacci number, to show that fn is the integer closest to

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

B. Determine for which n f_n is greater than

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n$$

and for which n f_n is less than

$$\frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n.$$

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6.2.297 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 42 (Page No. 526) top

Show that if $a_n = a_{n-1} + a_{n-2}, a_0 = s$ and $a_1 = t,$ where s and t are constants, then $a_n = sf_{n-1} + tf_n$ for all positive integers n.

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6.2.298 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 43 (Page No. 526) top

Express the solution of the linear nonhomogenous recurrence relation $a_n = a_{n-1} + a_{n-2} + 1$ for $n \geq 2$ where $a_0 = 0$ and $a_1 = 1$ in terms of the Fibonacci numbers. [Hint: Let $b_n = a_{n+1}$ and apply question 42 to the sequence b_n.]

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6.2.299 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 44 (Page No. 526) top

(Linear algebra required) Let A_n be the $n \times n$ matrix with $2s$ on its main diagonal, $1s$ in all positions next to a diagonal element, and $0s$ everywhere else. Find a recurrence relation for d_n, the determinant of A_n. Solve this recurrence relation to find a formula for d_n.

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6.2.300 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 45 (Page No. 526) top

Suppose that each pair of a genetically engineered species of rabbits left on an island produces two new pairs of rabbits at the age of 1 month and six new pairs of rabbits at the age of 2 months and every month afterward. None of the rabbits ever die or leave the island.

A. Find a recurrence relation for the number of pairs of rabbits on the island n months after one newborn pair is left on the island.
B. By solving the recurrence relation in (A) determine the number of pairs of rabbits on the island n months after one pair is left on the island.

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6.2.301 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 46 (Page No. 526) top

Suppose that there are two goats on an island initially.The number of goats on the island doubles every year by natural reproduction, and some goats are either added or removed each year.

A. Construct a recurrence relation for the number of goats on the island at the start of the n^{th} year, assuming that during each year an extra 100 goats are put on the island.
B. Solve the recurrence relation from part (A) to find the number of goats on the island at the start of the n^{th} year.
C. Construct a recurrence relation for the number of goats on the island at the start of the n^{th} year, assuming that n goats are removed during the n^{th} year for each $n \geq 3$.

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6.2.302 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 47 (Page No. 526) top

A new employee at an exciting new software company starts with a salary of $\$50,000$ and is promised that at the end of each year her salary will be double her salary of the previous year, with an extra increment of $\$10,000$ for each year she has been with the company.

A. Construct a recurrence relation for her salary for her n^{th} year of employment.
B. Solve this recurrence relation to find her salary for her n^{th} year of employment.

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6.2.303 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 48 (Page No. 526) top

Some linear recurrence relations that do not have constant coefficients can be systematically solved. This is the case for recurrence relations of the form $f(n)a_n = g(n)a_{n-1} + h(n)$. Exercises 48– 50 illustrate this.

A. Show that the recurrence relation $f(n)a_n = g(n)a_{n-1} + h(n)$, for $n \geq 1$, and with $a_0 = C$, can be reduced to a recurrence relation of the form $b_n = b_{n-1} + Q(n)h(n)$, where $b_n = g(n+1)Q(n+1)a_n$, with $Q(n) = \dfrac{(f(1)f(2)\dots f(n-1))}{(g(1)g(2)\dots g(n))}$.
B. Use part (A) to solve the original recurrence relation to obtain $a_n = \dfrac{C + \sum_{i=1}^{n} Q(i)h(i)}{g(n+1)Q(n+1)}$

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6.2.304 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 49 (Page No. 527) top

Use question 48 to solve the recurrence relation $(n+1)a_n = (n+3)a_{n-1} + n$, for $n \geq 1$, with $a_0 = 1$

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6.2.305 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 5 (Page No. 524) top

How many different messages can be transmitted in n microseconds using the two signals described in question 19 in Section 8.1?

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6.2.306 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 50 (Page No. 527) top

It can be shown that Cn, the average number of comparisons made by the quick sort algorithm (described in preamble to question 50 in exercise 5.4), when sorting n elements in random order, satisfies the recurrence relation

$$C_n = 1 + n + \frac{2}{n}\sum_{k=0}^{n-1} C_k$$

for $n = 1, 2, \ldots,$ with initial condition $C_0 = 0$.

A. Show that $\{C_n\}$ also satisfies the recurrence relation $nC_n = (n+1)C_{n-1} + 2n$ for $n = 1, 2, \ldots$
B. Use question 48 to solve the recurrence relation in part (A) to find an explicit formula for C_n.

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6.2.307 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 51 (Page No. 527) top

Prove Theorem 4 : Let $c_1, c_2, \ldots, c_k$ be real numbers. Suppose that the characteristic equation

$$r^k - c_1 r^{k-1} - \ldots c_k = 0$$

has t distinct roots $r_1, r_2, \ldots, r_t$ with multiplicities $m_1, m_2, \ldots, m_t$, respectively, so that $m_i \geq 1$ for $i = 1, 2, \ldots, t$ and $m_1 + m_2 + \cdots + m_t = k$. Then a sequence $\{a_n\}$ is a solution of the recurrence relation.

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k}$$

if and only if

$$a_n = (\alpha_1, 0 + \alpha_{1,1n} + \cdots + \alpha_{1,m_1-1} n^{m_1-1}) r_1^n + (\alpha_1, 0 + \alpha_{2,1} n \ldots \alpha_{1,m_2-1} n^{m_2-1}) r_2^n + \cdots + (\alpha_t, 0 + \alpha_{t,1} n \ldots \alpha_{t,m_t-1} n^{m_t-}$$

for $n = 0, 1, 2, \ldots,$ where $\alpha_{i,j}$ are constants for $1 \leq i \leq t$ and $0 \leq j \leq m_i - 1$.

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6.2.308 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 52 (Page No. 527) top

Prove Theorem 6 :Suppose that $\{a_n\}$ satisfies the liner nonhomogeneous recurrence relation

$$a_n = c_1 a_{n-1} + c_2 a_{n-2} + \cdots + c_k a_{n-k} + F(n),$$

where $c_1 . c_2, \ldots, c_k$ are real numbers , and

$$F(n) = (b_t n^t + b_{t-1} n^{t-1}) + \cdots + b_1 n + b_0) s^n,$$

where $b_0, b_1, \ldots, b_t$ and s are real numbers. When s is is not a root of the characteristic equation of the associated linear homogeneous recurrence relation, there is a particular solution of the form

$$(p_t n^t + p_{t-1} n^{t-1} + \cdots + p_1 n + p_0) s^n.$$

When s is a root of this characteristic equation and its multiplicity is m, there is a particular solution of the form

$$n^m (p_t n^t + p_{t-1} n^{t-1} + \cdots + p_1 n + p_0) s^n.$$

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6.2.309 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 53 (Page No. 527) top

Solve the recurrence relation $T(n) = nT^2(n/2)$ with initial condition $T(1) = 6$ when $n = 2^k$ for some integer k. [Hint: Let $n = 2^k$ and then make the substitution $a_k = \log T(2^k)$ to obtain a linear nonhomogeneous recurrence relation.]

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6.2.310 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 6 (Page No. 524) top

How many different messages can be transmitted in n microseconds using three different signals if one signal requires 1 microsecond for transmittal, the other two signals require 2 microseconds each for transmittal, and a signal in a message is followed immediately by the next signal?

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6.2.311 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 7 (Page No. 524) top

In how many ways can a $2 \times n$ rectangular checkerboard be tiled using 1×2 and 2×2 pieces?

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6.2.312 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 8 (Page No. 524 - 525) top

A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

A. Find a recurrence relation for $\{L_n\}$, where L_n is the number of lobsters caught in year n, under the assumption for this model.
B. Find L_n if $100,000$ lobsters were caught in year 1 and $300,000$ were caught in year 2.

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6.2.313 Counting: Kenneth Rosen Edition 7 Exercise 8.2 Question 9 (Page No. 525) top

A deposit of $\$100,000$ is made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year.

A. Find a recurrence relation for $\{Pn\}$, where P_n is the amount in the account at the end of n years if no money is ever withdrawn.
B. How much is in the account after n years if no money has been withdrawn?

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6.2.314 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 1 (Page No. 535) top

How many comparisons are needed for a binary search in a set of 64 elements?

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Answer key

6.2.315 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 10 (Page No. 535) top

Find $f(n)$ when $n = 2^k$, where f satisfies the recurrence relation $f(n) = f(n/2) + 1$ with $f(1) = 1$.

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Answer key

6.2.316 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 11 (Page No. 535) top

Give a big-O estimate for the function f in question 10 if f is an increasing function.

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Answer key

6.2.317 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 12 (Page No. 535) top

Find $f(n)$ when $n = 3k$, where f satisfies the recurrence relation $f(n) = 2f(n/3) + 4$ with $f(1) = 1$.

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Answer key

6.2.318 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 13 (Page No. 535) top

Give a big-O estimate for the function f given below if f is an increasing function.

$f(n) = 2f(n/3) + 4$ with $f(1) = 1$.

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Answer key

6.2.319 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 14 (Page No. 535) top

Suppose that there are $n = 2^k$ teams in an elimination tournament, where there are $\frac{n}{2}$ games in the first round, with the $\frac{n}{2} = 2^{k-1}$ winners playing in the second round, and so on. Develop a recurrence relation for the number of rounds in the tournament.

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Answer key

6.2.320 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 15 (Page No. 535) top

How many rounds are in the elimination tournament described in question 14 when there are 32 teams?

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Answer key

6.2.321 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 16 (Page No. 535) top

Solve the recurrence relation for the number of rounds in the tournament described in question 14.

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Answer key

6.2.322 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 2 (Page No. 535) top

How many comparisons are needed to locate the maximum and minimum elements in a sequence with 128 elements using the algorithm in Example 2?

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6.2.323 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 3 (Page No. 535) top

Multiply $(1110)_2$ and $(1010)_2$ using the fast multiplication algorithm.

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6.2.324 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 4 (Page No. 535) top

Express the fast multiplication algorithm in pseudocode.

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Answer key

6.2.325 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 5 (Page No. 535) top

Determine a value for the constant C in Example 4 and use it to estimate the number of bit operations needed to multiply two 64-bit integers using the fast multiplication algorithm.

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6.2.326 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 6 (Page No. 535) top

How many operations are needed to multiply two 32×32 matrices using the algorithm referred to in Example 5?

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6.2.327 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 7 (Page No. 535) top

Suppose that $f(n) = f(n/3) + 1$ when n is a positive integer divisible by 3, and $f(1) = 1$. Find

A. $f(3)$
B. $f(27)$
C. $f(729)$

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Answer key

6.2.328 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 8 (Page No. 535) top

Suppose that $f(n) = 2f(n/2) + 3$ when n is an even positive integer, and $f(1) = 5$. Find

A. $f(2)$ B. $f(8)$ C. $f(64)$ D. (1024)

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Answer key

6.2.329 Counting: Kenneth Rosen Edition 7 Exercise 8.3 Question 9 (Page No. 535) top

Suppose that $f(n) = f(n/5) + 3n^2$ when n is a positive integer divisible by $5,$ and $f(1) = 4$. Find

A. $f(5)$
B. $f(125)$
C. $f(3125)$

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Answer key

6.3 Generating Functions (1) top

6.3.1 Generating Functions: Kenneth Rosen Edition 7 Exercise 8.4 Question 10 (Page No. 549) top

Find the coefficient of x^9 in the power series of each of these functions.

a) $(x^3 + x^5 + x^6).\,(x^3 + x^4).\,(x + x^2 + x^3 + x^4 + \cdots)$

b) $(1 + x + x^2)^3$

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Answer key

6.4 Pigeonhole Principle (47) top

6.4.1 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 1 (Page No. 405) top

Show that in any set of six classes, each meeting regularly once a week on a particular day of the week, there must be two that meet on the same day, assuming that no classes are held on weekends.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.2 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 10 (Page No. 405) top

Let $(x_i, y_i), i = 1, 2, 3, 4, 5,$ be a set of five distinct points with integer coordinates in the xy plane. Show that the midpoint of the line joining at least one pair of these points has integer coordinates.

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Answer key

6.4.3 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 11 (Page No. 405) top

Let $(x_i, y_i, z_i), i = 1, 2, 3, 4, 5, 6, 7, 8, 9,$ be a set of nine distinct points with integer coordinates in xyz space. Show that the midpoint of at least one pair of these points has integer coordinates.

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Answer key

6.4.4 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 12 (Page No. 405) top

How many ordered pairs of integers (a, b) are needed to guarantee that there are two ordered pairs (a_1, b_1) and (a_2, b_2) such that $a_1 \mod 5 = a_2 \mod 5$ and $b_1 \mod 5 = b_2 \mod 5$?

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Answer key

6.4.5 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 13 (Page No. 405) top

A. Show that if five integers are selected from the first eight positive integers, there must be a pair of these integers with a sum equal to 9.
B. Is the conclusion in part (A) true if four integers are selected rather than five?

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Answer key

6.4.6 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 14 (Page No. 405) top

A. Show that if seven integers are selected from the first 10 positive integers, there must be at least two pairs of these integers with the sum 11.
B. Is the conclusion in part (A) true if six integers are selected rather than seven?

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Answer key

6.4.7 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 15 (Page No. 405) top

How many numbers must be selected from the set $\{1, 2, 3, 4, 5, 6\}$ to guarantee that at least one pair of these numbers add up to 7?

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Answer key

6.4.8 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 16 (Page No. 405) top

How many numbers must be selected from the set $\{1, 3, 5, 7, 9, 11, 13, 15\}$ to guarantee that at least one pair of these numbers add up to 16?

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Answer key

6.4.9 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 17 (Page No. 405) top

A company stores products in a warehouse. Storage bins in this warehouse are specified by their aisle, location in the aisle, and shelf. There are 50 aisles, 85 horizontal locations in each aisle, and 5 shelves throughout the warehouse. What is the least number of products the company can have so that at least two products must be stored in the same bin?

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Answer key

6.4.10 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 18 (Page No. 405) top

Suppose that there are nine students in a discrete mathematics class at a small college.

A. Show that the class must have at least five male students or at least five female students.
B. Show that the class must have at least three male students or at least seven female students.

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Answer key

6.4.11 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 19 (Page No. 405 - 406) top

Suppose that every student in a discrete mathematics class of 25 students is a freshman, a sophomore, or a junior.

A. Show that there are at least nine freshmen, at least nine sophomores, or at least nine juniors in the class.
B. Show that there are either at least three freshmen, at least 19 sophomores, or at least five juniors in the class

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Answer key

6.4.12 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 2 (Page No. 405) top

Show that if there are 30 students in a class, then at least two have last names that begin with the same letter.

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Answer key

6.4.13 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 20 (Page No. 406) top

Find an increasing subsequence of maximal length and a decreasing subsequence of maximal length in the sequence $22, 5, 7, 2, 23, 10, 15, 21, 3, 17$.

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Answer key

6.4.14 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 21 (Page No. 406) top

Construct a sequence of 16 positive integers that has no increasing or decreasing subsequence of five terms.

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Answer key

6.4.15 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 22 (Page No. 406) top

Show that if there are 101 people of different heights standing in a line, it is possible to find 11 people in the order they are standing in the line with heights that are either increasing or decreasing.

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Answer key

6.4.16 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 23 (Page No. 406) top

Show that whenever 25 girls and 25 boys are seated around a circular table there is always a person both of whose neighbors are boys.

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6.4.17 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 24 (Page No. 406) top

Suppose that 21 girls and 21 boys enter a mathematics competition. Furthermore, suppose that each entrant solves at most six questions, and for every boy-girl pair, there is at least one question that they both solved. Show that there is a question that was solved by at least three girls and at least three boys.

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6.4.18 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 25 (Page No. 406) top

Describe an algorithm in pseudocode for producing the largest increasing or decreasing subsequence of a sequence of distinct integers.

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6.4.19 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 26 (Page No. 406) top

Show that in a group of five people (where any two people are either friends or enemies), there are not necessarily three mutual friends or three mutual enemies.

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6.4.20 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 27 (Page No. 406) top

Show that in a group of 10 people (where any two people are either friends or enemies), there are either three mutual friends or four mutual enemies, and there are either three mutual enemies or four mutual friends.

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6.4.21 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 28 (Page No. 406) top

Use question 27 to show that among any group of 20 people (where any two people are either friends or enemies), there are either four mutual friends or four mutual enemies.

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6.4.22 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 29 (Page No. 406) top

Show that if n is an integer with $n \geq 2,$ then the Ramsey number $R(2, n)$ equals $n.$ (Recall that Ramsey numbers were discussed after Example 13 in Section 6.2.)

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6.4.23 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 3 (Page No. 405) top

A drawer contains a dozen brown socks and a dozen black socks, all unmatched. A man takes socks out at random in the dark.

A. How many socks must he take out to be sure that he has at least two socks of the same color?
B. How many socks must he take out to be sure that he has at least two black socks?

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Answer key

6.4.24 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 30 (Page No. 406) top

Show that if m and n are integers with $m \geq 2$ and $n \geq 2,$ then the Ramsey numbers $R(m, n)$ and $R(n, m)$ are equal. (Recall that Ramsey numbers were discussed after Example 13 in Section 6.2.)

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6.4.25 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 31 (Page No. 406) top

Show that there are at least six people in California (population: 37 million) with the same three initials who were born on the same day of the year (but not necessarily in the same year). Assume that everyone has three initials.

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6.4.26 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 32 (Page No. 406) top

Show that if there are $100,000,000$ wage earners in the United States who earn less than $1,000,000$ dollars (but at least a penny), then there are two who earned exactly the same amount of money, to the penny, last year.

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6.4.27 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 33 (Page No. 406) top

In the 17^{th} century, there were more than $800,000$ inhabitants of Paris. At the time, it was believed that no one had more than $200,000$ hairs on their head. Assuming these numbers are correct and that everyone has at least one hair on their head (that is, no one is completely bald), use the pigeonhole principle to show, as the French writer Pierre Nicole did, that there had to be two Parisians with the same number of hairs on their heads. Then use the generalized pigeonhole principle to show that there had to be at least five Parisians at that time with the same number of hairs on their heads.

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6.4.28 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 34 (Page No. 406) top

Assuming that no one has more than $1,000,000$ hairs on the head of any person and that the population of New York City was $8,008,278 \text{ in } 2010,$ show there had to be at least nine people in NewYork City in 2010 with the same number of hairs on their heads.

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6.4.29 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 35 (Page No. 406) top

There are 38 different time periods during which classes at a university can be scheduled. If there are 677 different classes, how many different rooms will be needed?

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6.4.30 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 36 (Page No. 406) top

A computer network consists of six computers. Each computer is directly connected to at least one of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers.

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6.4.31 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 37 (Page No. 406) top

A computer network consists of six computers. Each computer is directly connected to zero or more of the other computers. Show that there are at least two computers in the network that are directly connected to the same number of other computers. [Hint: It is impossible to have a computer linked to none of the others and a computer linked to all the others.]

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6.4.32 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 38 (Page No. 406) top

Find the least number of cables required to connect eight computers to four printers to guarantee that for every choice of four of the eight computers, these four computers can directly access four different printers. Justify your answer.

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Answer key

6.4.33 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 39 (Page No. 406) top

Find the least number of cables required to connect 100 computers to 20 printers to guarantee that 2 every subset of 20computers can directly access 20 different printers. (Here, the assumptions about cables and computers are the same as in Example 9.) Justify your answer.

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6.4.34 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 4 (Page No. 405) top

A bowl contains 10 red balls and 10 blue balls. A woman selects balls at random without looking at them.

A. How many balls must she select to be sure of having at least three balls of the same color?
B. How many balls must she select to be sure of having at least three blue balls?

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.35 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 40 (Page No. 406) top

Prove that at a party where there are at least two people, there are two people who know the same number of other people there.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.36 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 41 (Page No. 406) top

An arm wrestler is the champion for a period of 75 hours. (Here, by an hour, we mean a period starting from an exact hour, such as 1 p.m., until the next hour.) The arm wrestler had at least one match an hour, but no more than 125 total matches. Show that there is a period of consecutive hours during which the arm wrestler had exactly 24 matches.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.37 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 42 (Page No. 406) top

Is the statement in question 41 true if 24 is replaced by

A. 2? B. 23? C. 25? D. 30?

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.38 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 43 (Page No. 406) top

Show that if f is a function from S to T, where S and T are nonempty finite sets and $m = \lceil \mid S \mid / \mid T \mid \rceil$, then there are at least m elements of S mapped to the same value of T. That is, show that there are distinct elements $s_1, s_2, \ldots, s_m$ of S such that $f(s_1) = f(s_2) = \cdots = f(s_m)$.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.39 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 44 (Page No. 406) top

There are 51 houses on a street. Each house has an address between 1000 and 1099, inclusive. Show that at least two houses have addresses that are consecutive integers.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.40 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 45 (Page No. 407) top

Let x be an irrational number. Show that for some positive integer j not exceeding the positive integer n, the absolute value of the difference between jx and the nearest integer to jx is less than $1/n$.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.41 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 46 (Page No. 407) top

Let $n_1, n_2, \ldots, n_t$ be positive integers. Show that if $n_1 + n_2 + \cdots + n_t - t + 1$ objects are placed into t boxes, then for some $i, i = 1, 2, \ldots, t$, the i^{th} box contains at least n_i objects.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.42 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 47 (Page No. 407) top

An alternative proof of Theorem 3 based on the generalized pigeonhole principle is outlined in this exercise. The notation used is the same as that used in the proof in the text.

A. Assume that $i_k \leq n$ for $k = 1, 2, \ldots, n^2 + 1$. Use the generalized pigeonhole principle to show that there are $n + 1$ terms $a_{k_1}, a_{k_2}, \ldots, a_{k_{n+1}}$ with $i_{k_1} = i_{k_2} = \cdots = i_{k_{n+1}}$, where $1 \leq k_1 < k_2 < \cdots < k_{n+1}$.
B. Show that $a_{k_j} > a_{k_{j+1}}$ for $j = 1, 2, \ldots, n$. [Hint:Assume that $a_{k_j} < a_{k_{j+1}}$, and show that this implies that $i_{k_j} > i_{k_{j+1}}$, which is a contradiction.]
C. Use parts (A) and (B) to show that if there is no increasing subsequence of length $n + 1$, then there must be a decreasing subsequence of this length.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

6.4.43 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 5 (Page No. 405) top

Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.44 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 6 (Page No. 405) top

Let d be a positive integer. Show that among any group of $d+1$ (not necessarily consecutive) integers there are two with exactly the same remainder when they are divided by d.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.45 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 7 (Page No. 405) top

Let n be a positive integer. Show that in any set of n consecutive integers there is exactly one divisible by n.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.46 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 8 (Page No. 405) top

Show that if f is a function from S to T, where S and T are finite sets with $|S|>|T|$, then there are elements s_1 and s_2 in S such that $f(s_1)=f(s_2)$, or in other words, f is not one-to-one.

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

6.4.47 Pigeonhole Principle: Kenneth Rosen Edition 7 Exercise 6.2 Question 9 (Page No. 405) top

What is the minimum number of students, each of whom comes from one of the 50 states, who must be enrolled in a university to guarantee that there are at least 100 who come from the same state?

kenneth-rosen discrete-mathematics counting pigeonhole-principle descriptive

Answer key

Answer Keys

6.1.1	N/A	6.1.2	N/A	6.1.3	N/A	6.1.4	N/A	6.1.5	N/A
6.1.6	N/A	6.1.7	N/A	6.1.8	N/A	6.1.9	N/A	6.1.10	N/A
6.1.11	N/A	6.1.12	N/A	6.1.13	N/A	6.1.14	N/A	6.1.15	N/A
6.1.16	N/A	6.1.17	N/A	6.1.18	N/A	6.1.19	N/A	6.1.20	N/A
6.1.21	N/A	6.1.22	N/A	6.1.23	N/A	6.1.24	N/A	6.1.25	N/A
6.1.26	N/A	6.1.27	N/A	6.1.28	N/A	6.1.29	N/A	6.1.30	N/A
6.1.31	N/A	6.1.32	N/A	6.1.33	N/A	6.1.34	N/A	6.1.35	N/A
6.1.36	N/A	6.1.37	N/A	6.1.38	N/A	6.1.39	N/A	6.2.1	N/A
6.2.2	N/A	6.2.3	N/A	6.2.4	N/A	6.2.5	N/A	6.2.6	N/A
6.2.7	N/A	6.2.8	N/A	6.2.9	N/A	6.2.10	N/A	6.2.11	N/A
6.2.12	N/A	6.2.13	N/A	6.2.14	N/A	6.2.15	N/A	6.2.16	N/A
6.2.17	N/A	6.2.18	N/A	6.2.19	N/A	6.2.20	N/A	6.2.21	N/A
6.2.22	N/A	6.2.23	N/A	6.2.24	N/A	6.2.25	N/A	6.2.26	N/A
6.2.27	N/A	6.2.28	N/A	6.2.29	N/A	6.2.30	N/A	6.2.31	N/A

6.2.32	N/A	6.2.33	N/A	6.2.34	N/A	6.2.35	N/A	6.2.36	N/A
6.2.37	N/A	6.2.38	N/A	6.2.39	N/A	6.2.40	N/A	6.2.41	N/A
6.2.42	N/A	6.2.43	N/A	6.2.44	N/A	6.2.45	N/A	6.2.46	N/A
6.2.47	N/A	6.2.48	N/A	6.2.49	N/A	6.2.50	N/A	6.2.51	N/A
6.2.52	N/A	6.2.53	N/A	6.2.54	N/A	6.2.55	N/A	6.2.56	N/A
6.2.57	N/A	6.2.58	N/A	6.2.59	N/A	6.2.60	N/A	6.2.61	N/A
6.2.62	N/A	6.2.63	N/A	6.2.64	N/A	6.2.65	N/A	6.2.66	N/A
6.2.67	N/A	6.2.68	N/A	6.2.69	N/A	6.2.70	N/A	6.2.71	N/A
6.2.72	N/A	6.2.73	N/A	6.2.74	N/A	6.2.75	N/A	6.2.76	N/A
6.2.77	N/A	6.2.78	N/A	6.2.79	N/A	6.2.80	N/A	6.2.81	N/A
6.2.82	N/A	6.2.83	N/A	6.2.84	N/A	6.2.85	N/A	6.2.86	N/A
6.2.87	N/A	6.2.88	N/A	6.2.89	N/A	6.2.90	N/A	6.2.91	N/A
6.2.92	N/A	6.2.93	N/A	6.2.94	N/A	6.2.95	N/A	6.2.96	N/A
6.2.97	N/A	6.2.98	N/A	6.2.99	N/A	6.2.100	N/A	6.2.101	N/A
6.2.102	N/A	6.2.103	N/A	6.2.104	N/A	6.2.105	N/A	6.2.106	N/A
6.2.107	N/A	6.2.108	N/A	6.2.109	N/A	6.2.110	N/A	6.2.111	N/A
6.2.112	N/A	6.2.113	N/A	6.2.114	N/A	6.2.115	N/A	6.2.116	N/A
6.2.117	N/A	6.2.118	N/A	6.2.119	N/A	6.2.120	N/A	6.2.121	N/A
6.2.122	Q-Q	6.2.123	N/A	6.2.124	N/A	6.2.125	N/A	6.2.126	N/A
6.2.127	N/A	6.2.128	N/A	6.2.129	N/A	6.2.130	N/A	6.2.131	N/A
6.2.132	N/A	6.2.133	N/A	6.2.134	N/A	6.2.135	N/A	6.2.136	N/A
6.2.137	N/A	6.2.138	N/A	6.2.139	N/A	6.2.140	N/A	6.2.141	N/A
6.2.142	N/A	6.2.143	N/A	6.2.144	N/A	6.2.145	N/A	6.2.146	N/A
6.2.147	N/A	6.2.148	N/A	6.2.149	N/A	6.2.150	N/A	6.2.151	N/A
6.2.152	N/A	6.2.153	N/A	6.2.154	N/A	6.2.155	N/A	6.2.156	N/A
6.2.157	N/A	6.2.158	N/A	6.2.159	N/A	6.2.160	N/A	6.2.161	N/A
6.2.162	N/A	6.2.163	N/A	6.2.164	N/A	6.2.165	N/A	6.2.166	N/A
6.2.167	N/A	6.2.168	N/A	6.2.169	N/A	6.2.170	N/A	6.2.171	N/A
6.2.172	N/A	6.2.173	N/A	6.2.174	N/A	6.2.175	N/A	6.2.176	N/A
6.2.177	N/A	6.2.178	N/A	6.2.179	N/A	6.2.180	N/A	6.2.181	N/A
6.2.182	N/A	6.2.183	N/A	6.2.184	N/A	6.2.185	N/A	6.2.186	N/A
6.2.187	N/A	6.2.188	N/A	6.2.189	N/A	6.2.190	N/A	6.2.191	N/A
6.2.192	N/A	6.2.193	N/A	6.2.194	N/A	6.2.195	N/A	6.2.196	N/A
6.2.197	N/A	6.2.198	N/A	6.2.199	N/A	6.2.200	N/A	6.2.201	N/A
6.2.202	N/A	6.2.203	N/A	6.2.204	N/A	6.2.205	N/A	6.2.206	N/A
6.2.207	N/A	6.2.208	N/A	6.2.209	N/A	6.2.210	N/A	6.2.211	N/A
6.2.212	N/A	6.2.213	N/A	6.2.214	N/A	6.2.215	N/A	6.2.216	N/A
6.2.217	N/A	6.2.218	N/A	6.2.219	N/A	6.2.220	N/A	6.2.222	N/A
6.2.223	N/A	6.2.224	N/A	6.2.225	N/A	6.2.226	N/A	6.2.227	N/A
6.2.228	N/A	6.2.229	N/A	6.2.230	N/A	6.2.231	N/A	6.2.232	N/A
6.2.233	N/A	6.2.234	N/A	6.2.235	N/A	6.2.236	N/A	6.2.237	N/A

6.2.238	N/A	6.2.239	N/A	6.2.240	N/A	6.2.241	N/A	6.2.242	N/A
6.2.243	N/A	6.2.244	N/A	6.2.245	N/A	6.2.246	N/A	6.2.247	N/A
6.2.248	N/A	6.2.249	N/A	6.2.250	N/A	6.2.251	N/A	6.2.252	N/A
6.2.253	N/A	6.2.254	N/A	6.2.255	N/A	6.2.256	N/A	6.2.257	N/A
6.2.258	N/A	6.2.259	N/A	6.2.260	N/A	6.2.261	N/A	6.2.262	N/A
6.2.263	N/A	6.2.264	N/A	6.2.265	N/A	6.2.266	N/A	6.2.267	N/A
6.2.268	N/A	6.2.269	N/A	6.2.270	N/A	6.2.271	N/A	6.2.272	N/A
6.2.273	N/A	6.2.274	N/A	6.2.275	N/A	6.2.276	N/A	6.2.277	N/A
6.2.278	N/A	6.2.279	N/A	6.2.280	N/A	6.2.281	N/A	6.2.282	N/A
6.2.283	N/A	6.2.284	N/A	6.2.285	N/A	6.2.286	N/A	6.2.287	N/A
6.2.288	N/A	6.2.289	N/A	6.2.290	N/A	6.2.291	N/A	6.2.292	N/A
6.2.293	N/A	6.2.294	N/A	6.2.295	N/A	6.2.296	N/A	6.2.297	N/A
6.2.298	N/A	6.2.299	N/A	6.2.300	N/A	6.2.301	N/A	6.2.302	N/A
6.2.303	N/A	6.2.304	N/A	6.2.305	N/A	6.2.306	N/A	6.2.307	N/A
6.2.308	N/A	6.2.309	N/A	6.2.310	N/A	6.2.311	N/A	6.2.312	N/A
6.2.313	N/A	6.2.314	N/A	6.2.315	N/A	6.2.316	N/A	6.2.317	N/A
6.2.318	N/A	6.2.319	N/A	6.2.320	N/A	6.2.321	N/A	6.2.322	N/A
6.2.323	N/A	6.2.324	N/A	6.2.325	N/A	6.2.326	N/A	6.2.327	N/A
6.2.328	N/A	6.2.329	N/A	6.3.1	Q-Q	6.4.1	N/A	6.4.2	N/A
6.4.3	N/A	6.4.4	N/A	6.4.5	N/A	6.4.6	N/A	6.4.7	N/A
6.4.8	N/A	6.4.9	N/A	6.4.10	N/A	6.4.11	N/A	6.4.12	N/A
6.4.13	N/A	6.4.14	N/A	6.4.15	N/A	6.4.16	N/A	6.4.17	N/A
6.4.18	N/A	6.4.19	N/A	6.4.20	N/A	6.4.21	N/A	6.4.22	N/A
6.4.23	N/A	6.4.24	N/A	6.4.25	N/A	6.4.26	N/A	6.4.27	N/A
6.4.28	N/A	6.4.29	N/A	6.4.30	N/A	6.4.31	N/A	6.4.32	N/A
6.4.33	N/A	6.4.34	N/A	6.4.35	N/A	6.4.36	N/A	6.4.37	N/A
6.4.38	N/A	6.4.39	N/A	6.4.40	N/A	6.4.41	N/A	6.4.42	N/A
6.4.43	N/A	6.4.44	N/A	6.4.45	N/A	6.4.46	N/A	6.4.47	N/A

7.0.1 Kenneth Rosen Edition 7 Exercise 10.8 Question 23 (Page No. 734) top

Find the edge chromatic numbers of
a) Cn, where n ≥ 3. (Cycle with n vertices)

b) Wn, where n ≥ 3 (Wheel with n vertices)

c)Complete graph with n vertices.

discrete-mathematics kenneth-rosen graph-theory

Answer key

Answer Keys

7.0.1	Q-Q

8.0.1 Kenneth Rosen Edition 7 Exercise 1.1 Question 1 (Page No. 12) top

Which of these sentences are propositions? What are the truth values of those that are propositions?

1. Boston is the capital of Massachusetts.
2. Miami is the capital of Florida.
3. 2 + 3 = 5.
4. 5 + 7 = 10.
5. x + 2 = 11.
6. Answer this question

kenneth-rosen mathematical-logic discrete-mathematics

Answer key

8.0.2 Kenneth Rosen Edition 7 Exercise 1.1 Question 7 (Page No. 13) top

Suppose that during the most recent fiscal year, the annual revenue of Acme Computer was 138 billion dollars and its net profit was 8 billion dollars, the annual revenue of Nadir Software was 87 billion dollars and its net profit was 5 billion dollars, and the annual revenue of Quixote Media was 111 billion dollars and its net profit was 13 billion dollars. Determine the truth value of each ofthese propositions for the most recent fiscal year.

A. Quixote Media had the largest annual revenue.
B. Nadir Software had the lowest net profit and Acme Computer had the largest annual revenue.
C. Acme Computer had the largest net profit or Quixote Media had the largest net profit.
D. If Quixote Media had the smallest net profit, then Acme Computer had the largest annual revenue.
E. Nadir Software had the smallest net profit if and only if Acme Computer had the largest annual revenue.

kenneth-rosen mathematical-logic discrete-mathematics

Answer key

8.0.3 Kenneth Rosen Edition 7 Exercise 2.3 Question 11 (Page No. 153) top

Determine whether each of these functions form $[a, b, c, d]$ to itself is onto?

a. $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
b. $f(a) = b, f(b) = b, f(c) = d, f(d) = c$
c. $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

kenneth-rosen discrete-mathematics set-theory&algebra

8.0.4 Kenneth Rosen Edition 7 Exercise 1.7 Question 3 (Page No. 91) top

Show that the square of an even number is an even number using a direct proof

kenneth-rosen discrete-mathematics mathematical-logic

8.0.5 Kenneth Rosen Edition 7 Exercise 1.7 Question 1 (Page No. 91) top

Use a direct proof to show that the sum of two odd integers is even.

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.6 Kenneth Rosen Edition 7 Exercise 1.7 Question 2 (Page No. 91) top

Use a direct proof to show that the sum of two even integers is even.

kenneth-rosen discrete-mathematics mathematical-logic

8.0.7 Kenneth Rosen Edition 7 Exercise 1.2 Question 23 (Page No. 23) top

Relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you can not determine what these people are, can you draw any conclusions?

A says "We are both knaves" and B says nothing.

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.8 Kenneth Rosen Edition 7 Exercise 1.2 Question 32 (Page No. 23) top

The police have three suspects for the murder of Mr. Cooper: Mr. Smith, Mr Jones, Mr. Williams. Smith Jones, and Williams each declare that they did not kill Cooper. Smith also states that Cooper was friend of Jones and that Williams disliked him. Jones also states that he did not know Cooper and that he was out of town the day Cooper was killed. Williams also states that he saw both Smith and Jones with Cooper the day of the killing and that either Smith or Jones must have killed him. Can you determine who the murderer was if

a. one of the three men is guilt, the two innocent men are telling the truth, but the statements of the guilty man may or may not b true?
b. innocent men do not lie?

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.9 Kenneth Rosen Edition 7 Exercise 1.2 Question 21 (Page No. 23) top

relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth always lie. You encounter two people. A and B. Determine, if possible, what A and B are if they address you in the ways described. If you can not determine what these people are, can you draw any conclusions?

A says "I am a knave or B is a knight" and B says nothing.

kenneth-rosen discrete-mathematics mathematical-logic descriptive

Answer key

8.0.10 Kenneth Rosen Edition 7 Exercise 1.2 Question 22 (Page No. 23) top

Relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you can not determine what these people are, can you draw any conclusions ?

Both A and B say "I am a knight."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.11 Kenneth Rosen Edition 7 Exercise 1.2 Question 20 (Page No. 23) top

relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth always lie. You encounter two people. A and B. Determine, if possible, what A and B are if they address you in the ways described. If you can not determine what these people are, can you draw any conclusions?

A says "The two of us are both knights " and B says "A is knave."

kenneth-rosen discrete-mathematics mathematical-logic descriptive

Answer key

8.0.12 Kenneth Rosen Edition 7 Exercise 1.2 Question 5 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
You are eligible to be President of the U.S.A. only if you are at least 35 years old, were born in the U.S.A, or at the time of your birth both of your parents were citizens, and you have lived at least 14 years in the country.

Express your answer in terms of
e: “You are eligible to be President of the U.S.A.,”
a: “You are at least 35 years old,”
b: “You were born in the U.S.A,” p: “At the time of your birth, both of your parents where citizens,”
and r: “You have lived at least 14 years in the U.S.A.”

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.13 Kenneth Rosen Edition 7 Exercise 1.1 Question 3 (Page No. 12) top

What is the negation of each of these propositions?

A. Mei has an MP3 player.
B. There is no pollution in New Jersey.
C. $2 + 1 = 3$.
D. The summer in Maine is hot and sunny.

mathematical-logic kenneth-rosen discrete-mathematics

Answer key

8.0.14 Kenneth Rosen Edition 7 Exercise 1.1 Question 4 (Page No. 12) top

What is the negation of each of these propositions?

A. Jennifer and Teja are friends.
B. There are 13 items in a baker’s dozen.
C. Abby sent more than 100 text messages every day.
D. 121 is a perfect square.

kenneth-rosen mathematical-logic discrete-mathematics

Answer key

8.0.15 Kenneth Rosen Edition 7 Exercise 1.1 Question 5 (Page No. 13) top

What is the negation of each of these propositions?

A. Steve has more than 100 GB free disk space on his laptop.
B. Zach blocks e-mails and texts from Jennifer.
C. $7 \cdot 11 \cdot 13 = 999$.
D. Diane rode her bicycle 100 miles on Sunday.

mathematical-logic kenneth-rosen discrete-mathematics

Answer key

8.0.16 Kenneth Rosen Edition 7 Exercise 1.1 Question 6 (Page No. 13) top

Suppose that SmartphoneA has 256 MB RAM and 32 GB ROM, and the resolution of its camera is 8 MP; Smartphone B has 288 MB RAM and 64 GB ROM, and the resolution of its camera is 4 MP; and Smartphone C has 128 MB RAM and 32 GB ROM, and the resolution of its camera is 5 MP. Determine the truth value of each of these propositions.

A. Smartphone B has the most RAM of these three smartphones.
B. Smartphone C has more ROM or a higher resolution camera than Smartphone B.
C. Smartphone B has more RAM, more ROM, and a higher resolution camera than Smartphone A.
D. If Smartphone B has more RAM and more ROM than Smartphone C, then it also has a higher resolution camera.
E. Smartphone A has more RAM than Smartphone B if and only if Smartphone B has more RAM than Smartphone A.

mathematical-logic kenneth-rosen discrete-mathematics

Answer key

8.0.17 Kenneth Rosen Edition 7 Exercise 1.1 Question 40 (Page No. 16) top

Explain, without using a truth table, why$(p \vee \neg q) \wedge (q \vee \neg r) \wedge (r \vee \neg p)$ is true when p, q, and r have the same truth value and it is false otherwise.

mathematical-logic discrete-mathematics kenneth-rosen descriptive

Answer key

8.0.18 Kenneth Rosen Edition 7 Exercise 1.5 Question 14 (Page No. 66) top

Use quantifiers and predicates with more than one variable to express these statements.

a. There is a student in this class who can speak Hindi.
b. Every student in this class plays some sport.c)Some student in this class has visited Alaska but has not visited Hawaii.
c. All students in this class have learned at least one programming language.
d. There is a student in this class who has taken every course offered by one of the departments in this school.
e. Some student in this class grew up in the same town as exactly one other student in this class.
f. Every student in this class has chatted with at least one other student in at least one chat group.

8.0.19 Kenneth Rosen Edition 7 Exercise 1.2 Question 9 (Page No. 22) top

Are these system specifications consistent? “The system is in multi-user state if and only if it is operating normally. If the system is operating normally, the kernel is functioning. The kernel is not functioning or the system is in interrupt mode. If the system is not in multiuser state, then it is in interrupt mode. The system is not in interrupt mode.

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.20 Kenneth Rosen Edition 7 Exercise 1.2 Question 17 (Page No. 23) top

When three professors are seated in a restaurant, the hostess asks them: “Does everyone want coffee ?” The first professor says: “I do not know.” The second professor then says: “I do not know.” Finally, the third professor says: “No, not everyone wants coffee.” The hostess comes back and gives coffee to the professors who want it. How did she figure out who wanted coffee?

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.21 Kenneth Rosen Edition 7 Exercise 1.2 Question 15 (Page No. 23) top

Each inhabitant of a remote village always tells the truth or always lies. A villager will give only a “Yes” or a “No” response to a question a tourist asks. Suppose you are a tourist visiting this area and come to a fork in the road. One branch leads to the ruins you want to visit; The other branch leads deep into the jungle. A villager is standing at the fork in the road. What one question can you ask the villager to determine which branch to take?

kenneth-rosen discrete-mathematics mathematical-logic difficult

8.0.22 Kenneth Rosen Edition 7 Exercise 1.2 Question 13 (Page No. 23) top

What Boolean search would you use to look for Web pages about beaches in New Jersey? what if you wanted to find Web pages about beaches on the isle of Jersey(in the English Channel)

kenneth-rosen discrete-mathematics mathematical-logic

8.0.23 Kenneth Rosen Edition 7 Exercise 1.2 Question 16 (Page No. 23) top

An explorer is captured by a group of cannibals. There are two types of cannibals—those who always tell the truth and those who always lie. The cannibals will barbecue the explorer unless he can determine whether a particular cannibal always lies or always tells the truth. He is allowed to ask the cannibal exactly one question..

a) Explain why the question “Are you a liar?” does not work.

b) Find a question that the explorer can use to determine whether the cannibal always lies or always tells the truth.

in the below link, it mentioned double negation will work. I am not getting what is double negation here. ow the cannibal will consider as two separate question.

https://math.stackexchange.com/questions/1078866/is-this-a-correct-solution-to-determining-which-of-two-people-is-the-liar-using

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.24 Kenneth Rosen Edition 7 Exercise 1.2 Question 12 (Page No. 23) top

Are these system specifications consistent? "If the file system is not locked, then new messages will be queued. If the file system is not locked, then the system is functioning normally, and conversely. If new messages are not queued, then they will be sent to the message buffer. If the file system is not locked, then new messages will be sent to the message buffer. New messages will not be sent to the message buffer."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.25 Kenneth Rosen Edition 7 Exercise 1.2 Question 10 (Page No. 23) top

Are these system specifications consistent? "Whenever the system software is being upgraded, users cannot access the file system. If users can access the file system, then they can save new files. If users cannot save new files, then the system software is not being upgraded."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.26 Kenneth Rosen Edition 7 Exercise 1.2 Question 8 (Page No. 22) top

Express these system specifications using the propositions p "The user enters a valid password," q "Access is granted," and r "The user has paid the subscription fee" and logical connectives (including negations).

a. "The user has paid the subscription fee, but does not enter a valid password."
b. "Access is granted whenever the user has paid the subscription fee and enters a valid password."
c. "Access is denied if the user has not paid the subscription fee."
d. "If the user has not entered a valid password but has paid the subscription fee, then access is granted."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.27 Kenneth Rosen Edition 7 Exercise 1.1 Question 41 (Page No. 16) top

Explain, without using a truth table, why $(p \vee q \vee r) \wedge (\neg p \vee \neg q \vee \neg r)$ is true when at least one of p, q, and r is true and at least one is false, but is false when all three variables have the same truth value.

kenneth-rosen discrete-mathematics mathematical-logic descriptive

Answer key

8.0.28 Kenneth Rosen Edition 7 Exercise 1.2 Question 7 (Page No. 22) top

Express these system specifications using the propositions p "The message is scanned for viruses" and q "The message was sent from an unknown system" together with logical connectives (including negations).

a. "The message is scanned for viruses whenever the message was sent from an unknown system."
b. "The message was sent from an unknown system but it was not scanned for viruses."
c. "It is necessary to scan the message for viruses whenever it was sent from an unknown system."
d. "When a message is not sent from an unknown system it is not scanned for viruses."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.29 Kenneth Rosen Edition 7 Exercise 1.2 Question 6 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
You can upgrade your operating system only if you have a 32-bit processor running at 1 GHz or faster, at least 1 GB

RAM, and 16 GB free hard disk space, or a 64-bit processor running at 2 GHz or faster, at least 2 GB RAM, and at least 32 GB free hard disk space. Express you answer in terms of
u: "You can upgrade your operating system,"
b_{32}: "You have a 32-bit processor,"
b_{64}:"You have a 64-bit processor," g1: "Your processor runs at 1 GHz or faster,"
g_2: "Your processor runs at 2 GHz or faster,"
r_1: "Your processor has at least 1 GB RAM,"
r_2: "Your processor has at least 2 GB RAM,"
h_{16}: "You have at least 16 GB free hard disk space,"
and h_{32}: "You have at least 32 GB free hard disk space."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.30 Kenneth Rosen Edition 7 Exercise 1.2 Question 4 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
To use the wireless network in the airport you must pay the daily fee unless you are a subscriber to the service.Express your answer in terms of

w: "You can use the wireless network in the airport,"
d: "You pay the daily fee,"
and s: "You are a subscriber to the service."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.31 Kenneth Rosen Edition 7 Exercise 1.2 Question 3 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
You can graduate only if you have completed the requirements of your major and you do not owe money to the university and you do not have an overdue library book. Express your answer in terms of
g: "You can graduate,"
m: "You owe money to the university,"
r: "You have completed the requirements of your major,"
and b: "You have an overdue library book."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.32 Kenneth Rosen Edition 7 Exercise 1.2 Question 2 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
You can see the movie only if you are over 18 years old or you have the permission of a parent. Express your answer in terms of
m: "You can see the movie,"
e: "You are over 18 years old,"
and p: "You have the permission of a parent."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.33 Kenneth Rosen Edition 7 Exercise 1.2 Question 1 (Page No. 22) top

Translate the given statement into propositional logic using the propositions provided.
You cannot edit a protected Wikipedia entry unless you are an administrator.
Express your answer in terms of e: "You can edit a protected Wikipedia entry"
and a: "You are an administrator."

kenneth-rosen discrete-mathematics mathematical-logic

Answer key

8.0.34 Kenneth Rosen Edition 7 Exercise 2.1 Question 16 (Page No. 126) top

Use a Venn diagram to illustrate the relationships $A \subset B$ and $A \subset C$.

kenneth-rosen discrete-mathematics set-theory&algebra

8.0.35 Kenneth Rosen Edition 7 Exercise 1.1 Question 2 (Page No. 12) top

Which of these are propositions?What are the truth values of those that are propositions?

A. Do not pass go.
B. What time is it?
C. There are no black flies in Maine.
D. 4 + x = 5.
E. The moon is made of green cheese.
F. $2^n \geq 100$

kenneth-rosen mathematical-logic discrete-mathematics

Answer key

8.0.36 Kenneth Rosen Edition 7 Exercise 2.3 Question 51 (Page No. 154) top

Show that if x is a real number and n is an integer, then

a. $x < n$ if and only if $\lfloor x \rfloor < n$
b. $n < x$ if and only if $n <= \lfloor x \rfloor$

kenneth-rosen discrete-mathematics set-theory&algebra

8.1 First Order Logic (1) top

8.1.1 First Order Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 31 (Page No. 54) top

Suppose that the domain of $Q(x, y, z)$ consists of triples x, y, z, where $x = 0, 1$ or 2 , $y = 0$ or 1, and $z = 0$ or 1. Write out these propositions using disjunctions and conjunctions.

$a) \ \forall y \quad Q(0, y, 0)$
$b) \ \exists x \quad Q(x, 1, 1)$
$c) \ \exists z \ \neg Q(0, 0, z)$
$d) \ \exists x \ \neg Q(x, 0, 1)$

kenneth-rosen discrete-mathematics mathematical-logic first-order-logic

8.2 Logical Reasoning (1) top

8.2.1 Logical Reasoning: Kenneth Rosen Edition 7 Exercise 1.2 Question 19 (Page No. 23) top

Relate to inhabitants of the island of knights and knaves created by Smullyan, where knights always tell the truth and knaves always lie. You encounter two people, A and B. Determine, if possible, what A and B are if they address you in the ways described. If you can not determine what these people are, can you draw any conclusions ?

A says “At least one of us is a knave ” and B says nothing.

kenneth-rosen discrete-mathematics mathematical-logic descriptive logical-reasoning

Answer key

8.3 Propositional Logic (184) top

8.3.1 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.2 Question 1 (Page No. 22) top

You cannot edit a protected Wikipedia entry unless you are an administrator. Express your answer in terms of e: “You can edit a protected Wikipedia entry” and a: “You are an administrator.”

the answer given is e----> a

This question seems to be silly. But I am getting so much confused. why can't the answer be a--->e.
If the user is an Administrator, he can edit the wikipedia entry.

please clarify.

discrete-mathematics kenneth-rosen propositional-logic

Answer key

8.3.2 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.2 Question 33 (Page No. 24) top

Steve would like to determine the relative salaries of three coworkers using two facts. First, he knows that if Fred is not the highest paid of the three, then Janice is. Second, he knows that if Janice is not the lowest paid, then Maggie is paid the most. Is it possible to determine the relative salaries of Fred, Maggie, and Janice from what Steve knows? If so, who is paid the most and who the least? Explain your reasoning.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.3 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.2 Question 34 (Page No. 24) top

Five friends have access to a chat room. Is it possible to determine who is chatting if the following information is known? Either Kevin or Heather, or both, are chatting. Either Randy or Vijay, but not both, are chatting. If Abby is chatting, so is Randy. Vijay and Kevin are either both chatting or neither is. If Heather is chatting, then so are Abby and Kevin. Explain your reasoning.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.4 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.2 Question 35 (Page No. 24) top

A detective has interviewed four witnesses to a crime. From the stories of the witnesses the detective has concluded that if the butler is telling the truth then so is the cook; the cook and the gardener cannot both be telling the truth; the gardener and the handyman are not both lying; and if the handyman is telling the truth then the cook is lying. For each of the four persons can the detective determine whether that person is telling the truth or lying ? Explain your reasoning.

kenneth-rosen discrete-mathematics propositional-logic

Answer key

8.3.5 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 1 (Page No. 34) top

Use truth tables to verify these equivalences.

a. $P \wedge T \equiv P$
b. $P \vee F \equiv P$
c. $P \wedge F \equiv F$
d. $P \vee T \equiv T$
e. $P \vee P \equiv P$
f. $P \wedge P \equiv P$

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8.3.6 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 14 (Page No. 35) top

Determine whether $(\sim p \wedge (p \rightarrow q)) \rightarrow \sim q$ is a tautology.

kenneth-rosen discrete-mathematics propositional-logic mathematical-logic

Answer key

8.3.7 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 15 (Page No. 34) top

Determine whether $(\sim q \wedge (p \rightarrow q)) \rightarrow \sim p$ is a tautology.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.8 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 16 (Page No. 35) top

Show that $p \leftrightarrow q$ and $(p \wedge q) \vee (\sim p \wedge \sim q)$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.9 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 17 (Page No. 35) top

Show that $\sim(p \leftrightarrow q)$ and $p \leftrightarrow \sim q$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.10 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 18 (Page No. 35) top

show that $p \rightarrow q$ and $\sim q \rightarrow \sim p$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.11 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 19 (Page No. 35) top

Show that $\sim p \leftrightarrow q$ and $p \leftrightarrow \sim q$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.12 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 2 (Page No. 34) top

Show that $\sim(\sim p)$ and p are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.13 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 20 (Page No. 35) top

Show that $\sim(p \oplus q)$ and $p \leftrightarrow q$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.14 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 21 (Page No. 35) top

Show that $\sim(p \leftrightarrow q)$ and $\sim p \leftrightarrow q$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.15 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 22 (Page No. 35) top

Show that $(p \rightarrow q) \wedge (p \rightarrow r)$ and $p \rightarrow (q \wedge r)$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.16 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 23 (Page No. 35) top

Show that $(p \rightarrow r) \wedge (q \rightarrow r)$ and $(p \vee q) \rightarrow r$ are logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.17 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 24 (Page No. 35) top

Show that $(p \rightarrow q) \vee (p \rightarrow r)$ and $p \rightarrow (q \vee r)$ are logically equivalent.

kenneth-rosen discrete-mathematics propositional-logic mathematical-logic

Answer key

8.3.18 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 25 (Page No. 35) top

Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent.

kenneth-rosen discrete-mathematics propositional-logic mathematical-logic

Answer key

8.3.19 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 26 (Page No. 35) top

Show that $\sim p \to (q \to r)$ and $q \to (p \vee r)$ are logically equivalent.

kenneth-rosen discrete-mathematics propositional-logic mathematical-logic

Answer key

8.3.20 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 27 (Page No. 35) top

Show that $p \leftrightarrow q$ and $(p \to q) \wedge (q \to p)$ are logically equivalent.

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Answer key

8.3.21 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 28 (Page No. 35) top

Show that $p \leftrightarrow q$ and $\sim p \leftrightarrow \sim q$ are logically equivalent.

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Answer key

8.3.22 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 3 (Page No. 34) top

Use truth tables to verify the commutative laws

a. $p \vee q \equiv q \vee p$
b. $p \wedge q \equiv q \wedge p$

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.23 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 31 (Page No. 35) top

Show that $(p \to q) \to r$ and $p \to (q \to r)$ are not logically equivalent.

kenneth-rosen discrete-mathematics propositional-logic mathematical-logic

Answer key

8.3.24 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 32 (Page No. 35) top

Show that $(p \wedge q) \to r$ and $(p \to r) \wedge (q \to r)$ are not logically equivalent.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

8.3.25 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 33 (Page No. 35) top

Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.

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Answer key

8.3.26 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 34 (Page No. 35) top

Find the dual of each of these compound propositions.

a. $p \vee \sim q$

b. $p \wedge (q \vee (r \wedge T))$
c. $(p \wedge \sim q) \vee (q \wedge F)$

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Answer key

8.3.27 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 35 (Page No. 35) top

Find the dual of each of these compound propositions.

a. $p \wedge \sim q \wedge \sim r$
b. $(p \wedge q \wedge r) \vee s$
c. $(p \vee F) \wedge (q \vee T)$

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Answer key

8.3.28 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 39 (Page No. 35) top

Why are the duals of two equivalent compound propositions also equivalent, where these compound propositions contain only the operators $\wedge, \vee, \sim$?

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8.3.29 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 4 (Page No. 34) top

Use truth tables to verify the associative laws.

a. $(p \vee q) \vee r \equiv p \vee (q \vee r)$.
b. $(p \wedge q) \wedge \equiv p \wedge (q \wedge r)$.

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8.3.30 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 44 (Page No. 36) top

Show that $\sim$ and $\wedge$ form a functionally complete collection of logical operators. [Hint: First use a De Morgan law to show that $p \vee q$ is logically equivalent to $(\sim (\sim p \wedge \sim q))$.]

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic difficult

8.3.31 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 45 (Page No. 36) top

Show that $\sim$ and $\vee$ form a functionally complete collection of logical operators.

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic difficult descriptive

8.3.32 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 47 (Page No. 36) top

Show that $p \uparrow q$ is logically equivalent to $\sim (p \wedge q)$.

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Answer key

8.3.33 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 49 (Page No. 36) top

Show that $p \downarrow q$ is logically equivalent to $\sim (p \vee q)$.

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Answer key

8.3.34 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 5 (Page No. 34) top

Use a truth table to verify the distributive law

$p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r).$

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8.3.35 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 6 (Page No. 34) top

Use a truth table to verify the first De Morgan law

$\sim (p \wedge q) \equiv \sim p \vee \sim q$

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8.3.36 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 7 (Page No. 34) top

Use De Morgan's laws to find negation of each of the following statements.

a. Jan is rich and happy.
b. Carlos will bicycle or run tomorrow.
c. Mei walks or takes the bus to class.
d. Ibrahim is smart and hard working.

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8.3.37 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 8 (Page No. 35) top

Use De Morgan's laws to find the negation of each of the following statements.

a. Kwame will take a job in industry or go to graduate school.
b. Yoshiko knows Java and calculus.
c. James is young and strong.
d. Rita will move to Oregon or Washington.

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Answer key

8.3.38 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.3 Question 9 (Page No. 35) top

Show that each of these conditional statements is a tautology by using truth tables.

a. $(p \wedge q) \rightarrow p$
b. $p \rightarrow (p \vee q)$
c. $\sim p \rightarrow (p \rightarrow q)$
d. $(p \wedge q) \rightarrow (p \rightarrow q)$
e. $\sim (p \rightarrow q) \rightarrow p$
f. $\sim (p \rightarrow p) \rightarrow \sim q$

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Answer key

8.3.39 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 1 (Page No. 53) top

Let $P(x)$ denote the statement "$x <= 4$" . What are these truth values?

a. $P(0)$
b. $P(4)$
c. $P(6)$

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Answer key

8.3.40 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 10 (Page No. 53) top

Let $C(x)$ be the statement "x has a cat," let $D(x)$ be the statement "x has a dog," and let $F(x)$ be the statement "x has a ferret." Express each of these statements in terms of $C(x), D(x), F(x)$, quantifiers, and logical connectives.

Let the domain consist of all students in your class.

a. A student in your class has a cat, a dog, and a ferret.
b. All students in your class have a cat, a dog, or a ferret.
c. Some student in your class has a cat and a ferret, but not a dog.
d. No student in your class has a cat, a dog, and a ferret.
e. For each of the three animals, cats,dogs, and ferrets, there is a student in your class who has this animal as a pet.

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8.3.41 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 11 (Page No. 53) top

Let $P(x)$ be the statement "$x = x^2$". If the domain consists of the integers, what are these truth values?

a. $P(0)$ b. $P(1)$ c. $P(2)$ d. $P(-1)$ e. $\exists x P(x)$

f. $\forall x P(x)$

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Answer key

8.3.42 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 12 (Page No. 53) top

Let $Q(x)$ be the statement "$x + 1 > 2x$." If the domain consists of all integers, what are these truth values?

a. $Q(0)$
b. $Q(-1)$
c. $Q(1)$
d. $\exists x Q(x)$
e. $\forall x Q(x)$
f. $\exists x \sim Q(x)$
g. $\forall x \sim Q(x)$

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8.3.43 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 13 (Page No. 53) top

Determine the truth value of each of these statements if the domain consists of all integers.

a. $\forall n(n + 1 > n)$
b. $\exists n(2n = 3n)$
c. $\exists n(n = -n)$
d. $\forall n(3n <= 4n)$

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8.3.44 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 14 (Page No. 53) top

Determine the truth value of each of these statements if the domain consists of all real numbers.

a. $\exists x(x^3 = -1)$
b. $existsx(x^4 < x^2)$
c. $\forall x((-x)^2 = x^2)$
d. $\forall x(2x > x)$

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Answer key

8.3.45 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 15 (Page No. 53) top

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a. $\forall n(n^2 >= 0)$
b. $\exists n(n^2 = 2)$
c. $\forall n(n^2 >= n)$
d. $\exists n(n^2 < 0)$

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Answer key

8.3.46 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 16 (Page No. 53) top

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a. $\exists x(x^2 = 2)$
b. $\exists x(x^2 = -1)$
c. $\exists x(x^2 + 2 >= 1)$
d. $\forall x(x^2 \neq x)$

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8.3.47 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 17 (Page No. 53) top

Suppose that the domain of the propositional function $P(x)$ consists of the integers $0, 1, 2, 3, 4$. Write out each of these propositions using disjunctions, conjunctions, and negations.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\exists x \sim P(x)$
d. $\forall x \sim P(x)$
e. $\sim \exists x P(x)$
f. $\sim \forall x P(x)$

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Answer key

8.3.48 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 18 (Page No. 53) top

Suppose that the domain of the propositional function $P(x)$ consists of the integers $-2, -1, 0, 1, 2$. Write out each of these propositions using disjunctions, conjunctions, and negations.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\exists x \sim p(x)$
d. $\forall x \sim P(x)$
e. $\sim \exists x P(x)$
f. $\sim \forall x P(x)$

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8.3.49 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 19 (Page No. 54) top

Suppose that the domain of the propositional function $P(x)$ consists of the integers $1, 2, 3, 4, 5$. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\sim \exists x P(x)$
d. $\sim \forall x P(x)$
e. $\forall x((x \neq 3) \to P(x)) \vee \exists x \sim P(x)$

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8.3.50 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 2 (Page No. 53) top

Let $P(x)$ be the statement "The word x contains the letter a." what are these truth values?

a. P(orange)
b. P(lemon)
c. P(true)
d. P(false)

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Answer key

8.3.51 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 20 (Page No. 54) top

Suppose that the domain of the propositional function $P(x)$ consists of $-5, -3, -1, 1, 3, 5$. Express these statements without using quantifiers, instead using only negations, disjunctions, and conjunctions.

a. $\exists x p(x)$
b. $\forall x p(x)$
c. $\forall x((x \neq 1) \to p(x))$
d. $\exists x((x >= 0) \wedge P(x))$
e. $\exists x(\sim p(x)) \wedge \forall x((x < 0) \to p(x))$

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8.3.52 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 21 (Page No. 54) top

For each fo these statements find a domain for which the statements is true and a domain for which the statement is false.

a. Everyone is studying discrete mathematics.
b. Everyone is older than 21 years.
c. Everyone two people have the same mother.
d. No two different people have the same grandmother.

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Answer key

8.3.53 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 22 (Page No. 54) top

For each of these statements find a domain for which the statement is true and a domain for which the statement is false.

a. Everyone speak Hindi.
b. There is someone older than 21 years.
c. Everyone two people have the same first name.
d. Someone knows more than two other people.

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8.3.54 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 26 (Page No. 54) top

Translate each of these statements into logical expression in three different ways by varying the domain and by using predicates with one and with two variables.

a. Someone in your school has visited Uzbekistan.
b. Everyone in your class has studied calculus and C++.
c. No one in your school owns both a bicycle and a motorcycle.
d. There is a person in your school who is not happy.
e. Everyone in your school was born in the twentieth century.

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8.3.55 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 27 (Page No. 54) top

Translate each of these statements into logical expression in three different ways by varying the domain and by using predicates with one and with two variables.

a. A student in your school has lived in Vietnam.
b. There is a student in your school who can not speak Hindi.
c. A student in your school knows Java, Prolog, and C++.
d. Everyone in your class enjoys Thai food.
e. Someone in your class does not play hockey.

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8.3.56 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 28 (Page No. 54) top

Translate each of these statements into logical expression using predicates, quantifiers, and logical connectives.

a. Something is not in the correct place.
b. All tools are in the correct place and are in excellent condition.
c. Everyone is in the correct place and in excellent condition.
d. Nothing is in the correct place and is in excellent condition.
e. One of your tools is not in the correct, but it is in excellent condition.

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8.3.57 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 30 (Page No. 54) top

Suppose the domain of the propositional function $P(x,y)$ consists of pairs x and y , where x is 1,2 or 3 and y is 1,2 or 3 . Write out these propositions using disjunctions and conjunctions.

a. $\exists x P(x,3)$
b. $\forall y P(1,y)$
c. $\exists y \sim p(2,y)$
d. $\forall x \sim P(x,2)$

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8.3.58 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 32 (Page No. 55) top

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a. All dogs have fleas.
b. There is horse that can add.
c. Every koala can climb.
d. No monkey can speak French.
e. There exists a pig that can swim and catch fish.

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8.3.59 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 33 (Page No. 55) top

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of quantifier. Next, express the negation in simple English. (Do not simply use the phrase “It is not the case that.”)

a. Some old dogs can learn new tricks.
b. No rabbit knows calculus.
c. Every bird can fly.
d. There is no dog that can talk.
e. There is no one in this class who knows French and Russian.

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8.3.60 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 33 (Page No. 55) top

Express the negation of these propositions using quantifiers, and then express the negation in English.

a. Some drivers do not obey the speed limit.
b. All Swedish movies are serious.
c. No one can keep a secret.
d. There is someone in this class who does not have a good attitude.

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8.3.61 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 35 (Page No. 55) top

Find a counterexample, if possible, to these universallyquantified statements, where the domain for all variablesconsists of all integers.

a. $\forall x(x^2 >= x)$
b. $\forall x(x > 0 \vee x < 0)$
c. $\forall x(x = 1)$

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8.3.62 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 36 (Page No. 55) top

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all real numbers.

a. $\forall x(x^2 \neq x)$
b. $\forall x(x^2 \neq 2)$
c. $\forall x(|x| > 0)$

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8.3.63 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 37 (Page No. 55) top

Express each of these statements using predicates and quantifiers.

a. A passenger on an airline qualifies as an elite flyer if the passenger flies more than 25,000 miles in a year or takes more than 25 flights during that year.
b. A man qualifies for the marathon if his best previous time is less than 3 hours and a woman qualifies for the marathon if her best previous time is less than3.5 hours.
c. A student must take at least 60 course hours, or at least 45 course hours and write a master's thesis, and receive a grade no lower than a B in all required courses,to receive a master's degree.
d. There is a student who has taken more than 21 credit hours in a semester and received all A's.

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8.3.64 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 38 (Page No. 55) top

Translate these system specifications into English where the predicate $S(x, y)$ is "x is in state y " and where the domain for x and y consists of all system and all possible states, respectively.

a. $\exists x S(x, open)$
b. $\forall x S(x, malfunctioning) \vee S(x, diagnostic)$
c. $\exists x S(x, open) \vee \exists x S(x, diagnostic)$
d. $\exists x \sim S(x, available)$
e. $\forall x \sim S(x, working)$

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8.3.65 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 39 (Page No. 55) top

Translate these specifications into English where $F(p)$ is"Printer p is out of service," $B(p)$ is "Printer p is busy," $L(j)$ is "Print job j is lost," and $Q(j)$is "Print job j is queued."

a. $\exists p(F(p) \wedge B(p)) \rightarrow \exists jL(j)$
b. $\forall B(p) \rightarrow \exists jQ(j)$
c. $\exists j(Q(j) \wedge L(j)) \rightarrow \exists pF(p)$
d. $(\forall pB(p) \wedge \forall jQ(j)) \rightarrow \exists jL(j)$

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8.3.66 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 40 (Page No. 55) top

Express each of these system specifications using predicates, quantifiers, and logical connectives.

a. When there is less than 30 megabytes free on the hard disk, a warning message is sent to all users.
b. No directories in the file system can be opened and no files can be closed when system errors have been detected.
c. The file system cannot be backed up if there is a user currently logged on.
d. Video on demand can be delivered when there are at least 8 megabytes of memory available and the connection speed is at least 56 kilobits per second.

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8.3.67 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 41 (Page No. 55) top

Express each of these system specifications using predicates, quantifiers, and logical connectives.

a. At least one mail message, among the nonempty set of messages, can be saved if there is a disk with more than 10 kilobytes of free space.
b. Whenever there is an active alert, all queued messages are transmitted.
c. The diagnostic monitor tracks the status of all systems except the main console.
d. Each participant on the conference call whom the host of the call did not put on a special list was billed.

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8.3.68 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 42 (Page No. 55) top

Express each of these system specifications using predicates, quantifiers, and logical connectives.

a. Every user has access to an electronic mailbox.
b. The system mailbox can be accessed by everyone in the group if the file system is locked.
c. The firewall is in a diagnostic state only if the proxy server is in a diagnostic state.
d. At least one router is functioning normally if the throughput is between 100 kbps and 500 kbps and the proxy server is not in diagnostic mode

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8.3.69 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 43 (Page No. 56) top

Determine whether $\forall x(P(x) \rightarrow Q(x))$ and $\forall xP(x) \rightarrow \forall xQ(x)$ are logically equivalent . Justify your answer.

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Answer key

8.3.70 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 44 (Page No. 56) top

Determine whether $\forall x(P(x) \leftrightarrow Q(x))$ and $\forall xP(x) \leftrightarrow \forall xQ(x)$ are logically equivalent . Justify your answer.

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8.3.71 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 45 (Page No. 56) top

Show that $\exists x(P(x) \vee Q(x))$ and $\exists xP(x) \vee \exists xQ(x)$ are logically equivalent.

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Answer key

8.3.72 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 46 (Page No. 56) top

Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

a. $(\forall x P(x)) \lor A \equiv \forall x(P(x) \lor A)$
b. $(\exists x P(x)) \lor A \equiv \exists x(P(x) \lor A)$

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8.3.73 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 47 (Page No. 56) top

Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

a. $(\forall x P(x)) \land A \equiv \forall x(P(x) \land A)$
b. $(\exists x P(x)) \land A \equiv \exists x(P(x) \land A)$

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Answer key

8.3.74 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 49 (Page No. 56) top

Establish these logical equivalences, where x does not occur as a free variable in A. Assume that the domain is nonempty.

a. $\forall x P(x) \rightarrow A \equiv \exists x P(x) \rightarrow A$
b. $\exists x P(x) \rightarrow A \equiv \forall x(P(x) \rightarrow A$

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8.3.75 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 5 (Page No. 53) top

Let $P(x)$ be the statement “x spends more than five hours every weekday in class.” where the domain for x consists of all students. Express each of these qualifications in English.

a. $\exists x P(x)$
b. $\forall x P(x)$
c. $\exists x \sim p(x)$
d. $\forall x \sim P(x)$

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Answer key

8.3.76 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 50 (Page No. 56) top

Show that $\forall x P(x) \lor \forall x Q(x)$ and $\forall x(P(x) \lor Q(x))$ are not logically equivalent.

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Answer key

8.3.77 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 51 (Page No. 56) top

Show that $\exists x P(x) \land \exists x Q(x)$ and $\exists x(P(x) \land Q(x))$ are not logically equivalent.

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8.3.78 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 52 (Page No. 56) top

As mentioned in the text, the notation $\exists \sim x P(x)$ denotes “There exists a unique x such that $P(x)$ is true.”If the domain consists of all integers, what are the truth values of these statements?

a. $\exists \sim x(x > 1)$
b. $\exists \sim x(x^2 = 1)$
c. $\exists x(x + 3 = 2x)$
d. $\exists \sim x(x = x + 1)$

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Answer key

8.3.79 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 53 (Page No. 56) top

What are the truth values of these statements?

a. $\exists \sim xP(x) \rightarrow \exists xP(x)$
b. $\forall xP(x) \rightarrow \exists \sim xP(x)$
c. $\exists \sim x \sim P(x) \rightarrow \sim \forall xP(x)$

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8.3.80 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 57 (Page No. 56) top

Suppose that Prolog facts are used to define the predicates mother (M, Y) and father (F, X), which represent that M is the mother of Y and F is the father of X, respectively. Give a Prolog rule to define the predicate sibling (X, Y), which represent that X, Y are siblings (that is, have the same mother and the same father.)

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8.3.81 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 58 (Page No. 56) top

Suppose that Prolog facts are used to define the predicates mother (M, Y) and father (F,X), which represent that M is the mother of Y and F is the father of X respectively. Give a Prolog rule to define the predicate grandfather (X, Y), which represent that X is the grandfather of Y.

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8.3.82 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 59 (Page No. 56) top

Let $P(x), Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively.Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x)$, and $R(x)$, where the domain consists of all people.

a. No professors are ignorant.
b. All ignorant people are vain.
c. No professors are vain.
d. Does (c) follow from (a) and (b)?

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8.3.83 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 6 (Page No. 53) top

Let $N(x)$ be the statements "x has visited North Dakota," where the domain consists of the students in your school. Express each of these quantifications in English.

a. $\exists xN(x)$
b. $\forall xN(x)$
c. $\sim \exists xN(x)$
d. $\exists x \sim N(x)$
e. $\sim \forall xN(x)$
f. $\forall x \sim N(x)$

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Answer key

8.3.84 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 60 (Page No. 56) top

Let$P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a baby," "x is logical," "x is able to manage a crocodile,"and "$x$is despised," respectively. Suppose that the domain consists of all people. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

a. Babies are illogical.
b. Nobody is despised who can manage a crocodile.
c. Illogical persons are despised.
d. Babies cannot manage crocodiles.
e. Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion

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8.3.85 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 62 (Page No. 56) top

Let $P(x), Q(x), R(x)$, and $S(x)$ be the statements "x is a duck," "x is one of my poultry," "x is an officer,"and "x is willing to waltz," respectively. Express each of these statements using quantifiers; logical connectives; and $P(x), Q(x), R(x)$, and $S(x)$.

a. No ducks are willing to waltz.
b. No officers ever decline to waltz.
c. All my poultry are ducks.
d. My poultry are not officers.
e. Does (d) follow from (a), (b), and (c)? If not, is there a correct conclusion?

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8.3.86 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 7 (Page No. 53) top

Translate these statements into English, where $C(x)$ is "x is comedian" and $F(x)$ is "x is funny" and the domain consists of all poeple.

a. $\forall x(C(x) \rightarrow F(x))$
b. $\forall x(C(x) \wedge F(x))$
c. $\exists x(C(x) \rightarrow F(x))$
d. $\exists x(C(x) \wedge F(x))$

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8.3.87 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.4 Question 8 (Page No. 53) top

Translate these statements into English, where $R(x)$ is "x is a rabbit" and $H(x)$ is "x hops" and the domain consists of all animals.

a. $\forall x(R(x) \rightarrow H(x))$
b. $\forall x(R(x) \wedge H(x))$
c. $\exists x(R(x) \rightarrow H(x))$
d. $\exists x(R(x) \wedge H(x))$

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Answer key

8.3.88 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 1 (Page No. 64) top

Translate these statements into English, Where the domain for each variable consists of all real numbers.

a. $\forall x \exists y(x < y)$
b. $\forall x \forall y((x >= 0) \wedge (y >= 0) \rightarrow (xy >= 0))$
c. $\forall x \forall y \exists z(xy = z)$

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8.3.89 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 10 (Page No. 65) top

Let $F(x, y)$ be the statement "x can fooly," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

a. Everybody can fool Fred.
b. Evelyn can fool everybody.Everybody can fool somebody.
c. There is no one who can fool everybody.
d. Everyone can be fooled by somebody.
e. No one can fool both Fred and Jerry.
f. Nancy can fool exactly two people.
g. There is exactly one person whom everybody can fool.
h. No one can fool himself or herself.
i. There is someone who can fool exactly one person besides himself or herself.

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8.3.90 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 11 (Page No. 65) top

Let$S(x)$ be the predicate "x is a student,"$F(x)$ the predicate "x is a faculty member," and $A(x, y)$ the predicate"x has asked y a question," where the domain consists of all people associated with your school. Use quantifiers to express each of these statements.

a. Lois has asked Professor Michaels a question.
b. Every student has asked Professor Gross a question.
c. Every faculty member has either asked Professor Miller a question or been asked a question by Professor Miller.
d. Some student has not asked any faculty member a question
e. There is a faculty member who has never been asked a question by a student.
f. Some student has asked every faculty member a question.
g. There is a faculty member who has asked every other faculty member a question.
h. Some student has never been asked a question by a faculty member

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8.3.91 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 12 (Page No. 65) top

Let $I(x)$ be the statement "x has an Internet connection"and $C(x, y)$ be the statement "x and y have chatted over the Internet," where the domain for the variables x and y consists of all students in your class. Use quantifiers to express each of these statements.

a. Jerry does not have an Internet connection.
b. Rachel has not chatted over the Internet with Chelsea.
c. Jan and Sharon have never chatted over the Internet.
d. No one in the class has chatted with Bob.
e. Sanjay has chatted with everyone except Joseph.
f. Someone in your class does not have an Internet connection.
g. Not everyone in your class has an Internet connection.
h. Exactly one student in your class has an Internet connection.
i. Everyone except one student in your class has an Internet connection.
j. Everyone in your class with an Internet connection has chatted over the Internet with at least one other student in your class.
k. Someone in your class has an Internet connection but has not chatted with anyone else in your class.
l. There are two students in your class who have not chatted with each other over the Internet.
m. There is a student in your class who has chatted with everyone in your class over the Internet.
n. There are at least two students in your class who have not chatted with the same person in your class.
o. There are two students in the class who between them have chatted with everyone else in the class.

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8.3.92 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 13 (Page No. 66) top

Let $M(x, y)$ be "x has sent y an e-mail message" and $T(x, y)$ be "x has telephoned y," where the domain consists of all students in your class. Use quantifiers to express each of these statements. (Assume that all e-mail messages that were sent are received, which is not the way things often work.)

a. Chou has never sent an e-mail message to Koko.
b. Arlene has never sent an e-mail message to or telephoned Sarah.
c. José has never received an e-mail message from Deborah.
d. Every student in your class has sent an e-mail message to Ken.
e. No one in your class has telephoned Nina.
f. Everyone in your class has either telephoned Avi orsent him an e-mail message.
g. There is a student in your class who has sent everyone else in your class an e-mail message.
h. There is someone in your class who has either sent an e-mail message or telephoned everyone else in your class.
i. There are two different students in your class who have sent each other e-mail messages.
j. There is a student who has sent himself or herself ane-mail message.
k. There is a student in your class who has not received an e-mail message from anyone else in the class and who has not been called by any other student in the class.
l. Every student in the class has either received an e-mail message or received a telephone call from an-other student in the class.
m. There are at least two students in your class such that one student has sent the other e-mail and the second student has telephoned the first student.
n. There are two different students in your class who between them have sent an e-mail message to or telephoned everyone else in the class.

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8.3.93 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 15 (Page No. 66) top

Use quantifiers and predicates with more than one variable to express these statements.

a. Every computer science student needs a course in discrete mathematics
b. There is a student in this class who owns a personal computer.
c. Every student in this class has taken at least one computer science course.
d. There is a student in this class who has taken at least one course in computer science.e)Every student in this class has been in every building on campus.
e. There is a student in this class who has been in every room of at least one building on campus.
f. Every student in this class has been in at least one room of every building on campus.

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8.3.94 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 16 (Page No. 66) top

A discrete mathematics class contains 1 mathematics ma-jor who is a freshman, 12 mathematics majors who are sophomores, 15 computer science majors who are sophomores, 2 mathematics majors who are juniors, 2 computer science majors who are juniors, and 1 computer science major who is a senior. Express each of these statements in terms of quantifiers and then determine its truth value.

a. There is a student in the class who is a junior.
b. Every student in the class is a computer science major.
c. There is a student in the class who is neither a mathematics major nor a junior.
d. Every student in the class is either a sophomore or a computer science major.
e. There is a major such that there is a student in the class in every year of study with that major

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8.3.95 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 17 (Page No. 66) top

Express each of these system specifications using predicates, quantifiers, and logical connectives, if necessary.

a. Every user has access to exactly one mailbox.
b. There is a process that continues to run during all error conditions only if the kernel is working correctly.
c. All users on the campus network can access all web-sites whose url has a .edu extension.
d. There are exactly two systems that monitor every re-mote server.

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8.3.96 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 18 (Page No. 66) top

Express each of these system specifications using predicates, quantifiers, and logical connectives, if necessary.

a. At least one console must be accessible during every fault condition.
b. The e-mail address of every user can be retrieved whenever the archive contains at least one message sent by every user on the system.
c. For every security breach there is at least one mechanism that can detect that breach if and only if there is a process that has not been compromised.
d. There are at least two paths connecting every two dis-tinct endpoints on the network.
e. No one knows the password of every user on the system except for the system administrator, who knows all passwords.

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Express each of these statements using mathematical and logical operators, predicates, and quantifiers, where the domain consists of all integers.

a. The sum of two negative integers is negative.
b. The difference of two positive integers is not necessarily positive.
c. The sum of the squares of two integers is greater than or equal to the square of their sum.

d. The absolute value of the product of two integers is the product of their absolute values.

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8.3.98 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 2 (Page No. 64) top

Translate these statements into English, where the domain for each variable consists of all real numbers.

a. $\exists x \forall y(xy = y)$
b. $\forall x \forall y(((x >= 0) \wedge (y < 0)) \rightarrow (x - y > 0))$
c. $\forall x \forall y \exists z(x = y + z)$

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8.3.99 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 20 (Page No. 66) top

Express each of these statements using predicates, quantifiers, logical connectives, and mathematical operators where the domain consists of all integers.

a. The product of two negative integers is positive.
b. The average of two positive integers is positive.
c. The difference of two negative integers is not necessarily negative.
d. The absolute value of the sum of two integers does not exceed the sum of the absolute values of these integers.

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8.3.100 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 23 (Page No. 66) top

Express each of these mathematical statements using predicates, quantifiers, logical connectives, and mathematical operators.

a. The product of two negative real numbers is positive.
b. The difference of a real number and itself is zero.
c. Every positive real number has exactly two square roots.
d. A negative real number does not have a square root that is a real number

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8.3.101 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 24 (Page No. 66) top

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a. $\exists x \forall y(y = y)$
b. $\forall x \forall y((x >= 0) \wedge (y < 0)) \rightarrow (x - y > 0)$
c. $\exists x \exists y(((x <= 0) \wedge (y <= 0)) \wedge (x - y > 0))$
d. $\forall x \forall y((x \neq 0) \wedge (y \neq 0) \leftrightarrow (xy \neq 0))$

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8.3.102 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 25 (Page No. 66) top

Translate each of these nested quantifications into an English statement that expresses a mathematical fact. The domain in each case consists of all real numbers.

a. $\exists x \forall y(xy = y)$
b. $\forall x \forall y(((x < 0) \wedge (y < 0)) \rightarrow (xy > 0))$
c. $\exists x \exists y((x^2 > y) \wedge (x < y))$
d. $\forall x \forall y \exists z(x + y = z)$

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8.3.103 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 26 (Page No. 66) top

Let $Q(x, y)$ be the statement " $x + y = x - y$. " If the domain for both variables consists of all integers, what are the truth values?

a. $Q(1,1)$
b. $Q(2,0)$
c. $\forall y Q(1,y)$
d. $\exists x Q(x,2)$
e. $\exists x \exists y Q(x,y)$
f. $\forall x \exists y Q(x,y)$
g. $\exists x \forall y Q(x,y)$
h. $\forall y \exists x Q(x,y)$
i. $\forall x \forall y Q(x,y)$

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Answer key

8.3.104 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 27 (Page No. 66) top

Determine the truth value of each of these statements if the domain for all variables consists of all integers.

a. $\forall n \exists m (n^2 < m)$
b. $\exists n \forall m (n < m^2)$
c. $\forall n \exists m (n + m = 0)$
d. $\exists n \forall m (nm = m)$
e. $\exists n \exists m (n^2 + m^2 = 5)$
f. $\exists n \exists m (n^2 + m^2 = 6)$
g. $\exists n \exists m (n + m = 4 \wedge n - m = 1)$
h. $\exists n \exists m (n + m = 4 \wedge n - m = 2)$
i. $\forall n \forall m \exists p (p = (m + n)/2)$

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8.3.105 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 28 (Page No. 66) top

Determine the truth value of each of these statements if the domain of each variable consists of all real numbers.

a. $\forall x \exists y (x^2 = y)$
b. $\forall x \exists y (x = y^2)$
c. $\exists x \forall y (xy = 0)$
d. $\exists x \exists y (x + y \neq y + x)$
e. $\forall x (x \neq 0 \rightarrow \exists y (xy = 1))$
f. $\exists x \forall y (y \neq 0 \rightarrow xy = 1)$
g. $\forall x \exists y (x + y = 1)$
h. $\exists x \exists y (x + 2y = 2 \wedge 2x + 4y = 5)$
i. $\forall x \exists y (x + y = 2 \wedge 2x - y = 1)$
j. $\forall x \forall y \exists z (z = (x + y)/2)$

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8.3.106 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 29 (Page No. 66) top

Suppose the domain of the propositions function $P(x, y)$ consists of pairs x and y, where x is $1, 2$ or 3 and y is $1, 2$ or 3. Write out these propositions using disjunctions and conjunctions.

a. $\forall x \forall y P(x,y)$
b. $\exists x \exists y P(x,y)$
c. $\exists x \forall y P(x,y)$
d. $\forall y \exists x P(x,y)$

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8.3.107 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 3 (Page No. 64) top

Let $Q(x, y)$ be the statement "x has sent an e-mail message toy," where the domain for both x and y consists of all students in your class. Express each of these quantifications in English.

a. $\exists x \exists y Q(x,y)$
b. $\exists x \forall y Q(x,y)$
c. $\forall x \exists y Q(x,y)$
d. $\exists y \forall x Q(x,y)$
e. $\forall y \exists x Q(x,y)$
f. $\forall x \forall y Q(x,y)$

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8.3.108 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 30 (Page No. 66) top

Rewrite each of these statements so that negations appear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a. $\sim \exists y \exists x P(x, y)$
b. $\sim \forall x \exists y P(x, y)$
c. $\sim \exists y(Q(y) \wedge \forall x \sim R(x, y))$
d. $\sim \exists y(\exists x R(x, y) \vee \forall x S(x, y))$
e. $\sim \exists y(\forall x \exists z T(x, y, z) \vee \exists x \forall z U(x, y, z))$

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8.3.109 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 31 (Page No. 66) top

Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a. $\forall x \exists y \forall z T(x, y, z)$
b. $\forall x \exists y P(x, y) \vee \forall x \exists y Q(x, y)$
c. $\forall x \exists y(P(x, y) \wedge \exists z R(x, y, z))$
d. $\forall x \exists y(P(x, y) \rightarrow Q(x, y))$

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8.3.110 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 32 (Page No. 66) top

Express the negations of each of these statements so that all negation symbols immediately precede predicates.

a. $\exists z \forall y \forall x T(x, y, z)$
b. $\exists x \exists y P(x, y) \wedge \forall x \forall y Q(x, y)$
c. $\exists x \exists y(Q(x, y) \leftrightarrow Q(y, x))$
d. $\forall y \exists x \exists z(T(x, y, z) \vee Q(x, y))$

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8.3.111 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 33 (Page No. 66) top

Rewrite each of these statements so that negations ap-pear only within predicates (that is, so that no negation is outside a quantifier or an expression involving logical connectives).

a. $\sim \forall x \forall y P(x, y)$
b. $\sim \forall y \exists x P(x, y)$
c. $\sim \forall y \forall x(P(x, y) \wedge \forall x \forall y Q(x, y))$
d. $\sim \forall x(\exists y \forall z P(x, y, z) \wedge \exists z \forall y P(x, y, z))$

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8.3.112 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 34 (Page No. 66) top

Find a common domain for the variables x, y, and z for which the statement $\forall x \forall y((x \neq y) \rightarrow \forall z(z=x) \vee(z=y))$ is true and another domain for which it is false.

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8.3.113 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 36 (Page No. 68) top

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a. No one has lost more than one thousand dollars playing the lottery.
b. There is a student in this class who has chatted with exactly one other student.
c. No student in this class has sent e-mail to exactly two other students in this class.
d. Some student has solved every exercise in this book.
e. No student has solved at least one exercise in every section of this book.

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8.3.114 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 37 (Page No. 68) top

Express each of these statements using quantifiers. Then form the negation of the statement so that no negation is to the left of a quantifier. Next, express the negation in simple English. (Do not simply use the phrase "It is not the case that.")

a. Every student in this class has taken exactly two mathematics classes at this school.
b. Someone has visited every country in the world except Libya.
c. No one has climbed every mountain in the Himalayas.
d. Every movie actor has either been in a movie with Kevin Bacon or has been in a movie with someone who has been in a movie with Kevin Bacon

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8.3.115 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 39 (Page No. 68) top

Find a counterexample, if possible, to these universally quantified statements, where the domain for all variables consists of all integers.

a. $\forall x \forall y (x^2 = y^2 \rightarrow x = y)$
b. $\forall x \exists y (y^2 = x)$
c. $\forall x \forall y (xy >= x)$

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8.3.116 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 4 (Page No. 64) top

Let $P(x, y)$ be the statement "Student x has taken classy," where the domain for x consists of all students in your class and for y consists of all computer science courses at your school. Express each of these quantifications in English.

a. $\exists x \exists y P(x, y)$
b. $\exists x \forall y P(x, y)$
c. $\forall x \exists y P(x, y)$
d. $\exists y \exists x P(x, y)$
e. $\forall y \exists x P(x, y)$
f. $\forall x \forall y P(x, y)$

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8.3.117 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 40 (Page No. 68) top

Find a counterexample, if possible ,to these universally quantified statements, where the domain for all variables consists of all integers.

a. $\forall x \exists y (x = 1/y)$
b. $\forall x \exists y (y^2 - x < 100)$
c. $\forall x \forall y (x^2 \neq y^3)$

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8.3.118 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 45 (Page No. 68) top

Determine the truth value of the statement $\forall x \exists y (xy = 1)$ if the domain for the variables consists of

a. the nonzero real numbers.
b. the nonzero integers.
c. the positive real numbers.

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8.3.119 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 46 (Page No. 68) top

Determine the truth value of the statement $\exists x \forall y (x <= y^2)$ if the domain for the variables consists of

a. he positive real numbers.
b. the integers.
c. the nonzero real numbers.

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8.3.120 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 47 (Page No. 68) top

Show that the two statements $\sim \exists x \forall y P(x, y)$ and $\forall x \exists y \sim P(x, y)$, where both quantifiers over the first variable in $P(x, y)$ have the same domain , and both quantifiers over the second variable in $P(x, y)$ have the same domain, are logically equivalent.

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8.3.121 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 48 (Page No. 68) top

Show that $\forall x P(x) \vee \forall x Q(x)$ and $\forall x \forall y (P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain, are logically equivalent . (The new variable y is used to combine the quantifications correctly.)

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8.3.122 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 49 (Page No. 68) top

a. Show that $\forall x P(x) \wedge \exists x Q(x)$ is logically equivalent to $\forall x \exists y (P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.
b. Show that $\forall x P(x) \vee \exists x Q(x)$ is equivalent to $\forall x \exists y (P(x) \vee Q(y))$, where all quantifiers have the same nonempty domain.

A statement is in **prenex normal form (PNF)** if and only if it is of the form

$Q_1 x_1 Q_2 x_2 \ldots Q_k x_k P(x_1, x_2, \ldots\ldots, x_k)$,

where each $Q_i, i = 1, 2, \ldots, k,$ is either the existential quantifier or the universal quantifier, and $P(x_1, \ldots, x_k)$ is a predicate involving no quantifiers. For example $\exists x \forall y (P(x, y) \wedge Q(y))$ is in prenex normal form, whereas $\exists x P(x) \vee \forall x Q(x)$ is not (because the quantifiers do not all occur first).Every statement formed from propositional variables,predicates,**T**, and **F** using logical connectives and quantifiers is equivalent to a statement in prenex normal form.

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8.3.123 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.5 Question 9 (Page No. 65) top

Let $L(x, y)$ be the statement "x loves y," where the domain for both x and y consists of all people in the world. Use quantifiers to express each of these statements.

a. Everybody loves Jerry.
b. Everybody loves somebody.
c. There is somebody whom everybody loves.
d. Nobody loves everybody.
e. There is somebody whom Lydia does not love.
f. There is somebody whom no one loves.
g. There is exactly one person whom everybody loves.
h. There are exactly two people whom Lynn loves.
i. Everyone loves himself or herself.
j. There is someone who loves no one besides himself or herself.

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8.3.124 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 13 (Page No. 79) top

For each of these arguments, explain which rules of inference are used for each step.

a. “Doug, a student in this class, knows how to write programs in JAVA. Everyone who knows how to write programs in JAVA can get a high-paying job. There-fore, someone in this class can get a high-paying job.”
b. “Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is a person in this class who cares about ocean pollution.”
c. “Each of the 93 students in this class owns a personal computer. Everyone who owns a personal computer can use a word processing program. Therefore, Zeke,a student in this class, can use a word processing pro-gram.”
d. “Everyone in New Jersey lives within 50 miles of the ocean. Someone in New Jersey has never seen the ocean. Therefore, someone who lives within 50 miles of the ocean has never seen the ocean.”

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8.3.125 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 14 (Page No. 79) top

For each of these arguments, explain which rules of inference are used for each step.

a. “Linda, a student in this class, owns a red convertible.Everyone who owns a red convertible has gotten at least one speeding ticket. Therefore, someone in this class has gotten a speeding ticket.”
b. “Each of five roommates, Melissa, Aaron, Ralph, Veneesha, and Keeshawn, has taken a course in discrete mathematics. Every student who has taken a course indiscrete mathematics can take a course in algorithms.Therefore, all five roommates can take a course in algorithms next year.”
c. “All movies produced by John Sayles are wonderful. John Sayles produced a movie about coal miners.Therefore, there is a wonderful movie about coal miners.”
d. “There is someone in this class who has been to France. Everyone who goes to France visits the Louvre. Therefore, someone in this class has visited the Louvre.

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8.3.126 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 15 (Page No. 79) top

For each of these arguments determine whether the argument is correct or incorrect and explain why.

a. All students in this class understand logic. Xavier is a student in this class. Therefore, Xavier understands logic.
b. Every computer science major takes discrete mathematics. Natasha is taking discrete mathematics.Therefore, Natasha is a computer science major.
c. All parrots like fruit. My pet bird is not a parrot. Therefore, my pet bird does not like fruit.
d. Everyone who eats granola every day is healthy. Linda is not healthy. Therefore, Linda does not eat granola every day.

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8.3.127 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 16 (Page No. 79) top

For each of these arguments determine whether the argument is correct or incorrect and explain why.

a. Everyone enrolled in the university has lived in a dormitory. Mia has never lived in a dormitory. Therefore,Mia is not enrolled in the university.
b. A convertible car is fun to drive. Isaac’s car is not a convertible. Therefore, Isaac’s car is not fun to drive.
c. Quincy likes all action movies. Quincy likes the movie Eight Men Out. Therefore,Eight Men Out is an action movie.
d. All lobstermen set at least a dozen traps. Hamilton is a lobsterman. Therefore, Hamilton sets at least a dozen traps

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8.3.128 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 17 (Page No. 79) top

What is wrong with this argument? Let $H(x)$ be “x is happy.” Given the premise $\exists x H(x)$, we conclude that $H(Lola)$. Therefore, Lola is happy.

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8.3.129 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 18 (Page No. 79) top

What is wrong with this argument? Let $S(x, y)$ be “x is shorter than y.” Given the premise $\exists s S(s, Max)$, it follows that $S(Max, Max)$. Then by existential generalization it follows that $\exists x S(x, x)$, so that someone is shorter than himself.

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8.3.130 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 25 (Page No. 80) top

Justify the rule of universal modus tollens by showing that the premises $\forall x(P(x) \rightarrow Q(x))$ and $\sim Q(a)$ for a particular element a in the domain, imply $\sim P(a)$

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8.3.131 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 26 (Page No. 80) top

Justify the rule of universal transitivity, which states that if $\forall x(P(x) \rightarrow Q(x))$ and $\forall x(Q(x) \rightarrow R(x))$ are true, then $\forall x(P(x) \rightarrow R(x))$ is true, where the domains of all quantifiers are the same.

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8.3.132 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 27 (Page No. 80) top

Use rules of inference to show that if $\forall x(P(x) \rightarrow (Q(x) \wedge S(x)))$ and $\forall x(P(x) \wedge R(x))$ are true, then $\forall x(R(x) \wedge S(x))$ is true.

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8.3.133 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 28 (Page No. 80) top

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$ and $\forall x((\sim P(x) \wedge Q(x)) \rightarrow R(x))$ are true, then $\forall x(\sim R(x) \rightarrow P(x))$ is also true, where the domains of all quantifiers are the same.

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8.3.134 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 29 (Page No. 80) top

Use rules of inference to show that if $\forall x(P(x) \vee Q(x))$, $\forall x(\sim Q(x) \vee S(x)), \forall x(R(x) \rightarrow \sim S(x))$, and $\exists x \sim P(x)$ are true, then $\exists x \sim R(x)$ is true.

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8.3.135 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 3 (Page No. 78) top

What rule of inference is used in each of these arguments?

a. Alice is a mathematics major. Therefore, Alice is either a mathematics major or a computer science major.
b. Jerry is a mathematics major and a computer science major. Therefore, Jerry is a mathematics major.
c. If it is rainy, then the pool will be closed. It is rainy.Therefore, the pool is closed.
d. If it snows today, the university will close. The university is not closed today. Therefore, it did not snow today.
e. If I go swimming, then I will stay in the sun too long.If I stay in the sun too long, then I will sunburn. Therefore, if I go swimming, then I will sunburn

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8.3.136 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 30 (Page No. 80) top

Use resolution to show the hypotheses “Allen is a bad boy or Hillary is a good girl” and “Allen is a good boy or David is happy” imply the conclusion “Hillary is a good girl or David is happy.”

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Answer key

8.3.137 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 31 (Page No. 80) top

Use resolution to show that the hypotheses “It is not raining or Yvette has her umbrella,” “Yvette does not have her umbrella or she does not get wet,” and “It is raining or Yvette does not get wet” imply that “Yvette does not get wet.”

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8.3.138 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 32 (Page No. 80) top

Show that the equivalence $p \wedge \sim p \equiv F$ can be derived using resolution together with the fact that a conditional statement with a false hypothesis is true. [Hint:Let $q = r = F$ in resolution.]

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8.3.139 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 33 (Page No. 80) top

Use resolution to show that the compound proposition $(p \vee q) \wedge (\sim p \vee q) \wedge (p \vee \sim q) \wedge (\sim p \vee \sim q)$ is not satisfiable.

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Answer key

8.3.140 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 34 (Page No. 80) top

The Logic Problem, taken from WFF'N PROOF, The Game of Logic, has these two assumptions:1. "Logic is difficult or not many students like logic."2. "If mathematics is easy, then logic is not difficult."By translating these assumptions into statements involving propositional variables and logical connectives, deter-mine whether each of the following are valid conclusions of these assumptions:

a. That mathematics is not easy, if many students like logic.
b. That not many students like logic, if mathematics is not easy.
c. That mathematics is not easy or logic is difficult.
d. That logic is not difficult or mathematics is not easy.
e. That if not many students like logic, then either mathematics is not easy or logic is not difficult.

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8.3.141 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 35 (Page No. 80) top

Determine whether this argument, taken from Kalish and Montague [KaMo64], is valid.

If Superman were able and willing to prevent evil,he would do so. If Superman were unable to prevent evil, he would be impotent; if he were unwilling to prevent evil, he would be malevolent. Superman does not prevent evil. If Superman exists, he is neither impotent nor malevolent. Therefore, Superman does not exist.

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8.3.142 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 4 (Page No. 78) top

What rule of inference is used in each of these arguments?

a. Kangaroos live in Australia and are marsupials. There-fore, kangaroos are marsupials.
b. It is either hotter than 100 degrees today or the pollution is dangerous. It is less than 100 degrees outside today. Therefore, the pollution is dangerous.
c. Linda is an excellent swimmer. If Linda is an excellent swimmer, then she can work as a lifeguard. Therefore,Linda can work as a lifeguard.
d. Steve will work at a computer company this summer.Therefore, this summer Steve will work at a computer company or he will be a beach bum.
e. If I work all night on this homework, then I can answer all the exercises. If I answer all the exercises, I will understand the material . Therefore ,If I work all night on this homework, Then I will understand the material.

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8.3.143 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 5 (Page No. 78) top

Use rules of inference to show that the hypotheses "Randy works hard," "If Randy works hard, then he is a dull boy,"and "If Randy is a dull boy, then he will not get the job"imply the conclusion "Randy will not get the job."

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8.3.144 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 7 (Page No. 78) top

What rules of inference are used in this famous argument? “All men are mortal. Socrates is a man. Therefore,Socrates is mortal.”

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8.3.145 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 8 (Page No. 78) top

What rules of inference are used in this argument? “No man is an island. Manhattan is an island. Therefore, Manhattan is not a man.”

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8.3.146 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.6 Question 9 (Page No. 78) top

For each of these collections of premises, what relevant conclusion or conclusions can be drawn? Explain the rules of inference used to obtain each conclusion from the premises.

a. “If I take the day off, it either rains or snows.” “I took Tuesday off or I took Thursday off.” “It was sunny on Tuesday.” “It did not snow on Thursday.”
b. “If I eat spicy foods, then I have strange dreams.” “I have strange dreams if there is thunder while I sleep.”“I did not have strange dreams.”
c. “I am either clever or lucky.” “I am not lucky.” “If I am lucky, then I will win the lottery.”
d. “Every computer science major has a personal computer.” “Ralph does not have a personal computer.”“Ann has a personal computer.”
e. “What is good for corporations is good for the United States.” “What is good for the United States is good for you.” “What is good for corporations is for you to buy lots of stuff.”
f. “All rodents gnaw their food.” “Mice are rodents.”“Rabbits do not gnaw their food.” “Bats are not ro-dents.”

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8.3.147 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 10 (Page No. 91) top

Use a direct proof to show that the product of two rational numbers is rational.

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Answer key

8.3.148 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 11 (Page No. 91) top

Prove or disprove that the product of two irrational numbers is irrational.

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Answer key

8.3.149 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 12 (Page No. 91) top

Prove or disprove that the product of a nonzero rational number and an irrational number is irrational.

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8.3.150 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 13 (Page No. 91) top

Prove that if x is irrational, then $1/x$ is irrational.

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8.3.151 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 14 (Page No. 91) top

Prove that if x is rational and $x \neq 0$, then $1/x$ is rational.

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8.3.152 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 15 (Page No. 91) top

Use a proof by contraposition to show that if $x + y \geq 2$,where x and y are real numbers, then $x \geq 1$ or $y \geq 1$.

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Answer key

8.3.153 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 16 (Page No. 91) top

Prove that if m and n are integers and mn is even, then m is even or n is even.

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Answer key

8.3.154 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 17 (Page No. 91) top

Show that if n is an integer and $n^3 + 5$ is odd, then n is even using.

a. a proof by contraposition.
b. a proof by contradiction.

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Answer key

8.3.155 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 19 (Page No. 91) top

Prove the position $P(0)$, where $P(n)$ is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?

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8.3.156 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 20 (Page No. 91) top

Prove the position $P(1)$, where $P(n)$ is the proposition "If n is a positive integer greater than 1, then $n^2 > n$." What kind of proof did you use?

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8.3.157 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 21 (Page No. 91) top

Let $P(n)$ be the proposition "If a and b are positive real numbers, then $(a+b)n \geq a^n + b^n$." Prove that $P(1)$ is true. What kind of proof did you use?

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8.3.158 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 22 (Page No. 91) top

Show that if you pick three socks from a drawer containing just blue socks and black socks, you must get either a pair of blue socks or a pair of black socks.

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8.3.159 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 23 (Page No. 91) top

Show that at least ten of any 64 days chosen must fall on the same day of the week.

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8.3.160 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 24 (Page No. 91) top

Show that at least three of any 25 days chosen must fall in the same month of the year.

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8.3.161 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 25 (Page No. 91) top

Use a proof by contradiction to show that there is no rational number r for which $r^3 + r + 1 = 0$. [Hint:Assume that $r = a/b$ is a root, where a and b are integers and a/b is in lowest terms. Obtain an equation involving integer s by multiplying by b^3. Then look at whether a and b are each odd or even.]

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8.3.162 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 26 (Page No. 91) top

Prove that if n is a positive integer, then n is even if and only if $7n + 4$ is even.

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8.3.163 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 27 (Page No. 91) top

Prove that if n is a positive integer, then n is odd if and only if $5n + 6$ is odd.

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8.3.164 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 28 (Page No. 91) top

Prove that $m^2 = n^2$ if and only if $m = n$ or m = -n.

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8.3.165 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 29 (Page No. 91) top

Prove or disprove that if m and n are integers such that $mn = 1$, then either $m = 1$ and $n = 1$, or else $m = -1$ and $n = -1$.

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8.3.166 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 30 (Page No. 91) top

Show that these three statements are equivalent, where a and b are real numbers:

a. a is less than b,
b. the average of a and b is greater than a, and
c. the average of a and b is less than b.

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8.3.167 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 31 (Page No. 91) top

Show that these statements about the integer x are equivalent:

a. $3x + 2$ is even,
b. $x + 5$ is odd,
c. x^2 is even

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8.3.168 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 32 (Page No. 91) top

Show that these statements about the real number x are equivalent:

a. x is rational,
b. $x/2$ is rational,
c. $3x - 1$ is rational.

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8.3.169 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 33 (Page No. 91) top

Show that these statements about the real number x are equivalent:

x is irrational,

$3x + 2$ is irrational,

$x/2$ is irrational.

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8.3.170 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 34 (Page No. 91) top

Is this reasoning for finding the solutions of the equation $\sqrt{2x^2 - 1 = x}$ correct?

a. $\sqrt{2x^2 - 1 = x}$ is given;
b. $2x^2 - 1 = x^2$, obtained by squaring both sides of (1);
c. $x^2 - 1 = 0$, obtained by subtracting x^2 from both sides of (2);
d. $(x - 1)(x + 1) = 0$, obtained by factoring the left-hand side of $x^2 - 1$;
e. $x = 1$ or $x = -1$,which follows because $ab = 0$ implies that $a = 0$ or $b = 0$

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8.3.171 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 35 (Page No. 91) top

Are these steps for finding the solutions of $\sqrt{x + 3 = 3 - x}$ correct?

a. $\sqrt{x + 3 = 3 - x}$ is given;
b. $x + 3 = x2 - 6x + 9$, obtained by squaring both sides of(1);
c. $0 = x2 - 7x + 6$, obtained by subtracting $x + 3$ from both sides of(2);
d. $0 = (x - 1)(x - 6)$, obtained by factoring the right-hand side of(3);
e. $x = 1$ or $x = 6$,which follows from(4) because $ab = 0$ implies that $a = 0$ or $b = 0$.

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8.3.172 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 36 (Page No. 91) top

Show that the propositions $p1, p2, p3$, and $p4$can be shown to be equivalent by showing that $p1 \leftrightarrow p4, p2 \leftrightarrow p3$, and $p1 \leftrightarrow p3$.

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8.3.173 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 37 (Page No. 91) top

Show that the propositions $p1, p2, p3, p4,$ and $p5$ can be shown to be equivalent by proving that the conditional statements $p1 \rightarrow p4$, $p3 \rightarrow p1$,$p4 \rightarrow p2$,$p2 \rightarrow p5$, and $p5 \rightarrow p3$ are true.

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8.3.174 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 38 (Page No. 92) top

Find a counterexample to the statement that every positive integer can be written as the sum of the squares of three integers

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8.3.175 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 39 (Page No. 92) top

Prove that at least one of the real numbers $a_1, a_2, \ldots, a_n$ is greater than or equal to the average of these numbers.What kind of proof did you use?

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8.3.176 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 4 (Page No. 91) top

Show that the additive inverse, or negative, of an even number is an even number using a direct proof.

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8.3.177 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 41 (Page No. 92) top

Prove that if n is an integer, these four statements are equivalent:

a. n is even, b. $n + 1$ is odd, c. $3n + 1$isodd, d. $3n$ is even.

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8.3.178 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 42 (Page No. 92) top

Prove that these four statements about the integer n are equivalent:

n^2 is odd,

$1-n$ is even,

n^3 is odd,

n^2+1 is even.

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8.3.179 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 5 (Page No. 91) top

Prove that if $m+n$ and $n+p$ are even integers, where $m,n,$and p are integers, then $m+p$ is even. What kind of proof did you use?

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Answer key

8.3.180 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 6 (Page No. 91) top

Use a direct proof to show that the product of two odd numbers is odd.

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8.3.181 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 7 (Page No. 91) top

Use a direct proof to show that every odd integer is the difference of two squares.

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Answer key

8.3.182 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 8 (Page No. 91) top

Prove that if n is a perfect square, then $n+2$ is not a perfect square

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8.3.183 Propositional Logic: Kenneth Rosen Edition 7 Exercise 1.7 Question 9 (Page No. 91) top

Use a proof by contradiction to prove that the sum of an irrational number and a rational number is irrational.

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8.3.184 Propositional Logic: Kenneth Rosen Edition 7 Exercise 2.1 Question 9 (Page No. 125) top

Determine whether each of these statements is true or false.

a. $0 \epsilon \phi$
b. $\phi \epsilon \{0\}$
c. $\{0\} \subset \{\phi\}$
d. $\phi \subset \{0\}$
e. $\{0\} \epsilon \{0\}$
f. $\{0\} \subset \{0\}$
g. $\{\phi\} \subseteq \{\phi\}$

kenneth-rosen discrete-mathematics mathematical-logic propositional-logic

Answer key

Answer Keys

8.0.1	Q-Q	8.0.2	Q-Q	8.0.3	Q-Q	8.0.4	Q-Q	8.0.5	Q-Q
8.0.6	Q-Q	8.0.7	Q-Q	8.0.8	Q-Q	8.0.9	N/A	8.0.10	Q-Q
8.0.11	N/A	8.0.12	Q-Q	8.0.13	Q-Q	8.0.14	Q-Q	8.0.15	Q-Q
8.0.16	Q-Q	8.0.17	N/A	8.0.18	Q-Q	8.0.19	Q-Q	8.0.20	Q-Q

8.0.21	Q-Q
8.0.26	Q-Q
8.0.31	Q-Q
8.0.36	Q-Q
8.3.3	Q-Q
8.3.8	Q-Q
8.3.13	Q-Q
8.3.18	Q-Q
8.3.23	Q-Q
8.3.28	Q-Q
8.3.33	Q-Q
8.3.38	Q-Q
8.3.43	Q-Q
8.3.48	Q-Q
8.3.53	Q-Q
8.3.58	Q-Q
8.3.63	Q-Q
8.3.68	Q-Q
8.3.73	Q-Q
8.3.78	Q-Q
8.3.83	Q-Q
8.3.88	Q-Q
8.3.93	Q-Q
8.3.98	Q-Q
8.3.103	Q-Q
8.3.108	Q-Q
8.3.113	Q-Q
8.3.118	Q-Q
8.3.123	Q-Q
8.3.128	Q-Q
8.3.133	Q-Q
8.3.138	Q-Q
8.3.143	Q-Q
8.3.148	Q-Q
8.3.153	Q-Q
8.3.158	Q-Q
8.3.163	Q-Q
8.3.168	Q-Q
8.3.173	Q-Q
8.3.178	Q-Q
8.3.183	Q-Q

8.0.22	Q-Q
8.0.27	N/A
8.0.32	Q-Q
8.1.1	Q-Q
8.3.4	Q-Q
8.3.9	Q-Q
8.3.14	Q-Q
8.3.19	Q-Q
8.3.24	Q-Q
8.3.29	Q-Q
8.3.34	Q-Q
8.3.39	Q-Q
8.3.44	Q-Q
8.3.49	Q-Q
8.3.54	Q-Q
8.3.59	Q-Q
8.3.64	Q-Q
8.3.69	Q-Q
8.3.74	Q-Q
8.3.79	Q-Q
8.3.84	Q-Q
8.3.89	Q-Q
8.3.94	Q-Q
8.3.99	Q-Q
8.3.104	Q-Q
8.3.109	Q-Q
8.3.114	Q-Q
8.3.119	Q-Q
8.3.124	Q-Q
8.3.129	Q-Q
8.3.134	Q-Q
8.3.139	Q-Q
8.3.144	Q-Q
8.3.149	Q-Q
8.3.154	Q-Q
8.3.159	Q-Q
8.3.164	Q-Q
8.3.169	Q-Q
8.3.174	Q-Q
8.3.179	Q-Q
8.3.184	Q-Q

8.0.23	Q-Q
8.0.28	Q-Q
8.0.33	Q-Q
8.2.1	N/A
8.3.5	Q-Q
8.3.10	Q-Q
8.3.15	Q-Q
8.3.20	Q-Q
8.3.25	Q-Q
8.3.30	Q-Q
8.3.35	Q-Q
8.3.40	Q-Q
8.3.45	Q-Q
8.3.50	Q-Q
8.3.55	Q-Q
8.3.60	Q-Q
8.3.65	Q-Q
8.3.70	Q-Q
8.3.75	Q-Q
8.3.80	Q-Q
8.3.85	Q-Q
8.3.90	Q-Q
8.3.95	Q-Q
8.3.100	Q-Q
8.3.105	Q-Q
8.3.110	Q-Q
8.3.115	Q-Q
8.3.120	Q-Q
8.3.125	Q-Q
8.3.130	Q-Q
8.3.135	Q-Q
8.3.140	Q-Q
8.3.145	Q-Q
8.3.150	Q-Q
8.3.155	Q-Q
8.3.160	Q-Q
8.3.165	Q-Q
8.3.170	Q-Q
8.3.175	Q-Q
8.3.180	Q-Q

8.0.24	Q-Q
8.0.29	Q-Q
8.0.34	Q-Q
8.3.1	Q-Q
8.3.6	Q-Q
8.3.11	Q-Q
8.3.16	Q-Q
8.3.21	Q-Q
8.3.26	Q-Q
8.3.31	N/A
8.3.36	Q-Q
8.3.41	Q-Q
8.3.46	Q-Q
8.3.51	Q-Q
8.3.56	Q-Q
8.3.61	Q-Q
8.3.66	Q-Q
8.3.71	Q-Q
8.3.76	Q-Q
8.3.81	Q-Q
8.3.86	Q-Q
8.3.91	Q-Q
8.3.96	Q-Q
8.3.101	Q-Q
8.3.106	Q-Q
8.3.111	Q-Q
8.3.116	Q-Q
8.3.121	Q-Q
8.3.126	Q-Q
8.3.131	Q-Q
8.3.136	Q-Q
8.3.141	Q-Q
8.3.146	Q-Q
8.3.151	Q-Q
8.3.156	Q-Q
8.3.161	Q-Q
8.3.166	Q-Q
8.3.171	Q-Q
8.3.176	Q-Q
8.3.181	Q-Q

8.0.25	Q-Q
8.0.30	Q-Q
8.0.35	Q-Q
8.3.2	Q-Q
8.3.7	Q-Q
8.3.12	Q-Q
8.3.17	Q-Q
8.3.22	Q-Q
8.3.27	Q-Q
8.3.32	Q-Q
8.3.37	Q-Q
8.3.42	Q-Q
8.3.47	Q-Q
8.3.52	Q-Q
8.3.57	Q-Q
8.3.62	Q-Q
8.3.67	Q-Q
8.3.72	Q-Q
8.3.77	Q-Q
8.3.82	Q-Q
8.3.87	Q-Q
8.3.92	Q-Q
8.3.97	Q-Q
8.3.102	Q-Q
8.3.107	Q-Q
8.3.112	Q-Q
8.3.117	Q-Q
8.3.122	Q-Q
8.3.127	Q-Q
8.3.132	Q-Q
8.3.137	Q-Q
8.3.142	Q-Q
8.3.147	Q-Q
8.3.152	Q-Q
8.3.157	Q-Q
8.3.162	Q-Q
8.3.167	Q-Q
8.3.172	Q-Q
8.3.177	Q-Q
8.3.182	Q-Q

9 Discrete Mathematics: Set Theory & Algebra (236)

9.0.1 Kenneth Rosen Edition 7 Exercise 1.2 Question 11 (Page No. 23) top

Are these system specifications consistent? “The router can send packets to the edge system only if it supports the new address space. For the router to support the new address space it is necessary that the latest software release be installed. The router can send packets to the edge system if the latest software release is installed, The router does not support the new address space.”

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

Answer key

9.0.2 Kenneth Rosen Edition 7 Exercise 2.1 Question 27 (Page No. 126) top

Let $A = \{a, b, c, d\}$ and $B = \{y, z\}$ Find

a. $A \times B$.
b. $B \times A$.

kenneth-rosen discrete-mathematics set-theory&algebra easy

Answer key

9.0.3 Kenneth Rosen Edition 7 Exercise 2.2 Question 49 (Page No. 137) top

Find $\bigcup\limits_{i=1}^{\infty} A_i \, and \bigcup_{i=1}^{\infty} A_i$ if for every positive integer i,

a) Ai = {i, i + 1, i + 2, . . .}.
b) Ai = {0, i}.
c) Ai = (0, i), that is, the set of real numbers x with
0 < x < i.
d) Ai = (i,∞), that is, the set of real numbers x with
x > i.

kenneth-rosen discrete-mathematics

9.0.4 Kenneth Rosen Edition 7 Exercise 2.1 Question 1 (Page No. 125) top

List the numbers of these sets.

a. { x | x is a real number such that $x^2 = 1$ }
b. { x | x is a positive integer less than 12 }
c. { x | x is the square of an integer and $x < 100$ }
d. { x | x is an integer such that $x^2 = 2$ }

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9.0.5 Kenneth Rosen Edition 7 Exercise 2.1 Question 2 (Page No. 125) top

Use set builder notation to give a description of each of these sets.

a. { $0, 3, 6, 9, 12$ }
b. { $-3, -2, -1, 0, 1, 2, 3$ }
c. { m, n, o, p }

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.6 Kenneth Rosen Edition 7 Exercise 2.1 Question 3 (Page No. 125) top

For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first, or neither is a subset of the other.

a. the set of airline flights from New York to New Delhi, the set of nonstop airline flights from New York to New Delhi
b. the set of people who speak English, the set of people who speak Chinese
c. he set of flying squirrels, the set of living creatures that can fly.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.7 Kenneth Rosen Edition 7 Exercise 2.1 Question 4 (Page No. 125) top

For each of these pairs of sets, determine whether the first is a subset of the second, the second is a subset of the first,or neither is a subset of the other.

a. the set of people who speak English, the set of people who speak English with an Australian accent
b. the set of fruits, the set of citrus fruits
c. the set of students studying discrete mathematics, the set of students studying data structures

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.8 Kenneth Rosen Edition 7 Exercise 2.1 Question 5 (Page No. 125) top

Determine whether each of these pairs of sets are equal.

a. $\{ 1,3,3,3,5,5,5,5,5 \}$, $\{ 5,3,1 \}$
b. $\{\{ 1 \}\}$, $\{ 1 , \{ 1 \}\}$
c. ϕ, $\{ \phi \}$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.9 Kenneth Rosen Edition 7 Exercise 2.1 Question 6 (Page No. 125) top

Suppose that $A = \{ 2,4,6 \}$, $B = \{ 2,6 \}$, $C = \{ 4,6 \}$, and $D = \{ 4,6,8 \}$. Determine which of these sets are subsets of which other of these sets.

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.10 Kenneth Rosen Edition 7 Exercise 2.1 Question 7 (Page No. 125) top

For each of the following sets, determine whether 2 is an element of that set.

1. $\{x \in R \mid x \text{ is an integer greater than}\}$
2. $\{x \in R \mid x \text{ is the square of an integer}\}$
3. $\{2, \{2\}\}$
4. $\{\{2\}, \{\{2\}\}\}$
5. $\{\{2\}, \{2, \{2\}\}\}$
6. $\{\{\{2\}\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.11 Kenneth Rosen Edition 7 Exercise 2.1 Question 10 (Page No. 125) top

Determine whether each of these statements is true or false.

a. $\phi \epsilon \{\phi\}$
b. $\phi \epsilon \{\phi, \{ \phi\}\}$
c. $\{\phi\} \epsilon \{\phi\}$
d. $\{\phi\} \epsilon \{\{\phi\}\}$
e. $\{\phi\} \subset \{0\}$
f. $\{0\} \subset \{\phi , \{ \phi \}\}$
g. $\{\phi\} \subset \{\{\phi \}, \{ \phi\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.12 Kenneth Rosen Edition 7 Exercise 2.1 Question 11 (Page No. 125) top

Determine whether each of these statements is true or false.

a. $x \in \{x\}$
b. $\{x\} \subset \{x\}$
c. $\{x\} \in \{x\}$
d. $\{x\} \in \{\{x\}\}$
e. $\phi \subseteq \{x\}$
f. $\phi \in \{x\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.13 Kenneth Rosen Edition 7 Exercise 2.1 Question 13 (Page No. 126) top

Use a Venn diagram to illustrate the set of all months of the year whose names do not contain the letter R in the set of all months of the year.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.14 Kenneth Rosen Edition 7 Exercise 2.1 Question 14 (Page No. 126) top

Use a Venn diagram to illustrate the relationship $A \subseteq B$ and $B \subseteq C$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.15 Kenneth Rosen Edition 7 Exercise 1.2 Question 18 (Page No. 23) top

When planning a party you want to know whom to invite.
Among the people you would like to invite are three
touchy friends.You know that if Jasmine attends, she will
become unhappy if Samir is there, Samir will attend only
if Kanti will be there, and Kanti will not attend unless Jasmine
also does.Which combinations of these three friends
can you invite so as not to make someone unhappy?

I was trying by converting the english language to logical equivalent statments.
j-> not s
s->k
not j -> not k
How to approach this question?

kenneth-rosen discrete-mathematics mathematical-logic

9.0.16 Kenneth Rosen Edition 7 Exercise 2.1 Question 15 (Page No. 126) top

Use a Venn diagram to illustrate the relationships $A \subset B$ and $B \subset C$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.17 Kenneth Rosen Edition 7 Exercise 2.1 Question 17 (Page No. 126) top

Suppose that A, B, and C are sets such that $A \subseteq B$ and $B \subseteq C$. show that $A \subseteq C$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.18 Kenneth Rosen Edition 7 Exercise 2.1 Question 18 (Page No. 126) top

Find two sets A and B such that $A \in B$ and $A \subset B$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.19 Kenneth Rosen Edition 7 Exercise 2.1 Question 19 (Page No. 126) top

What is the cardinality of each of these sets?

a. $\{a\}$
b. $\{\{a\}\}$
c. $\{a, \{a\}\}$
d. $\{a,\{a\},\{a, \{a\}\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.20 Kenneth Rosen Edition 7 Exercise 2.1 Question 20 (Page No. 126) top

What is the cardinality of each of these sets?

a. ϕ
b. $\{\phi\}$
c. $\{\phi, \{\phi\}\}$
d. $\{\phi, \{\phi\}, \{\phi, \{\phi\}\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.21 Kenneth Rosen Edition 7 Exercise 2.1 Question 21 (Page No. 126) top

Find the power set of each of these sets, where a and b are distinct elements

a. $\{a\}$
b. $\{a, b\}$
c. $\{\phi, \{\phi\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.22 Kenneth Rosen Edition 7 Exercise 2.1 Question 22 (Page No. 126) top

Can you conclude that $A = B$ if A and B are two sets with the same power set?

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.23 Kenneth Rosen Edition 7 Exercise 2.1 Question 23 (Page No. 126) top

How many elements does each of these sets have where a and b are distinct elements?

1. $P(\{a, b, \{a, b\}\})$
2. $P\left\{\phi, a, \{a\}, \{\{a\}\}\right\}$
3. $P(P(\phi))$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.24 Kenneth Rosen Edition 7 Exercise 2.1 Question 24 (Page No. 126) top

Determine whether each of these sets is the power set of a set, where a and b are distinct elements.

a. ϕ
b. $\{\phi, \{a\}\}$
c. $\{\phi, \{a\}, \{\phi, a\}\}$
d. $\{\phi, \{a\}, \{b\}, \{a, b\}\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.25 Kenneth Rosen Edition 7 Exercise 2.1 Question 25 (Page No. 126) top

Prove that $P(A) \subseteq P(B)$ if and only if $A \subseteq B$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.26 Kenneth Rosen Edition 7 Exercise 2.1 Question 26 (Page No. 126) top

Show that if $A \subseteq C$ and $B \subseteq D$, then $A \times B \subseteq C \times D$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.27 Kenneth Rosen Edition 7 Exercise 2.2 Question 11 (Page No. 136) top

Let A and B be sets. Prove the commutative laws from Table 1 by showing that

1. $A \cup B = B \cup A$.
2. $A \cap B = B \cap A$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.28 Kenneth Rosen Edition 7 Exercise 7.1 Question 14 (Page No. 451) top

What is the probability that a five-card poker hand contains two pairs (that is, two of each of two different kinds and a fifth card of a third kind)?

kenneth-rosen discrete-mathematics probability

Answer key

9.0.29 Kenneth Rosen Edition 7 Exercise 2.1 Question 12 (Page No. 125) top

Use a Venn diagram to illustrate the subset of odd integers in the set of all positive integers not exceeding 10.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.30 Kenneth Rosen Edition 7 Exercise 2.3 Question 40 (Page No. 154) top

Let f be a function from the set A to the set B. Let S adn$T$ be subsets of A .Show that

a. $f(S \cup T) = f(S) \cup f(T)$
b. $f(S \cap T) = f(S) \cap f(T)$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.31 Kenneth Rosen Edition 7 Exercise 2.1 Question 44 (Page No. 126) top

Find the truth set of each of these predicates where the domain is the set of integers.

1. $P(x) : x^3 >= 1$
2. $Q(x) : x^2 = 2$
3. $R(x) : x < x^2$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.32 Kenneth Rosen Edition 7 Exercise 2.1 Question 32 (Page No. 126) top

Let $A = \{a, b, c\}$, $B = \{x, y\}$, and $C = \{0, 1\}$. Find

1. $A \times B \times C$
2. $C \times B \times A$
3. $C \times A \times B$
4. $B \times B \times B$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.33 Kenneth Rosen Edition 7 Exercise 2.1 Question 33 (Page No. 126) top

Find A^2 if

a. $A = \{0, 1, 3\}$
b. $A = \{1, 2, a, b\}$

kenneth-rosen discrete-mathematics set-theory&algebra

Answer key

9.0.34 Kenneth Rosen Edition 7 Exercise 2.1 Question 34 (Page No. 126) top

Find A^3 if

a. $A = \{a\}$
b. $A = \{0, a\}$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.35 Kenneth Rosen Edition 7 Exercise 2.1 Question 35 (Page No. 126) top

How many different elements does $A \times B$ have if A has m elements and B has n elements?

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.36 Kenneth Rosen Edition 7 Exercise 2.1 Question 36 (Page No. 126) top

How many different elements does $A \times B \times C$ have it A has m elemetns, B has n elements, and C has p elements?

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.37 Kenneth Rosen Edition 7 Exercise 2.1 Question 37 (Page No. 126) top

How many different elements does A^n have when A has m elements and n is a positive integer?

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.38 Kenneth Rosen Edition 7 Exercise 2.1 Question 38 (Page No. 126) top

Show that $A \times B \neq B \times A$, when A and B are nonempty. unless $A = B$.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.39 Kenneth Rosen Edition 7 Exercise 2.1 Question 39 (Page No. 126) top

Explain why $A \times B \times C$ and $(A \times B) \times C$ are not the same.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.40 Kenneth Rosen Edition 7 Exercise 2.1 Question 40 (Page No. 126) top

Explain why $(A \times B) \times (C \times D)$ and $A \times (B \times C) \times D$ are not the same.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.41 Kenneth Rosen Edition 7 Exercise 2.1 Question 41 (Page No. 126) top

Translate each of these quantifications into English and determine is truth value.

a. $\forall x \in (x^2 \neq -1)$
b. $\exists x \in Z(x^2 = 2)$
c. $\forall x \in (x^2 > 0)$
d. $\forall x \in R(x^2 = x)$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.42 Kenneth Rosen Edition 7 Exercise 2.1 Question 43 (Page No. 126) top

Find the truth set of each of these predicates where the domain is the set of integers.

a. $P(x) : x^2 < 3$
b. $Q(x) : x^2 > x$
c. $R(x) : 2x + 1 = 0$

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.43 Kenneth Rosen Edition 7 Exercise 2.1 Question 29 (Page No. 126) top

What is the Cartesian product $A \times B \times C$, where A is the set of all airlines and B and C are both the set of all cities in the United States? Give an example of how this Cartesian product can be used.

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.44 Kenneth Rosen Edition 7 Exercise 2.1 Question 45 (Page No. 126) top

The defining property of an ordered pair is that two ordered pairs are equal if and only if their first elements are equal and their second elements are equal. Surprisingly, instead of taking the ordered pair as a primitive concept, we can construct ordered pairs using basic notions from set theory. Show that if we define the ordered pair (a, b) to be $\{\{a\}, \{a, b\}\}$, then $(a, b) = (c, d)$ if and only if $a = c$ and $b = d$. [Hint: First show that $\{\{a\}, \{a, b\}\} = \{\{c\}, \{c, d\}\}$ if and only if $a = c$ and $b = d$.]

kenneth-rosen discrete-mathematics set-theory&algebra difficult

9.0.45 Kenneth Rosen Edition 7 Exercise 2.2 Question 12 (Page No. 136) top

Prove the first absorption law from Table 1 by showing that if A and B are sets, then $A \cup (A \cup B) = A$

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9.0.46 Kenneth Rosen Edition 7 Exercise 2.2 Question 1 (Page No. 136) top

Let A be the set of students who live within one mile of school and let B be the set of students who walk to classes. Describe the students in each of these sets.

a. $A \cap B$ b. $A \cup B$ c. $A - B$ d. $B - A$

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9.0.47 Kenneth Rosen Edition 7 Exercise 2.2 Question 2 (Page No. 136) top

Suppose that A is the set of sophomores at your school and B is the set of students in discrete mathematics at your school. Express each of these sets in terms of A and B .

a. the set of sophomores taking discrete mathematics in your school
b. the set of sophomores at your school who are not taking discrete mathematics
c. the set of students at your school who either are sophomores or are taking discrete mathematics
d. the set of students at your school who either are not sophomores or are not taking discrete mathematics

kenneth-rosen discrete-mathematics set-theory&algebra

9.0.48 Kenneth Rosen Edition 7 Exercise 2.2 Question 3 (Page No. 136) top

Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{0, 3, 6\}$ .Find

a. $A \cup B$ b. $A \cap B$ c. $A - B$ d. $B - A$

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Answer key

9.0.49 Kenneth Rosen Edition 7 Exercise 2.2 Question 4 (Page No. 136) top

Let $A = \{a, b, c, d, e\}$ and $B = \{a, b, c, d, e, f, g, h\}$. Find

a. $A \cup B$ b. $A \cap B$ c. $A - B$ d. $B - A$

kenneth-rosen discrete-mathematics set-theory&algebra easy

Answer key

9.0.50 Kenneth Rosen Edition 7 Exercise 2.2 Question 5 (Page No. 136) top

Prove the complement law in Table 1 by showing That $\sim\sim A = A$.

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9.0.51 Kenneth Rosen Edition 7 Exercise 2.2 Question 6 (Page No. 136) top

Prove the identity laws in Table 1 by showing that

a. $A \cup \phi = A$.
b. $A \cap U = A$.

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9.0.52 Kenneth Rosen Edition 7 Exercise 2.2 Question 7 (Page No. 136) top

Prove the domination laws in Table 1 by showing that

a. $A \cup U = U$.
b. $A \cap \phi = \phi$.

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9.0.53 Kenneth Rosen Edition 7 Exercise 2.2 Question 8 (Page No. 136) top

Prove the idempotent laws in Table 1 by showing that

a. $A \cup A = A$.
b. $A \cap A = A$.

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9.0.54 Kenneth Rosen Edition 7 Exercise 2.2 Question 9 (Page No. 136) top

Prove the complement laws in Table 1 by showing that

a. $A \cup \sim A = U$.
b. $A \cap \sim A = \phi$.

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9.0.55 Kenneth Rosen Edition 7 Exercise 2.2 Question 10 (Page No. 136) top

Show that

1. $A - \phi = A$.
2. $\phi - A = \phi$

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9.0.56 Kenneth Rosen Edition 7 Exercise 2.1 Question 30 (Page No. 126) top

Suppose that $A \times B = \phi$, where A and B are sets. What can you conclude?

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9.0.57 Kenneth Rosen Edition 7 Exercise 2.1 Question 28 (Page No. 126) top

What is the Cartesian product $A \times B$, where A is the set of courses offered by the mathematics department at a university and B is the set of mathematics professors at this university? Give an example of how this Cartesian product can be used.

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9.0.58 Kenneth Rosen Edition 7 Exercise 2.2 Question 13 (Page No. 136) top

Prove the second absorption law from Table 1 by showing that if A and B are sets, then $A \cap (A \cup B) = A$

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9.0.59 Kenneth Rosen Edition 7 Exercise 2.3 Question 25 (Page No. 153) top

Let $f : R \rightarrow R$ and let $f(x) > 0$ for all $x \epsilon R$. Show that $f(x)$ is strictly decreasing if and only if the function $g(x) = 1/f(x)$ is strictly increasing.

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9.0.60 Kenneth Rosen Edition 7 Exercise 2.1 Question 42 (Page No. 126) top

Translate each of these quantifications into English and determine is truth value.

a. $\exists x \in R(x^3 = -1)$
b. $\exists x \in Z(x + 1 > x)$
c. $\forall x \in (x - 1) \in Z$
d. $\forall x \in Z(x^2 \in Z)$

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9.0.61 Kenneth Rosen Edition 7 Exercise 2.2 Question 28 (Page No. 136) top

Draw the Venn diagrams for each of these combinations of the sets A, B, C, and D.

a. $(A \cap B) \cup (C \cap D)$
b. $\sim A \cup \sim B \cup \sim C \cup \sim D$
c. $A - (B \cap C \cap D)$

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9.0.62 Kenneth Rosen Edition 7 Exercise 2.2 Question 16 (Page No. 136) top

Let A and B be sets. Show that

a. $(A \cap B) \subseteq A$.
b. $A \subseteq (A \subseteq B.)$
c. $A - B \subseteq A$.
d. $A \cap (B - A) = \phi$

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9.0.63 Kenneth Rosen Edition 7 Exercise 2.2 Question 17 (Page No. 136) top

Show that if A, B, and C are sets, then $\sim (A \cap B \cap C) = \sim A \cup \sim B \cup \sim C$

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9.0.64 Kenneth Rosen Edition 7 Exercise 2.2 Question 18 (Page No. 136) top

Let A, B, and C be sets, Show that

a. $(A \cup B) \subseteq (A \cup B \cup C)$,
b. $(A \cap B \cap C) \subseteq (A \cap B.)$
c. $(A - B) - C \subseteq A - C$.
d. $(A - C) \cap (C - B) = \phi$.
e. $(B - A) \cup (C - A) = (B \cup C) - A$.

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9.0.65 Kenneth Rosen Edition 7 Exercise 2.2 Question 19 (Page No. 136) top

Show that if A and B are sets, then

a. $A - B = A \cap \sim B$.
b. $(A \cap B) \cup (A \cap \sim B) = A$.

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9.0.66 Kenneth Rosen Edition 7 Exercise 2.2 Question 20 (Page No. 136) top

Show that if A and B are sets with $A \subseteq B$, then

a. $A \cup B = B$
b. $A \cap B = A$

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9.0.67 Kenneth Rosen Edition 7 Exercise 2.2 Question 21 (Page No. 136) top

Prove the first associative law from Table 1 by showing that if A, B, and C are sets, then $A \cup (B \cup C) = (A \cup B) \cup C$.

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9.0.68 Kenneth Rosen Edition 7 Exercise 2.2 Question 22 (Page No. 136) top

Prove the second associative law from Table 1 by showing that if $A, B,$ and C are sets, then $A \cup (B \cap C) = (A \cap B) \cap C$.

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9.0.69 Kenneth Rosen Edition 7 Exercise 2.2 Question 23 (Page No. 136) top

Prove the first distributive law from Table 1 by showing that if $A, B,$ and C are sets, then $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$.

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9.0.70 Kenneth Rosen Edition 7 Exercise 2.2 Question 24 (Page No. 136) top

Let A, B and C be sets. Show that $(A - B) - C = (A - C) - (B - C)$.

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9.0.71 Kenneth Rosen Edition 7 Exercise 2.2 Question 25 (Page No. 136) top

Let $A = \{0, 2, 4, 6, 8, 10\}$, $B = \{0, 1, 2, 3, 4, 5, 6\}$, and $C = \{4, 5, 6, 7, 8, 9, 10\}$. Find

a. $A \cap B \cap C$.
b. $A \cup B \cup C$.
c. $(A \cup B) \cap C$.
d. $(A \cap B) \cup C$.

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9.0.72 Kenneth Rosen Edition 7 Exercise 2.2 Question 26 (Page No. 136) top

Draw the Venn diagrams for each of these combination sof the sets A, B, and C.

a. $A \cap (B \cup C)$
b. $\sim A \cap \sim B \cap \sim C$
c. $(A - B) \cup (A - C) \cup (B - C)$

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9.0.73 Kenneth Rosen Edition 7 Exercise 2.3 Question 6 (Page No. 152) top

Find the domain and range of these functions.

a. the function that assigns to each pair of positive integers the first integer of the pair
b. the function that assigns to each positive integer its largest decimal digit
c. the function that assigns to a bit string the number of ones minus the number of zeros in the string
d. the function that assigns to each positive integer the largest integer not exceeding the square root of the integer
e. the function that assigns to a bit string the longest string of ones in the string

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9.0.74 Kenneth Rosen Edition 7 Exercise 2.2 Question 27 (Page No. 136) top

Draw the Venn diagrams for each of these combinations of the sets A, B, and C.

a. $A \cap (B - C)$
b. $(A \cap B) \cup (A \cap C)$
c. $(A \cap \sim B) \cup (A \cap \sim C)$

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9.0.75 Kenneth Rosen Edition 7 Exercise 2.2 Question 29 (Page No. 136) top

What can you say about the sets A and B if we know that

a. $A \cup B = A$?
b. $A \cap B = A$?
c. $A - B = A$?
d. $A \cap B = B \cap A$?
e. $A - B = B - A$?

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9.0.76 Kenneth Rosen Edition 7 Exercise 2.2 Question 30 (Page No. 137) top

Can you conclude that $A = B$ if A, B, and C are sets such that

1. $A \cup C = B \cup C$?
2. $A \cap C = B \cap C$?
3. $A \cup C = B \cup C$ and $A \cap C = B \cap C$?

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9.0.77 Kenneth Rosen Edition 7 Exercise 2.2 Question 31 (Page No. 137) top

Let A and B be sbusets of a universal set U. Show that $A \subseteq B$ if and only if $\sim B \subseteq \sim A$.

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9.0.78 Kenneth Rosen Edition 7 Exercise 2.2 Question 32 (Page No. 137) top

Find the symmetric difference of $\{1, 3, 5\}$ and $\{1, 2, 3\}$.

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Answer key

9.0.79 Kenneth Rosen Edition 7 Exercise 2.2 Question 35 (Page No. 137) top

Show that $A \oplus B = (A \cup B) - (A \cap B)$.

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Answer key

9.0.80 Kenneth Rosen Edition 7 Exercise 2.2 Question 36 (Page No. 137) top

Show that $A \oplus B = (A - B) \cup (B - A)$.

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Answer key

9.0.81 Kenneth Rosen Edition 7 Exercise 2.2 Question 37 (Page No. 137) top

Show that if A is a subset of a universal set U, then

a. $A \oplus A = \phi$.
b. $A \oplus \phi = A$.
c. $A \oplus U = \sim A$.
d. $A \oplus \sim A = U$.

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Answer key

9.0.82 Kenneth Rosen Edition 7 Exercise 2.3 Question 1 (Page No. 152) top

Why is f not a function from R to R if

a. $f(x) = 1/x$?
b. $f(x) = \sqrt{x}$?
c. $f(x) = \pm\sqrt{(x^2 + 1)}$?

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9.0.83 Kenneth Rosen Edition 7 Exercise 2.3 Question 2 (Page No. 152) top

Determine whether f is a function from Z to R if

a. $f(n) = + - n.$
b. $f(n) = \sqrt{n^2 + 1}.$
c. $f(n) = 1/(n^2 - 4).$

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9.0.84 Kenneth Rosen Edition 7 Exercise 2.3 Question 3 (Page No. 152) top

Determine whether f is a function from the set of all bit strings to the set of integers if

a. $f(S)$ is the position of a 0 bit in S.
b. $f(S)$ is the number of 1 bits in S.
c. $f(S)$ is the smallest integer i such that the i th bit of S is 1 and $f(S) = 0$ when S is the empty string, the string with no bits.

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9.0.85 Kenneth Rosen Edition 7 Exercise 2.3 Question 4 (Page No. 152) top

Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a. the function that assigns to each nonnegative integer its last digit
b. the function that assigns the next largest integer to a positive integer
c. the function that assigns to a bit string the number of one bits in the string
d. the function that assigns to a bit string the number of bits in the string

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9.0.86 Kenneth Rosen Edition 7 Exercise 2.3 Question 5 (Page No. 152) top

Find the domain and range of these functions. Note that in each case, to find the domain, determine the set of elements assigned values by the function.

a. the function that assigns to each bit string the number of ones in the string minus the number of zeros in the string
b. the function that assigns to each bit string twice the number of zeros in that string
c. the function that assigns the number of bits left over when a bit string is split into bytes (which are blocks of 8 bits)
d. the function that assigns to each positive integer the largest perfect square not exceeding this integer

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9.0.87 Kenneth Rosen Edition 7 Exercise 2.2 Question 15 (Page No. 136) top

rove the second De Morgan law in Table 1 by showing that if A and B are sets, then $\sim(A \cup B) = (\sim A \cap \sim B)$

a. by showing each side is a subset of the other side.
b. using a membership table.

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9.0.88 Kenneth Rosen Edition 7 Exercise 2.2 Question 14 (Page No. 136) top

Find the sets A and B if $A - B = \{1, 5, 7, 8\}$, $B - A = \{2, 10\}$, and $A \cap B = \{3, 6, 9\}$.

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Answer key

9.0.89 Kenneth Rosen Edition 7 Exercise 2.3 Question 10 (Page No. 153) top

Determine whether each of these functions form $[a, b, c, d]$ to itself is one-to-one.

a. $f(a) = b, f(b) = a, f(c) = c, f(d) = d$
b. $f(a) = b, f(b) = b, f(c) = d, f(d) = c$

c. $f(a) = d, f(b) = b, f(c) = c, f(d) = d$

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Answer key

9.0.90 Kenneth Rosen Edition 7 Exercise 2.3 Question 8 (Page No. 153) top

Find the values.

a. $\lfloor 1.1 \rfloor$
b. $\lceil 1.1 \rceil$
c. $\lfloor -0.1 \rfloor$
d. $\lceil -0.1 \rceil$
e. $\lceil 2.99 \rceil$
f. $(\lfloor -2.99 \rfloor)$
g. $\lfloor 1/2 + \lceil 1/2 \rceil \rfloor$
h. $\lceil \lfloor 1/2 \rfloor + \lceil 1/2 \rceil + 1/2 \rceil$

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9.0.91 Kenneth Rosen Edition 7 Exercise 2.3 Question 12 (Page No. 153) top

Determine whether each of these functions from Z to Z is one-to-one.

a. $f(n) = n - 1$
b. $f(n) = n^2 + 1$
c. $f(n) = n^3$
d. $f(n) = \lceil n/2 \rceil$

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9.0.92 Kenneth Rosen Edition 7 Exercise 2.3 Question 13 (Page No. 153) top

Determine whether each of these functions from Z to Z is onto??

a. $f(n) = n - 1$
b. $f(n) = n^2 + 1$
c. $f(n) = n^3$
d. $f(n) = \lceil n/2 \rceil$

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9.0.93 Kenneth Rosen Edition 7 Exercise 2.3 Question 14 (Page No. 153) top

Determine whether $f : Z \times Z \to Z$ is onto if

a. $f(m, n) = 2m - n.$
b. $f(m, n) = m^2 - n^2.$
c. $f(m, n) = m + n + 1.$
d. $f(m, n) = |m| - |n|.$
e. $f(m, n) = m^2 - 4.$

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9.0.94 Kenneth Rosen Edition 7 Exercise 2.3 Question 15 (Page No. 153) top

Determine whether the function $f : Z \times Z \to Z$ is onto if

a. $f(m, n) = m + n$
b. $f(m, n) = m^2 + n^2.$
c. $f(m, n) = m.$
d. $f(m, n) = |n|.$
e. $f(m, n) = m - n.$

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9.0.95 Kenneth Rosen Edition 7 Exercise 2.3 Question 16 (Page No. 153) top

Consider these functions from the set of students in a discrete mathematics class. Under what conditions is the function one-to-one if it assigns to a student his or her

a. mobile phone number.
b. student identification number.
c. final grade in the class.
d. home town.

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9.0.96 Kenneth Rosen Edition 7 Exercise 2.3 Question 17 (Page No. 153) top

Consider these functions from the set of teachers in a school. Under what conditions is the function one-to-one if it assigns to a teacher his or her

a. office.
b. assigned bus to chaperone in a group of buses taking students on a field trip.
c. salary.
d. social security number

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9.0.97 Kenneth Rosen Edition 7 Exercise 2.3 Question 20 (Page No. 153) top

Give an example of a function from N to N that is

a. one-to-one but not onto.
b. onto but not one-to-one.
c. both onto and one-to-one (but different from the identity function).
d. neither one-to-one nor onto.

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9.0.98 Kenneth Rosen Edition 7 Exercise 2.3 Question 21 (Page No. 153) top

Give an explicit formula for a function from the set of integers to the set of positive integers that is

a. one-to-one, but not onto.
b. onto, but not one-to-one.
c. one-to-one and onto.
d. neither one-to-one nor onto.

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9.0.99 Kenneth Rosen Edition 7 Exercise 2.3 Question 22 (Page No. 153) top

Determine whether each of these functions is a bijection from R to R.

a. $f(x) = -3x + 4$
b. $f(x) = -3x^2 + 7$
c. $f(x) = (x + 1)/(x + 2)$
d. $f(x) = x^5 + 1$

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Answer key

9.0.100 Kenneth Rosen Edition 7 Exercise 2.3 Question 23 (Page No. 153) top

Determine whether each of these functions is a bijection from R to R.

a. $f(x) = 2x + 1$
b. $f(x) = x^2 + 1$
c. $f(x) = x^3$
d. $f(x) = (x^2 + 1)/(x^2 + 2)$

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Answer key

9.0.101 Kenneth Rosen Edition 7 Exercise 2.3 Question 24 (Page No. 153) top

Let $f : R \to R$ and let $f(x) > 0$ for all $x \epsilon R$. Show that $f(x)$ is strictly increasing if and only if the function $g(x) = 1/f(x)$ is strictly decreasing.

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9.0.102 Kenneth Rosen Edition 7 Exercise 2.3 Question 26 (Page No. 153) top

a. Prove that a strictly increasing function from R to it-self is one-to-one.
b. Give an example of an increasing function from R to itself that is not one-to-one

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9.0.103 Kenneth Rosen Edition 7 Exercise 2.3 Question 27 (Page No. 153) top

a. Prove that a strictly decreasing function from R to it-self is one-to-one.
b. Give an example of an decreasing function from R to itself that is not one-to-one

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Answer key

9.0.104 Kenneth Rosen Edition 7 Exercise 2.3 Question 1 (Page No. 153) top

Find the domain and range of these functions.

a. the function that assigns to each pair of positive integers the maximum of these two integers
b. the function that assigns to each positive integer the number of the digits $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$ that do not appear as decimal digits of the integer
c. the function that assigns to a bit string the number of times the block 11 appears
d. the function that assigns to a bit string the numerical position of the first 1 in the string and that assigns the value 0 to a bit string consisting of all 0s

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9.0.105 Kenneth Rosen Edition 7 Exercise 2.3 Question 28 (Page No. 153) top

Show that the function $f(x) = e^x$ from the set of real numbers to the set of real numbers is not invertible, but if the codomain is restricted to the set of positive real numbers, the resulting function is invertible

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9.0.106 Kenneth Rosen Edition 7 Exercise 2.3 Question 29 (Page No. 154) top

Show that the function $f(x) = |x|$ from the set of real numbers to the set of nonnegative real numbers is not invertible, but if the domain is restricted to the set of nonnegative real numbers, the resulting function is invertible.

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Answer key

9.0.107 Kenneth Rosen Edition 7 Exercise 2.3 Question 30 (Page No. 154) top

Let $S = \{-1, 0, 2, 4, 7\}$. Find $f(S)$ if

a. $f(x) = 1$
b. $f(x) = 2x + 1$
c. $f(x) = \lceil x/5 \rceil$
d. $f(x) = \lfloor (x^2 + 1)/3 \rfloor$

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9.0.108 Kenneth Rosen Edition 7 Exercise 2.3 Question 31 (Page No. 154) top

Let $f(x) = \lfloor x^2/3 \rfloor$. Find $f(S)$ if

a. $S = \{-2, -1, 0, 1, 2, 3\}$
b. $S = \{0, 1, 2, 3, 4, 5\}$
c. $S = \{1, 5, 7, 11\}$
d. $S = \{2, 6, , 10, 14\}$

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9.0.109 Kenneth Rosen Edition 7 Exercise 2.3 Question 32 Page No. 154) top

Let $f(x) = 2x$ where the domain is the set of real numbers. What is

a. $f(Z)$
b. $f(N)$
c. $f(R)$

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9.0.110 Kenneth Rosen Edition 7 Exercise 2.3 Question 33 (Page No. 154) top

Suppose that g is a function from A to B and f is a function from B to C.

Show that if both f and g are one-to-one functions,then fog is also one-to-one.

Show that if both f and g are onto functions, then fog is also onto.

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9.0.111 Kenneth Rosen Edition 7 Exercise 2.3 Question 34 (Page No. 154) top

If f and fog are one-to-one, does it follow that g is one-to-one? Justify your answer.

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9.0.112 Kenneth Rosen Edition 7 Exercise 2.3 Question 35 (Page No. 154) top

If f and fog are onto, does it follow that g is onto?Justify your answer.

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Answer key

9.0.113 Kenneth Rosen Edition 7 Exercise 2.3 Question 36 (Page No. 154) top

Find fog and gof. Where $f(x) = x^2 + 1$ and $g(x) = x + 2$, are functions from R to R.

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Answer key

9.0.114 Kenneth Rosen Edition 7 Exercise 2.3 Question 38 (Page No. 154) top

Let $f(x) = ax + b$ and $g(x) = cx + d$, where a,b,c, and d ar constants. Determine neccessary and sufficient conditions on the constants a,b,c, and d so that $fog = gof$

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9.0.115 Kenneth Rosen Edition 7 Exercise 2.3 Question 39 (Page No. 154) top

Show that the function $f(x) = ax + b$ from R to R is invertible, where a and $b$are constants, with $a = 0$, and find the inverse of f.

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9.0.116 Kenneth Rosen Edition 7 Exercise 2.3 Question 9 (Page No. 153) top

Find the values.

a. $\lceil 3/4 \rceil$
b. $\lfloor 7/8 \rfloor$
c. $\lceil -3/4 \rceil$
d. $\lfloor -7/8 \rfloor$
e. $\lceil 3 \rceil$
f. $\lfloor -1 \rfloor$
g. $\lfloor 1/2 + \lceil 3/2 \rceil \rfloor$
h. $\lfloor 1/2 . \lfloor 5/2 \rfloor \rfloor$

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9.0.117 Kenneth Rosen Edition 7 Exercise 2.1 Question 31 (Page No. 126) top

Let A be a set. Show that $\phi \times A = A \times \phi = \phi$

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9.0.118 Kenneth Rosen Edition 7 Exercise 2.6 Question 22 (Page No. 177) top

Suppose that A is a countable set. Show that the set B is also countable if there is an onto function f from A to B.

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Answer key

9.0.119 Kenneth Rosen Edition 7 Exercise 2.4 Question 13 (Page No. 168) top

Is the sequence $\{a_n\}$ a solution of the recurrence relation $a_n = 8a_{n-1} - 16a_{n-2}$ if

A. $a_n = 0$
B. $an = 1$
C. $a_n = 2^n$
D. $a_n = 4^n$
E. $a_n = n4^n$
F. $a_n = 2 \cdot 4^n + 3n4^n$
G. $a_n = (-4)^n$
H. $a_n = n^2 4^n$

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9.0.120 Kenneth Rosen Edition 7 Exercise 2.3 Question 59 (Page No. 155) top

How many bytes are required to encode n bits of data where n equals

a. 7 b. 17 c. 1001 d. 28800

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9.0.121 Kenneth Rosen Edition 7 Exercise 2.3 Question 46 (Page No. 154) top

Show that $\lfloor x + 1/2 \rfloor$ is the closest integer to the number x,except when x is midway between two integers, when it is the larger of these two integers

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9.0.122 Kenneth Rosen Edition 7 Exercise 2.3 Question 47 Page No. 154) top

Show that $\lceil x - 1/2 \rceil$ is the closest integer to the number x,except when x is midway between two integers, when it is the smaller of these two integers

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9.0.123 Kenneth Rosen Edition 7 Exercise 2.3 Question 48 (Page No. 154) top

Show that if x is a real number, then $\lceil x \rceil - \lfloor x \rfloor = 1$ if x is not an integer and $\lceil x \rceil - \lfloor x \rfloor = 0$ if x is an integer.

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9.0.124 Kenneth Rosen Edition 7 Exercise 2.3 Question 49 (Page No. 154) top

Show that if x is a real number, then $x - 1 < \lfloor x \rfloor <= x <= \lceil x \rceil < x + 1$.

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9.0.125 Kenneth Rosen Edition 7 Exercise 2.3 Question 50 (Page No. 154) top

Show that if x is a real number, and m is an integer, then $\lceil x + m \rceil = \lceil x \rceil + m$.

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Answer key

9.0.126 Kenneth Rosen Edition 7 Exercise 2.3 Question 52 (Page No. 154) top

Show that if x is a real number and n is an integer, then

a. $x \leq n$ if and only if $\lceil x \rceil \leq n$
b. $n \leq x$ if and only if $n \leq \lfloor x \rfloor$

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9.0.127 Kenneth Rosen Edition 7 Exercise 2.3 Question 53 (Page No. 154) top

Prove that if n is an integer, then $\lfloor n/2 \rfloor = n/2$ if n is even and $(n-1)/2$ if n is odd.

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9.0.128 Kenneth Rosen Edition 7 Exercise 2.3 Question 54 (Page No. 154) top

Prove that if x is a reall number , then $\lfloor -x \rfloor = -\lceil x \rceil$ and $\lceil -x \rceil = -\lfloor x \rfloor$

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9.0.129 Kenneth Rosen Edition 7 Exercise 2.3 Question 55 (Page No. 154) top

The function INT is found on some calculators, where INT(x) = $\lfloor x \rfloor$ when x nonnegative real number and INT(x) = $\lceil x \rceil$ when x is a negative real number. Show that this INT function satisfies the identity INT $(-x)$=$-$ INT(x)

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9.0.130 Kenneth Rosen Edition 7 Exercise 2.3 Question 56 (Page No. 154) top

Let a and b be real numbers with $a < b$. Use the floor and / or ceiling functions to express the number of integers n that satisfy the inequality $a \leq n \leq b$.

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9.0.131 Kenneth Rosen Edition 7 Exercise 2.3 Question 57 (Page No. 154) top

Let a and b be real numbers with $a < b$. Use the floor and / or ceiling functions to express the number of integers n that satisfy the inequality $a < n < b$.

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9.0.132 Kenneth Rosen Edition 7 Exercise 2.3 Question 72 (Page No. 155) top

Suppose that f is a function from A to B, where A and B are finite sets with $|A| = |B|$. Show that f is one-to-one if and only if it is onto.

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Answer key

9.0.133 Kenneth Rosen Edition 7 Exercise 2.3 Question 58 (Page No. 154) top

How many bytes are required to encode n bits of data where n equals

a. 4
b. 10
c. 500
d. 3000

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9.0.134 Kenneth Rosen Edition 7 Exercise 2.3 Question 60 (Page No. 155) top

How many ATM cells (described in Example 28) can be transmitted in 10 seconds over a link operating at the following rates?

a. 128 kilobits per second (1 kilobit= 1000 bits)
b. 300 kilobits per second
c. 1 megabit per second (1 megabit=$1,000,000$ bits)

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9.0.135 Kenneth Rosen Edition 7 Exercise 2.3 Question 61 (Page No. 155) top

Data are transmitted over a particular Ethernet network in blocks of 1500 octets (blocks of 8 bits). How many blocks are required to transmit the following amounts of data over this Ethernet network? (Note that a byte is a synonym for an octet, a kilobyte is 1000 bytes, and a megabyte is $1,000,000$ bytes.)

a. 150 kilobytes of data
b. 384 kilobytes of data
c. 1.544 megabytes of data
d. 45.3 megabytes of data

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9.0.136 Kenneth Rosen Edition 7 Exercise 2.3 Question 62 (Page No. 155) top

Draw the graph of the function $f(n) = 1 - n^2$ from Z to Z

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9.0.137 Kenneth Rosen Edition 7 Exercise 2.3 Question 63 (Page No. 155) top

Draw the graph of the function $f(n) = \lfloor 2x \rfloor$ from R to R

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9.0.138 Kenneth Rosen Edition 7 Exercise 2.3 Question 64 (Page No. 155) top

Draw the graph of the function $f(n) = \lfloor x/2 \rfloor$ from R to R

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9.0.139 Kenneth Rosen Edition 7 Exercise 2.3 Question 65 (Page No. 155) top

Draw the graph of the function $f(n) = \lfloor x \rfloor + \lfloor x/2 \rfloor$ from R to R

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9.0.140 Kenneth Rosen Edition 7 Exercise 2.3 Question 66 (Page No. 155) top

Draw the graph of the function $f(n) = \lceil x \rceil + \lceil x/2 \rceil$ from R to R

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9.0.141 Kenneth Rosen Edition 7 Exercise 2.3 Question 67 (Page No. 155) top

Draw graphs of each of these functions.

a. $f(x) = \lfloor x + 1/2 \rfloor$
b. $f(x) = \lfloor 2x + 1 \rfloor$
c. $f(x) = \lceil x/3 \rceil$
d. $f(x) = \lceil 1/x \rceil$
e. $f(x) = \lceil x - 2 \rceil + \lfloor x + 2 \rfloor$
f. $f(x) = \lfloor 2x \rfloor \lceil x/2 \rceil$
g. $f(x) = \lceil \lfloor x - 12 \rfloor + 1/2 \rceil$

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9.0.142 Kenneth Rosen Edition 7 Exercise 2.3 Question 68 (Page No. 155) top

Draw graphs of each of these functions.

a. $f(x) = \lceil 3x - 2 \rceil$
b. $f(x) = \lceil 0.2x \rceil$
c. $f(x) = \lfloor -1/x \rfloor$
d. $f(x) = \lfloor x^2 \rfloor$
e. $f(x) = \lceil x/2 \rceil \lfloor x/2 \rfloor$
f. $f(x) = \lfloor x/2 \rfloor + \lceil x/2 \rceil$
g. $f(x) = \lfloor 2 \lceil x/2 \rceil + 1/2 \rfloor$

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9.0.143 Kenneth Rosen Edition 7 Exercise 2.3 Question 69 (Page No. 155) top

Find the inverse function of $f(x) = x^3 + 1$.

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Answer key

9.0.144 Kenneth Rosen Edition 7 Exercise 2.3 Question 70 (Page No. 155) top

Suppose that f is an invertible function from Y to Z and g is an invertible function from X to Y. Show that the inverse of the composition fog is given by $(fog)^{-1} = g^{-1} o f^{-1}$.

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9.0.145 Kenneth Rosen Edition 7 Exercise 2.3 Question 71 (Page No. 155) top

Let S be a subset of a universal set U. The characteristic function f_s of S is the function from U to the set $\{0, 1\}$ such that $f_S(x) = 1$ if x belongs to S and $f_S(x) = 0$ if x does not belong to S. Let A and B be sets. Show that for all $x \epsilon U$,

a. $f_{A \cap B}(x) = f_A(x). f_B(x)$
b. $f_{A \cup B}(x) = f_A(x) + f_B(x) - f_A(x). f_B(x)$
c. $f_{\sim A} = 1 - f_A(x)$
d. $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2f_A(x)f_B(x)$

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9.0.146 Kenneth Rosen Edition 7 Exercise 2.3 Question 45 (Page No. 154) top

Let f be a function from A to B. Let S be a subset of B. Show that $f^{-1} \sim (S) = \sim f^{-1}(S)$.

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9.0.147 Kenneth Rosen Edition 7 Exercise 2.4 Question 28 (Page No. 169) top

Let a_n be the nth term of the sequence $1, 2, 2, 3, 3, 3, 4, 4, 4, 4, 5, 5, 5, 5, 5, 6, 6, 6, 6, 6, 6, \ldots,$ constructed by including the integer k exactly k times. Show that $a_n = \lfloor \sqrt{2n} + \frac{1}{2} \rfloor$.

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9.0.148 Kenneth Rosen Edition 7 Exercise 2.4 Question 29 (Page No. 169) top

What are the values of these sums?

A. $\sum_{k=1}^{5}(k+1)$
B. $\sum_{j=0}^{4}(-2)^j$
C. $\sum_{i=1}^{10} 3$
D. $\sum_{j=0}^{8}(2^{j+1} - 2^j)$

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9.0.149 Kenneth Rosen Edition 7 Exercise 2.3 Question 74 (Page No. 155) top

Prove or disprove each of these statements about the floor and ceiling functions.

a. $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for all real numbers x.
b. $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$ for all real numbers x.
c. $\lceil \lceil x/2 \rceil /2 \rceil \lceil x/4 \rceil$ for all real numbers x.
d. $\left\lfloor \lceil x \rceil^{-1/2} \right\rfloor = \lfloor x \rfloor^{-1/2}$ for all positive real numbers x.
e. $\lfloor x \rfloor + \lfloor y \rfloor + \lfloor x + y \rfloor <= \lfloor 2x \rfloor = \lfloor 2y \rfloor$ for all real numbers x and y.

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9.0.150 Kenneth Rosen Edition 7 Exercise 2.4 Question 2 (Page No. 167) top

What is the term a_8 of the sequence $\{a_n\}$, if a_n equals

A. 2^{n-1} B. 7 C. $1 + (-1)^n$ D. $-(-2)^n$

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Answer key

9.0.151 Kenneth Rosen Edition 7 Exercise 2.4 Question 3 (Page No. 167) top

What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

A. $2^n + 1$
B. $(n+1)^{n+1}$
C. $\lfloor n/2 \rfloor$
D. $\lfloor n/2 \rfloor + \lceil n/2 \rceil$

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Answer key

9.0.152 Kenneth Rosen Edition 7 Exercise 2.4 Question 4 (Page No. 167) top

What are the terms a_0, a_1, a_2, and a_3 of the sequence $\{a_n\}$, where a_n equals

A. $(-2)^n$ B. 3 C. $7+4^n$ D. $2^n+(-2)^n$

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Answer key

9.0.153 Kenneth Rosen Edition 7 Exercise 2.4 Question 5 (Page No. 167) top

List the first 10 terms of each of these sequences.

A. the sequence that begins with 2 and in which each successive term is 3 more than the preceding term
B. the sequence that lists each positive integer three times, in increasing order
C. the sequence that lists the odd positive integers in increasing order, listing each odd integer twice
D. the sequence whose nth term is $n!-2^n$
E. the sequence that begins with 3, where each succeeding term is twice the preceding term
F. the sequence whose first term is 2, second term is 4, and each succeeding term is the sum of the two preceding terms
G. the sequence whose nth term is the number of bits in the binary expansion of the number n (defined in Section 4.2)
H. the sequence where the nth term is the number of letters in the English word for the index n

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Answer key

9.0.154 Kenneth Rosen Edition 7 Exercise 2.4 Question 6 (Page No. 167 - 168) top

List the first 10 terms of each of these sequences.

A. the sequence obtained by starting with 10 and obtaining each term by subtracting 3 from the previous term
B. the sequence whose nth term is the sum of the first n positive integers
C. the sequence whose nth term is 3^n-2^n
D. the sequence whose nth term is $\lfloor\sqrt{n}\rfloor$
E. the sequence whose first two terms are 1 and 5 and each succeeding term is the sum of the two previous terms
F. the sequence whose nth term is the largest integer whose binary expansion (defined in Section 4.2) has n bits (Write your answer in decimal notation.)
G. the sequence whose terms are constructed sequentially as follows: start with 1, then add 1, then multiply by 1, then add 2, then multiply by 2, and so on
H. the sequence whose nth term is the largest integer k such that $k!\leq n$

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Answer key

9.0.155 Kenneth Rosen Edition 7 Exercise 2.4 Question 7 (Page No. 168) top

Find at least three different sequences beginning with the terms $1, 2, 4$ whose terms are generated by a simple formula or rule.

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Answer key

9.0.156 Kenneth Rosen Edition 7 Exercise 2.4 Question 8 (Page No. 168) top

Find at least three different sequences beginning with the terms $3, 5, 7$ whose terms are generated by a simple formula or rule.

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Answer key

9.0.157 Kenneth Rosen Edition 7 Exercise 2.4 Question 9 (Page No. 168) top

Find the first five terms of the sequence defined by each of these recurrence relations and initial conditions.

A. $a_n=6a_{n-1}, a_0=2$

B. $a_n = a_{n-1}^2, a_1 = 2$
C. $a_n = a_{n-1} + 3a_{n-2}, a_0 = 1, a_1 = 2$
D. $a_n = na_{n-1} + n^2 a_{n-2}, a_0 = 1, a_1 = 1$
E. $a_n = a_{n-1} + a_{n-3}, a_0 = 1, a_1 = 2, a_2 = 0$

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9.0.158 Kenneth Rosen Edition 7 Exercise 2.4 Question 10 (Page No. 168) top

Find the first six terms of the sequence defined by each of these recurrence relations and initial conditions.

A. $a_n = -2a_{n-1}, a_0 = -1$
B. $a_n = a_{n-1} - a_{n-2}, a_0 = 2, a_1 = -1$
C. $a_n = 3a_{n-1}^2, a_0 = 1$
D. $a_n = na_{n-1} + a_{n-2}^2, a_0 = -1, a_1 = 0$
E. $a_n = a_{n-1} - a_{n-2} + a_{n-3}, a_0 = 1, a_1 = 1, a_2 = 2$

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9.0.159 Kenneth Rosen Edition 7 Exercise 2.4 Question 11 (Page No. 168) top

Let $a_n = 2^n + 5 \cdot 3^n$ for $n = 0, 1, 2, \ldots$

A. Find $a_0 a_1, a_2, a_3,$ and a_4.
B. Show that $a_2 = 5a_1 - 6a_0, a_3 = 5a_2 - 6a_1,$ and $a_4 = 5a_3 - 6a_2$.
C. Show that $an = 5a_{n-1} - 6a_{n-2}$ for all integers n with $n \geq 2$.

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9.0.160 Kenneth Rosen Edition 7 Exercise 2.4 Question 12 (Page No. 168) top

Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = -3a_{n-1} + 4a_{n-2}$ if

A. $a_n = 0$.
B. $an = 1$.
C. $a_n = (-4)^n$.
D. $a_n = 2(-4)^n + 3$.

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9.0.161 Kenneth Rosen Edition 7 Exercise 2.4 Question 14 (Page No. 168) top

For each of these sequences find a recurrence relation satisfied by this sequence. (The answers are not unique because there are infinitely many different recurrence relations satisfied by any sequence.)

A. $a_n = 3$
B. $a_n = 2n$
C. $a_n = 2n + 3$
D. $a_n = 5^n$
E. $a_n = n^2$
F. $a_n = n^2 + n$
G. $a_n = n + (-1)^n$
H. $a_n = n!$

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9.0.162 Kenneth Rosen Edition 7 Exercise 2.4 Question 15 (Page No. 168) top

Show that the sequence $\{a_n\}$ is a solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2} + 2n - 9$ if

A. $a_n = -n + 2$.
B. $a_n = 5(-1)^n - n + 2$.
C. $a_n = 3(-1)^n + 2^n - n + 2$.
D. $a_n = 7 \cdot 2^n - n + 2$.

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9.0.163 Kenneth Rosen Edition 7 Exercise 2.3 Question 73 (Page No. 155) top

Prove or disprove each of these statements about the floor and ceiling functions.

a. $\lceil \lfloor x \rfloor \rceil = \lfloor x \rfloor$ for all real number x.
b. $\lfloor 2x \rfloor = 2 \lfloor x \rfloor$ whenever x is a real number.
c. $\lceil x \rceil + \lceil y \rceil - \lceil x + y \rceil = 0$ or 1 whenever x and y are real numbers.
d. $\lceil xy \rceil = \lceil x \rceil \lceil y \rceil$ for all real numbers x and y.

e. $\lceil x/2 \rceil = \lfloor x + 1/2 \rfloor$ for all real numbers x.

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9.0.164 Kenneth Rosen Edition 7 Exercise 2.4 Question 16 (Page No. 168) top

Find the solution to each of these recurrence relations with the given initial conditions. Use an iterative approach such as that used in Example 10.

A. $a_n = -a_{n-1}, a_0 = 5$
B. $a_n = a_{n-1} + 3, a_0 = 1$
C. $a_n = a_{n-1} - n, a_0 = 4$
D. $a_n = 2a_{n-1} - 3, a_0 = -1$
E. $a_n = (n+1)a_{n-1}, a_0 = 2$
F. $a_n = 2na_{n-1}, a_0 = 3$
G. $a_n = -a_{n-1} + n - 1, a_0 = 7$

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9.0.165 Kenneth Rosen Edition 7 Exercise 2.4 Question 17 (Page No. 168) top

Find the solution to each of these recurrence relations and initial conditions. Use an iterative approach such as that used in Example 10.

A. $a_n = 3a_{n-1}, a_0 = 2$
B. $a_n = a_{n-1} + 2, a_0 = 3$
C. $a_n = a_{n-1} + n, a_0 = 1$
D. $a_n = a_{n-1} + 2n + 3, a_0 = 4$
E. $a_n = 2a_{n-1} - 1, a_0 = 1$
F. $a_n = 3a_{n-1} + 1, a_0 = 1$
G. $a_n = na_{n-1}, a_0 = 5$
H. $a_n = 2na_{n-1}, a_0 = 1$

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9.0.166 Kenneth Rosen Edition 7 Exercise 2.4 Question 18 (Page No. 168) top

A person deposits $\$1000$ in an account that yields 9% interest compounded annually.

A. Set up a recurrence relation for the amount in the account at the end of n years.
B. Find an explicit formula for the amount in the account at the end of n years.
C. How much money will the account contain after 100 years?

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9.0.167 Kenneth Rosen Edition 7 Exercise 2.4 Question 19 (Page No. 168) top

Suppose that the number of bacteria in a colony triples every hour.

A. Set up a recurrence relation for the number of bacteria after n hours have elapsed.
B. If 100 bacteria are used to begin a new colony, how many bacteria will be in the colony in 10 hours?

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9.0.168 Kenneth Rosen Edition 7 Exercise 2.4 Question 20 (Page No. 168) top

Assume that the population of the world in 2010 was 6.9 billion and is growing at the rate of 1.1% a year.

A. Set up a recurrence relation for the population of the world n years after 2010.
B. Find an explicit formula for the population of the world n years after 2010.
C. What will the population of the world be in 2030?

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9.0.169 Kenneth Rosen Edition 7 Exercise 2.4 Question 21 (Page No. 168) top

A factory makes custom sports cars at an increasing rate. In the first month, only one car is made, in the second month, two cars are made, and so on, with n cars made in the nth month.

A. Set up a recurrence relation for the number of cars produced in the first n months by this factory.
B. How many cars are produced in the first year?
C. Find an explicit formula for the number of cars produced in the first n months by this factory

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9.0.170 Kenneth Rosen Edition 7 Exercise 2.4 Question 22 (Page No. 168 - 169) top

An employee joined a company in 2009 with a starting salary of $\$50,000$. Every year this employee receives a raise of $\$1000$ plus 5% of the salary of the previous year.

A. Set up a recurrence relation for the salary of this employee n years after 2009.
B. What will the salary of this employee be in 2017?
C. Find an explicit formula for the salary of this employee n years after 2009.

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9.0.171 Kenneth Rosen Edition 7 Exercise 2.4 Question 23 (Page No. 169) top

Find a recurrence relation for the balance $B(k)$ owed at the end of k months on a loan of $\$5000$ at a rate of 7% if a payment of $\$100$ is made each month. $[$Hint: Express $B(k)$ in terms of $B(k-1)$; the monthly interest is $(0.07/12)B(k-1)$. $]$

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9.0.172 Kenneth Rosen Edition 7 Exercise 2.4 Question 24 (Page No. 169) top

A. Find a recurrence relation for the balance $B(k)$ owed at the end of k months on a loan at a rate of r if a payment P is made on the loan each month. $[$Hint: Express $B(k)$ in terms of $B(k-1)$ and note that the monthly interest rate is $r/12$.$]$
B. Determine what the monthly payment P should be so that the loan is paid off after T months.

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9.0.173 Kenneth Rosen Edition 7 Exercise 2.4 Question 25 (Page No. 169) top

For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list.Assuming that your formula or rule is correct, determine the next three terms of the sequence.

A. $1, 0, 1, 1, 0, 0, 1, 1, 1, 0, 0, 0, 1, \dots$
B. $1, 2, 2, 3, 4, 4, 5, 6, 6, 7, 8, 8, \dots$
C. $1, 0, 2, 0, 4, 0, 8, 0, 16, 0, \dots$
D. $3, 6, 12, 24, 48, 96, 192, \dots$
E. $15, 8, 1, -6, -13, -20, -27, \dots$
F. $3, 5, 8, 12, 17, 23, 30, 38, 47, \dots$
G. $2, 16, 54, 128, 250, 432, 686, \dots$
H. $2, 3, 7, 25, 121, 721, 5041, 40321, \dots$

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9.0.174 Kenneth Rosen Edition 7 Exercise 2.4 Question 26 (Page No. 169) top

For each of these lists of integers, provide a simple formula or rule that generates the terms of an integer sequence that begins with the given list.Assuming that your formula or rule is correct, determine the next three terms of the sequence.

A. $3, 6, 11, 18, 27, 38, 51, 66, 83, 102, \dots$
B. $7, 11, 15, 19, 23, 27, 31, 35, 39, 43, \dots$
C. $1, 10, 11, 100, 101, 110, 111, 1000, 1001, 1010, 1011, \dots$
D. $1, 2, 2, 2, 3, 3, 3, 3, 3, 5, 5, 5, 5, 5, 5, 5, \dots$
E. $0, 2, 8, 26, 80, 242, 728, 2186, 6560, 19682, \dots$
F. $1, 3, 15, 105, 945, 10395, 135135, 2027025, 34459425, \dots$
G. $1, 0, 0, 1, 1, 1, 0, 0, 0, 0, 1, 1, 1, 1, 1, \dots$
H. $2, 4, 16, 256, 65536, 4294967296, \dots$

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9.0.175 Kenneth Rosen Edition 7 Exercise 2.4 Question 1 (Page No. 167) top

Find these terms of the sequence $\{a_n\}$, where $a_n = 2 \cdot (-3)^n + 5^n$.

A. a_0 B. a_1 C. a_4 D. a_5

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Answer key

9.0.176 Kenneth Rosen Edition 7 Exercise 2.4 Question 27 (Page No. 169) top

Show that if a_n denotes the nth positive integer that is not a perfect square, then $a_n = n + \{\sqrt{n}\}$, where $\{x\}$ denotes the integer closest to the real number x.

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9.0.177 Kenneth Rosen Edition 7 Exercise 2.3 Question 43 (Page No. 154) top

Let $g(x) = \lfloor x \rfloor$. Find

a. $g^{-1}(\{0\})$
b. $g^{-1}(\{-1, 0, 1\})$
c. $g^{-1}(\{x | 0 < x < 1\})$

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9.0.178 Kenneth Rosen Edition 7 Exercise 2.3 Question 42 (Page No. 154) top

Let f be the function from R to R defined by $f(x) = x^2$. Find

a. $f^{-1}(\{1\})$
b. $f^{-1}(\{x | 0 < x < 1\})$
c. $f^{-1}(\{x | x > 4\})$

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9.0.179 Kenneth Rosen Edition 7 Exercise 2.5 Question 11 (Page No. 176) top

Give an example of two uncountable sets A and B such that $A \cap B$ is

A. finite.
B. countably infinite.
C. uncountable

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9.0.180 Kenneth Rosen Edition 7 Exercise 2.4 Question 32 (Page No. 169) top

Find the value of each of these sums.

A. $\sum_{j=0}^{8}(1 + (-1)^j)$ B. $\sum_{j=0}^{8}(3^j - 2^j)$

C. $\sum_{j=0}^{8}(2 \cdot 3^j + 3 \cdot 2^j)$ D. $\sum_{j=0}^{8}(2^{j+1} - 2^j)$

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9.0.181 Kenneth Rosen Edition 7 Exercise 2.4 Question 33 (Page No. 169) top

Compute each of these double sums.

A. $\sum_{i=1}^{2}\sum_{j=1}^{3}(i + j)$ B. $\sum_{i=0}^{2}\sum_{j=0}^{3}(2i + 3j)$

C. $\sum_{i=1}^{3}\sum_{j=0}^{2} i$ D. $\sum_{i=0}^{2}\sum_{j=1}^{3} ij$

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9.0.182 Kenneth Rosen Edition 7 Exercise 2.4 Question 34 (Page No. 169) top

Compute each of these double sums.

1. $\sum_{i=1}^{3}\sum_{j=1}^{2}(i-j)$
2. $\sum_{i=0}^{3}\sum_{j=0}^{2}(3i+2j)$
3. $\sum_{i=1}^{3}\sum_{j=0}^{2} j$
4. $\sum_{i=0}^{2}\sum_{j=0}^{3} i^2 j^3$

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9.0.183 Kenneth Rosen Edition 7 Exercise 2.4 Question 35 (Page No. 169) top

Show that $\sum_{j=1}^{n}(a_j - a_{j-1}) = a_n - a_{0,}$ where $a_0, a_1, \dots, a_n$ is a sequence of real numbers. This type of sum is called telescoping.

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9.0.184 Kenneth Rosen Edition 7 Exercise 2.4 Question 36 (Page No. 169) top

Version1 : Use the identity $\dfrac{1}{k(k+1)} = \dfrac{\left(\frac{1}{k-1}\right)}{(k+1)}$ and question 35 to compute $\sum_{k=1}^{n}\dfrac{1}{k(k+1)}$.

Version2 : Use the identity $1/(k(k+1)) = 1/k - 1/(k+1)$ and question 35 to compute $\sum_{k=1}^{n} 1/(k(k+1))$.

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9.0.185 Kenneth Rosen Edition 7 Exercise 2.4 Question 37 (Page No. 169) top

Sum both sides of the identity $k^2 - (k-1)^2 = 2k - 1$ from $k = 1$ to $k = n$ and use question 35 to find

A. a formula for $\sum_{k=1}^{n}(2k-1)$ (the sum of the first n odd natural numbers).

B. a formula for $\sum_{k=1}^{n} k$.

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9.0.186 Kenneth Rosen Edition 7 Exercise 2.4 Question 38 (Page No. 169) top

Use the technique given in question 35, together with the result of question $37b$, to derive the formula for $\sum_{k=1}^{n} k^2$ given in Table 2. [Hint: Take $a_k = k^3$ in the telescoping sum in question 35.]

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9.0.187 Kenneth Rosen Edition 7 Exercise 2.4 Question 39 (Page No. 169) top

Find $\sum_{k=100}^{200} k$. (Use Table 2.)

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9.0.188 Kenneth Rosen Edition 7 Exercise 2.4 Question 40 (Page No. 169) top

Find $\sum_{k=99}^{200} k^3$. (Use Table 2.)

9.0.189 Kenneth Rosen Edition 7 Exercise 2.4 Question 41 (Page No. 169) top

Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt{k} \rfloor$, when m is a positive integer.

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9.0.190 Kenneth Rosen Edition 7 Exercise 2.4 Question 42 (Page No. 169) top

Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt{k} \rfloor$, when m is a positive integer.

There is also a special notation for products. The product of $a_m, a_{m+1}, \ldots, a_n$ is represented by $\prod_{j=m}^{n} a_j$, read as the product from $j = m$ to $j = n$ of a_j.

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9.0.191 Kenneth Rosen Edition 7 Exercise 2.4 Question 44 (Page No. 170) top

Recall that the value of the factorial function at a positive integer n, denoted by $n!$, is the product of the positive integers from 1 to n, inclusive. Also, we specify that $0! = 1$.

A. Express $n!$ using product notation.

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9.0.192 Kenneth Rosen Edition 7 Exercise 2.4 Question 45 (Page No. 170) top

Recall that the value of the factorial function at a positive integer n, denoted by $n!$, is the product of the positive integers from 1 to n, inclusive. Also, we specify that $0! = 1$.

B. $\sum_{i=0}^{4} j\,!$

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9.0.193 Kenneth Rosen Edition 7 Exercise 2.4 Question 43 (Page No. 170) top

What are the values of the following products?

A. $\prod_{i=0}^{10} i$

B. $\prod_{i=5}^{8} i$

C. $\prod_{i=1}^{100} (-1)^i$

D. $\prod_{i=1}^{10} 2$

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9.0.194 Kenneth Rosen Edition 7 Exercise 2.4 Question 46 (Page No. 170) top

Recall that the value of the factorial function at a positive integer n, denoted by $n!$, is the product of the positive integers from 1 to n, inclusive. Also, we specify that $0! = 1$.

C. $\prod_{i=0}^{4} j\,!$

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9.0.195 Kenneth Rosen Edition 7 Exercise 2.5 Question 1 (Page No. 176) top

Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

A. the negative integers
B. the even integers
C. the integers less than 100
D. the real numbers between 0 and $\frac{1}{2}$
E. the positive integers less than $1,000,000,000$
F. the integers that are multiples of 7

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Answer key

9.0.196 Kenneth Rosen Edition 7 Exercise 2.5 Question 2 (Page No. 176) top

Determine whether each of these sets is finite, countably infinite, or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

A. the integers greater than 10
B. the odd negative integers
C. the integers with absolute value less than $1,000,000$
D. the real numbers between 0 and 2
E. the set $A \times Z^{+}$ where $A = \{2, 3\}$
F. the integers that are multiples of 10

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Answer key

9.0.197 Kenneth Rosen Edition 7 Exercise 2.5 Question 3 (Page No. 176) top

Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

A. all bit strings not containing the bit 0
B. all positive rational numbers that cannot be written with denominators less than 4
C. the real numbers not containing 0 in their decimal representation
D. the real numbers containing only a finite number of $1s$ in their decimal representation

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9.0.198 Kenneth Rosen Edition 7 Exercise 2.5 Question 4 (Page No. 176) top

Determine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one correspondence between the set of positive integers and that set.

A. integers not divisible by 3
B. integers divisible by 5 but not by 7
C. the real numbers with decimal representations consisting of all $1s$
D. the real numbers with decimal representations of all $1s$ or $9s$

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9.0.199 Kenneth Rosen Edition 7 Exercise 2.5 Question 5 (Page No. 176) top

Show that a finite group of guests arriving at Hilbert's fully occupied Grand Hotel can be given rooms without evicting any current guest.

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9.0.200 Kenneth Rosen Edition 7 Exercise 2.5 Question 6 (Page No. 176) top

Suppose that Hilbert's Grand Hotel is fully occupied, but the hotel closes all the even numbered rooms for maintenance. Show that all guests can remain in the hotel.

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9.0.201 Kenneth Rosen Edition 7 Exercise 2.5 Question 7 (Page No. 176) top

Suppose that Hilbert's Grand Hotel is fully occupied on the day the hotel expands to a second building which also contains a countably infinite number of rooms. Show that the current guests can be spread out to fill every room of the two buildings of the hotel.

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9.0.202 Kenneth Rosen Edition 7 Exercise 2.5 Question 8 (Page No. 176) top

Show that a countably infinite number of guests arriving at Hilbert's fully occupied Grand Hotel can be given rooms without evicting any current guest.

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9.0.203 Kenneth Rosen Edition 7 Exercise 2.5 Question 9 (Page No. 176) top

Suppose that a countably infinite number of buses, each containing a countably infinite number of guests, arrive at Hilbert's fully occupied Grand Hotel. Show that all the arriving guests can be accommodated without evicting any current guest.

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9.0.204 Kenneth Rosen Edition 7 Exercise 2.5 Question 10 (Page No. 176) top

Give an example of two uncountable sets A and B such that $A - B$ is

A. finite.
B. countably infinite.
C. uncountable.

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9.0.205 Kenneth Rosen Edition 7 Exercise 2.3 Question 44 (Page No. 154) top

Let f be a function from A to B. Let S and T be subsets of B. Show that

a. $f^{-1}(S \cup T) = f^{-1}(S) \cup f^{-1}(T)$
b. $f^{-1}(S \cap T) = f^{-1}(S) \cap f^{-1}(T)$

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9.0.206 Kenneth Rosen Edition 7 Exercise 2.4 Question 31 (Page No. 169) top

What is the value of each of these sums of terms of a geometric progression?

A. $\sum_{j=0}^{8} 3 \cdot 2^j$
B. $\sum_{j=1}^{8} 2^j$
C. $\sum_{j=2}^{8} (-3)^j$
D. $\sum_{j=0}^{8} 2 \cdot (-3)^j$

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9.0.207 Kenneth Rosen Edition 7 Exercise 2.4 Question 30 (Page No. 169) top

What are the values of these sums, where $S = \{1, 3, 5, 7\}$?

A. $\sum_{j \in S} j$

B. $\sum_{j \in S} j^2$

C. $\sum_{j \in S} (1/j)$

D. $\sum_{j \in S} 1$

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9.0.208 Kenneth Rosen Edition 7 Exercise 2.5 Question 26 (Page No. 177) top

Use question 25 to provide a proof different from that in the text that the set of rational numbers is countable. [Hint: Show that you can express a rational number as a string of digits with a slash and possibly a minus sign.]

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9.0.209 Kenneth Rosen Edition 7 Exercise 2.5 Question 13 (Page No. 176) top

Explain why the set A is countable if and only if $|A| \leq |Z^+|$.

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9.0.210 Kenneth Rosen Edition 7 Exercise 2.5 Question 15 (Page No. 176) top

Show that if A and B are sets, A is uncountable, and $A \subseteq B$, then B is uncountable

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9.0.211 Kenneth Rosen Edition 7 Exercise 2.5 Question 16 (Page No. 176) top

Show that a subset of a countable set is also countable.

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9.0.212 Kenneth Rosen Edition 7 Exercise 2.5 Question 17 (Page No. 176) top

If A is an uncountable set and B is a countable set, must $A - B$ be uncountable?

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9.0.213 Kenneth Rosen Edition 7 Exercise 2.5 Question 18 (Page No. 177) top

Show that if A and B are sets $|A| = |B|$, then $|P(A)| = |P(B)|$.

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9.0.214 Kenneth Rosen Edition 7 Exercise 2.5 Question 19 (Page No. 177) top

Show that if A, B, C, and D are sets with $|A| = |B|$ and $|C| = |D|$, then $|A \times C| = |B \times D|$.

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9.0.215 Kenneth Rosen Edition 7 Exercise 2.5 Question 20 (Page No. 177) top

Show that if $|A| = |B|$ and $|B| = |C|$, then $|A| = |C|$.

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9.0.216 Kenneth Rosen Edition 7 Exercise 2.5 Question 21 (Page No. 177) top

Show that if A, B, and C are sets such that $|A| \leq |B|$ and $|B| \leq |C|$, then $|A| \leq |C|$.

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9.0.217 Kenneth Rosen Edition 7 Exercise 2.5 Question 22 (Page No. 177) top

Suppose that A is a countable set. Show that the set B is also countable if there is an onto function f from A to B.

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9.0.218 Kenneth Rosen Edition 7 Exercise 2.5 Question 23 (Page No. 177) top

Show that if A is an infinite set, then it contains a countably infinite subset.

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9.0.219 Kenneth Rosen Edition 7 Exercise 2.5 Question 24 (Page No. 177) top

Show that there is no infinite set A such that $|A| < |Z^+| = \aleph_0$.

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9.0.220 Kenneth Rosen Edition 7 Exercise 2.5 Question 25 (Page No. 177) top

Prove that if it is possible to label each element of an infinite set S with a finite string of keyboard characters, from a finite list characters, where no two elements of S have the same label, then S is a countably infinite set.

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9.0.221 Kenneth Rosen Edition 7 Exercise 2.5 Question 27 (Page No. 177) top

Show that the union of a countable number of countable sets is countable.

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9.0.222 Kenneth Rosen Edition 7 Exercise 2.5 Question 28 (Page No. 177) top

Show that the set $Z^+ \times Z^+$ is countable.

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9.0.223 Kenneth Rosen Edition 7 Exercise 2.5 Question 12 (Page No. 176) top

Show that if A and B are sets and $A \subset B$ then $|A| \leq |B|$.

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9.0.224 Kenneth Rosen Edition 7 Exercise 2.5 Question 29 (Page No. 177) top

Show that the set of all finite bit strings is countable.

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9.0.225 Kenneth Rosen Edition 7 Exercise 2.5 Question 30 (Page No. 177) top

Show that the set of real numbers that are solutions of quadratic equations $ax^2 + bx + c = 0$, where $a, b,$ and c are integers, is countable.

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9.0.226 Kenneth Rosen Edition 7 Exercise 2.5 Question 31 (Page No. 177) top

Show that $Z^+ \times Z^+$ is countable by showing that the polynomial function $f : Z^+ \times Z^+ \to Z^+$ with $f(m, n) = \dfrac{(m+n-2)(m+n-1)}{2} + m$ is one-to one and onto.

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9.0.227 Kenneth Rosen Edition 7 Exercise 2.5 Question 32 (Page No. 177) top

Show that when you substitute $(3n+1)^2$ for each occurrence of n and $(3m+1)^2$ for each occurrence of m in the right-hand side of the formula for the function $f(m, n)$ in question 31, you obtain a one-to-one polynomial function $Z \times Z \to Z$. It is an open question whether there is a one-to-one polynomial function $Q \times Q \to Q$.

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9.0.228 Kenneth Rosen Edition 7 Exercise 2.5 Question 33 (Page No. 177) top

Use the Schröder-Bernstein theorem to show that $(0, 1)$ and $[0, 1]$ have the same cardinality.

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9.0.229 Kenneth Rosen Edition 7 Exercise 2.5 Question 34 (Page No. 177) top

Show that $(0, 1)$ and R have the same cardinality. [Hint: Use the Schröder-Bernstein theorem.]

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9.0.230 Kenneth Rosen Edition 7 Exercise 2.5 Question 35 (Page No. 177) top

Show that there is no one-to-one correspondence from the set of positive integers to the power set of the set of positive integers. [Hint: Assume that there is such a one-to-one correspondence. Represent a subset of the set of positive integers as an infinite bit string with ith bit 1 if i belongs to the subset and 0 otherwise. Suppose that you can list these infinite strings in a sequence indexed by the positive integers. Construct a new bit string with its ith bit equal to the complement of the ith bit of the ith string in the list. Show that this new bit string cannot appear in the list.]

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

9.0.231 Kenneth Rosen Edition 7 Exercise 2.5 Question 36 (Page No. 177) top

Show that there is a one-to-one correspondence from the set of subsets of the positive integers to the set real numbers between 0 and 1. Use this result and question 34 and 35 to conclude that $\aleph_0 <| P(Z^+) |=| R |.$ [Hint: Look at the first part of the hint for Exercise 35.]

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

9.0.232 Kenneth Rosen Edition 7 Exercise 2.5 Question 37 (Page No. 177) top

Show that the set of all computer programs in a particular programming language is countable. [Hint: A computer program written in a programming language can be thought of as a string of symbols from a finite alphabet.]

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

9.0.233 Kenneth Rosen Edition 7 Exercise 2.5 Question 38 (Page No. 177) top

Show that the set of functions from the positive integers to the set $\{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\}$ is uncountable. [Hint: First set up a one-to-one correspondence between the set of real numbers between 0 and 1 and a subset of these functions. Do this by associating to the real number $0.\ d_1 d_2 \dots d_n \dots$ the function f with $f(n) = dn.$]

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

Answer key

9.0.234 Kenneth Rosen Edition 7 Exercise 2.5 Question 39 (Page No. 177) top

We say that a function is computable if there is a computer program that finds the values of this function. Use question 37 and 38 to show that there are functions that are not computable.

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

9.0.235 Kenneth Rosen Edition 7 Exercise 2.5 Question 14 (Page No. 176) top

Show that if A and B are sets with the same cardinality, then $|A| \leq |B|$ and $|B| \leq |A|$.

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

9.0.236 Kenneth Rosen Edition 7 Exercise 2.5 Question 40 (Page No. 177) top

Show that if S is a set, then there does not exist an onto function f from S to $P(S)$, the power set of S. Conclude that $|S| < |P(S)|$. This result is known as Cantor's theorem. [Hint: Suppose such a function f existed. Let $T = \{s \in S \mid s \notin f(s)\}$ and show that no element s can exist for which $f(s) = T.$]

kenneth-rosen discrete-mathematics set-theory&algebra descriptive

Answer key

Answer Keys

9.0.1	N/A	9.0.2	Q-Q	9.0.3	Q-Q	9.0.4	Q-Q	9.0.5	Q-Q
9.0.6	Q-Q	9.0.7	Q-Q	9.0.8	Q-Q	9.0.9	Q-Q	9.0.10	Q-Q
9.0.11	Q-Q	9.0.12	Q-Q	9.0.13	Q-Q	9.0.14	Q-Q	9.0.15	Q-Q
9.0.16	Q-Q	9.0.17	Q-Q	9.0.18	Q-Q	9.0.19	Q-Q	9.0.20	Q-Q
9.0.21	Q-Q	9.0.22	Q-Q	9.0.23	Q-Q	9.0.24	D	9.0.25	Q-Q
9.0.26	Q-Q	9.0.27	Q-Q	9.0.28	Q-Q	9.0.29	Q-Q	9.0.30	Q-Q
9.0.31	Q-Q	9.0.32	Q-Q	9.0.33	Q-Q	9.0.34	Q-Q	9.0.35	Q-Q
9.0.36	Q-Q	9.0.37	Q-Q	9.0.38	Q-Q	9.0.39	Q-Q	9.0.40	Q-Q
9.0.41	Q-Q	9.0.42	Q-Q	9.0.43	Q-Q	9.0.44	Q-Q	9.0.45	Q-Q
9.0.46	Q-Q	9.0.47	Q-Q	9.0.48	Q-Q	9.0.49	Q-Q	9.0.50	Q-Q
9.0.51	Q-Q	9.0.52	Q-Q	9.0.53	Q-Q	9.0.54	Q-Q	9.0.55	Q-Q
9.0.56	Q-Q	9.0.57	Q-Q	9.0.58	Q-Q	9.0.59	Q-Q	9.0.60	Q-Q
9.0.61	Q-Q	9.0.62	Q-Q	9.0.63	Q-Q	9.0.64	Q-Q	9.0.65	Q-Q
9.0.66	Q-Q	9.0.67	Q-Q	9.0.68	Q-Q	9.0.69	Q-Q	9.0.70	Q-Q
9.0.71	Q-Q	9.0.72	Q-Q	9.0.73	Q-Q	9.0.74	Q-Q	9.0.75	Q-Q
9.0.76	Q-Q	9.0.77	Q-Q	9.0.78	Q-Q	9.0.79	Q-Q	9.0.80	Q-Q
9.0.81	Q-Q	9.0.82	Q-Q	9.0.83	Q-Q	9.0.84	Q-Q	9.0.85	Q-Q
9.0.86	Q-Q	9.0.87	Q-Q	9.0.88	Q-Q	9.0.89	Q-Q	9.0.90	Q-Q
9.0.91	Q-Q	9.0.92	Q-Q	9.0.93	Q-Q	9.0.94	Q-Q	9.0.95	Q-Q
9.0.96	Q-Q	9.0.97	Q-Q	9.0.98	Q-Q	9.0.99	Q-Q	9.0.100	Q-Q
9.0.101	Q-Q	9.0.102	Q-Q	9.0.103	Q-Q	9.0.104	Q-Q	9.0.105	Q-Q
9.0.106	Q-Q	9.0.107	Q-Q	9.0.108	Q-Q	9.0.109	Q-Q	9.0.110	Q-Q
9.0.111	Q-Q	9.0.112	Q-Q	9.0.113	Q-Q	9.0.114	Q-Q	9.0.115	Q-Q
9.0.116	Q-Q	9.0.117	Q-Q	9.0.118	Q-Q	9.0.119	N/A	9.0.120	Q-Q
9.0.121	Q-Q	9.0.122	Q-Q	9.0.123	Q-Q	9.0.124	Q-Q	9.0.125	Q-Q
9.0.126	Q-Q	9.0.127	Q-Q	9.0.128	Q-Q	9.0.129	Q-Q	9.0.130	Q-Q
9.0.131	Q-Q	9.0.132	Q-Q	9.0.133	Q-Q	9.0.134	Q-Q	9.0.135	Q-Q
9.0.136	Q-Q	9.0.137	Q-Q	9.0.138	Q-Q	9.0.139	Q-Q	9.0.140	Q-Q
9.0.141	Q-Q	9.0.142	Q-Q	9.0.143	Q-Q	9.0.144	Q-Q	9.0.145	Q-Q
9.0.146	Q-Q	9.0.147	N/A	9.0.148	N/A	9.0.149	Q-Q	9.0.150	N/A
9.0.151	N/A	9.0.152	N/A	9.0.153	N/A	9.0.154	N/A	9.0.155	N/A
9.0.156	N/A	9.0.157	N/A	9.0.158	N/A	9.0.159	N/A	9.0.160	N/A
9.0.161	N/A	9.0.162	N/A	9.0.163	Q-Q	9.0.164	N/A	9.0.165	N/A
9.0.166	N/A	9.0.167	N/A	9.0.168	N/A	9.0.169	N/A	9.0.170	N/A
9.0.171	N/A	9.0.172	N/A	9.0.173	N/A	9.0.174	N/A	9.0.175	N/A
9.0.176	N/A	9.0.177	Q-Q	9.0.178	Q-Q	9.0.179	N/A	9.0.180	N/A
9.0.181	N/A	9.0.182	N/A	9.0.183	N/A	9.0.184	N/A	9.0.185	N/A
9.0.186	N/A	9.0.187	N/A	9.0.188	Q-Q	9.0.189	N/A	9.0.190	N/A
9.0.191	N/A	9.0.192	N/A	9.0.193	N/A	9.0.194	N/A	9.0.195	N/A
9.0.196	N/A	9.0.197	N/A	9.0.198	N/A	9.0.199	N/A	9.0.200	N/A

9.0.201	N/A
9.0.206	N/A
9.0.211	N/A
9.0.216	N/A
9.0.221	N/A
9.0.226	N/A
9.0.231	N/A
9.0.236	N/A

9.0.202	N/A
9.0.207	N/A
9.0.212	N/A
9.0.217	N/A
9.0.222	N/A
9.0.227	N/A
9.0.232	N/A

9.0.203	N/A
9.0.208	N/A
9.0.213	N/A
9.0.218	N/A
9.0.223	N/A
9.0.228	N/A
9.0.233	N/A

9.0.204	N/A
9.0.209	N/A
9.0.214	N/A
9.0.219	N/A
9.0.224	N/A
9.0.229	N/A
9.0.234	N/A

9.0.205	Q-Q
9.0.210	N/A
9.0.215	N/A
9.0.220	N/A
9.0.225	N/A
9.0.230	N/A
9.0.235	N/A

10.0.1 Kenneth Rosen Edition 7 Exercise 7.3 Example 2 (Page No. 471) top

Suppose that one person in 10,000 people has a rare genetic disease. There is an excellent test for the disease; 99.9% of people with the disease test positive and only 0.02% who do not have the disease test positive.

a) What is the probability that someone who tests positive has the genetic disease?

b) What is the probability that someone who tests negative does not have the disease?

My Answers :-

P(D) = 0.00001

P(D') = 0.99999

P(P/D) = 0.999

P(P'/D) = 0.001

P(P/D') = 0.0002

P(P'/D') = 0.9998

Answer a) $\frac{(0.999)(0.00001)}{(0.999)(0.00001)+(0.0002)(0.99999)}$

Answer b) $\frac{(0.9998)(0.99999)}{(0.9998)(0.99999)+(0.001)(0.00001)}$

Correct me if I am wrong?

discrete-mathematics kenneth-rosen probability

10.0.2 Probability - Gravner-15.b top

A tennis tournament has $2n$ participants, n Swedes and n Norwegians. First, n people are chosen at random from the $2n$ (with no regard to nationality) and then paired randomly with the other n people. Each pair proceeds to play one match. An outcome is a set of n (ordered) pairs, giving the winner and the loser in each of the n matches.

(b) What do you need to assume to conclude that all outcomes are equally likely?

probability gravner engineering-mathematics

10.0.3 Probability - Gravner-7 top

A bag has 6 pieces of paper, each with one of the letters, $E. E, P, P, P,$ and R, on it. Pull 6 pieces at random out of the bag (1) without , and (2) with replacement. What is the probability that these pieces, in order, spell $PEPPER$?

probability gravner engineering-mathematics

10.0.4 Probability - Gravner-8 top

Sit 3 men and 3 women at random (1) in a row of chairs and (2) around a table. Compute Probability (all women sit together). In case (2), also compute Probability (men and women alternate).

probability gravner engineering-mathematics

10.0.5 Probability - Gravner-9 top

A group consists of 3 Norwegians, 4 Swedes, and 5 Finns, and they sit at random around a table. What is the probability that all groups end up sitting together?

probability gravner engineering-mathematics

Answer key

10.0.6 Probability - Gravner-28 top

A group of 20 Scandinavians consists of 7 Swedes, 3 Finns, and 10 Norwegians. A committee of five people is chosen at random from this group. What is the probability that at least one of the three nations is not represented on the committee?

probability gravner engineering-mathematics

10.0.7 Probability - Gravner-10 top

A fair coin is tossed 10 times. What is the probability that we get exactly 5 Heads?

probability gravner engineering-mathematics

10.0.8 Probability - Gravner-6.b top

Shuffle a deck of cards. Compute the probability:

Probability for(all cards of the same suit end up next to each other)

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10.0.9 Probability - Gravner-6.c top

Shuffle a deck of cards. Compute the probability:

Probability for(hearts are together)

probability gravner engineering-mathematics

10.0.10 Probability - Gravner-15.c top

A tennis tournament has $2n$ participants, n Swedes and n Norwegians. First, n people are chosen at random from the $2n$ (with no regard to nationality) and then paired randomly with the other n people. Each pair proceeds to play one match. An outcome is a set of n (ordered) pairs, giving the winner and the loser in each of the n matches.

(c) Under this assumption, compute the probability that all Swedes are the winners.

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10.0.11 Probability - Gravner-5 top

Roll a die 4 times. What is the probability that you get different numbers?

probability gravner engineering-mathematics

10.0.12 Probability - Gravner-20 top

A group of 3 Norwegians, 4 Swedes, and 5 Finns is seated at random around a table. Compute the probability that at least one of the three groups ends up sitting together.

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10.0.13 Probability - Gravner-26.a top

Roll a single die 10 times. Computer the following probabilities:

a) that you get at least one 6.

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10.0.14 Probability - Gravner-26.b top

Roll a single die 10 times. Computer the following probabilities:

b) that you get at least one 6 and at least one 5.

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10.0.15 Probability - Gravner-26.c top

Roll a single die 10 times. Computer the following probabilities:

c) that you get three $1's$ two $2's$ and five $3's$

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10.0.16 Probability - Gravner-15.a top

A tennis tournament has $2n$ participants, n Swedes and n Norwegians. First, n people are chosen at random from the $2n$ (with no regard to nationality) and then paired randomly with the other n people. Each pair proceeds to play one match. An outcome is a set of n (ordered) pairs, giving the winner and the loser in each of the n matches.

(a) Determine the number of outcomes.

probability gravner engineering-mathematics

Answer key

10.0.17 Kenneth Rosen Edition 7 Exercise 7.2 Example 2 (Page No. 455) top

Why is it 7 in the denominator?

EXAMPLE 2 Suppose that a die is biased (or loaded) so that 3 appears twice as often as each other number but that the other five outcomes are equally likely. What is the probability that an odd number appears when we roll this die?

7.2 Probability Theory 455

Solution: We want to find the probability of the event $E = \{1, 3, 5\}$. By Exercise 2, we have

$$p(1) = p(2) = p(4) = p(5) = p(6) = 1/7;\ p(3) = 2/7.$$

It follows that

$$p(E) = p(1) + p(3) + p(5) = 1/7 + 2/7 + 1/7 = 4/7.$$

kenneth-rosen engineering-mathematics probability

Answer key

10.0.18 Probability - Gravner-55 top

An urn contains 20 balls numbers $1, \dots\dots\dots 20$. Select 5 balls at random, without replacement. Let X be the largest number among selected balls. Determine its p.m.f. and the probability that at least one the selected numbers is 15 or more.

probability gravner engineering-mathematics

Answer key

10.0.19 Probability - Gravner-6.a top

Shuffle a deck of cards. Compute the probability :

Probability for(top card is an Ace)

probability gravner engineering-mathematics

10.0.20 Probability - Gravner-4 top

Roll Two dice. What is the most likely sum?

probability gravner engineering-mathematics

10.0.21 Probability - Gravner-65.c top

A biologist needs at least 3 mature specimens of certain plant. The plant needs a year to reach maturity; once a seed is planted, any plant will survive for the year with probability $1/1000$ (independently of other plants). The biologist plants 3000 seeds. A year is deemed a success if three or more plants from these seeds reach maturity.

(c) The biologist plans to do this year after year. What is the probability that he has at least 2 success in 10 years?

10.0.22 Probability - Gravner-43 top

A mathematician carries two matchboxes, each originally containing n matches. Each time he needs a match, he is equally likely to take it from either box. what is the probability that, upon reaching for a box and finding it empty, there are exactly k matches still in the other box? Here, $0 \leq k \leq n$.

probability gravner engineering-mathematics

10.0.23 Probability - Gravner-30.b top

Roll a fair die 10 times.

b) Compute the probability that at least one number occurs exactly once.

probability gravner engineering-mathematics

10.0.24 Probability - Gravner-36 top

We have a fair coin and unfair coin,which always comes out Heads. Choose one at random, toss it twice. It comes out Heads both times. What is the probability that the coin is fair?

probability gravner engineering-mathematics

Answer key

10.0.25 Probability - Gravner-30.a top

Roll a fair die 10 times.

a) Compute the probability that at least one number occurs exactly 6 times

gravner probability engineering-mathematics

10.0.26 Probability - Gravner-38 top

Assume 10 % of people have a certain disease. A test gives the correct diagnosis with probability of 0.8; that is, if the person is sick, the test will be positive with probability 0.8, but if the person is not sick, the test will be positive with probability 0.2. A random person from the population has tested positive for the disease. What is the probability that he is actually sick?(No, it is not 0.8!)

probability gravner engineering-mathematics

10.0.27 Probability - Gravner-40 top

You roll a die, your friend tosses a coin.

- If you roll 6, you win outright.
- If you do not roll 6 and your friend tosses Heads, you lose outright.
- If neither, the game is repeated until decided.

What is the probability that you win?

probability gravner engineering-mathematics

Answer key

10.0.28 Probability - Gravner-41 top

Many casinos allow you to bet even money of the following game. Two dice are rolled and the sum S is observed.

- If $S \epsilon \{7, 11\}$, you win immediately.
- If $S \epsilon \{2, 3, 12\}$, you lose immediately.
- If $S \epsilon \{4, 5, 6, 8, 9, 10\}$, the pair of dice rolled repeatedly until one of the following happens:

1. S repeats, in which case you win.
2. 7 repeats , in which case you lose.

What is the winning probability?

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10.0.29 Probability - Gravner-42 top

Assume that two equally matched teams, A and B, play a series of games and that the first team that wins four games and that the first wins four games is the overall winner of the series. As it happens, team A lost the first game. What is the probability it will win the series? Assume that the games are Bernoulli trials with success probability 1/2.

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10.0.30 Probability - Gravner-44.a top

Each day, you independently decide, with probability p, to flip a fair coin. Otherwise, you do nothing.

(a) What is the probability exactly 10 Heads in the first 20 days?

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10.0.31 Probability - Gravner-3 top

Toss three fair coins. What is the probability of exactly one Heads(H) ?

probability gravner engineering-mathematics

Answer key

10.0.32 Probability - Gravner-44.b top

Each day, you independently decide, with probability p, to flip a fair coin. Otherwise, you do nothing.

(b) What is the probability of getting 10 Heads before 5 Tails?

probability gravner engineering-mathematics

10.0.33 Probability - Gravner-45.a top

You roll a die and your score is the number shown on the die. Your friends rolls five dice his score is the number of $6's$ shown. Compute

(a) the probability of even A that the two scores are equal

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10.0.34 Probability - Gravner-45.b top

You roll a die and your score is the number shown on the die. Your friends rolls five dice his score is the number of $6's$ shown. Compute.

(b) The probability of event B that your friend's score is strictly larger than yours.

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10.0.35 Probability - Gravner-39 top

Roll a four sided fair die, that is , choose one of the numbers $1, 2, 3, 4$ at random. Let A=$\{1,2\}$, B=$\{1,3\}$, C=$\{1,4\}$. Check that these are pairwise independent (each pair is independent), but not independent.

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10.0.36 Probability - Gravner-46 top

Consider the following game. Pick one card at random from a full deck of 52 cards. If you pull an Ace, you win outright.If not, then you look at the value of the card(K,Q, and J count as 10). If the number is 7 or less you lose outright. Otherwise you select (at random, without replacement) that number of additional cards from the deck.(For example, if you picked a 9 the first time, you select 9 more cards.) If you get at least one Ace, you win. What are your chances of winning this game?

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10.0.37 Probability - Gravner-29 top

Choose each digit of a 5 digit number at random from digits $1, \ldots 9$. Compute the probability that no digit appears more than twice..

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10.0.38 Probability - Gravner-27 top

Three married couples take seats around a table at random . Compute Probability(no wife sits next to her husband).

probability gravner engineering-mathematics

10.0.39 Probability - Gravner-2 top

In a family with 4 children, what is the probability of a $2:2$ boy-girl split?

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Answer key

10.0.40 Probability - Gravner-49.c top

You have 16 balls, 4 green, and 9 red. You also have 3 urns. For each of the 16 balls. you select an urn at random and put the ball into it.(Urns are large enough to accommodate any number of balls.)

(c) What is the probability that each urn contains all three colors?

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Answer key

10.0.41 Probability - Gravner-49.b top

You have 16 balls, 4 green, and 9 red. You also have 3 urns. For each of the 16 balls. you select an urn at random and put the ball into it.(Urns are large enough to accommodate any number of balls.)

(b) What is the probability that each urn contains 3 red balls?

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10.0.42 Probability - Gravner-51.a top

Ten fair dice are rolled. What is the probability that:

a) At least one 1 appears.

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Answer key

10.0.43 Probability - Gravner-49.a top

You have 16 balls, 3 blue 4 green, and 9 red. You also have 3 urns. For each of the 16 balls. you select an urn at random and put the ball into it.(Urns are large enough to accommodate any number of balls.)

(a) What is the probability that no urn is empty?

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10.0.44 Probability - Gravner-56 top

Let X be a random variable with $P(X = 1) = 0.2, P(X = 2) = 0.3$, and $P(X = 3) = 0.5$. What is the expected value of X?

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Answer key

10.0.45 Probability - Gravner-57 top

An urn contains 11 balls, 3 white , 3 red, and 5 blue balls. Take out 3 balls at random, without replacement. You win 1 for each red ball you select and lose a 1 for each white ball you select. Determine the p.m.f. of X , the amount you win.

probability gravner engineering-mathematics

Answer key

10.0.46 Probability - Gravner-54 top

Let X be the number of Heads in 2 fair coin tosses. Determine its p.m.f.

probability gravner engineering-mathematics

Answer key

10.0.47 Probability - Gravner-53.b top

Consider the following game. A player rolls a die. If he rolls 3 or less, he loses immediately. Otherwise he selects, at random, as many cards from a full deck as the number that came up on the die. The player wins if all four Aces are among the selected cards.

(b) Smith tells you that he recently played this game once and won. That is the probability that he rolled a 6 on the die?

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10.0.48 Probability - Gravner-53.a top

Consider the following game. A player rolls a die. If he rolls 3 or less, he loses immediately. Otherwise he selects, at random, as many cards from a full deck as the number that came up on the die. The player wins if all four Aces are among the selected cards.

(a) Compute the winning probability for this game.

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10.0.49 Probability - Gravner-52.b top

Five married couples are seated at random around a round table.

(b) Compute the probability that at most one wife does not sit next to her husband.

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10.0.50 Probability - Gravner-52.a top

Five married couples are seated at random around a round table.

(a) Compute the probability that all couples sit together(i.e., every husband-wife pair occupies adjacent seats).

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10.0.51 Probability - Gravner-51.b top

Ten fair dice are rolled. What is the probability that:

(b) Each of the number $1, 2, 3$ appears exactly twice, while the number 4 appears four times.

probability engineering-mathematics gravner

10.0.52 Probability - Gravner-51.c top

Ten fair dice are rolled. What is the probability that:

(c) Each of the number $1, 2, 3$ appears at least once.

probability engineering-mathematics gravner

10.0.53 Probability - Gravner-50 top

Assume that you have an n-element set U and that you select r independent random subsets $A_{1}, \dots \dots \dots A_r \subset U$. All A_i are chosen so that all 2^n choices are equally likely. Compute (in a simple closed form) the probability that the A_i are pairwise disjoint.

probability gravner engineering-mathematics

10.1 Conditional Probability (8) top

10.1.1 Conditional Probability: Kenneth Rosen Edition 7 Exercise 7.2 Question 24 (Page No. 467) top

What is the conditional probability that exactly four heads appear when a fair coin is flipped five times, given that the first flip came up heads?

shouldn't the answer be 1/16?

my approach
probability that we get 4 head after 4 toss=(1/2)^4
probability that outcome of first toss is head=½
P(A/B)=P(A ∩ B)/P(B)=$\frac{(1/2)^4 * 1/2}{1/2}$

answer is given ¼?

where am I going wrong?

kenneth-rosen discrete-mathematics probability conditional-probability

10.1.2 Conditional Probability: Probability - Gravner-32.a top

Toss a coin 10 times.

a) that exactly 7 Heads ar tossed.

probability gravner engineering-mathematics conditional-probability

10.1.3 Conditional Probability: Probability - Gravner-32.b top

Toss a coin 10 times.

b) at least 7 Heads are tossed , what is the probability that your first toss is Heads?

probability gravner engineering-mathematics conditional-probability

Answer key

10.1.4 Conditional Probability: Probability - Gravner-33.a top

An urn contains 10 black and 10 white balls. Draw 3

a) without replacement

probability gravner engineering-mathematics conditional-probability

Answer key

10.1.5 Conditional Probability: Probability - Gravner-33.b top

An urn contains 10 black and 10 white balls. Draw 3

b) with replacement . what is probability that all three are white

probability gravner engineering-mathematics conditional-probability

Answer key

10.1.6 Conditional Probability: Probability - Gravner-34 top

Flip a fair coin. If you toss Heads, roll 1 die. If you toss Tails, roll 2 dice. Compute the probability that you roll exactly one 6.

gravner probability engineering-mathematics conditional-probability

Answer key

10.1.7 Conditional Probability: Probability - Gravner-35 top

Roll a die, then select at random, without replacement, as many cards from the deck as the number shown on the die. What is the probability that you get at least one Ace?

gravner probability engineering-mathematics conditional-probability

Answer key

10.1.8 Conditional Probability: Probability - Gravner-37 top

A factory has three machines, M_1, M_2, and M_3, that produce items (say, light bulbs). It is impossible to tell which machine produced a particular item, but some are defective. Here are the known numbers:

machine	proportion of items made	prob. any made item is defective
M_1	0.2	0.001
M2	0.3	0.002
M_3	0.5	0.003

You pick an item, test it, and find it is defective. What is the probability that it was made by M_2?

probability gravner engineering-mathematics conditional-probability

Answer key

10.2 Normal Distribution (1) top

10.2.1 Normal Distribution: Probability - Gravner-74 top

Let X be a $N(\mu, \sigma^2)$ random variable and let $Y = \alpha X + \beta$, with $\alpha > 0$. How is Y distributed?

probability gravner engineering-mathematics random-variable normal-distribution

Answer key

10.3 Random Variable (27) top

10.3.1 Random Variable: Probability - Gravner-59 top

Let X be the number shown on a rolled fair die. Compute EX,E(X^2), and Var(X).

probability gravner engineering-mathematics random-variable

10.3.2 Random Variable: Probability - Gravner-60 top

Denote by d the dominant gene and by r the recessive gene at a single locus. Then dd is called the pure dominant genotype, dr is called hybrid, and rr the pure recessive genotype. The two genotypes with at least one dominant gene, dd and dr, result in the phenotype of the dominant gene, while rr results in a recessive phenotype.

probability gravner engineering-mathematics random-variable

10.3.3 Random Variable: Probability - Gravner-61 top

Suppose that the probability that a person is killed by lighting in a year is, independently, $1/(500)$ million. Assume that the US population is 300 million.

probability gravner engineering-mathematics random-variable

10.3.4 Random Variable: Probability - Gravner-62 top

Assume a crime has been committed. It is known that the particular has certain characteristics, which occur with a small frequency p (say,10^{-8}) in a population of size n say (10^8). A person who matches these characteristics has been found at random(e.g., at routine traffic stop or by airport security) and , since p is so small charged with the crime. There is no other evidence. What should the defense be?

random-variable probability gravner engineering-mathematics

10.3.5 Random Variable: Probability - Gravner-63 top

You roll a die, your opponent tosses a coin. If you roll 6 you win; if you do not roll 6 and your opponent tosses Heads you lose; otherwise, this round ends and the game repeats. On the average, how many rounds does the game last?

probability gravner engineering-mathematics random-variable

10.3.6 Random Variable: Probability - Gravner-64.a top

Roll a fair die repeatedly. Let X be the number of 6's in the first 10 rolls and let Y the number of rolls needed to obtain a 3.

(a) Write down the probability mass function of X.

gravner probability engineering-mathematics random-variable

10.3.7 Random Variable: Probability - Gravner-64.b top

Roll a fair die repeatedly. Let X be the number of 6's in the first 10 rolls and let Y the number of rolls needed to obtain a 3.

(b) Write down the probability mass function of Y.

gravner random-variable probability engineering-mathematics

10.3.8 Random Variable: Probability - Gravner-64.c top

Roll a fair die repeatedly. Let X be the number of 6's in the first 10 rolls and let Y the number of rolls needed to obtain a 3.

(c) Find an expression $P(X \geqslant 6)$.

probability gravner engineering-mathematics random-variable

10.3.9 Random Variable: Probability - Gravner-64.d top

Roll a fair die repeatedly. Let X be the number of 6's in the first 10 rolls and let Y the number of rolls needed to obtain a 3.

(d) Find an expression for $P(Y > 10)$.

probability gravner engineering-mathematics random-variable

10.3.10 Random Variable: Probability - Gravner-65.a top

A biologist needs at least 3 mature specimens of certain plant. The plant needs a year to reach maturity; once a seed is planted, any plant will survive for the year with probability $1/1000$ (independently of other plants). The biologist plants 3000 seeds. A year is deemed a success if three or more plants from these seeds reach maturity.

(a) Write down the exact expression for the probability that the biologist will indeed end up with at least 3 mature plants.

gravner probability engineering-mathematics random-variable

10.3.11 Random Variable: Probability - Gravner-65.b top

A biologist needs at least 3 mature specimens of certain plant. The plant needs a year to reach maturity; once a seed is planted, any plant will survive for the year with probability $1/1000$ (independently of other plants). The biologist plants 3000 seeds. A year is deemed a success if three or more plants from these seeds reach maturity.

(b) Write down a relevant approximate expression for the probability from(a).Justify briefly the approximation.

gravner probability engineering-mathematics random-variable

10.3.12 Random Variable: Probability - Gravner-65.d top

A biologist needs at least 3 mature specimens of certain plant. The plant needs a year to reach maturity; once a seed is planted, any plant will survive for the year with probability $1/1000$ (independently of other plants). The biologist plants 3000 seeds. A year is deemed a success if three or more plants from these seeds reach maturity.

(d) Devise a method to determine the number of seeds the biologist should plant in order to get at least 3 mature plants in a year with probability at least 0.999.

gravner probability engineering-mathematics random-variable

10.3.13 Random Variable: Probability - Gravner-66 top

You are dealt one card at random form a full deck and your opponent is dealt 2 cards (Without any replacement). If you get an Ace, he pays you 10 dollar, if you get a King, he pays you 5 dollar (regardless of his cards). If you have neither an Ace nor a King, but your card is red and your opponent has no red cards, he pays you 1 dollar. In all other cases you pay him 1 dollar . Determine your expected earnings . Are they positive?

probability gravner engineering-mathematics random-variable

Answer key

10.3.14 Random Variable: Probability - Gravner-67.a top

You and your opponent both roll a fair die. If you both roll the same number, the game is repeated, otherwise whoever rolls the larger number wins. Let N be the number of times the two dice have to be rolled before the game is decided.

(a)Determine the probability mass function of N.

probability gravner engineering-mathematics random-variable

10.3.15 Random Variable: Probability - Gravner-67.b top

You and your opponent both roll a fair die. If you both roll the same number, the game is repeated, otherwise whoever rolls the larger number wins. Let N be the number of times the two dice have to be rolled before the game is decided.

(b) Compute Probability you win

probability gravner engineering-mathematics random-variable

Answer key

10.3.16 Random Variable: Probability - Gravner-67.c top

You and your opponent both roll a fair die. If you both roll the same number, the game is repeated, otherwise whoever rolls the larger number wins. Let N be the number of times the two dice have to be rolled before the game is decided.

(c) Assume that you get paid 10 dollar for winning in the first round, 1 dollar for winning in any other round, and nothing otherwise.Compute your expected winnings .

probability gravner engineering-mathematics random-variable

Answer key

10.3.17 Random Variable: Probability - Gravner-68.a top

Each of 50 students in class belongs to exactly one the four groups A, B, C or D. The membership numbers for the four groups are as follows: $A : 5, B : 5, C : 15, D : 20$. First choose one of the 50 students at random and let X be the size of that student's group . Next, choose one the four groups at random and let Y be its size.

(a) Write down the probability mass functions for X and Y.

gravner probability engineering-mathematics random-variable

10.3.18 Random Variable: Probability - Gravner-68.b top

Each of 50 students in class belongs to exactly one the four groups A, B, C or D. The membership numbers for the four groups are as follows: $A: 5, B: 5, C: 15, D: 20$. First choose one of the 50 students at random and let X be the size of that student's group . Next, choose one the four groups at random and let Y be its size.

(b) Compute EX and EY.

gravner probability engineering-mathematics random-variable

10.3.19 Random Variable: Probability - Gravner-68.c top

Each of 50 students in class belongs to exactly one the four groups A, B, C or D. The membership numbers for the four groups are as follows: $A: 5, B: 5, C: 15, D: 20$. First choose one of the 50 students at random and let X be the size of that student's group . Next, choose one the four groups at random and let Y be its size.

(c) Compute Var(X) and Var(Y).

gravner probability engineering-mathematics random-variable

10.3.20 Random Variable: Probability - Gravner-68.d top

Each of 50 students in class belongs to exactly one the four groups A, B, C or D. The membership numbers for the four groups are as follows: $A: 5, B: 5, C: 15, D: 20$. First choose one of the 50 students at random and let X be the size of that student's group . Next, choose one the four groups at random and let Y be its size.

(d) Assume you have a students divided into n groups with memberships $s_1, \ldots\ldots\ldots s_n$, and X be the size of the group of a randomly chosen student, while Y is the size of the randomly chosen group.

Let EY= μ and Var(Y) = σ^2 . Express EX with s, n, μ and σ.

gravner probability engineering-mathematics random-variable

10.3.21 Random Variable: Probability - Gravner-69.a top

$$f(x) = \begin{cases} cx & if(0 < x < 4) \\ 0 & otherwise \end{cases}$$

(a) Determine c.

gravner probability engineering-mathematics random-variable

Answer key

10.3.22 Random Variable: Probability - Gravner-69.b top

$$f(x) = \begin{cases} cx & if(0 < x < 4) \\ 0 & otherwise \end{cases}$$

(c) Determine EX and Var(X).

probability gravner engineering-mathematics random-variable

Answer key

10.3.23 Random Variable: Probability - Gravner-69.b top

$$f(x) = \begin{cases} cx & if(0 < x < 4) \\ 0 & otherwise \end{cases}$$

(b) Compute $P(1 \leqslant X \leqslant 2)$

probability gravner engineering-mathematics random-variable

Answer key

10.3.24 Random Variable: Probability - Gravner-70 top

Assume that X has density

$$fx(x) = \left\{ \begin{array}{cc} 3x^2 & if(x\epsilon[0,1]), \\ 0 & otherwise \end{array} \right\}$$

Compute the density f_y of Y= $1\text{-}X^4$

probability gravner engineering-mathematics random-variable

10.3.25 Random Variable: Probability - Gravner-71 top

Assume that X is uniform on [0,1]. What is $P(X\epsilon Q)$? What is the probability that the binary expansion of X starts with 0.010?

probability gravner engineering-mathematics random-variable

10.3.26 Random Variable: Probability - Gravner-72 top

A uniform random number X divides $[0,1]$ into two segments. Let R be the ratio of the smaller versus the larger segment. Compute the density of R.

probability gravner engineering-mathematics random-variable

10.3.27 Random Variable: Probability - Gravner-73 top

Assume that a light bulb lasts on average 100 hours. Assuming exponential distribution, compute the probability that it lasts more than 200 hours and the probability that it lasts less than 50 hours.

probability gravner engineering-mathematics random-variable

Answer key

Answer Keys

10.0.1	Q-Q	10.0.2	Q-Q	10.0.3	Q-Q	10.0.4	Q-Q	10.0.5	Q-Q
10.0.6	Q-Q	10.0.7	Q-Q	10.0.8	Q-Q	10.0.9	Q-Q	10.0.10	Q-Q
10.0.11	Q-Q	10.0.12	Q-Q	10.0.13	Q-Q	10.0.14	Q-Q	10.0.15	Q-Q
10.0.16	Q-Q	10.0.17	Q-Q	10.0.18	Q-Q	10.0.19	Q-Q	10.0.20	Q-Q
10.0.21	Q-Q	10.0.22	Q-Q	10.0.23	Q-Q	10.0.24	Q-Q	10.0.25	Q-Q
10.0.26	Q-Q	10.0.27	Q-Q	10.0.28	Q-Q	10.0.29	Q-Q	10.0.30	Q-Q
10.0.31	Q-Q	10.0.32	Q-Q	10.0.33	Q-Q	10.0.34	Q-Q	10.0.35	Q-Q
10.0.36	Q-Q	10.0.37	Q-Q	10.0.38	Q-Q	10.0.39	Q-Q	10.0.40	Q-Q
10.0.41	Q-Q	10.0.42	Q-Q	10.0.43	Q-Q	10.0.44	Q-Q	10.0.45	Q-Q
10.0.46	Q-Q	10.0.47	Q-Q	10.0.48	Q-Q	10.0.49	Q-Q	10.0.50	Q-Q
10.0.51	Q-Q	10.0.52	Q-Q	10.0.53	Q-Q	10.1.1	Q-Q	10.1.2	Q-Q
10.1.3	Q-Q	10.1.4	Q-Q	10.1.5	Q-Q	10.1.6	Q-Q	10.1.7	Q-Q
10.1.8	Q-Q	10.2.1	Q-Q	10.3.1	Q-Q	10.3.2	Q-Q	10.3.3	Q-Q
10.3.4	Q-Q	10.3.5	Q-Q	10.3.6	Q-Q	10.3.7	Q-Q	10.3.8	Q-Q
10.3.9	Q-Q	10.3.10	Q-Q	10.3.11	Q-Q	10.3.12	Q-Q	10.3.13	Q-Q
10.3.14	Q-Q	10.3.15	Q-Q	10.3.16	Q-Q	10.3.17	Q-Q	10.3.18	Q-Q
10.3.19	Q-Q	10.3.20	Q-Q	10.3.21	Q-Q	10.3.22	Q-Q	10.3.23	Q-Q
10.3.24	Q-Q	10.3.25	Q-Q	10.3.26	Q-Q	10.3.27	Q-Q		

11 Operating System (600)

11.0.1 Galvin Edition 9 Exercise 1 Question 1 (Page No. 49) top

What are the three main purposes of an operating system ?

galvin operating-system descriptive

11.0.2 Galvin Edition 9 Exercise 1 Question 28 (Page No. 52) top

What are some advantages of peer-to-peer systems over client-server systems ?

galvin operating-system descriptive

Answer key

11.0.3 Galvin Edition 9 Exercise 1 Question 4 (Page No. 49) top

Keeping in mind the various definitions of operating system, consider whether the operating system should include applications such as web browsers and mail programs. Argue both that it should and that it should not, and support your answers.

galvin operating-system descriptive

11.0.4 Galvin Edition 9 Exercise 1 Question 3 (Page No. 49) top

What is the main difficulty that a programmer must overcome in writing an operating system for a real-time environment ?

galvin operating-system descriptive

11.0.5 Galvin Edition 9 Exercise 1 Question 9 (Page No. 50) top

Timers could be used to compute the current time. Provide a short description of how this could be accomplished.

galvin operating-system descriptive

11.0.6 Galvin Edition 9 Exercise 2 Question 7 (Page No. 95) top

What is the purpose of system programs ?

galvin operating-system descriptive

11.0.7 Galvin Edition 9 Exercise 1 Question 19 (Page No. 51) top

What is the purpose of interrupts ? How does an interrupt differ from a trap ? Can traps be generated intentionally by a user program ? If so, for what purpose ?

galvin operating-system descriptive

11.0.8 Galvin Edition 9 Exercise 1 Question 21 (Page No. 51) top

Some computer systems do not provide a privileged mode of operation in hardware. Is it possible to construct a secure operating system for these computer systems ? Give arguments both that it is and that it is not possible.

galvin operating-system descriptive

Answer key

11.0.9 Galvin Edition 9 Exercise 2 Question 5 (Page No. 94) top

What is the purpose of the command interpreter ? Why is it usually separate from the kernel ?

galvin operating-system descriptive

11.0.10 Galvin Edition 9 Exercise 2 Question 4 (Page No. 94) top

What are the three major activities of an operating system with regard to secondary-storage management?

galvin operating-system descriptive

11.0.11 Galvin Edition 9 Exercise 2 Question 3 (Page No. 94) top

What are the three major activities of an operating system with regard to memory management?

galvin operating-system descriptive

11.0.12 Galvin Edition 9 Exercise 2 Question 2 (Page No. 94) top

What are the five major activities of an operating system with regard to process management ?

galvin operating-system descriptive

11.0.13 Galvin Edition 9 Exercise 2 Question 1 (Page No. 94) top

What is the purpose of system calls ?

galvin operating-system descriptive

11.0.14 Galvin Edition 9 Exercise 1 Question 23 (Page No. 51) top

Consider an SMP system similar to the one shown in Figure 1.6. Illustrate with an example how data residing in memory could in fact have a different value in each of the local caches.

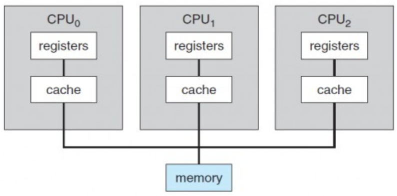

Figure 1.6 Symmetric multiprocessing architecture.

galvin operating-system descriptive

11.0.15 Galvin Edition 9 Exercise 1 Question 20 (Page No. 51) top

Direct memory access is used for high-speed I/O devices in order to avoid increasing the CPU's execution load.
a. How does the CPU interface with the device to coordinate the transfer ?
b. How does the CPU know when the memory operations are complete ?
c. The CPU is allowed to execute other programs while the DMA controller is transferring data. Does this process interfere with the execution of the user programs ? If so, describe what forms of interference are caused.

galvin operating-system descriptive

11.0.16 Galvin Edition 9 Exercise 1 Question 30 (Page No. 52) top

Identify several advantages and several disadvantages of open-source operating systems. Include the types of people who would find each aspect to be an advantage or a disadvantage.

galvin operating-system descriptive

11.0.17 Galvin Edition 9 Exercise 1 Question 27 (Page No. 52) top

Describe some of the challenges of designing operating systems for mobile devices compared with designing operating systems for traditional PCs.

galvin operating-system descriptive

11.0.18 Galvin Edition 9 Exercise 1 Question 6 (Page No. 50) top

Which of the following instructions should be privileged ?

a. Set value of timer.
b. Read the clock.
c. Clear memory.

d. Issue a trap instruction.
e. Turn off interrupts.
f. Modify entries in device-status table.
g. Switch from user to kernel mode.
h. Access I/O device.

galvin operating-system

Answer key

11.0.19 Galvin Edition 9 Exercise 1 Question 26 (Page No. 52) top

Which network configuration—LAN or WAN—would best suit the following environments ?
a. A campus student union
b. Several campus locations across a statewide university system
c. A neighborhood

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11.0.20 Galvin Edition 9 Exercise 1 Question 25 (Page No. 52) top

Describe a mechanism for enforcing memory protection in order to prevent a program from modifying the memory associated with other programs.

galvin operating-system descriptive

11.0.21 Galvin Edition 9 Exercise 1 Question 24 (Page No. 51) top

Discuss, with examples, how the problem of maintaining coherence of cached data manifests itself in the following processing environments:
a. Single-processor systems
b. Multiprocessor systems
c. Distributed systems

galvin operating-system descriptive

11.0.22 Galvin Edition 9 Exercise 1 Question 22 (Page No. 51) top

Many SMP(Symmetric Multiprocessing) systems have different levels of caches; one level is local to each processing core, and another level is shared among all processing cores. Why are caching systems designed this way ?

galvin operating-system descriptive

11.0.23 Galvin Edition 9 Exercise 1 Question 29 (Page No. 52) top

Describe some distributed applications that would be appropriate for a peer-to-peer system.

galvin operating-system descriptive

11.0.24 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 22 (Page No. 82) top

Can the

count = write(fd, buffer, nbytes);

call return any value in count other than nbytes? If so, why?

tanenbaum operating-system descriptive

11.0.25 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 11 (Page No. 81) top

A 255-GB disk has $65,536$ cylinders with 255 sectors per track and 512 bytes per sector. How many platters and heads does this disk have? Assuming an average cylinder seek time of 11 ms, average rotational delay of 7 msec and reading rate of 100 MB/sec, calculate the average time it will take to read 400 KB from one sector.

tanenbaum operating-system descriptive

11.0.26 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 9 (Page No. 81) top

There are several design goals in building an operating system, for example, resource utilization, timeliness, robustness, and so on. Give an example of two design goals that may contradict one another.

tanenbaum operating-system descriptive

11.0.27 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 8 (Page No. 81) top

One reason GUIs were initially slow to be adopted was the cost of the hardware needed to support them. How much video RAM is needed to support a $25-\text{line} \times 80-\text{row}$ character monochrome text screen? How much for a $1200 \times 900-$ pixel $24-bit$ color bitmap? What was the cost of this RAM at 1980 prices $(\$5/KB)$? How much is it now?

tanenbaum operating-system descriptive

11.0.28 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 7 (Page No. 81) top

The family-of-computers idea was introduced in the $1960s$ with the IBM System$/360$ mainframes. Is this idea now dead as a doornail or does it live on?

tanenbaum operating-system descriptive

11.0.29 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 26 (Page No. 83) top

In the example given in Fig. $1-17$, the library procedure is called read and the system call itself is called read. Is it essential that both of these have the same name? If not, which one is more important?

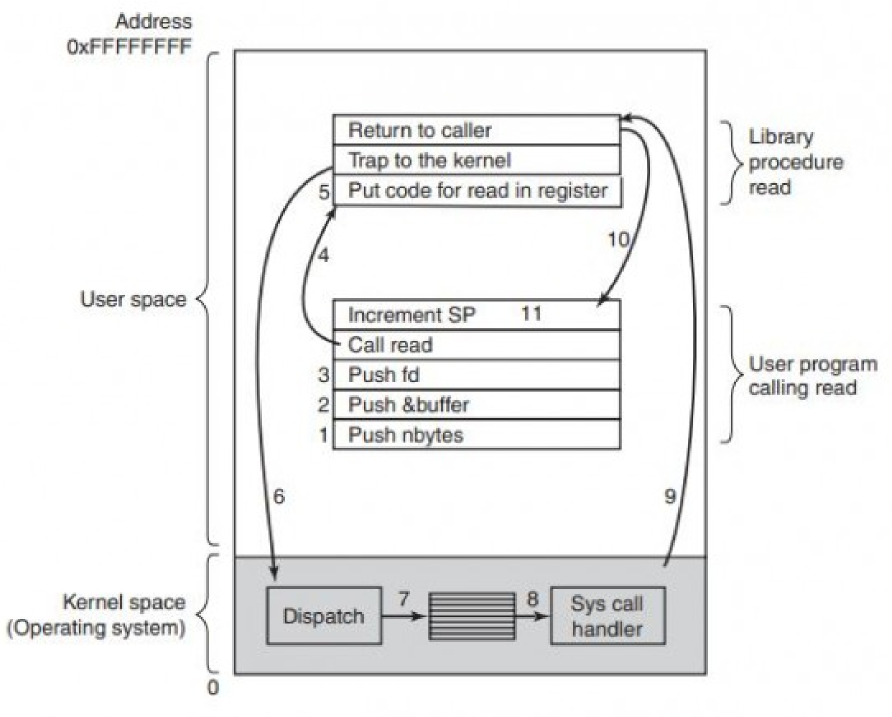

Figure 1-17. The 11 steps in making the system call read(fd, buffer, nbytes).

tanenbaum operating-system descriptive

11.0.30 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 25 (Page No. 83) top

What is the essential difference between a block special file and a character special file?

tanenbaum operating-system descriptive

Answer key

11.0.31 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 34 (Page No. 83) top

Write a shell that is similar to Fig. $1-19$ but contains enough code that it actually works so you can test it. You might also add some features such as redirection of input and output, pipes, and background jobs.

```
#define TRUE 1

while (TRUE) {                                  /* repeat forever */
      type_prompt( );                           /* display prompt on the screen */
      read_command(command, parameters);        /* read input from terminal */

      if (fork( ) != 0) {                       /* fork off child process */
          /* Parent code. */
          waitpid(−1, &status, 0);              /* wait for child to exit */
      } else {
          /* Child code. */
          execve(command, parameters, 0);       /* execute command */
      }
}
```

Figure 1-19. A stripped-down shell. Throughout this book, *TRUE* is assumed to be defined as 1.

tanenbaum operating-system descriptive

11.0.32 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 31 (Page No. 83) top

Explain how separation of policy and mechanism aids in building microkernel-based operating systems.

tanenbaum operating-system descriptive

11.0.33 Galvin Edition 9 Exercise 1 Question 5 (Page No. 50) top

How does the distinction between kernel mode and user mode function as a rudimentary form of protection (security) system ?

galvin operating-system descriptive

11.0.34 Galvin Edition 9 Exercise 1 Question 7 (Page No. 50) top

Some early computers protected the operating system by placing it in a memory partition that could not be modified by either the user job or the operating system itself. Describe two difficulties that you think could arise with such a scheme.

galvin operating-system descriptive

11.0.35 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 33 (Page No. 83) top

Here are some questions for practicing unit conversions:

a. How long is a nanoyear in seconds?
b. Micrometers are often called microns. How long is a megamicron?
c. How many bytes are there in a 1-PB memory?
d. The mass of the earth is 6000 yottagrams. What is that in kilograms?

tanenbaum operating-system descriptive

11.0.36 Galvin Edition 9 Exercise 2 Question 13 (Page No. 95) top

Describe three general methods for passing parameters to the operating system.

galvin operating-system descriptive

11.0.37 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 2 (Page No. 81) top

In Section 1.4, nine different types of operating systems are described. Give a list of applications for each of these systems (one per operating systems type).

tanenbaum operating-system descriptive

Answer key

11.0.38 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 1 (Page No. 81) top

What are the two main functions of an operating system?

tanenbaum operating-system descriptive

Answer key

11.0.39 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 3 (Page No. 81) top

What is the difference between timesharing and multiprogramming systems?

tanenbaum operating-system descriptive

Answer key

11.0.40 Galvin Edition 9 Exercise 2 Question 24 (Page No. 96) top

Explain why Java programs running on Android systems do not use the standard Java API and virtual machine ?

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11.0.41 Galvin Edition 9 Exercise 2 Question 23 (Page No. 96) top

How are iOS and Android similar ? How are they different ?

galvin operating-system descriptive

11.0.42 Galvin Edition 9 Exercise 2 Question 22 (Page No. 95) top

What are the advantages of using loadable kernel modules ?

galvin operating-system descriptive

11.0.43 Galvin Edition 9 Exercise 2 Question 21 (Page No. 95) top

What is the main advantage of the micro kernel approach to system design ? How do user programs and system services interact micro kernel architecture ? What are the disadvantages of using the micro kernel approach ?

galvin operating-system descriptive

11.0.44 Galvin Edition 9 Exercise 2 Question 20 (Page No. 95) top

It is sometimes difficult to achieve a layered approach if two components of the operating system are dependent on each other. Identify a scenario in which it is unclear how to layer two system components that require tight coupling of their functionalities.

galvin operating-system descriptive

11.0.45 Galvin Edition 9 Exercise 2 Question 19 (Page No. 95) top

Why is the separation of mechanism and policy desirable ?

galvin operating-system descriptive

11.0.46 Galvin Edition 9 Exercise 2 Question 18 (Page No. 95) top

What are the two models of inter process communication ? What are the strengths and weaknesses of the two approaches ?

galvin operating-system descriptive

11.0.47 Galvin Edition 9 Exercise 2 Question 25 (Page No. 96) top

The experimental Synthesis operating system has an assembler incorporated in the kernel. To optimize system-call performance, the kernel assembles routines within kernel space to minimize the path that the system call must take through the kernel. This approach is the antithesis of the layered approach, in which the path through the kernel is extended to make building the operating system easier. Discuss the pros and cons of the Synthesis approach to kernel design and system-performance optimization.

galvin operating-system descriptive

11.0.48 Galvin Edition 9 Exercise 2 Question 17 (Page No. 95) top

Would it be possible for the user to develop a new command interpreter using the system-call interface provided by the operating system?

galvin operating-system descriptive

11.0.49 Galvin Edition 9 Exercise 2 Question 15 (Page No. 95) top

What are the five major activities of an operating system with regard to file management ?

galvin operating-system descriptive

11.0.50 Galvin Edition 9 Exercise 2 Question 14 (Page No. 95) top

Describe how you could obtain a statistical profile of the amount of time spent by a program executing different sections of its code. Discuss the importance of obtaining such a statistical profile.

galvin operating-system descriptive

11.0.51 Galvin Edition 9 Exercise 2 Question 12 (Page No. 95) top

The services and functions provided by an operating system can be divided into two main categories. Briefly describe the two categories,and discuss how they differ.

galvin operating-system descriptive

11.0.52 Galvin Edition 9 Exercise 1 Question 8 (Page No. 50) top

Some CPUs provide for more than two modes of operation. What are two possible uses of these multiple modes ?

galvin operating-system descriptive

11.0.53 Galvin Edition 9 Exercise 1 Question 15 (Page No. 51) top

Describe the differences between symmetric and asymmetric multiprocessing. What are three advantages and one disadvantage of multiprocessor systems ?

galvin operating-system descriptive

11.0.54 Galvin Edition 9 Exercise 1 Question 10 (Page No. 50) top

Give two reasons why caches are useful. What problems do they solve ? What problems do they cause ? If a cache can be made as large as the device for which it is caching (for instance, a cache as large as a disk), why not make it that large and eliminate the device ?

galvin operating-system descriptive

11.0.55 Galvin Edition 9 Exercise 1 Question 18 (Page No. 51) top

How are network computers different from traditional personal computers ? Describe some usage scenarios in which it is advantageous to use network computers.

galvin operating-system descriptive

11.0.56 Galvin Edition 9 Exercise 1 Question 11 (Page No. 50) top

Distinguish between the client–server and peer-to-peer models of distributed systems.

galvin operating-system descriptive

11.0.57 Galvin Edition 9 Exercise 1 Question 12 (Page No. 50) top

In a multiprogramming and time-sharing environment, several users share the system simultaneously. This situation can result in various security problems.
a. What are two such problems ?
b. Can we ensure the same degree of security in a time-shared machine as in a dedicated machine ? Explain your answer.

galvin operating-system descriptive

11.0.58 Galvin Edition 9 Exercise 1 Question 13 (Page No. 50) top

The issue of resource utilization shows up in different forms in different types of operating systems. List what resources must be managed carefully in the following settings:
a. Mainframe or minicomputer systems
b. Workstations connected to servers
c. Mobile computers

galvin operating-system descriptive

11.0.59 Galvin Edition 9 Exercise 1 Question 14 (Page No. 51) top

Under what circumstances would a user be better off using a timesharing system than a PC or a single-user workstation ?

galvin operating-system descriptive

11.0.60 Galvin Edition 9 Exercise 1 Question 16 (Page No. 51) top

How do clustered systems differ from multiprocessor systems ? What is required for two machines belonging to a cluster to cooperate to provide a highly available service ?

galvin operating-system descriptive

11.0.61 Galvin Edition 9 Exercise 2 Question 11 (Page No. 95) top

How could a system be designed to allow a choice of operating systems from which to boot ? What would the bootstrap program need to do ?

galvin operating-system descriptive

11.0.62 Galvin Edition 9 Exercise 1 Question 17 (Page No. 51) top

Consider a computing cluster consisting of two nodes running a database. Describe two ways in which the cluster software can manage access to the data on the disk. Discuss the benefits and disadvantages of each.

galvin operating-system descriptive

11.0.63 Galvin Edition 9 Exercise 2 Question 6 (Page No. 95) top

What system calls have to be executed by a command interpreter or shell in order to start a new process ?

galvin operating-system descriptive

11.0.64 Galvin Edition 9 Exercise 2 Question 8 (Page No. 95) top

What is the main advantage of the layered approach to system design ?What are the disadvantages of the layered approach ?

galvin operating-system descriptive

11.0.65 Galvin Edition 9 Exercise 2 Question 9 (Page No. 95) top

List five services provided by an operating system, and explain how each creates convenience for users. In which cases would it be impossible for user-level programs to provide these services ? Explain your answer.

galvin operating-system descriptive

11.0.66 Galvin Edition 9 Exercise 2 Question 16 (Page No. 95) top

What are the advantages and disadvantages of using the same system call interface for manipulating both files and devices?

galvin operating-system descriptive

11.0.67 Galvin Edition 9 Exercise 2 Question 10 (Page No. 95) top

Why do some systems store the operating system in firmware, while others store it on disk ?

galvin operating-system descriptive

11.0.68 Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 30 (Page No. 83) top

A portable operating system is one that can be ported from one system architecture to another without any modification. Explain why it is infeasible to build an operating system that is completely portable. Describe two high-level layers that you will have in designing an operating system that is highly portable.

tanenbaum operating-system descriptive

11.0.69 Galvin Edition 9 Exercise 1 Question 2 (Page No. 49) top

We have stressed the need for an operating system to make efficient use of the computing hardware. When is it appropriate for the operating system to forsake this principle and to “waste” resources ? Why is such a system not really wasteful ?

galvin operating-system descriptive

11.1 Bankers Algorithm (1) top

11.1.1 Bankers Algorithm: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 41 (Page No. 470) top

Program a simulation of the banker’s algorithm. Your program should cycle through each of the bank clients asking for a request and evaluating whether it is safe or unsafe. Output a log of requests and decisions to a file.

tanenbaum operating-system deadlock-prevention-avoidance-detection bankers-algorithm descriptive

11.2 Bitmaps (1) top

11.2.1 Bitmaps: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 29 (Page No. 335) top

Suppose that file 21 in Fig. $4-25$ was not modified since the last dump. In what way would the four bitmaps of Fig. $4-26$ be different?

310 FILE SYSTEMS CHAP. 4

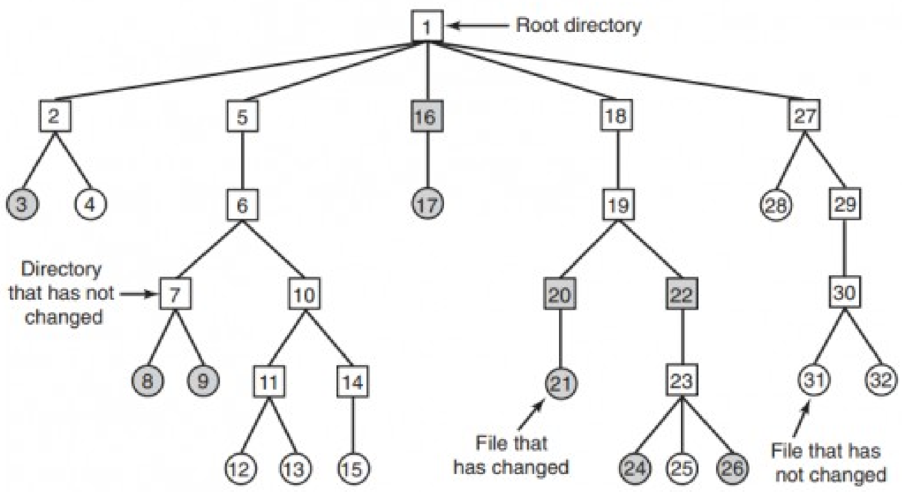

Figure 4-25. A file system to be dumped. The squares are directories and the circles are files. The shaded items have been modified since the last dump. Each directory and file is labeled by its i-node number.

SEC. 4.4 FILE-SYSTEM MANAGEMENT AND OPTIMIZATION 311

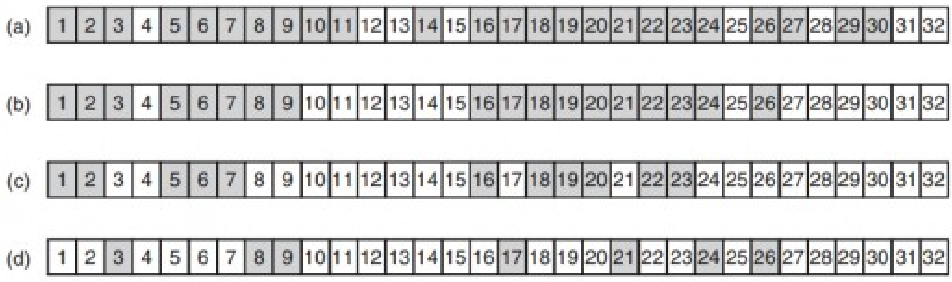

Figure 4-26. Bitmaps used by the logical dumping algorithm.

tanenbaum operating-system file-system bitmaps descriptive

11.3 Cache Memory (1) top

11.3.1 Cache Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 4 (Page No. 81) top

To use cache memory, main memory is divided into cache lines, typically 32 or 64 bytes long. An entire cache line is cached at once. What is the advantage of caching an entire line instead of a single byte or word at a time?

tanenbaum operating-system cache-memory descriptive

Answer key

11.4 Contiguous Allocation (1) top

11.4.1 Contiguous Allocation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 15 (Page No. 333) top

Some digital consumer devices need to store data, for example as files. Name a modern device that requires file storage and for which contiguous allocation would be a fine idea.

tanenbaum operating-system file-system memory-management contiguous-allocation descriptive

11.5 Cylinders (2) top

11.5.1 Cylinders: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 28 (Page No. 431) top

Consider a magnetic disk consisting of 16 heads and 400 cylinders. This disk has four 100-cylinder zones with the cylinders in different zones containing $160, 200, 240.$ and 280 sectors, respectively. Assume that each sector contains 512 bytes, average seek time between adjacent cylinders is $1\ msec$, and the disk rotates at 7200 RPM. Calculate the

a. disk capacity,
b. optimal track skew, and
c. maximum data transfer rate.

tanenbaum operating-system input-output disk cylinders descriptive

Answer key

11.5.2 Cylinders: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 29 (Page No. 432) top

A disk manufacturer has two 5.25-inch disks that each have $10,000$ cylinders. The newer one has double the linear recording density of the older one. Which disk properties are better on the newer drive and which are the same? Are any worse on the newer one?

tanenbaum operating-system input-output disk cylinders descriptive

11.6 Deadlock Prevention Avoidance Detection (62) top

11.6.1 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 1 (Page No. 465) top

Give an example of a deadlock taken from politics.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.2 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 10 (Page No. 466) top

Consider Fig. 6-4. Suppose that in step (o) C requested S instead of requesting R. Would this lead to deadlock? Suppose that it requested both S and R.

442 DEADLOCKS CHAP. 6

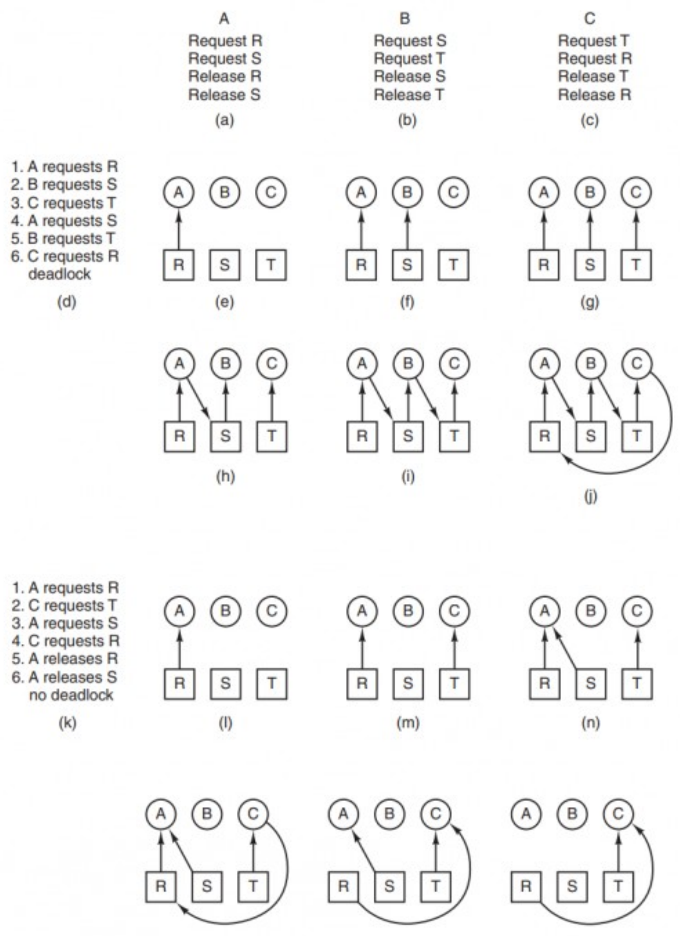

Figure 6-4. An example of how deadlock occurs and how it can be avoided.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.3 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 11 (Page No. 466) top

Suppose that there is a resource deadlock in a system. Give an example to show that the set of processes deadlocked can include processes that are not in the circular chain in the corresponding resource allocation graph.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.4 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 12 (Page

No. 466) top

In order to control traffic, a network router, A periodically sends a message to its neighbor, B, telling it to increase or decrease the number of packets that it can handle. At some point in time, Router A is flooded with traffic and sends B a message telling it to cease sending traffic. It does this by specifying that the number of bytes B may send ($A's$ window size) is 0. As traffic surges decrease, A sends a new message, telling B to restart transmission. It does this by increasing the window size from 0 to a positive number. That message is lost. As described, neither side will ever transmit. What type of deadlock is this?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.5 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 13 (Page No. 466) top

The discussion of the ostrich algorithm mentions the possibility of process-table slots or other system tables filling up. Can you suggest a way to enable a system administrator to recover from such a situation?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.6 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 14 (Page No. 466) top

Consider the following state of a system with four processes , $P1, P2, P3,$ and $P4,$ and five types of resources, $RS1, RS2, RS3, RS4,$ and $RS5$:

$$C = \begin{bmatrix} 0 & 1 & 1 & 1 & 2 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 2 & 1 & 0 & 0 & 0 \end{bmatrix} \quad R = \begin{bmatrix} 1 & 1 & 0 & 2 & 1 \\ 0 & 1 & 0 & 2 & 1 \\ 0 & 2 & 0 & 3 & 1 \\ 0 & 2 & 1 & 1 & 0 \end{bmatrix} \quad \begin{matrix} E = (24144) \\ A = (01021) \end{matrix}$$

Using the deadlock detection algorithm described in Section $6.4.2$, show that there is a deadlock in the system. Identify the processes that are deadlocked.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.7 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 15 (Page No. 467) top

Explain how the system can recover from the deadlock in previous problem using

a. recovery through preemption.
b. recovery through rollback.
c. recovery through killing processes.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.8 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 16 (Page No. 467) top

Suppose that in Fig. 6-6 $C_{ij} + R_{ij} > E_j$ for some i. What implications does this have for the system?

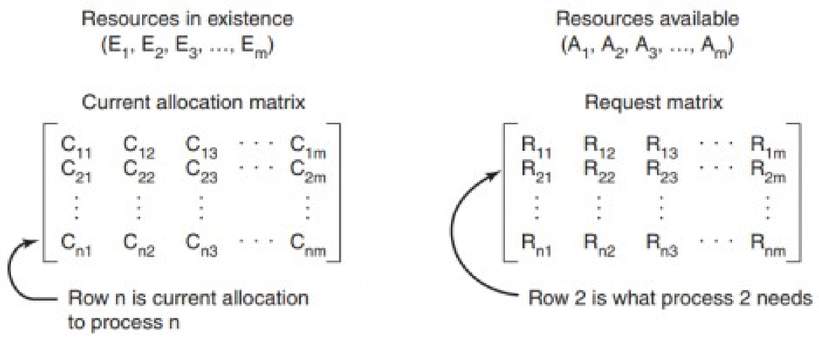

Figure 6-6. The four data structures needed by the deadlock detection algorithm.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.9 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 17 (Page No. 467) top

All the trajectories in Fig. 6-8 are horizontal or vertical. Can you envision any circumstances in which diagonal trajectories are also possible?

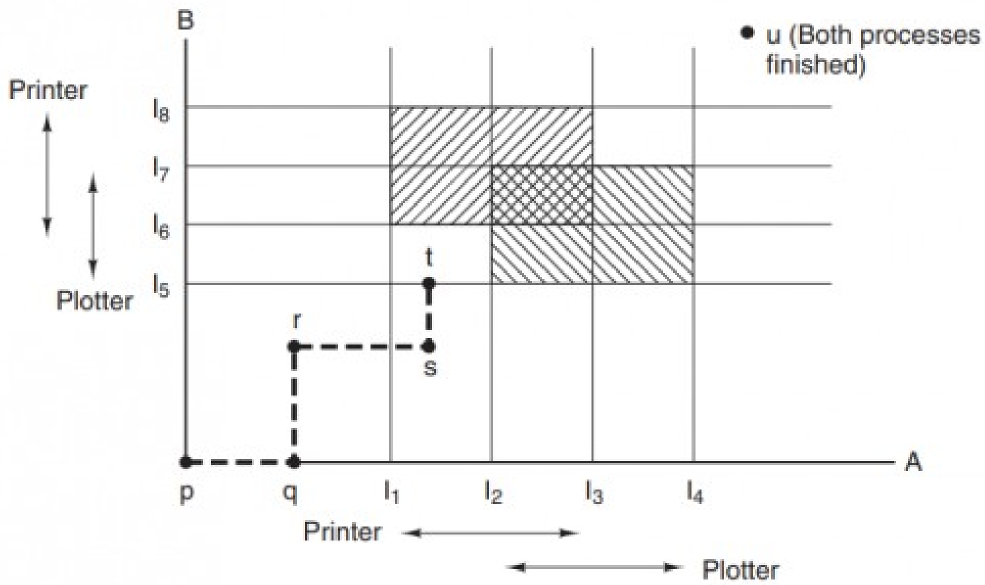

Figure 6-8. Two process resource trajectories.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.10 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 18 (Page No. 467) top

Can the resource trajectory scheme of Fig. 6-8 also be used to illustrate the problem of deadlocks with three processes and three resources? If so, how can this be done? If not, why not?

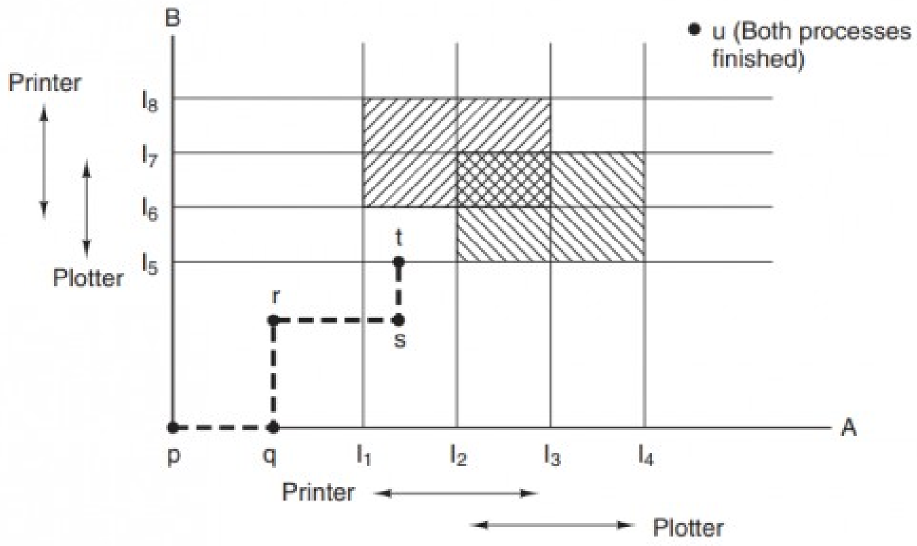

Figure 6-8. Two process resource trajectories.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.11 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 19 (Page No. 467) top

In theory, resource trajectory graphs could be used to avoid deadlocks. By clever scheduling, the operating system could avoid unsafe regions. Is there a practical way of actually doing this?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.12 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 2 (Page No. 465) top

Students working at individual PCs in a computer laboratory send their files to be printed by a server that spools the files on its hard disk. Under what conditions may a deadlock occur if the disk space for the print spool is limited? How may the deadlock be avoided?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.13 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 20 (Page No. 467) top

Can a system be in a state that is neither deadlocked nor safe? If so, give an example. If not, prove that all states are either deadlocked or safe.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.14 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 21 (Page No. 467) top

Take a careful look at Fig. 6-11(b). If D asks for one more unit, does this lead to a safe state or an unsafe one? What if the request came from C instead of D?

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(a)

	Has	Max
A	0	6
B	0	5
C	0	4
D	0	7

Free: 10

(b)

	Has	Max
A	1	6
B	1	5
C	2	4
D	4	7

Free: 2

(c)

	Has	Max
A	1	6
B	2	5
C	2	4
D	4	7

Free: 1

Figure 6-11. Three resource allocation states: (a) Safe. (b) Safe. (c) Unsafe.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.15 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 22 (Page No. 467) top

A system has two processes and three identical resources. Each process needs a maximum of two resources. Is deadlock possible? Explain your answer.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.16 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 23 (Page No. 467) top

Consider the previous problem again, but now with p processes each needing a maximum of m resources and a total of r resources available. What condition must hold to make the system deadlock free?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.17 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 24 (Page No. 467) top

Suppose that process A in Fig. 6-12 requests the last tape drive. Does this action lead to a deadlock?

Process	Tape drives	Plotters	Printers	Blu-rays
A	3	0	1	1
B	0	1	0	0
C	1	1	1	0
D	1	1	0	1
E	0	0	0	0

Resources assigned

Process	Tape drives	Plotters	Printers	Blu-rays
A	1	1	0	0
B	0	1	1	2
C	3	1	0	0
D	0	0	1	0
E	2	1	1	0

Resources still assigned

E = (6342)
P = (5322)
A = (1020)

Figure 6-12. The banker's algorithm with multiple resources.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.18 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 25 (Page No. 467) top

The banker's algorithm is being run in a system with m resource classes and n processes. In the limit of large m and n, the number of operations that must be performed to check a state for safety is proportional to $m^a n^b$. What are the values of a and b?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.19 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 26 (Page No. 467) top

A system has four processes and five allocatable resources. The current allocation and maximum needs are as follows:

	Allocated	*Maximum*	*Available*
Process A	1 0 2 1 1	1 1 2 1 3	0 0 x 1 1
Process B	2 0 1 1 0	2 2 2 1 0	
Process C	1 1 0 1 0	2 1 3 1 0	
Process D	1 1 1 1 0	1 1 2 2 1	

What is the smallest value of x for which this is a safe state?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.20 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 27 (Page No. 467) top

One way to eliminate circular wait is to have rule saying that a process is entitled only to a single resource at any moment. Give an example to show that this restriction is unacceptable in many cases.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.21 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 28 (Page No. 468) top

Two processes, A and B, each need three records, $1, 2,$ and 3, in a database. If A asks for them in the order $1, 2, 3,$ and B asks for them in the same order, deadlock is not possible. However, if B asks for them in the order $3, 2, 1$, then deadlock is possible. With three resources, there are $3!$ or six possible combinations in which each process can request them. What fraction of all the combinations is guaranteed to be deadlock free?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.22 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 29 (Page No. 468) top

A distributed system using mailboxes has two IPC primitives, send and receive. The latter primitive specifies a process to receive from and blocks if no message from that process is available, even though messages may be waiting from other processes. There are no shared resources, but processes need to communicate frequently about other matters. Is deadlock possible? Discuss.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.23 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 30 (Page No. 468) top

In an electronic funds transfer system, there are hundreds of identical processes that work as follows. Each process reads an input line specifying an amount of money, the account to be credited, and the account to be debited. Then it locks both accounts and transfers the money, releasing the locks when done. With many processes running in parallel, there is a very real danger that a process having locked account x will be unable to lock y because y has been locked by a process now waiting for x. Devise a scheme that avoids deadlocks. Do not release an account record until you have completed the transactions. (In other words, solutions that lock one account and then release it immediately if the other is locked are not allowed.)

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.24 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 31 (Page No. 468) top

One way to prevent deadlocks is to eliminate the hold-and-wait condition. In the text it was proposed that before asking for a new resource, a process must first release whatever resources it already holds (assuming that is possible). However, doing so introduces the danger that it may get the new resource but lose some of the existing ones to competing processes. Propose an improvement to this scheme.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.25 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 32 (Page No. 468) top

A computer science student assigned to work on deadlocks thinks of the following brilliant way to eliminate deadlocks. When a process requests a resource, it specifies a time limit. If the process blocks because the resource is not available, a timer is started. If the time limit is exceeded, the process is released and allowed to run again. If you were the professor, what grade would you give this proposal and why?

tanenbaum deadlock-prevention-avoidance-detection descriptive

11.6.26 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 33 (Page No. 468) top

Main memory units are preempted in swapping and virtual memory systems. The processor is preempted in time-sharing environments. Do you think that these preemption methods were developed to handle resource deadlock or for other purposes? How high is their overhead?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.27 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 34 (Page No. 468) top

Explain the differences between deadlock, livelock, and starvation.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.28 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 35 (Page No. 468) top

Assume two processes are issuing a seek command to reposition the mechanism to access the disk and enable a read command. Each process is interrupted before executing its read, and discovers that the other has moved the disk arm. Each then reissues the seek command, but is again interrupted by the other. This sequence continually repeats. Is this a resource deadlock or a livelock? What methods would you recommend to handle the anomaly?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.29 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 36 (Page No. 468 - 469) top

Local Area Networks utilize a media access method called CSMA/CD, in which stations sharing a bus can sense the medium and detect transmissions as well as collisions. In the Ethernet protocol, stations requesting the shared channel do not transmit frames if they sense the medium is busy. When such transmission has terminated, waiting stations each transmit their frames. Two frames that are transmitted at the same time will collide. If stations immediately and repeatedly retransmit after collision detection, they will continue to collide indefinitely

a. Is this a resource deadlock or a livelock?
b. Can you suggest a solution to this anomaly?
c. Can starvation occur with this scenario?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.30 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 37 (Page No. 469) top

A program contains an error in the order of cooperation and competition mechanisms, resulting in a consumer process locking a mutex (mutual exclusion semaphore) before it blocks on an empty buffer. The producer process blocks on the mutex before it can place a value in the empty buffer and awaken the consumer. Thus, both processes are blocked forever, the producer waiting for the mutex to be unlocked and the consumer waiting for a signal from the producer. Is this a resource deadlock or a communication deadlock? Suggest methods for its control.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.31 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 38 (Page No. 469) top

Cinderella and the Prince are getting divorced. To divide their property, they have agreed on the following algorithm. Every morning, each one may send a letter to the other's lawyer requesting one item of property. Since it takes a day for letters to be delivered, they have agreed that if both discover that they have requested the same item on the same day, the next day they will send a letter canceling the request. Among their property is their dog, Woofer, Woofer's doghouse, their canary, Tweeter, and Tweeter's cage. The animals love their houses, so it has been agreed that any division of property separating an animal from its house is invalid, requiring the whole division to start over from scratch. Both Cinderella and the Prince desperately want Woofer. So that they can go on (separate) vacations, each spouse has programmed a personal computer to handle the negotiation. When they come back from vacation, the computers are still negotiating. Why? Is deadlock possible? Is starvation possible? Discuss your answer.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.32 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 4 (Page No. 465) top

In Fig. 6-1 the resources are returned in the reverse order of their acquisition. Would giving them back in the other order be just as good?

```
typedef int semaphore;
semaphore resource_1;

void process_A(void) {
      down(&resource_1);
      use_resource_1( );
      up(&resource_1);
}
```

(a)

```
typedef int semaphore;
semaphore resource_1;
semaphore resource_2;

void process_A(void) {
      down(&resource_1);
      down(&resource_2);
      use_both_resources( );
      up(&resource_2);
      up(&resource_1);
}
```

(b)

Figure 6-1. Using a semaphore to protect resources. (a) One resource. (b) Two resources.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.33 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 42 (Page No. 470) top

Write a program to implement the deadlock detection algorithm with multiple resources of each type. Your program should read from a file the following inputs: the number of processes, the number of resource types, the number of resources of each type in existence $(\text{vector } E)$, the current allocation matrix C (first row, followed by the second row, and so on), the request matrix R (first row, followed by the second row, and so on). The output of your program should indicate whether there is a deadlock in the system. In case there is, the program should print out the identities of all processes that are deadlocked.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.34 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 44 (Page No. 470) top

In certain countries, when two people meet they bow to each other. The protocol is that one of them bows first and stays down until the other one bows. If they bow at the same time, they will both stay bowed forever. Write a program that does not deadlock.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.35 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 5 (Page No. 465) top

The four conditions (mutual exclusion, hold and wait, no preemption and circular wait) are necessary for a resource deadlock to occur. Give an example to show that these conditions are not sufficient for a resource deadlock to occur. When are these conditions sufficient for a resource deadock to occur?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.36 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 6 (Page No. 465 - 466) top

City streets are vulnerable to a circular blocking condition called gridlock, in which intersections are blocked by cars that then block cars behind them that then block the cars that are trying to enter the previous intersection, etc. All intersections around a city block are filled with vehicles that block the oncoming traffic in a circular manner. Gridlock is a resource deadlock and a problem in competition synchronization. New York City's prevention algorithm, called "don't block the box," prohibits cars from entering an intersection unless the space following the intersection is also available. Which prevention algorithm is this? Can you provide any other prevention algorithms for gridlock?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.37 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 7 (Page No. 466) top

Suppose four cars each approach an intersection from four different directions simultaneously. Each corner of the intersection has a stop sign. Assume that traffic regulations require that when two cars approach adjacent stop signs at the same time, the car on the left must yield to the car on the right. Thus, as four cars each drive up to their individual stop signs, each waits (indefinitely) for the car on the left to proceed. Is this anomaly a communication deadlock? Is it a resource deadlock?

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

11.6.38 Deadlock Prevention Avoidance Detection: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 8 (Page No. 466) top

Is it possible that a resource deadlock involves multiple units of one type and a single unit of another? If so, give an example.

tanenbaum operating-system deadlock-prevention-avoidance-detection descriptive

Answer key

11.6.39 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 1 (Page No. 339) top

List three examples of deadlocks that are not related to a computer system environment.

galvin operating-system deadlock-prevention-avoidance-detection descriptive

11.6.40 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 10 (Page No. 341) top

Is it possible to have a deadlock involving only one single-threaded process ? Explain your answer

galvin operating-system deadlock-prevention-avoidance-detection

11.6.41 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 11 (Page No. 341-342) top

Consider the traffic deadlock depicted in Figure 7.10.

a. Show that the four necessary conditions for deadlock hold in this example.
b. State a simple rule for avoiding deadlocks in this system

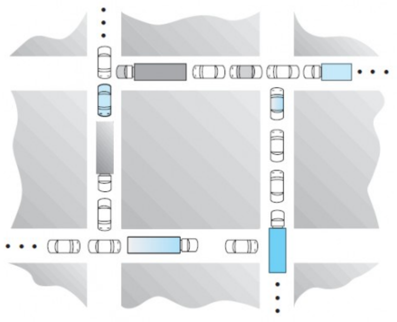

Figure 7.10 Traffic deadlock for Exercise 7.11.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.42 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 12 (Page No. 341) top

Assume a multithreaded application uses only reader–writer locks for synchronization. Applying the four necessary conditions for deadlock, is deadlock still possible if multiple reader–writer locks are used ?

galvin operating-system deadlock-prevention-avoidance-detection

11.6.43 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 13 (Page No. 341) top

```
/* thread one runs in this function */
void *do work one(void *param)
{
pthread mutex lock(&first mutex);
pthread mutex lock(&second mutex);
/**
* Do some work
*/
pthread mutex unlock(&second mutex);
pthread mutex unlock(&first mutex);
pthread exit(0);
}
/* thread two runs in this function */
void *do work two(void *param)
{
pthread mutex lock(&second mutex);
pthread mutex lock(&first mutex);
/**
* Do some work
*/
pthread mutex unlock(&first mutex);
pthread mutex unlock(&second mutex);
pthread exit(0);
}
```

The program example shown above doesn't always lead to deadlock. Describe what role the CPU scheduler plays and how it can contribute to deadlock in this program.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.44 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 15 (Page No. 342) top

Compare the circular-wait scheme with the various deadlock-avoidance schemes (like the banker's algorithm) with respect to the following issues:

a. Runtime overheads
b. System throughput

galvin operating-system deadlock-prevention-avoidance-detection

11.6.45 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 16 (Page No. 342-343) top

In a real computer system, neither the resources available nor the demands of processes for resources are consistent over long periods (months). Resources break or are replaced, new processes come and go, and new resources are bought and added to the system. If deadlock is controlled by the banker's algorithm, which of the following changes can be made safely (without introducing the possibility of deadlock), and under what circumstances ?

a. Increase $Available$ (new resources added).

b. Decrease $Available$ (resource permanently removed from system).

c. Increase Max for one process (the process needs or wants more resources than allowed).

d. Decrease Max for one process (the process decides it does not need that many resources).

e. Increase the number of processes.

f. Decrease the number of processes.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.46 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 17 (Page No. 343) top

Consider a system consisting of four resources of the same type that are shared by three processes, each of which needs at most two resources. Show that the system is deadlock free.

galvin operating-system deadlock-prevention-avoidance-detection

Answer key

11.6.47 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 18 (Page No. 343) top

Consider a system consisting of m resources of the same type being shared by n processes.Aprocess can request or release only one resource at a time. Show that the system is deadlock free if the following two conditions hold:

a. The maximum need of each process is between one resource and m resources.

b. The sum of all maximum needs is less than $m + n$.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.48 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 19 (Page No. 343) top

Consider the version of the dining-philosophers problem in which the chopsticks are placed at the center of the table and any two of them can be used by a philosopher. Assume that requests for chopsticks are made one at a time. Describe a simple rule for determining whether a particular request can be satisfied without causing deadlock given the current allocation of chopsticks to philosophers.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.49 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 2 (Page No. 339) top

Suppose that a system is in an unsafe state. Show that it is possible for the processes to complete their execution without entering a deadlocked state.

galvin operating-system deadlock-prevention-avoidance-detection descriptive

11.6.50 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 20 (Page No. 343) top

Consider again the setting in the preceding question. Assume now that each philosopher requires three chopsticks to eat. Resource requests are still issued one at a time. Describe some simple rules for determining whether a particular request can be satisfied without causing deadlock given the current allocation of chopsticks to philosophers.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.51 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 21 (Page No. 343) top

We can obtain the banker's algorithm for a single resource type from the general banker's algorithm simply by reducing the dimensionality of the various arrays by 1. Show through an example that we cannot implement the multiple-resource-type banker's scheme by applying the single-resource-type scheme to each resource type individually.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.52 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 22 (Page No. 343) top

Consider the following snapshot of a system:

	Allocation	Max
	A B C D	A B C D
P_0	3 0 1 4	5 1 1 7
P_1	2 2 1 0	3 2 1 1
P_2	3 1 2 1	3 3 2 1
P_3	0 5 1 0	4 6 1 2
P_4	4 2 1 2	6 3 2 5

Using the banker's algorithm, determine whether or not each of the following states is unsafe. If the state is safe, illustrate the order in which the processes may complete.Otherwise, illustrate why the state is unsafe.

$a.\ Available = (0, 3, 0, 1)$
$b.\ Available = (1, 0, 0, 2)$

galvin operating-system deadlock-prevention-avoidance-detection

Answer key

11.6.53 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 23 (Page No. 344) top

Consider the following snapshot of a system:

	Allocation	Max	Available
	A B C D	A B C D	A B C D
P_0	2 0 0 1	4 2 1 2	3 3 2 1
P_1	3 1 2 1	5 2 5 2	
P_2	2 1 0 3	2 3 1 6	
P_3	1 3 1 2	1 4 2 4	
P_4	1 4 3 2	3 6 6 5	

Answer the following questions using the banker's algorithm:

a. Illustrate that the system is in a safe state by demonstrating an order in which the processes may complete.

b. If a request from process P_1 arrives for $(1, 1, 0, 0)$, can the request be granted immediately ?

c. If a request from process P_4 arrives for $(0, 0, 2, 0)$, can the request be granted immediately?

galvin operating-system deadlock-prevention-avoidance-detection

Answer key

11.6.54 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 24 (Page No. 344) top

What is the optimistic assumption made in the deadlock-detection algorithm ? How can this assumption be violated ?

galvin operating-system deadlock-prevention-avoidance-detection descriptive

11.6.55 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 25 (Page No. 344) top

A single-lane bridge connects the two Vermont villages of North Tunbridge and South Tunbridge. Farmers in the two villages use this bridge to deliver their produce to the neighboring town. The bridge can become deadlocked if a northbound and a southbound farmer get on the bridge at the same time. (Vermont farmers are stubborn and are unable to back up.) Using semaphores and/or mutex lock design an algorithm in pseudo code that prevents deadlock. Initially, do not be concerned about starvation (the situation in which northbound farmers prevent southbound farmers from using the bridge, or vice versa).

galvin operating-system deadlock-prevention-avoidance-detection descriptive

11.6.56 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 26 (Page No. 344) top

Modify your solution to previous question so that it is starvation-free.

galvin operating-system deadlock-prevention-avoidance-detection descriptive

11.6.57 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 3 (Page No. 340) top

Consider the following snapshot of a system:

	Allocation	Max	Available
	A B C D	A B C D	A B C D
P_0	0 0 1 2	0 0 1 2	1 5 2 0
P_1	1 0 0 0	1 7 5 0	
P_2	1 3 5 4	2 3 5 6	
P_3	0 6 3 2	0 6 5 2	
P_4	0 0 1 4	0 6 5 6	

Answer the following questions using the banker's algorithm:

a. What is the content of the matrix $Need$?

b. Is the system in a safe state ?

c. If a request from process P_1 arrives for $(0, 4, 2, 0)$, can the request be granted immediately ?

galvin operating-system deadlock-prevention-avoidance-detection

Answer key

11.6.58 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 5 (Page No. 340) top

Prove that the safety algorithm requires an order of $m \times n^2$ operations where n is the number of processes in the system and m is the number of resource types.

galvin operating-system deadlock-prevention-avoidance-detection

Answer key

11.6.59 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 6 (Page No. 340) top

Consider a computer system that runs 5,000 jobs per month and has no deadlock-prevention or deadlock-avoidance scheme. Deadlocks occur about twice per month, and the operator must terminate and re run about ten jobs per deadlock. Each job is worth about two dollars (in CPU time), and the jobs terminated tend to be about half done when they are aborted.

A systems programmer has estimated that a deadlock-avoidance algorithm (like the banker's algorithm) could be installed in the system

with an increase of about 10 percent in the average execution time per job. Since the machine currently has 30 percent idle time, all 5,000 jobs per month could still be run, although turnaround time would increase by about 20 percent on average.

a. What are the arguments for installing the deadlock-avoidance algorithm ?

b. What are the arguments against installing the deadlock-avoidance algorithm?

galvin operating-system deadlock-prevention-avoidance-detection

11.6.60 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 7 (Page No. 341) top

Can a system detect that some of its processes are starving? If you answer "yes," explain how it can. If you answer "no," explain how the system can deal with the starvation problem.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.61 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 8 (Page No. 341) top

Consider the following resource-allocation policy. Requests for and releases of resources are allowed at any time. If a request for resources cannot be satisfied because the resources are not available, then we check any processes that are blocked waiting for resources. If a blocked process has the desired resources, then these resources are taken away from it and are given to the requesting process. The vector of resources for which the blocked process is waiting is increased to include the resources that were taken away.

For example, a system has three resource types, and the vector Available is initialized to $(4, 2, 2)$. If process P_0 asks for

$(2,2,1)$, it gets them. If P_1 asks for $(1,0,1)$, it gets them. Then, if P_0 asks for $(0,0,1)$, it is blocked (resource not available). If P_2 now asks for $(2,0,0)$, it gets the available one $(1,0,0)$, as well as one that was allocated to P_0 (since P_0 is blocked). P_0 Allocation vector goes down to $(1,2,1)$, and its Need vector goes up to $(1,0,1)$.

a. Can deadlock occur ? If you answer "yes," give an example. If you answer "no," specify which necessary condition cannot occur.
b. Can indefinite blocking occur ? Explain your answer.

galvin operating-system deadlock-prevention-avoidance-detection

11.6.62 Deadlock Prevention Avoidance Detection: Galvin Edition 9 Exercise 7 Question 9 (Page No. 341) top

Suppose that you have coded the deadlock-avoidance safety algorithm and now have been asked to implement the deadlock-detection algorithm. Can you do so by simply using the safety algorithm code and redefining $Max_i = Waiting_i + Allocation_i$, where $Waiting_i$ is a vector specifying the resources for which process i is waiting and $Allocation_i$ specifies the resources currently allocated to process P_i ? Explain your answer.

galvin operating-system deadlock-prevention-avoidance-detection

11.7 Dining Philosophers Problem (1) top

11.7.1 Dining Philosophers Problem: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 54 (Page No. 178) top

In the solution to the dining philosophers problem (Fig. $2-47$), why is the state variable set to HUNGRY in the procedure *take_forks*?

tanenbaum operating-system process-and-threads dining-philosophers-problem descriptive

11.8 Disk (14) top

11.8.1 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 24 (Page No. 83) top

Suppose that a 10-MB file is stored on a disk on the same track (track 50) in consecutive sectors. The disk arm is currently situated over track number 100. How long will it take to retrieve this file from the disk? Assume that it takes about 1 ms to move the arm from one cylinder to the next and about 5 ms for the sector where the beginning of the file is stored to rotate under the head. Also, assume that reading occurs at a rate of 200 MB/s.

tanenbaum operating-system disk descriptive

Answer key

11.8.2 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 14 (Page No. 333) top

In light of the answer to the previous question, does compacting the disk ever make any sense?

tanenbaum operating-system file-system disk descriptive

11.8.3 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 19 (Page No. 334) top

It has been suggested that efficiency could be improved and disk space saved by storing the data of a short file within the i-node. For the i-node of Fig. $4-13$, how many bytes of data could be stored inside the i-node?

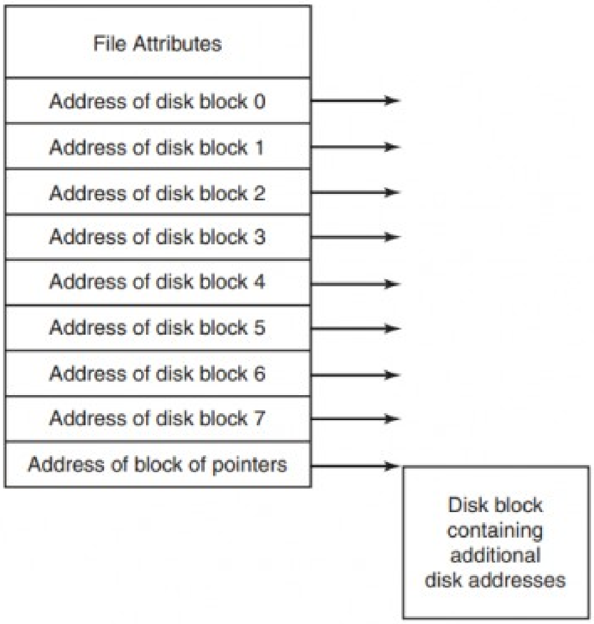

Figure 4-13. An example i-node.

tanenbaum operating-system file-system disk descriptive

11.8.4 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 36 (Page No. 335) top

Consider the idea behind Fig. $4-21$, but now for a disk with a mean seek time of $6\,msec$, a rotational rate of $15,000\,rpm$, and $1,048,576$ bytes per track. What are the data rates for block sizes of $1\,KB, 2\,KB,$ and $4\,KB$, respectively?

302 FILE SYSTEMS CHAP. 4

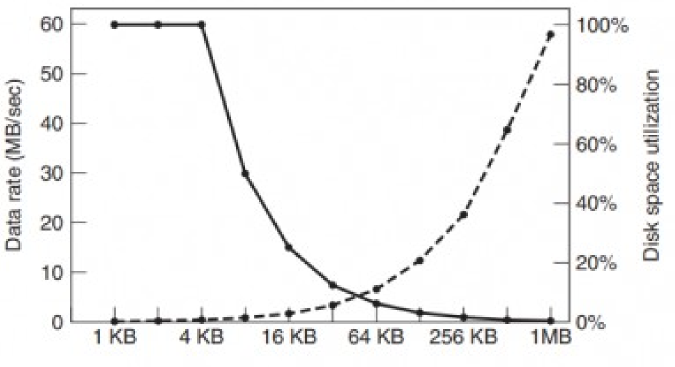

Figure 4-21. The dashed curve (left-hand scale) gives the data rate of a disk. The solid curve (right-hand scale) gives the disk-space efficiency. All files are 4 KB.

tanenbaum operating-system file-system disk descriptive

Answer key

11.8.5 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 37 (Page No. 335) top

A certain file system uses $4-KB$ disk blocks. The median file size is $1KB$. If all files were exactly $1KB$, what fraction of the disk space would be wasted? Do you think the wastage for a real file system will be higher than this number or lower than it? Explain your answer.

tanenbaum operating-system file-system disk descriptive

Answer key

11.8.6 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 38 (Page No. 336) top

Given a disk-block size of $4\ KB$ and block-pointer address value of 4 bytes, what is the largest file size (in bytes) that can be accessed using 10 direct addresses and one indirect block?

tanenbaum operating-system file-system disk descriptive

Answer key

11.8.7 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 40 (Page No. 336) top

A UNIX file system has $4-KB$ blocks and $4-$byte disk addresses. What is the maximum file size if i-nodes contain 10 direct entries, and one single, double, and triple indirect entry each?

tanenbaum operating-system file-system disk descriptive

11.8.8 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 41 (Page No. 336) top

How many disk operations are needed to fetch the i-node for a file with the path name */usr/ast/courses/os/handout.t*? Assume that the i-node for the root directory is in memory, but nothing else along the path is in memory. Also assume that all directories fit in one disk block.

tanenbaum operating-system file-system disk descriptive

Answer key

11.8.9 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 42 (Page No. 336) top

In many UNIX systems, the i-nodes are kept at the start of the disk. An alternative design is to allocate an i-node when a file is created and put the i-node at the start of the first block of the file. Discuss the pros and cons of this alternative.

tanenbaum operating-system file-system disk descriptive

11.8.10 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 43 (Page No. 336) top

Write a program that reverses the bytes of a file, so that the last byte is now first and the first byte is now last. It must work with an arbitrarily long file, but try to make it reasonably efficient.

tanenbaum operating-system file-system disk descriptive

11.8.11 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 44 (Page No. 336) top

Write a program that starts at a given directory and descends the file tree from that point recording the sizes of all the files it finds. When it is all done, it should print a histogram of the file sizes using a bin width specified as a parameter (e.g., with 1024, file sizes of 0 to 1023 go in one bin, 1024 to 2047 go in the next bin, etc.).

tanenbaum operating-system file-system disk descriptive

11.8.12 Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 48 (Page No. 336) top

Implement a simulated file system that will be fully contained in a single regular file stored on the disk. This disk file will contain directories, i-nodes, free-block information, file data blocks, etc. Choose appropriate algorithms for maintaining free-block information and for allocating data blocks (contiguous, indexed, linked). Your program will accept system commands from the user to create/delete directories, create/delete/open files, read/write from/to a selected file, and to list directory contents.

tanenbaum operating-system file-system disk descriptive

11.8.13 Disk: Galvin Edition 9 Exercise 10 Question 23 (Page No. 501) top

Discuss the reasons why the operating system might require accurate information on how blocks are stored on a disk. How could the operating system improve file-system performance with this knowledge ?

galvin operating-system file-system disk

11.8.14 Disk: Galvin Edition 9 Exercise 10 Question 7 (Page No. 497-498) top

It is sometimes said that tape is a sequential-access medium, whereas a magnetic disk is a random-access medium. In fact, the suitability of a storage device for random access depends on the transfer size. The term "streaming transfer rate" denotes the rate for a data transfer that is underway, excluding the effect of access latency. In contrast, the "effective transfer rate" is the ratio of total bytes per total seconds, including overhead time such as access latency.

Suppose we have a computer with the following characteristics: the level-2 cache has an access latency of 8 nanoseconds and a streaming transfer rate of 800 megabytes per second, the main memory has an access latency of 60 nanoseconds and a streaming transfer rate of 80 megabytes per second, the magnetic disk has an access latency of 15 milliseconds and a streaming transfer rate of 5 megabytes per second, and a tape drive has an access latency of 60 seconds and a streaming transfer rate of 2 megabytes per second.

a. Random access causes the effective transfer rate of a device to decrease, because no data are transferred during the access time. For the disk described, what is the effective transfer rate if an average access is followed by a streaming transfer of $(1)512$ bytes,
$(2)8$ kilobytes, $(3)1$ megabyte, and $(4)16$ megabytes ?

b. The utilization of a device is the ratio of effective transfer rate to streaming transfer rate. Calculate the utilization of the disk drive for each of the four transfer sizes given in part a.

c. Suppose that a utilization of 25 percent (or higher) is considered acceptable. Using the performance figures given, compute the smallest transfer size for disk that gives acceptable utilization.

d. Complete the following sentence: A disk is a random-access device for transfers larger than bytes and is a sequential access device for smaller transfers.

e. Compute the minimum transfer sizes that give acceptable utilization for cache, memory, and tape.

f. When is a tape a random-access device, and when is it a sequential-access device ?

galvin operating-system disk descriptive

11.9 Disk Controller (1) top

11.9.1 Disk Controller: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 26 (Page No. 431) top

If a disk controller writes the bytes it receives from the disk to memory as fast as it receives them, with no internal buffering, is interleaving conceivably useful? Discuss your answer.

tanenbaum operating-system input-output disk-controller descriptive

Answer key

11.10 Disk Scheduling (7) top

11.10.1 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 1 (Page No. 497) top

Is disk scheduling, other than $FCFS$ scheduling, useful in a single-user environment ? Explain your answer.

galvin operating-system disk-scheduling descriptive

Answer key

11.10.2 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 10 (Page No. 498) top

Explain why $SSDs$(Solid State Drives) often use an $FCFS$ disk-scheduling algorithm.

galvin operating-system disk-scheduling descriptive

11.10.3 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 11 (Page No. 498-499) top

Suppose that a disk drive has $5,000$ cylinders, numbered 0 to $4,999$. The drive is currently serving a request at cylinder $2,150$, and the previous request was at cylinder $1,805$. The queue of pending requests, in $FIFO$ order, is:
$2,069, 1,212, 2,296, 2,800, 544, 1,618, 356, 1,523, 4,965, 3681$

Starting from the current head position, what is the total distance (in cylinders) that the disk arm moves to satisfy all the pending requests for each of the following disk-scheduling algorithms ?

$a. FCFS$
$b. SSTF$
$c. SCAN$
$d. LOOK$
$e. C - SCAN$
$f. C - LOOK$

galvin operating-system disk-scheduling descriptive

11.10.4 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 15 (Page No. 499-500) top

Compare the performance of $C - SCAN$ and $SCAN$ scheduling, assuming a uniform distribution of requests. Consider the average response time (the time between the arrival of a request and the completion of that request's service), the variation in response time, and the effective bandwidth. How does performance depend on the relative sizes of seek time and rotational latency ?

galvin operating-system disk-scheduling descriptive

11.10.5 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 2 (Page No. 497) top

Explain why $SSTF$ scheduling tends to favor middle cylinders over the innermost and outermost cylinders.

galvin operating-system disk-scheduling descriptive

11.10.6 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 3 (Page No. 497) top

Why is rotational latency usually not considered in disk scheduling ? How would you modify $SSTF$, $SCAN$, and $C - SCAN$ to include latency optimization ?

galvin operating-system disk-scheduling descriptive

Answer key

11.10.7 Disk Scheduling: Galvin Edition 9 Exercise 10 Question 9 (Page No. 498) top

None of the disk-scheduling disciplines, except $FCFS$, is truly fair (starvation may occur).
a. Explain why this assertion is true.
b. Describe a way to modify algorithms such as $SCAN$ to ensure fairness.
c. Explain why fairness is an important goal in a time-sharing system.
d. Give three or more examples of circumstances in which it is important that the operating system be unfair in serving I/O requests.

galvin operating-system disk-scheduling descriptive

Answer key

11.11 File Allocation Table (2) top

11.11.1 File Allocation Table: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 35 (Page No. 335) top

Consider a disk that has 10 data blocks starting from block 14 through 23. Let there be 2 files on the disk: $f1$ and $f2$. The directory structure lists that the first data blocks of $f1$ and $f2$ are respectively 22 and 16. Given the FAT table entries as below, what are the data blocks allotted to $f1$ and $f2$?$(14, 18); (15, 17); (16, 23); (17, 21); (18, 20); (19, 15); (20, -1); (21, -1); (22, 19); (23, 14)$. In the above notation, (x, y) indicates that the value stored in table entry x points to data block y.

tanenbaum operating-system file-system file-allocation-table descriptive

Answer key

11.11.2 File Allocation Table: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 39 (Page No. 336) top

Files in $MS - DOS$ have to compete for space in the $FAT - 16$ table in memory. If one file uses k entries, that is k entries that are not available to any other file, what constraint does this place on the total length of all files combined?

tanenbaum operating-system file-system file-allocation-table descriptive

Answer key

11.12 File Organization (1) top

11.12.1 File Organization: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 12 (Page No. 333) top

Describe the effects of a corrupted data block for a given file for:

a. contiguous,
b. linked, and
c. indexed (or table based).

tanenbaum operating-system file-system file-organization descriptive

11.13 File System (55) top

11.13.1 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 19 (Page No. 82) top

Is there any reason why you might want to mount a file system on a nonempty directory? If so, what is it?

tanenbaum operating-system file-system descriptive

11.13.2 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 1 (Page No. 332) top

Give five different path names for the file /etc/passwd. (Hint: Think about the directory entries ‘‘.’’ and ‘‘..’’.)

tanenbaum operating-system file-system descriptive

11.13.3 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 10 (Page No. 333) top

Consider the directory tree of Fig. $4-8$. If /usr/jim is the working directory, what is the absolute path name for the file whose relative path name is ../ast/x?

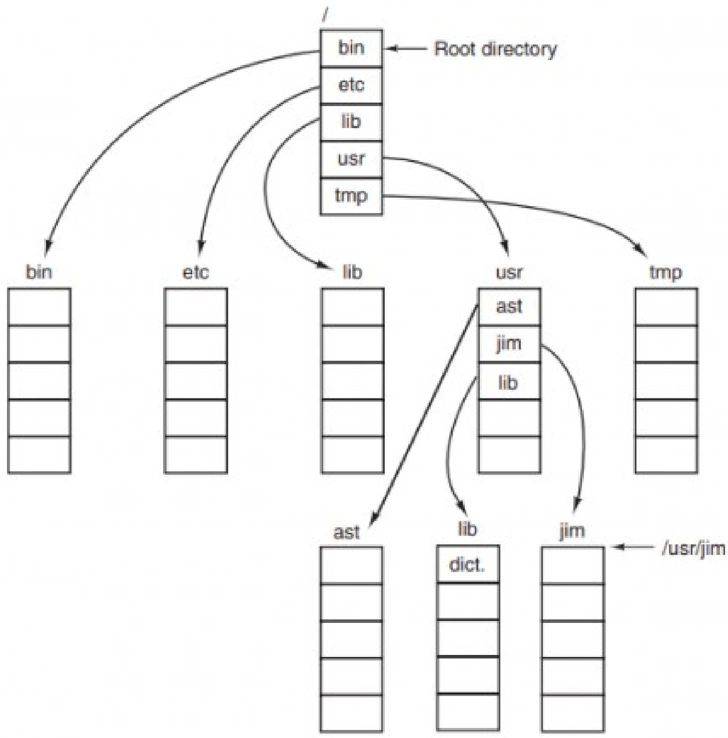

Figure 4-8. A UNIX directory tree.

tanenbaum operating-system file-system descriptive

Answer key

11.13.4 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 13 (Page No. 333) top

One way to use contiguous allocation of the disk and not suffer from holes is to compact the disk every time a file is removed. Since all files are contiguous, copying a file requires a seek and rotational delay to read the file, followed by the transfer at full speed. Writing the file back requires the same work. Assuming a seek time of $5\ msec$, a rotational delay of $4\ msec$, a transfer rate of $80\ MB/sec$, and an average file size of $8\ KB$, how long does it take to read a file into main memory and then write it back to the disk at a new location? Using these numbers, how long would it take to compact half of a $16 - GB$ disk?

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.5 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 17 (Page No. 334) top

For a given class, the student records are stored in a file. The records are randomly accessed and updated. Assume that each student's record is of fixed size. Which of the three allocation schemes (contiguous, linked and table/indexed) will be most appropriate?

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.6 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 18 (Page No. 334) top

Consider a file whose size varies between $4\ KB$ and $4\ MB$ during its lifetime. Which of the three allocation schemes (contiguous, linked and table/indexed) will be most appropriate?

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.7 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 2 (Page No. 332) top

In Windows, when a user double clicks on a file listed by Windows Explorer, a program is run and given that file as a parameter. List two different ways the operating system could know which program to run.

tanenbaum operating-system file-system descriptive

11.13.8 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 20 (Page No. 334) top

Two computer science students, Carolyn and Elinor, are having a discussion about i-nodes. Carolyn maintains that memories have gotten so large and so cheap that when a file is opened, it is simpler and faster just to fetch a new copy of the i-node into the i-node table, rather than search the entire table to see if it is already there. Elinor disagrees. Who is right?

tanenbaum operating-system file-system descriptive

11.13.9 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 21 (Page No. 334) top

Name one advantage of hard links over symbolic links and one advantage of symbolic links over hard links.

tanenbaum operating-system file-system descriptive

11.13.10 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 22 (Page No. 334) top

Explain how hard links and soft links differ with respective to i-node allocations.

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.11 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 23 (Page No. 334) top

Consider a $4 - TB$ disk that uses $4 - KB$ blocks and the free-list method. How many block addresses can be stored in one block?

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.12 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 24 (Page No. 334) top

Free disk space can be kept track of using a free list or a bitmap. Disk addresses require D bits. For a disk with B blocks, F of which are free, state the condition under which the free list uses less space than the bitmap. For D having the value 16 bits, express your answer as a percentage of the disk space that must be free.

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.13 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 25 (Page No. 334) top

The beginning of a free-space bitmap looks like this after the disk partition is first formatted : $1000\ 0000\ 0000\ 0000$ (the first block is used by the root directory). The system always searches for free blocks starting at the lowest-numbered block, so after writing file A, which uses six blocks, the bitmap looks like this : $1111\ 1110\ 0000\ 0000$. Show the bitmap after each of the following additional actions:

a. File B is written, using five blocks.
b. File A is deleted.
c. File C is written, using eight blocks.
d. File B is deleted.

tanenbaum operating-system file-system memory-management descriptive

Answer key

11.13.14 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 26 (Page No. 334) top

What would happen if the bitmap or free list containing the information about free disk blocks was completely lost due to a crash? Is there any way to recover from this disaster, or is it bye-bye disk? Discuss your answers for $UNIX$ and the $FAT-16$ file system separately.

tanenbaum operating-system file-system memory-management descriptive

11.13.15 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 27 (Page No. 335) top

Oliver Owl's night job at the university computing center is to change the tapes used for overnight data backups. While waiting for each tape to complete, he works on writing his thesis that proves Shakespeare's plays were written by extraterrestrial visitors. His text processor runs on the system being backed up since that is the only one they have. Is there a problem with this arrangement?

tanenbaum operating-system file-system descriptive

11.13.16 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 31 (Page No. 335) top

Consider Fig. $4-27$. Is it possible that for some particular block number the counters in both lists have the value 2? How should this problem be corrected?

SEC. 4.4 FILE-SYSTEM MANAGEMENT AND OPTIMIZATION **313**

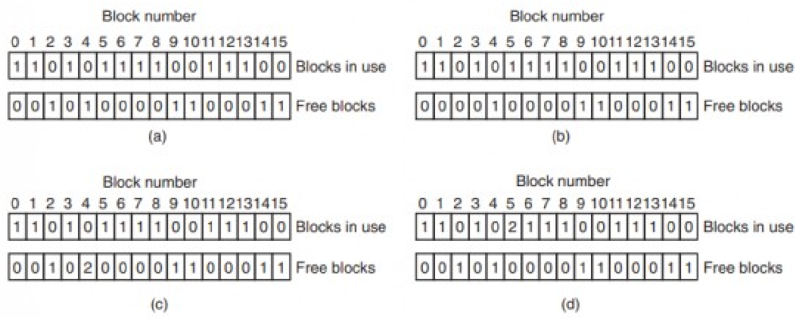

Figure 4-27. File-system states. (a) Consistent. (b) Missing block. (c) Duplicate block in free list. (d) Duplicate data block.

tanenbaum operating-system file-system descriptive

11.13.17 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 32 (Page No. 335) top

The performance of a file system depends upon the cache hit rate (fraction of blocks found in the cache). If it takes $1\ msec$ to satisfy a request from the cache, but $40\ msec$ to satisfy a request if a disk read is needed, give a formula for the mean time required to satisfy a request if the hit rate is h. Plot this function for values of h varying from 0 to 1.0.

tanenbaum operating-system file-system cache-memory descriptive

11.13.18 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 34 (Page No. 335) top

Consider an application where students' records are stored in a file. The application takes a student ID as input and subsequently reads, updates, and writes the corresponding student record; this is repeated till the application quits. Would the "block read ahead" technique be useful here?

tanenbaum operating-system file-system descriptive

11.13.19 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 5 (Page No. 333) top

Systems that support sequential files always have an operation to rewind files. Do systems that support random-access files need this, too?

tanenbaum operating-system file-system descriptive

11.13.20 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 6 (Page No. 333) top

Some operating systems provide a system call *rename* to give a file a new name. Is there any difference at all between using this call to rename a file and just copying the file to a new file with the new name, followed by deleting the old one?

tanenbaum operating-system file-system descriptive

11.13.21 File System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 8 (Page No. 333) top

A simple operating system supports only a single directory but allows it to have arbitrarily many files with arbitrarily long file names. Can something approximating a hierarchical file system be simulated? How?

tanenbaum operating-system file-system descriptive

11.13.22 File System: Galvin Edition 9 Exercise 11 Question 1 (Page No. 539) top

Some systems automatically delete all user files when a user logs off or
a job terminates, unless the user explicitly requests that they be kept.
Other systems keep all files unless the user explicitly deletes them.
Discuss the relative merits of each approach.

operating-system galvin file-system descriptive

11.13.23 File System: Galvin Edition 9 Exercise 11 Question 10 (Page No. 540) top

The open-file table is used to maintain information about files that are
currently open. Should the operating system maintain a separate table
for each user or maintain just one table that contains references to files
that are currently being accessed by all users? If the same file is being
accessed by two different programs or users, should there be separate
entries in the open-file table? Explain.

operating-system galvin descriptive file-system

11.13.24 File System: Galvin Edition 9 Exercise 11 Question 11 (Page No. 540) top

What are the advantages and disadvantages of providing mandatory
locks instead of advisory locks whose use is left to users' discretion?

operating-system galvin descriptive file-system

11.13.25 File System: Galvin Edition 9 Exercise 11 Question 12 (Page No. 540) top

Provide examples of applications that typically access files according
to the following methods:

• Sequential
• Random

operating-system galvin descriptive file-system

11.13.26 File System: Galvin Edition 9 Exercise 11 Question 15 (Page No. 540) top

Give an example of an application that could benefit from operatingsystem support for random access to indexed files.

operating-system galvin descriptive file-system

11.13.27 File System: Galvin Edition 9 Exercise 11 Question 16 (Page No. 540) top

Discuss the advantages and disadvantages of supporting links to files that cross mount points (that is, the file link refers to a file that is stored in a different volume).

operating-system galvin file-system descriptive

11.13.28 File System: Galvin Edition 9 Exercise 11 Question 17 (Page No. 540) top

Some systems provide file sharing by maintaining a single copy of a file. Other systems maintain several copies, one for each of the users sharing the file. Discuss the relative merits of each approach

operating-system galvin descriptive file-system

11.13.29 File System: Galvin Edition 9 Exercise 11 Question 18 (Page No. 541) top

Discuss the advantages and disadvantages of associating with remote file systems (stored on file servers) a set of failure semantics different from that associated with local file systems.

operating-system galvin descriptive file-system

11.13.30 File System: Galvin Edition 9 Exercise 11 Question 2 (Page No. 539) top

Why do some systems keep track of the type of a file, while others leave it to the user and others simply do not implement multiple file types? Which system is "better"?

3- Similarly, some systems support many types of structures for a file's data, while others simply support a stream of bytes. What are the advantages and disadvantages of each approach?

operating-system galvin file-system descriptive

11.13.31 File System: Galvin Edition 9 Exercise 11 Question 4 (Page No. 539) top

Could you simulate a multilevel directory structure with a single-level directory structure in which arbitrarily long names can be used? If your answer is yes, explain how you can do so, and contrast this scheme with the multilevel directory scheme. If your answer is no, explain what prevents your simulation's success. How would your answer change if file names were limited to seven characters?

operating-system galvin descriptive file-system

11.13.32 File System: Galvin Edition 9 Exercise 11 Question 5 (Page No. 539) top

Explain the purpose of the open() and close() operations.

operating-system galvin descriptive file-system

11.13.33 File System: Galvin Edition 9 Exercise 11 Question 6 (Page No. 539) top

In some systems, a subdirectory can be read and written by an authorized user, just as ordinary files can be.
a. Describe the protection problems that could arise.
b. Suggest a scheme for dealing with each of these protection problems.

operating-system galvin descriptive file-system

11.13.34 File System: Galvin Edition 9 Exercise 11 Question 7 (Page No. 539) top

Consider a system that supports 5,000 users. Suppose that you want to allow 4,990 of these users to be able to access one file.
a. How would you specify this protection scheme in UNIX?
b. Can you suggest another protection scheme that can be used more effectively for this purpose than the scheme provided by UNIX?

operating-system galvin descriptive file-system

11.13.35 File System: Galvin Edition 9 Exercise 11 Question 8 (Page No. 539) top

Researchers have suggested that, instead of having an access list associated with each file (specifying which users can access the file, and how), we should have a user control list associated with each user (specifying which files a user can access, and how). Discuss the relative merits of these two schemes.

operating-system galvin descriptive file-system

11.13.36 File System: Galvin Edition 9 Exercise 11 Question 9 (Page No. 540) top

Consider a file system in which a file can be deleted and its disk space reclaimed while links to that file still exist.What problems may occur if a new file is created in the same storage area or with the same absolute path name? How can these problems be avoided?

operating-system galvin descriptive file-system

11.13.37 File System: Galvin Edition 9 Exercise 12 Question 1 (Page No. 581) top

Consider a file currently consisting of 100 blocks. Assume that the filecontrol block (and the index block, in the case of indexed allocation) is already in memory. Calculate how many disk I/O operations are required for contiguous, linked, and indexed (single-level) allocation strategies, if, for one block, the following conditions hold. In the contiguous-allocation case, assume that there is no room to grow at the beginning but there is room to grow at the end. Also assume that the block information to be added is stored in memory.
a. The block is added at the beginning.
b. The block is added in the middle.
c. The block is added at the end.
d. The block is removed from the beginning.
e. The block is removed from the middle.
f. The block is removed from the end.

operating-system galvin file-system

11.13.38 File System: Galvin Edition 9 Exercise 12 Question 10 (Page No. 582) top

Contrast the performance of the three techniques for allocating disk blocks (contiguous, linked, and indexed) for both sequential and random file access.

operating-system galvin descriptive file-system

11.13.39 File System: Galvin Edition 9 Exercise 12 Question 11 (Page No. 582) top

What are the advantages of the variant of linked allocation that uses a FAT to chain together the blocks of a file?

operating-system galvin descriptive file-system

11.13.40 File System: Galvin Edition 9 Exercise 12 Question 12 (Page No. 582) top

Consider a system where free space is kept in a free-space list.
a. Suppose that the pointer to the free-space list is lost. Can the system reconstruct the free-space list? Explain your answer.
b. Consider a file system similar to the one used by UNIX with indexed allocation. How many disk I/O operations might be required to read the contents of a small local file at /a/b/c? Assume that none of the disk blocks is currently being cached.
c. Suggest a scheme to ensure that the pointer is never lost as a result of memory failure.

operating-system galvin descriptive file-system

11.13.41 File System: Galvin Edition 9 Exercise 12 Question 13 (Page No. 582) top

Some file systems allow disk storage to be allocated at different levels of granularity. For instance, a file system could allocate 4 KB of disk space as a single 4-KB block or as eight 512-byte blocks. How could we take advantage of this flexibility to improve performance? What modifications would have to be made to the free-space management scheme in order to support this feature?

operating-system galvin file-system descriptive

11.13.42 File System: Galvin Edition 9 Exercise 12 Question 14 (Page No. 582) top

Discuss how performance optimizations for file systems might result in difficulties in maintaining the consistency of the systems in the event of computer crashes

operating-system galvin descriptive file-system

11.13.43 File System: Galvin Edition 9 Exercise 12 Question 15 (Page No. 583) top

Consider a file system on a disk that has both logical and physical block sizes of 512 bytes. Assume that the information about each file is already in memory. For each of the three allocation strategies (contiguous, linked, and indexed), answer these questions:
a. How is the logical-to-physical address mapping accomplished in this system? (For the indexed allocation, assume that a file is always less than 512 blocks long.)
b. If we are currently at logical block 10 (the last block accessed was block 10) and want to access logical block 4, how many physical blocks must be read from the disk?

operating-system galvin file-system

11.13.44 File System: Galvin Edition 9 Exercise 12 Question 16 (Page No. 583) top

Consider a file system that uses inodes to represent files. Disk blocks are 8 KB in size, and a pointer to a disk block requires 4 bytes. This file system has 12 direct disk blocks, as well as single, double, and triple indirect disk blocks. What is the maximum size of a file that can be stored in this file system?

operating-system galvin file-system

11.13.45 File System: Galvin Edition 9 Exercise 12 Question 17 (Page No. 582) top

Fragmentation on a storage device can be eliminated by recompaction of the information. Typical disk devices do not have relocation or base registers (such as those used when memory is to be compacted), so how can we relocate files? Give three reasons why recompacting and relocation of files are often avoided.

operating-system galvin descriptive file-system

11.13.46 File System: Galvin Edition 9 Exercise 12 Question 18 (Page No. 583) top

Assume that in a particular augmentation of a remote-file-access protocol, each client maintains a name cache that caches translations from file names to corresponding file handles. What issues should we take into account in implementing the name cache?

operating-system galvin file-system descriptive

11.13.47 File System: Galvin Edition 9 Exercise 12 Question 19 (Page No. 583) top

Explain why logging metadata updates ensures recovery of a file system after a file-system crash.

operating-system galvin file-system descriptive

11.13.48 File System: Galvin Edition 9 Exercise 12 Question 2 (Page No. 581) top

What problems could occur if a system allowed a file system to be mounted simultaneously at more than one location?

operating-system galvin file-system

11.13.49 File System: Galvin Edition 9 Exercise 12 Question 3 (Page No. 581) top

Why must the bit map for file allocation be kept on mass storage, rather than in main memory?

galvin operating-system file-system

11.13.50 File System: Galvin Edition 9 Exercise 12 Question 4 (Page No. 581) top

Consider a system that supports the strategies of contiguous, linked, and indexed allocation. What criteria should be used in deciding which strategy is best utilized for a particular file?

galvin operating-system file-system

11.13.51 File System: Galvin Edition 9 Exercise 12 Question 5 (Page No. 581) top

One problem with contiguous allocation is that the user must preallocate enough space for each file. If the file grows to be larger than the space allocated for it, special actions must be taken. One solution to this problem is to define a file structure consisting of an initial contiguous area (of a specified size). If this area is filled, the operating system automatically defines an overflow area that is linked to the initial contiguous area. If the overflow area is filled, another overflow area is allocated. Compare this implementation of a file with the standard contiguous and linked implementations.

operating-system galvin file-system

11.13.52 File System: Galvin Edition 9 Exercise 12 Question 6 (Page No. 582) top

How do caches help improve performance? Why do systems not use more or larger caches if they are so useful?

operating-system galvin descriptive file-system

11.13.53 File System: Galvin Edition 9 Exercise 12 Question 7 (Page No. 582) top

Why is it advantageous to the user for an operating system to dynamically allocate its internal tables? What are the penalties to the operating system for doing so?

operating-system galvin descriptive file-system

11.13.54 File System: Galvin Edition 9 Exercise 12 Question 8 (Page No. 582) top

Explain how the VFS layer allows an operating system to support multiple types of file systems easily.

operating-system galvin descriptive file-system

11.13.55 File System: Galvin Edition 9 Exercise 12 Question 9 (Page No. 582) top

Consider a file system that uses a modifed contiguous-allocation scheme with support for extents. A file is a collection of extents, with each extent corresponding to a contiguous set of blocks. A key issue in such systems is the degree of variability in the size of the extents. What are the advantages and disadvantages of the following schemes?

a. All extents are of the same size, and the size is predetermined.
b. Extents can be of any size and are allocated dynamically.
c. Extents can be of a few fixed sizes, and these sizes are predetermined.

galvin operating-system descriptive file-system

11.14 Fragmentation (1) top

11.14.1 Fragmentation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 11 (Page No. 333) top

Contiguous allocation of files leads to disk fragmentation, as mentioned in the text, because some space in the last disk block will be wasted in files whose length is not an integral number of blocks. Is this internal fragmentation or external fragmentation? Make an analogy with something discussed in the previous chapter.

tanenbaum operating-system file-system fragmentation disk descriptive

11.15 Hard Disk (1) top

11.15.1 Hard Disk: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 33 (Page No. 335) top

For an external USB hard drive attached to a computer, which is more suitable: a write through cache or a block cache?

tanenbaum operating-system file-system hard-disk descriptive

11.16 I Node (1) top

11.16.1 I Node: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 16 (Page No. 334) top

Consider the i-node shown in Fig. $4-13$. If it contains 10 direct addresses and these were 8 bytes each and all disk blocks were $1024\ KB$, what would the largest possible file be?

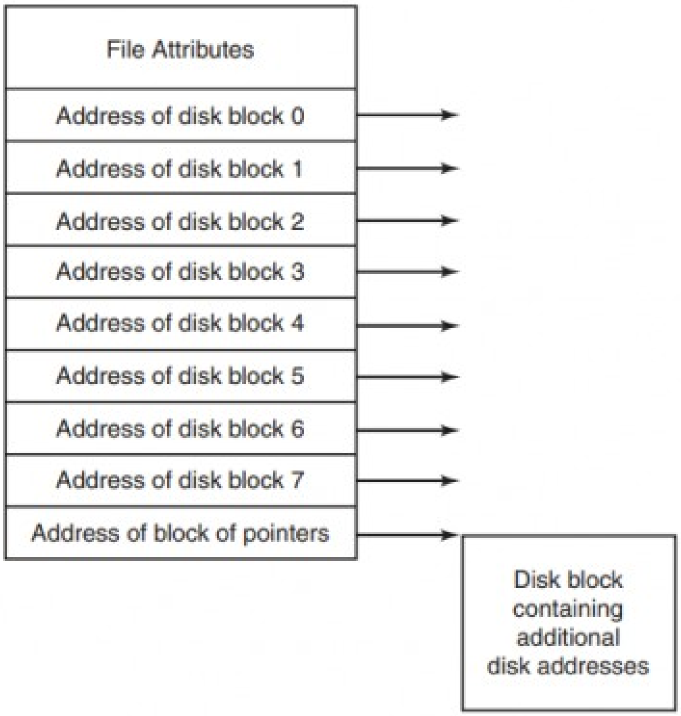

Figure 4-13. An example i-node.

tanenbaum operating-system file-system disk i-node descriptive

11.17 Input Output (50) top

11.17.1 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 1 (Page No. 429) top

Advances in chip technology have made it possible to put an entire controller, including all the bus access logic, on an inexpensive chip. How does that affect the model of Fig. $1-6$?

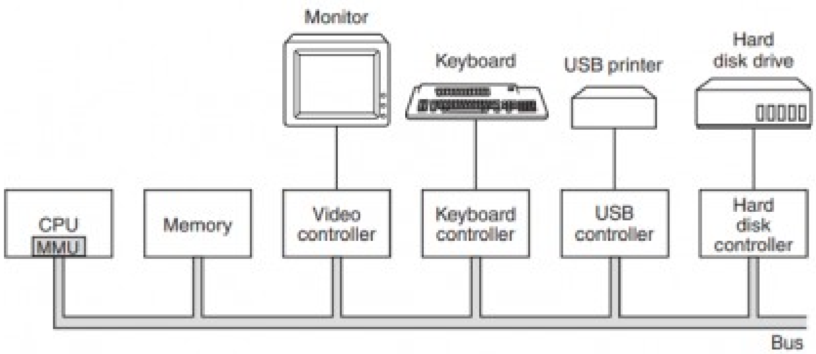

Figure 1-6. Some of the components of a simple personal computer.

tanenbaum operating-system input-output descriptive

11.17.2 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 10 (Page No. 430) top

In Fig. 5-9(b), the interrupt is not acknowledged until after the next character has been output to the printer. Could it have equally well been acknowledged right at the start of the interrupt service procedure? If so, give one reason for doing it at the end, as in the text. If not, why not?

```
copy_from_user(buffer, p, count);
enable_interrupts( );
while (*printer_status_reg != READY) ;
*printer_data_register = p[0];
scheduler( );
```

(a)

```
if (count == 0) {
     unblock_user( );
} else {
     *printer_data_register = p[i];
     count = count – 1;
     i = i + 1;
}
acknowledge_interrupt( );
return_from_interrupt( );
```

(b)

Figure 5-9. Writing a string to the printer using interrupt-driven I/O. (a) Code executed at the time the print system call is made. (b) Interrupt service procedure for the printer.

tanenbaum operating-system input-output interrupts descriptive

11.17.3 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 13 (Page No. 430) top

Explain how an OS can facilitate installation of a new device without any need for recompiling the OS.

tanenbaum operating-system input-output descriptive

11.17.4 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 14 (Page No. 430) top

In which of the four I/O software layers is each of the following done.

a. Computing the track, sector, and head for a disk read.
b. Writing commands to the device registers.
c. Checking to see if the user is permitted to use the device.
d. Converting binary integers to ASCII for printing.

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.5 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 15 (Page No. 430 - 431) top

A local area network is used as follows. The user issues a system call to write data packets to the network. The operating system then copies the data to a kernel buffer. Then it copies the data to the network controller board. When all the bytes are safely inside the controller, they are sent over the network at a rate of 10 megabits/sec. The receiving network controller stores each bit a microsecond after it is sent. When the last bit arrives, the destination CPU is interrupted, and the kernel copies the newly arrived packet to a kernel buffer to inspect it. Once it has figured out which user the packet is for, the kernel copies the data to the user space. If we assume that each interrupt and its associated processing takes $1 \; msec$, that packets are 1024 bytes (ignore the headers), and that copying a byte takes $1 \; \mu sec$, what is the maximum rate at which one process can pump data to another? Assume that the sender is blocked until the work is finished at the receiving side and an acknowledgement comes back. For simplicity, assume that the time to get the acknowledgement back is so small it can be ignored.

tanenbaum operating-system input-output descriptive

11.17.6 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 16 (Page No. 431) top

Why are output files for the printer normally spooled on disk before being printed?

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.7 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 17 (Page No. 431) top

How much cylinder skew is needed for a 7200-RPM disk with a track-to-track seek time of $1 \; msec$? The disk has 200 sectors of 512 bytes each on each track.

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.8 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 18 (Page No. 431) top

A disk rotates at 7200 RPM. It has 500 sectors of 512 bytes around the outer cylinder. How long does it take to read a sector?

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.9 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 19 (Page No. 431) top

Calculate the maximum data rate in bytes/sec for the disk described in the previous problem.

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.10 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 2 (Page No. 429) top

Given the speeds listed in Fig. $5-1$, is it possible to scan documents from a scanner and transmit them over an $802.11g$ network at full speed? Defend your answer.

Device	Data rate
Keyboard	10 bytes/sec
Mouse	100 bytes/sec
56K modem	7 KB/sec
Scanner at 300 dpi	1 MB/sec
Digital camcorder	3.5 MB/sec
4x Blu-ray disc	18 MB/sec
802.11n Wireless	37.5 MB/sec
USB 2.0	60 MB/sec
FireWire 800	100 MB/sec
Gigabit Ethernet	125 MB/sec
SATA 3 disk drive	600 MB/sec
USB 3.0	625 MB/sec
SCSI Ultra 5 bus	640 MB/sec
Single-lane PCIe 3.0 bus	985 MB/sec
Thunderbolt 2 bus	2.5 GB/sec
SONET OC-768 network	5 GB/sec

Figure 5-1. Some typical device, network, and bus data rates.

tanenbaum operating-system input-output descriptive

11.17.11 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 20 (Page No. 431) top

RAID level 3 is able to correct single-bit errors using only one parity drive. What is the point of RAID level 2? After all, it also can only correct one error and takes more drives to do so.

tanenbaum operating-system input-output disk descriptive

11.17.12 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 21 (Page No. 431) top

A RAID can fail if two or more of its drives crash within a short time interval. Suppose that the probability of one drive crashing in a given hour is p. What is the probability of a k-drive RAID failing in a given hour?

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.13 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 22 (Page No. 431) top

Compare RAID level 0 through 5 with respect to read performance, write performance, space overhead, and reliability.

tanenbaum operating-system input-output disk descriptive

11.17.14 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 23 (Page No. 431) top

How many pebibytes are there in a zebibyte?

tanenbaum operating-system input-output easy descriptive

Answer key

11.17.15 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 24 (Page No. 431) top

Why are optical storage devices inherently capable of higher data density than magnetic storage devices? Note: This problem requires some knowledge of high-school physics and how magnetic fields are generated.

tanenbaum operating-system input-output disk descriptive

11.17.16 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 25 (Page No. 431) top

What are the advantages and disadvantages of optical disks versus magnetic disks?

tanenbaum operating-system input-output disk descriptive

11.17.17 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 27 (Page No. 431) top

If a disk has double interleaving, does it also need cylinder skew in order to avoid missing data when making a track-to-track seek? Discuss your answer.

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.18 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 30 (Page No. 432) top

A computer manufacturer decides to redesign the partition table of a Pentium hard disk to provide more than four partitions. What are some consequences of this change?

tanenbaum operating-system input-output disk descriptive

11.17.19 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 31 (Page No. 432) top

Disk requests come in to the disk driver for cylinders $10, 22, 20, 2, 40, 6,$ and $38,$ in that order. A seek takes $6\;msec$ per cylinder. How much seek time is needed for

a. First-come, first served.
b. Closest cylinder next.
c. Elevator algorithm (initially moving upward).

In all cases, the arm is initially at cylinder 20.

tanenbaum operating-system input-output disk disk-scheduling descriptive

Answer key

11.17.20 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 32 (Page No. 432) top

A slight modification of the elevator algorithm for scheduling disk requests is to always scan in the same direction. In what respect is this modified algorithm better than the elevator algorithm?

tanenbaum operating-system input-output disk-scheduling descriptive

Answer key

11.17.21 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 33 (Page No. 432) top

A personal computer salesman visiting a university in South-West Amsterdam remarked during his sales pitch that his company had devoted substantial effort to making their version of UNIX very fast. As an example, he noted that their disk driver used the elevator algorithm and also queued multiple requests within a cylinder in sector order. A student, Harry Hacker, was impressed and bought one. He took it home and wrote a program to randomly read $10,000$ blocks spread across the disk. To his amazement, the performance that he measured was identical to what would be expected from first-come, first-served. Was the salesman lying?

tanenbaum operating-system input-output unix disk descriptive

11.17.22 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 34 (Page No. 432) top

In the discussion of stable storage using nonvolatile RAM, the following point was glossed over. What happens if the stable write completes but a crash occurs before the operating system can write an invalid block number in the nonvolatile RAM? Does this race condition ruin the abstraction of stable storage? Explain your answer.

tanenbaum operating-system input-output disk descriptive

11.17.23 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 35 (Page No. 432) top

In the discussion on stable storage, it was shown that the disk can be recovered to a consistent state (a write either completes or does not take place at all) if a CPU crash occurs during a write. Does this property hold if the CPU crashes again during a recovery procedure. Explain your answer.

tanenbaum operating-system input-output disk descriptive

11.17.24 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 36 (Page No. 432) top

In the discussion on stable storage, a key assumption is that a CPU crash that corrupts a sector leads to an incorrect ECC. What problems might arise in the five crash-recovery scenarios shown in Figure 5-27 if this assumption does not hold?

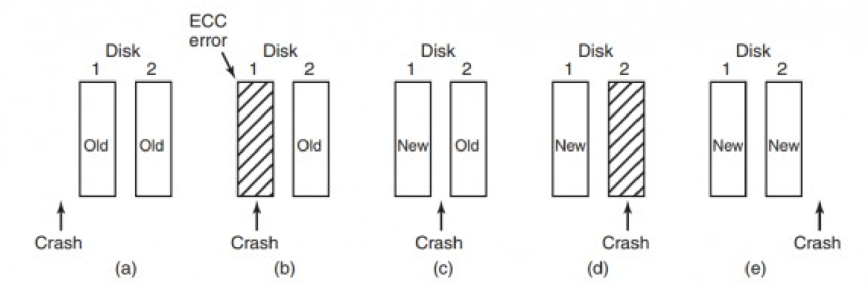

Figure 5-27. Analysis of the influence of crashes on stable writes.

tanenbaum operating-system input-output disk descriptive

11.17.25 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 37 (Page No. 432) top

The clock interrupt handler on a certain computer requires $2\ msec$ (including process switching overhead) per clock tick. The clock runs at $60\ Hz$. What fraction of the CPU is devoted to the clock?

tanenbaum operating-system input-output interrupts descriptive

Answer key

11.17.26 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 38 (Page No. 432) top

A computer uses a programmable clock in square-wave mode. If a $500\ MHz$ crystal is used, what should be the value of the holding register to achieve a clock resolution of

a. a millisecond (a clock tick once every millisecond)?
b. 100 microseconds?

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.27 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 39 (Page No. 433) top

A system simulates multiple clocks by chaining all pending clock requests together as shown in Fig. 5-30. Suppose the current time is 5000 and there are pending clock requests for time $5008, 5012, 5015, 5029, 4$ and 5037. Show the values of Clock header, Current time, and Next signal at times $5000, 5005,$ and 5013. Suppose a new (pending) signal arrives at time 5017 for 5033. Show the values of Clock header, Current time and Next signal at time 5023.

392 INPUT/OUTPUT CHAP. 5

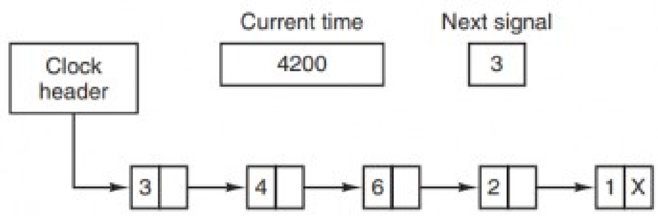

Figure 5-30. Simulating multiple timers with a single clock.

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.28 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 4 (Page No. 429) top

Explain the tradeoffs between precise and imprecise interrupts on a superscalar machine.

tanenbaum operating-system input-output interrupts descriptive

11.17.29 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 40 (Page No. 433) top

Many versions of UNIX use an unsigned 32-bit integer to keep track of the time as the number of seconds since the origin of time. When will these systems wrap around (year and month)? Do you expect this to actually happen?

tanenbaum operating-system input-output disk unix descriptive

Answer key

11.17.30 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 41 (Page No. 433) top

A bitmap terminal contains 1600 by 1200 pixels. To scroll a window, the CPU (or controller) must move all the lines of text upward by copying their bits from one part of the video RAM to another. If a particular window is 80 lines high by 80 characters wide $(6400$ characters, total$)$, and a character's box is 8 pixels wide by 16 pixels high, how long does it take to scroll the whole window at a copying rate of $50\ nsec$ per byte? If all lines are 80 characters long, what is the equivalent baud rate of the terminal? Putting a character on the screen takes $5\ \mu sec$. How many lines per second can be displayed?

tanenbaum operating-system input-output disk descriptive

Answer key

11.17.31 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 42 (Page No. 433) top

After receiving a DEL (SIGINT) character, the display driver discards all output currently queued for that display. Why?

tanenbaum operating-system input-output descriptive

11.17.32 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 43 (Page No. 433) top

A user at a terminal issues a command to an editor to delete the word on line 5 occupying character positions 7 through and including 12. Assuming the cursor is not on line 5 when the command is given, what ANSI escape sequence should the editor emit to delete the word?

tanenbaum operating-system input-output descriptive

Answer key

11.17.33 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 44 (Page No. 433) top

The designers of a computer system expected that the mouse could be moved at a maximum rate of $20\ cm/sec$. If a mickey is $0.1\ mm$ and each mouse message is 3 bytes, what is the maximum data rate of the mouse assuming that each mickey is reported separately?

tanenbaum operating-system input-output descriptive

Answer key

11.17.34 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 45 (Page No. 433) top

The primary additive colors are red, green, and blue, which means that any color can be constructed from a linear superposition of these colors. Is it possible that someone could have a color photograph that cannot be represented using full 24-bit color?

tanenbaum operating-system input-output descriptive

11.17.35 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 46 (Page No. 433) top

One way to place a character on a bitmapped screen is to use *BitBlt* from a font table. Assume that a particular font uses characters that are 16×24 pixels in true RGB color.

a. How much font table space does each character take?
b. If copying a byte takes $100\ nsec$, including overhead, what is the output rate to the screen in characters/sec?

tanenbaum operating-system input-output descriptive

Answer key

11.17.36 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 48 (Page No. 433) top

In Fig. 5-36 there is a class to *RegisterClass*. In the corresponding X Window code, in Fig. 5-34, there is no such call or anything like it. Why not?

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```
#include <X11/Xlib.h>
#include <X11/Xutil.h>

main(int argc, char *argv[])
{
      Display disp;                                  /* server identifier */
      Window win;                                    /* window identifier */
      GC gc;                                         /* graphic context identifier */
      XEvent event;                                  /* storage for one event */
      int running = 1;

      disp = XOpenDisplay("display_name");           /* connect to the X server */
      win = XCreateSimpleWindow(disp, ... ); /* allocate memory for new window */
      XSetStandardProperties(disp, ...);      /* announces window to window mgr */
      gc = XCreateGC(disp, win, 0, 0);        /* create graphic context */
      XSelectInput(disp, win, ButtonPressMask | KeyPressMask | ExposureMask);
      XMapRaised(disp, win);                  /* display window; send Expose event */

      while (running) {
            XNextEvent(disp, &event);                /* get next event */
            switch (event.type) {
                case Expose:       ...;   break;     /* repaint window */
                case ButtonPress:  ...;   break;     /* process mouse click */
                case Keypress:     ...;   break;     /* process keyboard input */
            }
      }

      XFreeGC(disp, gc);                      /* release graphic context */
      XDestroyWindow(disp, win);              /* deallocate window's memory space */
      XCloseDisplay(disp);                    /* tear down network connection */
}
```

Figure 5-34. A skeleton of an X Window application program.

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```
#include <windows.h>

int WINAPI WinMain(HINSTANCE h, HINSTANCE, hprev, char *szCmd, int iCmdShow)
{
      WNDCLASS wndclass;                    /* class object for this window */
      MSG msg;                              /* incoming messages are stored here */
      HWND hwnd;                            /* handle (pointer) to the window object */

      /* Initialize wndclass */
      wndclass.lpfnWndProc = WndProc;     /* tells which procedure to call */
      wndclass.lpszClassName = "Program name";     /* text for title bar */
      wndclass.hIcon = LoadIcon(NULL, IDI_APPLICATION);     /* load program icon */
      wndclass.hCursor = LoadCursor(NULL, IDC_ARROW);      /* load mouse cursor */

      RegisterClass(&wndclass);             /* tell Windows about wndclass */
      hwnd = CreateWindow ( ... )           /* allocate storage for the window */
      ShowWindow(hwnd, iCmdShow);           /* display the window on the screen */
      UpdateWindow(hwnd);                   /* tell the window to paint itself */

     while (GetMessage(&msg, NULL, 0, 0)) {          /* get message from queue */
           TranslateMessage(&msg);         /* translate the message */
           DispatchMessage(&msg);          /* send msg to the appropriate procedure */
     }
     return(msg.wParam);
}

long CALLBACK WndProc(HWND hwnd, UINT message, UINT wParam, long lParam)
{
     /* Declarations go here. */

     switch (message) {
           case WM_CREATE:     ... ;   return ... ;     /* create window */
           case WM_PAINT:      ... ;   return ... ;     /* repaint contents of window */
           case WM_DESTROY:    ... ;   return ... ;     /* destroy window */
     }
     return(DefWindowProc(hwnd, message, wParam, lParam)); /* default */
}
```

Figure 5-36. A skeleton of a Windows main program.

tanenbaum operating-system input-output descriptive

11.17.37 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 49 (Page No. 433 - 434) top

In the text we gave an example of how to draw a rectangle on the screen using the Windows GDI:

- *Rectangle(hdc, xleft, ytop, xright, ybottom);*

Is there any real need for the first parameter (hdc), and if so, what? After all, the coordinates of the rectangle are explicitly specified as parameters.

tanenbaum operating-system input-output descriptive

11.17.38 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 5 (Page No. 429) top

A DMA controller has five channels. The controller is capable of requesting a 32-bit word every 40 $nsec$. A response takes equally long. How fast does the bus have to be to avoid being a bottleneck?

tanenbaum operating-system input-output dma descriptive

Answer key

11.17.39 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 50 (Page No. 434) top

A thin-client terminal is used to display a Web page containing an animated cartoon of size 400 pixels $\times$ 160 pixels running at 10 frames/sec. What fraction of a 100-Mbps Fast Ethernet is consumed by displaying the cartoon?

tanenbaum operating-system input-output descriptive

Answer key

11.17.40 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 51 (Page No. 434) top

It has been observed that a thin-client system works well with a 1-Mbps network in a test. Are any problems likely in a multiuser situation? (Hint: Consider a large number of users watching a scheduled TV show and the same number of users browsing the World Wide Web.)

tanenbaum operating-system input-output descriptive

Answer key

11.17.41 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 52 (Page No. 434) top

Describe two advantages and two disadvantages of thin client computing?

tanenbaum operating-system input-output descriptive

Answer key

11.17.42 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 53 (Page No. 434) top

If a CPU's maximum voltage, V, is cut to V/n, its power consumption drops to $1/n^2$ of its original value and its clock speed drops to $1/n$ of its original value. Suppose that a user is typing at $1\ char/sec$, but the CPU time required to process each character is $100\ msec$. What is the optimal value of n and what is the corresponding energy saving in percent compared to not cutting the voltage? Assume that an idle CPU consumes no energy at all.

tanenbaum operating-system input-output descriptive

Answer key

11.17.43 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 54 (Page No. 434) top

A notebook computer is set up to take maximum advantage of power saving features including shutting down the display and the hard disk after periods of inactivity. A user sometimes runs UNIX programs in text mode, and at other times uses the X Window System. She is surprised to find that battery life is significantly better when she uses text-only programs. Why?

tanenbaum operating-system input-output unix descriptive

11.17.44 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 55 (Page No. 434) top

Write a program that simulates stable storage. Use two large fixed-length files on your disk to simulate the two disks.

tanenbaum operating-system input-output descriptive

11.17.45 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 56 (Page No. 434) top

Write a program to implement the three disk-arm scheduling algorithms. Write a driver program that generates a sequence of cylinder numbers $(0–999)$ at random, runs the three algorithms for this sequence and prints out the total distance (number of cylinders) the arm needs to traverse in the three algorithms.

tanenbaum operating-system input-output disk-scheduling descriptive

11.17.46 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 57 (Page No. 434) top

Write a program to implement multiple timers using a single clock. Input for this program consists of a sequence of four types of commands $(S < int >, T < int >, E < int >, P < int >) : S < int >$ sets the current time to $< int >; T$ is a clock tick; and $E < int >$ schedules a signal to occur at time $< int >; P$ prints out the values of Current time, Next signal, and Clock header. Your program should also print out a statement whenever it is time to raise a signal.

tanenbaum operating-system input-output descriptive

11.17.47 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 6 (Page No. 429 - 430) top

Suppose that a system uses DMA for data transfer from disk controller to main memory. Further assume that it takes $t_1\ nsec$ on average to acquire the bus and $t_2\ nsec$ to transfer one word over the bus $(t_1 >> t_2)$. After the CPU has programmed the DMA controller, how long will it take to transfer 1000 words from the disk controller to main memory, if

a. word-at-a-time mode is used,
b. burst mode is used?

Assume that commanding the disk controller requires acquiring the bus to send one word and acknowledging a transfer also requires acquiring the bus to send one word.

tanenbaum operating-system input-output dma descriptive

Answer key

11.17.48 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 7 (Page No. 430) top

One mode that some DMA controllers use is to have the device controller send the word to the DMA controller, which then issues a second bus request to write to memory. How can this mode be used to perform memory to memory copy? Discuss any advantage or disadvantage of using this method instead of using the CPU to perform memory to memory copy.

tanenbaum operating-system input-output dma descriptive

Answer key

11.17.49 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 8 (Page No. 430) top

Suppose that a computer can read or write a memory word in $5nsec.$ Also suppose that when an interrupt occurs, all 32 CPU registers, plus the program counter and PSW are pushed onto the stack. What is the maximum number of interrupts per second this machine can process?

tanenbaum operating-system input-output interrupts descriptive

11.17.50 Input Output: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 9 (Page No. 430) top

CPU architects know that operating system writers hate imprecise interrupts. One way to please the OS folks is for the CPU to stop issuing new instructions when an interrupt is signaled, but allow all the instructions currently being executed to finish, then force the interrupt. Does this approach have any disadvantages? Explain your answer.

tanenbaum operating-system input-output interrupts descriptive

11.18 Instruction Format (1) top

11.18.1 Instruction Format: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 6 (Page No. 81) top

Instructions related to accessing I/O devices are typically privileged instructions, that is, they can be executed in kernel mode but not in user mode. Give a reason why these instructions are privileged.

tanenbaum operating-system instruction-format descriptive

11.19 Interrupt Driven (1) top

11.19.1 Interrupt Driven: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 12 (Page No. 430) top

A typical printed page of text contains 50 lines of 80 characters each. Imagine that a certain printer can print 6 pages per minute and that the time to write a character to the printer's output register is so short it can be ignored. Does it make sense to run this printer using interrupt-driven I/O if each character printed requires an interrupt that takes 50 μsec all-in to service?

tanenbaum operating-system input-output interrupt-driven descriptive

11.20 Inverted Page Table (1) top

11.20.1 Inverted Page Table: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 25 (Page No. 256) top

A computer with an $8-KB$ page, a $256-KB$ main memory, and a $64-GB$ virtual address space uses an inverted page table to implement its virtual memory. How big should the hash table be to ensure a mean hash chain length of less than 1? Assume that the hash table size is a power of two.

tanenbaum operating-system memory-management virtual-memory inverted-page-table descriptive

11.21 Io System (17) top

11.21.1 Io System: Galvin Edition 9 Exercise 11 Question 13 (Page No. 540) top

Some systems automatically open a file when it is referenced for the first time and close the file when the job terminates. Discuss the advantages and disadvantages of this scheme compared with the more traditional one, where the user has to open and close the file explicitly

operating-system galvin io-system descriptive

11.21.2 Io System: Galvin Edition 9 Exercise 11 Question 14 (Page No. 540) top

If the operating system knew that a certain application was going to access file data in a sequential manner, how could it exploit this information to improve performance?

operating-system galvin io-system descriptive

Answer key

11.21.3 Io System: Galvin Edition 9 Exercise 13 Question 1 (Page No. 619) top

State three advantages of placing functionality in a device controller, rather than in the kernel. State three disadvantages.

galvin operating-system descriptive io-system

11.21.4 Io System: Galvin Edition 9 Exercise 13 Question 10 (Page No. 619) top

Consider the following I/O scenarios on a single-user PC:
a. A mouse used with a graphical user interface
b. A tape drive on a multitasking operating system (with no device preallocation available)
c. A disk drive containing user files
d. A graphics card with direct bus connection, accessible through memory-mapped I/O
For each of these scenarios, would you design the operating system to use buffering, spooling, caching, or a combination? Would you use polled I/O or interrupt-driven I/O? Give reasons for your choices.

operating-system galvin descriptive io-system

11.21.5 Io System: Galvin Edition 9 Exercise 13 Question 11 (Page No. 619) top

In most multiprogrammed systems, user programs access memory through virtual addresses, while the operating system uses raw physical addresses to access memory. What are the implications of this design for the initiation of I/O operations by the user program and their execution by the operating system?

operating-system galvin descriptive io-system

11.21.6 Io System: Galvin Edition 9 Exercise 13 Question 12 (Page No. 619) top

What are the various kinds of performance overhead associated with servicing an interrupt?

operating-system galvin descriptive io-system

11.21.7 Io System: Galvin Edition 9 Exercise 13 Question 14 (Page No. 620) top

Typically, at the completion of a device I/O, a single interrupt is raised and appropriately handled by the host processor. In certain settings, however, the code that is to be executed at the completion of the I/O can be broken into two separate pieces. The first piece executes immediately after the I/O completes and schedules a second interrupt for the remaining piece of code to be executed at a later time. What is the purpose of using this strategy in the design of interrupt handlers?

operating-system galvin descriptive io-system

11.21.8 Io System: Galvin Edition 9 Exercise 13 Question 15 (Page No. 620) top

Describe three circumstances under which blocking I/O should be used. Describe three circumstances under which nonblocking I/O should be used. Why not just implement nonblocking I/O and have processes busy-wait until their devices are ready?

galvin operating-system descriptive io-system

11.21.9 Io System: Galvin Edition 9 Exercise 13 Question 15 (Page No. 620) top

Some DMA controllers support direct virtual memory access, where the targets of I/O operations are specified as virtual addresses and a translation from virtual to physical address is performed during the DMA. How does this design complicate the design of the DMA controller? What are the advantages of providing such functionality?

operating-system galvin descriptive io-system

11.21.10 Io System: Galvin Edition 9 Exercise 13 Question 16 (Page No. 620) top

UNIX coordinates the activities of the kernel I/O components by manipulating shared in-kernel data structures, whereas Windows uses object-oriented message passing between kernel I/O components. Discuss three pros and three cons of each approach

operating-system galvin descriptive io-system

11.21.11 Io System: Galvin Edition 9 Exercise 13 Question 3 (Page No. 619) top

Why might a system use interrupt-driven I/O to manage a single serial port and polling I/O to manage a front-end processor, such as a terminal concentrator?

galvin operating-system descriptive io-system

11.21.12 Io System: Galvin Edition 9 Exercise 13 Question 4 (Page No. 619) top

Polling for an I/O completion can waste a large number of CPU cycles if the processor iterates a busy-waiting loop many times before the I/O completes. But if the I/O device is ready for service, polling can be much more efficient than is catching and dispatching an interrupt. Describe a hybrid strategy that combines polling, sleeping, and interrupts for I/O device service. For each of these three strategies (pure polling, pure interrupts, hybrid), describe a computing environment in which that strategy is more efficient than is either of the others.

operating-system galvin descriptive io-system

11.21.13 Io System: Galvin Edition 9 Exercise 13 Question 5 (Page No. 619) top

How does DMA increase system concurrency? How does it complicate hardware design?

operating-system galvin descriptive io-system

11.21.14 Io System: Galvin Edition 9 Exercise 13 Question 6 (Page No. 619) top

Why is it important to scale up system-bus and device speeds as CPU speed increases?

operating-system galvin io-system descriptive

11.21.15 Io System: Galvin Edition 9 Exercise 13 Question 7 (Page No. 619) top

Distinguish between a STREAMS driver and a STREAMS module.

operating-system galvin descriptive io-system

11.21.16 Io System: Galvin Edition 9 Exercise 13 Question 8 (Page No. 619) top

When multiple interrupts from different devices appear at about the same time, a priority scheme could be used to determine the order in which the interrupts would be serviced. Discuss what issues need to be considered in assigning priorities to different interrupts.

operating-system galvin descriptive io-system

11.21.17 Io System: Galvin Edition 9 Exercise 13 Question 9 (Page No. 619) top

What are the advantages and disadvantages of supporting memorymapped I/O to device control registers?

operating-system galvin descriptive io-system

11.22 Kernel Mode (1) top

11.22.1 Kernel Mode: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 12 (Page No. 82) top

Which of the following instructions should be allowed only in kernel mode?

a. Disable all interrupts.
b. Read the time-of-day clock.
c. Set the time-of-day clock.
d. Change the memory map

tanenbaum operating-system kernel-mode easy

11.23 Kernel User Mode (1) top

11.23.1 Kernel User Mode: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 10 (Page No. 81) top

What is the difference between kernel and user mode? Explain how having two distinct modes aids in designing an operating system.

tanenbaum operating-system kernel-user-mode descriptive

11.24 Least Recently Used (1) top

11.24.1 Least Recently Used: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 31 (Page No. 257) top

Give a simple example of a page reference sequence where the first page selected for replacement will be different for the clock and LRU page replacement algorithms. Assume that a process is allocated $3 = \text{three}$ frames, and the reference string contains page numbers from the set $0, 1, 2, 3$.

tanenbaum operating-system memory-management page-replacement least-recently-used descriptive

11.25 Memory (1) top

11.25.1 Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 7 (Page No. 333) top

In some systems it is possible to map part of a file into memory. What restrictions must such systems impose? How is this partial mapping implemented?

tanenbaum operating-system file-system memory descriptive

11.26 Memory Management (37) top

11.26.1 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 27 (Page No. 83) top

Modern operating systems decouple a process address space from the machine's physical memory. List two advantages of this design.

tanenbaum operating-system memory-management descriptive

11.26.2 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 1 (Page No. 254) top

The IBM 360 had a scheme of locking $2 - KB$ blocks by assigning each one a $4 - bit$ key and having the CPU compare the key on every memory reference to the $4 - bit$ key in the PSW. Name two drawbacks of this scheme not mentioned in the text.

tanenbaum operating-system memory-management descriptive

Answer key

11.26.3 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 10 (Page No. 255) top

Copy on write is an interesting idea used on server systems. Does it make any sense on a smartphone?

tanenbaum operating-system memory-management descriptive

11.26.4 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 2 (Page No. 254) top

In Fig. $3-3$ the base and limit registers contain the same value, $16,384$. Is this just an accident, or are they always the same? It is just an accident, why are they the same in this example?

188 MEMORY MANAGEMENT CHAP. 3

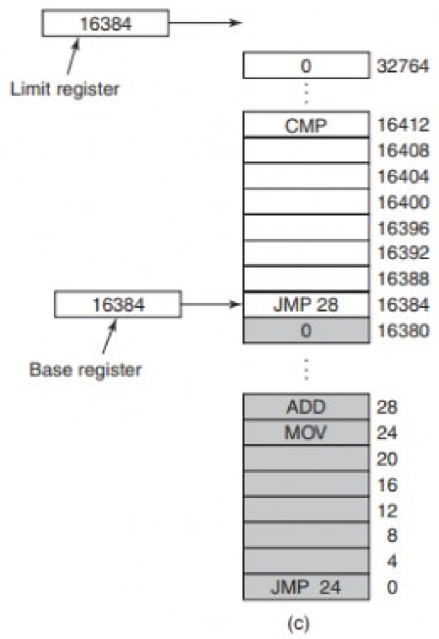


Figure 3-3. Base and limit registers can be used to give each process a separate address space.

tanenbaum operating-system memory-management descriptive

Answer key

11.26.5 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 3 (Page No. 254) top

A swapping system eliminates holes by compaction. Assuming a random distribution of many holes and many data segments and a time to read or write a $32-bit$ memory word of $4\,nsec$, about how long does it take to compact $4\,GB$? For simplicity, assume that word 0 is part of a hole and that the highest word in memory contains valid data.

tanenbaum operating-system memory-management descriptive

Answer key

11.26.6 Memory Management: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 4 (Page No. 254) top

Consider a swapping system in which memory consists of the following hole sizes in memory order: $10\,MB, 4\,MB, 20\,MB, 18\,MB, 7\,MB, 9\,MB, 12\,MB,$ and $15\,MB$. Which hole is taken for successive segment requests of

a. $12\,MB$
b. $10\,MB$
c. $9\,MB$

for first fit? Now repeat the question for best fit, worst fit, and next fit.

tanenbaum operating-system memory-management descriptive

Answer key

11.26.7 Memory Management: Galvin Edition 9 Exercise 8 Question 1 (Page No. 390) top

Name two differences between logical and physical addresses.

galvin operating-system memory-management descriptive

11.26.8 Memory Management: Galvin Edition 9 Exercise 8 Question 10 (Page No. 391) top

Consider the following process for generating binaries. A compiler is used to generate the object code for individual modules, and a linkage editor is used to combine multiple object modules into a single program binary. How does the linkage editor change the binding of instructions and data to memory addresses ? What information needs to be passed from the compiler to the linkage editor to facilitate the memory-binding tasks of the linkage editor ?

galvin operating-system memory-management descriptive

11.26.9 Memory Management: Galvin Edition 9 Exercise 8 Question 11 (Page No. 391) top

Given six memory partitions of $300\ KB$, $600\ KB$, $350\ KB$, $200\ KB$, $750KB$ and $125KB$ (in order), how would the $first-fit$, $best-fit$, and $worst-fit$ algorithms place processes of size $115\ KB$, $500\ KB$, $358KB$, $200\ KB$, and $375\ KB$ (in order) ? Rank the algorithms in terms of how efficiently they use memory.

galvin operating-system memory-management

Answer key

11.26.10 Memory Management: Galvin Edition 9 Exercise 8 Question 12 (Page No. 391) top

Most systems allow a program to allocate more memory to its address space during execution. Allocation of data in the heap segments of programs is an example of such allocated memory. What is required to support dynamic memory allocation in the following schemes ?

a. Contiguous memory allocation
b. Pure segmentation
c. Pure paging

galvin operating-system memory-management

Answer key

11.26.11 Memory Management: Galvin Edition 9 Exercise 8 Question 13 (Page No. 391) top

Compare the memory organization schemes of contiguous memory allocation, pure segmentation, and pure paging with respect to the following issues:

a. External fragmentation
b. Internal fragmentation
c. Ability to share code across processes

galvin operating-system memory-management

11.26.12 Memory Management: Galvin Edition 9 Exercise 8 Question 14 (Page No. 391) top

On a system with paging, a process cannot access memory that it does not own. Why ? How could the operating system allow access to other memory ? Why should it or should it not ?

galvin operating-system memory-management descriptive

11.26.13 Memory Management: Galvin Edition 9 Exercise 8 Question 15 (Page No. 392) top

Explain why mobile operating systems such as iOS and Android do not support swapping ?

galvin operating-system memory-management descriptive

11.26.14 Memory Management: Galvin Edition 9 Exercise 8 Question 16 (Page No. 392) top

Although Android does not support swapping on its boot disk, it is possible to set up a swap space using a separate SD nonvolatile memory card. Why would Android disallow swapping on its boot disk yet allow it on a secondary disk ?

galvin operating-system memory-management descriptive

11.26.15 Memory Management: Galvin Edition 9 Exercise 8 Question 17 (Page No. 392) top

Compare paging with segmentation with respect to how much memory the address translation structures require to convert virtual addresses to physical addresses.

galvin operating-system memory-management descriptive

11.26.16 Memory Management: Galvin Edition 9 Exercise 8 Question 18 (Page No. 392) top

Explain why address space identifiers ($ASIDs$) are used.

galvin operating-system memory-management descriptive

11.26.17 Memory Management: Galvin Edition 9 Exercise 8 Question 19 (Page No. 392) top

Program binaries in many systems are typically structured as follows. Code is stored starting with a small, fixed virtual address, such as 0. The code segment is followed by the data segment that is used for storing the program variables. When the program starts executing, the stack is allocated at the other end of the virtual address space and is allowed to grow toward lower virtual addresses. What is the significance of this structure for the following schemes ?

a. Contiguous memory allocation
b. Pure segmentation
c. Pure paging

galvin operating-system memory-management descriptive

11.26.18 Memory Management: Galvin Edition 9 Exercise 8 Question 2 (Page No. 390) top

Consider a system in which a program can be separated into two parts: code and data. The CPU knows whether it wants an instruction (instruction fetch) or data (data fetch or store). Therefore, two base–limit register pairs are provided: one for instructions and one for data.The instruction base–limit register pair is automatically read-only, so programs can be shared among different users. Discuss the advantages and disadvantages of this scheme.

galvin operating-system memory-management descriptive

11.26.19 Memory Management: Galvin Edition 9 Exercise 8 Question 20 (Page No. 392) top

Assuming a $1\ KB$ page size, what are the page numbers and offsets for the following address references (provided as decimal numbers):

$a.$ 3085
$b.$ 42095
$c.$ 215201
$d.$ 650000
$e.$ 2000001

galvin operating-system memory-management

11.26.20 Memory Management: Galvin Edition 9 Exercise 8 Question 21 (Page No. 392) top

The BTV operating system has a $21-bit$ virtual address, yet on certain embedded devices, it has only a $16-bit$ physical address. It also has a $2-KB$ page size. How many entries are there in each of the following ?

a. A conventional, single-level page table
b. An inverted page table

galvin operating-system memory-management

Answer key

11.26.21 Memory Management: Galvin Edition 9 Exercise 8 Question 22 (Page No. 392) top

What is the maximum amount of physical memory ?

galvin operating-system memory-management

11.26.22 Memory Management: Galvin Edition 9 Exercise 8 Question 23 (Page No. 392) top

Consider a logical address space of 256 pages with a $4-KB$ page size, mapped onto a physical memory of 64 frames.

a. How many bits are required in the logical address ?
b. How many bits are required in the physical address ?

galvin operating-system memory-management

Answer key

11.26.23 Memory Management: Galvin Edition 9 Exercise 8 Question 25 (Page No. 393) top

Consider a paging system with the page table stored in memory.

a. If a memory reference takes 50 nanoseconds, how long does a paged memory reference take ?

b. If we add $TLBs$, and 75 percent of all page-table references are found in the $TLBs$, what is the effective memory reference time ? (Assume that finding a page-table entry in the $TLBs$ takes 2 nanoseconds, if the entry is present.)

galvin operating-system memory-management

Answer key

11.26.24 Memory Management: Galvin Edition 9 Exercise 8 Question 26 (Page No. 393) top

Why are segmentation and paging sometimes combined into one scheme ?

galvin operating-system memory-management descriptive

11.26.25 Memory Management: Galvin Edition 9 Exercise 8 Question 27 (Page No. 393) top

Explain why sharing a reentrant module is easier when segmentation is used than when pure paging is used.

galvin operating-system memory-management descriptive

11.26.26 Memory Management: Galvin Edition 9 Exercise 8 Question 28 (Page No. 393) top

Consider the following segment table:

$Segment$	$Base$	$Length$
0	219	600
1	2300	14
2	90	100
3	1327	580
4	1952	96

What are the physical addresses for the following logical addresses ?
$a.\, 0, 430$
$b.\, 1, 10$
$c.\, 2, 500$
$d.\, 3, 400$
$e.\, 4, 112$

galvin operating-system memory-management

Answer key

11.26.27 Memory Management: Galvin Edition 9 Exercise 8 Question 29 (Page No. 393) top

What is the purpose of paging the page tables ?

galvin operating-system memory-management

Answer key

11.26.28 Memory Management: Galvin Edition 9 Exercise 8 Question 3 (Page No. 390) top

Why are page sizes always powers of 2 ?

galvin operating-system memory-management descriptive

11.26.29 Memory Management: Galvin Edition 9 Exercise 8 Question 30 (Page No. 393) top

Consider the hierarchical paging scheme used by the VAX architecture. How many memory operations are performed when a user program executes a memory-load operation ?

galvin operating-system memory-management

11.26.30 Memory Management: Galvin Edition 9 Exercise 8 Question 31 (Page No. 393) top

Compare the segmented paging scheme with the hashed page table scheme for handling large address spaces. Under what circumstances is one scheme preferable to the other ?

galvin operating-system memory-management descriptive

Answer key

11.26.31 Memory Management: Galvin Edition 9 Exercise 8 Question 32 (Page No. 393-394) top

Consider the Intel address-translation scheme shown in Figure 8.22.

a. Describe all the steps taken by the Intel Pentium in translating a logical address into a physical address.
b. What are the advantages to the operating system of hardware that provides such complicated memory translation ?

c. Are there any disadvantages to this address-translation system? If so, what are they? If not, why is this scheme not used by every manufacturer ?

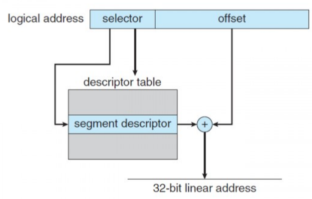


Figure 8.22 IA-32 segmentation.

galvin operating-system memory-management

11.26.32 Memory Management: Galvin Edition 9 Exercise 8 Question 4 (Page No. 390) top

Consider a logical address space of 64 pages of $1,024$ words each, mapped onto a physical memory of 32 frames.
a. How many bits are there in the logical address ?
b. How many bits are there in the physical address ?

galvin operating-system memory-management

Answer key

11.26.33 Memory Management: Galvin Edition 9 Exercise 8 Question 5 (Page No. 390) top

What is the effect of allowing two entries in a page table to point to the same page frame in memory? Explain how this effect could be used to decrease the amount of time needed to copy a large amount of memory from one place to another. What effect would updating some byte on the one page have on the other page ?

galvin operating-system memory-management descriptive

11.26.34 Memory Management: Galvin Edition 9 Exercise 8 Question 6 (Page No. 390) top

Describe a mechanism by which one segment could belong to the address space of two different processes.

galvin operating-system memory-management descriptive

11.26.35 Memory Management: Galvin Edition 9 Exercise 8 Question 7 (Page No. 390) top

Sharing segments among processes without requiring that they have the same segment number is possible in a dynamically linked segmentation system.

a. Define a system that allows static linking and sharing of segments without requiring that the segment numbers be the same.

b. Describe a paging scheme that allows pages to be shared without requiring that the page numbers be the same.

galvin operating-system memory-management descriptive

11.26.36 Memory Management: Galvin Edition 9 Exercise 8 Question 8 (Page No. 390-391) top

In the IBM/370, memory protection is provided through the use of keys. A key is a 4-bit quantity. Each 2-K block of memory has a key (the storage key) associated with it. The CPU also has a key (the protection key) associated with it.
A store operation is allowed only if both keys are equal or if either is 0. Which of the following memory-management schemes could be used successfully with this hardware?
a. Bare machine
b. Single-user system
c. Multiprogramming with a fixed number of processes
d. Multiprogramming with a variable number of processes
e. Paging
f.Segmentation

galvin operating-system memory-management descriptive

11.26.37 Memory Management: Galvin Edition 9 Exercise 8 Question 9 (Page No. 391) top

Explain the difference between internal and external fragmentation.

galvin operating-system memory-management descriptive

11.27 Memory Mapped (2) top

11.27.1 Memory Mapped: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 3 (Page No. 429) top

Figure $5-3(b)$ shows one way of having memory-mapped I/O even in the presence of separate buses for memory and I/O devices, namely, to first try the memory bus and if that fails try the I/O bus. A clever computer science student has thought of an improvement on this idea: try both in parallel, to speed up the process of accessing I/O devices. What do you think of this idea?

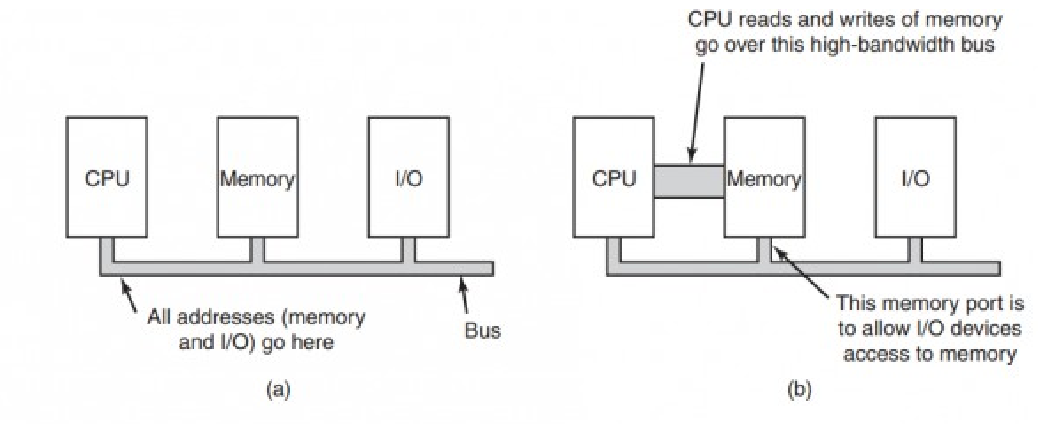


Figure 5-3. (a) A single-bus architecture. (b) A dual-bus memory architecture.

tanenbaum operating-system input-output memory-mapped descriptive

11.27.2 Memory Mapped: Andrew S. Tanenbaum (OS) Edition 4 Exercise 5 Question 47 (Page No. 433) top

Assuming that it takes $2\,nsec$ to copy a byte, how much time does it take to completely rewrite the screen of an $80\ \text{character} \times 25$ line text mode memory-mapped screen? What about a 1024×768 pixel graphics screen with 24-bit color?

tanenbaum operating-system input-output memory-mapped descriptive

Answer key

11.28 Monitors (1) top

11.28.1 Monitors: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 59 (Page No. 179) top

Solve the dining philosophers problem using monitors instead of semaphores.

tanenbaum operating-system process-and-threads semaphore monitors descriptive

11.29 Multi Programming (1) top

11.29.1 Multi Programming: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 5 (Page No. 81) top

On early computers, every byte of data read or written was handled by the CPU (i.e., there was no DMA). What implications does this have for multiprogramming?

tanenbaum operating-system multi-programming dma descriptive

Answer key

11.30 Multilevel Paging (1) top

11.30.1 Multilevel Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 19 (Page No. 256) top

A computer with a $32-bit$ address uses a two-level page table. Virtual addresses are split into a $9-bit$ top-level page table field, an $11-bit$ second-level page table field, and an offset. How large are the pages and how many are there in the address space?

tanenbaum operating-system memory-management multilevel-paging paging descriptive

Answer key

11.31 Multiplexing (1) top

11.31.1 Multiplexing: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 21 (Page No. 82) top

What type of multiplexing (time, space, or both) can be used for sharing the following resources: CPU, memory, disk, network card, printer, keyboard, and display?

tanenbaum operating-system multiplexing descriptive

11.32 Multiprocessors (2) top

11.32.1 Multiprocessors: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 28 (Page No. 176) top

When a computer is being developed, it is usually first simulated by a program that runs one instruction at a time. Even multiprocessors are simulated strictly sequentially like this. Is it possible for a race condition to occur when there are no simultaneous events like this?

tanenbaum operating-system process-and-threads multiprocessors descriptive

11.32.2 Multiprocessors: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 29 (Page No. 176) top

The producer-consumer problem can be extended to a system with multiple producers and consumers that write (or read) to (from) one shared buffer. Assume that each producer and consumer runs in its own thread. Will the solution presented in Fig. $2-28$, using semaphores, work for this system?

```
#define N 100                           /* number of slots in the buffer */
typedef int semaphore;                  /* semaphores are a special kind of int */
semaphore mutex = 1;                    /* controls access to critical region */
semaphore empty = N;                    /* counts empty buffer slots */
semaphore full = 0;                     /* counts full buffer slots */

void producer(void)
{
     int item;

     while (TRUE) {                     /* TRUE is the constant 1 */
          item = produce_item( );       /* generate something to put in buffer */
          down(&empty);                 /* decrement empty count */
          down(&mutex);                 /* enter critical region */
          insert_item(item);            /* put new item in buffer */
          up(&mutex);                   /* leave critical region */
          up(&full);                    /* increment count of full slots */
     }
}

void consumer(void)
{
     int item;

     while (TRUE) {                     /* infinite loop */
          down(&full);                  /* decrement full count */
          down(&mutex);                 /* enter critical region */
          item = remove_item( );        /* take item from buffer */
          up(&mutex);                   /* leave critical region */
          up(&empty);                   /* increment count of empty slots */
          consume_item(item);           /* do something with the item */
     }
}
```

Figure 2-28. The producer-consumer problem using semaphores.

tanenbaum operating-system process-and-threads multiprocessors descriptive

11.33 Multithreaded (2) top

11.33.1 Multithreaded: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 12 (Page No. 174) top

In Fig. $2-8$, a multithreaded Web server is shown. If the only way to read from a file is the normal blocking read system call, do you think user-level threads or kernel-level threads are being used for the Web server? Why?

tanenbaum operating-system process-and-threads multithreaded descriptive

11.33.2 Multithreaded: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 64 (Page No. 179 - 180) top

The objective of this exercise is to implement a multithreaded solution to find if a given number is a perfect number. N is a perfect number if the sum of all its factors, excluding itself, is N; examples are 6 and 28. The input is an integer, N. The output is true if the number is a perfect number and false otherwise. The main program will read the numbers N and P from the command line. The main process will spawn a set of P threads. The numbers from 1 to N will be partitioned among these threads so that two threads do not work on the name number. For each number in this set, the thread will determine if the number is a factor of N. If it is, it adds the number to a shared buffer that stores factors of N. The parent process waits till all the threads complete. Use the appropriate synchronization primitive here. The parent will then determine if the input number is perfect, that is, if N is a sum of all its factors and then report accordingly. (**Note:** You can make the computation faster by restricting the numbers searched from 1 to the square root of N.)

tanenbaum operating-system process-and-threads multithreaded semaphore descriptive

11.34 Mutual Exclusion (1) top

11.34.1 Mutual Exclusion: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 30 (Page No. 176) top

Consider the following solution to the mutual-exclusion problem involving two processes $P0$ and $P1$. Assume that the variable turn is initialized to 0. Process $P0's$ code is presented below.

```
/* Other code */

while (turn != 0) { } /* Do nothing and wait. */

Critical Section /* . . . */
```

```
tur n = 0;

/* Other code */
```

For process $P1$, replace 0 by 1 in above code. Determine if the solution meets all the required conditions for a correct mutual-exclusion solution.

tanenbaum operating-system process-and-threads mutual-exclusion descriptive

11.35 Page Fault (10) top

11.35.1 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 13 (Page No. 255) top

If an instruction takes $1 \, nsec$ and a page fault takes an additional $n \, nsec$, give a formula for the effective instruction time if page faults occur every k instructions.

tanenbaum operating-system memory-management paging page-fault descriptive

Answer key

11.35.2 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 39 (Page No. 259) top

You have been hired by a cloud computing company that deploys thousands of servers at each of its data centers. They have recently heard that it would be worthwhile to handle a page fault at server A by reading the page from the RAM memory of some other server rather than its local disk drive.

a. How could that be done?
b. Under what conditions would the approach be worthwhile? Be feasible?

tanenbaum operating-system memory-management paging page-fault descriptive

11.35.3 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 42 (Page No. 259) top

It has been observed that the number of instructions executed between page faults is directly proportional to the number of page frames allocated to a program. If the available memory is doubled, the mean interval between page faults is also doubled. Suppose that a normal instruction takes $1 \, microsec$, but if a page fault occurs, it takes $2001 \, \mu sec (i.e., 2 \, msec)$ to handle the fault. If a program takes $60 \, sec$ to run, during which time it gets $15,000$ page faults, how long would it take to run if twice as much memory were available?

tanenbaum operating-system memory-management paging page-fault descriptive

Answer key

11.35.4 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 44 (Page No. 259) top

A machine-language instruction to load a $32-bit$ word into a register contains the $32-bit$ address of the word to be loaded. What is the maximum number of page faults this instruction can cause?

tanenbaum operating-system memory-management paging page-fault descriptive

11.35.5 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 47 (Page No. 259 - 260) top

We consider a program which has the two segments shown below consisting of instructions in segment 0, and read/write data in segment 1. Segment 0 has read/execute protection, and segment 1 has just read/write protection. The memory system is a demand- paged virtual memory system with virtual addresses that have a $4-bit$ page number, and a $10-bit$ offset. The page tables and protection are as follows (all numbers in the table are in decimal):

Segment 0		Segment 1	
Read/Execute		Read/Write	
Virtual Page #	Page frame #	Virtual Page #	Page frame #
0	2	0	On Disk
1	On Disk	1	14
2	11	2	9
3	5	3	6
4	On Disk	4	On Disk
5	On Disk	5	13
6	4	6	8
7	3	7	12

For each of the following cases, either give the real (actual) memory address which results from dynamic address translation or identify the type of fault which occurs (either page or protection fault).

a. Fetch from segment 1, page 1, offset 3
b. Store into segment 0, page 0, offset 16
c. Fetch from segment 1, page 4, offset 28
d. Jump to location in segment 1, page 3, offset 32

tanenbaum operating-system memory-management paging page-fault descriptive

11.35.6 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 51 (Page No. 260) top

Write a program that simulates a paging system using the aging algorithm. The number of page frames is a parameter. The sequence of page references should be read from a file. For a given input file, plot the number of page faults per 1000 memory references as a function of the number of page frames available.

tanenbaum operating-system memory-management paging page-fault descriptive

11.35.7 Page Fault: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 52 (Page No. 260 - 261) top

Write a program that simulates a toy paging system that uses the WSClock algorithm. The system is a toy in that we will assume there are no write references (not very realistic), and process termination and creation are ignored (eternal life). The inputs will be:

- The reclamation age threshhold
- The clock interrupt interval expressed as number of memory references
- A file containing the sequence of page references

a. Describe the basic data structures and algorithms in your implementation.
b. Show that your simulation behaves as expected for a simple (but nontrivial) input example.
c. Plot the number of page faults and working set size per 1000 memory references.
d. Explain what is needed to extend the program to handle a page reference stream that also includes writes.

tanenbaum operating-system memory-management paging page-fault descriptive

11.35.8 Page Fault: Galvin Edition 9 Exercise 9 Question 19 (Page No. 453) top

Assume that we have a demand-paged memory. The page table is held in registers. It takes 8 milliseconds to service a page fault if an empty frame is available or if the replaced page is not modified and 20 milliseconds if the replaced page is modified. Memory-access time is 100 nanoseconds. Assume that the page to be replaced is modified 70 percent of the time. What is the maximum acceptable page-fault rate for an effective access time of no more than 200 nanoseconds ?

galvin operating-system virtual-memory page-fault

Answer key

11.35.9 Page Fault: Galvin Edition 9 Exercise 9 Question 20 (Page No. 453) top

When a page fault occurs, the process requesting the page must block while waiting for the page to be brought from disk into physical memory. Assume that there exists a process with five user-level threads and that the mapping of user threads to kernel threads is one to one. If one user thread incurs a page fault while accessing its stack, would the other user threads belonging to the same process also be affected by the page fault—that is, would they also have to wait for the faulting page to be brought into memory? Explain.

galvin operating-system virtual-memory page-fault descriptive

11.35.10 Page Fault: Galvin Edition 9 Exercise 9 Question 21 (Page No. 453) top

Consider the following page reference string:

$7, 2, 3, 1, 2, 5, 3, 4, 6, 7, 7, 1, 0, 5, 4, 6, 2, 3, 0, 1.$

Assuming demand paging with three frames, how many page faults would occur for the following replacement algorithms ?

- $LRU\ replacement$
- $FIFO\ replacement$
- $Optimal\, replacement$

galvin operating-system virtual-memory page-fault

Answer key

11.36 Page Replacement (14) top

11.36.1 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 26 (Page No. 256) top

A student in a compiler design course proposes to the professor a project of writing a compiler that will produce a list of page references that can be used to implement the optimal page replacement algorithm. Is this possible? Why or why not? Is there anything that could be done to improve paging efficiency at run time?

tanenbaum operating-system memory-management paging page-replacement descriptive

11.36.2 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 27 (Page No. 256 - 257) top

Suppose that the virtual page reference stream contains repetitions of long sequences of page references followed occasionally by a random page reference. For example, the sequence: $0, 1, \dots, 511, 431, 0, 1, \dots, 511, 332, 0, 1, \dots$ consists of repetitions of the sequence $0, 1, \dots, 511$ followed by a random reference to pages 431 and $332.$

a. Why will the standard replacement algorithms $(\text{LRU}, \text{FIFO}, \text{clock})$ not be effective in handling this workload for a page allocation that is less than the sequence length?
b. If this program were allocated 500 page frames, describe a page replacement approach that would perform much better than the LRU, FIFO, or clock algorithms.

tanenbaum operating-system memory-management page-replacement descriptive

11.36.3 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 28 (Page No. 257) top

If $FIFO$ page replacement is used with four page frames and eight pages, how many page faults will occur with the reference string 0172327103 if the four frames are initially empty? Now repeat this problem for $LRU.$

tanenbaum operating-system memory-management page-replacement descriptive

11.36.4 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 29 (Page No. 257) top

Consider the page sequence of Fig. $3-15(b)$. Suppose that the R bits for the pages B through A are 11011011, respectively. Which page will second chance remove?

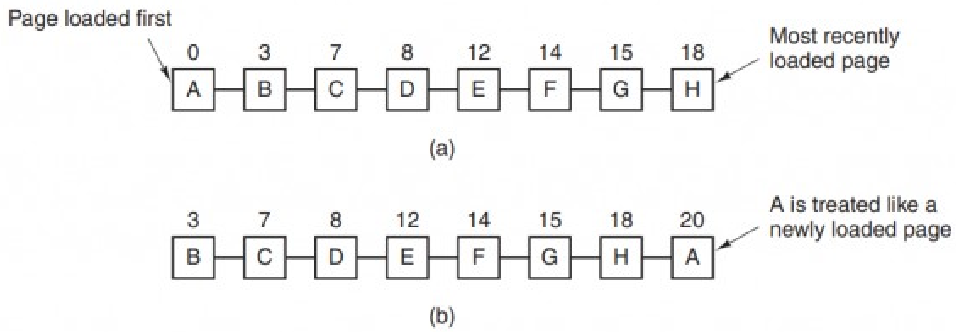


Figure 3-15. Operation of second chance. (a) Pages sorted in FIFO order. (b) Page list if a page fault occurs at time 20 and A has its R bit set. The numbers above the pages are their load times.

tanenbaum operating-system memory-management page-replacement descriptive

11.36.5 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 30 (Page No. 257) top

A small computer on a smart card has four page frames. At the first clock tick, the R bits are 0111 (page 0 is 0, the rest are 1). At subsequent clock ticks, the values are $1011, 1010, 1101, 0010, 1010, 1100,$ and 0001. If the aging algorithm is used with an $8-bit$ counter, give the values of the four counters after the last tick.

tanenbaum operating-system memory-management page-replacement descriptive

11.36.6 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 32 (Page No. 257) top

In the WSClock algorithm of Fig. $3-20(c)$, the hand points to a page with $R=0$. If $\tau=400$, will this page be removed? What about if $\tau=1000$?

220 MEMORY MANAGEMENT CHAP. 3

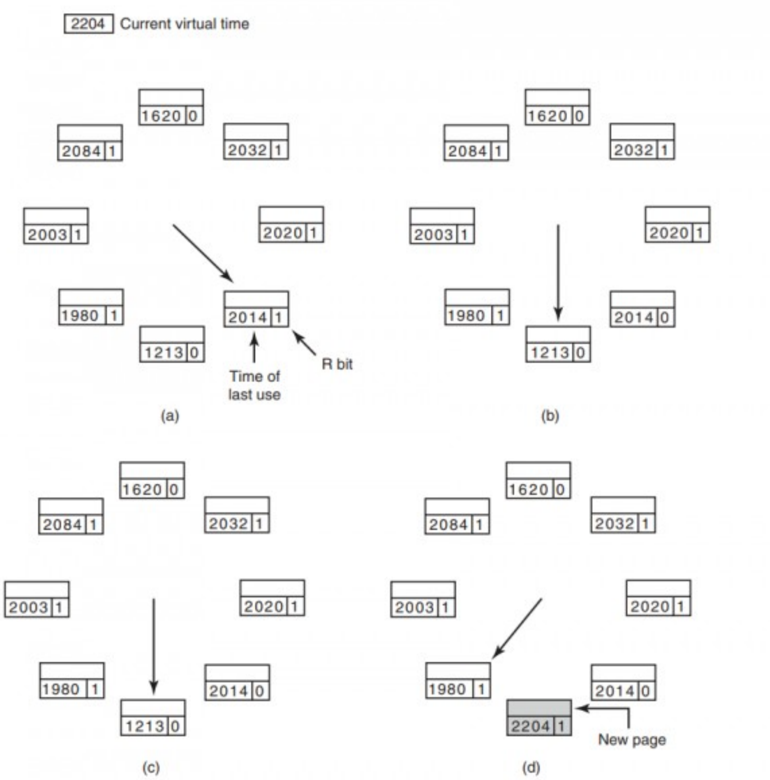


Figure 3-20. Operation of the WSClock algorithm. (a) and (b) give an example of what happens when $R = 1$. (c) and (d) give an example of $R = 0$.

tanenbaum operating-system memory-management page-replacement descriptive

Answer key

11.36.7 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 33 (Page No. 257) top

Suppose that the WSClock page replacement algorithm uses a τ of two ticks, and the system state is the following:

Page	Time stamp	V	R	M
0	6	1	0	1
1	9	1	1	0
2	9	1	1	1
3	7	1	0	0
4	4	0	0	0

where the three flag bits V, R, and M stand for Valid, Referenced, and Modified, respectively.

a. If a clock interrupt occurs at tick 10, show the contents of the new table entries. Explain. (You can omit entries that are unchanged.)
b. Suppose that instead of a clock interrupt, a page fault occurs at tick 10 due to a read request to page 4. Show the contents of the new table entries. Explain. (You can omit entries that are unchanged.)

tanenbaum operating-system memory-management page-replacement descriptive

11.36.8 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 34 (Page No. 257) top

A student has claimed that "in the abstract, the basic page replacement algorithms (FIFO, LRU, optimal) are identical except for the attribute used for selecting the page to be replaced.''

a. What is that attribute for the FIFO algorithm? LRU algorithm? Optimal algorithm?
b. Give the generic algorithm for these page replacement algorithms.

tanenbaum operating-system memory-management page-replacement descriptive

11.36.9 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 35 (Page No. 258) top

How long does it take to load a $64-KB$ program from a disk whose average seek time is $5\ msec$, whose rotation time is $5msec$, and whose tracks hold $1\ MB$

a. for a $2-KB$ page size?
b. for a $4-KB$ page size?

The pages are spread randomly around the disk and the number of cylinders is so large that the chance of two pages being on the same cylinder is negligible.

tanenbaum operating-system memory-management paging page-replacement descriptive

11.36.10 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 36 (Page No. 258) top

A computer has four page frames. The time of loading, time of last access, and the R and M bits for each page are as shown below (the times are in clock ticks):

Page	Loaded	Last ref.	R	M
0	126	280	1	0
1	230	265	0	1
2	140	270	0	0
3	110	285	1	1

a. Which page will NRU replace?
b. Which page will FIFO replace?
c. Which page will LRU replace?
d. Which page will second chance replace?

tanenbaum operating-system memory-management page-replacement descriptive

Answer key

11.36.11 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 37 (Page No. 258) top

Suppose that two processes A and B share a page that is not in memory. If process A faults on the shared page, the page table entry for process A must be updated once the page is read into memory.

a. Under what conditions should the page table update for process B be delayed even though the handling of process $A's$ page fault will bring the shared page into memory? Explain.
b. What is the potential cost of delaying the page table update?

tanenbaum operating-system memory-management page-replacement descriptive

11.36.12 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 38 (Page No. 258) top

Consider the following two-dimensional array:

```
int X[64][64];
```

Suppose that a system has four page frames and each frame is 128 words (an integer occupies one word). Programs that manipulate the X array fit into exactly one page and always occupy page 0. The data are swapped in and out of the other three frames. The X array is stored in row-major order $(i.e., X[0][1]$ follows $X[0][0]$ in memory$)$. Which of the two code fragments shown below will generate the lowest number of page faults? Explain and compute the total number of page faults.

```
Fragment A
for (int j = 0; j < 64; j++)
     for (int i = 0; i < 64; i++) X[i][j] = 0;

Fragment B
for (int i = 0; i < 64; i++)
     for (int j = 0; j < 64; j++) X[i][j] = 0;
```

tanenbaum operating-system memory-management page-replacement descriptive

Answer key

11.36.13 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 40 (Page No. 259) top

One of the first timesharing machines, the $DEC\ PDP-1$, had a (core) memory of $4K$ $18-bit$ words. It held one process at a time in its memory. When the scheduler decided to run another process, the process in memory was written to a paging drum, with $4K$ $18-bit$ words around the circumference of the drum. The drum could start writing (or reading) at any word, rather than only at word 0. Why do you suppose this drum was chosen?

tanenbaum operating-system memory-management paging page-replacement descriptive

11.36.14 Page Replacement: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 54 (Page No. 261) top

Write a program that will demonstrate the difference between using a local page replacement policy and a global one for the simple case of two processes. You will need a routine that can generate a page reference string based on a statistical model. This model has N states numbered from 0 to $N-1$ representing each of the possible page references and a probability p_i associated with each state i representing the chance that the next reference is to the same page. Otherwise, the next page reference will be one of the other pages with equal probability.

a. Demonstrate that the page reference string-generation routine behaves properly for some small N.
b. Compute the page fault rate for a small example in which there is one process and a fixed number of page frames. Explain why the behavior is correct.
c. Repeat part (b) with two processes with independent page reference sequences and twice as many page frames as in part (b).
d. Repeat part (c) but using a global policy instead of a local one. Also, contrast the per-process page fault rate with that of the local policy approach.

tanenbaum operating-system memory-management page-replacement descriptive

11.37 Paging (14) top

11.37.1 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 14 (Page No. 255) top

A machine has a $32-bit$ address space and an $8-KB$ page. The page table is entirely in hardware, with one

$32 - bit$ word per entry. When a process starts, the page table is copied to the hardware from memory, at one word every $100\,nsec$. If each process runs for $100\,msec$ (including the time to load the page table), what fraction of the CPU time is devoted to loading the page tables?

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.2 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 15 (Page No. 255) top

Suppose that a machine has $48 - bit$ virtual addresses and $32 - bit$ physical addresses.

a. If pages are $4\,KB$, how many entries are in the page table if it has only a single level? Explain.
b. Suppose this same system has a TLB (Translation Lookaside Buffer) with 32 entries. Furthermore, suppose that a program contains instructions that fit into one page and it sequentially reads long integer elements from an array that spans thousands of pages. How effective will the TLB be for this case?

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.3 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 17 (Page No. 255) top

Suppose that a machine has $438 - bit$ virtual addresses and $32 - bit$ physical addresses.

a. What is the main advantage of a multilevel page table over a single-level one?
b. With a two-level page table, $16 - KB$ pages, and $4 - byte$ entries, how many bits should be allocated for the top-level page table field and how many for the next level page table field? Explain.

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.4 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 18 (Page No. 256) top

Section $3.3.4$ states that the Pentium Pro extended each entry in the page table hierarchy to 64 bits but still could only address only $4\,GB$ of memory. Explain how this statement can be true when page table entries have 64 bits.

tanenbaum operating-system memory-management paging descriptive

11.37.5 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 20 (Page No. 256) top

A computer has $32 - bit$ virtual addresses and $4 - KB$ pages. The program and data together fit in the lowest page $(0\text{–}4095)$ The stack fits in the highest page. How many entries are needed in the page table if traditional (one-level) paging is used? How many page table entries are needed for two-level paging, with 10 bits in each part?

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.6 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 21 (Page No. 256) top

Below is an execution trace of a program fragment for a computer with $512 - byte$ pages. The program is located at address 1020, and its stack pointer is at 8192 (the stack grows toward 0). Give the page reference string generated by this program. Each instruction occupies 4 bytes (1word) including immediate constants. Both instruction and data references count in the reference string.

- Load word 6144 into register 0
- Push register 0 onto the stack
- Call a procedure at 5120, stacking the return address
- Subtract the immediate constant 16 from the stack pointer
- Compare the actual parameter to the immediate constant 4
- Jump if equal to 5152

tanenbaum operating-system memory-management paging descriptive

11.37.7 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 24 (Page No. 256) top

A machine has $48-bit$ virtual addresses and $32-bit$ physical addresses. Pages are $8\ KB$. How many entries are needed for a single-level linear page table?

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.8 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 41 (Page No. 259) top

A computer provides each process with $65,536$ bytes of address space divided into pages of 4096 bytes each. A particular program has a text size of $32,768$ bytes, a data size of $16,386$ bytes, and a stack size of $15,870$ bytes. Will this program fit in the machine's address space? Suppose that instead of 4096 bytes, the page size were 512 bytes, would it then fit? Each page must contain either text, data, or stack, not a mixture of two or three of them.

tanenbaum operating-system memory-management paging descriptive

Answer key

11.37.9 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 43 (Page No. 259) top

A group of operating system designers for the Frugal Computer Company are thinking about ways to reduce the amount of backing store needed in their new operating system. The head guru has just suggested not bothering to save the program text in the swap area at all, but just page it in directly from the binary file whenever it is needed. Under what conditions, if any, does this idea work for the program text? Under what conditions, if any, does it work for the data?

tanenbaum operating-system memory-management paging descriptive

11.37.10 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 48 (Page No. 260) top

Can you think of any situations where supporting virtual memory would be a bad idea, and what would be gained by not having to support virtual memory? Explain.

tanenbaum operating-system memory-management virtual-memory paging descriptive

Answer key

11.37.11 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 49 (Page No. 260) top

Virtual memory provides a mechanism for isolating one process from another. What memory management difficulties would be involved in allowing two operating systems to run concurrently? How might these difficulties be addressed?

tanenbaum operating-system memory-management paging virtual-memory descriptive

11.37.12 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 50 (Page No. 260) top

Plot a histogram and calculate the mean and median of the sizes of executable binary files on a computer to which you have access. On a Windows system, look at all .exe and .dll files; on a UNIX system look at all executable files in /bin, /usr/bin, and /local/bin that are not scripts (or use the file utility to find all executables). Determine the optimal page size for this computer just considering the code (not data). Consider internal fragmentation and page table size, making some reasonable assumption about the size of a page table entry. Assume that all programs are equally likely to be run and thus should be weighted equally.

tanenbaum operating-system memory-management paging descriptive

11.37.13 Paging: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 9 (Page No. 255) top

What kind of hardware support is needed for a paged virtual memory to work?

tanenbaum operating-system memory-management virtual-memory paging descriptive

Answer key

11.37.14 Paging: Galvin Edition 9 Exercise 9 Question 38 (Page No. 456) top

Consider a system that allocates pages of different sizes to its processes. What are the advantages of such a paging scheme ? What modifications to the virtual memory system provide this functionality ?

galvin operating-system memory-management paging

11.38 Preemptable Nonpreemptable (1) top

11.38.1 Preemptable Nonpreemptable: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 3 (Page No. 465) top

In the preceding question, which resources are preemptable and which are nonpreemptable?

tanenbaum operating-system deadlock-prevention-avoidance-detection preemptable-nonpreemptable descriptive

11.39 Process (18) top

11.39.1 Process: Galvin Edition 9 Exercise 3 Question 1 (Page No. 149) top

```c
#include <sys/types.h>
#include <stdio.h>
#include <unistd.h>
int value = 5;
int main()
{
    pid t pid;
    pid = fork();
    if (pid == 0) { /* child process */
        value += 15;
        return 0;
    }
    else if (pid > 0) { /* parent process */
        wait(NULL);
        printf("PARENT: value = %d",value); /* LINE A */
        return 0;
    }
}
```

Explain what the output will be at LINE A in this program.

galvin operating-system process programming fork-system-call

11.39.2 Process: Galvin Edition 9 Exercise 3 Question 10 (Page No. 151-152) top

Construct a process tree similar to Figure 3.8. To obtain process information for the UNIX or Linux system, use the command ps -ael.Use the command man ps to get more information about the ps command.The task manager on Windows systems does not provide the
parent process ID, but the process monitor tool, available from technet.microsoft.com, provides a process-tree tool.

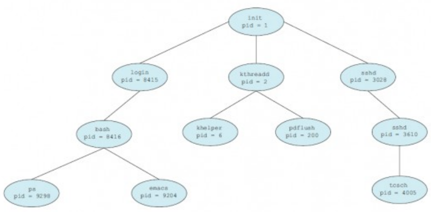


Figure 3.8 A tree of processes on a typical Linux system.

galvin operating-system process descriptive

11.39.3 Process: Galvin Edition 9 Exercise 3 Question 11 (Page No. 152) top

Explain the role of the init process on UNIX and Linux systems in regard to process termination.

galvin operating-system process descriptive

11.39.4 Process: Galvin Edition 9 Exercise 3 Question 12 (Page No. 152) top

Including the initial parent process, how many processes are created by the program shown below-

#include <stdio.h>
#include <unistd.h>
int main()
{
int i;

```
for (i = 0; i < 4; i++)
fork();
return 0;
}
```

galvin operating-system process programming

Answer key

11.39.5 Process: Galvin Edition 9 Exercise 3 Question 13 (Page No. 152) top

Explain the circumstances under which which the line of code marked printf("LINE J") in following program will be reached.

```
#include <sys/types.h>
#include <stdio.h>
#include <unistd.h>
int main()
{
pid t pid;
/* fork a child process */
pid = fork();
if (pid < 0) { /* error occurred */
fprintf(stderr, "Fork Failed");
return 1;
}
else if (pid == 0) { /* child process */
execlp("/bin/ls","ls",NULL);
printf("LINE J");
}
else { /* parent process */
/* parent will wait for the child to complete */
wait(NULL);
printf("Child Complete");
}
return 0;
}
```

galvin operating-system process programming

Answer key

11.39.6 Process: Galvin Edition 9 Exercise 3 Question 14 (Page No. 152) top

Using the program in Figure 3.34, identify the values of pid at lines A, B, C, and D. (Assume that the actual pids of the parent and child are 2600 and 2603, respectively.)

```
#include <sys/types.h>
#include <stdio.h>
#include <unistd.h>
int main()
{
pid t pid, pid1;
/* fork a child process */
pid = fork();
if (pid < 0) { /* error occurred */
fprintf(stderr, "Fork Failed");
return 1;
}
else if (pid == 0) { /* child process */
pid1 = getpid();
printf("child: pid = %d",pid); /* A */
printf("child: pid1 = %d",pid1); /* B */
}
else { /* parent process */
pid1 = getpid();
```

```
printf("parent: pid = %d",pid); /* C */
printf("parent: pid1 = %d",pid1); /* D */
wait(NULL);
}
return 0;
}
```

galvin operating-system process programming

11.39.7 Process: Galvin Edition 9 Exercise 3 Question 15 (Page No. 153)

top

Give an example of a situation in which ordinary pipes are more suitable than named pipes and an example of a situation in which named pipes are more suitable than ordinary pipes.

galvin operating-system process descriptive

11.39.8 Process: Galvin Edition 9 Exercise 3 Question 16 (Page No. 153)

top

Consider the RPC mechanism. Describe the undesirable consequences that could arise from not enforcing either the “at most once” or “exactly once” semantic. Describe possible uses for a mechanism that has neither of these guarantees.

galvin operating-system process descriptive

11.39.9 Process: Galvin Edition 9 Exercise 3 Question 17 (Page No. 153)

top

Using the program shown below, explain what the output will be at lines X and Y.

```
#include <sys/types.h>
#include <stdio.h>
#include <unistd.h>
#define SIZE 5
int nums[SIZE] = {0,1,2,3,4};
int main()
{
int i;
pid t pid;
pid = fork();
if (pid == 0) {
for (i = 0; i < SIZE; i++) {
nums[i] *= -i;
printf("CHILD: %d ",nums[i]); /* LINE X */
}
}
else if (pid > 0) {
wait(NULL);
for (i = 0; i < SIZE; i++)
printf("PARENT: %d ",nums[i]); /* LINE Y */
}
return 0;
}
```

galvin operating-system process programming

11.39.10 Process: Galvin Edition 9 Exercise 3 Question 18 (Page No. 153)

top

What are the benefits and the disadvantages of each of the following ? Consider both the system level and the programmer level.

a. Synchronous and asynchronous communication
b. Automatic and explicit buffering
c. Send by copy and send by reference
d. Fixed-sized and variable-sized messages

galvin operating-system process descriptive

11.39.11 Process: Galvin Edition 9 Exercise 3 Question 2 (Page No. 149-150) top

Including the initial parent process, how many processes are created by the following program.

```
#include <stdio.h>
#include <unistd.h>
int main()
{
fork();
fork();
fork();
return 0;
}
```

galvin operating-system process programming

Answer key

11.39.12 Process: Galvin Edition 9 Exercise 3 Question 3 (Page No. 150) top

Original versions of Apple's mobile iOS operating system provided no means of concurrent processing. Discuss three major complications that concurrent processing adds to an operating system.

galvin operating-system process descriptive

11.39.13 Process: Galvin Edition 9 Exercise 3 Question 4 (Page No. 150) top

The Sun UltraSPARC processor has multiple register sets. Describe what happens when a context switch occurs if the new context is already loaded into one of the register sets. What happens if the new context is in memory rather than in a register set and all the register sets are in use ?

galvin operating-system process descriptive

11.39.14 Process: Galvin Edition 9 Exercise 3 Question 5 (Page No. 150) top

When a process creates a new process using the fork() operation, which of the following states is shared between the parent process and the child process ?
a. Stack
b. Heap
c. Shared memory segments

galvin operating-system process descriptive

11.39.15 Process: Galvin Edition 9 Exercise 3 Question 6 (Page No. 150) top

Consider the "exactly once"semantic with respect to the RPC mechanism. Does the algorithm for implementing this semantic execute correctly even if the ACK message sent back to the client is lost due to a network problem? Describe the sequence of messages, and discuss whether "exactly once" is still preserved.

galvin operating-system process descriptive

11.39.16 Process: Galvin Edition 9 Exercise 3 Question 7 (Page No. 150) top

Assume that a distributed system is susceptible to server failure. What mechanisms would be required to guarantee the "exactly once" semantic for execution of RPCs?

galvin operating-system process descriptive

11.39.17 Process: Galvin Edition 9 Exercise 3 Question 8 (Page No. 151) top

Describe the differences among short-term, medium-term, and long term scheduling.

galvin operating-system process descriptive

Answer key

11.39.18 Process: Galvin Edition 9 Exercise 3 Question 9 (Page No. 151) top

Describe the actions taken by a kernel to context-switch between processes.

galvin operating-system process descriptive

Answer key

11.40 Process And Threads (54) top

11.40.1 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 1 (Page No. 174) top

In Fig. $2-2$, three process states are shown. In theory, with three states, there could be six transitions, two out of each state. However, only four transitions are shown. Are there any circumstances in which either or both of the missing transitions might occur?

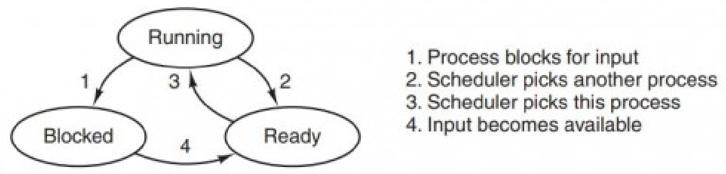


Figure 2-2. A process can be in running, blocked, or ready state. Transitions between these states are as shown.

tanenbaum operating-system process-and-threads descriptive

11.40.2 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 10 (Page No. 174) top

In the text it was stated that the model of Fig. $2-11(a)$ was not suited to a file server using a cache in memory. Why not? Could each process have its own cache?

tanenbaum operating-system process-and-threads cache-memory descriptive

11.40.3 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 11 (Page No. 174) top

If a multithreaded process forks, a problem occurs if the child gets copies of all the parent's threads. Suppose that one of the original threads was waiting for keyboard input. Now two threads are waiting for keyboard input, one in each process. Does this problem ever occur in single-threaded processes?

tanenbaum operating-system process-and-threads threads descriptive

Answer key

11.40.4 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 13 (Page No. 174) top

In the text, we described a multithreaded Web server, showing why it is better than a single-threaded server and a finite-state machine server. Are there any circumstances in which a single-threaded server might be better? Give an example.

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.5 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 14 (Page No. 175) top

In Fig. $2-12$ the register set is listed as a per-thread rather than a per-process item. Why? After all, the machine has only one set of registers.

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.6 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 15 (Page No. 175) top

Why would a thread ever voluntarily give up the CPU by calling thread yield? After all, since there is no periodic clock interrupt, it may never get the CPU back.

tanenbaum operating-system process-and-threads interrupts descriptive

Answer key

11.40.7 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 16 (Page No. 175) top

Can a thread ever be preempted by a clock interrupt? If so, under what circumstances? If not, why not?

tanenbaum operating-system process-and-threads interrupts descriptive

Answer key

11.40.8 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 17 (Page No. 175) top

In this problem, you are to compare reading a file using a single-threaded file server and a multithreaded server. It takes 12 msec to get a request for work, dispatch it, and do the rest of the necessary processing, assuming that the data needed are in the block cache. If a disk operation is needed, as is the case one-third of the time, an additional 75 msec is required, during which time the thread sleeps. How many requests/sec can the server handle if it is single threaded? If it is multithreaded?

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.9 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 18 (Page No. 175) top

What is the biggest advantage of implementing threads in user space? What is the biggest disadvantage?

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.10 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 19 (Page No. 175) top

In Fig. $2-15$ the thread creations and messages printed by the threads are interleaved at random. Is there a way to force the order to be strictly thread 1 created, thread 1 prints message, thread 1 exits, thread 2 created, thread 2 prints message, thread 2 exists, and so on? If so, how? If not, why not?

tanenbaum operating-system process-and-threads descriptive

11.40.11 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 2 (Page No. 174) top

Suppose that you were to design an advanced computer architecture that did process switching in hardware, instead of having interrupts. What information would the CPU need? Describe how the hardware process switching might work.

tanenbaum operating-system process-and-threads interrupts descriptive

Answer key

11.40.12 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 20 (Page No. 175) top

In the discussion on global variables in threads, we used a procedure create global to allocate storage for a pointer to the variable, rather than the variable itself. Is this essential, or could the procedures work with the values themselves just as well?

tanenbaum operating-system process-and-threads descriptive

11.40.13 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 21 (Page No. 175) top

Consider a system in which threads are implemented entirely in user space, with the run-time system getting a clock interrupt once a second. Suppose that a clock interrupt occurs while some thread is executing in the run-time system. What problem might occur? Can you suggest a way to solve it?

tanenbaum operating-system process-and-threads interrupts descriptive

Answer key

11.40.14 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 22 (Page No. 175) top

Suppose that an operating system does not have anything like the select system call to see in advance if it is safe to read from a file, pipe, or device, but it does allow alarm clocks to be set that interrupt blocked system calls. Is it possible to implement a threads package in user space under these conditions? Discuss.

tanenbaum operating-system process-and-threads interrupts descriptive

11.40.15 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 23 (Page No. 175) top

Does the busy waiting solution using the turn variable (Fig. $2 - 23$) work when the two processes are running on a shared-memory multiprocessor, that is, two CPUs sharing a common memory?

tanenbaum operating-system process-and-threads descriptive

11.40.16 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 24 (Page No. 175) top

Does Peterson's solution to the mutual-exclusion problem shown in Fig. $2 - 24$ work when process scheduling is preemptive? How about when it is nonpreemptive?

```
#define FALSE  0
#define TRUE   1
#define N      2                        /* number of processes */

int turn;                               /* whose turn is it? */
int interested[N];                      /* all values initially 0 (FALSE) */

void enter_region(int process);         /* process is 0 or 1 */
{
     int other;                         /* number of the other process */

     other = 1 – process;               /* the opposite of process */
     interested[process] = TRUE;        /* show that you are interested */
     turn = process;                    /* set flag */
     while (turn == process && interested[other] == TRUE)  /* null statement */ ;
}

void leave_region(int process)          /* process: who is leaving */
{
     interested[process] = FALSE;       /* indicate departure from critical region */
}
```

Figure 2-24. Peterson's solution for achieving mutual exclusion.

tanenbaum operating-system process-and-threads process-scheduling descriptive

11.40.17 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 25 (Page No. 175) top

Can the priority inversion problem discussed in Sec. $2.3.4$ happen with user-level threads? Why or why not?

tanenbaum operating-system process-and-threads descriptive

11.40.18 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 26 (Page No. 175) top

In Sec. $2.3.4$, a situation with a high-priority process, H, and a low-priority process, L, was described, which led to H looping forever. Does the same problem occur if round-robin scheduling is used instead of priority scheduling? Discuss.

tanenbaum operating-system process-and-threads process-scheduling descriptive

11.40.19 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 27 (Page No. 175) top

In a system with threads, is there one stack per thread or one stack per process when user-level threads are used? What about when kernel-level threads are used? Explain.

tanenbaum operating-system process-and-threads threads descriptive

Answer key

11.40.20 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 3 (Page No. 174) top

On all current computers, at least part of the interrupt handlers are written in assembly language. Why?

tanenbaum operating-system process-and-threads interrupts descriptive

Answer key

11.40.21 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 31 (Page No. 176) top

How could an operating system that can disable interrupts implement semaphores?

tanenbaum operating-system process-and-threads interrupts semaphore descriptive

Answer key

11.40.22 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 32 (Page No. 176) top

Show how counting semaphores (i.e., semaphores that can hold an arbitrary value) can be implemented using only binary semaphores and ordinary machine instructions.

tanenbaum operating-system process-and-threads machine-instructions descriptive

11.40.23 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 33 (Page No. 176) top

If a system has only two processes, does it make sense to use a barrier to synchronize them? Why or why not?

tanenbaum operating-system process-and-threads process-synchronization descriptive

Answer key

11.40.24 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 34 (Page No. 176) top

Can two threads in the same process synchronize using a kernel semaphore if the threads are implemented by the kernel? What if they are implemented in user space? Assume that no threads in any other processes have access to the semaphore. Discuss your answers.

tanenbaum operating-system process-and-threads semaphore descriptive

11.40.25 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 35 (Page No. 176) top

Synchronization within monitors uses condition variables and two special operations, wait and signal. A more general form of synchronization would be to have a single primitive, waituntil, that had an arbitrary Boolean predicate as parameter. Thus, one could say, for example,

$$\text{waituntil}\, x < 0 \text{ or } y + z < n$$

The signal primitive would no longer be needed. This scheme is clearly more general than that of Hoare or Brinch Hansen, but it is not used. Why not? (Hint: Think about the implementation.)

tanenbaum operating-system process-and-threads process-synchronization semaphore descriptive

Answer key

11.40.26 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 36 (Page No. 176) top

A fast-food restaurant has four kinds of employees: (1) order takers, who take customers' orders; (2) cooks, who prepare the food; (3) packaging specialists, who stuff the food into bags; and (4) cashiers, who give the bags to customers and take their money. Each employee can be regarded as a communicating sequential process. What form of interprocess communication do they use? Relate this model to processes in UNIX.

tanenbaum operating-system process-and-threads descriptive

11.40.27 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 38 (Page No. 177) top

The CDC 6600 computers could handle up to 10 I/O processes simultaneously using an interesting form of round-robin scheduling called processor sharing. A process switch occurred after each instruction, so instruction 1 came from process 1, instruction 2 came from process 2, etc. The process switching was done by special hardware, and the overhead was zero. If a process needed T sec to complete in the absence of competition, how much time would it need if processor sharing was used with n processes?

tanenbaum operating-system process-and-threads process-synchronization descriptive

Answer key

11.40.28 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 39 (Page No. 177) top

Consider the following piece of C code:

```
void main( ) {

     fork( );
     fork( );
     exit( );

}
```

How many child processes are created upon execution of this program?

tanenbaum operating-system process-and-threads fork-system-call descriptive

Answer key

11.40.29 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 40 (Page No. 177) top

Round-robin schedulers normally maintain a list of all runnable processes, with each process occurring exactly once in the list. What would happen if a process occurred twice in the list? Can you think of any reason for allowing this?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.30 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 41 (Page No. 177) top

Can a measure of whether a process is likely to be CPU bound or I/O bound be determined by analyzing source code? How can this be determined at run time?

tanenbaum operating-system process-and-threads process descriptive

Answer key

11.40.31 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 44 (Page No. 177) top

Five jobs are waiting to be run. Their expected run times are $9, 6, 3, 5,$ and X. In what order should they be run to minimize average response time? (Your answer will depend on X.)

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.32 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 45 (Page No. 177 - 178) top

Five batch jobs. A through E, arrive at a computer center at almost the same time. They have estimated running times of $10, 6, 2, 4,$ and 8 minutes. Their (externally determined) priorities are $3, 5, 2, 1,$ and 4, respectively, with 5 being the highest priority. For each of the following scheduling algorithms, determine the mean process turnaround time. Ignore process switching overhead.

a. Round robin.
b. Priority scheduling.
c. First-come, first-served (run in order $10, 6, 2, 4, 8$).
d. Shortest job first.

For (a), assume that the system is multiprogrammed, and that each job gets its fair share of the CPU. For (b) through (d), assume that only one job at a time runs, until it finishes. All jobs are completely CPU bound.

tanenbaum operating-system process-and-threads process-scheduling descriptive

11.40.33 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 46 (Page No. 178) top

A process running on $CTSS$ needs 30 quanta to complete. How many times must it be swapped in, including the very first time (before it has run at all)?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.34 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 47 (Page No. 178) top

Consider a real-time system with two voice calls of periodicity 5 msec each with CPU time per call of 1 msec, and one video stream of periodicity 33 ms with CPU time per call of 11 msec. Is this system schedulable?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.35 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 48 (Page No. 178) top

For the above problem, can another video stream be added and have the system still be schedulable?

tanenbaum operating-system process-and-threads process-scheduling descriptive

11.40.36 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 49 (Page No. 178) top

The aging algorithm with $a = 1/2$ is being used to predict run times. The previous four runs, from oldest to most recent, are $40, 20, 40,$ and 15 msec. What is the prediction of the next time?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.37 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 5 (Page No. 174) top

A computer system has enough room to hold five programs in its main memory. These programs are idle waiting for I/O half the time. What fraction of the CPU time is wasted?

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.38 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 50 (Page No. 178) top

A soft real-time system has four periodic events with periods of $50, 100, 200,$ and 250 msec each. Suppose that the four events require $35, 20, 10,$ and x msec of CPU time, respectively. What is the largest value of x for which the system is schedulable?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.39 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 51 (Page No. 178) top

In the dining philosophers problem, let the following protocol be used: An even-numbered philosopher always picks up his left fork before picking up his right fork; an odd-numbered philosopher always picks up his right fork before picking up his left fork. Will this protocol guarantee deadlock-free operation?

tanenbaum operating-system process-and-threads deadlock-prevention-avoidance-detection descriptive

11.40.40 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 52 (Page No. 178) top

A real-time system needs to handle two voice calls that each run every 6 msec and consume 1 msec of CPU time per burst, plus one video at 25 frames/sec, with each frame requiring 20 msec of CPU time. Is this system schedulable?

tanenbaum operating-system process-and-threads process-scheduling descriptive

Answer key

11.40.41 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 53 (Page No. 178) top

Consider a system in which it is desired to separate policy and mechanism for the scheduling of kernel threads. Propose a means of achieving this goal.

tanenbaum operating-system process-and-threads process-scheduling threads descriptive

11.40.42 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 55 (Page No. 178) top

Consider the procedure put forks in Fig. $2-47$. Suppose that the variable $state[i]$ was set to THINKING after the two calls to test, rather than before. How would this change affect the solution?

tanenbaum operating-system process-and-threads descriptive

11.40.43 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 56 (Page No. 178) top

The readers and writers problem can be formulated in several ways with regard to which category of processes can be started when. Carefully describe three different variations of the problem, each one favoring (or not favoring) some category of processes. For each variation, specify what happens when a reader or a writer becomes ready to access the database, and what happens when a process is finished.

tanenbaum operating-system process-and-threads process-synchronization descriptive

11.40.44 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 57 (Page No. 178 - 179) top

Write a shell script that produces a file of sequential numbers by reading the last number in the file, adding 1 to it, and then appending it to the file. Run one instance of the script in the background and one in the foreground, each accessing the same file. How long does it take before a race condition manifests itself? What is the critical region? Modify the script to prevent the race.

(Hint: use ln file file.lock to lock the data file.)

tanenbaum operating-system process-and-threads descriptive

11.40.45 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 58 (Page No. 179) top

Assume that you have an operating system that provides semaphores. Implement a message system. Write the procedures for sending and receiving messages.

tanenbaum operating-system process-and-threads semaphore descriptive

11.40.46 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 6 (Page No. 174) top

A computer has 4 GB of RAM of which the operating system occupies 512 MB. The processes are all 256 MB (for simplicity) and have the same characteristics. If the goal is 99% CPU utilization, what is the maximum I/O wait that can be tolerated?

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.47 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 60 (Page No. 179) top

Suppose that a university wants to show off how politically correct it is by applying the U.S. Supreme Court's "Separate but equal is inherently unequal" doctrine to gender as well as race, ending its long-standing practice of gender-segregated bathrooms on campus. However, as a concession to tradition, it decrees that when a woman is in a bathroom, other women may enter, but no men, and vice versa. A sign with a sliding marker on the door of each bathroom indicates which of three possible states it is currently in:

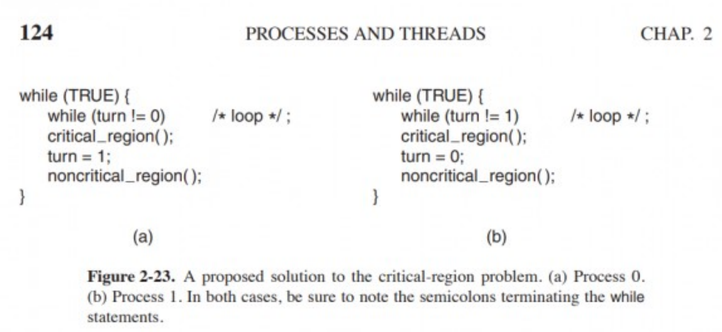
- Empty
- Women present
- Men present

In some programming language you like, write the following procedures: woman_ *wants_to_enter, man_wants_to_enter, woman_leaves, man_leaves*. You may use whatever counters and synchronization techniques you like.

tanenbaum operating-system process-and-threads process-synchronization semaphore descriptive

11.40.48 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 61 (Page No. 179) top

Rewrite the program of Fig. $2-23$ to handle more than two processes.


124 PROCESSES AND THREADS CHAP. 2

```
while (TRUE) {
    while (turn != 0)      /* loop */ ;
    critical_region( );
    turn = 1;
    noncritical_region( );
}
```

(a)

```
while (TRUE) {
    while (turn != 1)      /* loop */ ;
    critical_region( );
    turn = 0;
    noncritical_region( );
}
```

(b)

Figure 2-23. A proposed solution to the critical-region problem. (a) Process 0. (b) Process 1. In both cases, be sure to note the semicolons terminating the while statements.

tanenbaum operating-system process-and-threads descriptive

11.40.49 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 62 (Page No. 179) top

Write a producer-consumer problem that uses threads and shares a common buffer. However, do not use semaphores or any other synchronization primitives to guard the shared data structures. Just let each thread access them when it wants to. Use sleep and wakeup to handle the full and empty conditions. See how long it takes for a fatal race condition to occur. For example, you might have the producer print a number once in a while. Do not print more than one number every minute because the I/O could affect the race conditions.

tanenbaum operating-system process-and-threads semaphore process-synchronization descriptive

11.40.50 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 63 (Page No. 179) top

A process can be put into a round-robin queue more than once to give it a higher priority. Running multiple instances of a program each working on a different part of a data pool can have the same effect. First write a program that tests a list of numbers for primality. Then devise a method to allow multiple instances of the program to run at once in such a way that no two instances of the program will work on the same number. Can you in fact get through the list faster by running multiple copies of the program? Note that your results will depend upon what else your computer is doing; on a personal computer running only instances of this program you would not expect an improvement, but on a system with other processes, you should be able to grab a bigger share of the CPU this way.

tanenbaum operating-system process-and-threads process-scheduling descriptive

11.40.51 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 65 (Page No. 180) top

Implement a program to count the frequency of words in a text file. The text file is partitioned into N segments. Each segment is processed by a separate thread that outputs the intermediate frequency count for its segment. The main process waits until all the threads complete; then it computes the consolidated word-frequency data based on the individual threads' output.

tanenbaum operating-system process-and-threads descriptive

11.40.52 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 7 (Page No. 174) top

Multiple jobs can run in parallel and finish faster than if they had run sequentially. Suppose that two jobs, each needing 20 minutes of CPU time, start simultaneously. How long will the last one take to complete if they run sequentially? How long if they run in parallel? Assume 50% I/O wait.

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.53 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 8 (Page No. 174) top

Consider a multiprogrammed system with degree of 6 (i.e., six programs in memory at the same time). Assume that each process spends 40% of its time waiting for I/O. What will be the CPU utilization?

tanenbaum operating-system process-and-threads descriptive

Answer key

11.40.54 Process And Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 9 (Page No. 174) top

Assume that you are trying to download a large 2-GB file from the Internet. The file is available from a set of mirror servers, each of which can deliver a subset of the file's bytes; assume that a given request specifies the starting and ending bytes of the file. Explain how you might use threads to improve the download time.

tanenbaum operating-system process-and-threads threads descriptive

11.41 Process Scheduling (28) top

11.41.1 Process Scheduling: Galvin Edition 9 Exercise 6 Question 1 (Page No. 305) top

A CPU-scheduling algorithm determines an order for the execution of its scheduled processes. Given n processes to be scheduled on one processor, how many different schedules are possible? Give a formula in terms of n.

galvin operating-system process-scheduling descriptive

Answer key

11.41.2 Process Scheduling: Galvin Edition 9 Exercise 6 Question 10 (Page No. 307) top

Why is it important for the scheduler to distinguish $I/O - bound$ programs from $CPU - bound$ programs?

galvin operating-system process-scheduling descriptive

11.41.3 Process Scheduling: Galvin Edition 9 Exercise 6 Question 11 (Page No. 307) top

Discuss how the following pairs of scheduling criteria conflict in certain settings.
a. CPU utilization and response time
b. Average turnaround time and maximum waiting time
c. I/O device utilization and CPU utilization

galvin operating-system process-scheduling descriptive

11.41.4 Process Scheduling: Galvin Edition 9 Exercise 6 Question 12 (Page No. 307) top

One technique for implementing $lotteryscheduling$ works by assigning processes lottery tickets, which are used for allocating CPU time.Whenever a scheduling decision has to be made, a lottery ticket is chosen at random, and the process holding that ticket gets the CPU. The BTV operating system implements lottery scheduling by holding a lottery 50 times each second, with each lottery winner getting 20 milliseconds of CPU time (20 milliseconds $\times$ $50 = 1$ second). Describe how the BTV scheduler can ensure that higher-priority threads receive more attention from the CPU than lower-priority threads.

galvin operating-system process-scheduling descriptive

11.41.5 Process Scheduling: Galvin Edition 9 Exercise 6 Question 15 (Page No. 308) top

A variation of the round-robin scheduler is the $regressive\ round - robin\ scheduler$. This scheduler assigns each process a time quantum and a priority. The initial value of a time quantum is 50 milliseconds. However, every time a process has been allocated the CPU and uses its entire time quantum (does not block for I/O), 10 milliseconds is added to its time quantum, and its priority level is boosted. (The time quantum for a process can be increased to a maximum of 100 milliseconds.) When a process blocks before using its entire time quantum, its time quantum is reduced by 5 milliseconds, but its priority remains the same. What type of process ($CPU - bound$ or $I/Obound$) does the regressive round-robin scheduler favor ? Explain.

galvin operating-system process-scheduling descriptive

11.41.6 Process Scheduling: Galvin Edition 9 Exercise 6 Question 16 (Page No. 308) top

Consider the following set of processes, with the length of the CPU burst given in milliseconds:

$Process$	$Burst\ Time$	$Priority$
P_1	2	2
P_2	1	1
P_3	8	4
P_4	4	2
P_5	5	3

The processes are assumed to have arrived in the order P_1, P_2, P_3, P_4, P_5, all at time 0.

a. Draw four Gantt charts that illustrate the execution of these processes using the following scheduling algorithms: $FCFS$, SJF, non preemptive priority (a larger priority number implies a higher priority), and RR (quantum = 2).

b. What is the turnaround time of each process for each of the scheduling algorithms in part a ?

c. What is the waiting time of each process for each of these scheduling algorithms ?

d. Which of the algorithms results in the minimum average waiting time (over all processes)?

galvin operating-system process-scheduling descriptive

11.41.7 Process Scheduling: Galvin Edition 9 Exercise 6 Question 18 (Page No. 309) top

The nice command is used to set the nice value of a process on Linux, as well as on other UNIX systems. Explain why some systems may allow any user to assign a process a nice value $>= 0$ yet allow only the root user to assign nice values < 0.

galvin operating-system process-scheduling descriptive

11.41.8 Process Scheduling: Galvin Edition 9 Exercise 6 Question 19 (Page No. 309) top

Which of the following scheduling algorithms could result in starvation ?

$a. First - come, first - served$

$b. Shortest\ job\ first$

$c. Round\ robin$

$d. Priority$

galvin operating-system process-scheduling

Answer key

11.41.9 Process Scheduling: Galvin Edition 9 Exercise 6 Question 2 (Page No. 306) top

Explain the difference between preemptive and nonpreemptive scheduling.

galvin operating-system process-scheduling descriptive

Answer key

11.41.10 Process Scheduling: Galvin Edition 9 Exercise 6 Question 20 (Page No. 309) top

Consider a variant of the RR scheduling algorithm in which the entries in the ready queue are pointers to the $PCBs$.
a. What would be the effect of putting two pointers to the same process in the ready queue ?
b. What would be two major advantages and two disadvantages of this scheme?
c. How would you modify the basic RR algorithm to achieve the same effect without the duplicate pointers?

galvin operating-system process-scheduling descriptive

11.41.11 Process Scheduling: Galvin Edition 9 Exercise 6 Question 21 (Page No. 309-310) top

Consider a system running ten $I/O - bound$ tasks and one $CPU - bound$ task. Assume that the $I/O - bound$ tasks issue an I/O operation once for every millisecond of CPU computing and that each I/O operation takes 10 milliseconds to complete. Also assume that the context-switching overhead is 0.1 millisecond and that all processes are long-running tasks. Describe the CPU utilization for a round-robin scheduler when:

a. The time quantum is 1 millisecond
b. The time quantum is 10 milliseconds

galvin operating-system process-scheduling descriptive

11.41.12 Process Scheduling: Galvin Edition 9 Exercise 6 Question 22 (Page No. 310) top

Consider a system implementing multilevel queue scheduling. What strategy can a computer user employ to maximize the amount of CPU time allocated to the user's process ?

galvin operating-system process-scheduling descriptive

11.41.13 Process Scheduling: Galvin Edition 9 Exercise 6 Question 23 (Page No. 310) top

Consider a preemptive priority scheduling algorithm based on dynamically changing priorities. Larger priority numbers imply higher priority. When a process is waiting for the CPU (in the ready queue, but not running), its priority changes at a rate β. When it is running, its priority changes at a rate α. All processes are given a priority of 0 when they enter the ready queue. The parameters β and α can be set to give many different scheduling algorithms.

a. What is the algorithm that results from $\beta > \alpha > 0$?
b. What is the algorithm that results from $\alpha < \beta < 0$?

galvin operating-system process-scheduling descriptive

Answer key

11.41.14 Process Scheduling: Galvin Edition 9 Exercise 6 Question 24 (Page No. 310) top

Explain the differences in how much the following scheduling algorithms discriminate in favor of short processes:
$a. FCFS$
$b. RR$
$c. Multilevel feedback queues$

galvin operating-system process-scheduling descriptive

11.41.15 Process Scheduling: Galvin Edition 9 Exercise 6 Question 25 (Page No. 310) top

Using the Windows scheduling algorithm, determine the numeric priority of each of the following threads.
a. A thread in the $REALTIME\_PRIORITY\_CLASS$ with a relative priority of $NORMAL$
b. A thread in the $ABOVE\_NORMAL\_PRIORITY\_CLASS$ with a relative priority of $HIGHEST$
c. A thread in the $BELOW\_NORMAL\_PRIORITY\_CLASS$ with a relative priority of $ABOVE\_NORMAL$

galvin operating-system process-scheduling descriptive

11.41.16 Process Scheduling: Galvin Edition 9 Exercise 6 Question 26 (Page No. 310) top

Assuming that no threads belong to the $REALTIME\_PRIORITY\_CLASS$ and that none may be assigned a $TIME\_CRITICAL$ priority, what combination of priority class and priority corresponds to the highest possible relative priority in Windows scheduling?

galvin operating-system process-scheduling descriptive

11.41.17 Process Scheduling: Galvin Edition 9 Exercise 6 Question 27 (Page No. 310) top

Consider the scheduling algorithm in the Solaris operating system for time-sharing threads.
a. What is the time quantum $(in milliseconds)$ for a thread with priority 15? With priority 40?
b. Assume that a thread with priority 50 has used its entire time quantum without blocking. What new priority will the scheduler assign this thread?
c. Assume that a thread with priority 20 blocks for I/O before its time quantum has expired. What new priority will the scheduler assign this thread

galvin operating-system process-scheduling descriptive

11.41.18 Process Scheduling: Galvin Edition 9 Exercise 6 Question 28 (Page No. 311) top

Assume that two tasks A and B are running on a $Linux\ system.$ The nice values of A and B are -5 and $+5,$ respectively. Using the CFS scheduler as a guide, describe how the respective values of $vruntime$ vary between the two processes given each of the following scenarios:
• Both A and B are $CPU - bound.$
• A is $I/O - bound$, and B is $CPU - bound.$
• A is $CPU - bound$, and B is $I/O - bound.$

operating-system galvin process-scheduling descriptive

11.41.19 Process Scheduling: Galvin Edition 9 Exercise 6 Question 29 (Page No. 311) top

Discuss ways in which the priority inversion problem could be addressed in a real-time system. Also discuss whether the solutions could be implemented within the context of a proportional share scheduler

operating-system galvin process-scheduling descriptive

11.41.20 Process Scheduling: Galvin Edition 9 Exercise 6 Question 3 (Page No. 306) top

Suppose that the following processes arrive for execution at the times indicated. Each process will run for the amount of time listed. In answering the questions, use non preemptive scheduling, and base all decisions on the information you have at the time the decision must be made.

$Process$	$Arrival\ Time$	$Burst\,Time$
P_1	0.0	8
P_2	0.4	4
P_3	1.0	1

a. What is the average turnaround time for these processes with the $FCFS$ scheduling algorithm ?

b. What is the average turnaround time for these processes with the SJF scheduling algorithm ?

c. The SJF algorithm is supposed to improve performance, but notice that we chose to run process P_1 at time 0 because we did not know that two shorter processes would arrive soon. Compute what the average turnaround time will be if the CPU is left idle for the first 1 unit and then SJF scheduling is used. Remember that processes P_1 and P_2 are waiting during this idle time, so their waiting time may increase. This algorithm could be called future-knowledge scheduling.

galvin operating-system process-scheduling descriptive

11.41.21 Process Scheduling: Galvin Edition 9 Exercise 6 Question 30 (Page No. 311) top

Under what circumstances is $rate\text{-}monotonic\ scheduling$ inferior to $earliest\text{-}deadline\text{-}first\ scheduling$ in meeting the deadlines associated with processes ?

galvin operating-system process-scheduling descriptive

11.41.22 Process Scheduling: Galvin Edition 9 Exercise 6 Question 32 (Page No. 311) top

Explain why interrupt and dispatch latency times must be bounded in a hard real-time system ?

galvin operating-system process-scheduling descriptive

11.41.23 Process Scheduling: Galvin Edition 9 Exercise 6 Question 4 (Page No. 306) top

What advantage is there in having different time-quantum sizes at different levels of a multilevel queueing system ?

galvin operating-system process-scheduling descriptive

11.41.24 Process Scheduling: Galvin Edition 9 Exercise 6 Question 5 (Page No. 306) top

Many CPU-scheduling algorithms are parametrized. For example, the RR algorithm requires a parameter to indicate the time slice. Multilevel feedback queues require parameters to define the number of queues, the scheduling algorithm for each queue, the criteria used to move processes between queues, and so on. These algorithms are thus really sets of algorithms (for example, the set of RR algorithms for all time slices, and so on). One set of algorithms may include another (for example, the $FCFS$ algorithm is the RR algorithm with an infinite time quantum). What (if any) relation holds between the following pairs of algorithm sets ?

a. $Priority$ and SJF
b. $Multilevel feedbackqueues$ and $FCFS$
c. $Priority$ and $FCFS$
d. RR and SJF

galvin operating-system process-scheduling descriptive

11.41.25 Process Scheduling: Galvin Edition 9 Exercise 6 Question 6 (Page No. 306-307) top

Suppose that a scheduling algorithm (at the level of short-term CPU scheduling) favors those processes that have used the least processor time in the recent past. Why will this algorithm favor $I/O - bound$ programs and yet not permanently starve $CPU - bound$ programs ?

galvin operating-system process-scheduling descriptive

11.41.26 Process Scheduling: Galvin Edition 9 Exercise 6 Question 7 (Page No. 307) top

Distinguish between PCS(Process Contention Scope) and SCS(Source Contention Scope) scheduling.

galvin operating-system process-scheduling descriptive

11.41.27 Process Scheduling: Galvin Edition 9 Exercise 6 Question 8 (Page No. 307) top

Assume that an operating system maps user-level threads to the kernel using the many-to-many model and that the mapping is done through the use of LWP(Light Weight Processes). Furthermore, the system allows program developers to create real-time threads. Is it necessary to bind a real-time thread to an LWP?

galvin operating-system process-scheduling descriptive

11.41.28 Process Scheduling: Galvin Edition 9 Exercise 6 Question 9 (Page No. 307) top

The traditional $UNIX$ scheduler enforces an inverse relationship between priority numbers and priorities: the higher the number, the lower the priority. The scheduler recalculates process priorities once per second using the following function:
Priority = (recent CPU usage / 2) + base
where base = 60 and recent CPU usage refers to a value indicating how often a process has used the CPU since priorities were last recalculated.
Assume that recent CPU usage is 40 for process P_1, 18 for process P_2, and 10 for process P_3. What will be the new priorities for these three processes when priorities are recalculated? Based on this information, does the traditional UNIX scheduler raise or lower the relative priority of a $CPU - bound$ process?

galvin operating-system process-scheduling descriptive

11.42 Process Synchronization (31) top

11.42.1 Process Synchronization: Galvin Edition 9 Exercise 5 Question 1 (Page No. 242) top

disabling interrupts frequently can affect the system's clock. Explain why this can occur and how such effects can be minimized.

galvin operating-system process-synchronization descriptive

11.42.2 Process Synchronization: Galvin Edition 9 Exercise 5 Question 10 (Page No. 243) top

Explain why implementing synchronization primitives by disabling interrupts is not appropriate in a single-processor system if the synchronization primitives are to be used in user-level programs.

galvin operating-system process-synchronization descriptive

11.42.3 Process Synchronization: Galvin Edition 9 Exercise 5 Question 11 (Page No. 244) top

Explain why interrupts are not appropriate for implementing synchronization primitives in multiprocessor systems.

galvin operating-system process-synchronization descriptive

11.42.4 Process Synchronization: Galvin Edition 9 Exercise 5 Question 12 (Page No. 244) top

The Linux kernel has a policy that a process cannot hold a spin lock while attempting to acquire a semaphore. Explain why this policy is in place.

galvin operating-system process-synchronization descriptive

11.42.5 Process Synchronization: Galvin Edition 9 Exercise 5 Question 13 (Page No. 244) top

Describe two kernel data structures in which race conditions are possible.Be sure to include a description of how a race condition can occur.

galvin operating-system process-synchronization descriptive

11.42.6 Process Synchronization: Galvin Edition 9 Exercise 5 Question 14 (Page No. 244) top

Describe how the compare_and_swap() instruction can be used to provide mutual exclusion that satisfies the bounded-waiting requirement.

galvin operating-system process-synchronization descriptive

11.42.7 Process Synchronization: Galvin Edition 9 Exercise 5 Question 15 (Page No. 244) top

Consider how to implement a mutex lock using an atomic hardware instruction. Assume that the following structure defining the mutex
lock is available:
typedef struct {
int available;
} lock;
(available == 0) indicates that the lock is available, and a value of 1 indicates that the lock is unavailable. Using this struct, illustrate how the following functions can be implemented using the test and set() and compare and swap() instructions:
• $voidacquire(lock * mutex)$
•$voidrelease(lock * mutex)$
Be sure to include any initialization that may be necessary.

galvin operating-system process-synchronization descriptive

11.42.8 Process Synchronization: Galvin Edition 9 Exercise 5 Question 17 (Page No. 245) top

Assume that a system has multiple processing cores. For each of the following scenarios, describe which is a better locking mechanism—a spinlock or a mutex lock where waiting processes sleep while waiting for the lock to become available:
• The lock is to be held for a short duration.
• The lock is to be held for a long duration.
• A thread may be put to sleep while holding the lock.

galvin operating-system process-synchronization descriptive

11.42.9 Process Synchronization: Galvin Edition 9 Exercise 5 Question 18 (Page No. 246) top

Assume that a context switch takes T time. Suggest an upper bound (in terms of T) for holding a spinlock. If the spinlock is held for any longer, a mutex lock (where waiting threads are put to sleep) is a better alternative.

galvin operating-system process-synchronization descriptive

11.42.10 Process Synchronization: Galvin Edition 9 Exercise 5 Question 19 (Page No. 246) top

A multithreaded web server wishes to keep track of the number of requests it services (known as hits). Consider the two following
strategies to prevent a race condition on the variable hits. The first strategy is to use a basic mutex lock when updating hits:

```
int hits;
mutex lock hit lock;
hit lock.acquire();
hits++;
hit lock.release();
A second strategy is to use an atomic integer:
atomic t hits;
atomic inc(&hits);
```

Explain which of these two strategies is more efficient.

galvin operating-system process-synchronization descriptive

11.42.11 Process Synchronization: Galvin Edition 9 Exercise 5 Question 2 (Page No. 242) top

Explain why Windows, Linux, and Solaris implement multiple locking mechanisms. Describe the circumstances under which they use spin locks,mutex locks, semaphores, adaptive mutex locks, and condition variables. In each case, explain why the mechanism is needed.

galvin operating-system process-synchronization descriptive

11.42.12 Process Synchronization: Galvin Edition 9 Exercise 5 Question 20 (Page No. 246-247) top

Consider the code example for allocating and releasing processes shown below:

```
#define MAX PROCESSES 255
int number of processes = 0;
/* the implementation of fork() calls this function */
int allocate process() {
int new pid;
if (number of processes == MAX PROCESSES)
return -1;
else {
/* allocate necessary process resources */
++number of processes;
return new pid;
}
}
/* the implementation of exit() calls this function */
void release process() {
/* release process resources */
--number of processes;
}
```

a. Identify the race condition(s).
b. Assume you have a mutex lock named mutex with the operations $acquire()$ and $release()$. Indicate where the locking needs to be placed to prevent the race condition(s).
c. Could we replace the integer variable int number of processes = 0 with the atomic integer atomic t number of processes = 0 to prevent the race condition(s)?

galvin operating-system process-synchronization descriptive

11.42.13 Process Synchronization: Galvin Edition 9 Exercise 5 Question 21 (Page No. 247) top

Servers can be designed to limit the number of open connections. For example, a server may wish to have only N socket connections at any point in time. As soon as N connections are made, the server will not accept another incoming connection until an existing connection is released. Explain how semaphores can be used by a server to limit the number of concurrent connections.

galvin operating-system process-synchronization descriptive

11.42.14 Process Synchronization: Galvin Edition 9 Exercise 5 Question 22 (Page No. 247) top

Windows Vista provides a lightweight synchronization tool called slim reader–writer locks. Whereas most implementations of reader–writer locks favor either readers or writers, or perhaps order waiting threads using a $FIFO$ policy, slim reader–writer locks favor neither readers nor writers, nor are waiting threads ordered in a $FIFO$ queue. Explain the benefits of providing such a synchronization tool.

galvin operating-system process-synchronization descriptive

11.42.15 Process Synchronization: Galvin Edition 9 Exercise 5 Question 23 (Page No. 247) top

Show how to implement the $wait()$ and $signal()$ semaphore operations in multiprocessor environments using the $testandset()$instruction. The solution should exhibit minimal busy waiting.

galvin operating-system process-synchronization descriptive

11.42.16 Process Synchronization: Galvin Edition 9 Exercise 5 Question 25 (Page No. 247) top

Demonstrate that monitors and semaphores are equivalent in so far as they can be used to implement solutions to the same types of synchronization problems.

galvin operating-system process-synchronization descriptive

11.42.17 Process Synchronization: Galvin Edition 9 Exercise 5 Question 26 (Page No. 247) top

Design an algorithm for a bounded-buffer monitor in which the buffers (portions) are embedded within the monitor itself.

galvin operating-system process-synchronization descriptive

11.42.18 Process Synchronization: Galvin Edition 9 Exercise 5 Question 28 (Page No. 247) top

Discuss the tradeoff between fairness and throughput of operations in the readers–writers problem. Propose a method for solving the readers–writers problem without causing starvation.

galvin operating-system process-synchronization descriptive

11.42.19 Process Synchronization: Galvin Edition 9 Exercise 5 Question 29 (Page No. 248) top

How does the $signal()$ operation associated with monitors differ from the corresponding operation defined for semaphores?

galvin operating-system process-synchronization descriptive

11.42.20 Process Synchronization: Galvin Edition 9 Exercise 5 Question 3 (Page No. 243) top

What is the meaning of the term busy waiting ? What other kinds of waiting are there in an operating system ? Can busy waiting be avoided altogether ? Explain your answer.

galvin operating-system process-synchronization descriptive

11.42.21 Process Synchronization: Galvin Edition 9 Exercise 5 Question 31 (Page No. 248) top

Consider a system consisting of processes $P_1, P_2, \dots, P_n$, each of which has a unique priority number. Write a monitor that allocates three identical printers to these processes, using the priority numbers for deciding the order of allocation.

galvin operating-system process-synchronization descriptive

11.42.22 Process Synchronization: Galvin Edition 9 Exercise 5 Question 32 (Page No. 248) top

A file is to be shared among different processes, each of which has a unique number. The file can be accessed simultaneously by several processes, subject to the following constraint: the sum of all unique numbers associated with all the processes currently accessing the file must be less than n.Write a monitor to coordinate access to the file.

galvin operating-system process-synchronization descriptive

11.42.23 Process Synchronization: Galvin Edition 9 Exercise 5 Question 33 (Page No. 248) top

When a signal is performed on a condition inside a monitor, the signaling process can either continue its execution or transfer control to the process that is signaled. How would the solution to the preceding exercise differ with these two different ways in which signaling can be performed?

galvin operating-system process-synchronization descriptive

11.42.24 Process Synchronization: Galvin Edition 9 Exercise 5 Question 34 (Page No. 248) top

Suppose we replace the $wait()$ and $signal()$ operations of monitors with a single construct a $wait(B)$, where B is a general Boolean expression that causes the process executing it to wait until B becomes true.

a. Write a monitor using this scheme to implement the readers–writers problem.
b. Explain why, in general, this construct cannot be implemented efficiently.
c. What restrictions need to be put on the await statement so that it can be implemented efficiently ?

galvin operating-system process-synchronization descriptive

11.42.25 Process Synchronization: Galvin Edition 9 Exercise 5 Question 35 (Page No. 248) top

Design an algorithm for a monitor that implements an alarm clock that enables a calling program to delay itself for a specified number of time units (ticks). You may assume the existence of a real hardware clock that invokes a function $tick()$ in your monitor at regular intervals.

galvin operating-system process-synchronization descriptive

11.42.26 Process Synchronization: Galvin Edition 9 Exercise 5 Question 4 (Page No. 243) top

Explain why spin locks are not appropriate for single-processor systems yet are often used in multiprocessor systems.

galvin operating-system process-synchronization descriptive

11.42.27 Process Synchronization: Galvin Edition 9 Exercise 5 Question 5 (Page No. 243) top

Show that, if the $wait()$ and $signal()$ semaphore operations are not executed atomically, then mutual exclusion may be violated.

galvin operating-system process-synchronization descriptive

11.42.28 Process Synchronization: Galvin Edition 9 Exercise 5 Question 6 (Page No. 243) top

Illustrate how a binary semaphore can be used to implement mutual exclusion among n processes.

galvin operating-system process-synchronization descriptive

Answer key

11.42.29 Process Synchronization: Galvin Edition 9 Exercise 5 Question 7 (Page No. 243) top

Race conditions are possible in many computer systems. Consider a banking system that maintains an account balance with two functions: $deposit(amount)$ and $withdraw(amount)$. These two functions are passed the amount that is to be deposited or withdrawn from the bank account balance. Assume that a husband and wife share a bank account. Concurrently, the husband calls the $withdraw()$ function and the wife calls $deposit()$. Describe how a race condition is possible and what might be done to prevent the race condition from occurring.

galvin operating-system process-synchronization descriptive

11.42.30 Process Synchronization: Galvin Edition 9 Exercise 5 Question 8 (Page No. 243-244) top

The first known correct software solution to the critical-section problem for two processes was developed by Dekker. The two processes, P0 and P1, share the following variables:
boolean $flag[2]$; /* initially false */
int turn;
The structure of process Pi (i == 0 or 1) is shown below. The other process is Pj (j == 1 or 0). Prove that the algorithm satisfies all three requirements for the critical-section problem.

do {
$flag[i]$ = true;
while ($flag[j]$) {
if (turn == j) {
$flag[i]$ = false;
while (turn == j)
; /* do nothing */
$flag[i]$ = true;
}
}
/* critical section */
turn = j;
$flag[i]$ = false;
/* remainder section */
} while (true);

galvin operating-system process-synchronization descriptive

11.42.31 Process Synchronization: Galvin Edition 9 Exercise 5 Question 9 (Page No. 243-245) top

The first known correct software solution to the critical-section problem for n processes with a lower bound on waiting of n − 1 turns was presented by Eisenberg and McGuire. The processes share the following variables:
enum pstate $idle, wantin, incs$;
pstate $flag[n]$;
int turn;
All the elements of flag are initially idle. The initial value of turn is immaterial (between 0 and n-1). The structure of process Pi is shown below. Prove that the algorithm satisfies all three requirements for the critical-section problem.

$do {
while (true) {
flag[i] = want in;
j = turn;
while (j != i) {

```
if (flag[j] != idle) {
j = turn;
else
j = (j + 1) % n;
}
flag[i] = in cs;
j = 0;
while ( (j < n) && (j == i || flag[j] != in cs))
j++;
if ( (j >= n) && (turn == i || flag[turn] == idle))
break;
}
/* critical section */
j = (turn + 1) % n;
while (flag[j] == idle)
j = (j + 1) % n;
turn = j;
flag[i] = idle;
/* remainder section */
} while (true);$
```

galvin operating-system process-synchronization descriptive

Answer key

11.43 Program (2) top

11.43.1 Program: Galvin Edition 9 Exercise 13 Question 17 (Page No. 620) top

Write (in pseudocode) an implementation of virtual clocks, including the queueing and management of timer requests for the kernel and applications. Assume that the hardware provides three timer channels.

operating-system galvin program io-system

11.43.2 Program: Galvin Edition 9 Exercise 13 Question 18 (Page No. 620) top

Write (in pseudocode) an implementation of virtual clocks, including the queueing and management of timer requests for the kernel and applications. Assume that the hardware provides three timer channels.

operating-system galvin program io-system

11.44 Race Conditions (1) top

11.44.1 Race Conditions: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 37 (Page No. 177) top

Suppose that we have a message-passing system using mailboxes. When sending to a full mailbox or trying to receive from an empty one, a process does not block. Instead, it gets an error code back. The process responds to the error code by just trying again, over and over, until it succeeds. Does this scheme lead to race conditions?

tanenbaum operating-system process-and-threads race-conditions descriptive

11.45 Resource Allocation (2) top

11.45.1 Resource Allocation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 43 (Page No. 470) top

Write a program that detects if there is a deadlock in the system by using a resource allocation graph. Your program should read from a file the following inputs: the number of processes and the number of resources. For each process if should read four numbers: the number of resources it is currently holding, the IDs of resources it is holding, the number of resources it is currently requesting, the IDs of resources it is requesting. The output of program should indicate if there is a deadlock in the system. In case there is, the program should print out the identities of all processes that are deadlocked.

tanenbaum operating-system deadlock-prevention-avoidance-detection resource-allocation descriptive

11.45.2 Resource Allocation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 9 (Page No. 466) top

Fig. 6-3 shows the concept of a resource graph. Do illegal graphs exist, that is, graphs that structurally violate the model we have used of resource usage? If so, give an example of one.

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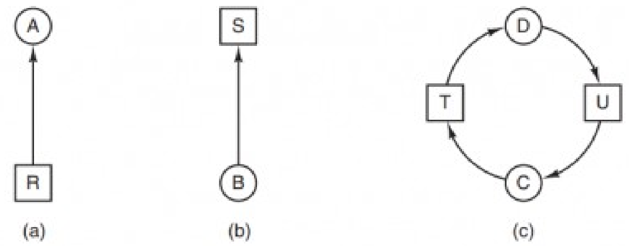

Figure 6-3. Resource allocation graphs. (a) Holding a resource. (b) Requesting a resource. (c) Deadlock.

tanenbaum operating-system deadlock-prevention-avoidance-detection resource-allocation descriptive

Answer key

11.46 Round Robin Scheduling (2) top

11.46.1 Round Robin Scheduling: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 42 (Page No. 177) top

Explain how time quantum value and context switching time affect each other, in a round-robin scheduling algorithm.

tanenbaum operating-system process-and-threads context-switch process-scheduling round-robin-scheduling descriptive

Answer key

11.46.2 Round Robin Scheduling: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 43 (Page No. 177) top

Measurements of a certain system have shown that the average process runs for a time T before blocking on I/O. A process switch requires a time S, which is effectively wasted (overhead). For round-robin scheduling with quantum Q, give a formula for the CPU efficiency for each of the following:

a. $Q = \infty$ b. $Q > T$ c. $S < Q < T$ d. $Q = S$ e. Q nearly 0

tanenbaum operating-system process-and-threads process-scheduling round-robin-scheduling descriptive

Answer key

11.47 Segmentation (2) top

11.47.1 Segmentation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 45 (Page No. 259) top

Explain the difference between internal fragmentation and external fragmentation. Which one occurs in paging systems? Which one occurs in systems using pure segmentation?

tanenbaum operating-system memory-management paging fragmentation segmentation descriptive

Answer key

11.47.2 Segmentation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 46 (Page No. 259) top

When segmentation and paging are both being used, as in $MULTICS$, first the segment descriptor must be looked up, then the page descriptor. Does the TLB also work this way, with two levels of lookup?

tanenbaum operating-system memory-management paging segmentation descriptive

11.48 Semaphore (1) top

11.48.1 Semaphore: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 39 (Page No. 469) top

A student majoring in anthropology and minoring in computer science has embarked on a research project to see if African baboons can be taught about deadlocks. He locates a deep canyon and fastens a rope across it, so the baboons can cross hand-overhand. Several baboons can cross at the same time, provided that they are all going in the same direction. If eastward-moving and westward-moving baboons ever get onto the rope at the same time, a deadlock will result (the baboons will get stuck in the middle) because it is impossible for one baboon to climb over another one while suspended over the canyon. If a baboon wants to cross the canyon, he must check to see that no other baboon is currently crossing in the opposite direction. Write a program using semaphores that avoids deadlock. Do not worry about a series of eastward-moving baboons holding up the westward-moving baboons indefinitely.

tanenbaum operating-system deadlock-prevention-avoidance-detection semaphore descriptive

11.49 Shared System (1) top

11.49.1 Shared System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 35 (Page No. 83 - 84) top

If you have a personal UNIX-like system (Linux, MINIX 3, FreeBSD, etc.) available that you can safely crash and reboot, write a shell script that attempts to create an unlimited number of child processes and observe what happens. Before running the experiment, type sync to the shell to flush the file system buffers to disk to avoid ruining the file system. You can also do the experiment safely in a virtual machine.

Note: Do not try this on a shared system without first getting permission from the system administrator. The consequences will be instantly obvious so you are likely to be caught and sanctions may follow.

tanenbaum operating-system shared-system descriptive

11.50 Starvation (1) top

11.50.1 Starvation: Andrew S. Tanenbaum (OS) Edition 4 Exercise 6 Question 40 (Page No. 469) top

Repeat the previous problem, but now avoid starvation. When a baboon that wants to cross to the east arrives at the rope and finds baboons crossing to the west, he waits until the rope is empty, but no more westward-moving baboons are allowed to start until at least one baboon has crossed the other way.

tanenbaum operating-system deadlock-prevention-avoidance-detection starvation descriptive

11.51 System Call (7) top

11.51.1 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 16 (Page No. 82) top

When a user program makes a system call to read or write a disk file, it provides an indication of which file it wants, a pointer to the data buffer, and the count. Control is then transferred to the operating system, which calls the appropriate driver. Suppose that the driver starts the disk and terminates until an interrupt occurs. In the case of reading from the disk, obviously the caller will have to be blocked (because there are no data for it). What about the case of writing to the disk? Need the caller be blocked awaiting completion of the disk transfer?

tanenbaum operating-system system-call interrupts descriptive

11.51.2 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 20 (Page No. 82) top

For each of the following system calls, give a condition that causes it to fail: fork, exec, and unlink.

tanenbaum operating-system system-call descriptive

11.51.3 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 23 (Page No. 82 - 83) top

A file whose file descriptor is fd contains the following sequence of bytes : $3, 1, 4, 1, 5, 9, 2, 6, 5, 3, 5.$ The following system calls are made:

- lseek(fd, 3, SEEK SET);
- read(fd, &buffer, 4);

where the lseek call makes a seek to byte 3 of the file. What does buffer contain after the read has completed?

tanenbaum operating-system system-call descriptive

11.51.4 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 28 (Page No. 83) top

To a programmer, a system call looks like any other call to a library procedure. Is it important that a programmer know which library procedures result in system calls? Under what circumstances and why?

tanenbaum operating-system system-call descriptive

11.51.5 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 29 (Page No. 83) top

Figure $1-23$ shows that a number of UNIX system calls have no Win32 API equivalents. For each of the calls listed as having no Win32 equivalent, what are the consequences for a programmer of converting a UNIX program to run under Windows?

UNIX	Win32	Description
fork	CreateProcess	Create a new process
waitpid	WaitForSingleObject	Can wait for a process to exit
execve	(none)	CreateProcess = fork + execve
exit	ExitProcess	Terminate execution
open	CreateFile	Create a file or open an existing file
close	CloseHandle	Close a file
read	ReadFile	Read data from a file
write	WriteFile	Write data to a file
lseek	SetFilePointer	Move the file pointer
stat	GetFileAttributesEx	Get various file attributes
mkdir	CreateDirectory	Create a new directory
rmdir	RemoveDirectory	Remove an empty directory
link	(none)	Win32 does not support links
unlink	DeleteFile	Destroy an existing file
mount	(none)	Win32 does not support mount
umount	(none)	Win32 does not support mount, so no umount
chdir	SetCurrentDirectory	Change the current working directory
chmod	(none)	Win32 does not support security (although NT does)
kill	(none)	Win32 does not support signals
time	GetLocalTime	Get the current time

Figure 1-23. The Win32 API calls that roughly correspond to the UNIX calls of Fig. 1-18. It is worth emphasizing that Windows has a very large number of other system calls, most of which do not correspond to anything in UNIX.

tanenbaum operating-system unix system-call descriptive

11.51.6 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 2 Question 4 (Page No. 174) top

When an interrupt or a system call transfers control to the operating system, a kernel stack area separate from the stack of the interrupted process is generally used. Why?

tanenbaum operating-system process-and-threads system-call threads descriptive

Answer key

11.51.7 System Call: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 9 (Page No. 333) top

In UNIX and Windows, random access is done by having a special system call that moves the "current position" pointer associated with a file to a given byte in the file. Propose an alternative way to do random access without having this system call.

tanenbaum operating-system file-system system-call descriptive

11.52 Threads (15) top

11.52.1 Threads: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 13 (Page No. 82) top

Consider a system that has two CPUs, each CPU having two threads (hyperthreading). Suppose three programs, $P0, P1$, and $P2$, are started with run times of $5, 10$ and 20 msec, respectively. How long will it take to complete the execution of these programs? Assume that all three programs are 100% CPU bound, do not block during execution, and do not change CPUs once assigned.

tanenbaum operating-system threads descriptive

11.52.2 Threads: Galvin Edition 9 Exercise 4 Question 1 (Page No. 191) top

Provide two programming examples in which multithreading provides better performance than a single-threaded solution.

galvin operating-system threads descriptive

11.52.3 Threads: Galvin Edition 9 Exercise 4 Question 11 (Page No. 192) top

Is it possible to have concurrency but not parallelism ? Explain.

galvin operating-system threads descriptive

Answer key

11.52.4 Threads: Galvin Edition 9 Exercise 4 Question 12 (Page No. 192) top

Using Amdahl's Law, calculate the speedup gain of an application that has a 60 percent parallel component for (a) two processing cores and (b) four processing cores.

galvin operating-system threads descriptive

Answer key

11.52.5 Threads: Galvin Edition 9 Exercise 4 Question 14 (Page No. 192-193) top

A system with two dual-core processors has four processors available for scheduling. A CPU-intensive application is running on this system. All input is performed at program start-up, when a single file must be opened. Similarly, all output is performed just before the program terminates, when the program results must be written to a single file. Between startup and termination, the program is entirely CPU bound.Your task is to improve the performance of this application by multithreading it. The application runs on a system that uses the one-to-one threading model (each user thread maps to a kernel thread).

- How many threads will you create to perform the input and output ? Explain.
- How many threads will you create for the CPU-intensive portion of the application ? Explain.

galvin operating-system threads descriptive

11.52.6 Threads: Galvin Edition 9 Exercise 4 Question 15 (Page No. 193) top

Consider the following code segment:

```
pid t pid;
pid = fork();
if (pid == 0) { /* child process */
fork();
thread create( . . .);
}
fork();
```

a. How many unique processes are created?
b. How many unique threads are created?

galvin operating-system threads programming

Answer key

11.52.7 Threads: Galvin Edition 9 Exercise 4 Question 2 (Page No. 191) top

What are two differences between user-level threads and kernel-level threads ? Under what circumstances is one type better than the other ?

galvin operating-system threads descriptive

Answer key

11.52.8 Threads: Galvin Edition 9 Exercise 4 Question 3 (Page No. 191) top

Describe the actions taken by a kernel to context-switch between kernel level threads.

galvin operating-system threads descriptive

11.52.9 Threads: Galvin Edition 9 Exercise 4 Question 4 (Page No. 191) top

What resources are used when a thread is created ? How do they differ from those used when a process is created ?

galvin operating-system threads descriptive

11.52.10 Threads: Galvin Edition 9 Exercise 4 Question 5 (Page No. 192) top

Assume that an operating system maps user-level threads to the kernel using the many-to-many model and that the mapping is done through LWPs(Light Weight Processes). Furthermore, the system allows developers to create real-time threads for use in real-time systems. Is it necessary to bind a real-time thread to an LWP(Light Weight Process) ?Explain.

galvin operating-system threads descriptive

11.52.11 Threads: Galvin Edition 9 Exercise 4 Question 6 (Page No. 192) top

Provide two programming examples in which multithreading does not provide better performance than a single-threaded solution.

galvin operating-system threads descriptive

11.52.12 Threads: Galvin Edition 9 Exercise 4 Question 7 (Page No. 192) top

Under what circumstances does a multithreaded solution using multiple kernel threads provide better performance than a single-threaded solution on a single-processor system ?

galvin operating-system threads descriptive

11.52.13 Threads: Galvin Edition 9 Exercise 4 Question 8 (Page No. 192) top

Which of the following components of program state are shared across threads in a multithreaded process ?
a. Register values
b. Heap memory
c. Global variables
d. Stack memory

galvin operating-system threads

Answer key

11.52.14 Threads: Galvin Edition 9 Exercise 4 Question 9 (Page No. 192) top

Can a multithreaded solution using multiple user-level threads achieve better performance on a multiprocessor system than on a single processor system ? Explain.

galvin operating-system threads descriptive

Answer key

11.52.15 Threads: Galvin Edition 9 Exercise 9 Question 36 (Page No. 456) top

A system provides support for user-level and kernel-level threads. The mapping in this system is one to one (there is a corresponding kernel thread for each user thread). Does a multithreaded process consist of (a) a working set for the entire process or (b) a working set for each thread ? Explain

galvin operating-system threads descriptive

Answer key

11.53 Timesharing System (1) top

11.53.1 Timesharing System: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 18 (Page No. 82) top

Why is the process table needed in a timesharing system? Is it also needed in personal computer systems running UNIX or Windows with a single user?

tanenbaum operating-system timesharing-system descriptive

11.54 Translation Lookaside Buffer (8) top

11.54.1 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 11 (Page No. 255) top

Consider the following C program:

```
int X[N];
int step = M; /* M is some predefined constant */
for (int i = 0; i < N; i += step) X[i] = X[i] + 1;
```

a. If this program is run on a machine with a $4-KB$ page size and 64-entry TLB, what values of M and N will cause a TLB miss for every execution of the inner loop?
b. Would your answer in part (a) be different if the loop were repeated many times? Explain.

tanenbaum operating-system memory-management paging translation-lookaside-buffer descriptive

Answer key

11.54.2 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 16 (Page No. 255) top

You are given the following data about a virtual memory system:

a. The TLB can hold 1024 entries and can be accessed in 1 clock cycle $(1\ nsec)$.
b. A page table entry can be found in 100 clock cycles or $100\ nsec$.
c. The average page replacement time is $6\ msec$.

If page references are handled by the TLB 99% of the time, and only 0.01% lead to a page fault, what is the effective address-translation time?

tanenbaum operating-system memory-management virtual-memory translation-lookaside-buffer descriptive

Answer key

11.54.3 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 22 (Page No. 256) top

A computer whose processes have 1024 pages in their address spaces keeps its page tables in memory. The overhead required for reading a word from the page table is $5\ nsec$. To reduce this overhead, the computer has a TLB, which holds 32 (virtual page, physical page frame) pairs, and can do a lookup in $1\ nsec$. What hit rate is needed to reduce the mean overhead to $2\ nsec$?

tanenbaum operating-system memory-management paging translation-lookaside-buffer descriptive

Answer key

11.54.4 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 23 (Page No. 256) top

How can the associative memory device needed for a TLB be implemented in hardware, and what are the implications of such a design for expandability?

tanenbaum operating-system memory-management paging translation-lookaside-buffer descriptive

11.54.5 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 53 (Page No. 261) top

Write a program that demonstrates the effect of TLB misses on the effective memory access time by measuring the per-access time it takes to stride through a large array.

a. Explain the main concepts behind the program, and describe what you expect the output to show for some practical virtual memory architecture.
b. Run the program on some computer and explain how well the data fit your expectations.
c. Repeat part (b) but for an older computer with a different architecture and explain any major differences in the output.

tanenbaum operating-system memory-management virtual-memory translation-lookaside-buffer descriptive

11.54.6 Translation Lookaside Buffer: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 55 (Page No. 261 - 262) top

Write a program that can be used to compare the effectiveness of adding a tag field to TLB entries when control is toggled between two programs. The tag field is used to effectively label each entry with the process id. Note that a nontagged TLB can be simulated by requiring that all TLB entries have the same tag at any one time. The inputs will be:

- The number of TLB entries available
- The clock interrupt interval expressed as number of memory references
- A file containing a sequence of (process, page references) entries
- The cost to update one TLB entry

a. Describe the basic data structures and algorithms in your implementation.
b. Show that your simulation behaves as expected for a simple (but nontrivial) input example.
c. Plot the number of TLB updates per 1000 references.

tanenbaum operating-system memory-management paging translation-lookaside-buffer descriptive

11.54.7 Translation Lookaside Buffer: Galvin Edition 9 Exercise 9 Question 14 (Page No. 452) top

Assume that a program has just referenced an address in virtual memory. Describe a scenario in which each of the following can occur. (If no such scenario can occur, explain why.)

• TLB miss with no page fault
• TLB miss and page fault
• TLB hit and no page fault
• TLB hit and page fault

galvin operating-system virtual-memory translation-lookaside-buffer descriptive

Answer key

11.54.8 Translation Lookaside Buffer: Galvin Edition 9 Exercise 9 Question 15 (Page No. 452) top

A simplified view of thread states is $Ready$, $Running$, and $Blocked$,where a thread is either ready and waiting to be scheduled, is running on the processor, or is blocked (for example, waiting for I/O). This is illustrated in Figure 9.31. Assuming a thread is in the Running state, answer the following questions, and explain your answer:

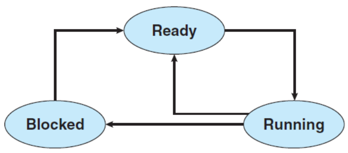

Figure 9.31 Thread state diagram for Exercise 9.15.

a. Will the thread change state if it incurs a page fault? If so, to what state will it change?
b. Will the thread change state if it generates a TLB miss that is resolved in the page table ? If so, to what state will it change?
c. Will the thread change state if an address reference is resolved in the page table? If so, to what state will it change?

galvin operating-system virtual-memory translation-lookaside-buffer descriptive

Answer key

11.55 Trap Instruction (1) top

11.55.1 Trap Instruction: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 17 (Page No. 82) top

What is a trap instruction? Explain its use in operating systems.

tanenbaum operating-system trap-instruction descriptive

Answer key

11.56 Unix (8) top

11.56.1 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 36 (Page No. 84) top

Examine and try to interpret the contents of a UNIX-like or Windows directory with a tool like the UNIX od program. (Hint: How you do this will depend upon what the OS allows. One trick that may work is to create a directory on a USB stick with one operating system and then read the raw device data using a different operating system that allows such access.)

tanenbaum operating-system unix descriptive

11.56.2 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 28 (Page No. 335) top

We discussed making incremental dumps in some detail in the text. In Windows it is easy to tell when to dump a file because every file has an archive bit. This bit is missing in $UNIX.$ How do $UNIX$ backup programs know which files to dump?

tanenbaum operating-system file-system unix descriptive

11.56.3 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 3 (Page No. 333) top

In early UNIX systems, executable files (a.out files) began with a very specific magic number, not one chosen at random. These files began with a header, followed by the text and data segments. Why do you think a very specific number was chosen for executable files, whereas other file types had a more-or-less random magic number as the first word?

tanenbaum operating-system file-system unix descriptive

11.56.4 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 30 (Page No. 335) top

It has been suggested that the first part of each UNIX file be kept in the same disk block as its i-node. What good would this do?

tanenbaum operating-system file-system unix descriptive

11.56.5 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 4 (Page No. 333) top

Is the open system call in $UNIX$ absolutely essential? What would the consequences be of not having it?

tanenbaum operating-system file-system unix descriptive

Answer key

11.56.6 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 45 (Page No. 336) top

Write a program that scans all directories in a $UNIX$ file system and finds and locates all i-nodes with a hard link count of two or more. For each such file, it lists together all file names that point to the file.

tanenbaum operating-system file-system disk unix descriptive

11.56.7 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 46 (Page No. 336) top

Write a new version of the UNIX ls program. This version takes as an argument one or more directory names and for each directory lists all the files in that directory, one line per file. Each field should be formatted in a reasonable way given its type. List only the first disk address, if any.

tanenbaum operating-system file-system unix descriptive

11.56.8 Unix: Andrew S. Tanenbaum (OS) Edition 4 Exercise 4 Question 47 (Page No. 336) top

Implement a program to measure the impact of application-level buffer sizes on read time. This involves writing to and reading from a large file (say, 2 GB). Vary the application buffer size (say, from 64 bytes to 4 KB). Use timing measurement routines (such as *gettimeofday* and *getitimer* on UNIX) to measure the time taken for different buffer sizes. Analyze the results and report your findings: does buffer size make a difference to the overall write time and per-write time?

tanenbaum operating-system file-system unix descriptive

11.57 Virtual Machines (1) top

11.57.1 Virtual Machines: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 32 (Page No. 83) top

Virtual machines have become very popular for a variety of reasons. Nevertheless, they have some downsides. Name one.

tanenbaum operating-system virtual-machines descriptive

11.58 Virtual Memory (34) top

11.58.1 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 1 Question 15 (Page No. 82) top

Consider a computer system that has cache memory, main memory (RAM) and disk, and an operating system that uses virtual memory. It takes 1 nsec to access a word from the cache, 10 nsec to access a word from the RAM, and 10 ms to access a word from the disk. If the cache hit rate is 95% and main memory hit rate (after a cache miss) is 99%, what is the average time to access a word?

tanenbaum operating-system virtual-memory descriptive

Answer key

11.58.2 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 12 (Page No. 255) top

The amount of disk space that must be available for page storage is related to the maximum number of processes, n, the number of bytes in the virtual address space, v, and the number of bytes of RAM, r. Give an expression for the worst-case disk-space requirements. How realistic is this amount?

tanenbaum operating-system memory-management virtual-memory descriptive

Answer key

11.58.3 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 5 (Page No. 254) top

What is the difference between a physical address and a virtual address?

tanenbaum operating-system memory-management virtual-memory descriptive

Answer key

11.58.4 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 6 (Page No. 254) top

For each of the following decimal virtual addresses, compute the virtual page number and offset for a $4-KB$ page and for an $8KB$ page: $20000, 32768, 60000.$

tanenbaum operating-system memory-management virtual-memory descriptive

11.58.5 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 7 (Page No. 254) top

Using the page table of Fig. $3-9$, give the physical address corresponding to each of the following virtual addresses:

a. 20
b. 4100
c. 8300

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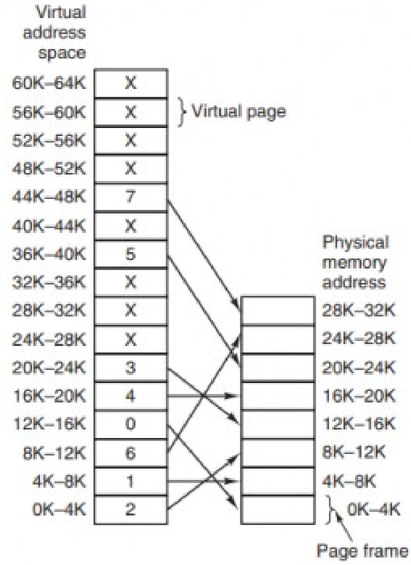


Figure 3-9. The relation between virtual addresses and physical memory addresses is given by the **page table**. Every page begins on a multiple of 4096 and ends 4095 addresses higher, so 4K–8K really means 4096–8191 and 8K to 12K means 8192–12287.

tanenbaum operating-system memory-management virtual-memory descriptive

11.58.6 Virtual Memory: Andrew S. Tanenbaum (OS) Edition 4 Exercise 3 Question 8 (Page No. 254) top

The Intel 8086 processor did not have an MMU or support virtual memory. Nevertheless, some companies sold systems that contained an unmodified 8086 CPU and did paging. Make an educated guess as to how they did it. (Hint: Think about the logical location of the MMU.)

tanenbaum operating-system memory-management virtual-memory descriptive

11.58.7 Virtual Memory: Galvin Edition 9 Exercise 9 Question 1 (Page No. 449) top

Under what circumstances do page faults occur? Describe the actions taken by the operating system when a page fault occurs.

galvin operating-system virtual-memory descriptive

11.58.8 Virtual Memory: Galvin Edition 9 Exercise 9 Question 10 (Page No. 451) top

You have devised a new page-replacement algorithm that you think may be optimal. In some contorted test cases, Belady's anomaly occurs. Is the new algorithm optimal ? Explain your answer.

galvin operating-system virtual-memory descriptive

11.58.9 Virtual Memory: Galvin Edition 9 Exercise 9 Question 11 (Page No. 451) top

Segmentation is similar to paging but uses variable-sized "pages." Define two segment-replacement algorithms, one based on the FIFO page replacement scheme and the other on the LRU page-replacement scheme. Remember that since segments are not the same size, the segment that is chosen for replacement may be too small to leave enough consecutive locations for the needed segment. Consider strategies for systems where segments cannot be relocated and strategies for systems where they can.

galvin operating-system virtual-memory descriptive

11.58.10 Virtual Memory: Galvin Edition 9 Exercise 9 Question 12 (Page No. 451) top

Consider a demand-paged computer system where the degree of multiprogramming is currently fixed at four. The system was recently measured to determine utilization of the CPU and the paging disk. Three alternative results are shown below. For each case, what is happening? Can the degree of multiprogramming be increased to increase the CPU utilization? Is the paging helping?

$a.\ CPU\ utilization\ 13\ percent;\ disk\ utilization\ 97\ percent$
$b.\ CPU\ utilization\ 87\ percent;\ disk\ utilization\ 3\ percent$
$c. CPU\ utilization\ 13\ percent;\ disk\ utilization\ 3\ percent$

galvin operating-system virtual-memory descriptive

11.58.11 Virtual Memory: Galvin Edition 9 Exercise 9 Question 13 (Page No. 452) top

We have an operating system for a machine that uses base and limit registers, but we have modified the machine to provide a page table. Can the page tables be set up to simulate base and limit registers ? How can they be, or why can they not be?

galvin operating-system virtual-memory descriptive

11.58.12 Virtual Memory: Galvin Edition 9 Exercise 9 Question 16 (Page No. 452-453) top

Consider a system that uses pure demand paging.

a. When a process first starts execution, how would you characterize the page-fault rate ?

b. Once the working set for a process is loaded into memory, how would you characterize the page-fault rate ?

c. Assume that a process changes its locality and the size of the new working set is too large to be stored in available free memory. Identify some options system designers could choose from to handle this situation.

galvin operating-system virtual-memory descriptive

11.58.13 Virtual Memory: Galvin Edition 9 Exercise 9 Question 17 (Page No. 453) top

What is the copy-on-write feature, and under what circumstances is its use beneficial ? What hardware support is required to implement this feature ?

galvin operating-system virtual-memory descriptive

11.58.14 Virtual Memory: Galvin Edition 9 Exercise 9 Question 18 (Page No. 453) top

A certain computer provides its users with a virtual memory space of 2^{32} bytes. The computer has 2^{22} bytes of physical memory. The virtual memory is implemented by paging, and the page size is $4,096$ bytes. A user process generates the virtual address 11123456. Explain how the system establishes the corresponding physical location. Distinguish between software and hardware operations.

galvin operating-system virtual-memory descriptive

11.58.15 Virtual Memory: Galvin Edition 9 Exercise 9 Question 2 (Page No. 449) top

Assume that you have a page-reference string for a process with m frames (initially all empty). The page-reference string has length p, and n distinct page numbers occur in it. Answer these questions for any page-replacement algorithms:

a. What is a $lower\ bound$ on the number of $page\ faults$?
b. What is an $upper\ bound$ on the number of $page\ faults$?

galvin operating-system virtual-memory descriptive

11.58.16 Virtual Memory: Galvin Edition 9 Exercise 9 Question 23 (Page No. 454) top

Assume that you are monitoring the rate at which the pointer in the clock algorithm moves. (The pointer indicates the candidate page for replacement.) What can you say about the system if you notice the following behavior:

a. Pointer is moving fast.
b. Pointer is moving slow.

galvin operating-system virtual-memory descriptive

11.58.17 Virtual Memory: Galvin Edition 9 Exercise 9 Question 24 (Page No. 454) top

Discuss situations in which the least frequently used (LFU) page replacement algorithm generates fewer page faults than the least recently used (LRU) page-replacement algorithm. Also discuss under what circumstances the opposite holds.

galvin operating-system virtual-memory descriptive

Answer key

11.58.18 Virtual Memory: Galvin Edition 9 Exercise 9 Question 25 (Page No. 454) top

Discuss situations in which the most frequently used (MFU) page replacement algorithm generates fewer page faults than the least recently used (LRU) page-replacement algorithm. Also discuss under what circumstances the opposite holds.

galvin operating-system virtual-memory descriptive

Answer key

11.58.19 Virtual Memory: Galvin Edition 9 Exercise 9 Question 26 (Page No. 455) top

The VAX/VMS system uses a FIFO replacement algorithm for resident pages and a free-frame pool of recently used pages. Assume that the free-frame pool is managed using the LRU replacement policy. Answer the following questions:
a. If a page fault occurs and the page does not exist in the free-frame pool, how is free space generated for the newly requested page ?
b. If a page fault occurs and the page exists in the free-frame pool, how is the resident page set and the free-frame pool managed to make space for the requested page ?
c. What does the system degenerate to if the number of resident pages is set to one ?
d. What does the system degenerate to if the number of pages in the free-frame pool is zero?

galvin operating-system virtual-memory descriptive

11.58.20 Virtual Memory: Galvin Edition 9 Exercise 9 Question 27 (Page No. 455) top

Consider a demand-paging system with the following time-measured utilizations:

$$CPU\ utilization\ 20\%$$
$$Paging\ disk\ 97.7\%$$
$$Other\ I/O\ devices\ 5\%$$

For each of the following, indicate whether it will (or is likely to) improve CPU utilization. Explain your answers.
a. Install a faster CPU.
b. Install a bigger paging disk.
c. Increase the degree of multiprogramming.
d. Decrease the degree of multiprogramming.
e. Install more main memory.
f. Install a faster hard disk or multiple controllers with multiple hard disks.
g. Add prepaging to the page-fetch algorithms.
h. Increase the page size.

galvin operating-system virtual-memory descriptive

11.58.21 Virtual Memory: Galvin Edition 9 Exercise 9 Question 28 (Page No. 455) top

Suppose that a machine provides instructions that can access memory locations using the one-level indirect addressing scheme. What sequence of page faults is incurred when all of the pages of a program are currently nonresident and the first instruction of the program is an indirect memory-load operation ? What happens when the operating system is using a per-process frame allocation technique and only two pages are allocated to this process ?

galvin operating-system virtual-memory descriptive

11.58.22 Virtual Memory: Galvin Edition 9 Exercise 9 Question 29 (Page No. 455) top

Suppose that your replacement policy (in a paged system) is to examine each page regularly and to discard that page if it has not been used since the last examination. What would you gain and what would you lose by using this policy rather than LRU or second-chance replacement ?

galvin operating-system virtual-memory descriptive

Answer key

11.58.23 Virtual Memory: Galvin Edition 9 Exercise 9 Question 30 (Page No. 455) top

A page-replacement algorithm should minimize the number of page faults. We can achieve this minimization by distributing heavily used pages evenly over all of memory, rather than having them compete for a small number of page frames. We can associate with each page frame a counter of the number of pages associated with that frame. Then,to replace a page, we can search for the page frame with the smallest counter

a. Define a page-replacement algorithm using this basic idea. Specifically address these problems:
i. What is the initial value of the counters ?
ii. When are counters increased ?
iii. When are counters decreased ?
iv. How is the page to be replaced selected ?

b. How many page faults occur for your algorithm for the following reference string with four page frames ?
$1, 2, 3, 4, 5, 3, 4, 1, 6, 7, 8, 7, 8, 9, 7, 8, 9, 5, 4, 5, 4, 2.$

c. What is the minimum number of page faults for an optimal page replacement strategy for the reference string in part b with four page frames?

galvin operating-system virtual-memory descriptive

11.58.24 Virtual Memory: Galvin Edition 9 Exercise 9 Question 31 (Page No. 455) top

Consider a demand-paging system with a paging disk that has an average access and transfer time of 20 milliseconds. Addresses are translated through a page table in main memory, with an access time of 1 microsecond per memory access. Thus, each memory reference through the page table takes two accesses. To improve this time, we have added an associative memory that reduces access time to one memory reference if the page-table entry is in the associative memory.

Assume that 80 percent of the accesses are in the associative memory and that, of those remaining, 10 percent (or 2 percent of the total) cause page faults. What is the effective memory access time?

galvin operating-system virtual-memory descriptive

11.58.25 Virtual Memory: Galvin Edition 9 Exercise 9 Question 32 (Page No. 455) top

What is the cause of thrashing ? How does the system detect thrashing ? Once it detects thrashing, what can the system do to eliminate this problem ?

galvin operating-system virtual-memory descriptive

Answer key

11.58.26 Virtual Memory: Galvin Edition 9 Exercise 9 Question 33 (Page No. 455) top

Is it possible for a process to have two working sets, one representing data and another representing code ? Explain.

galvin operating-system virtual-memory descriptive

Answer key

11.58.27 Virtual Memory: Galvin Edition 9 Exercise 9 Question 34 (Page No. 455) top

Consider the parameter$\triangle$ used to define the working-set window in the working-set model. When $\triangle$ is set to a small value, what is the effect on the page-fault frequency and the number of active (non suspended) processes currently executing in the system ? What is the effect when $\triangle$ is set to a very high value ?

galvin operating-system virtual-memory descriptive

Answer key

11.58.28 Virtual Memory: Galvin Edition 9 Exercise 9 Question 37 (Page No. 456) top

The slab-allocation algorithm uses a separate cache for each different object type. Assuming there is one cache per object type, explain why this scheme doesn't scale well with multiple CPUs. What could be done to address this scalability issue?

galvin operating-system virtual-memory descriptive

11.58.29 Virtual Memory: Galvin Edition 9 Exercise 9 Question 4 (Page No. 450) top

Consider the following page-replacement algorithms. Rank these algorithms on a five-point scale from “bad” to “perfect” according to their page-fault rate. Separate those algorithms that suffer from Belady's anomaly from those that do not.

a. LRU replacement
b. $FIFO$ replacement
c. Optimal replacement
d. Second-chance replacement

galvin operating-system virtual-memory descriptive

11.58.30 Virtual Memory: Galvin Edition 9 Exercise 9 Question 5 (Page No. 450) top

Discuss the hardware support required to support demand paging.

galvin operating-system virtual-memory descriptive

11.58.31 Virtual Memory: Galvin Edition 9 Exercise 9 Question 6 (Page No. 450) top

An operating system supports a paged virtual memory. The central processor has a cycle time of 1 microsecond. It costs an additional 1 microsecond to access a page other than the current one. Pages have $1,000$ words, and the paging device is a drum that rotates at $3,000$ revolutions per minute and transfers 1 million words per second. The following statistical measurements were obtained from the system:

• One percent of all instructions executed accessed a page other than the current page.

• Of the instructions that accessed another page, 80 percent accessed a page already in memory.

• When a new page was required, the replaced page was modified 50 percent of the time.

Calculate the effective instruction time on this system, assuming that the system is running one process only and that the processor is idle during drum transfers.

galvin operating-system virtual-memory descriptive

11.58.32 Virtual Memory: Galvin Edition 9 Exercise 9 Question 7 (Page No. 450-451) top

Consider the two-dimensional array A:

$$intA[][]= new\ int[100][100];$$

where $A[0][0]$ is at location 200 in a paged memory system with pages of size 200. A small process that manipulates the matrix resides in page 0 (locations 0 to 199). Thus, every instruction fetch will be from page 0.For three page frames, how many page faults are generated by the following array-initialization loops? Use LRU replacement, and assume that page frame 1 contains the process and the other two are initially empty.

a. $for(intj = 0; j < 100; j++)$
$for(inti = 0; i < 100; i++)$
$A[i][j] = 0;$

b. $for(inti = 0; i < 100; i++)$
$for(intj = 0; j < 100; j++)$
$A[i][j] = 0;$

galvin operating-system virtual-memory descriptive

11.58.33 Virtual Memory: Galvin Edition 9 Exercise 9 Question 8 (Page No. 451) top

Consider the following page reference string:
$1, 2, 3, 4, 2, 1, 5, 6, 2, 1, 2, 3, 7, 6, 3, 2, 1, 2, 3, 6.$
How many page faults would occur for the following replacement algorithms, assuming one, two, three, four, five, six, and seven frames ?
Remember that all frames are initially empty, so your first unique pages will cost one fault each.

•LRU replacement
• $FIFO$ replacement
• $Optimal$ replacement

galvin operating-system virtual-memory

11.58.34 Virtual Memory: Galvin Edition 9 Exercise 9 Question 9 (Page No. 451) top

Suppose that you want to use a paging algorithm that requires a reference bit (such as second-chance replacement or working-set model), but the hardware does not provide one. Sketch how you could simulate a reference bit even if one were not provided by the hardware, or explain why it is not possible to do so. If it is possible, calculate what the cost would be.

galvin operating-system virtual-memory

Answer Keys

11.0.1	N/A	11.0.2	N/A	11.0.3	N/A	11.0.4	N/A	11.0.5	N/A
11.0.6	N/A	11.0.7	N/A	11.0.8	N/A	11.0.9	N/A	11.0.10	N/A
11.0.11	N/A	11.0.12	N/A	11.0.13	N/A	11.0.14	N/A	11.0.15	N/A
11.0.16	N/A	11.0.17	N/A	11.0.18	Q-Q	11.0.19	N/A	11.0.20	N/A
11.0.21	N/A	11.0.22	N/A	11.0.23	N/A	11.0.24	N/A	11.0.25	N/A
11.0.26	N/A	11.0.27	N/A	11.0.28	N/A	11.0.29	N/A	11.0.30	N/A
11.0.31	N/A	11.0.32	N/A	11.0.33	N/A	11.0.34	N/A	11.0.35	N/A
11.0.36	N/A	11.0.37	N/A	11.0.38	N/A	11.0.39	N/A	11.0.40	N/A
11.0.41	N/A	11.0.42	N/A	11.0.43	N/A	11.0.44	N/A	11.0.45	N/A
11.0.46	N/A	11.0.47	N/A	11.0.48	N/A	11.0.49	N/A	11.0.50	N/A
11.0.51	N/A	11.0.52	N/A	11.0.53	N/A	11.0.54	N/A	11.0.55	N/A
11.0.56	N/A	11.0.57	N/A	11.0.58	N/A	11.0.59	N/A	11.0.60	N/A
11.0.61	N/A	11.0.62	N/A	11.0.63	N/A	11.0.64	N/A	11.0.65	N/A
11.0.66	N/A	11.0.67	N/A	11.0.68	N/A	11.0.69	N/A	11.1.1	N/A
11.2.1	N/A	11.3.1	N/A	11.4.1	N/A	11.5.1	N/A	11.5.2	N/A
11.6.1	N/A	11.6.2	N/A	11.6.3	N/A	11.6.4	N/A	11.6.5	N/A
11.6.6	N/A	11.6.7	N/A	11.6.8	N/A	11.6.9	N/A	11.6.10	N/A
11.6.11	N/A	11.6.12	N/A	11.6.13	N/A	11.6.14	N/A	11.6.15	N/A
11.6.16	N/A	11.6.17	N/A	11.6.18	N/A	11.6.19	N/A	11.6.20	N/A
11.6.21	N/A	11.6.22	N/A	11.6.23	N/A	11.6.24	N/A	11.6.25	N/A
11.6.26	N/A	11.6.27	N/A	11.6.28	N/A	11.6.29	N/A	11.6.30	N/A
11.6.31	N/A	11.6.32	N/A	11.6.33	N/A	11.6.34	N/A	11.6.35	N/A
11.6.36	N/A	11.6.37	N/A	11.6.38	N/A	11.6.39	N/A	11.6.40	Q-Q
11.6.41	Q-Q	11.6.42	Q-Q	11.6.43	Q-Q	11.6.44	Q-Q	11.6.45	Q-Q
11.6.46	Q-Q	11.6.47	Q-Q	11.6.48	Q-Q	11.6.49	N/A	11.6.50	Q-Q
11.6.51	Q-Q	11.6.52	Q-Q	11.6.53	Q-Q	11.6.54	N/A	11.6.55	N/A
11.6.56	N/A	11.6.57	Q-Q	11.6.58	Q-Q	11.6.59	Q-Q	11.6.60	Q-Q
11.6.61	Q-Q	11.6.62	Q-Q	11.7.1	N/A	11.8.1	N/A	11.8.2	N/A
11.8.3	N/A	11.8.4	N/A	11.8.5	N/A	11.8.6	N/A	11.8.7	N/A
11.8.8	N/A	11.8.9	N/A	11.8.10	N/A	11.8.11	N/A	11.8.12	N/A
11.8.13	Q-Q	11.8.14	N/A	11.9.1	N/A	11.10.1	N/A	11.10.2	N/A
11.10.3	N/A	11.10.4	N/A	11.10.5	N/A	11.10.6	N/A	11.10.7	N/A
11.11.1	N/A	11.11.2	N/A	11.12.1	N/A	11.13.1	N/A	11.13.2	N/A

11.13.3	N/A	11.13.4	N/A	11.13.5	N/A	11.13.6	N/A	11.13.7	N/A
11.13.8	N/A	11.13.9	N/A	11.13.10	N/A	11.13.11	N/A	11.13.12	N/A
11.13.13	N/A	11.13.14	N/A	11.13.15	N/A	11.13.16	N/A	11.13.17	N/A
11.13.18	N/A	11.13.19	N/A	11.13.20	N/A	11.13.21	N/A	11.13.22	N/A
11.13.23	N/A	11.13.24	N/A	11.13.25	N/A	11.13.26	N/A	11.13.27	N/A
11.13.28	N/A	11.13.29	N/A	11.13.30	N/A	11.13.31	N/A	11.13.32	N/A
11.13.33	N/A	11.13.34	N/A	11.13.35	N/A	11.13.36	N/A	11.13.37	Q-Q
11.13.38	N/A	11.13.39	N/A	11.13.40	N/A	11.13.41	N/A	11.13.42	N/A
11.13.43	Q-Q	11.13.44	Q-Q	11.13.45	N/A	11.13.46	N/A	11.13.47	N/A
11.13.48	Q-Q	11.13.49	Q-Q	11.13.50	Q-Q	11.13.51	Q-Q	11.13.52	N/A
11.13.53	N/A	11.13.54	N/A	11.13.55	N/A	11.14.1	N/A	11.15.1	N/A
11.16.1	N/A	11.17.1	N/A	11.17.2	N/A	11.17.3	N/A	11.17.4	N/A
11.17.5	N/A	11.17.6	N/A	11.17.7	N/A	11.17.8	N/A	11.17.9	N/A
11.17.10	N/A	11.17.11	N/A	11.17.12	N/A	11.17.13	N/A	11.17.14	N/A
11.17.15	N/A	11.17.16	N/A	11.17.17	N/A	11.17.18	N/A	11.17.19	N/A
11.17.20	N/A	11.17.21	N/A	11.17.22	N/A	11.17.23	N/A	11.17.24	N/A
11.17.25	N/A	11.17.26	N/A	11.17.27	N/A	11.17.28	N/A	11.17.29	N/A
11.17.30	N/A	11.17.31	N/A	11.17.32	N/A	11.17.33	N/A	11.17.34	N/A
11.17.35	N/A	11.17.36	N/A	11.17.37	N/A	11.17.38	N/A	11.17.39	N/A
11.17.40	N/A	11.17.41	N/A	11.17.42	N/A	11.17.43	N/A	11.17.44	N/A
11.17.45	N/A	11.17.46	N/A	11.17.47	N/A	11.17.48	N/A	11.17.49	N/A
11.17.50	N/A	11.18.1	N/A	11.19.1	N/A	11.20.1	N/A	11.21.1	N/A
11.21.2	N/A	11.21.3	N/A	11.21.4	N/A	11.21.5	N/A	11.21.6	N/A
11.21.7	N/A	11.21.8	N/A	11.21.9	N/A	11.21.10	N/A	11.21.11	N/A
11.21.12	N/A	11.21.13	N/A	11.21.14	N/A	11.21.15	N/A	11.21.16	N/A
11.21.17	N/A	11.22.1	Q-Q	11.23.1	N/A	11.24.1	N/A	11.25.1	N/A
11.26.1	N/A	11.26.2	N/A	11.26.3	N/A	11.26.4	N/A	11.26.5	N/A
11.26.6	N/A	11.26.7	N/A	11.26.8	N/A	11.26.9	Q-Q	11.26.10	Q-Q
11.26.11	Q-Q	11.26.12	N/A	11.26.13	N/A	11.26.14	N/A	11.26.15	N/A
11.26.16	N/A	11.26.17	N/A	11.26.18	N/A	11.26.19	Q-Q	11.26.20	Q-Q
11.26.21	Q-Q	11.26.22	Q-Q	11.26.23	Q-Q	11.26.24	N/A	11.26.25	N/A
11.26.26	Q-Q	11.26.27	Q-Q	11.26.28	N/A	11.26.29	Q-Q	11.26.30	N/A
11.26.31	Q-Q	11.26.32	Q-Q	11.26.33	N/A	11.26.34	N/A	11.26.35	N/A
11.26.36	N/A	11.26.37	N/A	11.27.1	N/A	11.27.2	N/A	11.28.1	N/A
11.29.1	N/A	11.30.1	N/A	11.31.1	N/A	11.32.1	N/A	11.32.2	N/A
11.33.1	N/A	11.33.2	N/A	11.34.1	N/A	11.35.1	N/A	11.35.2	N/A
11.35.3	N/A	11.35.4	N/A	11.35.5	N/A	11.35.6	N/A	11.35.7	N/A
11.35.8	Q-Q	11.35.9	N/A	11.35.10	Q-Q	11.36.1	N/A	11.36.2	N/A
11.36.3	N/A	11.36.4	N/A	11.36.5	N/A	11.36.6	N/A	11.36.7	N/A
11.36.8	N/A	11.36.9	N/A	11.36.10	N/A	11.36.11	N/A	11.36.12	N/A
11.36.13	N/A	11.36.14	N/A	11.37.1	N/A	11.37.2	N/A	11.37.3	N/A

11.37.4	N/A	11.37.5	N/A	11.37.6	N/A	11.37.7	N/A	11.37.8	N/A
11.37.9	N/A	11.37.10	N/A	11.37.11	N/A	11.37.12	N/A	11.37.13	N/A
11.37.14	Q-Q	11.38.1	N/A	11.39.1	Q-Q	11.39.2	N/A	11.39.3	N/A
11.39.4	Q-Q	11.39.5	Q-Q	11.39.6	Q-Q	11.39.7	N/A	11.39.8	N/A
11.39.9	Q-Q	11.39.10	N/A	11.39.11	Q-Q	11.39.12	N/A	11.39.13	N/A
11.39.14	N/A	11.39.15	N/A	11.39.16	N/A	11.39.17	N/A	11.39.18	N/A
11.40.1	N/A	11.40.2	N/A	11.40.3	N/A	11.40.4	N/A	11.40.5	N/A
11.40.6	N/A	11.40.7	N/A	11.40.8	N/A	11.40.9	N/A	11.40.10	N/A
11.40.11	N/A	11.40.12	N/A	11.40.13	N/A	11.40.14	N/A	11.40.15	N/A
11.40.16	N/A	11.40.17	N/A	11.40.18	N/A	11.40.19	N/A	11.40.20	N/A
11.40.21	N/A	11.40.22	N/A	11.40.23	N/A	11.40.24	N/A	11.40.25	N/A
11.40.26	N/A	11.40.27	N/A	11.40.28	N/A	11.40.29	N/A	11.40.30	N/A
11.40.31	N/A	11.40.32	N/A	11.40.33	N/A	11.40.34	N/A	11.40.35	N/A
11.40.36	N/A	11.40.37	N/A	11.40.38	N/A	11.40.39	N/A	11.40.40	N/A
11.40.41	N/A	11.40.42	N/A	11.40.43	N/A	11.40.44	N/A	11.40.45	N/A
11.40.46	N/A	11.40.47	N/A	11.40.48	N/A	11.40.49	N/A	11.40.50	N/A
11.40.51	N/A	11.40.52	N/A	11.40.53	N/A	11.40.54	N/A	11.41.1	N/A
11.41.2	N/A	11.41.3	N/A	11.41.4	N/A	11.41.5	N/A	11.41.6	N/A
11.41.7	N/A	11.41.8	Q-Q	11.41.9	N/A	11.41.10	N/A	11.41.11	N/A
11.41.12	N/A	11.41.13	N/A	11.41.14	N/A	11.41.15	N/A	11.41.16	N/A
11.41.17	N/A	11.41.18	N/A	11.41.19	N/A	11.41.20	N/A	11.41.21	N/A
11.41.22	N/A	11.41.23	N/A	11.41.24	N/A	11.41.25	N/A	11.41.26	N/A
11.41.27	N/A	11.41.28	N/A	11.42.1	N/A	11.42.2	N/A	11.42.3	N/A
11.42.4	N/A	11.42.5	N/A	11.42.6	N/A	11.42.7	N/A	11.42.8	N/A
11.42.9	N/A	11.42.10	N/A	11.42.11	N/A	11.42.12	N/A	11.42.13	N/A
11.42.14	N/A	11.42.15	N/A	11.42.16	N/A	11.42.17	N/A	11.42.18	N/A
11.42.19	N/A	11.42.20	N/A	11.42.21	N/A	11.42.22	N/A	11.42.23	N/A
11.42.24	N/A	11.42.25	N/A	11.42.26	N/A	11.42.27	N/A	11.42.28	N/A
11.42.29	N/A	11.42.30	N/A	11.42.31	N/A	11.43.1	Q-Q	11.43.2	Q-Q
11.44.1	N/A	11.45.1	N/A	11.45.2	N/A	11.46.1	N/A	11.46.2	N/A
11.47.1	N/A	11.47.2	N/A	11.48.1	N/A	11.49.1	N/A	11.50.1	N/A
11.51.1	N/A	11.51.2	N/A	11.51.3	N/A	11.51.4	N/A	11.51.5	N/A
11.51.6	N/A	11.51.7	N/A	11.52.1	N/A	11.52.2	N/A	11.52.3	N/A
11.52.4	N/A	11.52.5	N/A	11.52.6	Q-Q	11.52.7	N/A	11.52.8	N/A
11.52.9	N/A	11.52.10	N/A	11.52.11	N/A	11.52.12	N/A	11.52.13	Q-Q
11.52.14	N/A	11.52.15	N/A	11.53.1	N/A	11.54.1	N/A	11.54.2	N/A
11.54.3	N/A	11.54.4	N/A	11.54.5	N/A	11.54.6	N/A	11.54.7	N/A
11.54.8	N/A	11.55.1	N/A	11.56.1	N/A	11.56.2	N/A	11.56.3	N/A
11.56.4	N/A	11.56.5	N/A	11.56.6	N/A	11.56.7	N/A	11.56.8	N/A
11.57.1	N/A	11.58.1	N/A	11.58.2	N/A	11.58.3	N/A	11.58.4	N/A
11.58.5	N/A	11.58.6	N/A	11.58.7	N/A	11.58.8	N/A	11.58.9	N/A

11.58.10	N/A	11.58.11	N/A	11.58.12	N/A	11.58.13	N/A	11.58.14	N/A
11.58.15	N/A	11.58.16	N/A	11.58.17	N/A	11.58.18	N/A	11.58.19	N/A
11.58.20	N/A	11.58.21	N/A	11.58.22	N/A	11.58.23	N/A	11.58.24	N/A
11.58.25	N/A	11.58.26	N/A	11.58.27	N/A	11.58.28	N/A	11.58.29	N/A
11.58.30	N/A	11.58.31	N/A	11.58.32	N/A	11.58.33	Q-Q	11.58.34	Q-Q

12.0.1 Michael Sipser Edition 3 Exercise 0 Question 14 (Page No. 28) top

Ramsey's theorem: Let G be a graph. A clique in G is a sub-graph in which every two nodes are connected by an edge. An anti-clique also called an independent set, is a sub-graph in which every two nodes are not connected by an edge. Show that every graph with n nodes contains either a clique or an anti-clique with at least $\frac{1}{2}log_2n$ nodes.

michael-sipser theory-of-computation graph-theory proof

12.0.2 Michael Sipser Edition 3 Exercise 0 Question 8 (Page No. 26) top

Consider the undirected graph G= (V, E) where V , the set of nodes, is {1, 2, 3, 4} and E, the set of edges, is {{1, 2}, {2, 3}, {1, 3}, {2, 4}, {1, 4}}. Draw the graph G. What are the degrees of each node? Indicate a path from node 3 to node 4 on your drawing of G.

michael-sipser theory-of-computation graph-theory easy

12.0.3 Michael Sipser Edition 3 Exercise 0 Question 15 (Page No. 28) top

Use Theorem 0.25 to derive a formula for calculating the size of the monthly payment for a mortgage in terms of the principal P, the interest rate I, and the number of payments, t. Assume that after t payments have been made, the loan amount is reduced to 0. Use the formula to calculate the dollar amount of each monthly payment for a $30-$year mortgage with 360 monthly payments on an initial loan amount of $\$100,000$ with a 5% annual interest rate.

Hint: Theorem 0.25 are explained in page number $24-25$.

michael-sipser theory-of-computation descriptive

12.0.4 Michael Sipser Edition 3 Exercise 0 Question 12 (Page No. 27) top

Find the error in the following proof that all horses are the same color.
CLAIM: In any set of h horses, all horses are the same color.
PROOF: By induction on h.
Basis: For $h = 1$. In any set containing just one horse, all horses clearly are the same color.

Induction step: For $k \geq 1$, assume that the claim is true for $h = k$ and prove that it is true for $h = k + 1$. Take any set H of $k + 1$ horses. We show that all the horses in this set are the same color. Remove one horse from this set to obtain the set H_1 with just k horses. By the induction hypothesis, all the horses in H_1 are the same color. Now replace the removed horse and remove a different one to obtain the set H_2. By the same argument, all the horses in H_2 are the same color. Therefore, all the horses in H must be the same color, and the proof is complete.

michael-sipser theory-of-computation descriptive proof

Answer key

12.0.5 Michael Sipser Edition 3 Exercise 0 Question 11 (Page No. 27) top

Let $S(n) = 1 + 2 + \cdots + n$ be the sum of the first n natural numbers and let $C(n) = 1^3 + 2^3 + \cdots + n^3$ be the sum of the first n cubes. Prove the following equalities by induction on n, to arrive at the curious conclusion that $C(n) = S^2(n)$ for every n.

a. $S(n) = \frac{1}{2}n(n + 1)$.
b. $C(n) = \frac{1}{4}(n^4 + 2n^3 + n^2 = \frac{1}{4}n^2(n + 1)^2$.

michael-sipser theory-of-computation proof

Answer key

12.0.6 Michael Sipser Edition 3 Exercise 0 Question 13 (Page No. 27) top

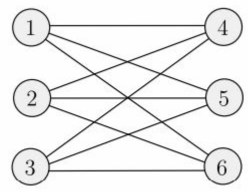
Show that every graph with two or more nodes contains two nodes that have equal degrees.

michael-sipser theory-of-computation graph-theory proof

Answer key

12.0.7 Michael Sipser Edition 3 Exercise 0 Question 9 (Page No. 27) top

Write a formal description of the following graph.



michael-sipser theory-of-computation graph-theory easy

12.0.8 Michael Sipser Edition 3 Exercise 0 Question 10 (Page No. 27) top

Find the error in the following proof that $2 = 1$.Consider the equation $a = b$. Multiply both sides by a to obtain $a^2 = ab$. Subtract b^2 from both sides to get $a^2 - b^2 = ab - b^2$. Now factor each side ,$(a + b)(a - b) = b(a - b)$, and divide each side by $(a - b)$ to get $a + b = b$. Finally, let a and b equal 1, which shows that $2 = 1$.

michael-sipser theory-of-computation proof

12.0.9 Michael Sipser Edition 3 Exercise 0 Question 2 (Page No. 25) top

Write formal descriptions of the following sets.
a. The set containing the numbers 1, 10, and 100
b. The set containing all integers that are greater than 5
c. The set containing all natural numbers that are less than 5
d. The set containing the string aba
e. The set containing the empty string
f. The set containing nothing at all

michael-sipser theory-of-computation easy

12.0.10 Michael Sipser Edition 3 Exercise 0 Question 1 (Page No. 25) top

Examine the following formal descriptions of sets so that you understand which members they contain. Write a short informal English description of each set.
a. {1, 3, 5, 7, . . . }
b. { . . . , −4, −2, 0, 2, 4, . . . }
c. {n| n = 2m for some m in N }
d. {n| n = 2m for some m in N , and n = 3k for some k in N }
e. {w| w is a string of 0s and 1s and w equals the reverse of w}
f. {n| n is an integer and n = n + 1}

michael-sipser theory-of-computation easy

12.1 Ambiguous Grammar (1) top

12.1.1 Ambiguous Grammar: Michael Sipser Edition 3 Exercise 2 Question 27 (Page No. 157) top

***2.27** Let $G = (V, \Sigma, R, \langle \text{STMT} \rangle)$ be the following grammar.

$$
\begin{aligned}
\langle \text{STMT} \rangle &\to \langle \text{ASSIGN} \rangle \mid \langle \text{IF-THEN} \rangle \mid \langle \text{IF-THEN-ELSE} \rangle \\
\langle \text{IF-THEN} \rangle &\to \texttt{if condition then } \langle \text{STMT} \rangle \\
\langle \text{IF-THEN-ELSE} \rangle &\to \texttt{if condition then } \langle \text{STMT} \rangle \texttt{ else } \langle \text{STMT} \rangle \\
\langle \text{ASSIGN} \rangle &\to \texttt{a:=1}
\end{aligned}
$$

$$\Sigma = \{\texttt{if}, \texttt{condition}, \texttt{then}, \texttt{else}, \texttt{a:=1}\}$$
$$V = \{\langle \text{STMT} \rangle, \langle \text{IF-THEN} \rangle, \langle \text{IF-THEN-ELSE} \rangle, \langle \text{ASSIGN} \rangle\}$$

G is a natural-looking grammar for a fragment of a programming language, but G is ambiguous.

a. Show that G is ambiguous.
b. Give a new unambiguous grammar for the same language .

michael-sipser theory-of-computation context-free-grammar ambiguous-grammar

Answer key

12.2 Closure Property (31) top

12.2.1 Closure Property: Michael Sipser Edition 3 Exercise 2 Question 49 (Page No. 159) top

We defined the rotational closure of language A to be $RC(A) = \{yx \mid xy \in A\}$.Show that the class of CFLs is closed under rotational closure.

michael-sipser theory-of-computation context-free-language closure-property descriptive

Answer key

12.2.2 Closure Property: Michael Sipser Edition 3 Exercise 2 Question 50 (Page No. 159) top

We defined the CUT of language A to be $CUT(A) = \{yxz|xyz \in A\}$. Show that the class of $CFLs$ is not closed under CUT.

michael-sipser theory-of-computation context-free-language closure-property descriptive

12.2.3 Closure Property: Michael Sipser Edition 3 Exercise 2 Question 53 (Page No. 159) top

Show that the class of DCFLs is not closed under the following operations:

a. Union b. Intersection c. Concatenation d. Star e. Reversal

michael-sipser theory-of-computation context-free-language closure-property descriptive

Answer key

12.2.4 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 10 (Page No. 109) top

Let $L_1 = L(a^*baa^*)$ and $L_2 = L(aba^*)$. Find L_1/L_2.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.5 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 11 (Page No. 109) top

Show that $L_1 = L_1L_2/L_2$ is not true for all languages L_1 and L_2.

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12.2.6 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 12 (Page No. 109) top

Suppose we know that $L_1 \cup L_2$ is regular and that L_1 is finite. Can we conclude from this that L_2 is regular?

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12.2.7 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 13 (Page No. 109) top

If L is a regular language, prove that $L_1 = \{uv : u \in L, |v| = 2\}$ is also regular.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

Answer key

12.2.8 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 14 (Page No. 109) top

If L is a regular language, prove that the language $\{uv : u \in L, v \in L^R\}$ is also regular.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

Answer key

12.2.9 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 15 (Page No. 110) top

The left quotient of a language L_1 with respect to L_2 is defined as

$$L_2/L_1 = \{y : x \in L_2, xy \in L_1\}$$

Show that the family of regular languages is closed under the left quotient with a regular language.

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12.2.10 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 16 (Page No. 110) top

Show that if the statement "If L_1 is regular and $L_1 \cup L_2$ is also regular, then L_2 must be regular" were true for all L_1 and L_2, then all languages would be regular.

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12.2.11 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 17 (Page No. 110) top

The $tail$ of a language is defined as the set of all suffixes of its strings, that is,

$$tail(L) = \{y : xy \in L \text{ for some } x \in \Sigma^*\}$$

Show that if L is regular, so is $tail(L)$.

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12.2.12 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 18 (Page No. 110) top

The $head$ of a language is the set of all prefixes of its strings, that is,

$$head(L) = \{x : xy \in L \text{ for some } y \in \Sigma^*\}$$

Show that the family of regular languages is closed under this operation.

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12.2.13 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 19 (Page No. 110) top

Define an operation $third$ on strings and languages as

$$third(a_1a_2a_3a_4a_5a_6 \dots) = a_3a_6 \dots$$

with the appropriate extension of this definition to languages. Prove the closure of the family of regular languages under this operation.

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12.2.14 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 2 (Page No. 108) top

Find nfa's that accept
(a) $L((a+b)a^*) \cap L(baa^*)$.
(b) $L(ab^*a^*) \cap L(a^*b^*a)$.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

Answer key

12.2.15 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 20 (Page No. 110) top

For a string $a_1a_2 \dots a_n$ define the operation $shift$ as

$$shift(a_1a_2 \dots a_n) = a_2a_3 \dots a_na_1$$

From this, we can define the operation on a language as

$$shift(L) = \{v : v = shift(w \text{ for some } w \in L\}$$

Show that regularity is preserved under the shift operation.

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12.2.16 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 21 (Page No. 110) top

Define

$exchange(a_1a_2a_3 \dots a_{n-1}a_n) = a_na_2a_3 \dots a_{n-1}a_1$,

and

$exchange(L) = \{v : v = exchange(w)$ for some $w \in L\}$

Show that the family of regular languages is closed under exchange.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.17 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 22 (Page No. 110) top

The *shuffle* of two languages L_1 and L_2 is defined as

$$shuffle(L_1, L_2) = \{w_1v_1w_2v_2w_3v_3 \dots w_mv_m : w_1w_2w_3 \dots . w_m \in L_1, v_1v_2 \dots v_m \in L_2, \text{ for all } w_i, v_i \in \Sigma^*\}.$$

Show that the family of regular languages is closed under the $shuffle$ operation.

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12.2.18 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 23 (Page No. 111) top

Define an operation $minus5$ on a language L as the set of all strings of L with the fifth symbol from the left removed (strings of length less than five are left unchanged). Show that the family of regular languages is closed under the $minus5$ operation.

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12.2.19 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 24 (Page No. 111) top

Define the operation $leftside$ on L by

$$leftside(L) = \{w : ww^R \in L\}$$

Is the family of regular languages closed under this operation?

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.20 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 25 (Page No. 111) top

The min of a language L is defined as

$$min(L) = \{w \in L : \text{there is no } u \in L, v \in \Sigma^+, \text{ such that } w = uv\}$$

Show that the family of regular languages is closed under the min operation.

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12.2.21 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 26 (Page No. 110) top

Let G_1 and G_2 be two regular grammars. Show how one can derive regular grammars for the languages
(a) $L(G_1) \cup L(G_2)$.
(b) $L(G_1)L(G_2)$.
(b) $L(G_1)^*$.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.22 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 3 (Page No. 108) top

"The family of regular languages is closed under difference."

Provide constructive proof for this argument.

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12.2.23 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 5 (Page No. 109) top

Show that the family of regular languages is closed under finite union and intersection, that is, if $L_1, L_2, \ldots, L_n$ are regular, then

$$L_U = \bigcup_{i=1,2,..,n} L_i$$

and

$$L_I = \bigcap_{i=1,2,3,\ldots,n} L_i$$

are also regular.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.24 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 6 (Page No. 109) top

The symmetric difference of two sets S_1 and S_2 is defined as $S_1 \theta S_2 = \{x : x \in S_1$ or $x \in S_2$, but x is not in both S_1 and $S_2\}$.
Show that the family of regular languages is closed under symmetric difference.

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12.2.25 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 7 (Page No. 109) top

The *nor* of two languages is

$$nor(L_1, L_2) = \{w : w \notin L_1 \text{ and } w \notin L_2\}$$

Show that the family of regular languages is closed under the *nor* operation.

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

12.2.26 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 8 (Page No. 109) top

Define the $complementary$ or (cor) of two languages by

$$cor(L_1, L_2) = \{w : w \in \overline{L_1} \text{ or } w \in \overline{L_2}\}$$

Show that the family of regular languages is closed under the *cor* operation.

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12.2.27 Closure Property: Peter Linz Edition 4 Exercise 4.1 Question 9 (Page No. 109) top

Which of the following are true for all regular languages and all homomorphisms?
(a) $h(L_1 \cup L_2) = h(L_1) \cap h(L_2)$.
(b) $h(L_1 \cap L_2) = h(L_1) \cap h(L_2)$.
(c) $h(L_1 L_2) = h(L_1) h(L_2)$.

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12.2.28 Closure Property: Peter Linz Edition 4 Exercise 4.3 Question 21 (Page No. 124) top

Let P be an infinite but countable set, and associate with each $p \in P$ a language L_p. The smallest set containing every L_p is the union over the infinite set P; it will be denoted by $U_{p \in p} L_p$. Show by example that the family of regular languages is not closed under infinite union.

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12.2.29 Closure Property: Peter Linz Edition 4 Exercise 4.3 Question 22 (Page No. 124) top

Consider the argument that the language associated with any generalized transition graph is regular. The language associated with such a graph is

$$L = \bigcup_{p \in P} L(r_p),$$

where P is the set of all walks through the graph and r_p is the expression associated with a walk p. The set of walks is generally infinite, so that in light of Exercise 21, it does not immediately follow that L is regular. Show that in this case, because of the special nature of P, the infinite union is regular.

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12.2.30 Closure Property: Peter Linz Edition 4 Exercise 4.3 Question 23 (Page No. 124) top

Is the family of regular languages closed under infinite intersection?

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

Answer key

12.2.31 Closure Property: Peter Linz Edition 4 Exercise 4.3 Question 24 (Page No. 124) top

Suppose that we know that $L_1 \cup L_2$ and L_1 are regular. Can we conclude from this that L_2 is regular?

peter-linz peter-linz-edition4 theory-of-computation regular-language closure-property

Answer key

12.3 Computability (1) top

12.3.1 Computability: Michael Sipser Edition 3 Exercise 5 Question 16 (Page No. 240) top

Let $\Gamma = \{0, 1, \sqcup\}$ be the tape alphabet for all TMs in this problem. Define the busy **beaver function** $BB : N \rightarrow N$ as follows. For each value of k, consider all $k-$state TMs that halt when started with a blank tape. Let $BB(k)$ be the maximum number of $1s$ that remain on the tape among all of these machines. Show that BB is not a computable function.

michael-sipser theory-of-computation turing-machine computability proof

12.4 Conjunctive Normal Form (3) top

12.4.1 Conjunctive Normal Form: Michael Sipser Edition 3 Exercise 2 Question 14 (Page No. 156) top

Convert the following CFG into an equivalent CFG in Chomsky normal form,using the procedure given in Theorem 2.9.

- $A \rightarrow BAB \mid B \mid \epsilon$
- $B \rightarrow 00 \mid \epsilon$

michael-sipser theory-of-computation context-free-grammar conjunctive-normal-form

Answer key

12.4.2 Conjunctive Normal Form: Michael Sipser Edition 3 Exercise 2 Question 26 (Page No. 157) top

Show that if G is a CFG in Chomsky normal form, then for any string $w \in L(G)$ of length $n \geq 1$, exactly $2n - 1$ steps are required for any derivation of w.

michael-sipser theory-of-computation context-free-grammar conjunctive-normal-form proof

Answer key

12.4.3 Conjunctive Normal Form: Michael Sipser Edition 3 Exercise 2 Question 35 (Page No. 157) top

Let G be a CFG in Chomsky normal form that contains b variables. Show that if G generates some string with a derivation having at least 2^b steps, $L(G)$ is infinite.

michael-sipser theory-of-computation context-free-language conjunctive-normal-form proof

Answer key

12.5 Context Free Grammar (100) top

12.5.1 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 13 (Page No. 156) top

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}; \Sigma = \{0, \#\};$ and R is the set of rules:

- $S \rightarrow TT \mid U$
- $T \rightarrow 0T \mid T0 \mid \#$
- $U \rightarrow 0U00 \mid \#$

a. Describe $L(G)$ in English.
b. Prove that $L(G)$ is not regular.

michael-sipser theory-of-computation context-free-grammar regular-language

Answer key

12.5.2 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 15 (Page No. 156) top

Give a counterexample to show that the following construction fails to prove that the class of context-free languages is closed under star. Let A be a CFL that is generated by the CFG $G = (V, \Sigma, R, S)$. Add the new rule $S \rightarrow SS$ and call the resulting grammar G'. This grammar is supposed to generate A^*.

michael-sipser theory-of-computation context-free-language context-free-grammar

12.5.3 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 19 (Page No. 156) top

Let $CFG \quad G$ be the following grammar.

- $S \rightarrow aSb \mid bY \mid Ya$
- $Y \rightarrow bY \mid aY \mid \epsilon$

Give a simple description of $L(G)$ in English. Use that description to give a CFG for $\overline{L(G)}$, the complement of $L(G)$.

michael-sipser theory-of-computation context-free-grammar context-free-language

12.5.4 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 21 (Page No. 156) top

Let $\Sigma = \{a, b\}$. Give a CFG generating the language of strings with twice as many $a's$ as $b's$. Prove that your grammar is correct.

michael-sipser theory-of-computation context-free-grammar context-free-language

Answer key

12.5.5 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 22 (Page No. 156) top

Let $C = \{x\#y \mid x, y \in \{0,1\}^* \text{ and } x \neq y\}$. Show that C is a context-free language.

michael-sipser theory-of-computation context-free-grammar

Answer key

12.5.6 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 28 (Page No. 157) top

Give unambiguous $CFG's$ for the following languages.

a. $\{w|$ in every prefix of w the number of $a's$ is at least the number of $b's\}$
b. $\{w|$ the number of $a's$ and the number of $b's$ in w are equal$\}$
c. $\{w|$ the number of $a's$ is at least the number of $b's$ in $w\}$

michael-sipser theory-of-computation context-free-grammar

Answer key

12.5.7 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 3 (Page No. 155) top

Answer each part for the following context-free grammar G.

$$R \rightarrow XRX|S$$
$$S \rightarrow aTb|bTa$$
$$T \rightarrow XTX|X|\epsilon$$
$$X \rightarrow a|b$$

a. What are the variables of G?
b. What are the terminals of G?
c. Which is the start variable of G?
d. Give three strings in $L(G)$.

e. Give three strings not in $L(G)$.
f. True or False: $T \Rightarrow aba$.
g. True or False: $T \stackrel{*}{\Rightarrow} aba$.
h. True or False: $T \Rightarrow T$.
i. True or False: $T \stackrel{*}{\Rightarrow} T$.
j. True or False: $XXX \stackrel{*}{\Rightarrow} aba$.
k. True or False: $X \stackrel{*}{\Rightarrow} aba$.
l. True or False: $T \stackrel{*}{\Rightarrow} XX$.
m. True or False: $T \stackrel{*}{\Rightarrow} XXX$.
n. True or False: $S \stackrel{*}{\Rightarrow} \epsilon$.
o. Give a description in English of $L(G)$.

michael-sipser theory-of-computation context-free-grammar descriptive

12.5.8 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 4 (Page No. 155) top

Give context-free grammars that generate the following languages. In all parts, the alphabet $\sum$ is $\{0, 1\}$.

a. {w| w contains at least three 1's}
b. {w| w starts and ends with the same symbol}
c. {w| the length of w is odd}
d. {w| the length of w is odd and its middle symbol is a 0}
e. $\{w|w = w^R$,that is, w is a palindrome$\}$
f. The empty set.

michael-sipser theory-of-computation context-free-language context-free-grammar descriptive

Answer key

12.5.9 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 46 (Page No. 158) top

Consider the following CFG G :

- $S \rightarrow SS \mid T$
- $T \rightarrow aTb \mid ab$

Describe $L(G)$ and show that G is ambiguous. Give an unambiguous grammar H where $L(H) = L(G)$ and sketch a proof that H is unambiguous.

michael-sipser theory-of-computation context-free-grammar ambiguous proof

12.5.10 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 51 (Page No. 159) top

Show that every DCFG is an unambiguous CFG.

michael-sipser theory-of-computation context-free-grammar ambiguous proof

12.5.11 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 54 (Page No. 159) top

Let G be the following grammar:

- $S \rightarrow T \dashv$
- $T \rightarrow TaTb \mid TbTa|\epsilon$

a. Show that $L(G) = \{w \dashv \mid w \text{ contains equal numbers of a's and b's}\}$. Use a proof by induction on the length of w.
b. Use the DK-test to show that G is a DCFG.
c. Describe a DPDA that recognizes L(G).

michael-sipser theory-of-computation context-free-grammar descriptive

12.5.12 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 55 (Page No. 159) top

Let G_1 be the following grammar that we introduced in Example 2.45. Use the DK-test to show that G_1 is not a DCFG.

- $R \rightarrow S \mid T$
- $S \rightarrow aSb \mid ab$
- $T \rightarrow aTbb \mid abb$

michael-sipser theory-of-computation context-free-grammar descriptive

12.5.13 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 59 (Page No. 160) top

If we disallow ϵ-rules in CFGs, we can simplify the DK-test. In the simplified test,we only need to check that each of DK's accept states has a single rule. Prove that a CFG without ϵ-rules passes the simplified DK-test iff it is a DCFG.

michael-sipser theory-of-computation context-free-grammar descriptive

12.5.14 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 6 (Page No. 155) top

Give context-free grammars generating the following languages.

a. The set of strings over the alphabet $\{a, b\}$ with more $a's$ than $b's$
b. The complement of the language $\{a^n b^n \mid n \geq 0\}$
c. $\{w\#x \mid w^R$ is a substring of x for $w, x \in \{0,1\}^*\}$
d. $\{x_1\#x_2\#\ldots\#x_k \mid k \geq 1,$ each $x_i \in \{a, b\}^*$, and for some i and $j, x_i = x_j^R\}$

michael-sipser theory-of-computation context-free-language context-free-grammar

Answer key

12.5.15 Context Free Grammar: Michael Sipser Edition 3 Exercise 2 Question 8 (Page No. 155) top

Show that the string the girl touches the boy with the flower has two different leftmost derivations in grammar G_2 on page 103. Describe in English the two different meanings of this sentence.

michael-sipser theory-of-computation context-free-language context-free-grammar

12.5.16 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 14 (Page No. 211) top

Let $\Sigma = \{0, 1\}$. Show that the problem of determining whether a CFG generates some string in 1^* is decidable. In other words, show that $\{\langle G \rangle \mid$ G is a CFG over $\{0,1\}$ and $1^* \cap L(G) \neq \phi\}$ is a decidable language.

michael-sipser theory-of-computation context-free-grammar decidability proof

12.5.17 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 15 (Page No. 212) top

Show that the problem of determining whether a CFG generates all strings in 1^* is decidable. In other words, show that $\{\langle G \rangle \mid$ G is a CFG over $\{0,1\}$ and $1^* \subseteq L(G)\}$ is a decidable language.

michael-sipser theory-of-computation context-free-grammar decidability proof

12.5.18 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 28 (Page No. 212) top

Let $C = \{\langle G, x \rangle \mid$ G is a CFG x is a substring of some $y \in L(G)\}$. Show that C is decidable. (Hint: An elegant solution to this problem uses the decider for E_{CFG} .)

michael-sipser theory-of-computation context-free-grammar decidability proof

12.5.19 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 29 (Page No. 212) top

Let $C_{CFG} = \{\langle G, k \rangle \mid$ G is a CFG and L(G) contains exactly k strings where $k \geq 0$ or $k = \infty\}$. Show that C_{CFG} is decidable.

michael-sipser theory-of-computation context-free-grammar decidability proof

12.5.20 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 31 (Page No. 212) top

Say that a variable A in $CFL\ G$ is usable if it appears in some derivation of some string $w \in G$. Given a $CFG\ G$ and a variable A, consider the problem of testing whether A is usable. Formulate this problem as a language and show that it is decidable.

michael-sipser theory-of-computation context-free-language context-free-grammar decidability proof

12.5.21 Context Free Grammar: Michael Sipser Edition 3 Exercise 4 Question 4 (Page No. 211) top

Let $A\varepsilon_{CFG} = \{\langle G \rangle \mid G \text{ is a CFG that generates } \epsilon\}$.Show that $A\varepsilon_{CFG}$ is decidable.

michael-sipser theory-of-computation turing-machine context-free-grammar decidability proof

12.5.22 Context Free Grammar: Michael Sipser Edition 3 Exercise 5 Question 1 (Page No. 239) top

Show that EQ_{CFG} is undecidable.

michael-sipser theory-of-computation context-free-grammar decidability proof

12.5.23 Context Free Grammar: Michael Sipser Edition 3 Exercise 5 Question 32 (Page No. 241) top

Prove that the following two languages are undecidable.

a. $OVERLAP_{CFG} = \{\langle G, H \rangle \mid \text{G and H are CFGs where } L(G) \cap L(H) \neq \emptyset\}$.
b. $PREFIX-FREE_{CFG} = \{\langle G \rangle \mid \text{G is a CFG where L(G) is prefix-free}\}$.

michael-sipser theory-of-computation context-free-grammar turing-machine decidability proof

12.5.24 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 1 (Page No. 133) top

Find the language generated by following grammar:

The grammar G, with productions

$S \rightarrow abB$

$A \rightarrow aaBb$

$B \rightarrow bbAa$

$A \rightarrow \lambda$

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Answer key

12.5.25 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 10 (Page No. 134) top

Find a context-free grammar for $head(L)$, where L is the language $L = \{a^n b^m : n \leq m + 3\}$. For the definition of $head$ see Exercise 18, Section 4.1 .

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12.5.26 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 11 (Page No. 134) top

Find a context-free grammar for $\Sigma = \{a, b\}$ for the language $L = \{a^n w w^R b^n : w \in \Sigma^*, n \geq 1\}$.

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Answer key

12.5.27 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 12 (Page No. 134) top

Given a context-free grammar G for a language L, show how one can create from G a grammar $\widehat{G}$ so that $L(\widehat{G}) =$ head (L).

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12.5.28 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 13 (Page No. 134) top

Let $L = \{a^n b^n : n \geq 0\}$.
(a) https://gateoverflow.in/305106/peter-linz-edition-4-exercise-5-1-question-13-a-page-no-134
(b) Show that L^k is context-free for any given $k \geq 1$.
(c) Show that $\overline{L}$ and L^* are context-free.

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Answer key

12.5.29 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 13.a (Page No. 134) top

L=$\{a^n b^n | n \geq 0\}$
please show how L^2 is CFL

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Answer key

12.5.30 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 14 (Page No. 134) top

Let L_1 be the language $L_1 = \{a^n b^m c^k : n = m \text{ or } m \leq k\}$ and L_2 the language $L_2 = \{a^n b^m c^k : n + 2m = k\}$. Show that $L_1 \cup L_2$ is a context-free language.

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Answer key

12.5.31 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 15 (Page No. 134) top

Show that the following language is context-free.

$$L = \{uvwv^R : u, v, w \in \{a, b\}^+, |u| = |w| = 2\}.$$

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Answer key

12.5.32 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 16 (Page No. 134) top

Show that the complement of the language $L = \{ww^R : w \in \{a, b\}^*\}$ is context-free.

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Answer key

12.5.33 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 17 (Page No. 134) top

Show that the complement of the language $L = \{a^n b^m c^k : k = n + m\}$ is context-free.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar context-free-language

12.5.34 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 18 (Page No. 134) top

Show that the language $L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^+, w_1 \neq w_2^R\}$, with $\Sigma = \{a, b, c\}$,is context-free.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar context-free-language

Answer key

12.5.35 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 19 (Page No. 134) top

Show a derivation tree for the string $aabbbb$ with the grammar

$S \rightarrow AB|\lambda,$

$A \rightarrow aB,$

$B \rightarrow Sb.$

Give a verbal description of the language generated by this grammar.

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Answer key

12.5.36 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 2 (Page No. 133) top

Draw the derivation tree corresponding to the derivation in ***Example 5.1***.

Example 5.1
The grammar $G = (\{S\}, \{a, b\}, S, P)$, with productions

$S \rightarrow aSa$

$S \rightarrow bSb$

$S \rightarrow \lambda$

is context-free. A typical derivation in this grammar is

$S \Rightarrow aSa \Rightarrow aaSaa \Rightarrow aabSbaa \Rightarrow aabbaa.$

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12.5.37 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 20 (Page No. 135) top

Consider the grammar with productions

$S \rightarrow aaB,$

$A \rightarrow bBb|\lambda,$

$B \rightarrow Aa.$

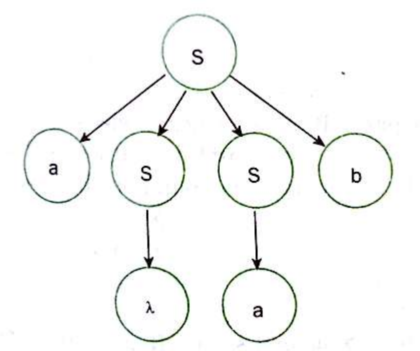
Show that the string $aabbabba$ is not in the language generated by this grammar.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar

Answer key

12.5.38 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 21 (Page No. 135) top

Consider the derivation tree below.



Find a grammar G for which this is the derivation tree of the string aab. Then find two more sentences of $L(G)$. Find a sentence in $L(G)$ that has a derivation tree of height five or larger.

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Answer key

12.5.39 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 22 (Page No. 135) top

Define what one might mean by properly nested parenthesis structures involving two kinds of parentheses, say ( ) and [ ]. Intuitively, properly nested strings in this situation are ([ ]), ([[ ]])[( )], but not ([ )] or (( ]]. Using your definition, give a context-free grammar for generating all properly nested parentheses.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar

Answer key

12.5.40 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 23 (Page No. 135) top

Find a context-free grammar for the set of all regular expressions on the alphabet $\{a, b\}$.

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12.5.41 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 24 (Page No. 135) top

Find a context-free grammar that can generate all the production rules for context-free grammars with $T = \{a, b\}$ and $V = \{A, B, C\}$.

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12.5.42 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 25 (Page No. 135) top

Prove that if G is a context-free grammar, then every $w \in L(G)$ has a leftmost and rightmost derivation. Give an algorithm for finding such derivations from a derivation tree.

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12.5.43 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 26 (Page No. 135) top

Find a linear grammar for the language $L = \{a^n b^m : n \neq m\}$.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar context-free-language

12.5.44 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 27 (Page No. 135) top

Let $G = (V, T, S, P)$ be a context-free grammar such that every one of its productions is of the form $A \rightarrow v$, with $|v| = k > 1$. Show that the derivation tree for any $w \in L(G)$ has a height h such that

$$\log_k |w| \leq h \leq \frac{(|w|-1)}{k-1}.$$

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12.5.45 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 3 (Page No. 133) top

Give a derivation tree for $w = abbbaabbaba$ for the grammar G, with productions

$S \rightarrow abB$

$A \rightarrow aaBb$

$B \rightarrow bbAa$

$A \rightarrow \lambda.$

Use the derivation tree to find a leftmost derivation.

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12.5.46 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 4 (Page No. 133) top

Show that the grammar with productions $S \rightarrow aSb|SS|\lambda$ does in fact generate the language
$L = \{w \in \{a, b\}^* : n_a(w) = n_b(w)$ and $n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$.

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12.5.47 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 6 (Page No. 133) top

Give the Complete proof of ***Theorem 5.1*** by showing that the yield of every partial derivation tree with root S is a sentential form of G.

Theorem 5.1
Let $G = (V, T, S, P)$ be a context-free grammar. Then for every $w \in L(G)$, there exists a derivation tree of G whose yield is w. Conversely, the yield of any derivation tree is in $L(G)$. Also, if t_G is any partial derivation tree for G whose root is labeled S, then the yield of t_G is a sentential form of G.

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12.5.48 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 7 (Page No. 133) top

Find context-free grammars for the following languages (with $n \geq 0, m \geq 0$).
(a) $L = \{a^n b^m : n \leq m + 3\}$.
(b) $L = \{a^n b^m : n \neq m - 1\}$.
(c) https://gateoverflow.in/208410/peter-linz-edition-4-exercise-5-1-question-7-c-page-no-133
(d) $L = \{a^n b^m : 2n \leq m \leq 3n\}$.
(e) $L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$.
(f) $L = \{w \in \{a, b\}^* : n_a(v) \geq n_b(v)$, where v is any prefix of $w\}$.
(g) $L = \{w \in \{a, b\}^* : n_a(w) = 2n_b(w) + 1\}$.

peter-linz peter-linz-edition4 theory-of-computation context-free-language context-free-grammar

Answer key

12.5.49 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 8 (Page No. 134) top

Find context-free grammars for the following languages (with $n \geq 0, m \geq 0, k \geq 0$).
(a) $L = \{a^n b^m c^k : n = m$ or $m \leq k\}$.
(b) $L = \{a^n b^m c^k : n = m$ or $m \neq k\}$.
(c) $L = \{a^n b^m c^k : k = n + m\}$.
(d) $L = \{a^n b^m c^k : n + 2m = k\}$.
(e) $L = \{a^n b^m c^k : k = |n - m|\}$.
(f) $L = \{w \in \{a, b, c\}^* : n_a(w) + n_b(w) \neq n_c(w)\}$.
(g) $L = \{a^n b^m c^k, k \neq n + m\}$.
(h) $L = \{a^n b^m c^k : k \geq 3\}$.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar context-free-language

Answer key

12.5.50 Context Free Grammar: Peter Linz Edition 4 Exercise 5.1 Question 9 (Page No. 134) top

Show that $L = \{w \in \{a, b, c\}^* : |w| = 3n_a(w)\}$ is a context-free language.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar context-free-language

Answer key

12.5.51 Context Free Grammar: Peter Linz Edition 4 Exercise 5.2 Question 17 (Page No. 145) top

Use the exhaustive search parsing method to parse the string *abbbbbb* with the grammar with productions

$S \to aAB,$

$A \to bBb,$

$B \to A|\lambda.$

In general, how many rounds will be needed to parse any string w in this language?

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar parsing

Answer key

12.5.52 Context Free Grammar: Peter Linz Edition 4 Exercise 5.2 Question 18 (Page No. 145) top

Is the string *aabbababb* in the language generated by the grammar $S \to aSS|b$?

Show that the grammar with productions

$S \to aAb|\lambda,$

$A \to aAb|\lambda$ is unambiguous.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar ambiguous

Answer key

12.5.53 Context Free Grammar: Peter Linz Edition 4 Exercise 5.2 Question 19 (Page No. 145) top

Prove the following result. Let $G = (V, T, S, P)$ be a context-free grammar in which every $A \in V$ occurs on the left side of at most one production. Then G is unambiguous.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar ambiguous

12.5.54 Context Free Grammar: Peter Linz Edition 4 Exercise 5.2 Question 20 (Page No. 145) top

Find a grammar equivalent to $S \rightarrow aAB, A \rightarrow bBb, B \rightarrow A|\lambda$ that satisfies the conditions of ***Theorem 5.2***.

Theorem 5.2
Suppose that $G = (V, T, S, P)$ is a context-free grammar that does not have any rules of the form $A \rightarrow \lambda$, or $A \rightarrow B$, where $A, B \in V$. Then the exhaustive search parsing method can be made into an algorithm which, for any $w \in \Sigma^*$, either produces a parsing of w or tells us that no parsing is possible.

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Answer key

12.5.55 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 1 (Page No. 161) top

Complete the proof of ***Theorem 6.1*** by showing that

$$S \overset{*}{\Rightarrow}_{\widehat{G}} w$$

implies

$$S \overset{*}{\Rightarrow}_{G} w.$$

Theorem 6.1
Let $G = (V, T, S, P)$ be a context-free grammar. Suppose that P contains a production of the form $A \rightarrow x_1 B x_2$.
Assume that A and B are different variables and that $B \rightarrow y_1 | y_2 | \ldots | y_n$ is the set of all productions in P that have B as the left side. Let $\widehat{G} = (V, T, S, \widehat{P})$ be the grammar in which $\widehat{P}$ is constructed by deleting $A \rightarrow x_1 B x_2$ from P, and adding to it $A \rightarrow x_1 y_1 x_2 | x_1 y_2 x_2 | \ldots | x_1 y_n x_2$.
Then, $L(\widehat{G}) = L(G)$.

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12.5.56 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 10 (Page No. 162) top

Complete the proof of ***Theorem 6.3***.

Theorem 6.3
Let G be any context-free grammar with λ not in $L(G)$. Then there exists an equivalent grammar $\widehat{G}$ having no λ-productions.

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12.5.57 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 11 (Page No. 162) top

Complete the proof of ***Theorem 6.4***.

Theorem 6.4
Let $G = (V, T, S, P)$ be any context-free grammar without λ-productions. Then there exists a context-free grammar $\widehat{G} = (\widehat{V}, \widehat{T}, S, \widehat{P})$ that does not have any unit-productions and that is equivalent to G.

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12.5.58 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 12 (Page No. 162) top

Remove λ-productions from the grammar with productions $S \rightarrow aSb|SS|\lambda$.
What language does the resulting grammar generate?

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Answer key

12.5.59 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 14 (Page No. 162) top

Suppose that G is a context-free grammar for which $\lambda \in L(G)$. Show that if we apply the construction in Theorem 6.3, we obtain a new grammar $\widehat{G}$ such that $L(\widehat{G}) = L(G) - \{\lambda\}$.

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12.5.60 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 16 (Page No. 162) top

Let G be a grammar without λ-productions, but possibly with some unit-productions. Show that the construction of ***Theorem 6.4*** does not then introduce any λ-productions.

Theorem 6.4

Let $G = (V, T, S, P)$ be any context-free grammar without λ-productions. Then there exists a context-free grammar $\widehat{G} = (\widehat{V}, \widehat{T}, \widehat{S}, \widehat{P})$ that does not have any unit-productions and that is equivalent to G.

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12.5.61 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 17 (Page No. 162) top

Show that if a grammar has no λ-productions and no unit-productions, then the removal of useless productions by the construction of ***Theorem 6.2*** does not introduce any such productions.

Theorem 6.2

Let $G = (V, T, S, P)$ be a context-free grammar. Then there exists an equivalent grammar $\widehat{G} = (\widehat{V}, \widehat{T}, \widehat{S}, \widehat{P})$ that does not contain any useless variables or productions.

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12.5.62 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 18 (Page No. 162) top

Justify the claim made in the proof of **Theorem 6.1** that the variable B can be replaced as soon as it appears.

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12.5.63 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 19 (Page No. 163) top

Suppose that a context-free grammar $G = (V, T, S, P)$ has a production of the form $A \to xy$, where $x, y \in (V \cup T)^+$. Prove that if this rule is replaced by $A \to By, B \to x$, where $B \notin V$, then the resulting grammar is equivalent to the original one.

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12.5.64 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 2 (Page No. 161) top

Show a derivation tree for the string *ababbac*, using grammar with productions

$A \to a|aaA|abBC,$

$B \to abbA|b.$

also show the derivation tree for grammar with productions

$A \to a|aaA|ababbAc|abbc,$

$B \to abbA|b.$

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12.5.65 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 20 (Page No. 163) top

Consider the procedure suggested in ***Theorem 6.2*** for the removal of useless productions. Reverse the order of the two parts, first eliminating variables that cannot be reached from S, then removing those that do not yield a terminal string. Does the new procedure still work correctly? If so, prove it. If not, give a counterexample.

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12.5.66 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 21 (Page No. 163) top

It is possible to define the term *simplification* precisely by introducing the concept of **complexity** of a grammar. This can be done in many ways; one of them is through the length of all the strings giving the production rules.

For example, we might use

$$complexity(G) = \sum_{A \to V \in P} \{1 + |v|\}$$

Show that the removal of useless productions always reduces the complexity in this sense. What can you say about the removal of λ-productions and unit-productions?

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12.5.67 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 22 (Page No. 163) top

A context-free grammar G is said to be minimal for a given language L if $complexity(G) \leq complexity(\widehat{G})$ for any $\widehat{G}$ generating L. Show by example that the removal of useless productions does not necessarily produce a minimal grammar.

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12.5.68 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 23 (Page No. 163) top

Prove the following result. Let $G = (V, T, S, P)$ be a context-free grammar. Divide the set of productions whose left sides are some given variable (say, A), into two disjoint subsets

$$A \to Ax_1|Ax_2|\dots|Ax_n,$$

$$A \to y_1|y_2|\dots|y_m,$$

where x_i, y_i are in $(V \cup T)^*$, but A is not a prefix of any y_i. Consider the grammar $\widehat{G} = (V \cup \{Z\}, T, S, \widehat{P})$, where $Z \notin V$ and $\widehat{P}$ is obtained by replacing all productions that have A on the left by

$$A \to y_i|y_iZ, \quad i = 1, 2, 3, \dots, m$$

$$Z \to x_i|x_iZ, \quad i = 1, 2, 3, \dots, n.$$

Then $L(G) = L(\widehat{G})$.

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12.5.69 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 24 (Page No. 164) top

Use the result of the preceding exercise to rewrite the grammar

$A \to Aa|aBc|\lambda,$

$B \to Bb|bc$

so that it no longer has productions of the form $A \to Ax$ or $B \to Bx$.

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12.5.70 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 25 (Page No. 164) top

Prove the following counterpart of Exercise 23. Let the set of productions involving the variable A on the left be divided into two disjoint subsets

$$A \to x_1A|x_2A|\dots|x_nA,$$

and, $$A \to y_1|y_2|\dots|y_m,$$

where A is not a suffix of any y_i. Show that the grammar obtained by replacing these productions with

$$A \to y_i|Zy_i, \quad i = 1, 2, 3, \dots, m$$

$$Z \to x_i|Zx_i, \quad i = 1, 2, 3, \dots, n.$$

is equivalent to the original grammar.

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12.5.71 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 3 (Page No. 161) top

Show that the two grammars

$S \to abAB|ba,$

$A \to aaa,$

$B \to aA|bb$
and

$S \to abAaA|abAbb|ba,$

$A \to aaa$ are equivalent.

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12.5.72 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 4 (Page No. 161) top

In Theorem 6.1, why is it necessary to assume that A and B are different variables?

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12.5.73 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 5 (Page No. 161) top

Eliminate all useless productions from the grammar

$S \to aS|AB,$

$A \to bA,$

$B \to AA.$
What language does this grammar generate?

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Answer key

12.5.74 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 6 (Page No. 161) top

Eliminate useless productions from

$S \to a|aA|B|C,$

$A \to aB|\lambda,$

$B \to Aa,$

$C \to cCD,$

$D \to ddd.$

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Answer key

12.5.75 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 7 (Page No. 162) top

Eliminate all λ-productions from

$S \to AaB|aaB,$

$A \to \lambda,$

$B \to bbA|\lambda.$

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Answer key

12.5.76 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 8 (Page No. 162) top

Remove all unit-productions, all useless productions, and all λ-productions from the grammar

$S \to aA|aBB,$

$A \to aaA|\lambda,$

$B \to bB|bbC,$

$C \to B.$

What language does this grammar generate?

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Answer key

12.5.77 Context Free Grammar: Peter Linz Edition 4 Exercise 6.1 Question 9 (Page No. 162) top

Eliminate all unit-productions from the grammar

$S \to a|aA|B|C,$

$A \to aB|\lambda,$

$B \to aA,$

$C \to aCD,$

$D \to ddd$

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12.5.78 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 1 (Page No. 169) top

Provide the details of the proof of ***Theorem 6.6***.

Theorem 6.6

Any context-free grammar $G = (V, T, S, P)$ with $\lambda \notin L(G)$ has an equivalent grammar $\widehat{G} = (\widehat{V}, \widehat{T}, S, \widehat{P})$ in Chomsky normal form.

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12.5.79 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 15 (Page No. 170) top

A context-free grammar is said to be in two-standard form if all production rules satisfy the following pattern

$$A \to aBC,$$

$$A \to aB,$$

$$A \to a,$$

where $A, B, C \in V$ and $a \in T$.

Convert the grammar $G = (\{S, A, B, C\}, \{a, b\}, S, P)$ with P given as

$$S \to aSA,$$

$$A \to bABC,$$

$$B \to b,$$

$$C \to aBC$$

into two-standard form.

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12.5.80 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 16 (Page No. 170) top

"Two-standard form is general; for any context-free grammar G with $\lambda \notin L(G)$, there exists an equivalent grammar in two-standard form." Prove this.

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12.5.81 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 2 (Page No. 169) top

Convert the grammar $S \rightarrow aSb|ab$ into Chomsky normal form.

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12.5.82 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 3 (Page No. 169) top

Transform the grammar $S \rightarrow aSaA|A, A \rightarrow abA|b$ into Chomsky normal form.

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12.5.83 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 4 (Page No. 169) top

Transform the grammar with productions
$S \rightarrow abAB$,

$A \rightarrow bAB|\lambda$,

$B \rightarrow BAa|A|\lambda$ into Chomsky normal form.

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12.5.84 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 5 (Page No. 169) top

Convert the grammar with productions

$S \rightarrow AB|aB$,

$A \rightarrow aab|\lambda$,

$B \rightarrow bbA$ into Chomsky normal form.

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12.5.85 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 6 (Page No. 169) top

Let $G = (V, T, S, P)$ be any context-free grammar without any λ-productions or unit-productions.
Let k be the maximum number of symbols on the right of any production in P. Show that there is an equivalent grammar in Chomsky normal form with no more than $(k-1)|P| + |T|$ production rules.

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12.5.86 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 7 (Page No. 169) top

Draw the dependency graph for the grammar in ***Exercise 4***.

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12.5.87 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 8 (Page No. 169) top

A linear language is one for which there exists a linear grammar

[A linear grammar is a grammar in which at most one variable can occur on the right side of any production, without restriction on the position of this variable. Clearly, a regular grammar is always linear, but not all linear grammars are regular].

Let L be any linear language not containing λ. Show that there exists a grammar $G = (V, T, S, P)$ all of whose productions have one of the forms

$$A \rightarrow aB,$$

$$A \to Ba,$$
$$A \to a,$$
where $a \in T$, $A, B \in V$, such that $L = L(G)$.

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12.5.88 Context Free Grammar: Peter Linz Edition 4 Exercise 6.2 Question 9 (Page No. 170) top

Show that for every context-free grammar $G = (V, T, S, P)$ there is an equivalent one in which all productions have the form
$$A \to aBC,$$
or
$$A \to \lambda,$$
where $a \in \Sigma \cup \{\lambda\}$, $A, B, C \in V$.

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12.5.89 Context Free Grammar: Peter Linz Edition 4 Exercise 6.3 Question 1 (Page No. 172) top

Use the CYK algorithm to determine whether the strings $aabb$, $aabba$, and $abbbb$ are in the language generated by the grammar with productions

$$S \to AB,$$

$$A \to BB|a,$$

$$B \to AB|b.$$

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12.5.90 Context Free Grammar: Peter Linz Edition 4 Exercise 6.3 Question 2 (Page No. 173) top

Use the CYK algorithm to find a parsing of the string aab, using the grammar with productions

$$S \to AB,$$

$$A \to BB|a,$$

$$B \to AB|b.$$

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12.5.91 Context Free Grammar: Peter Linz Edition 4 Exercise 6.3 Question 3 (Page No. 173) top

Use the approach employed in ***Exercise 2*** to show how the CYK membership algorithm can be made into a parsing method.

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12.5.92 Context Free Grammar: Peter Linz Edition 4 Exercise 6.3 Question 4 (Page No. 173) top

Use the CYK method to determine if the string $w = aaabbbbab$ is in the language generated by the grammar $S \to aSb|b$.

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12.5.93 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 1 (Page No. 204) top

Show that the grammar $S_0 \to aSbS, S \to aSbS|\lambda$ is an LL grammar and that it is equivalent to the grammar $S \to SS|aSb|ab$.

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12.5.94 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 2 (Page No. 204) top

Show that the grammar for L = $\{w : n_a(w) = n_b(w)\}$ which is, $S \to SS, S \to \lambda, S \to aSb, S \to bSa$ is not an LL grammar.

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Answer key

12.5.95 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 3 (Page No. 204) top

Find an LL grammar for the language L = $\{w : n_a(w) = n_b(w)\}$.

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12.5.96 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 4 (Page No. 204) top

Construct an LL grammar for the language L (a*ba) ∪ L (abbb*).

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Answer key

12.5.97 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 5 (Page No. 204) top

Show that any LL grammar is unambiguous.

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12.5.98 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 6 (Page No. 204) top

Show that if G is an LL (k) grammar, then L (G) is a deterministic context-free language.

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12.5.99 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 8 (Page No. 204) top

Let G be a context-free grammar in Greibach normal form. Describe an algorithm which, for any given k, determines whether or not G is an LL (k) grammar.

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12.5.100 Context Free Grammar: Peter Linz Edition 4 Exercise 7.4 Question 9 (Page No. 204) top

Give LL grammars for the following languages, assuming $\Sigma = \{a, b, c\}$.

(i) $L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$.

(ii) $L = \{a^{n+2} b^m c^{n+m} : n \geq 0, m \geq 0\}$.

(iii) $L = \{a^n b^{n+2} c^m : n \geq 0, m > 1\}$.

(iv) $L = \{w : n_a(w) < n_b(w)\}$.

(v) $L = \{w : n_a(w) + n_b(w) \neq n_c(w)\}$.

peter-linz peter-linz-edition4 theory-of-computation context-free-language context-free-grammar

12.6 Context Free Language (36) top

12.6.1 Context Free Language: Michael Sipser Edition 3 Exercise 1 Question 73 (Page No. 93) top

Let $\sum = \{0, 1, \#\}$. Let $C = \{x\#x^R\#x | x \in \{0,1\}^*\}$. Show that $\overline{C}$ is a CFL.

michael-sipser theory-of-computation context-free-language proof descriptive

12.6.2 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 16 (Page No. 156) top

Show that the class of context-free languages is closed under the regular operations, union, concatenation, and star.

michael-sipser theory-of-computation context-free-language

12.6.3 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 17 (Page No. 156) top

Use the results of Question 16 to give another proof that every regular language is context free , by showing how to convert a regular expression directly to an equivalent context-free grammar.

michael-sipser theory-of-computation regular-language context-free-language

12.6.4 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 18 (Page No. 156) top

a. Let C be a context-free language and R be a regular language. Prove that the language $C \cap R$ is context-free.
b. Let $A = \{w \mid w \in \{a, b, c\}^* \text{ and } w \text{ contains equal numbers of } a's, b's, \text{and } c's\}$. Use part (a) to show that A is not a CFL.

michael-sipser theory-of-computation context-free-language regular-language

Answer key

12.6.5 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 2 (Page No. 154) top

a. Use the languages $A = \{a^m b^n c^n | m, n \geq 0\}$ and $B = \{a^n b^n c^m | m, n \geq 0\}$ together with Example 2.36 to show that the class of context-free languages is not closed under intersection.
b. Use part (a) and DeMorgan's law (Theorem 0.20) to show that the class of context-free languages is not closed under complementation.

michael-sipser theory-of-computation context-free-language

Answer key

12.6.6 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 20 (Page No. 156) top

Let $A/B = \{w \mid wx \in A \text{ for some } x \in B\}$. Show that if A is context free and B is regular, then A/B is context free.

michael-sipser theory-of-computation context-free-language regular-language

12.6.7 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 23 (Page No. 157) top

Let $D = \{xy \mid x, y \in \{0,1\}^* \text{ and } |x| = |y| \text{ but } x \neq y\}$. Show that D is a context-free language.

michael-sipser theory-of-computation context-free-language proof

12.6.8 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 24 (Page No. 157) top

Let $E = \{a^i b^j \mid i \neq j \text{ and } 2i \neq j\}$. Show that E is a context-free language.

michael-sipser theory-of-computation context-free-language proof

Answer key

12.6.9 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 31 (Page No. 157) top

Let B be the language of all palindromes over $\{0,1\}$ containing equal numbers of $0's$ and $1's$. Show that B is not context free.

michael-sipser theory-of-computation context-free-language

Answer key

12.6.10 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 32 (Page No. 157) top

L e t $\Sigma = \{1, 2, 3, 4\}$ and $C = \{w \in \Sigma^* |$ in w, the number of $1's$ equals the number of $2's$, and the number of $3's$ equals the number of $4's\}$. Show that C is not context free.

michael-sipser theory-of-computation context-free-language proof

Answer key

12.6.11 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 33 (Page No. 157) top

Show that $F = \{a^i b^j \mid i = kj \text{ for some positive integer } k\}$ is not context free.

michael-sipser theory-of-computation context-free-language

Answer key

12.6.12 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 40 (Page No. 158) top

Say that a language is prefix-closed if all prefixes of every string in the language are also in the language. Let C be an infinite, prefix-closed, context-free language. Show that C contains an infinite regular subset.

michael-sipser theory-of-computation context-free-language

12.6.13 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 41 (Page No. 158) top

Read the definitions of NOPREFIX(A) and NOEXTEND(A) in Question 1.40.

a. Show that the class of CFLs is not closed under NOPREFIX.
b. Show that the class of CFLs is not closed under NOEXTEND.

michael-sipser theory-of-computation context-free-language

12.6.14 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 42 (Page No. 158) top

Let $Y = \{w \mid w = t_1\#t_2\# \dots t_k \text{ for } k \geq 0, \text{each } t_i \in 1^*, \text{and } t_i \neq t_j \text{whenever } i \neq j\}$ .Here $\Sigma = \{1, \#\}$.
Prove that Y is not context free.

michael-sipser theory-of-computation context-free-language proof

12.6.15 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 43 (Page No. 158) top

For strings $w4and$t,4 write $w \stackrel{\circ}{=} t$ if the symbols of w are a permutation of the symbols of t. In other words, $w \stackrel{\circ}{=} t$ if t and $w4 have the same symbols in the same quantities, but possibly in a different order.

For any string w, define $SCRAMBLE(w) = \{t \mid t \stackrel{\circ}{=} w\}$. For any language A, let $SCRAMBLE(A) = \{t \mid t in SCRAMBLE(w) \text{for some} w \in A\}$.

a. Show that if $\Sigma = \{0, 1\}$, then the SCRAMBLE of a regular language is context free.
b. What happens in part (a) if Σ contains three or more symbols? Prove your answer.

michael-sipser theory-of-computation context-free-language proof

12.6.16 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 45 (Page No. 158) top

Let $A = \{wtw^R \mid w, t \in \{0, 1\}^* \text{ and } \mid w \mid = \mid t \mid\}$. Prove that A is not a CFL.

michael-sipser theory-of-computation context-free-language proof

12.6.17 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 48 (Page No. 159) top

Let $\Sigma = \{0, 1\}$. Let C_1 be the language of all strings that contain a 1 in their middle third. Let C_2 be the language of all strings that contain two $1s$ in their middle third.So $C_1 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^*, \text{where} \mid x \mid = \mid z \mid \geq \mid y \mid\}$ and $C_2 = \{xyz \mid x, z \in \Sigma^* \text{ and } y \in \Sigma^* 1 \Sigma^* 1 \Sigma^*, \text{where} \mid x \mid = \mid z \mid \geq \mid y \mid\}$.

a. Show that C_1 is a CFL.
b. Show that C_2 is not a CFL.

michael-sipser theory-of-computation context-free-language descriptive

12.6.18 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 56 (Page No. 160) top

Let $A = L(G_1)$ where G_1 is defined in Question 2.55. Show that A is not a DCFL.(Hint: Assume that A is a DCFL and consider its DPDA P. Modify P so that its input alphabet is $\{a, b, c\}$. When it first enters an accept state, it pretends that $c's$ are $b's$ in the input from that point on. What language would the modified P accept?)

michael-sipser theory-of-computation context-free-language descriptive

12.6.19 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 57 (Page No. 160) top

Let $B = \{a^i b^j c^k \mid i, j, k \geq 0 \text{ and } i = j \text{ or } i = k\}$. Prove that B is not a DCFL.

michael-sipser theory-of-computation context-free-language descriptive

Answer key

12.6.20 Context Free Language: Michael Sipser Edition 3 Exercise 2 Question 58 (Page No. 160) top

Let $C = \{ww^R \mid win\{0, 1\}^*\}$. Prove that C is not a DCFL. (Hint: Suppose that when some DPDA P is started in state q with symbol x on the top of its stack, P never pops its stack below x, no matter what input string P reads from that point on. In that case, the contents of $P's$ stack at that point cannot affect its subsequent behavior, so $P's$ subsequent behavior can depend only on q and x.)

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12.6.21 Context Free Language: Peter Linz Edition 4 Exercise 5.1 Question 7.c (Page No. 133) top

Find CFG for the following language

L = $\{a^n b^m : n \neq 2m\}$

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Answer key

12.6.22 Context Free Language: Peter Linz Edition 4 Exercise 6.1 Question 15 (Page No. 162) top

Give an example of a situation in which the removal of λ-productions introduces previously nonexistent unit-productions.

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12.6.23 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 1 (Page No. 200) top

Show that $L = \{a^n b^{2n} : n \geq 0\}$ is a deterministic context-free language.

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12.6.24 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 10 (Page No. 200) top

While the language in **Exercise 9** is deterministic, the closely related language $L = \{ww^R : w \in \{a, b\}^*\}$ is known to be nondeterministic. Give arguments that make this statement plausible.

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Answer key

12.6.25 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 11 (Page No. 200) top

Show that $L = \{w \in \{a, b\}^* : n_a(w) \neq n_b(w)\}$ is a deterministic context-free language.

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12.6.26 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 15 (Page No. 200) top

Show that every regular language is a deterministic context-free language.

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Answer key

12.6.27 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 16 (Page No. 200) top

Show that if L_1 is deterministic context-free and L_2 is regular, then the language $L_1 \cup L_2$ is deterministic context-free.

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Answer key

12.6.28 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 17 (Page No. 200) top

Show that under the conditions of **Exercise 16**, $L_1 \cap L_2$ is a deterministic context-free language.

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12.6.29 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 18 (Page No. 200) top

Give an example of a deterministic context-free language whose reverse is not deterministic.

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12.6.30 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 2 (Page No. 200) top

Show that $L = \{a^n b^m : m \geq n+2\}$ is deterministic.

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12.6.31 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 3 (Page No. 200) top

Is the language $L = \{a^n b^n : n \geq 1\} \cup \{b\}$ deterministic?

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Answer key

12.6.32 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 4 (Page No. 200) top

Is the language $L = \{a^n b^n : n \geq 1\} \cup \{a\}$ deterministic?

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Answer key

12.6.33 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 6 (Page No. 200) top

For the language $L = \{a^n b^{2n} : n \geq 0\}$, show that L^* is a deterministic context-free language.

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Answer key

12.6.34 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 7 (Page No. 200) top

Give reasons why one might conjecture that the following language is not deterministic.

$$L = \{\, a^n b^m c^k : n = m \text{ or } m = k\}.$$

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12.6.35 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 8 (Page No. 200) top

Is the language $L = \{a^n b^m : n = m \text{ or } n = m+2\}$ deterministic?

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12.6.36 Context Free Language: Peter Linz Edition 4 Exercise 7.3 Question 9 (Page No. 200) top

Is the language $\{wcw^R : w \in \{a,b\}^*\}$ deterministic?

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Answer key

12.7 Countable Uncountable Set (2) top

12.7.1 Countable Uncountable Set: Michael Sipser Edition 3 Exercise 4 Question 8 (Page No. 211) top

Let $T = \{(i, j, k) \mid i, j, k \in N\}$. Show that T is countable.

michael-sipser theory-of-computation turing-machine countable-uncountable-set proof

12.7.2 Countable Uncountable Set: Michael Sipser Edition 3 Exercise 4 Question 9 (Page No. 211) top

Review the way that we define sets to be the same size in Definition 4.12 (page 203). Show that "is the same size" is an equivalence relation.

michael-sipser theory-of-computation turing-machine countable-uncountable-set proof

12.8 Decidability (70) top

12.8.1 Decidability: Michael Sipser Edition 3 Exercise 3 Question 15 (Page No. 189) top

Show that the collection of decidable languages is closed under the operation of

a. union. b. concatenation. c. star. d. complementation. e. intersection.

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12.8.2 Decidability: Michael Sipser Edition 3 Exercise 3 Question 18 (Page No. 190) top

Show that a language is decidable iff some enumerator enumerates the language in the standard string order.

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12.8.3 Decidability: Michael Sipser Edition 3 Exercise 3 Question 22 (Page No. 190) top

Let A be the language containing only the single string s, where

$$s = \begin{cases} 0 \text{ if life never will be found on Mars} \\ 1 \text{ if life will be found on Mars someday} \end{cases}$$

Is A decidable? Why or why not? For the purposes of this problem, assume that the question of whether life will be found on Mars has an unambiguous YES or NO answer.

michael-sipser theory-of-computation turing-machine decidability descriptive

12.8.4 Decidability: Michael Sipser Edition 3 Exercise 4 Question 10 (Page No. 211) top

Let $INFINITE_{DFA} = \{\langle A \rangle \mid \text{ A is a DFA and L(A) is an infinite language}\}$. Show that $INFINITE_{DFA}$ is decidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.5 Decidability: Michael Sipser Edition 3 Exercise 4 Question 11 (Page No. 211) top

Let $INFINITE_{PDA} = \{\langle M \rangle \mid \text{M is a PDA and L(M) is an infinite language}\}$. Show that $INFINITE_{PDA}$ is decidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.6 Decidability: Michael Sipser Edition 3 Exercise 4 Question 12 (Page No. 211) top

Let $A = \{\langle M \rangle \mid \text{M is a DFA that doesn't accept any string containing an odd number of 1s}\}$. Show that A is decidable.

michael-sipser theory-of-computation decidability proof

12.8.7 Decidability: Michael Sipser Edition 3 Exercise 4 Question 13 (Page No. 211) top

Let $A = \{\langle R, S \rangle \mid \text{R and S are regular expressions and } L(R) \subseteq L(S)\}$. Show that A is decidable.

michael-sipser theory-of-computation decidability proof

12.8.8 Decidability: Michael Sipser Edition 3 Exercise 4 Question 16 (Page No. 212) top

$A = \{\langle R \rangle |$R is a regular expression describing a language containing at least one string w that has 111 as a substri
Show that A is decidable.

michael-sipser theory-of-computation decidability proof

12.8.9 Decidability: Michael Sipser Edition 3 Exercise 4 Question 17 (Page No. 212) top

Prove that EQ_{DFA} is decidable by testing the two DFAs on all strings up to a certain size. Calculate a size that works.

michael-sipser theory-of-computation finite-automata decidability proof

12.8.10 Decidability: Michael Sipser Edition 3 Exercise 4 Question 19 (Page No. 212) top

Prove that the class of decidable languages is not closed under homomorphism.

michael-sipser theory-of-computation decidability proof

12.8.11 Decidability: Michael Sipser Edition 3 Exercise 4 Question 2 (Page No. 211) top

Consider the problem of determining whether a DFA and a regular expression are equivalent. Express this problem as a language and show that it is decidable.

michael-sipser theory-of-computation finite-automata regular-expression decidability proof

12.8.12 Decidability: Michael Sipser Edition 3 Exercise 4 Question 21 (Page No. 212) top

Let $S = \{\langle M \rangle \mid$ M is a DFA that accepts w^R whenever it accepts $w\}$. Show that S is decidable.

michael-sipser theory-of-computation decidability proof

12.8.13 Decidability: Michael Sipser Edition 3 Exercise 4 Question 22 (Page No. 212) top

Let $PREFIX - FREE_{REX} = \{\langle R \rangle \mid$ R is a regular expression and L(R) is prefix-free$\}$. Show that $PREFIXFREE_{REX}$ is decidable. Why does a similar approach fail to show that $PREFIX - FREE_{CFG}$ is decidable?

michael-sipser theory-of-computation regular-expression decidability proof

12.8.14 Decidability: Michael Sipser Edition 3 Exercise 4 Question 23 (Page No. 212) top

Say that an NFA is ambiguous if it accepts some string along two different computation branches. Let $AMBIG_{NFA} = \{\langle N \rangle \mid$ N is an ambiguous NFA$\}$. Show that $AMBIG_{NFA}$ is decidable. (Suggestion: One elegant way to solve this problem is to construct a suitable DFA and then run E_{DFA} on it.)

michael-sipser theory-of-computation finite-automata decidability proof

12.8.15 Decidability: Michael Sipser Edition 3 Exercise 4 Question 25 (Page No. 212) top

Let $BAL_{DFA} = \{\langle M \rangle \mid$ M is a DFA that accepts some string containing an equal number of 0s and 1s$\}$.

Show that BAL_{DFA} is decidable. (Hint: Theorems about $CFLs$ are helpful here.)

michael-sipser theory-of-computation finite-automata decidability proof

12.8.16 Decidability: Michael Sipser Edition 3 Exercise 4 Question 26 (Page No. 212) top

Let $PAL_{DFA} = \{\langle M \rangle \mid$ M is a DFA that accepts some palindrome$\}$. Show that PAL_{DFA} is decidable. (Hint: Theorems about $CFLs$ are helpful here.)

michael-sipser theory-of-computation finite-automata decidability proof

Answer key

12.8.17 Decidability: Michael Sipser Edition 3 Exercise 4 Question 27 (Page No. 212) top

Let $E = \{\langle M \rangle \mid \text{ M is a DFA that accepts some string with more 1s than 0s}\}$. Show that E is decidable. (Hint: Theorems about $CFLs$ are helpful here.)

michael-sipser theory-of-computation finite-automata decidability proof

12.8.18 Decidability: Michael Sipser Edition 3 Exercise 4 Question 3 (Page No. 211) top

Let $ALL_{DFA} = \{\langle A \rangle \mid A \text{ is a DFA and } L(A) = \Sigma^*\}$. Show that ALL_{DFA} is decidable.

michael-sipser theory-of-computation turing-machine finite-automata decidability proof

12.8.19 Decidability: Michael Sipser Edition 3 Exercise 4 Question 7 (Page No. 211) top

Let B be the set of all infinite sequences over $\{0, 1\}$. Show that B is uncountable using a proof by diagonalization.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.20 Decidability: Michael Sipser Edition 3 Exercise 5 Question 10 (Page No. 239) top

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape when it is run on input w. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.21 Decidability: Michael Sipser Edition 3 Exercise 5 Question 11 (Page No. 239) top

Consider the problem of determining whether a two-tape Turing machine ever writes a nonblank symbol on its second tape during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.22 Decidability: Michael Sipser Edition 3 Exercise 5 Question 12 (Page No. 239) top

Consider the problem of determining whether a single-tape Turing machine ever writes a blank symbol over a nonblank symbol during the course of its computation on any input string. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.23 Decidability: Michael Sipser Edition 3 Exercise 5 Question 13 (Page No. 239) top

A useless state in a Turing machine is one that is never entered on any input string. Consider the problem of determining whether a Turing machine has any useless states. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.24 Decidability: Michael Sipser Edition 3 Exercise 5 Question 14 (Page No. 240) top

Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left when its head is on the left-most tape cell. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.25 Decidability: Michael Sipser Edition 3 Exercise 5 Question 15 (Page No. 240) top

Consider the problem of determining whether a Turing machine M on an input w ever attempts to move its head left at any point during its computation on w. Formulate this problem as a language and show that it is decidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.26 Decidability: Michael Sipser Edition 3 Exercise 5 Question 20 (Page No. 240) top

Prove that there exists an undecidable subset of $\{1\}^*$.

michael-sipser theory-of-computation decidability proof

12.8.27 Decidability: Michael Sipser Edition 3 Exercise 5 Question 26 (Page No. 240) top

Define a **two-headed finite automaton** $(2DFA)$ to be a deterministic finite automaton that has two read-only, bidirectional heads that start at the left-hand end of the input tape and can be independently controlled to move in either direction. The tape of a $2DFA$ is finite and is just large enough to contain the input plus two additional blank tape cells, one on the left-hand end and one on the right-hand end, that serve as delimiters. A $2DFA$ accepts its input by entering a special accept state. For example, a $2DFA$ can recognize the language $\{a^nb^nc^n \mid n \geq 0\}$.

a. Let $A_{2DFA} = \{\langle M, x\rangle \mid \text{M is a 2DFA and M accepts } x\}$. Show that A_{2DFA} is decidable.
b. Let $E_{2DFA} = \{\langle M\rangle \mid \text{M is a 2DFA and } L(M) = \emptyset\}$. Show that E_{2DFA} is not decidable.

michael-sipser theory-of-computation finite-automata turing-machine decidability proof

12.8.28 Decidability: Michael Sipser Edition 3 Exercise 5 Question 27 (Page No. 241) top

A **two-dimensional finite automaton** $(2DIM - DFA)$ is defined as follows. The input is an $m \times n$ rectangle, for any $m, n \geq 2$. The squares along the boundary of the rectangle contain the symbol $\#$ and the internal squares contain symbols over the input alphabet Σ. The transition function $\delta : Q \times (\Sigma \cup \#) \rightarrow Q \times \{L, R, U, D\}$ indicates the next state and the new head position (Left, Right, Up, Down). The machine accepts when it enters one of the designated accept states. It rejects if it tries to move off the input rectangle or if it never halts. Two such machines are equivalent if they accept the same rectangles. Consider the problem of determining whether two of these machines are equivalent. Formulate this problem as a language and show that it is undecidable.

michael-sipser theory-of-computation finite-automata turing-machine decidability proof

12.8.29 Decidability: Michael Sipser Edition 3 Exercise 5 Question 31 (Page No. 241) top

Let

$$f(x) = \begin{cases} 3x+1 & \text{for odd } x \\ \dfrac{x}{2} & \text{for even } x \end{cases}$$

for any natural number x. If you start with an integer x and iterate f, you obtain a sequence, $x, f(x), f(f(x)), \ldots$ Stop if you ever hit 1. For example, if $x = 17$, you get the sequence $17, 52, 26, 13, 40, 20, 10, 5, 16, 8, 4, 2, 1$. Extensive computer tests have shown that every starting point between 1 and a large positive integer gives a sequence that ends in 1. But the question of whether all positive starting points end up at 1 is unsolved; it is called the $3x + 1$ problem. Suppose that ATM were decidable by a $TM\ H$. Use H to describe a TM that is guaranteed to state the answer to the $3x + 1$ problem.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.30 Decidability: Michael Sipser Edition 3 Exercise 5 Question 34 (Page No. 241) top

$X = \{\langle M, w\rangle \mid \text{M is a single-tape TM that never modifies the portion of the tape that contains the input } w\}$

Is X decidable? Prove your answer.

michael-sipser theory-of-computation turing-machine decidability proof

Answer key

12.8.31 Decidability: Michael Sipser Edition 3 Exercise 5 Question 9 (Page No. 239) top

Let $T = \{\langle M\rangle \mid \text{M is a TM that accepts } w^R \text{ whenever it accepts } w\}$. Show that T is undecidable.

michael-sipser theory-of-computation turing-machine decidability proof

12.8.32 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 1 (Page No. 307) top

If the halting problem were decidable, then every recursively enumerable language would be recursive. Consequently, the halting problem is undecidable.

Describe in detail how H in given Theorem can be modified to produce H'.

peter-linz peter-linz-edition5 decidability theory-of-computation

12.8.33 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 10 (Page No. 308) top

Let M be any Turing machine and x and y two possible instantaneous descriptions of it. Show that the problem of determining whether or not $x \vdash_M^* y$ is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.34 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 11 (Page No. 308) top

Let $\Gamma = \{0, 1, \square\}$. Consider the function $f(n)$ whose value is the maximum number of moves that can be made by any $n - state$ Turing machine that halts when started with a blank tape. This function, as it turns out, is not computable.

Give the values of $f(1)$ and $f(2)$.

peter-linz peter-linz-edition5 theory-of-computation decidability

12.8.35 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 12 (Page No. 308) top

Show that the problem of determining whether a Turing machine halts on any input is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.36 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 13 (Page No. 308) top

Let B be the set of all Turing machines that halt when started with a blank tape. Show that this set is recursively enumerable, but not recursive.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

Answer key

12.8.37 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 14 (Page No. 308) top

Consider the set of all $n-$state Turing machines with tape alphabet $\Gamma = \{0, 1, \square\}$. Give an expression for $m(n)$, the number of distinct Turing machines with this Γ.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.38 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 15 (Page No. 308) top

Let $\Gamma = \{0, 1, \square\}$ and let $b(n)$ be the maximum number of tape cells examined by any $n-$state Turing machine that halts when started with a blank tape. Show that $b(n)$ is not computable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.39 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 16 (Page No. 308) top

Determine whether or not the following statements is true: Any problem whose domain is finite is decidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.40 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 2 (Page No. 307) top

Definition: Let w_M be a string that describes a Turing machine $M = (Q, \Sigma, \Gamma, \delta, q_0, \square, F)$, and let w be a string in M's alphabet. We will assume that w_m and w_M and w are encoded as a string of $0'$s and $1'$s. A solution of the halting problem is a Turing machine H, which for any w_M and w performs the computation

$$q_0 w_M w \vdash ^* x_1 q_y x_2$$

If M applied to w halts, and

$$q_0 w_M w \vdash ^* y_1 q_n y_2$$

If M applied to w does not halt. Here q_y and q_n are both final states of H

Theorem: There does not exist any Turing machine H that behave as required by Definition. The halting problem is therefore undecidable.

Suppose we change Definition to require that $q_0 w_M w \vdash^* q_y w$ or $q_0 w_M w \vdash^* q_n w$, depending on whether M applied to w halts or not. Reexamine the proof of Theorem to show that this difference in the definition does not affect the proof in any significant way.

peter-linz peter-linz-edition5 decidability theory-of-computation proof

12.8.41 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 3 (Page No. 307) top

Show that the following problem is undecidable. Given any Turing machine $M, a \in \Gamma$ and $w \in \Sigma^+$, determine whether or not the symbol a is ever written when M is applied to w.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.42 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 4 (Page No. 307) top

In the general halting problem, we ask for an algorithm that gives the correct answer for any M and w. We can relax this generality, for example by looking for an algorithm that works for all M but only a single w. We say that such a problem is decidable if for every w there exists a $($possibly different$)$ algorithm that determines whether or not (M, w) halts. Show that even in this restricted setting the problem is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.43 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 5 (Page No. 307) top

Show that there is no problem to decide whether or not an arbitrary Turing machine on all input.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.44 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 6 (Page No. 308) top

Consider the question: "Does a Turing machine in the course of a computation revisit the starting cell $($i.e the cell under the read-write head at the beginning of the computation$)$?" Is this a decidable question$?$

peter-linz peter-linz-edition5 theory-of-computation decidability proof

Answer key

12.8.45 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 7,8 (Page No. 308) top

$i)$ Show that there is no algorithm for deciding if any two Turing machines M_1 and M_2 accept the same language.

$ii)$ How is the conclusion of i affected if M_2 is a finite automaton$?$

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.46 Decidability: Peter Linz Edition 5 Exercise 12.1 Question 9 (Page No. 308) top

Definition: A pushdown automaton $M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$ is said to be deterministic if it is an automaton as defined as defined, subject to the restrictions that, for every $q \in Q, a \in \Sigma \cup \{\lambda\}$ and $b \in \Gamma$

1. $\delta(q, a, b)$ contains at most one element,

2. if $\delta(q, \lambda, b)$ is not empty then $\delta(q, c, b)$ must be empty for every $c \in \Sigma$

Is the halting problem solvable for deterministic pushdown automata; that is, given a pda as in Definition, can we always predict whether or not the automaton will halt on input w?

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.47 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 1 (Page No. 311) top

Theorem : Let M be any Turing machine. Then the question of whether or not $L(M)$ is finite is undecidable.

Show in detail how the machine $\widehat{M}$ in Theorem is constructed.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.48 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 2 (Page No. 311) top

Show that the two problems mentioned at the end of the preceding section, namely

(a) $L(M)$ contains any string of length five,

(b) $L(M)$ is regular,

are undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.49 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 3 (Page No. 311) top

Let M_1 and M_2 be arbitrary Turing machines. Show that the problem "$L(M_1) \subseteq L(M_2)$" is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.50 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 4 (Page No. 311) top

Let G be an unrestricted grammar. Does there exist an algorithm for determining whether or not $L(G)^R$ is recursive enumerable?

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.51 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 5 (Page No. 311) top

Let G be an unrestricted grammar. Does there exist an algorithm for determining whether or not $L(G) = L(G)^R$?

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.52 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 6 (Page No. 311) top

Let G_1 be an unrestricted grammar, and G_2 any regular grammar. Show that the problem

$$L(G_1) \cap L(G_2) = \phi$$

is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.53 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 7 (Page No. 311) top

Let G_1 be an unrestricted grammar, and G_2 any regular grammar. Show that the problem

$$L(G_1) \cap L(G_2) = \phi$$

is undecidable for any fixed G_2, as long as $L(G_2)$ is not empty.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.54 Decidability: Peter Linz Edition 5 Exercise 12.2 Question 8 (Page No. 311) top

For an unrestricted grammar G, show that the question "$Is\ L(G) = L(G)^*?$" is undecidable. Argue (a) from Rice's theorem and (b) from first principles.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.55 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 1 (Page No. 317) top

Let $A = \{001, 0011, 11, 101\}$ and $B = \{01, 111, 111, 010\}$. Does the pair (A, B) have a PC solution? Does it have an MPC solution?

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.56 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 2 (Page No. 317) top

Theorem : Let $G = (V, T, S, P)$ be an unrestricted grammar, with w any string in T^+. Let (A, B) be the correspondence pair constructed from G and w be the process exhibited in Figure. Then the pair (A, B) permits an MPC solution if and only if $w \in L(G)$.

A	B	
$FS \Rightarrow$	F	F is a symbol in $V \cup T$
a	a	for every $a \in T$
V_i	V_i	for every $V_i \in V$
E	$\Rightarrow wE$	E is a symbol not in $V \cup T$
y_i	x_i	for every $x_i \to y_i$ in P
$\Rightarrow$	$\Rightarrow$	

$FS \Rightarrow$ is to be taken as w_1 and the string F as v_1. The order of the rest of the strings is immaterial.

Provide the details of the proof of the Theorem.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.57 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 3 (Page No. 317) top

Show that for $|\Sigma| = 1$, the Post correspondence problem is decidable, that is, there is an algorithm that can decide whether or not (A, B) has a PC solution for any given (A, B) on a single-letter alphabet.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.58 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 4 (Page No. 317) top

Suppose we restrict the domain of the Post correspondence problem to include only alphabets with exactly two symbols. Is the resulting correspondence problem decidable?

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.59 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 5 (Page No. 317,318) top

Show that the following modifications of the Post correspondence problem are undecidable.

(a) There is an MPC solution if there is a sequence of integers such that $w_i w_j \dots w_k w_1 = v_i v_j \dots v_k v_i$.

(b) There is an MPC solution if there is a sequence of integers such that $w_1 w_2 w_i w_j \dots w_k = v_1 v_2 v_i v_j \dots v_k$.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.60 Decidability: Peter Linz Edition 5 Exercise 12.3 Question 6 (Page No. 318) top

The correspondence pair (A, B) is said to have an even PC solution if and only if there exists a nonempty sequence of even integers $i, j, .. k$ such that $w_i w_j \dots w_k = v_i v_j \dots v_k$. Show that the problem of deciding whether or not an arbitrary pair (A, B) has an even PC solution is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.61 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 1 (Page No. 320) top

Theorem : There exists no algorithm for deciding whether any given context-free grammar is ambiguous.

Prove the claim made in Theorem that G_A and G_B by themselves are unambiguous

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.62 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 2 (Page No. 321) top

Show that the problem of determining whether or not $L(G_1) \subseteq L(G_2)$ is undecidable for context-free grammars $G_1,\ G_2$.

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

12.8.63 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 3 (Page No. 321) top

Show that for arbitrary context-free grammars G_1 and G_2, the problem "$L(G_1) \cap L(G_2)$ is context-free" is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

12.8.64 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 4 (Page No. 321) top

Theorem : There exist no algorithms for deciding whether any given context-free grammar is ambiguous.

Show that if the language $L(G_A) \cap L(G_B)$ in Theorem is regular, then it must be empty. Use this to show that the problem "$L(G)$ is regular " is undecidable for context-free G.

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12.8.65 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 5 (Page No. 321) top

Let L_1 be a regular language and G a context-free grammar. Show that the problem "$L_1 \subseteq L(G)$" is undecidable.

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

Answer key

12.8.66 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 6 (Page No. 321) top

Let M be any Turing machine. We can assume without loss of generality that every computation involves an even number of moves. For any such computation

$$q_0w \vdash x_1 \vdash x_2 \vdash \ldots \vdash x_n \ ,$$

we can then construct the string

$$q_0w \vdash x_1^R \vdash x_2 \vdash x_3^R \vdash \ldots \vdash x_n \ .$$

This is called a valid computation.

Show that for every M we can construct three context-free grammars G_1, G_2, G_3 such that

(a) the set of all valid computations is $L(G_1) \cap L(G_2)$, and

(b) the set of all invalid computations (that is, the complement of the set of valid computations) is $L(G_3)$.

Use the results to show that "$L(G) = \Sigma^*$" is undecidable over the domain of all context-free grammars G.

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

12.8.67 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 7 (Page No. 321) top

Let G_1 be a context-free grammar and G_2 a regular grammar. Is the problem $L(G_1) \cap L(G_2) = \phi$ decidable?

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

Answer key

12.8.68 Decidability: Peter Linz Edition 5 Exercise 12.4 Question 8 (Page No. 321) top

Let G_1 and G_2 be grammars with G_1 regular. Is the problem $L(G_1) = L(G_2)$ decidable when

(a) G_2 is unrestricted,

(b) when G_2 is context-free,

(c) when G_2 is regular?

peter-linz peter-linz-edition5 theory-of-computation decidability proof difficult

Answer key

12.8.69 Decidability: Peter Linz Edition 5 Exercise 12.5 Question 1 (Page No. 323) top

Consider the language

$$L = \{ww : w \in \{a, b\}^+\}.$$

Discuss the construction and efficiency of algorithms for accepting L on

(a) a standard Turing machine,

(b) on a two-tape deterministic Turing machine,

(c) on a single-tape nondeterministic Turing machine,

(d) on a two-tape nondeterministic Turing machine.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.8.70 Decidability: Peter Linz Edition 5 Exercise 12.5 Question 2 (Page No. 323) top

Consider the language

$$L = \{www : w \in \{a, b\}^+\}.$$

Discuss the construction and efficiency of algorithms for accepting L on

(a) a standard Turing machine,

(b) on a two-tape deterministic Turing machine,

(c) on a single-tape nondeterministic Turing machine,

(d) on a two-tape nondeterministic Turing machine.

peter-linz peter-linz-edition5 theory-of-computation decidability proof

12.9 Dpda (1) top

12.9.1 Dpda: Michael Sipser Edition 3 Exercise 4 Question 32 (Page No. 213) top

The proof of Lemma 2.41 says that (q, x) is a looping situation for a $DPDA\ P$ if when P is started in state q with $x \in \Gamma$ on the top of the stack, it never pops anything below x and it never reads an input symbol. Show that F is decidable, where $F = \{\langle P, q, x\rangle \mid (q, x)$ is a looping situation for P$\}$.

michael-sipser theory-of-computation dpda decidability proof

12.10 Enumerated Language (1) top

12.10.1 Enumerated Language: Michael Sipser Edition 3 Exercise 3 Question 4 (Page No. 187) top

Give a formal definition of an enumerator. Consider it to be a type of two-tape Turing machine that uses its second tape as the printer. Include a definition of the enumerated language.

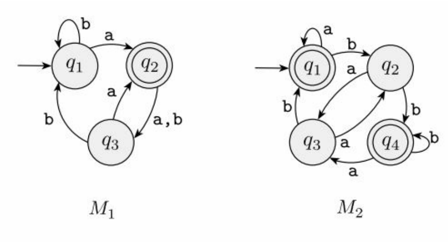
michael-sipser theory-of-computation turing-machine enumerated-language descriptive

Answer key

12.11 Finite Automata (19) top

12.11.1 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 1 (Page No. 83) top

The following are the state diagrams of two DFAs, M1, and M2. Answer the following questions about each of these machines.



a. What is the start state?
b. What is the set of accept states?
c. What sequence of states does the machine go through on input aabb?
d. Does the machine accept the string aabb?
e. Does the machine accept the string ϵ?

michael-sipser theory-of-computation finite-automata descriptive

12.11.2 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 11 (Page No. 85) top

Prove that every NFA can be converted to an equivalent one that has a single accept state.

michael-sipser theory-of-computation finite-automata proof

Answer key

12.11.3 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 12 (Page No. 85) top

Let $D = \{w \mid w$ contains an even number of a's and an odd number of b's and does not contain the substring $ab\}$. Give a DFA with five states that recognizes D and a regular expression that generates D. (Suggestion: Describe D more simply.)

michael-sipser theory-of-computation finite-automata

Answer key

12.11.4 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 13 (Page No. 85) top

Let F be the language of all strings over $\{0,1\}$ that do not contain a pair of $1's$ that are separated by an odd number of symbols. Give the state diagram of a DFA with five states that recognizes F. (You may find it helpful first to find a 4-state NFA for the complement of F.)

michael-sipser theory-of-computation finite-automata

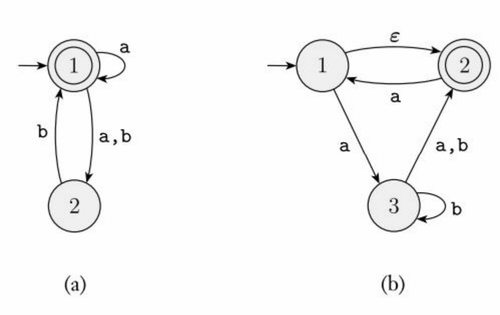
12.11.5 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 14 (Page No. 85) top

a. Show that if M is a DFA that recognizes language B, swapping the accept and not accept states in M yields a new DFA recognizing the complement of B. Conclude that the class of regular languages is closed under complement.
b. Show by giving an example that if M is an NFA that recognizes language C, swapping the accept and not accept states in M doesn't necessarily yield a new NFA that recognizes the complement of C. Is the class of languages recognized by $NFA's$ closed under complement? Explain your answer.

michael-sipser theory-of-computation finite-automata

12.11.6 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 16 (Page No. 86) top

Use the construction given in $\mathrm{Theorem\ 1.39}$ to convert the following two non-deterministic finite automata to equivalent deterministic finite automata.



michael-sipser theory-of-computation finite-automata

Answer key

12.11.7 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 17 (Page No. 86) top

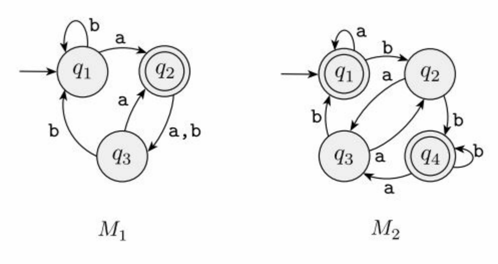
a. Give an NFA recognizing the language $(01 \cup 001 \cup 010)^*$.
b. Convert this NFA to an equivalent DFA. Give only the portion of the DFA that is reachable from the start state.

michael-sipser theory-of-computation finite-automata

Answer key

12.11.8 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 2 (Page No. 83) top

Give the formal description of the machines M1 and M2.



michael-sipser theory-of-computation finite-automata descriptive

12.11.9 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 28 (Page No. 88) top

Convert the following regular expressions to $NFA's$ using the procedure given in Theorem 1.54. In all parts, $\Sigma = \{a, b\}$.

a. $a(abb)^* \cup b$
b. $a^+ \cup (ab)^+$
c. $(a \cup b^+)a^+b^+$

michael-sipser theory-of-computation finite-automata

Answer key

12.11.10 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 39 (Page No. 89) top

The construction in Theorem 1.54 shows that every GNFA is equivalent to a GNFA with only two states. We can show that an opposite phenomenon occurs for DFAs. Prove that for every $k > 1$, a language $A_k \subseteq \{0,1\}^*$ exists that is recognized by a DFA with k states but not by one with only $k-1$ states.

michael-sipser theory-of-computation finite-automata proof

12.11.11 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 5 (Page No. 84) top

Each of the following languages is the complement of a simpler language. In each part, construct a DFA for the simpler language, then use it to give the state diagram of a DFA for the language given. In all parts, $\Sigma = a, b$.

a. {w| w does not contain the substring ab}
b. {w| w does not contain the substring baba}
c. {w| w contains neither the substrings ab nor ba}
d. {w| w is any string not in a*b*}
e. {w| w is any string not in $(ab^+)^*$}
f. {w| w is any string not in $a^* \cup b^*$}
g. {w| w is any string that doesn't contain exactly two a's}
h. {w| w is any string except a and b}

michael-sipser theory-of-computation finite-automata finite-automata descriptive

Answer key

12.11.12 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 51 (Page No. 90) top

Let x and y be strings and let L be any language. We say that x and y are distinguishable by L if some string z exists whereby exactly one of the strings xz and yz is a member of L; otherwise, for every string z, we have $xz \in L$ whenever $yz \in L$ and we say that x and y are indistinguishable by L. If x and y are indistinguishable by L, we write $x \equiv Ly$. Show that $\equiv L$ is an equivalence relation.
A palindrome is a string that reads the same forward and backward.

michael-sipser theory-of-computation finite-automata proof descriptive

12.11.13 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 52 (Page No. 91) top

Myhill–Nerode theorem. Refer to Question 51.Let L be a language and let X be a set of strings. Say that X is pairwise distinguishable by L if every two distinct strings in X are distinguishable by L. Define the index of L to be the maximum number of elements in any set that is pairwise distinguishable by L. The index of L may be finite or infinite.

a. Show that if L is recognized by a DFA with k states, L has index at most k.
b. Show that if the index of L is a finite number k, it is recognized by a DFA with k states.
c. Conclude that L is regular iff it has finite index. Moreover, its index is the size of the smallest DFA recognizing it.

michael-sipser theory-of-computation finite-automata finite-automata proof descriptive

12.11.14 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 60 (Page No. 92) top

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an a exactly k places from the right-hand end. Thus $C_k = \sum^* a \sum^{k-1}$. Describe an NFA with $k+1$ states that recognizes C_k in terms of both a state diagram and a formal description.

michael-sipser theory-of-computation finite-automata descriptive

Answer key

12.11.15 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 61 (Page No. 92) top

Let $\Sigma = \{a, b\}$. For each $k \geq 1$, let C_k be the language consisting of all strings that contain an a exactly k places from the right-hand end. Thus $C_k = \sum^* a \sum^{k-1}$. Prove that for each k, no DFA can recognize C_k with fewer than 2^k states.

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12.11.16 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 62 (Page No. 92) top

Let $\sum = \{a, b\}$. For each $k \geq 1$, let D_k be the language consisting of all strings that have at least one a among the last k symbols.Thus $D_k = \sum^* a(\sum \cup \epsilon)^{k-1}$.Describe a DFA with at most $k+1$ states that recognizes D_k in terms of both a state diagram and a formal description.

michael-sipser theory-of-computation finite-automata descriptive

12.11.17 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 64 (Page No. 92) top

Let N be an NFA with k states that recognizes some language A.

a. Show that if A is nonempty, A contains some string of length at most k.

b. Show, by giving an example, that part (a) is not necessarily true if you replace both $A's$ by $\overline{A}$.
c. Show that if $\overline{A}$ is nonempty, $\overline{A}$ contains some string of length at most 2^k.
d. Show that the bound given in part (c) is nearly tight; that is, for each k, demonstrate an NFA recognizing a language A_k where $\overline{A_k}$ is nonempty and where $\overline{A_k}$'s shortest member strings are of length exponential in k. Come as close to the bound in (c) as you can.

michael-sipser theory-of-computation finite-automata descriptive

12.11.18 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 65 (Page No. 93) top

Prove that for each $n > 0$, a language B_n exists where

a. B_n is recognizable by an NFA that has n states, and
b. if $B_n = A_1 \cup \ldots \cup A_k$, for regular languages A_i, then at least one of the A_i requires a DFA with exponentially many states.

michael-sipser theory-of-computation finite-automata descriptive

12.11.19 Finite Automata: Michael Sipser Edition 3 Exercise 1 Question 69 (Page No. 93) top

Let $\sum = \{0, 1\}$. Let $WW_k = \{ww | w \in \sum^*$ and w is of length $k\}$.

a. Show that for each k, no DFA can recognize WW_k with fewer than 2^k states.
b. Describe a much smaller NFA for $\overline{WW_k}$, the complement of WW_k.

michael-sipser theory-of-computation finite-automata proof descriptive

12.12 Finite State Transducer (2) top

12.12.1 Finite State Transducer: Michael Sipser Edition 3 Exercise 1 Question 25 (Page No. 87) top

Read the informal definition of the finite state transducer given in question 24. Give a formal definition of this model, following the pattern in Definition 1.5 (page 35). Assume that an FST has an input alphabet Σ and an output alphabet Γ but not a set of accept states. Include a formal definition of the computation of an FST. $(Hint:$ An FST is a 5-tuple. Its transition function is of the form $\delta : Q \times \Sigma \to Q \times \Gamma.)$

michael-sipser theory-of-computation finite-state-transducer

Answer key

12.12.2 Finite State Transducer: Michael Sipser Edition 3 Exercise 1 Question 50 (Page No. 90) top

Read the informal definition of the finite state transducer given in Question 24. Prove that no FST can output w^R for every input w if the input and output alphabets are $\{0, 1\}$.

michael-sipser theory-of-computation finite-automata finite-state-transducer proof descriptive

12.13 Fst (2) top

12.13.1 Fst: Michael Sipser Edition 3 Exercise 1 Question 26 (Page No. 87) top

Using the solution you gave to question 25, give a formal description of the machines T_1 and T_2 depicted in question 24.

michael-sipser theory-of-computation finite-automata fst descriptive

12.13.2 Fst: Michael Sipser Edition 3 Exercise 1 Question 27 (Page No. 88) top

Read the informal definition of the finite state transducer given in question 24. Give the state diagram of an FST with the following behavior. Its input and output alphabets are $\{0, 1\}$. Its output string is identical to the input string on the even positions but inverted on the odd positions. For example, on input 0000111 it should output 1010010.

michael-sipser theory-of-computation fst descriptive

Answer key

12.14 Functions (1) top

12.14.1 Functions: Michael Sipser Edition 3 Exercise 0 Question 6 (Page No. 26) top

Let X be the set {1, 2, 3, 4, 5} and Y be the set {6, 7, 8, 9, 10}. The unary function $f : X \to Y$ and the binary function $g : X \times Y \to Y$ are described in the following tables.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

g	6	7	8	9	10
1	10	10	10	10	10
2	7	8	9	10	6
3	7	7	8	8	9
4	9	8	7	6	10
5	6	6	6	6	6

a. What is the value of f(2)?
b. What are the range and domain of f?
c. What is the value of g(2, 10)?
d. What are the range and domain of g?
e. What is the value of g(4, f(4))?

michael-sipser theory-of-computation functions easy

12.15 Gnf (5) top

12.15.1 Gnf: Peter Linz Edition 4 Exercise 6.2 Question 10 (Page No. 170) top

Convert the grammar $S \to aSb|bSa|a|b$ into Greibach normal form.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar gnf

12.15.2 Gnf: Peter Linz Edition 4 Exercise 6.2 Question 11 (Page No. 170) top

Convert the following grammar into Greibach normal form.

$$S \to aSb|ab$$

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar gnf

12.15.3 Gnf: Peter Linz Edition 4 Exercise 6.2 Question 12 (Page No. 170) top

Convert the grammar $S \to ab|aS|aaS$ into Greibach normal form.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar gnf

12.15.4 Gnf: Peter Linz Edition 4 Exercise 6.2 Question 13 (Page No. 170) top

Convert the grammar

$$S \to ABb|a,$$

$$A \to aaA|B,$$

$$B \to bAb$$

into Greibach normal form.

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar gnf

12.15.5 Gnf: Peter Linz Edition 4 Exercise 6.2 Question 14 (Page No. 170) top

Can every linear grammar be converted to a form in which all productions look like $A \to ax$, where $a \in T$ and $x \in V \cup \{\lambda\}$?

peter-linz peter-linz-edition4 theory-of-computation context-free-grammar gnf

12.16 Grammar (28) top

12.16.1 Grammar: Michael Sipser Edition 3 Exercise 2 Question 9 (Page No. 155) top

Give a context-free grammar that generates the language $A = \{a^i b^j c^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$. Is your grammar ambiguous? Why or why not?

michael-sipser theory-of-computation context-free-language ambiguous grammar

Answer key

12.16.2 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 11 (Page No. 28) top

Find grammars for $\Sigma = \{a, b\}$ that generate the sets of
(a) all strings with exactly one a.
(b) all strings with at least one a.
(c) all strings with no more than three a's.
(d) all strings with at least three a's.

In each case, give convincing arguments that the grammar you give does indeed generate the indicated language.

peter-linz peter-linz-edition4 theory-of-computation grammar

12.16.3 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 12 (Page No. 28) top

Give a simple description of the language generated by the grammar with productions

$S \to aA$,

$A \to bS$,

$S \to \lambda$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.4 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 13 (Page No. 28) top

What language does the grammar with these productions generate?

$S \to Aa$,

$A \to B$,

$B \to Aa$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.5 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 14 (Page No. 28) top

Let $\Sigma = \{a, b\}$. For each of the following languages, find a grammar that generates it.
(a) $L_1 = \{a^n b^m : n \geq 0, m > n\}$.
(b) $L_2 = \{a^n b^{2n} : n \geq 0\}$.
(c) $L_3 = \{a^{n+2} b^n : n \geq 1\}$.
(d) $L_4 = \{a^n b^{n-3} : n \geq 3\}$.
(e) $L_1 L_2$.
(f) $L_1 \cup L_2$.
(g) L_1^3.
(h) L_1^*.
(i) $L_1 - \bar{L}_4$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.6 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 14.g (Page No. 29) top

$L = \{a^n b^m : n \geq 0, m > n\}$

Find a grammar that generates L^3

theory-of-computation peter-linz peter-linz-edition4 grammar

Answer key

12.16.7 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 15 (Page No. 29) top

Find grammars for the following languages on Σ = {a}.
(a) L = {w : |w| mod 3 = 0}.
(b) L = {w : |w| mod 3 > 0}.
(c) L = {w : |w| mod 3 ≠ |w| mod 2}.
(d) L = {w : |w| mod 3 ≥ |w| mod 2}.

peter-linz peter-linz-edition4 theory-of-computation grammar

12.16.8 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 15.d (Page No. 29) top

Find the grammar for the following language

$L = \{w : |w|\, mod 3 \geq |w|\, mod 2\}$

theory-of-computation grammar peter-linz peter-linz-edition4

Answer key

12.16.9 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 16 (Page No. 29) top

Find a grammar that generates the language:

L = $\{ww^R : w \in$ {a, b}+}

theory-of-computation grammar peter-linz peter-linz-edition4 context-free-language

Answer key

12.16.10 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 17 (Page No. 29) top

Give a verbal description of the language generated by the productions:

S → aSb

S → bSa

S → aa

theory-of-computation peter-linz peter-linz-edition4 grammar

Answer key

12.16.11 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 18 (Page No. 29) top

Assume $\sum = \{a, b\}$

1. $L = \{w : n_a(w) = n_b(w) + 1\}$
2. $L = \{w : n_a(w) > n_b(w)\}$
3. $L = \{w : n_a(w) = 2n_b(w)\}$
4. $L = \{w \in \{a, b\}^* : |n_a(w) - n_b(w)| = 1\}$

theory-of-computation peter-linz peter-linz-edition4 grammar

Answer key

12.16.12 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 21 (Page No. 29) top

Are the two grammars with respective productions

$$S \to aSb|ab|\lambda,$$

and

$$S \to aAb|ab,$$

$$A \to aAb|\lambda,$$

equivalent? Assume that S is the start symbol in both cases.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.13 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 22 (Page No. 30) top

Show that the grammar $G = (\{S\}, \{a, b\}, S, P)$, with productions

$$S \to SS|SSS|aSb|bSa|\lambda \ ,$$

is equivalent to the grammar

$$S \to SS \ ,$$
$$S \to \lambda \ ,$$
$$S \to aSb \ ,$$
$$S \to bSa \ .$$

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.14 Grammar: Peter Linz Edition 4 Exercise 1.2 Question 23 (Page No. 30) top

Show that the grammars

$$S \to aSb|bSa|SS|a$$

and

$$S \to aSb|bSa|a$$

are not equivalent.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.15 Grammar: Peter Linz Edition 4 Exercise 2.1 Question 21 (Page No. 48) top

Let L be the language accepted by the automaton $L = \{(a^n)b : n \geq 0\}$.

Find a dfa that accepts the language $L^2 - L$.

theory-of-computation regular-language peter-linz peter-linz-edition4 finite-automata grammar

Answer key

12.16.16 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 1 (Page No. 144) top

Find an s-grammar for $L(aaa^*b + b)$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.17 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 10 (Page No. 145) top

Give an unambiguous grammar that generates the set of all regular expressions on $\Sigma = \{a, b\}$.

peter-linz peter-linz-edition4 theory-of-computation grammar regular-expression

12.16.18 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 13 (Page No. 145) top

Show that the following grammar is ambiguous.

$$S \rightarrow aSbS|bSaS|\lambda$$

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

Answer key

12.16.19 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 14 (Page No. 145) top

Show that the grammar $S \rightarrow aSb|SS|\lambda$ is ambiguous, but that the language denoted by it is not.

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

Answer key

12.16.20 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 15 (Page No. 145) top

Show that the grammar with productions

$S \rightarrow SS,$

$S \rightarrow \lambda,$

$S \rightarrow aSb,$

$S \rightarrow bSa.$

is ambiguous.

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

Answer key

12.16.21 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 16 (Page No. 145) top

Show that the grammar with productions

$S \rightarrow aAB,$

$A \rightarrow bBb,$

$B \rightarrow A|\lambda.$

is unambiguous.

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

12.16.22 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 2 (Page No. 144) top

Find an s-grammar for $L = \{a^n b^n : n \geq 1\}$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.23 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 3 (Page No. 144) top

Find an s-grammar for $L = \{a^n b^{n+1} : n \geq 2\}$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.24 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 4 (Page No. 145) top

Show that every s-grammar is unambiguous.

peter-linz peter-linz-edition4 theory-of-computation ambiguous grammar

12.16.25 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 5 (Page No. 145) top

Let $G = (V, T, S, P)$ be an s-grammar. Give an expression for the maximum size of P in terms of $|V|$ and $|T|$.

peter-linz peter-linz-edition4 theory-of-computation grammar

Answer key

12.16.26 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 6 (Page No. 145) top

Show that the following grammar is ambiguous.

$S \rightarrow AB|aaB,$

$A \rightarrow a|Aa,$

$B \rightarrow b.$

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

Answer key

12.16.27 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 7 (Page No. 145) top

Construct an unambiguous grammar equivalent to the grammar in **Exercise 6**.

peter-linz peter-linz-edition4 theory-of-computation grammar ambiguous

Answer key

12.16.28 Grammar: Peter Linz Edition 4 Exercise 5.2 Question 8 (Page No. 145) top

Give the derivation tree for (((a + b) * c)) + a + b, using the grammar with productions

$E \rightarrow I,$

$E \rightarrow E + E,$

$E \rightarrow E * E,$

$E \rightarrow (E),$

$I \rightarrow a|b|c.$

peter-linz peter-linz-edition4 theory-of-computation grammar

12.17 Homomorphism (1) top

12.17.1 Homomorphism: Michael Sipser Edition 3 Exercise 1 Question 66 (Page No. 93) top

A homomorphism is a function $f : \Sigma \rightarrow \Gamma^*$ from one alphabet to strings overanother alphabet. We can extend f to operate on strings by defining $f(w) = f(w_1)f(w_2)\dots f(w_n)$, where $w = w_1w_2\dots w_n$ and each $w_i \in \sum$. We further extend f to operate on languages by defining $f(A) = \{f(w)|w \in A\}$, for any language A.

a. Show,by giving a formal construction, that the class of regular languages is closed under homomorphism. In other words, given a DFA M that recognizes B and a homomorphism f, construct a finite automaton M' that recognizes $f(B)$. Consider the machine M' that you constructed. Is it a DFA in every case ?
b. Show, by giving an example, that the class of non-regular languages is not closed under homomorphism.

michael-sipser theory-of-computation finite-automata homomorphism descriptive

12.18 Inherently Ambiguous (4) top

12.18.1 Inherently Ambiguous: Michael Sipser Edition 3 Exercise 2 Question 29 (Page No. 157) top

Show that the language $A = \{a^ib^jc^k \mid i = j \text{ or } j = k \text{ where } i, j, k \geq 0\}$ is inherently ambiguous.

michael-sipser theory-of-computation context-free-language inherently-ambiguous

12.18.2 Inherently Ambiguous: Peter Linz Edition 4 Exercise 5.2 Question 12 (Page No. 145) top

Show that the language $L = \{ww^R : w \in \{a,b\}^*\}$ is not inherently ambiguous.

peter-linz peter-linz-edition4 theory-of-computation inherently-ambiguous grammar

12.18.3 Inherently Ambiguous: Peter Linz Edition 4 Exercise 5.2 Question 9 (Page No. 145) top

Show that a regular language cannot be inherently ambiguous.

peter-linz peter-linz-edition4 theory-of-computation inherently-ambiguous grammar

12.18.4 Inherently Ambiguous: Peter Linz Edition 4 Exercise 7.4 Question 7 (Page No. 204) top

Show that a deterministic context-free language is never inherently ambiguous.

peter-linz peter-linz-edition4 theory-of-computation context-free-language inherently-ambiguous

12.19 Legitimate Turing Machine (1) top

12.19.1 Legitimate Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 7 (Page No. 188) top

Explain why the following is not a description of a legitimate Turing machine.

M_{bad} = " On input $\langle p \rangle$, a polynomial over variables $x_1, \dots, x_k$:

1. Try all possible settings of $x_1, \dots, x_k$ to integer values.
2. Evaluate p on all of these settings.
3. If any of these settings evaluates to 0, accept; otherwise, reject."

michael-sipser theory-of-computation turing-machine legitimate-turing-machine descriptive

12.20 Non Determinism (1) top

12.20.1 Non Determinism: Michael Sipser Edition 3 Exercise 3 Question 3 (Page No. 187) top

Modify the proof of Theorem 3.16 to obtain Corollary 3.19, showing that a language is decidable iff some nondeterministic Turing machine decides it. (You may assume the following theorem about trees. If every node in a tree has finitely many children and every branch of the tree has finitely many nodes, the tree itself has finitely many nodes.)

michael-sipser theory-of-computation non-determinism turing-machine descriptive

12.21 Npda (27) top

12.21.1 Npda: Peter Linz Edition 4 Exercise 7.1 Question 10 (Page No. 184) top

What language is accepted by the pda

$$M = (\{q_0, q_1, q_2, q_3, q_4, q_5\}, \{a, b\}, \{0, 1, a\}, \delta, q_0, z, \{q_5\}),$$

with

$$\delta(q_0, b, z) = \{(q_1, 1z)\},$$

$$\delta(q_0, b, 1) = \{(q_1, 11)\},$$

$$\delta(q_2, a, 1) = \{(q_3, \lambda)\},$$

$$\delta(q_3, a, 1) = \{(q_4, \lambda)\}$$

$$\delta(q_4, a, z) = \{(q_4, z), (q_5, z)\} \ ?$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.2 Npda: Peter Linz Edition 4 Exercise 7.1 Question 11 (Page No. 184) top

What language is accepted by the npda $M = (\{q_0, q_1, q_2\}, \{a, b\}, \{a, b, z\}, \delta, q_0, z, \{q_2\})$ with transitions

$$\delta(q_0, a, z) = \{(q_1, a), (q_2, \lambda)\},$$

$$\delta(q_1, b, a) = \{(q_1, b)\},$$

$$\delta(q_1, b, b) = \{(q_1, b)\},$$

$$\delta(q_1, a, b) = \{(q_2, \lambda)\} \text{ ?}$$

theory-of-computation peter-linz peter-linz-edition4 pushdown-automata npda

12.21.3 Npda: Peter Linz Edition 4 Exercise 7.1 Question 13 (Page No. 184) top

What language is accepted by the npda in **Exercise 11** if we use $F = \{q_0, q_1, q_2\}$?

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.4 Npda: Peter Linz Edition 4 Exercise 7.1 Question 14 (Page No. 184) top

Find an npda with no more than two internal states that accepts the language $L(aa^*ba^*)$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.5 Npda: Peter Linz Edition 4 Exercise 7.1 Question 16 (Page No. 184) top

We can define a restricted npda as one that can increase the length of the stack by at most one symbol in each move, changing ***Definition 7.1*** so that

$$\delta : Q \times (\sum \cup \{\lambda\}) \times \Gamma \rightarrow 2^{Q \times (\Gamma\Gamma \cup \Gamma \cup \lambda)}$$

The interpretation of this is that the range of δ consists of sets of pairs of the form $(q_i, ab), (q_i, a)$, or (q_i, λ). Show that for every npda M there exists such a restricted npda $\widehat{M}$ such that $L(M) = L(\widehat{M})$.

Definition 7.1: A nondeterministic pushdown accepter (npda) is defined by the septuple

$$M = (Q, \sum, \Gamma, \delta, q_0, z, F)$$

where,
Q is a finite set of internal states of the control unit,
Σ is the input alphabet,
Γ is a finite set of symbols called the stack alphabet,
$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \rightarrow$ set of finite subsets of $Q \times \Gamma^*$ is the transition function,
$q_0 \in Q$ is the initial state of the control unit,
$z \in \Gamma$ is the stack start symbol,
$F \subseteq Q$ is the set of final states.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.6 Npda: Peter Linz Edition 4 Exercise 7.1 Question 2 (Page No. 183) top

Prove that an npda for accepting the language $L = \{ ww^R : w \in \{a, b\}^+ \}$ does not accept any string not in $\{ww^R\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.7 Npda: Peter Linz Edition 4 Exercise 7.1 Question 3 (Page No. 183) top

Construct npda's that accept the following regular languages.
(a) $L_1 = L(aaa^*b)$.
(b) $L_1 = L(aab^*aba^*)$.
(**c & d here**)

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.8 Npda: Peter Linz Edition 4 Exercise 7.1 Question 3.c,3.d,4.f,4.j (Page No. 183) top

Q3) Given,

$L_1 = (aaa^*b)$

$L_2 = (aab^*aba^*)$

Find (c) the union of L_1 and L_2, and also find (d) $L_1 - L_2$.

Q4) Find the npda's of the following:

f) $L = \{a^n b^m : n \leq m \leq 3n\}$

j) $L = \{w : 2n_a(w) \leq n_b(w)) \leq 3n_a(w)\}$.

theory-of-computation context-free-language peter-linz peter-linz-edition4 pushdown-automata npda

Answer key

12.21.9 Npda: Peter Linz Edition 4 Exercise 7.1 Question 4 (Page No. 183) top

Construct npda's that accept the following languages on $\Sigma = \{a, b, c\}$.
(a) $L = \{a^n b^{2n} : n \geq 0\}$.
(b) $L = \{wcw^R : w \in \{a, b\}^*\}$.
(c) $L = \{a^n b^m c^{n+m} : n \geq 0, m \geq 0\}$.
(d) $L = \{a^n b^{n+m} c^m : n \geq 0, m \geq 1\}$.
(e) $L = \{a^3 b^n c^n : n \geq 0\}$.
(f) here
(g) $L = \{w : n_a(w) = n_b(w) + 1\}$.
(h) here
(i) $L = \{w : n_a(w) + n_b(w) = n_c(w)\}$.
(j) here.
(k) $L = \{w : n_a(w) < n_b(w)\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.10 Npda: Peter Linz Edition 4 Exercise 7.1 Question 4.h(Page No. 183) top

Construct npda for the following languages on $\sum = \{a, b, c\}$

$L = \{\ w : n_a(w) = 2 * n_b(w)\ \}$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

Answer key

12.21.11 Npda: Peter Linz Edition 4 Exercise 7.1 Question 5 (Page No. 183) top

Construct an npda that accepts the language $L = \{a^n b^m : n \geq 0, n \neq m\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.12 Npda: Peter Linz Edition 4 Exercise 7.1 Question 6 (Page No. 183) top

Find an npda on $\Sigma = \{a, b, c\}$ that accepts the language

$$L = \{w_1 c w_2 : w_1, w_2 \in \{a, b\}^*, w_1 \neq w_2^R\}.$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.13 Npda: Peter Linz Edition 4 Exercise 7.1 Question 7 (Page No. 183) top

Find an npda for the concatenation of $L(a^*)$ and the language in *Exercise 6*.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.14 Npda: Peter Linz Edition 4 Exercise 7.1 Question 8 (Page No. 183) top

Find an npda for the language $L = \{ab(ab)^n b(ba)^n : n \geq 0\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.15 Npda: Peter Linz Edition 4 Exercise 7.1 Question 9 (Page No. 183) top

Is it possible to find a dfa that accepts the same language as the pda

$$M = (\{q_0, q_1\}, \{a, b\}, \{z\}, \delta, q_0, z, \{q_1\}),$$

with

$$\delta(q_0, a, z) = \{(q_1, z)\},$$

$$\delta(q_0, b, z) = \{(q_0, z)\},$$

$$\delta(q_1, a, z) = \{(q_1, z)\},$$

$$\delta(q_1, b, z) = \{(q_0, z)\}\ ?$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

Answer key

12.21.16 Npda: Peter Linz Edition 4 Exercise 7.2 Question 10 (Page No. 195) top

Find an npda with two states that accepts $L = \{a^n b^{2n} : n \geq 1\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

Answer key

12.21.17 Npda: Peter Linz Edition 4 Exercise 7.2 Question 11,12,13 (Page No. 195) top

11. Show that the npda in *Example 7.8* accepts L (aa*b).
12. Find the grammar that generates *Example 7.8* and prove that this grammar generates the language L (aa*b).
13. show that the variable $(q_0 z q_1)$ is useless. (see page no. 191-193)

Example 7.8 : Consider the npda with transitions

$$\delta(q_0, a, z) = \{(q_0, Az)\},$$
$$\delta(q_0, a, A) = \{(q_0, A)\},$$
$$\delta(q_0, b, A) = \{(q_1, \lambda)\},$$
$$\delta(q_1, \lambda, z) = \{(q_2, \lambda)\}.$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.18 Npda: Peter Linz Edition 4 Exercise 7.2 Question 14 (Page No. 195) top

find an npda for the language $L = \{ ww^R : w \in \{a, b\}^+\}$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.19 Npda: Peter Linz Edition 4 Exercise 7.2 Question 15 (Page No. 195) top

Find a context-free grammar that generates the language accepted by the npda $M = (\{q_0, q_1\}, \{a, b\}, \{A, z\}, \delta, q_0, z, \{q_1\})$, with transitions

$$\delta(q_0, a, z) = \{(q_0, Az)\},$$
$$\delta(q_0, b, A) = \{(q_0, AA)\},$$
$$\delta(q_0, a, A) = \{(q_1, \lambda)\}.$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.20 Npda: Peter Linz Edition 4 Exercise 7.2 Question 17 (Page No. 196) top

Give full details of the proof of *Theorem 7.2* .

Theorem 7.2 : If L = L (M) for some npda M, then L is a context-free language.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.21 Npda: Peter Linz Edition 4 Exercise 7.2 Question 18 (Page No. 196) top

Give a construction by which an arbitrary context-free grammar can be used in the proof of *Theorem 7.1*.

Theorem 7.1: For any context-free language L, there exists an npda M such that L = L (M).

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.22 Npda: Peter Linz Edition 4 Exercise 7.2 Question 3 (Page No. 195) top

Construct an npda that accepts the language generated by the grammar

$$S \to aSbb|aab.$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.23 Npda: Peter Linz Edition 4 Exercise 7.2 Question 4 (Page No. 195) top

Construct an npda that accepts the language generated by the grammar $S \to aSSS|ab$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

Answer key

12.21.24 Npda: Peter Linz Edition 4 Exercise 7.2 Question 5 (Page No. 195) top

Construct an npda corresponding to the grammar

$$S \to aABB|aAA,$$

$$A \to aBB|a,$$

$$B \to bBB|A.$$

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.25 Npda: Peter Linz Edition 4 Exercise 7.2 Question 6 (Page No. 195) top

Construct a NPDA corresponding to the grammar.

$S \to AA|a$
$A \to SA|b$

also convert the given grammar to GNF.

theory-of-computation peter-linz peter-linz-edition4 pushdown-automata npda

12.21.26 Npda: Peter Linz Edition 4 Exercise 7.2 Question 7,8 (Page No. 195) top

7. Show that “For every npda M, there exists an npda $\widehat{M}$ with at most three states, such that $L(M) = L(\widehat{M})$.

8. Show how the number of states of $\widehat{M}$ in the above exercise can be reduced to two.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

12.21.27 Npda: Peter Linz Edition 4 Exercise 7.2 Question 9 (Page No. 195) top

Find an npda with two states for the language $L = \{a^n b^{n+1} : n \geq 0\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata npda

Answer key

12.22 Parse Trees (1) top

12.22.1 Parse Trees: Michael Sipser Edition 3 Exercise 2 Question 1 (Page No. 154) top

Recall the CFG G_4 that we gave in Example 2.4. For convenience,let's rename it's variable with single letters as follows,

$$E \to E + T|T$$
$$T \to T \times F|F$$
$$F \to (E)|a$$

Give parse trees and derivations for each string.

a. a b. $a + a$ c. $a + a + a$ d. $((a))$

michael-sipser theory-of-computation context-free-grammar parse-trees

12.23 Perfect Shuffle (2) top

12.23.1 Perfect Shuffle: Michael Sipser Edition 3 Exercise 1 Question 41 (Page No. 89) top

For languages A and B, let the perfect shuffle of A and B be the language

$\{w|w = a_1 b_1 \cdots a_k b_k,$ where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma\}$.

Show that the class of regular languages is closed under perfect shuffle.

michael-sipser theory-of-computation finite-automata regular-language perfect-shuffle

12.23.2 Perfect Shuffle: Michael Sipser Edition 3 Exercise 2 Question 38 (Page No. 158) top

For the definition of the perfect shuffle operation. For languages A and B, let the perfect shuffle of A and B be the language

$\{w|w = a_1 b_1 \cdots a_k b_k,$ where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma\}$.

Show that the class of context-free languages is not closed under perfect shuffle.

michael-sipser theory-of-computation context-free-language perfect-shuffle

12.24 Peter Linz (99) top

12.24.1 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 1 (Page No. 27) top

Use induction on n to show that $|u^n| = n|u|$ for all strings u and all n.

peter-linz peter-linz-edition4 theory-of-computation proof

12.24.2 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 10 (Page No. 28) top

Prove or disprove the following claims.
(a) $(L_1 \cup L_2)^R = L_1^R \cup L_2^R$ for all languages L_1 and L_2.
(b) $(L^R)^* = (L^*)^R$ for all languages L.

peter-linz peter-linz-edition4 theory-of-computation proof

12.24.3 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 2 (Page No. 27) top

The reverse of a string can be defined more precisely by the recursive rules

$a^R = a$,

$(wa)^R = aw^R$, for all $a \in \Sigma$, $w \in \Sigma^*$.

Use this to prove that $(uv)^R = v^R u^R$, for all $u, v \in \Sigma^+$.

peter-linz peter-linz-edition4 theory-of-computation proof

12.24.4 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 3 (Page No. 27) top

Prove that $(w^R)^R = w$ for all $w \in \Sigma^*$.

peter-linz peter-linz-edition4 theory-of-computation proof

12.24.5 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 4 (Page No. 28) top

Let L=$\{ab, aa, baa\}$.

Which of the following strings are in L^* :

$abaabaaabaa$, $aaaabaaaa$, $baaaaabaaaab$, $baaaaabaa$? Which strings are in L^4?

peter-linz peter-linz-edition4 theory-of-computation

Answer key

12.24.6 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 5 (Page No. 28) top

Let $\Sigma = \{a, b\}$ and $L = \{aa, bb\}$. Use set notation to describe L^c.

peter-linz peter-linz-edition4 theory-of-computation

Answer key

12.24.7 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 6 (Page No. 28) top

Let L be any language on a non-empty alphabet. Show that L and L^c cannot both be finite.

peter-linz peter-linz-edition4 theory-of-computation

Answer key

12.24.8 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 7 (Page No. 28) top

Are there languages for which $(L^c)^* = (L^*)^c$

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.9 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 8 (Page No. 28) top

Prove that $(L_1 L_2)^R = L_2^R L_1^R$
for all languages L_1 and L_2.

peter-linz peter-linz-edition4 theory-of-computation proof

12.24.10 Peter Linz: Peter Linz Edition 4 Exercise 1.2 Question 9 (Page No. 28) top

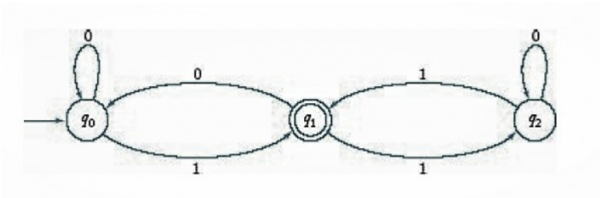
Show that $(L^*)^* = L^*$ for all languages.

peter-linz peter-linz-edition4 theory-of-computation proof

Answer key

12.24.11 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 1 (Page No. 47) top

Which of the strings 0001, 01001, 0000110 are accepted by the dfa



peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.12 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 10 (Page No. 48) top

Construct a dfa that accepts strings on $\{0,1\}$ if and only if the value of the string, interpreted as a binary representation of an integer, is zero modulo five. For example, 0101 and 1111, representing the integers 5 and 15, respectively, are to be accepted.

theory-of-computation peter-linz peter-linz-edition4 finite-automata

12.24.13 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 11 (Page No. 48) top

Show that the language $L = \{vwv : v, w \in \{a, b\}*, |v| = 2\}$ is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.14 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 12 (Page No. 48) top

Show that $L = \{a^n : n \geq 4\}$ is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.15 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 13 (Page No. 48) top

Show that the language $L = \{a^n : n \geq 0, n \neq 4\}$ is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

Answer key

12.24.16 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 14 (Page No. 48) top

Show that the language L= $\{a^n : n$ is either a multiple of 3 or a multiple of $5\}$ is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

Answer key

12.24.17 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 15 (Page No. 48) top

Show that the language $L = \{a^n : n$ is a multiple of 3, but not a multiple of $5\}$ is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

12.24.18 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 16 (Page No. 48) top

Show that the set of all real numbers in C is a regular language.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

12.24.19 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 17 (Page No. 48) top

Show that if L is regular, so is $L-$ $\{\lambda\}$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

Answer key

12.24.20 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 18 (Page No. 48) top

Show that if L is regular, so is $L \cup \{a\}$, for all $a \in \Sigma$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

Answer key

12.24.21 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 19 (Page No. 48) top

Show that

$$\delta^*(q, wv) = \delta^*(\delta^*(q, w), v)$$

for all $w, v \in \Sigma^*$.

(symbols have standard meaning)

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.22 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 2 (Page No. 47) top

For $\Sigma = \{a, b\}$, construct dfa's that accept the sets consisting of
(a) all strings with exactly one a,
(b) all strings with at least one a,
(c) all strings with no more than three a's

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.23 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 2.d, 2.e (Page No. 47) top

(d) all strings with at least one a and exactly two b's

(e) all the strings with exactly two a's and more than two b's.

theory-of-computation finite-automata peter-linz peter-linz-edition4

Answer key

12.24.24 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 20 (Page No. 48) top

Let L be the language accepted by the automaton in the following figure. Find a DFA that accepts L^2.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.25 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 22 (Page No. 49) top

Let, $L = \{awa : w \in \{a, b\}^*\}$.

Show that L^* is regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

Answer key

12.24.26 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 23 (Page No. 49) top

Let G_M be the transition graph for some dfa M. Prove the following:

(a) If $L(M)$ is infinite, then G_M must have at least one cycle for which there is a path from the initial vertex to some vertex in the cycle and a path from some vertex in the cycle to some final vertex.
(b) If $L(M)$ is finite, then no such cycle exists.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.27 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 24 (Page No. 49) top

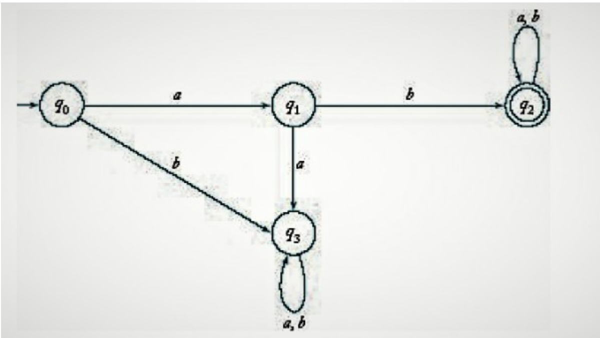
Let us define an operation $truncate$, which removes the rightmost symbol from any string. For example, $truncate(aaaba)$ is $aaab$. The operation can be extended to languages by
$truncate(L) = \{truncate(w) : w \in L\}$
Show how, given a dfa for any regular language L, one can construct a dfa for $truncate(L)$.
From this, prove that if L is a regular language not containing λ, then $truncate(L)$ is also regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.28 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 25 (Page No. 49) top

While the language accepted by a given dfa is unique, there are normally many dfa's that accept a language. Find a dfa with exactly six states that accepts the same language as the dfa in figure:



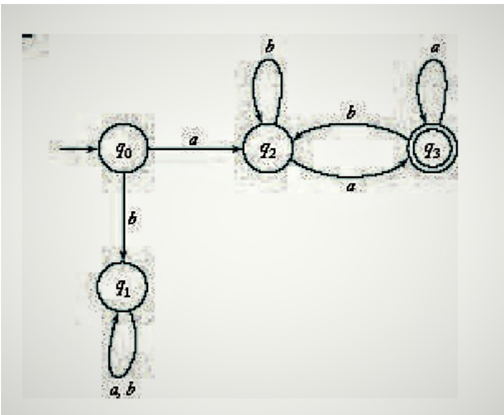
peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.29 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 3 (Page No. 47) top

$L = \{awa : w \in \{a, b\}^* \}$

Show that if we change the following figure, making q_3 a nonfinal state and making q_0, q_1, q_2 final states, the resulting dfa accepts $\bar{L}$



peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.30 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 5 (Page No. 47) top

Give dfa's for the languages

a. $L = \{ab^5wb^2 : w \in \{a, b\}^*\}$
b. $L = \{ab^na^m : n \geq 2, m \geq 3\}$
c. $L = \{w_1abw_2 : w_1 \in \{a, b\}^*, w_2 \in \{a, b\}^*\}$

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.31 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 6 (Page No. 47) top

With $\Sigma = \{a, b\}$, give a dfa for $L = \{w_1aw_2 : |w_1| \geq 3, |w_2| \leq 5\ \}$.

theory-of-computation peter-linz peter-linz-edition4 finite-automata

Answer key

12.24.32 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 6 (Page No. 47) top

With $\Sigma = \{a, b\}$, give a DFA for $L = \{w_1aw_2 : |w_1| \geq 3, |w_2| \leq 5\}$.

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Answer key

12.24.33 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 7 (Page No. 47) top

Can you please help me. I don't understand the question statement, please explain this question's statements.

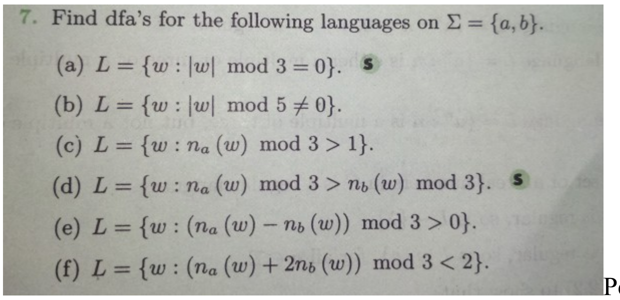
What they are trying to say in each statement

what does L={w:|w| mod 2=0} means?

Explain any one of them, please.

Chapter 2 Exercise

Question 7(a,b,c,d.e,f



7. Find dfa's for the following languages on $\Sigma = \{a, b\}$.

(a) $L = \{w : |w| \bmod 3 = 0\}$. S

(b) $L = \{w : |w| \bmod 5 \neq 0\}$.

(c) $L = \{w : n_a(w) \bmod 3 > 1\}$.

(d) $L = \{w : n_a(w) \bmod 3 > n_b(w) \bmod 3\}$. S

(e) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 > 0\}$.

(f) $L = \{w : (n_a(w) + 2n_b(w)) \bmod 3 < 2\}$.

Peter Linz

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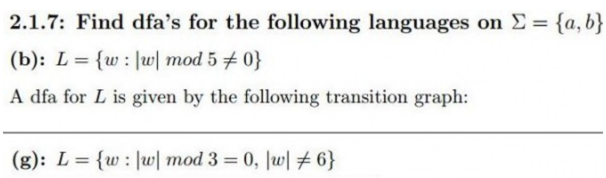
Answer key

12.24.34 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 7 (Page No. 47) top



2.1.7: Find dfa's for the following languages on $\Sigma = \{a, b\}$

(b): $L = \{w : |w|\ mod\ 5 \neq 0\}$

A dfa for L is given by the following transition graph:

(g): $L = \{w : |w|\ mod\ 3 = 0,\ |w| \neq 6\}$

Plez Tell someone briefly ....though i have already the anwers but i couldn't get it properlyyy

theory-of-computation peter-linz peter-linz-edition4 finite-automata

Answer key

12.24.35 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 7.e (Page No. 47) top

Please help in creating the DFA for (na (w)-nb (w))mod 3>0

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Answer key

12.24.36 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 8 (Page No. 47) top

A run in a string is a substring of length at least two, as long as possible and consisting entirely of the same symbol. For instance, the string $abbbaab$ contains a run of b's of length three and a run of a's of length two. Find dfa's for the following languages on $\{a, b\}$.

(a) $L = \{w : w$ contains no runs of length less than four$\}$.

(b) $L = \{w :$ every run of a's has length either two or three$\}$.

(c) $L = \{w :$ there are atmost two runs of a's of length $3\}$.

(d) $L = \{w :$ there are exactly two runs of a's of length $3\}$.

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12.24.37 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 8.c (Page No. 47) top

how to draw dfa for this?

L= {w: there are at most two runs of a's of length three}.

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12.24.38 Peter Linz: Peter Linz Edition 4 Exercise 2.1 Question 9 (Page No. 48) top

Consider the set of strings on $\{0, 1\}$ defined by the requirements below. For each, construct an accepting dfa.

(a) Every 00 is followed immediately by a 1. For example, the strings $101, 0010, 0010011001$ are in the language, but 0001 and 00100 are not.
(b) All strings containing 00 but not 000.
(c) The leftmost symbol differs from the rightmost one.
(d) Every substring of four symbols has at most two 0's. For example, 001110 and 011001 are in the language, but 10010 is not since one of its substrings, 0010, contains three zeros.
(e) All strings of length five or more in which the fourth symbol from the right end is different from the leftmost symbol.
(f) All strings in which the leftmost two symbols and the rightmost two symbols are identical.
(g) All strings of length four or greater in which the leftmost three symbols are the same, but different from the rightmost symbol.

theory-of-computation peter-linz peter-linz-edition4 finite-automata

Answer key

12.24.39 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 10 (Page No. 55) top

Find an NFA with three states that accepts the language
$L = \{a^n : n \geq 1\} \cup \{b^m a^k : m \geq 0, k \geq 0\}$

Also is this nfa possible with less than three states??

peter-linz peter-linz-edition4 finite-automata theory-of-computation

Answer key

12.24.40 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 11 (Page No. 55) top

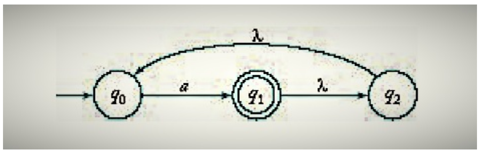
Find an nfa with four states for $L = \{a^n : n \geq 0\} \cup \{b^n a : n \geq 1\}$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.41 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 13 (Page No. 55) top

What is the complement of the language accepted by the nfa in the following figure:

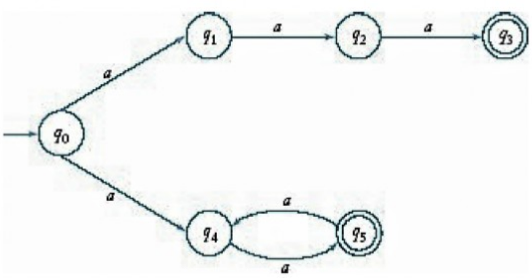


peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.42 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 14 (Page No. 55) top

Let L be the language accepted by the nfa in the following figure:



Find an nfa that accepts $L \cup \{a^5\}$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.43 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 16 (Page No. 55) top

Find an nfa that accepts $\{a\}^*$ and is such that if in its transition graph a single edge is removed (without any other changes), the resulting automaton accepts $\{a\}$.

Can this be solved using a dfa? If so, give the solution; if not, give convincing arguments for your conclusion. (Question 17)

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.44 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 18 (Page No. 55) top

An nfa with multiple initial states is defined by the quintuple $M = (Q, \Sigma, \delta, q0, F)$, where $Q_0 \subseteq Q$ is a set of possible initial states. The language accepted by such an automaton is defined as

$L(M) = \{w : \delta^*(q0, w)$ contains q_f, for any $q_0 \in Q_0, q_f \in F\}$

Show that for every nfa with multiple initial states there exists an nfa with a single initial state that accepts the same language.

Also, Suppose that we made the restriction $Q_0 \cap F = \emptyset$. Would this affect the conclusion? (Question 19)

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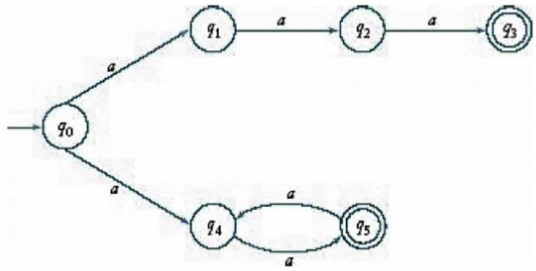
12.24.45 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 2 (Page No. 55) top

Find a dfa that accepts the language defined by the nfa in figure:



peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.46 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 20 (Page No. 56) top

Show that for any nfa for all $q \in Q$ and all $w, v \in \Sigma^*$:

$\delta^*(q, wv) = \cup_{p \epsilon \delta^*(q,w)} \delta^*(p, v)$

[***Use Definition:*** For an nfa, the extended transition function is defined so that $\delta^*(q_i, w)$ contains q_j if and only if there is a walk in the transition graph from q_i to q_j labeled w. This holds for all $q_i, q_j \in Q$, and $w \in \Sigma^*$.]

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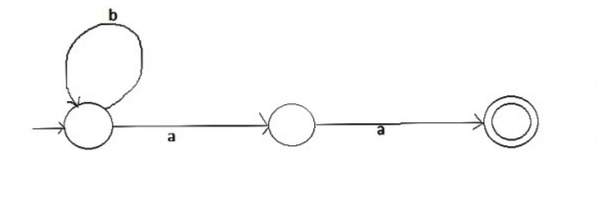
12.24.47 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 21 (Page No. 56) top

An nfa in which (a) there are no λ-transitions, and (b) for all $q \in Q$ and all $a \in \Sigma$, $\delta(q, a)$contains at most one element, is sometimes called an incomplete dfa. This is reasonable since the conditions make it such that there is never any choice of moves.
For $\Sigma = \{a, b\}$, convert the incomplete dfa below into a standard dfa

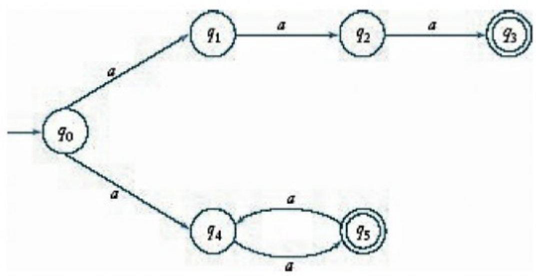


peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.48 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 3 (Page No. 55) top

Find a dfa that accepts the complement of the language defined by the nfa in figure:

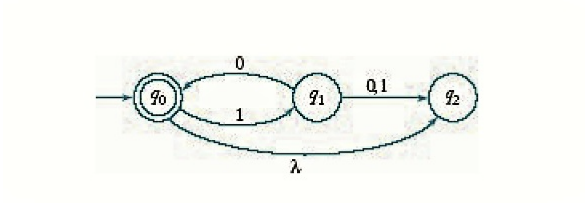


peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.49 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 4 (Page No. 55) top

In following figure, find $\delta^*(q_0, 1011)$ and $\delta^*(q_1, 01)$



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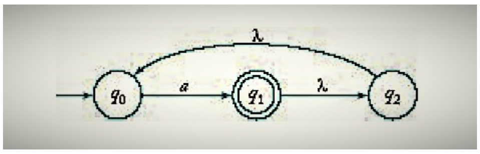
Answer key

12.24.50 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 5 (Page No. 55) top

In following figure, find $\delta^*(q_0, a)$ and $\delta^*(q_1, \lambda)$



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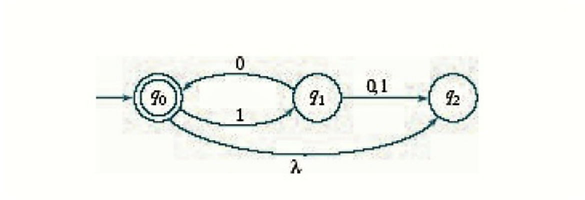
Answer key

12.24.51 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 6 (Page No. 55) top

For the nfa in following figure, find $\delta^*(q_0, 1010)$ and $\delta^*(q_1, 00)$



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Answer key

12.24.52 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 7 (Page No. 55) top

Design an nfa with no more than five states for the set $\{abab^n : n \geq 0\} \cup \{aba^n : n \geq 0\}$.

Do you think this can be solved with fewer than three states? (Question 9)

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Answer key

12.24.53 Peter Linz: Peter Linz Edition 4 Exercise 2.2 Question 8 (Page No. 55) top

Construct an nfa with three states that accepts the language $\{ab, abc\}*$.

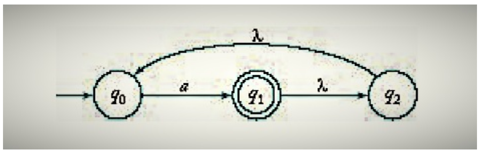
peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.54 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 1 (Page No. 62) top

Theorem: Let L be the language accepted by a nondeterministic finite accepter $M_N = (Q_N, \Sigma, \delta N, q0, F_N)$. Then there exists a deterministic finite accepter $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that
$L = L(M_D)$.

convert the nfa in following figure to a dfa:



Can you see a simpler answer more directly?

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Answer key

12.24.55 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 10 (Page No. 62) top

Define a dfa with multiple initial states in an analogous way to the corresponding nfa in Exercise 18, Section 2.2. Does there always exist an equivalent dfa with a single initial state?

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12.24.56 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 11 (Page No. 62) top

Prove that all finite languages are regular.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

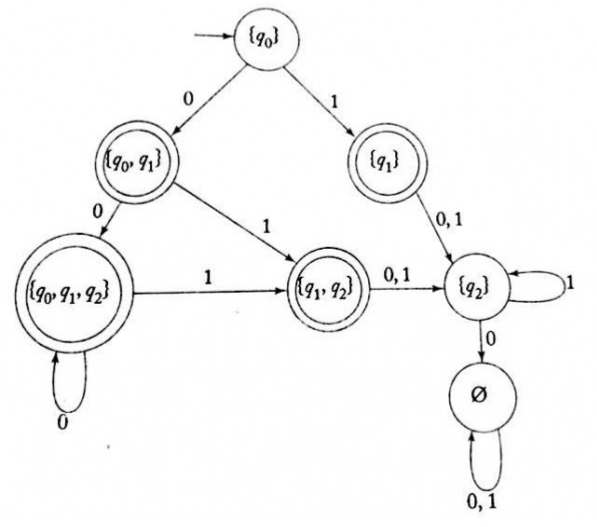
12.24.57 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 12 (Page No. 62) top

Show that if L is regular, so is L^R.

peter-linz peter-linz-edition4 theory-of-computation finite-automata regular-language

12.24.58 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 13 (Page No. 62) top

Give a simple verbal description of the language accepted by the dfa in following figure.



Use this to find another dfa, equivalent to the given one, but with fewer states.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.59 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 14 (Page No. 62) top

Let L be any language. Define $even(w)$ as the string obtained by extracting from w the letters in even-numbered positions; that is, if

$$w = a_1 a_2 a_3 a_4 \ldots,$$

then

$$even(w) = a_2 a_4 \ldots.$$

Corresponding to this, we can define a language

$$even(L) = \{even(w) : w \in L\}.$$

Prove that if L is regular, so is $even(L)$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.60 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 15 (Page No. 63) top

From a language L we create a new language $chop2(L)$ by removing the two leftmost symbols of every string in L. Specifically,

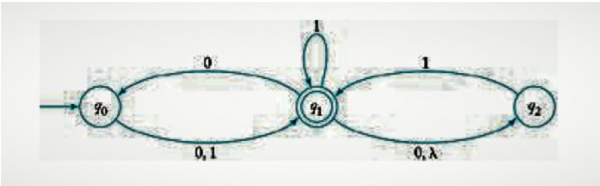
$$chop2(L) = \{w : vw \in L, \text{ with } |v| = 2\}.$$

Show that if L is regular, then $chop2(L)$ is also regular.

theory-of-computation peter-linz peter-linz-edition4 finite-automata regular-language

12.24.61 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 2 (Page No. 62) top

Convert the nfa in following figure, into an equivalent dfa.

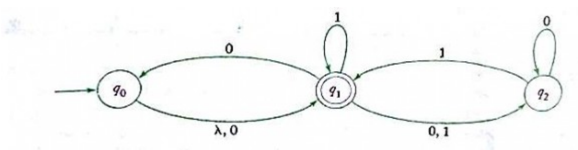


peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.62 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 3 (Page No. 62) top

Convert the following nfa into an equivalent dfa.



peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.63 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 4 (Page No. 62) top

Theorem: Let L be the language accepted by a nondeterministic finite accepter $M_N = (Q_N, \Sigma, \delta_N, q_0, F_N)$. Then there exists a deterministic finite accepter $M_D = (Q_D, \Sigma, \delta_D, \{q_0\}, F_D)$ such that $L = L(M_D)$.

Prove this Theorem.

Show in detail that if the label of $\delta_D^*(q_0, w)$ contains q_f, then $\delta_N^*(q_0, w)$ also contains q_f.

peter-linz peter-linz-edition4 theory-of-computation finite-automata proof

12.24.64 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 5 (Page No. 62) top

Is it true that for any nfa $M = (Q, \Sigma, \delta, q_0, F)$ the complement of $L(M)$ is equal to the set

$\{w \in \Sigma^* : \delta^*(q_0, w) \cap F = \varnothing\}$? If so, prove it. If not, give a counterexample.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.65 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 6 (Page No. 62) top

Is it true that for every nfa $M = (Q, \Sigma, \delta, q_0, F)$ the complement of $L(M)$ is equal to the set

$\{w \in \Sigma^* : \delta^*(q_0, w) \cap (Q - F) \neq \varnothing\}$? If so, prove it. If not, give a counterexample.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

12.24.66 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 7 (Page No. 62) top

Prove that for every nfa with an arbitrary number of final states there is an equivalent nfa with only one final state. Can we make a similar claim for dfa's?

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.67 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 8 (Page No. 62) top

Find an nfa without λ-transitions and with a single final state that accepts the set $\{a\} \cup \{b^n : n \geq 1\}$.

peter-linz peter-linz-edition4 theory-of-computation finite-automata

Answer key

12.24.68 Peter Linz: Peter Linz Edition 4 Exercise 2.3 Question 9 (Page No. 62) top

Let L be a regular language that does not contain λ. Show that there exists an nfa without λ-transitions and with a single final state that accepts L.

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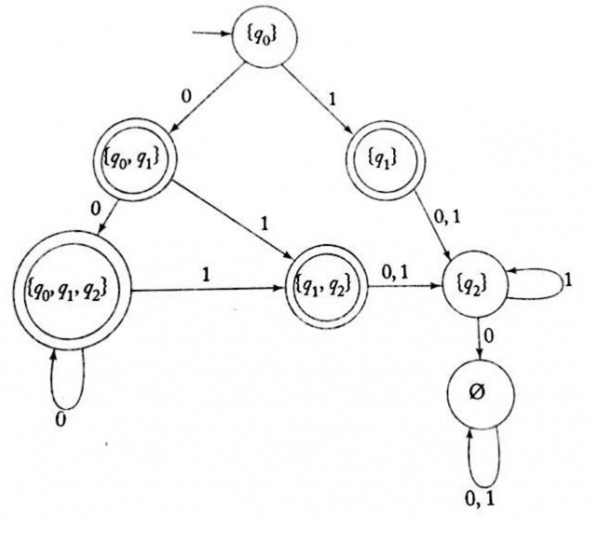
Answer key

12.24.69 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 1 (Page No. 68) top

Minimize the number of states in the dfa of following figure:



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12.24.70 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 10 (Page No. 69) top

Prove the following: If the states q_a and q_b are indistinguishable, and if q_a and q_c are distinguishable, then q_b and q_c must be distinguishable.

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12.24.71 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 2 (Page No. 68) top

Find minimal dfa's for the following languages. In each case prove that the result is minimal.
(a) $L = \{a^n b^m : n \geq 2, m \geq 1\}$.
(b) $L = \{a^n b : n \geq 0\} \cup \{b^n a : n \geq 1\}$
(c) $L = \{a^n : n \geq 0, n \neq 3\}$.
(d) $L = \{a^n : n \neq 2 \text{ and } n \neq 4\}$.
(e) $L = \{a^n : n \bmod 3 = 0\} \cup \{a^n : n \bmod 5 = 1\}$.

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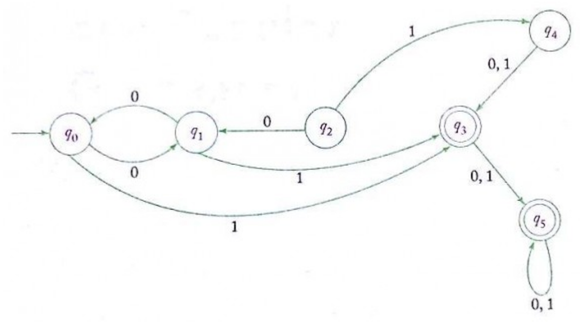
12.24.72 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 4 (Page No. 69) top

Minimize the states in the dfa depicted in the following diagram.



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12.24.73 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 5 (Page No. 69) top

Show that if L is a nonempty language such that any w in L has length at least n, then any dfa accepting L must have at least $n+1$ states.

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12.24.74 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 6 (Page No. 69) top

Prove or disprove the following conjecture.

If $M = (Q, \Sigma, \delta, q_0, F)$ is a minimal dfa for a regular language L, then $\widehat{M} = (Q, \Sigma, \delta, q_0, Q - F)$ is a minimal dfa for $\overline{L}$.

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12.24.75 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 7 (Page No. 69) top

Show that indistinguishability is an equivalence relation but that distinguishability is not.

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12.24.76 Peter Linz: Peter Linz Edition 4 Exercise 2.4 Question 9 (Page No. 69) top

Write a Computer program that produces a minimal dfa for any given dfa.

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12.24.77 Peter Linz: Peter Linz Edition 4 Exercise 3.2 Question 18 (Page No. 89) top

Find nfa's for $L(a\varnothing)$ and $L(\varnothing^*)$. Is the result consistent with the definition of these languages?

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12.24.78 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 1 (Page No. 113) top

Show that there exists an algorithm to determine whether or not $w \in L_1 - L_2$, for any given w and any regular languages L_1 and L_2.

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12.24.79 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 10 (Page No. 113) top

Show that there is an algorithm to determine if $L = shuffle(L, L)$ for any regular L.

peter-linz peter-linz-edition4 theory-of-computation regular-language

12.24.80 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 11 (Page No. 113) top

The operation $tail(L)$ is defined as $tail(L) = \{v : uv \in L, u, v \in \Sigma^*\}$.

Show that there is an algorithm for determining whether or not $L = tail(L)$ for any regular L.

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12.24.81 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 12 (Page No. 113) top

Let L be any regular language on $\Sigma = \{a, b\}$. Show that an algorithm exists for determining if L contains any strings of even length.

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12.24.82 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 13 (Page No. 114) top

Show that there exists an algorithm that can determine for every regular language L, whether or not $|L| \geq 5$.

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12.24.83 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 14 (Page No. 114) top

Find an algorithm for determining whether a regular language L contains an infinite number of even-length strings.

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Answer key

12.24.84 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 15 (Page No. 114) top

Describe an algorithm which, when given a regular grammar G, can tell us whether or not $L(G) = \Sigma^*$.

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12.24.85 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 2 (Page No. 113) top

Show that there exists an algorithm for determining if $L_1 \subseteq L_2$, for any regular languages L_1 and L_2.

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12.24.86 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 3 (Page No. 113) top

Show that there exists an algorithm for determining if $\lambda \in L$, for any regular language L.

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12.24.87 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 4 (Page No. 113) top

Show that for any regular L_1 and L_2 there is an algorithm to determine whether or not $L_1 = L_1/L_2$.

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12.24.88 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 5 (Page No. 113) top

A language is said to be a *palindrome* language if $L = L^R$. Find an algorithm for determining if a given regular language is a *palindrome* language.

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Answer key

12.24.89 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 6 (Page No. 113) top

Exhibit an algorithm for determining whether or not a regular language L contains any string w such that $w^R \in L$.

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12.24.90 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 7 (Page No. 113) top

Exhibit an algorithm that, given any three regular languages, L, L_1, L_2, determines whether or not $L = L_1 L_2$.

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12.24.91 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 8 (Page No. 113) top

Exhibit an algorithm that, given any regular language L, determines whether or not $L = L^*$.

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12.24.92 Peter Linz: Peter Linz Edition 4 Exercise 4.2 Question 9 (Page No. 113) top

Let L be a regular language on Σ and $\widehat{w}$ be any string in Σ^*. Find an algorithm to determine if L contains any w such that $\widehat{w}$ is a substring of it, that is, such that $w = u\widehat{w}v$ with $u, v \in \Sigma^*$.

peter-linz peter-linz-edition4 theory-of-computation regular-language

12.24.93 Peter Linz: Peter Linz Edition 4 Exercise 4.3 Question 17 (Page No. 124) top

Let L_1 and L_2 be regular languages. Is the language $L = \{w : w \in L_1, w^R \in L_2$ necessarily regular?

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12.24.94 Peter Linz: Peter Linz Edition 4 Exercise 4.3 Question 19 (Page No. 124) top

Are the following languages regular?

(a) $L = \{uww^Rv : u, v, w \in \{a, b\}^+\}$

(b) $L = \{uww^Rv : u, v, w \in \{a, b\}^+, |u| \geq |v|\}$

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12.24.95 Peter Linz: Peter Linz Edition 4 Exercise 4.3 Question 20 (Page No. 124) top

Is the following language regular?

$$L = \{ww^Rv : v, w \in \{a, b\}^+\}.$$

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12.24.96 Peter Linz: Peter Linz Edition 4 Exercise 4.3 Question 25 (Page No. 124) top

In the chain code language in Exercise 24, Section 3.1, let L be the set of all $w \in \{u, r, l, d\}^*$ that describe rectangles. Show that L is not a regular language.

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12.24.97 Peter Linz: Peter Linz Edition 4 Exercise 5.1 Question 5 (Page No. 133) top

Is the language $L(G) = \{ab(bbaa)^n bba(ba)^n : n \geq 0\}$ regular?

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12.24.98 Peter Linz: Peter Linz Edition 5 Exercise 2.2 Question 7 (Page No. 79) top

Design an nfa with no more than five states for the set $\{abab'' : n > 0\} \cup \{aba'' : n \geq 0\}$

peter-linz theory-of-computation peter-linz-edition5 finite-automata

12.24.99 Peter Linz: Peter Linz Edition 5 Exercise 2.4 Question 10 top

Show that given a regular language L, its minimal dfa is unique within a simple relabeling of the states.

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12.25 Pigeonhole Principle (1) top

12.25.1 Pigeonhole Principle: Peter Linz Edition 4 Exercise 4.3 Question 18 (Page No. 124) top

Apply the pigeonhole argument directly to the language in $L = \{ww^R : w \in \Sigma^+\}$.

peter-linz peter-linz-edition4 theory-of-computation regular-language pigeonhole-principle

12.26 Polynomials (1) top

12.26.1 Polynomials: Michael Sipser Edition 3 Exercise 3 Question 21 (Page No. 190) top

Let $c_1x^n + c_2x^{n-1} + \cdots + c_nx + c_{n+1}$ be a polynomial with a root at $x = x_0$. Let c_{max} be the largest absolute value of a c_i. Show that $|x_0| < (n+1)\frac{c_{max}}{|c_1|}$.

michael-sipser theory-of-computation turing-machine polynomials proof

12.27 Post Correspondence Problem (6) top

12.27.1 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 17 (Page No. 240) top

Show that the Post Correspondence Problem is decidable over the unary alphabet $\Sigma = \{1\}$.

michael-sipser theory-of-computation post-correspondence-problem decidability proof

12.27.2 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 18 (Page No. 240) top

Show that the Post Correspondence Problem is undecidable over the binary alphabet $\Sigma = \{0,1\}$.

michael-sipser theory-of-computation post-correspondence-problem decidability proof

12.27.3 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 19 (Page No. 240) top

In the *silly Post Correspondence Problem*, $SPCP$, the top string in each pair has the same length as the bottom string. Show that the $SPCP$ is decidable.

michael-sipser theory-of-computation post-correspondence-problem decidability proof

12.27.4 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 21 (Page No. 240) top

Let $AMBIG_{CFG} = \{\langle G \rangle \mid \text{G is an ambiguous CFG}\}$. Show that $AMBIG_{CFG}$ is undecidable.

(Hint: Use a reduction from PCP. Given an instance

$$P = \left\{ \left[\frac{t_1}{b_1} \right], \left[\frac{t_2}{b_2}, \right] \left[,\frac{t_k}{b_k} \right\} \right]$$

of the Post Correspondence Problem, construct a $CFG\ G$ with the rules

- $S \to T \mid B$
- $T \to t_1Ta_1 \mid \cdots \mid t_kTa_k \mid t_1a_1 \mid \cdots \mid t_kTa_k$
- $B \to b_1Ba_1 \mid \cdots \mid b_kBa_k \mid b_1a_1 \mid \cdots \mid b_kTa_k$,

where $a_1, \ldots, a_k$ are new terminal symbols. Prove that this reduction works.)

michael-sipser theory-of-computation context-free-grammar reduction post-correspondence-problem decidability proof

12.27.5 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 3 (Page No. 239) top

Find a match in the following instance of the Post Correspondence Problem.

$$\left\{ \left[\frac{ab}{abab} \right] \left[\frac{b}{a} \right], , \left[\frac{aba}{b} \right] \left[\frac{aa}{,a} \right\} \right]$$

michael-sipser theory-of-computation turing-machine post-correspondence-problem proof

12.27.6 Post Correspondence Problem: Michael Sipser Edition 3 Exercise 5 Question 8 (Page No. 239) top

In the proof of Theorem 5.15, we modified the Turing machine M so that it never tries to move its head off the left-hand end of the tape. Suppose that we did not make this modification to M. Modify the PCP construction to handle this case.

michael-sipser theory-of-computation turing-machine post-correspondence-problem proof

12.28 Prefix Free Property (2) top

12.28.1 Prefix Free Property: Michael Sipser Edition 3 Exercise 1 Question 40 (Page No. 89) top

Recall that string x is a prefix of string y if a string z exists where $xz = y$, and that x is a proper prefix of y if in addition $x \neq y$. In each of the following parts, we define an operation on a language A. Show that the class of regular languages is closed under that operation.

a. $\text{NOPREFIX(A)} = \{\text{w} \in \text{A} | \text{no proper prefix of w is a member of A}\}$.
b. $\text{NOEXTEND(A)} = \{\text{w} \in \text{A} | \text{ w is not the proper prefix of any string in A}\}$.

michael-sipser theory-of-computation finite-automata regular-language prefix-free-property

12.28.2 Prefix Free Property: Michael Sipser Edition 3 Exercise 2 Question 52 (Page No. 159) top

Show that every DCFG generates a prefix-free language.

michael-sipser theory-of-computation context-free-grammar prefix-free-property proof

12.29 Pumping Lemma (45) top

12.29.1 Pumping Lemma: Michael Sipser Edition 3 Exercise 1 Question 29 (Page No. 88) top

Use the pumping lemma to show that the following languages are not regular.

a. $A_1 = \{0^n 1^n 2^n | n \geq 0\}$
b. $A_2 = \{www | w \in \{a, b\}^*\}$
c. $A_3 = \{a^{2^n} | n \geq 0\}$ (Here, a^{2^n} means a strings of 2^n a's.)

michael-sipser theory-of-computation finite-automata regular-language pumping-lemma

Answer key

12.29.2 Pumping Lemma: Michael Sipser Edition 3 Exercise 1 Question 30 (Page No. 88) top

Describe the error in the following "proof" that 0^*1^* is not a regular language. (An error must exist because 0^*1^* is regular.) The proof is by contradiction. Assume that 0^*1^* is regular. Let p be the pumping length for 0^*1^* given by the pumping lemma. Chooses to be the string 0^p1^p. You know that s is a member of 0^*1^*,but example 1.73 shows that s cannot be pumped. Thus you have a contradiction. So 0^*1^* is not regular.

michael-sipser theory-of-computation finite-automata pumping-lemma proof

Answer key

12.29.3 Pumping Lemma: Michael Sipser Edition 3 Exercise 1 Question 54 (Page No. 91) top

Consider the language $F = \{a^i b^j c^k | i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}$.

a. Show that F is not regular.
b. Show that F acts like a regular language in the pumping lemma. In other words, give a pumping length p and demonstrate that F satisfies the three conditions of the pumping lemma for this value of p.
c. Explain why parts (a) and (b) do not contradict the pumping lemma.

michael-sipser theory-of-computation finite-automata regular-language pumping-lemma proof descriptive

Answer key

12.29.4 Pumping Lemma: Michael Sipser Edition 3 Exercise 1 Question 55 (Page No. 91) top

The pumping lemma says that every regular language has a pumping length p, such that every string in the language can be pumped if it has length p or more. If p is a pumping length for language A, so is any length $p' \geq p$. The minimum pumping length for A is the smallest p that is a pumping length for A.For example, if $A = 01^*$,the minimum pumping length is 2. The reason is that the string $s = 0$ is in A and has length 1 yet s cannot be pumped; but any string in A of length 2 or more contains a 1 and hence can be pumped by dividing it so that $x = 0, y = 1,$ and z is the rest. For each of the following languages, give the minimum pumping length and justify your answer.

a. 0001^* b. 0^*1^* c. $001 \cup 0^*1^*$ d. $0^*1^+0^+1^* \cup 10^*1$ e. $(01)^*$

f. ϵ g. $1^*01^*01^*$ h. $10(11^*0)^*0$ i. 1011 j. $\sum^*$

michael-sipser theory-of-computation regular-language pumping-lemma proof descriptive

Answer key

12.29.5 Pumping Lemma: Michael Sipser Edition 3 Exercise 2 Question 30 (Page No. 157) top

Use the pumping lemma to show that the following languages are not context free .

a. $\{0^n1^n0^n1^n \mid n \geq 0\}$
b. $\{0^n\#0^{2n}\#0^{3n} \mid n \geq 0\}$
c. $\{w\#t \mid w$ is a substring of t, where $w, t \in \{a, b\}^*\}$
d. $\{t_1\#t_2\#\ldots\#t_k \mid k \geq 2,$ each $t_i \in \{a, b\}^*$, and $t_i = t_j$ for some $i \neq j\}$

michael-sipser theory-of-computation context-free-language pumping-lemma

Answer key

12.29.6 Pumping Lemma: Michael Sipser Edition 3 Exercise 2 Question 34 (Page No. 157) top

Let $G = (V, \Sigma, R, S)$ be the following grammar. $V = \{S, T, U\}; \Sigma = \{0, \#\};$ and R is the set of rules:

- $S \rightarrow TT \mid U$
- $T \rightarrow 0T \mid T0 \mid \#$
- $U \rightarrow 0U00 \mid \#$

Consider the language $B = L(G)$, where G is the grammar which is given above.The pumping lemma for context-free languages, Theorem 2.34, states the existence of a pumping length p for B. What is the minimum value of p that works in the pumping lemma? Justify your answer.

michael-sipser theory-of-computation context-free-language pumping-lemma proof

Answer key

12.29.7 Pumping Lemma: Michael Sipser Edition 3 Exercise 2 Question 36 (Page No. 158) top

Give an example of a language that is not context free but that acts like a CFL in the pumping lemma. Prove that your example works. (See the analogous example for regular languages in Question 54.)

michael-sipser theory-of-computation context-free-language pumping-lemma proof

Answer key

12.29.8 Pumping Lemma: Michael Sipser Edition 3 Exercise 2 Question 37 (Page No. 158) top

Prove the following stronger form of the pumping lemma, where in both pieces v and y must be nonempty when the string s is broken up.If A is a context-free language, then there is a number k where, if s is any string in A of length at least k, then s may be divided into five pieces, $s = uvxyz$, satisfying the conditions:

a. for each $i \geq 0, uv^ixy^iz \in A$,
b. $v \neq \epsilon$ and $y \neq \epsilon$,and
c. $|vxy| \leq k$.

michael-sipser theory-of-computation context-free-language pumping-lemma

12.29.9 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 1 (Page No. 122) top

Prove the following version of the pumping lemma. If L is regular, then there is an m such that, every $w \in L$ of length greater than m can be decomposed as $w = xyz$, with $|yz| \leq m$ and $|y| \geq 1$, such that xy^iz is in L for all i.

peter-linz peter-linz-edition4 theory-of-computation pumping-lemma

12.29.10 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 10 (Page No. 123) top

Consider the language $L = \{a^n : n \text{ is not a perfect square}\}$.
(a) Show that this language is not regular by applying the pumping lemma directly.
(b) Then show the same thing by using the closure properties of regular languages.

peter-linz peter-linz-edition4 theory-of-computation pumping-lemma regular-language closure-property

12.29.11 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 11 (Page No. 123) top

Show that the language

$$L = \left\{a^{n!} : n \geq 1\right\} \text{ is not regular using pumping lemma}$$

theory-of-computation peter-linz peter-linz-edition4 regular-language pumping-lemma

Answer key

12.29.12 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 12 (Page No. 123) top

Apply the pumping lemma to show that $L = \{a^n b^k c^{n+k} : n \geq 0, k \geq 0\}$ is not regular.

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Answer key

12.29.13 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 13 (Page No. 123) top

Show that the following language is not regular.

$L = \{a^n b^k : n > k\} \cup \{a^n b^k : n \neq k - 1\}$.

peter-linz peter-linz-edition4 theory-of-computation pumping-lemma regular-language closure-property

Answer key

12.29.14 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 14 (Page No. 123) top

Prove or disprove the following statement: If L_1 and L_2 are non regular languages, then $L_1 \cup L_2$ is also non regular.

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Answer key

12.29.15 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 15 (Page No. 123) top

Consider the languages below. For each, make a conjecture whether or not it is regular. Then prove your conjecture.

(a) $L = \{a^n b^l a^k : n + k + l > 5\}$

(b) $L = \{a^n b^l a^k : n > 5, l > 3, k \leq l\}$

(c) $L = \{a^n b^l : n/l \text{ is an integer}\}$

(d) $L = \{a^n b^l : n + l \text{ is a prime number}\}$

(e) $L = \{a^n b^l : n \leq l \leq 2n\}$

(f) $L = \{a^n b^l : n \geq 100, l \leq 100\}$

(g) $L = \{a^n b^l : |n - l| = 2\}$

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12.29.16 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 16 (Page No. 123) top

Is the following language regular?

$L = \{w_1cw_2 : w_1, w_2 \in \{a,b\}^*, w_1 \neq w_2\}.$

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Answer key

12.29.17 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 2 (Page No. 122) top

Prove the following generalization of the pumping lemma.
If L is regular, then there exists an m, such that the following holds for every sufficiently long $w \in L$ and every one of its decompositions $w = u_1vu_2$, with $u_1, u_2 \in \Sigma^*, |v| \geq m.$ The middle string v can be written as $v = xyz,$ with $|xy| \leq m, |y| \geq 1,$ such that $u_1xy^izu_2 \in L$ for all $i = 0, 1, 2, \dots.$

peter-linz peter-linz-edition4 theory-of-computation pumping-lemma

12.29.18 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 26 (Page No. 124) top

Let $L = \{a^nb^m : n \geq 100, m \leq 50\}.$
(a) Can you use the pumping lemma to show that L is regular?
(b) Can you use the pumping lemma to show that L is not regular?

Explain your answers.

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12.29.19 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 3 (Page No. 122) top

Show that the language $L = \{w : n_a(w) = n_b(w)\}$ is not regular. Is L^* regular?

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Answer key

12.29.20 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 4 (Page No. 122) top

Prove that the following languages are not regular.
(a) $L = \{a^nb^la^k : k \geq n + l\}.$
(b) $L = \{a^nb^la^k : k \neq n + l\}.$
(c) $L = \{a^nb^la^k : n = l$ or $l \neq k\}.$
(d) $L = \{a^nb^l : n \leq l\}.$
(e) $L = \{w : n_a(w) \neq n_b(w)\}.$
(f) $L = \{ww : w \in \{a,b\}^*\}.$
(g) $L = \{wwww^R : w \in \{a,b\}^*\}.$

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12.29.21 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 5 (Page No. 122) top

Determine whether or not the following languages on $\Sigma = \{a\}$ are regular.
(a) $L = \{a^n : n \geq 2,$ is a prime number$\}.$
(b) $L = \{a^n : n$ is not a prime number$\}.$
(c) $L = \{a^n : n = k^3$ for some $k \geq 0\}.$
(d) $L = \{a^n : n = 2^k$ for some $k \geq 0\}.$
(e) $L = \{a^n : n$ is the product of two prime numbers$\}.$
(f) $L = \{a^n : n$ is either prime or the product of two or more prime numbers$\}.$
(g) L^*, where L is the language in part $(a).$

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12.29.22 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 6 (Page No. 122) top

Given $L_1 = \{a^nb^n | n \geqslant 1\}$, $L_2 = \{a^nb^m | n \geq 1, m \geq 1\}$, $L_3 = \{a^nb^{n+2} | n \geqslant 1\}$
if $L_1 \cup L_2$ is regular then why $L_1 \cup L_3$ is not regular?

also what is the language of $L_1 \cup L_3$?

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Answer key

12.29.23 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 7 (Page No. 123) top

Show that the language $L = \{a^n b^n : n \geq 0\} \cup \{a^n b^{n+1} : n \geq 0\} \cup \{a^n b^{n+2} : n \geq 0\}$ is not regular.

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12.29.24 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 8 (Page No. 123) top

Show that the language $L = \{a^n b^{n+k} : n \geq 0, k \geq 1\} \cup \{a^{n+k} b^n : n \geq 0, k \geq 3\}$ is not regular.

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Answer key

12.29.25 Pumping Lemma: Peter Linz Edition 4 Exercise 4.3 Question 9 (Page No. 123) top

Is the language $L = \{w \in \{a, b, c\}^* : |w| = 3n_a(w)\}$ regular?

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Answer key

12.29.26 Pumping Lemma: Peter Linz Edition 4 Exercise 8.1 Question 1 (Page No. 212) top

Show that the language $L = \{a^n b^n c^m, n \neq m\}$ is not context-free.

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Answer key

12.29.27 Pumping Lemma: Peter Linz Edition 4 Exercise 8.1 Question 5 (Page No. 212) top

Is the language L = $\{a^n b^m : n = 2^m\}$ context-free?

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12.29.28 Pumping Lemma: Peter Linz Edition 4 Exercise 8.1 Question 8 (Page No. 212) top

Determine whether or not the following languages are context-free.
(a) $L = \{a^n w w^R a^n : n \geq 0, w \in \{a, b\}^*\}$
(b) $L = \{a^n b^j a^n b^j : n \geq 0, j \geq 0\}$.
(C) $L = \{a^n b^j a^j b^n : n \geq 0, j \geq 0\}$.
(d) $L = \{a^n b^j a^k b^l : n + j \leq k + l\}$.
(e)$L = \{a^n b^j a^k b^l : n \leq k, j \leq l\}$.
(f) $L = \{a^n b^n c^j : n \leq j\}$.
(g) $L = \{w \in \{a, b, c\}^* : n_a(w) = n_b(w) = 2n_c(w)\}$.

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12.29.29 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 1 (Page No. 212) top

Show that the language

$$L = \{w \in \{a, b, c\}^* : n_a(w) = n_b(w) \leq n_c(w)\}$$

is not context-free.

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12.29.30 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 2 (Page No. 212) top

Show that the language $L = \{a^n : n \text{ is a prime number}\}$ is not context-free.

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12.29.31 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 3 (Page No. 212) top

Show that $L = \{ww^R w : w \in \{a, b\}^*\}$ is not a context-free language.

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12.29.32 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 4 (Page No. 212) top

Show that $L = \{w \in \{a, b, c\}^* : n_a^2(w) + n_b^2(w) = n_c^2(w)\}$ is not a context-free.

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12.29.33 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 5 (Page No. 212) top

Is the language $L = \{a^n b^m : n = 2^m\}$ context free?

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12.29.34 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 6 (Page No. 212) top

Show that the language $L = \left\{a^{n^2} : n \geq 0\right\}$ is not context free.

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12.29.35 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(a) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{a^n b^j : n \leq j^2\}.$$

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12.29.36 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(b) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{a^n b^j : n \geq (j-1)^3\}.$$

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12.29.37 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(c) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{a^n b^j c^k : k = jn\}.$$

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12.29.38 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(d) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{a^n b^j c^k : k > n, k > j\}.$$

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12.29.39 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(e) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{a^n b^j c^k : n < j, n \leq k \leq j\}.$$

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12.29.40 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(f) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{w : n_a(w) < n_b(w) < n_c(w)\}$$

.

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12.29.41 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(g) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{w : n_a(w)/n_b(w) = n_c(w)\}.$$

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12.29.42 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(h) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free.

$$L = \{w \in \{a, b, c\}^* : n_a(w) + n_b(w) = 2n_c(w), n_a(w) = n_b(w)\}.$$

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12.29.43 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(i) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free

$$L = \{a^n b^m : \text{n and m are both prime}\}.$$

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12.29.44 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(j) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free

$$L = \{a^n b^m : \text{n is prime or m is prime}\}.$$

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12.29.45 Pumping Lemma: Peter Linz Edition 5 Exercise 8.1 Question 7(k) (Page No. 212) top

Show that the following languages on $\Sigma = \{a, b, c\}$ are not context-free

$$L = \{a^n b^m : \text{n is prime and m is not prime}\}.$$

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12.30 Pushdown Automata (12) top

12.30.1 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 10 (Page No. 155) top

Give an informal description of a pushdown automaton that recognizes the language $A = \{a^i b^j c^k \mid i = j \text{ or } j = k$ where $i, j, k \geq 0\}$.

michael-sipser theory-of-computation context-free-language pushdown-automata

12.30.2 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 11 (Page No. 155) top

Convert the $CFG\ \ G_4$

- $E \to E + T \mid T$
- $T \to T \times F \mid F$
- $F \to (E) \mid a$

to an equivalent PDA, using the procedure given in Theorem 2.20.

michael-sipser theory-of-computation context-free-language pushdown-automata

12.30.3 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 12 (Page No. 156) top

Convert the $CFG\ \ G$

- $R \to XRX \mid S$

- $S \to aTb \mid bTa$
- $T \to XTX \mid X \mid \epsilon$
- $X \to a \mid b$

to an equivalent PDA, using the procedure given in Theorem 2.20.

michael-sipser theory-of-computation context-free-grammar pushdown-automata

12.30.4 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 47 (Page No. 159) top

Let $\Sigma = \{0, 1\}$ and let B be the collection of strings that contain at least one 1 in their second half. In other words, $B = \{uv \mid u \in \Sigma^*, v \in \Sigma^* 1 \Sigma^* \text{ and } \mid u \mid \geq \mid v \mid\}$.

A. Give a PDA that recognizes B.
B. Give a CFG that generates B.

michael-sipser theory-of-computation context-free-grammar pushdown-automata descriptive

12.30.5 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 5 (Page No. 155) top

Give informal descriptions and state diagrams of pushdown automata for the languages in the following languages In all parts, the alphabet $\sum$ is $\{0, 1\}$.

a. {w| w contains at least three 1's}
b. {w| w starts and ends with the same symbol}
c. {w| the length of w is odd}
d. {w| the length of w is odd and its middle symbol is a 0}
e. $\{w|w = w^R$,that is, w is a palindrome}
f. The empty set.

michael-sipser theory-of-computation context-free-language pushdown-automata

12.30.6 Pushdown Automata: Michael Sipser Edition 3 Exercise 2 Question 7 (Page No. 155) top

Give informal English descriptions of PDAs for the following languages.

a. The set of strings over the alphabet $\{a, b\}$ with more $a's$ than $b's$
b. The complement of the language $\{a^n b^n | n \geq 0\}$
c. $\{w\#x|w^R$ is a substring of x for $w, x \in \{0, 1\}^*\}$
d. $\{x_1\#x_2\#\ldots\#x_k|k \geq 1,$ each $x_i \in \{a, b\}^*$, and for some i and $j, x_i = x_j^R\}$

michael-sipser theory-of-computation context-free-language pushdown-automata

12.30.7 Pushdown Automata: Michael Sipser Edition 3 Exercise 3 Question 9 (Page No. 188) top

Let a $k - PDA$ be a pushdown automaton that has k stacks. Thus a $0 - PDA$ is an NFA and a $1 - PDA$ is a conventional PDA. You already know that $1 - PDAs$ are more powerful (recognize a larger class of languages) than $0 - PDAs$.

a. Show that $2 - PDAs$ are more powerful than $1 - PDAs$.
b. Show that $3 - PDAs$ are not more powerful than $2 - PDAs$.

(Hint: Simulate a Turing machine tape with two stacks.)

michael-sipser theory-of-computation pushdown-automata finite-automata descriptive

12.30.8 Pushdown Automata: Michael Sipser Edition 3 Exercise 4 Question 24 (Page No. 212) top

A useless state in a pushdown automaton is never entered on any input string. Consider the problem of determining whether a pushdown automaton has any useless states. Formulate this problem as a language and show that it is decidable.

michael-sipser theory-of-computation pushdown-automata decidability proof

12.30.9 Pushdown Automata: Michael Sipser Edition 3 Exercise 5 Question 33 (Page No. 241) top

Consider the problem of determining whether a PDA accepts some string of the form $\{ww \mid w \in \{0, 1\}^*\}$. Use the computation history method to show that this problem is undecidable.

michael-sipser theory-of-computation pushdown-automata decidability proof

Answer key

12.30.10 Pushdown Automata: Peter Linz Edition 4 Exercise 7.1 Question 1 (Page No. 183) top

Find a pda with fewer than four states that accepts the language $L = \{a^n b^n : n \geq 0\} \cup \{a\}$.

peter-linz peter-linz-edition4 theory-of-computation pushdown-automata

12.30.11 Pushdown Automata: Peter Linz Edition 4 Exercise 7.2 Question 1 (Page No. 195) top

Show that the pda constructed in ***Example 7.6*** accepts the string $aaabbbb$ that is in the language generated by the given grammar.

Example 7.6: Construct a pda that accepts the language generated by a grammar with productions

$$S \to aSbb|a.$$

theory-of-computation peter-linz peter-linz-edition4 pushdown-automata

12.30.12 Pushdown Automata: Peter Linz Edition 4 Exercise 7.2 Question 2 (Page No. 195) top

Prove that the pda in Example 7.6 accepts the language $L = \{a^{n+1}b^{2n} : n \geq 0\}$.

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12.31 Recursive And Recursively Enumerable Languages (53) top

12.31.1 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 11 (Page No. 189) top

A Turing machine with doubly infinite tape is similar to an ordinary Turing machine, but its tape is infinite to the left as well as to the right. The tape is initially filled with blanks except for the portion that contains the input. Computation is defined as usual except that the head never encounters an end to the tape as it moves leftward. Show that this type of Turing machine recognizes the class of Turing-recognizable languages.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages descriptive

12.31.2 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 12 (Page No. 189) top

A Turing machine with left reset is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, RESET\}$$

If $\delta(q, a) = (r, b, RESET)$, when the machine is in state q reading an a, the machine's head jumps to the left-hand end of the tape after it writes b on the tape and enters state r. Note that these machines do not have the usual ability to move the head one symbol left. Show that Turing machines with left reset recognize the class of Turing-recognizable languages.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages descriptive

12.31.3 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 13 (Page No. 189) top

A Turing machine with stay put instead of left is similar to an ordinary Turing machine, but the transition function has the form

$$\delta : Q \times \Gamma \to Q \times \Gamma \times \{R, S\}$$

At each point, the machine can move its head right or let it stay in the same position. Show that this Turing machine variant is not equivalent to the usual version. What class of languages do these machines recognize?

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages descriptive

12.31.4 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 14 (Page No. 189) top

A queue automaton is like a push-down automaton except that the stack is replaced by a queue. A queue is a tape allowing symbols to be written only on the left-hand end and read only at the right-hand end. Each write operation (we'll call it a push) adds a symbol to the left-hand end of the queue and each read operation (we'll call it a pull) reads and removes a symbol at the right-hand end. As with a PDA, the input is placed on a separate read-only input tape, and the head on the input tape can move only from left to right. The input tape contains a cell with a blank symbol following the input, so that the end of the input can be detected. A queue automaton accepts its input by entering a special accept state at any time. Show that a language can be recognized by a deterministic queue automaton iff the language is Turing-recognizable.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages descriptive

12.31.5 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 16 (Page No. 189) top

Show that the collection of Turing-recognizable languages is closed under the operation of

a. union.
b. concatenation.
c. star.
d. intersection.
e. homomorphism.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages

12.31.6 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 17 (Page No. 189) top

Let $B = \{\langle M_1 \rangle, \langle M_1 \rangle, \ldots\}$ be a Turing-recognizable language consisting of TM descriptions. Show that there is a decidable language C consisting of TM descriptions such that every machine described in B has an equivalent machine in C and vice versa.

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12.31.7 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 19 (Page No. 190) top

Show that every infinite Turing-recognizable language has an infinite decidable subset.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages proof

12.31.8 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 20 (Page No. 190) top

Show that single-tape TMs that cannot write on the portion of the tape containing the input string recognize only regular languages.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages proof

12.31.9 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 3 Question 6 (Page No. 188) top

In Theorem 3.21, we showed that a language is Turing-recognizable iff some enumerator enumerates it. Why didn't we use the following simpler algorithm for the forward direction of the proof? As before, $s_1, s_2, \ldots$ is a list of all strings in Σ^*.
$E =$" Ignore the input.

1. Repeat the following for $i = 1, 2, 3, \ldots$.
2. Run M on s_i.
3. If it accepts, print out s_i."

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages proof

12.31.10 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 4 Question 18 (Page No.

212) top

Let C be a language. Prove that C is Turing-recognizable iff a decidable language D exists such that $C = \{x \mid \exists y(\langle x, y\rangle \in D)\}$.

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages decidability proof

12.31.11 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 4 Question 20 (Page No. 212) top

Let A and B be two disjoint languages. Say that language C separates A and B if $A \subseteq C$ and $B \subseteq \overline{C}$. Show that any two disjoint co-Turing-recognizable languages are separable by some decidable language.

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages proof

12.31.12 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 4 Question 30 (Page No. 212) top

Let A be a Turing-recognizable language consisting of descriptions of Turing machines, $\{\langle M_1\rangle, \langle M_2\rangle, \ldots\}$, where every M_i is a decider. Prove that some decidable language D is not decided by any decider M_i whose description appears in A. (Hint: You may find it helpful to consider an enumerator for A.)

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages decidability proof

12.31.13 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 4 Question 5 (Page No. 211) top

Let $E_{TM} = \{\langle M\rangle \mid M \text{ is a TM and } L(M) = \phi\}$. Show that E_{TM}, the complement of E_{TM}, is Turing-recognizable.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages proof

12.31.14 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 5 Question 2 (Page No. 239) top

Show that EQ_{CFG} is co-Turing-recognizable.

michael-sipser theory-of-computation context-free-grammar recursive-and-recursively-enumerable-languages proof

12.31.15 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 5 Question 24 (Page No. 240) top

Let $J = \{w \mid \text{either } w = 0x \text{ for some } x \in A_{TM}, \text{ or } w = 1y \text{ for some } y \in \overline{A_{TM}}\}$. Show that neither J nor $\overline{J}$ is Turing-recognizable.

michael-sipser theory-of-computation turing-machine recursive-and-recursively-enumerable-languages proof

12.31.16 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 5 Question 35 (Page No. 242) top

Say that a variable A in $CFG\ G$ is **necessary** if it appears in every derivation of some string $w \in G$. Let $NECESSARY_{CFG} = \{\langle G, A\rangle \mid \text{A is a necessary variable in G}\}$.

a. Show that $NECESSARY_{CFG}$ is Turing-recognizable.
b. Show that $NECESSARY_{CFG}$ is undecidable.

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages decidability proof

12.31.17 Recursive And Recursively Enumerable Languages: Michael Sipser Edition 3 Exercise 5 Question 36 (Page No. 242) top

Say that a CFG is minimal if none of its rules can be removed without changing the language generated. Let $MIN_{CFG} = \{\langle G\rangle \mid \text{G is a minimal CFG}\}$.

a. Show that MIN_{CFG} is $T-$recognizable.
b. Show that MIN_{CFG} is undecidable.

michael-sipser theory-of-computation context-free-grammar recursive-and-recursively-enumerable-languages decidability proof

12.31.18 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 10 (Page No. 284) top

Is the family of recursive languages closed under concatenation?

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Answer key

12.31.19 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 11 (Page No. 284) top

Prove that the complement of a context-free language must be recursive.

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Answer key

12.31.20 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 12 (Page No. 284) top

Let L_1 be recursive and L_2 recursively enumerable. Show that $L_2 - L_1$ is necessarily recursively enumerable.

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Answer key

12.31.21 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 13 (Page No. 284) top

Suppose that L is such that there exists a Turing machine that enumerates the elements of L in proper order. Show that this means that L is recursive.

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12.31.22 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 14 (Page No. 284) top

If L is recursive, is it necessarily true that L^+ is also recursive?

theory-of-computation proof recursive-and-recursively-enumerable-languages turing-machine peter-linz peter-linz-edition5

Answer key

12.31.23 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 15 (Page No. 284) top

$\text{Theorem}:$ There exists a recursively enumerable language whose complement is not recursively enumerable.

Choose a particular encoding for Turing machines, and with it, find one element of the languages $\bar{L}$ in Theorem

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Answer key

12.31.24 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 16 (Page No. 284) top

Let S_1 be a countable set, S_2 a set that is not countable, and $S_1 \subset S_2$. Show that S_2 must then contain an infinite number of elements that are not in S_1.

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Answer key

12.31.25 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 17 (Page No. 284) top

Let S_1 be a countable set, S_2 a set that is not countable, and $S_1 \subset S_2$. Show that S_2 must then contain an infinite number of elements that are not in S_1.

Show that in fact $S_2 - S_1$ cannot be countable.

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12.31.26 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 18 (Page No. 284) top

Theorem : Let S be an infinite countable set. Then its powerset 2^S is not countable.

Why does the argument in Theorem fail when S is finite?

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12.31.27 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 19 (Page No. 284) top

Show that the set of all irrational numbers is not countable.

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Answer key

12.31.28 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 2 (Page No. 284) top

Prove that the set of all languages that are not recursively enumerable is not countable.

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12.31.29 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 3 (Page No. 284) top

Let L be a finite language. Show that then L^+ is recusively enumerable. Suggest an enumeration procedure for L^+

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12.31.30 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 4 (Page No. 284) top

Let L be a context-free language. Show that then L^+ is recusively enumerable. Suggest an enumeration procedure for it.

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12.31.31 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 5 (Page No. 284) top

Show that if a language is not recursively enumerable, its complement cannot be recursive.

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12.31.32 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 6 (Page No. 284) top

Show that the family of recursively enumerable languages is closed under union.

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12.31.33 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 7 (Page No. 284) top

Is the family of recursively enumerable languages closed under intersection?

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Answer key

12.31.34 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 8 (Page No.

284) top

Show that the family of recursive languages is closed under union and intersection.

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12.31.35 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.1 Question 9 (Page No. 284) top

Show that the families of recursively enumerable and recursive languages are closed under reversal.

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12.31.36 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 1 (Page No. 290) top

What language does the unrestricted grammar

$$S \to S_1B,$$

$$S_1 \to aS_1b,$$

$$bB \to bbbB,$$

$$aS_1b \to aa,$$

$$B \to \lambda$$

derive?

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12.31.37 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 2 (Page No. 290) top

What difficulties would arise if we allowed the empty string as the left side of a production in an unrestricted grammar?

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12.31.38 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 3 (Page No. 290) top

Consider a variation on grammars in which the starting point of any derivation can be a finite set of strings, rather than a single variable. Formalize this concept, then investigate how such grammars relate to the unrestricted grammars we have used here.

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12.31.39 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 4 (Page No. 290) top

Prove that constructed grammar cannot generate any sentence with a b in it.

$$S \to S_1B,$$

$$S_1 \to aS_1b,$$

$$bB \to bbbB,$$

$$aS_1b \to aa,$$

$$B \to \lambda$$

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12.31.40 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 5 (Page No.

290) top

$\text{Theorem}:$ For every recursively enumerable language L, there exists an unrestricted grammar G, such that $L = L(G)$.

Give the details of the proof of the Theorem.

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12.31.41 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 6 (Page No. 290) top

$\text{Theorem}:$ For every recursively enumerable language L, there exists an unrestricted grammar G, such that $L = L(G)$.

Construct a Turing machine for $L(01(01)^*)$, then find an unrestricted grammar for it using the construction in Theorem. Give a derivation for 0101 using the resulting grammar.

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12.31.42 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 7 (Page No. 290) top

Show that for every unrestricted grammar there exists an equivalent unrestricted grammar, all of whose productions have the form

$$u \to v,$$

with $u, v \in (V \cup T)^+$ and $|u| \leq |v|$, or

$$A \to \lambda$$

with $A \in V$

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12.31.43 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 8 (Page No. 290) top

Every unrestricted grammar there exists an equivalent unrestricted grammar, all of whose productions have the form

$$u \to v,$$

with $u, v \in (V \cup T)^+$ and $|u| \leq |v|$, or

$$A \to \lambda$$

with $A \in V$

Show that the conclusion still holds if we add the further conditions $|u| \leq 2$ and $|v| \leq 2$

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12.31.44 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.2 Question 9 (Page No. 290,291) top

A grammar $G = (V, T, S, P)$ is called unrestricted if all the production are of the form

$$u \to v,$$

where u is nit $(V \cup T)^+$ and v is int $(V \cup T)^*$

Some authors give a definition of unrestricted grammars that is not quite the same as our Definition. In this alternate definition, the productions of an unrestricted grammar are required to be of the form

$$x \to y,$$

where

$$x \in (V \cup T)^* V (V \cup T)^*,$$

and

$$y \in (V \cup T)^*$$

The difference is that here the left side must have at least one variable.

Show that this alternate definition is basically the same as the one we use, in the sense that for every grammar of one type, there is an equivalent grammar of the other type.

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12.31.45 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 1 (Page No. 296) top

Find the context-sensitive grammars for the following languages.

(a) $L = \{a^{n+1}b^nc^{n-1} : n \geq 1\}$.

(b) $L = \{a^nb^nc^{2n} : n \geq 1\}$.

(c) $L = \{a^nb^mc^nd^m : n \geq 1, m \geq 1\}$.

(d) $L = \{ww : w \in \{a,b\}^+\}$.

(e) $L = \{a^nb^nc^nd^m : n \geq 1\}$.

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12.31.46 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 2 (Page No. 296) top

Find context-sensitive grammars for the following languages.

(a) $L = \{w : n_a(w) = n_b(w) = n_c(w)\}$.

(b) $L = \{w : n_a(w) = n_b(w) < n_c(w)\}$.

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12.31.47 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 3 (Page No. 296) top

Show that the family of context-sensitive languages is closed under union.

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12.31.48 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 4 (Page No. 296) top

Show that the family of context-sensitive languages is closed under reversal.

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12.31.49 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 5 (Page No. 296) top

Theorem : Every context-sensitive language L is recursive.

For m in Theorem, give explicit bounds for m as a function of $|w|$ and $|V \cup T|$.

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12.31.50 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.3 Question 6 (Page No. 296) top

Without explicitly constructing it, show that there exists a context-sensitive grammar for the language $L = \{www^R : w, u \in \{a, b\}^+, |w| \geq |u|\}$.

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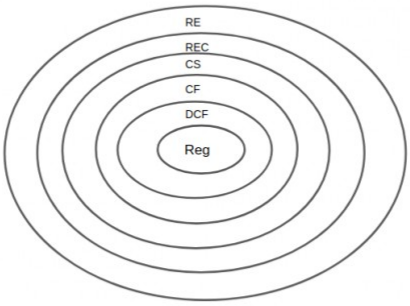
12.31.51 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.4 Question 1 (Page No. 298) top

Given examples that demonstrate that all the subset relations depicted in the figure are indeed proper ones.



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12.31.52 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.4 Question 2 (Page No. 298) top

Find two examples of languages that are linear but not deterministic context-free.

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12.31.53 Recursive And Recursively Enumerable Languages: Peter Linz Edition 5 Exercise 11.4 Question 3 (Page No. 298) top

Find two examples of languages that are deterministic context-free but not linear.

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12.32 Reduction (7) top

12.32.1 Reduction: Michael Sipser Edition 3 Exercise 5 Question 22 (Page No. 240) top

Show that A is Turing-recognizable iff $A \leq_m A_{TM}$.

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages reduction proof

12.32.2 Reduction: Michael Sipser Edition 3 Exercise 5 Question 23 (Page No. 240) top

Show that A is decidable iff $A \leq_m 0^*1^*$.

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12.32.3 Reduction: Michael Sipser Edition 3 Exercise 5 Question 25 (Page No. 240) top

Give an example of an undecidable language B, where $B \leq_m \overline{B}$.

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12.32.4 Reduction: Michael Sipser Edition 3 Exercise 5 Question 4 (Page No. 239) top

If $A \leq_m B$ and B is a regular language, does that imply that A is a regular language? Why or why not?

michael-sipser theory-of-computation regular-language reduction proof

12.32.5 Reduction: Michael Sipser Edition 3 Exercise 5 Question 5 (Page No. 239) top

Show that A_{TM} is not mapping reducible to E_{TM}. In other words, show that no computable function reduces A_{TM} to E_{TM}. (Hint: Use a proof by contradiction, and facts you already know about A_{TM} and E_{TM}.)

michael-sipser theory-of-computation turing-machine reduction proof

12.32.6 Reduction: Michael Sipser Edition 3 Exercise 5 Question 6 (Page No. 239) top

Show that $\leq_m$ is a transitive relation.

michael-sipser theory-of-computation turing-machine reduction proof

12.32.7 Reduction: Michael Sipser Edition 3 Exercise 5 Question 7 (Page No. 239) top

Show that if A is Turing-recognizable and $A \leq_m \overline{A}$, then A is decidable.

michael-sipser theory-of-computation recursive-and-recursively-enumerable-languages decidability reduction proof

12.33 Regular Expression (49) top

12.33.1 Regular Expression: Michael Sipser Edition 3 Exercise 1 Question 18 (Page No. 86) top

Give regular expressions generating the languages of the alphabet is $\{0, 1\}$.

a. $\{\text{w}|\text{ w begins with a 1 and ends with a 0}\}$
b. $\{\text{w}|\text{ w contains at least three 1s}\}$
c. $\{\text{w}|\text{ w contains the substring 0101 (i.e., w} = \text{x0101y for some x and y)}\}$
d. $\{\text{w}|\text{ w has length at least 3 and its third symbol is a 0}\}$
e. $\{\text{w}|\text{ w starts with 0 and has odd length, or starts with 1 and has even length}\}$
f. $\{\text{w}|\text{ w doesn't contain the substring 110}\}$
g. $\{\text{w}|\text{ the length of w is at most 5}\}$
h. $\{\text{w}|\text{ w is any string except 11 and 111}\}$
i. $\{\text{w}|\text{ every odd position of w is a 1}\}$
j. $\{\text{w}|\text{ w contains at least two 0's and at most one 1}\}$
k. $\{\epsilon, 0\}$
l. $\{\text{w}|\text{ w contains an even number of 0's, or contains exactly two 1's}\}$
m. The empty set
n. All strings except the empty string

michael-sipser theory-of-computation regular-expression

Answer key

12.33.2 Regular Expression: Michael Sipser Edition 3 Exercise 1 Question 19 (Page No. 86) top

Use the procedure described in Lemma 1.55 to convert the following regular expressions to non-deterministic finite automata.

a. $(0 \cup 1)^* 000 (0 \cup 1)^*$
b. $(((00)^*(11)) \cup 01)^*$
c. ϕ^*

michael-sipser theory-of-computation regular-expression finite-automata

Answer key

12.33.3 Regular Expression: Michael Sipser Edition 3 Exercise 1 Question 20 (Page No. 86) top

For each of the following languages, give two strings that are members and two strings that are not members—a total of four strings for each part. Assume the alphabet $\Sigma = \{a, b\}$ in all parts.

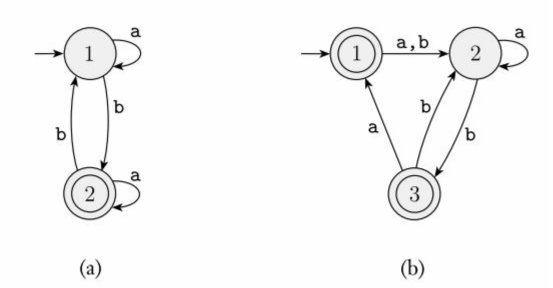
a. a^*b^*
b. $a(ba)^*b$
c. $a^* \cup b^*$
d. $(aaa)^*$
e. $\sum^* a \sum^* b \sum^* a \sum^*$
f. $aba \cup bab$
g. $(\epsilon \cup a)b$

h. $(a \cup ba \cup bb)\sum^*$

michael-sipser theory-of-computation regular-language regular-expression

12.33.4 Regular Expression: Michael Sipser Edition 3 Exercise 1 Question 21 (Page No. 86) top

Use the procedure described in Lemma 1.60 to convert the following finite automata to regular expressions.



michael-sipser theory-of-computation finite-automata regular-expression

Answer key

12.33.5 Regular Expression: Michael Sipser Edition 3 Exercise 1 Question 22 (Page No. 87) top

In certain programming languages, comments appear between delimiters such as $/\#$ and $\#/$. Let C be the language of all valid delimited comment strings. A member of C must begin with $/\#$ and end with $\#/$ but have no intervening $\#/$. For simplicity, assume that the alphabet for C is $\Sigma = \{\mathrm{a}, \mathrm{b}, /, \#\}$.
a. Give a DFA that recognizes C.
b. Give a regular expression that generates C.

michael-sipser theory-of-computation regular-expression finite-automata

Answer key

12.33.6 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 1 (Page No. 75) top

Find all strings in $L((a+b)b(a+ab)^*)$ of length less than four.

peter-linz peter-linz-edition4 theory-of-computation regular-expression

Answer key

12.33.7 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 10 (Page No. 76) top

Give a regular expression for

L = $\{a^n b^m; n \geq 1, m \geq 1, nm \geq 3\}$

theory-of-computation peter-linz peter-linz-edition4 regular-expression

Answer key

12.33.8 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 11 (Page No. 76) top

Find a regular expression for $L = \{ab^n w : n \geq 3, w \in \{a,b\}^+\}$.

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12.33.9 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 12 (Page No. 76) top

Find a regular expression for the complement of the language in $L(r) = \{a^{2n} b^{2m+1} : n \geq 0, m \geq 0\}$.

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12.33.10 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 13 (Page No. 76) top

Find a regular expression for $L = \{vwv : v, w \in \{a,b\}^*, |v| = 2\}$.

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12.33.11 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 14 (Page No. 76) top

Find a regular expression for $L = \{vwv : v, w \in \{a, b\}^*, |v| \leq 3\}$.

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12.33.12 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 15 (Page No. 76) top

Find a regular expression for
$L = \{w \in \{0, 1\}^* : w \text{ has exactly one pair of consecutive zeros}\}$.

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12.33.13 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 16 (Page No. 76) top

Give regular expressions for the following languages on $\Sigma = \{a, b, c\}$.
(a) all strings containing exactly one a,
(b) all strings containing no more than three a's,
(c) Peter Linz Edition 4 Exercise 3.1 Question 16.c (Page No. 76)
(d)Peter Linz Edition 4 Exercise 3.1 Question 16.d (Page No. 76)
(e) all strings in which all runs of a'shave lengths that are multiples of three.

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Answer key

12.33.14 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 16.c (Page No. 76) top

Give regular expression for the following language on $\sum = \{a, b, c\}$

All strings that contain at least one occurrence of each symbol in $\sum$

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Answer key

12.33.15 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 16.d (Page No. 76) top

Find a regular expression over Σ ={a,b,c} for all strings that contain no run of a's of length greater than 2.

Here a run in a string is a sub string of length at least two as long as possible and consisting entirely of the same symbol. For eg, the string **abbbaab** contains a run of b's of length three and a tun of a's of length two.

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Answer key

12.33.16 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 17 (Page No. 76) top

Write regular expressions for the following languages on $\{0, 1\}$.
(a) all strings ending in 01,
(b) all strings not ending in 01,
(c) all strings containing an even number of 0's,
(d) Peter Linz Edition 4 Exercise 3.1 Question 17.d (Page No. 76)
(e) all strings with at most two occurrences of the substring 00,
(f) all strings not containing the substring 101.

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12.33.17 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 17.d (Page No. 76) top

$\sum = \{0, 1\}$

Give a regular expression for all strings having at least two occurrences of the substring 00. (Note that with the usual interpretation of a substring, 000 counts two such occurences)

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Answer key

12.33.18 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 18 (Page No. 76) top

Find regular expressions for the following languages on $\{a, b\}$.
(a) $L = \{w : |w| \bmod 3 = 0\}$.
(b) $L = \{w : n_a(w) \bmod 3 = 0\}$.
(c) $L = \{w : n_a(w) \bmod 5 > 0\}$.

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Answer key

12.33.19 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 18 (Page No. 76) top

Find regular expressions for the following languages on {a, b}.

(a) L = {w : |w| mod 3 = 0}

(b) L = {w : na (w)mod 3 = 0}

(c) L = {w : na (w)mod 5 > 0}

Also Design DFA for the same.

regular-language regular-expression peter-linz peter-linz-edition4

Answer key

12.33.20 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 19 (Page No. 77) top

Repeat parts (a), (b), and (c) of Peter Linz Edition 4 Exercise 3.1 Question 18 (Page No. 76) with $\Sigma = \{a, b, c\}$.

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12.33.21 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 2 (Page No. 75) top

Does the expression $((0+1)(0+1)^*)^*00(0+1)^*$ denote the language in $L(r) = \{w \in \Sigma^* : w$ has at least one pair of consecutive zeros$\}$.?

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12.33.22 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 20 (Page No. 77) top

Determine whether or not the following claims are true for all regular expressions r_1 and r_2. The symbol $\equiv$ stands for equivalence of regular expressions in the sense that both expressions denote the same language.

(a) $(r_1^*)^* \equiv r_1^*$.

(b) $r_1^*(r_1 + r_2)^* \equiv (r_1 + r_2)^*$.

(c)$(r_1 + r_2)^* \equiv (r_1^* r_2^*)^*$.

(d)$(r_1 r_2)^* \equiv r_1^* r_2^*$.

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Answer key

12.33.23 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 21 (Page No. 77) top

Give a general method by which any regular expression r can be changed into $\hat{r}$ such that $(L(r))^R = L(\hat{r})$.

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Answer key

12.33.24 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 22 (Page No. 77) top

Prove rigorously that the expressions in $r = (1^*011^*)^*(0 + \lambda) + 1^*(0 + \lambda)$ do indeed denote the specified language.

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12.33.25 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 23 (Page No. 77) top

For the case of a regular expression r that does not involve λ or $\emptyset$, give a set of necessary and sufficient conditions that r must satisfy if $L(r)$ is to be infinite.

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12.33.26 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 24,25 (Page No. 77) top

24. Formal languages can be used to describe a variety of two-dimensional figures. Chain-code languages are defined on the alphabet $\Sigma = \{u, d, r, l\}$, where these symbols stand for unit-length straight lines in the directions up, down, right, and left, respectively. An example of this notation is $urdl$, which stands for the square with sides of unit length. Draw pictures of the figures denoted by the expressions $(rd)^*$, $(urddru)^*$, and $(ruldr)^*$.

25. In above, what are sufficient conditions on the expression so that the picture is a closed contour in the sense that the beginning and ending points are the same? Are these conditions also necessary?

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12.33.27 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 26 (Page No. 77) top

Find an nfa that accepts the language $L(aa^*(a+b))$.

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12.33.28 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 28 (Page No. 77) top

Find a regular expression for all bit strings, with leading bit 1, interpreted as a binary integer, with values not between 10 and 30.

theory-of-computation peter-linz peter-linz-edition4 regular-expression

Answer key

12.33.29 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 3 (Page No. 75) top

Show that $r = (1+01)^*(0+1^*)$ also denotes the language in $L = \{w \in \{0,1\}^* : w$ has no pair of consecutive zeros$\}$. Find two other equivalent expressions.

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12.33.30 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 4 (Page No. 75) top

Find a regular expression for the set $\{a^n b^m : n \geq 3, m$ is even$\}$.

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Answer key

12.33.31 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 6 (Page No. 75) top

Give regular expressions for the following languages.
(a) $L_1 = \{a^n b^m : n \geq 4, m \leq 3\}$.
(b) $L_2 = \{a^n b^m : n < 4, m \leq 3\}$.
(c) The complement of L_1.
(d) The complement of L_2.

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12.33.32 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 8 (Page No. 76) top

Give a simple verbal description of the language $L((aa)^*b(aa)^* + a(aa)^*ba(aa)^*)$.

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12.33.33 Regular Expression: Peter Linz Edition 4 Exercise 3.1 Question 9 (Page No. 76) top

Give a regular expression for L^R

L = $(a + bc)^*(c + \phi)$

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Answer key

12.33.34 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 1 (Page No. 87) top

Find an nfa that accepts the language $L(ab^*aa + bba^*ab)$.

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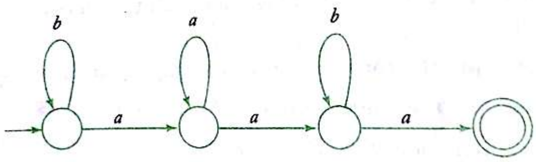
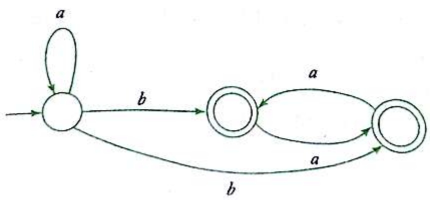
12.33.35 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 10 (Page No. 88) top

Find regular expressions for the languages accepted by the following automata:-



a.
b. https://gateoverflow.in/304714/peter-linz-edition-4-exercise-3-2-question-10-b-page-no-88



c.

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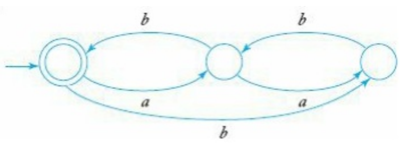
Answer key

12.33.36 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 10.b (Page No. 88) top

What is the regular expression for this 

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Answer key

12.33.37 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 13 (Page No. 88) top

Find a regular expression for the following languages on $\{a, b\}$.
(a) $L = \{w : n_a(w)$ and $n_b(w)$ are both even$\}$.
(b) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$.
(c) $L = \{w : (n_a(w) - n_b(w)) \bmod 3 = 0\}$.
(d) $L = \{w : 2n_a(w) + 3n_b(w)$ is even$\}$.

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Answer key

12.33.38 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 15 (Page No. 89) top

Write a regular expression for the set of all C real numbers.

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12.33.39 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 16 (Page No. 89) top

In some applications, such as programs that check spelling, we may not need an exact match of the pattern, only an approximate one. Once the notion of an approximate match has been made precise, automata theory can be applied to construct approximate pattern matchers. As an illustration of this, consider patterns derived from the original ones by insertion of one symbol. Let L be a regular language on Σ and define

$insert(L) = \{uav : a \in \Sigma, uv \in L\}$.

In effect, $insert(L)$ contains all the words created from L by inserting a spurious symbol anywhere in a word.

(a) Given an nfa for L, show how one can construct an nfa for $insert(L)$.

(b) Discuss how you might use this to write a pattern-recognition program for $insert(L)$, using as input a regular expression for L.

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12.33.40 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 17 (Page No. 89) top

Analogous to the previous exercise, consider all words that can be formed from L by dropping a single symbol of the string. Formally define this operation drop for languages. Construct an nfa for $drop(L)$, given an nfa for L.

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12.33.41 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 2 (Page No. 87) top

Find an nfa that accepts the complement of the language in $L(ab^*aa + bba^*ab)$.

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12.33.42 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 3 (Page No. 87) top

Give an nfa that accepts the language $L((a + b)^*b(a + bb)^*)$.

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12.33.43 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 4 (Page No. 87) top

Find dfa's that accept the following languages.

(a) $L(aa^* + aba^*b^*)$.

(b) $L(ab(a + ab)^*(a + aa))$.

(c) $L((abab)^* + (aaa^* + b)^*)$.

(d) $L(((aa^*)^*b)^*)$.

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12.33.44 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 5 (Page No. 87) top

Find dfa's that accept the following languages.

(a) $L = L(ab^*a^*) \cup L((ab)^*ba)$.

(b) $L = L(ab^*a^*) \cap L((ab)^*ba)$.

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Answer key

12.33.45 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 6 (Page No. 87) top

Find an nfa for ***all strings not containing the substring 101***. Use this to derive a regular expression for that language.

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12.33.46 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 7 (Page No. 87) top

Find the minimal dfa that accepts $L(a^*bb) \cup L(ab^*ba)$.

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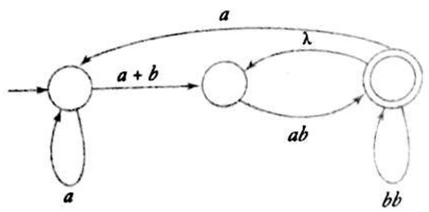
12.33.47 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 8 (Page No. 87) top

Consider the following generalized transition graph.



(a) Find an equivalent generalized transition graph with only two states.
(b) What is the language accepted by this graph?

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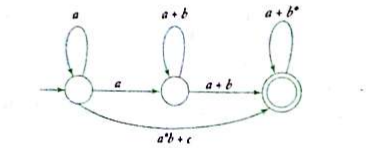
12.33.48 Regular Expression: Peter Linz Edition 4 Exercise 3.2 Question 9 (Page No. 88) top

What language is accepted by the following generalized transition graph?



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Answer key

12.33.49 Regular Expression: Peter Linz Edition 4 Exercise 4.1 Question 4 (Page No. 109) top

Theorem 4.3

"Let h be a homomorphism. If L is a regular language, then its homomorphic image h (L) is also regular. The family of regular languages is therefore closed under arbitrary homomorphisms."

Proof: Let L be a regular language denoted by some regular expression r. We find $h(r)$ by substituting $h(a)$ for each symbol $a \in \Sigma$ of r. It can be shown directly by an appeal to the definition of a regular expression that the result is a regular expression. It is equally easy to see that the resulting expression denotes $h(L)$. All we need to do is to show that for every $w \in L(r)$, the corresponding $h(w)$ is in $L(h(r))$ and conversely that every v in $L(h(r))$ there is a w in L, such that $v = h(w)$. we claim that $h(L)$ is regular.

In the proof of ***Theorem 4.3***, show that $h(r)$ is a regular expression. Then show that $h(r)$ denotes $h(L)$.

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12.34 Regular Grammar (17) top

12.34.1 Regular Grammar: Peter Linz Edition 4 Exercise 3.1 Question 27 (Page No. 77) top

Find a regular expression that denotes all bit strings whose value, when interpreted as a binary integer, is greater than or equal to 40.

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Answer key

12.34.2 Regular Grammar: Peter Linz Edition 4 Exercise 3.1 Question 5 (Page No. 75) top

what is the regular grammar for L={a^nb^m | n+m is even}

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Answer key

12.34.3 Regular Grammar: Peter Linz Edition 4 Exercise 3.1 Question 7 (Page No. 76) Exercise 3.3 Question 9 (Page No. 97) top

Regular Expression:-

Q1) What languages do the expression (∅*)* and a∅ denote?

Q2) Find a regular expression and finite automata for all bit strings, with leading bit 1 interpreted as a binary integer, with values not between 10 and 30.

Regular Grammar:-

Q1) Suggest a construction by which a left-linear grammar can be obtained from an nfa directly.

Q2) Find a regular grammar and draw the nfa or dfa that generates the language

L = { w ∈ {a, b}* / (number of a in w + 3*number of b) in w is even }

theory-of-computation regular-language regular-expression regular-grammar peter-linz peter-linz-edition4

Answer key

12.34.4 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 1 (Page No. 96) top

Construct a dfa that accepts the language generated by the grammar
$S \to abA, A \to baB, B \to aA|bb$.

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12.34.5 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 10 (Page No. 97) top

Find a left-linear grammar for the language $L((aab^*ab)^*)$.

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12.34.6 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 11 (Page No. 97) top

Find a regular grammar for the language $L = \{a^nb^m : n + m$ is even$\}$.

peter-linz peter-linz-edition4 theory-of-computation regular-grammar

12.34.7 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 12 (Page No. 97) top

Find a regular grammar that generates the language

$$L = \{w \in \{a,b\}^* : n_a(w) + 3n_b(w) \text{ is even } \}.$$

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12.34.8 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 13 (Page No. 97) top

Find regular grammars for the following languages on $\{a, b\}$.

(a) $L =\{w : n_a(w)$ and $n_b(w)$ are both even$\}$.

(b) $L =\{w : (n_a(w) - n_b(w)) \bmod 3 = 1\}$.

(c) $L =\{w : (n_a(w) - n_b(w)) \bmod 3 \neq 1\}$.

(d) $L =\{w : (n_a(w) - n_b(w)) \bmod 3 \neq 0\}$.

(e) $L =\{w : |n_a(w) - n_b(w)|$ is odd$\}$.

peter-linz peter-linz-edition4 theory-of-computation regular-grammar

12.34.9 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 14 (Page No. 97) top

Show that for every regular language not containing λ there exists a right-linear grammar whose productions are restricted to the forms

$$A \to aB,$$

or

$$A \to a,$$

where $A, B \in V$, and $a \in T$

peter-linz peter-linz-edition4 theory-of-computation regular-grammar

12.34.10 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 17 (Page No. 97) top

Let $G_1 = (V_1, \Sigma, S_1, P_1)$ be right-linear and $G_2 = (V_2, \Sigma, S_2, P_2)$ be a left-linear grammar, and assume that V_1 and V_2 are disjoint. Consider the linear grammar $G = (\{S\} \cup V_1 \cup V_2, \Sigma, S, P)$, where S is not in $V_1 \cup V_2$ and

$P = \{S \to S_1 | S_2\} \cup P_1 \cup P_2$. Show that $L(G)$ is regular.

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12.34.11 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 2 (Page No. 96) top

Find a regular grammar that generates the language $L(aa^*(ab+a)^*)$.

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12.34.12 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 3 (Page No. 96) top

Construct a left-linear grammar for the language generated by the grammar

$S \to abA,$
$A \to baB,$
$B \to aA|bb.$

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12.34.13 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 4 (Page No. 96) top

Construct right- and left-linear grammars for the language
$L = \{a^n b^m : n \geq 2, m \geq 3\}$.

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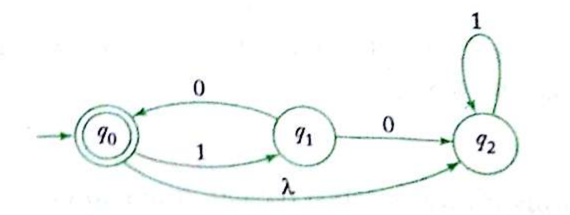
12.34.14 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 5 (Page No. 96) top

Find a left-linear grammar for the language accepted by
the nfa below.



peter-linz peter-linz-edition4 theory-of-computation regular-grammar

Answer key

12.34.15 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 6 (Page No. 97) top

Construct a right linear grammar for the language $L((aab^*ab)^*)$

is this grammar correct?

S->aaA | ε

A->bA | abA | S

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12.34.16 Regular Grammar: Peter Linz Edition 4 Exercise 3.3 Question 7 (Page No. 97) top

Find a regular grammar that generates the language on $\Sigma = \{a, b\}$ consisting of all strings with no more than three a's.

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12.34.17 Regular Grammar: Peter Linz Edition 4 Exercise 5.2 Question 11 (Page No. 145) top

Is it possible for a regular grammar to be ambiguous?

peter-linz peter-linz-edition4 theory-of-computation regular-grammar ambiguous

12.35 Regular Language (27) top

12.35.1 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 15 (Page No. 85) top

Give a counterexample to show that the following construction fails to prove Theorem 1.49, the closure of the class of regular languages under the star operation. Let $N_1 = (Q_1, \Sigma, \delta_1, q_1, F_1)$ recognize A_1. Construct $N = (Q_1, \Sigma, \delta, q_1, F)$ as
follows. N is supposed to recognize A_1^*.

a. The states of N are the states of N1.
b. The start state of N is the same as the start state of N1.
c. $F = \{q_1\} \cup F_1$. The accept states F are the old accept states plus its start state.
d. Define δ so that for any $q \in Q_1$ and any $a \in \Sigma\varepsilon$,

$$\delta(q, a) = \begin{cases} \delta_1(q, a) & q \notin F_1 \text{ or } a \neq \varepsilon \\ \delta_1(q, a) \cup \{q_1\} & q \in F_1 \text{ and } a = \varepsilon. \end{cases}$$

In other words, you must present a finite automaton, $N1$, for which the constructed automaton N does not recognize the star of $N_1^{'s}$ language.

michael-sipser theory-of-computation finite-automata regular-language

12.35.2 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 23 (Page No. 87) top

Let B be any language over the alphabet Σ. Prove that $B = B^+$ iff $BB \subseteq B$.

michael-sipser theory-of-computation regular-language proof

12.35.3 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 31 (Page No. 88) top

For any string $w = w_1 w_2 \cdots w_n$, the reverse of w, written w^R, is the string w in reverse order, $w_n \cdots w_2 w_1$. For any language A, let $A^R = \{w^R | w \in A\}$. Show that if A is regular, so is A^R.

michael-sipser theory-of-computation finite-automata regular-language

12.35.4 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 32 (Page No. 88) top

$$\text{Let } \sum_3 = \left\{ \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \dots\dots\dots, \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} \right\}.$$

$\sum_3$ contains all size 3 columns of $0's$ and $1's$. A string of symbols in $\sum_3$ gives three rows of $0's$ and $1's$. Consider each row to be a binary number and let

$$B = \{w \in \textstyle\sum_3^* \,|\text{the bottom row of } w \text{ is the sum of the top two rows}\}.$$

For example, $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\0\end{bmatrix}\begin{bmatrix}1\\1\\0\end{bmatrix} \in B$, but $\begin{bmatrix}0\\0\\1\end{bmatrix}\begin{bmatrix}1\\0\\1\end{bmatrix} \notin B$

Show that B is regular. (Hint:Working with B^R is easier. You may assume the result claimed in question 31)

michael-sipser theory-of-computation finite-automata regular-language

12.35.5 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 33 (Page No. 89) top

Let $\sum_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$.

Here $\sum_2$ contains all columns of $0's$ and $1's$ of height two . A string of symbols in $\sum_2$ gives two rows of $0's$ and $1's$. Consider each row to be a binary number and let

$C = \{w \in \sum_2^* \mid \text{the bottom row of } w \text{ is three times the top two rows}\}$.

For example, $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \in C$, but $\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} \notin C$

Show that C is regular. (You may assume the result claimed in question 31)

michael-sipser theory-of-computation finite-automata regular-language

12.35.6 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 34 (Page No. 89) top

Let $\sum_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$.

Here $\sum_2$ contains all columns of $0's$ and $1's$ of height two . A string of symbols in $\sum_2$ gives two rows of $0's$ and $1's$. Consider each row to be a binary number and let

$D = \{w \in \sum_2^* \mid \text{the top row of } w \text{ is larger number than is the bottom row}\}$.

For example, $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \in D$, but $\begin{bmatrix}0\\0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix}\begin{bmatrix}1\\1\end{bmatrix}\begin{bmatrix}0\\0\end{bmatrix} \notin D$

Show that D is regular.

michael-sipser theory-of-computation finite-automata regular-language

12.35.7 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 35 (Page No. 89) top

Let $\sum_2 = \left\{ \begin{bmatrix}0\\0\end{bmatrix}, \begin{bmatrix}0\\1\end{bmatrix}, \begin{bmatrix}1\\0\end{bmatrix}, \begin{bmatrix}1\\1\end{bmatrix} \right\}$.

Here $\sum_2$ contains all columns of $0's$ and $1's$ of height two . A string of symbols in $\sum_2$ gives two rows of $0's$ and $1's$. Consider each row to be a binary number and let

$E = \{w \in \sum_2^* \mid \text{the bottom row of } w \text{ is the reverse of the top row of } w\}$.

Show that E is not regular.

michael-sipser theory-of-computation finite-automata regular-language

12.35.8 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 36 (Page No. 89) top

Let $B_n = \{a^k \mid k \text{ is a multiple of } n\}$. Show that for each $n \geq 1$, the language B_n is regular.

michael-sipser theory-of-computation finite-automata regular-language

Answer key

12.35.9 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 37 (Page No. 89) top

Let $C_n = \{x|x \text{ is a binary number that is a multiple of } n\}$. Show that for each $n \geq 1$, the language C_n is regular.

michael-sipser theory-of-computation finite-automata regular-language

Answer key

12.35.10 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 38 (Page No. 89) top

An all-NFA M is a 5-tuple $(Q, \Sigma, \delta, q_0, F)$ that accepts $x \in \sum^*$ if every possible state that M could be in after reading input x is a state from F. Note , in contrast, that an ordinary NFA accepts a string if some state among these possible states is an accept state. Prove that all-NFAs recognize the class of regular languages .

michael-sipser theory-of-computation finite-automata regular-language

Answer key

12.35.11 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 42 (Page No. 89) top

For languages A and B, let the shuffle of A and B be the language $\{w|w = a_1b_1 \dots a_kb_k,$ where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$.

Show that the class of regular languages is closed under shuffle.

michael-sipser theory-of-computation finite-automata regular-language

12.35.12 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 43 (Page No. 90) top

Let A be any language. Define DROP-OUT(A) to be the language containing all strings that can be obtained by removing one symbol from a string in A. Thus, $\text{DROP-OUT(A)} = \{xz|xyz \in A \text{ where } x, z \in \sum^*, y \in \sum\}$. Show that the class of regular languages is closed under the DROP-OUT operation. Give both a proof by picture and a more formal proof by construction as in Theorem 1.47.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

12.35.13 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 44 (Page No. 90) top

Let B and C be languages over $\sum = \{0, 1\}$. Define $B \overset{1}{\leftarrow} C = \{w \in B|$ for some $y \in C$, strings w and y contain equal numbers of $1's\}$. Show that the class of regular languages is closed under the $\overset{1}{\leftarrow}$operation.

michael-sipser theory-of-computation finite-automata regular-language descriptive

12.35.14 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 45 (Page No. 90) top

Let $\mathrm{A/B} = \{\mathrm{w}|\ \mathrm{wx} \in \mathrm{A} \text{ for some } \mathrm{x} \in \mathrm{B}\}$.Show that if A is regular and B is any language, then $\mathrm{A/B}$ is regular.

michael-sipser theory-of-computation finite-automata regular-language descriptive

12.35.15 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 46 (Page No. 90) top

Prove that the following languages are not regular. You may use the pumping lemma and the closure of the class of regular languages under union, intersection,and complement.

a. $\{0^n1^m0^n|m, n \geq 0\}$
b. $\{0^m1^n|m \neq n\}$
c. $\{w|w \in \{0, 1\}^* \text{is not a palindrome}\}$
d. $\{wtw|w, t \in \{0, 1\}^+\}$

michael-sipser theory-of-computation finite-automata regular-language proof

12.35.16 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 47 (Page No. 90) top

Let $\sum = \{1, \#\}$ and let $Y = \{w|w = x_1\#x_2\#\dots\#x_k$ for $k \geq 0$, each $x_i \in 1^*$, and $x_i \neq x_j$ for $i \neq j\}$. Prove that Y is not regular.

michael-sipser theory-of-computation finite-automata regular-language proof

12.35.17 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 48 (Page No. 90) top

Let $\sum = \{0,1\}$ and let $D = \{w|w$ contains an equal number of occurrences of the sub strings 01 and 10$\}$. Thus $101 \in D$ because 101 contains a single 01 and a single 10, but $1010 \notin D$ because 1010 contains two $10's$ and one 01. Show that D is a regular language.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

12.35.18 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 49 (Page No. 90) top

a. Let $B = \{1^k y | y \in \{0,1\}^*$ and y contains at least k $1's$, for every $k \geq 1\}$. Show that B is a regular language.
b. Let $C = \{1^k y | y \in \{0,1\}^*$ and y contains at most k $1's$, for every $k \geq 1\}$. Show that C isn't a regular language.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

Answer key

12.35.19 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 53 (Page No. 91) top

Let $\sum = \{0,1,+,=\}$ and $ADD = \{x = y + z | x,y,z$ are binary integers,and x is the sum of y and $z\}$. Show that ADD is not a regular.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

12.35.20 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 56 (Page No. 91) top

I f A is a set of natural numbers and k is a natural number greater than 1, let $B_k(A) = \{\text{w}|\text{ w is the representation in base k of some number in A}\}$. Here, we do not allow leading $0's$ in the representation of a number. For example, $B_2(\{3,5\}) = \{11,101\}$ and $B_3(\{3,5\}) = \{10,12\}$. Give an example of a set A for which $B_2(A)$ is regular but $B_3(A)$ is not regular. Prove that your example works.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

12.35.21 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 57 (Page No. 92) top

I f A is any language,let $A_{\frac{1}{2}-}$ be the set of all first halves of strings in A so that $A_{\frac{1}{2}-} = \{\text{x}|\text{for some y,}|\text{x}|=|\text{y}|\text{ and xy} \in \text{A}\}$. Show that if A is regular,then so is $A_{\frac{1}{2}-}$.

michael-sipser theory-of-computation regular-language proof descriptive

12.35.22 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 58 (Page No. 92) top

If A is any language,let $A_{\frac{1}{2}-\frac{1}{3}}$ be the set of all strings in A with their ,middle thirds removed so that $A_{\frac{1}{2}-\frac{1}{3}} = \{\text{xz}|\text{for some y,}|\text{x}|=|\text{y}|=|\text{z}|\text{ and xyz} \in \text{A}\}$. Show that if A is regular,then $A_{\frac{1}{2}-\frac{1}{3}}$ is not necessarily regular.

michael-sipser theory-of-computation regular-language proof descriptive

12.35.23 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 63 (Page No. 92) top

a. Let A be an infinite regular language. Prove that A can be split into two infinite disjoint regular subsets.
b. Let B and D be two languages. Write $B \subsetneq D$ if $B \subseteq D$ and D contains infinitely many strings that are not in B. Show that if B and D are two regular languages where $B \subsetneq D$, then we can find a regular language C where $B \subsetneq C \subsetneq D$.

michael-sipser theory-of-computation finite-automata regular-language descriptive

12.35.24 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 70 (Page No. 93) top

We define the avoids operation for languages A and B to be $\text{A avoids B} = \{\text{w}|\text{ w} \in \text{A and w doesn't contain any string in B as a substring}\}$. Prove that the class of regular languages is closed under the *avoids* operation.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

Answer key

12.35.25 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 71 (Page No. 93) top

Let $\sum = \{0, 1\}$

a. Let $A = \{0^k u 0^k | k \geq 1 \text{ and } u \in \sum^*\}$. Show that A is regular.
b. Let $B = \{0^k 1 u 0^k | k \geq 1 \text{ and } u \in \sum^*\}$.Show that B is not regular.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

Answer key

12.35.26 Regular Language: Michael Sipser Edition 3 Exercise 1 Question 72 (Page No. 93) top

Let M_1 and M_2 be DFA's that have k_1 and k_2 states, respectively, and then let $U = L(M_1) \cup L(M_2)$.

a. Show that if $U \neq \phi$ then U contains some string s, where $|s| < max(k1, k2)$.
b. Show that if $U \neq \sum^*$, then U excludes some string s, where $|s| < k1k2$.

michael-sipser theory-of-computation finite-automata regular-language proof descriptive

12.35.27 Regular Language: Michael Sipser Edition 3 Exercise 2 Question 44 (Page No. 158) top

If A and B are languages, define $A \diamond B = \{xy \mid x \in A \text{ and } y \in B \text{ and } \mid x \mid = \mid y \mid\}$. Show that if A and B are regular languages, then $A \diamond B$ is a CFL.

michael-sipser theory-of-computation regular-language proof

12.36 Relations (1) top

12.36.1 Relations: Michael Sipser Edition 3 Exercise 0 Question 7 (Page No. 26) top

For each part, give a relation that satisfies the condition.

a. Reflexive and symmetric but not transitive
b. Reflexive and transitive but not symmetric
c. Symmetric and transitive but not reflexive

michael-sipser theory-of-computation relations easy

12.37 Rice Theorem (3) top

12.37.1 Rice Theorem: Michael Sipser Edition 3 Exercise 5 Question 28 (Page No. 241) top

Rice's theorem. Let P be any nontrivial property of the language of a Turing machine. Prove that the problem of determining whether a given Turing machine's language has property P is undecidable. In more formal terms, let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial—it contains some, but not all, TM descriptions. Second, P is a property of the $TM's$ language—whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here, M_1 and M_2 are any TMs. Prove that P is an undecidable language.

michael-sipser theory-of-computation turing-machine decidability rice-theorem proof

12.37.2 Rice Theorem: Michael Sipser Edition 3 Exercise 5 Question 29 (Page No. 241) top

Rice's theorem. Let P be any nontrivial property of the language of a Turing machine. Prove that the problem of determining whether a given Turing machine's language has property P is undecidable. In more formal terms, let P be a language consisting of Turing machine descriptions where P fulfills two conditions. First, P is nontrivial—it contains some, but not all, TM descriptions. Second, P is a property of the $TM's$ language—whenever $L(M_1) = L(M_2)$, we have $\langle M_1 \rangle \in P$ iff $\langle M_2 \rangle \in P$. Here, M_1 and M_2 are any TMs. Prove that P is an undecidable language.

Show that both conditions are necessary for proving that P is undecidable.

michael-sipser theory-of-computation turing-machine decidability rice-theorem proof

12.37.3 Rice Theorem: Michael Sipser Edition 3 Exercise 5 Question 30 (Page No. 241) top

Use Rice's theorem, to prove the undecidability of each of the following languages.

a. $INFINITE_{TM} = \{\langle M \rangle \mid \text{M is a TM and L(M) is an infinite language}\}$.
b. $\{\langle M \rangle \mid \text{M is a TM and } 1011 \in L(M)\}$.
c. $ALL_{TM} = \{\langle M \rangle \mid \text{ M is a TM and } L(M) = \Sigma^*\}$.

michael-sipser theory-of-computation turing-machine decidability rice-theorem proof

12.38 Rotational Closure Of Language (1) top

12.38.1 Rotational Closure Of Language: Michael Sipser Edition 3 Exercise 1 Question 67 (Page No. 93) top

Let the rotational closure of language A be $RC(A) = \{yx|xy \in A\}$.

a. Show that for any language A, we have $RC(A) = RC(RC(A))$.
b. Show that the class of regular languages is closed under rotational closure.

michael-sipser theory-of-computation regular-language rotational-closure-of-language descriptive

12.39 Scarnes Cut (1) top

12.39.1 Scarnes Cut: Michael Sipser Edition 3 Exercise 1 Question 68 (Page No. 93) top

In the traditional method for cutting a deck of playing cards, the deck is arbitrarily split two parts, which are exchanged beforereassembling the deck. In a more complex cut, called Scarne's cut, the deck is broken into three parts and the middle part in placed first in the reassembly. We'll take Scarne's cut as the inspiration for an operation on languages. For a language A, let $\text{CUT(A)} = \{\text{yxz}|\text{ xyz} \in \text{A}\}$.

a. Exhibit a language B for which $\text{CUT(B)} \neq \text{CUT(CUT(B))}$.
b. Show that the class of regular languages is closed under CUT.

michael-sipser theory-of-computation regular-language scarnes-cut proof descriptive

12.40 Set Theory (3) top

12.40.1 Set Theory: Michael Sipser Edition 3 Exercise 0 Question 3 (Page No. 26) top

Let A be the set {x, y, z} and B be the set {x, y}.

a. Is A a subset of B?

b. Is B a subset of A?

c. What is A ∪ B?

d. What is A ∩ B?

e. What is A × B?

f. What is the power set of B?

michael-sipser theory-of-computation set-theory easy

Answer key

12.40.2 Set Theory: Michael Sipser Edition 3 Exercise 0 Question 4 (Page No. 26) top

If A has a elements and B has b elements, how many elements are in A × B? Explain your answer.

michael-sipser theory-of-computation set-theory easy

Answer key

12.40.3 Set Theory: Michael Sipser Edition 3 Exercise 0 Question 5 (Page No. 26) top

If C is a set with c elements, how many elements are in the power set of C? Explain your answer.

michael-sipser theory-of-computation set-theory easy

Answer key

12.41 Shuffle (1) top

12.41.1 Shuffle: Michael Sipser Edition 3 Exercise 2 Question 39 (Page No. 158) top

For the definition of the shuffle operation. For languages A and B, let the shuffle of A and B be the language $\{w|w = a_1b_1 \ldots a_kb_k,$ where $a_1 \cdots a_k \in A$ and $b_1 \cdots b_k \in B$, each $a_i, b_i \in \Sigma^*\}$.

Show that the class of context-free languages is not closed under shuffle.

michael-sipser theory-of-computation context-free-language shuffle

12.42 Simplification (1) top

12.42.1 Simplification: Peter Linz Edition 4 Exercise 6.1 Question 13 (Page No. 162) top

Consider the grammar G with Productions

$S \rightarrow A|B,$
$A \rightarrow \lambda,$
$B \rightarrow aBb,$
$B \rightarrow b.$

Construct a Grammar $\hat{G}$ by applying the algorithm in Theorem 6.3.

Theorem 6.3

Let G be any context-free grammar with λ not in $L(G)$. Then there exists an equivalent grammar $\widehat{G}$ having no λ-productions.

Proof: We first find the set V_N of all nullable variables of G, using the following steps.

1. For all productions $A \rightarrow \lambda$, put A into V_N.
2. Repeat the following step until no further variables are added to V_N.
   For all productions

$$B \rightarrow A_1A_2 \ldots A_n,$$

   where $A_1, A_2, \ldots, A_n$ are in V_N, put B into V_N.

Once the set V_N has been found, we are ready to construct $\widehat{P}$. To do so, we look at all productions in P of the form

$$A \rightarrow x_1x_2 \cdots x_m, m \geq 1,$$

where each $x_i \in V \cup T$. For each such production of P, we put into $\widehat{P}$ that production as well as all those generated by replacing nullable variables with λ in all possible combinations. For example, if x_i and x_j are both nullable, there will be one production in $\widehat{P}$ with x_i replaced with λ, one in which x_j is replaced with λ, and one in which both x_i and x_j are replaced with λ. There is one exception: If all x_i are nullable, the production $A \rightarrow \lambda$ is not put into $\widehat{P}$.

theory-of-computation simplification peter-linz peter-linz-edition4

12.43 State Diagram (7) top

12.43.1 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 10 (Page No. 85) top

Use the construction in the proof of Theorem 1.49 to give the state diagrams of $NFA's$ recognizing the star of the languages described in

a. {w| w contains at least three 1s}
b. {w| w contains at least two 0's and at most one 1}
c. The empty set

michael-sipser theory-of-computation finite-automata state-diagram

Answer key

12.43.2 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 3 (Page No. 83) top

The formal description of a $\text{DFA}\ \ \text{M}$ is $(q1, q2, q3, q4, q5\ , u, d, \delta, q3, q3)$, where δ is given by the following table. Give the state diagram of this machine.

	u	d
q_1	q_1	q_2
q_2	q_1	q_3
q_3	q_2	q_4
q_4	q_3	q_5
q_5	q_4	q_5

michael-sipser theory-of-computation finite-automata state-diagram descriptive

12.43.3 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 4 (Page No. 83) top

Each of the following languages is the intersection of two simpler languages. In each part, construct DFAs for the simpler languages, then combine them using the construction discussed in footnote $3\ (\text{page } 46)$ to give the state diagram of a DFA for the language given. In all parts, $\Sigma = \{a, b\}$.

a. $\{\text{w}|\text{ w has at least three a's and at least two b's}\}$
b. $\{\text{w}|\text{ w has exactly two a's and at least two b's}\}$
c. $\{\text{w}|\text{ w has an even number of a's and one or two b's}\}$
d. $\{\text{w}|\text{ w has an even number of a's and each a is followed by at least one b}\}$
e. $\{\text{w}|\text{ w starts with an a and has at most one b}\}$
f. $\{\text{w}|\text{ w has an odd number of a's and ends with a b}\}$
g. $\{\text{w}|\text{ w has even length and an odd number of a's}\}$

michael-sipser theory-of-computation finite-automata state-diagram descriptive

12.43.4 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 6 (Page No. 84) top

Give state diagrams of DFAs recognizing the following languages. In all parts, the alphabet is $\{0, 1\}$.

a. $\{\text{w}|\text{ w begins with a 1 and ends with a 0}\}$
b. $\{\text{w}|\text{ w contains at least three 1s}\}$
c. $\{\text{w}|\text{ w contains the substring 0101 (i.e., w} = \text{x0101y for some x and y)}\}$
d. $\{\text{w}|\text{ w has length at least 3 and its third symbol is a 0}\}$
e. $\{\text{w}|\text{ w starts with 0 and has odd length, or starts with 1 and has even length}\}$
f. $\{\text{w}|\text{ w doesn't contain the substring 110}\}$
g. $\{\text{w}|\text{ the length of w is at most 5}\}$
h. $\{\text{w}|\text{ w is any string except 11 and 111}\}$
i. $\{\text{w}|\text{ every odd position of w is a 1}\}$
j. $\{\text{w}|\text{ w contains at least two 0's and at most one 1}\}$
k. $\{\epsilon, 0\}$
l. $\{\text{w}|\text{ w contains an even number of 0's, or contains exactly two 1's}\}$
m. The empty set
n. All strings except the empty string

michael-sipser theory-of-computation finite-automata state-diagram descriptive

Answer key

12.43.5 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 7 (Page No. 84) top

Give state diagrams of NFA's with the specified number of states recognizing each of the following languages. In all parts, the alphabet is $\{0, 1\}$.

a. The language $\{\text{w}|\text{ w ends with 00}\}$ with three states
b. $\{\text{w}|\text{ w contains the substring 0101 (i.e., w} = \text{x0101y for some x and y)}\}$ with five states
c. $\{\text{w}|\text{ w contains an even number of 0s, or contains exactly two 1s}\}$ with six states
d. The language $\{0\}$ with two states
e. The language $0^*1^*0^+$ with three states
f. The language $1^*(001^+)^*$ with three states

g. The language $\{\epsilon\}$ with one state
h. The language 0^* with one state

michael-sipser theory-of-computation finite-automata state-diagram descriptive

Answer key

12.43.6 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 8 (Page No. 84) top

Use the construction in the proof of Theorem 1.45 to give the state diagrams of NFA's recognizing the union of the languages described in

a. $L_1 = \{\text{w}|\text{ w begins with a 1 and ends with a 0}\} \cup L_2 = \{\text{w}|\text{ w contains at least three 1s}\}$
b. $L_1 = \{\text{w}|\text{ w contains the substring 0101 (i.e., w} = \text{x0101y for some x and y)}\} \cup L_2 = \{\text{w}|\text{ w doesn't contain the s}$

michael-sipser theory-of-computation finite-automata state-diagram descriptive

Answer key

12.43.7 State Diagram: Michael Sipser Edition 3 Exercise 1 Question 9 (Page No. 85) top

Use the construction in the proof of Theorem 1.47 to give the state diagrams of NFA's recognizing the concatenation of the languages described in and input alphabet is $\Sigma = \{0, 1\}$

a. L1={w| the length of w is at most 5} and L2={w| every odd position of w is a 1}
b. L1={w| w contains at least three 1s} and L2=The empty set

michael-sipser theory-of-computation finite-automata state-diagram

12.44 Suffix Operation (1) top

12.44.1 Suffix Operation: Michael Sipser Edition 3 Exercise 2 Question 25 (Page No. 157) top

For any language A, let $SUFFIX(A) = \{v \mid uv \in A \text{ for some string u}\}$. Show that the class of context-free languages is closed under the SUFFIX operation.

michael-sipser theory-of-computation context-free-language suffix-operation proof

12.45 Synchronizable Dfa (1) top

12.45.1 Synchronizable Dfa: Michael Sipser Edition 3 Exercise 1 Question 59 (Page No. 92) top

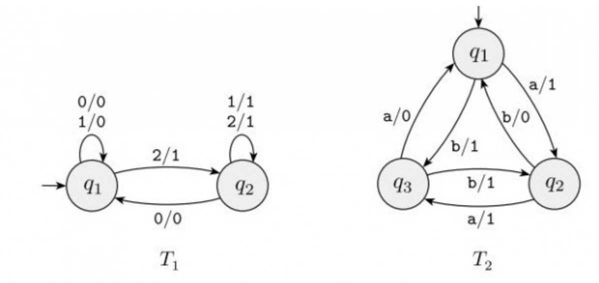
Let $M = (Q, \Sigma, \delta, q_0, F)$ be a DFA and let h be a state of M called its "home". A synchronizing sequence for M and h is a string $s \in \sum^*$ where $\delta(q, s) = h$ for every $q \in Q$. (Here we have extended δ to strings, so that $\delta(q, s)$ equals the state where M ends up when M starts at state q and reads input s.) Say that M is synchronizable if it has a synchronizing sequence for some state h. Prove that if M is a k-state synchronizable DFA, then it has a synchronizing sequence of length at most k^3.Can you improve upon this bound?

michael-sipser theory-of-computation finite-automata synchronizable-dfa descriptive

12.46 Transducer (1) top

12.46.1 Transducer: Michael Sipser Edition 3 Exercise 1 Question 24 (Page No. 87) top

A finite state transducer (FST) is a type of deterministic finite automaton whose output is a string and not just accept or reject. The following are state diagrams of finite state transducers $T1$ and $T2$.



Each transition of an FST is labeled with two symbols, one designating the input symbol for that transition and the other designating the output symbol. The two symbols are written with a slash, $/$, separating them. In T_1, the transition from q_1 to q_2 has input symbol 2 and output symbol 1. Some transitions may have multiple input-output pairs, such as the transition in T_1 from q_1 to itself. When an FST computes on an input string w, it takes the input symbols $w_1 \cdots w_n$ one by one and, starting at the start state, follows the transitions by matching the input labels with the sequence of symbols $w_1 \cdots w_n = w$. Every time it goes along a transition, it outputs the corresponding output symbol. For example, on input 2212011, machine T_1 enters the sequence of states $q_1, q_2, q_2, q_2, q_2, q_1, q_1, q_1$ and produces output 1111000. On input $abbb, T_2$
outputs 1011.
Give the sequence of states entered and the output produced in each of the following parts.

a. T_1 on input 011
b. T_1 on input 211
c. T_1 on input 121
d.T_1 on input 0202
e. T_2 on input b
f. T_2 on input bbab
g. T_2 on input bbbbbb
h. T_2 on input ϵ

michael-sipser theory-of-computation transducer finite-automata

Answer key

12.47 Turing Machine (103) top

12.47.1 Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 1 (Page No. 187) top

This exercise concerns $TM\ M_2$, whose description and state diagram appear in Example 3.7. In each of the parts, give the sequence of configurations that M_2 enters when started on the indicated input string.

a. 0 b. 00 c. 000 d. 000000

michael-sipser theory-of-computation turing-machine descriptive

12.47.2 Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 10 (Page No. 188) top

Say that a write-once Turing machine is a single-tape TM that can alter each tape square at most once (including the input portion of the tape). Show that this variant Turing machine model is equivalent to the ordinary Turing machine model. (Hint: As a first step, consider the case whereby the Turing machine may alter each tape square at most twice. Use lots of tape.)

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12.47.3 Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 2 (Page No. 187) top

This exercise concerns $TM\ M_1$, whose description and state diagram appear in Example 3.9. In each of the parts, give the sequence of configurations that M_1 enters when started on the indicated input string.

a. 11 b. 1#1 c. 1##1 d. 10#11 e. 10#10

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12.47.4 Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 5 (Page No. 188) top

Examine the formal definition of a Turing machine to answer the following questions, and explain your reasoning.

a. Can a Turing machine ever write the blank symbol $\sqcup$ on its tape?
b. Can the tape alphabet Γ be the same as the input alphabet Σ?

c. Can a Turing machine's head ever be in the same location in two successive steps?
d. Can a Turing machine contain just a single state?

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12.47.5 Turing Machine: Michael Sipser Edition 3 Exercise 3 Question 8 (Page No. 188) top

Give implementation-level descriptions of Turing machines that decide the following languages over the alphabet $\{0, 1\}$.

a. $\{w \mid w \text{contains an equal number of 0s and 1s}\}$
b. $\{w \mid w \text{contains twice as many 0s as 1s}\}$
c. $\{w \mid w \text{does not contain twice as many 0s as 1s}\}$

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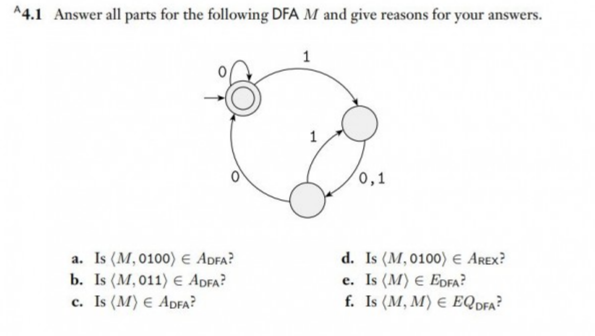
12.47.6 Turing Machine: Michael Sipser Edition 3 Exercise 4 Question 1 (Page No. 210) top


[A]4.1 Answer all parts for the following DFA M and give reasons for your answers.

a. Is $\langle M, 0100\rangle \in A_{DFA}$?
b. Is $\langle M, 011\rangle \in A_{DFA}$?
c. Is $\langle M\rangle \in A_{DFA}$?
d. Is $\langle M, 0100\rangle \in A_{REX}$?
e. Is $\langle M\rangle \in E_{DFA}$?
f. Is $\langle M, M\rangle \in EQ_{DFA}$?

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12.47.7 Turing Machine: Michael Sipser Edition 3 Exercise 4 Question 6 (Page No. 211) top

Let X be the set $\{1, 2, 3, 4, 5\}$ and Y be the set $\{6, 7, 8, 9, 10\}$. We describe the functions $f : X \to Y$ and $g : X \to Y$ in the following tables. Answer each part and give a reason for each negative answer.

n	$f(n)$
1	6
2	7
3	6
4	7
5	6

n	$g(n)$
1	10
2	9
3	8
4	7
5	6

a. Is f one-to-one?
b. Is f onto?
c. Is f a correspondence?
d. Is g one-to-one?
e. Is g onto?
f. Is g a correspondence?

michael-sipser theory-of-computation turing-machine proof

12.47.8 Turing Machine: Peter Linz Edition 4 Exercise 12.1 Question 14 (Page No. 306) top

Consider the set of all n-state Turing machines with tape alphabet $\Gamma = \{0,1, B\}$. Give an expression for m(n), the number of distinct Turing machines with this Γ.

turing-machine theory-of-computation peter-linz peter-linz-edition4

12.47.9 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 1 (Page No. 258) top

Give a formal definition of a Turing machine with a semi-infinite tape. Then prove that the class of Turing machines with semi-infinite tape. Then prove the class of Turing machines with semi-infinite tape is equivalent to the class of standard Turing machines.

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12.47.10 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 10 (Page No. 259) top

Consider a version of the standard Turing machine in which transitions can depend not only on the cell directly under the read-write head but also on the cells to the immediate right and left. Make a formal definition of such a machine, then sketch its simulation by a standard Turing machine.

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12.47.11 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 11 (Page No. 259) top

Consider a Turing machine with a different decision process in which transitions are made if the current tape symbol is not one of the specified set. For example,

$$\delta(q_i, \{a, b\}) = (q_j, c, R)$$

will allow the indicated move if the current tape symbol is neither a nor b. Formalize this concept and show that this modification is equivalent to a standard Turing machine.

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12.47.12 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 2 (Page No. 258) top

Give a formal definition of an off-line Turing machine.

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12.47.13 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 3 (Page No. 259) top

Give convincing arguments that any language accepted by an off-line Turing machine is also accepted by some standard machine.

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12.47.14 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 4 (Page No. 259) top

Consider a Turing machine that, on any particular move, can either change the top symbol or move the read-write head, but not both.

(a) Give a formal definition of such a machine.

(b) Show that the class of such machines is equivalent to the class of standard Turing machines.

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12.47.15 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 5 (Page No. 259) top

Consider a model of a Turing machine in which each move permits the read-write head to travel more than one cell to the left or right, the distance and direction of travel being one of the arguments of δ. Give a precise definition of such an automaton and sketch a simulation of it by a standard Turing machine.

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12.47.16 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 6 (Page No. 259) top

A nonerasing Turing machine is one that cannot change a nonblank symbol to a blank. This can be achieved by the restriction that if

$$\delta(q_i, a) = (q_j, \square, L \; or \; R),$$

then a must be $\square$. Show that no generality is lost by making much a restriction.

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12.47.17 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 7 (Page No. 259) top

Consider a Turing machine that cannot write blanks; that is, for all $\delta(q_i, a) = (q_j, b, L\ )$, b must be in $\Gamma - \{\square\}$. Show how such a machine can simulate a standard Turing machine.

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12.47.18 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 8 (Page No. 259) top

Suppose we make the requirement that a Turing machine can halt only in a final state, that is, we ask that $\delta(q, a)$ be defined for all pairs (q, a) with $a \in \Gamma$ and $q \notin F$. Does this restrict the power of the Turing machine?

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12.47.19 Turing Machine: Peter Linz Edition 5 Exercise 10.1 Question 9 (Page No. 259) top

Suppose we make the restriction that a Turing must always write a symbol different from the one it reads, that is, if

$$\delta(q_i, a) = (q_j, b, L \; or \; R),$$

then a and b must be different. Does this limitation reduce the power of the automaton ?

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12.47.20 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 1 (Page No. 264) top

Define what one might call a multitape off-line Turing machine and describe how it can be simulated by a standard Turing machine.

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12.47.21 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 10 (Page No. 264) top

Write out a detailed program for the computation in considering the language $\{a^n b^n\}$. We described a laborious method by which this language can be accepted by a Turing machine with one tape. Using a two-tape machine makes the job much easier. Assume that an initial string $a^n b^m$ is written on tape 1 at the beginning of the computation. We then read all the $a's$, we match the $b's$ on tape 1 against the copied $a's$ on tape 2. This way, we can determine whether there are an equal number of $a's$ and $b's$ without repeated back-and-forth movement of the read-write head.

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12.47.22 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 2 (Page No. 264) top

A multihead Turing machine can be visualized as a Turing machine with a single tape and single control unit but with multiple, independent read-write heads. Give a formal definition of a multihead Turing machine, and then show how much a machine can be simulated with a standard Turing machine.

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12.47.23 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 3 (Page No. 264) top

Give a formal definition of a multihead-multitape Turing machine. Then show how such a machine can be simulated by a standard Turing machine.

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12.47.24 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 4 (Page No. 264) top

Give a formal definition of a Turing machine with a single tape but multiple control units, each with a single read-write head. Show how such a machine can be simulated with a multitape machine.

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12.47.25 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 5 (Page No. 264) top

A queue automaton is an automaton in which the temporary storage is a queue. Assume that such a machine is an online machine, that is, it has no input file, with the string to be processed placed in the queue prior to the start of the computation. Give a formal definition of such an automaton, then investigate its power in relation to Turing machines.

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12.47.26 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 6,7 (Page No. 264) top

Exercise 6: Show that for every Turing machine there exists an equivalent standard Turing machine with no more than six states.

Exercise 7: Reduce the number of required states in Exercise 6 above as far as you can (Hint: The smallest possible number is three)

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12.47.27 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 8 (Page No. 264) top

A counter is a stack with an alphabet of exactly two symbols a stack start symbol and a counter symbol. Only the counter symbol can be put on the stack or removed from it. A counter automaton is a deterministic automaton with one or more counters as storage. Show that any Turing machines can be simulated using a counter automaton with four counters.

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12.47.28 Turing Machine: Peter Linz Edition 5 Exercise 10.2 Question 9 (Page No. 264) top

Show that every computation that can be done by a standard Turing machine can be done a multitape machine with a stay option and at most two states.

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12.47.29 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 1 (Page No. 267) top

Discuss in detail the simulation of a nondeterministic Turing machine by a deterministic one. Turing machine by a deterministic one. Indicate explicitly how new machines are created, how active machines are identified, and how machines that halt are removed from further consideration.

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12.47.30 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 2 (Page No. 267) top

Show how a two-dimensional nondeterministic Turing machine can be simulated by a deterministic machine.

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12.47.31 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 3 (Page No. 268) top

Write a program for a nondeterministic Turing machine that accepts the language.

$$L = \{ww : w \in \{a, b\}^+\}$$

Contrast this with a deterministic solution.

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12.47.32 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 4 (Page No. 268) top

Outline how one would write a program for a nondeterministic Turing machine to accept the language

$$L = \{ww^R w : w \in \{a, b\}^+\}.$$

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12.47.33 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 5 (Page No. 268) top

Write a simple program for a nondeterministic Turing machine that accepts the language

$$L = \left\{xww^R y : x, y, w \in \{a, b\}^+, |x| \geq |y|\right\}.$$

How would you solve this problem deterministically?

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12.47.34 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 6 (Page No. 268) top

Design a nondeterministic Turing machine that accepts the language.

$$L = \{a^n : \text{n is not a prime number}\}.$$

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12.47.35 Turing Machine: Peter Linz Edition 5 Exercise 10.3 Question 7 (Page No. 268) top

Definition: A nondeterministic pushdown acceptor (npda) is defined by the septuple

$$M = (Q, \Sigma, \Gamma, \delta, q_0, z, F)$$

where

Q is a finite set of internal states of the control unit,

Σ is the input alphabet,

Γ is a finite set of symbols called the stack alphabet,

$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \to$ set of finite subsets of $Q \times \Gamma^*$ is the transition function,

$q_0 \in Q$ is the initial state of the control unit,

$z \in \Gamma$ is the stack start symbol,

$F \subseteq Q$ is the set of final states.

A two-stack automaton is a nondeterministic pushdown automaton with two independent stacks. To define such an automaton, we modify Definition so that

$$\delta : Q \times (\Sigma \cup \{\lambda\}) \times \Gamma \times \Gamma \to \text{finite subsets of } Q \times \Gamma^* \times \Gamma^*.$$

A move depends on the tops of the two stacks and results in new values being pushed on these two stacks. Show that the class of two-stack automata is equivalent to the class of Turing machines.

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12.47.36 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 1 (Page No. 272) top

Sketch an algorithm that examines a string in $\{0, 1\}^+$ to determine whether or not it represents an encoded Turing machine.

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12.47.37 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 2 (Page No. 272) top

Give a complete encoding, using the suggested method, for the Turing machine with

$$\delta(q_1, a_1) = (q_1, a_1, R),$$

$$\delta(q_1, a_2) = (q_3, a_1, L),$$

$$\delta(q_3, a_1) = (q_2, a_2, L),$$

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12.47.38 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 3 (Page No. 272) top

Sketch a Turing machine program that enumerates the set $\{0,1\}^+$ in proper order.

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12.47.39 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 4 (Page No. 272) top

Exercise 3: Sketch a Turing machine program that enumerates the set {0,1}+{0,1}+ in proper order.

Exercise 4: What is the index of 0^i1^j in Exercise 3?

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12.47.40 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 5 (Page No. 272) top

Design a Turing machine that enumerates the following set in proper order

$$L = \{a^n b^n : n \geq 1\}.$$

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12.47.41 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 7 (Page No. 273) top

Show that the set of all triplets, (i, j, k) with i, j, k positive integers, is countable,

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12.47.42 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 8 (Page No. 273) top

Suppose that S_1 and S_2 are countable sets. Show that then $S_1 \cup S_2$ and $S_1 \times S_2$ are also countable.

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12.47.43 Turing Machine: Peter Linz Edition 5 Exercise 10.4 Question 9 (Page No. 273) top

Show that the Cartesian product of a finite number of countable sets is countable.

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12.47.44 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 1 (Page No. 275) top

Example: Find a linear bounded automaton that accepts the language

$$L = \{a^{n!} : n \geq 0\}$$

Give details for the solution of Example.

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12.47.45 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 2 (Page No. 275) top

Example: Find a linear bounded automaton that accepts the language

$$L = \{a^{n!} : n \geq 0\}$$

Find a solution for Example that does not require track as scratch space.

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12.47.46 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 3 (Page No. 275) top

Consider an offline Turing machine in which the input can be read only once, moving left to right, not rewritten. On its work tape, it can use at most n extra cells for work space, where n is fixed for all inputs. Show that such a machine is equivalent to a finite automaton.

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12.47.47 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(a) (Page No. 275) top

Find linear bounded automata for the following language.

$$L = \{a^n : n = m^2, m \geq 1\}$$

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12.47.48 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(b) (Page No. 275) top

Find linear bounded automata for the following language.

$$L = \{a^n : \text{n is a prime number}\}.$$

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12.47.49 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(c) (Page No. 275) top

Find linear bounded automata for the following language.

$$L = \{a^n : \text{n is not a prime number}\}.$$

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12.47.50 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(d) (Page No. 275) top

Find linear bounded automata for the following language.

$$L = \left\{ww : w \in \{a,b\}^+\right\}.$$

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12.47.51 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(e) (Page No. 276) top

Find linear bounded automata for the following languages.

$$L = \left\{w^n : w \in \{a,b\}^+, n \geq 2\right\}.$$

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12.47.52 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 4(f) (Page No. 276) top

Find linear bounded automata for the following languages.

$$L = \left\{www^R : w \in \{a,b\}^+\right\}.$$

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12.47.53 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 5 (Page No. 276) top

Example : Find a linear bounded automaton that accepts the language

$$L = \{a^{n!} : n \geq 0\}.$$

Find a lba for the complement of the language in Example, assuming that $\Sigma = \{a, b\}$.

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12.47.54 Turing Machine: Peter Linz Edition 5 Exercise 10.5 Question 6,7 (Page No. 276) top

Exercise : 6 Show that for every context-free language there exists an accepting pda, such that the number of symbols in the stack never exceeds the length of the input string by more than one.

Exercise : 7 Use the observation in the above exercise to show that any context-free language not containing λ is accepted by some linear bounded automaton.

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12.47.55 Turing Machine: Peter Linz Edition 5 Exercise 11.1 Question 1 (Page No. 284) top

Prove that the set of all real numbers is not countable.

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Answer key

12.47.56 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 1 (Page No. 238) top

Write a Turing machine simulator in some higher-level programming language. Such a simulator should accept as input the description of any Turing machine, together with an initial configuration, and should produce as output the result of the computation.

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12.47.57 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 10 (Page No. 239) top

Design a Turing machine that finds the middle of a string of even length. Specifically, if $w = a_1a_2 \dots a_na_{n+1} \dots a_{2n}$, with $a_i \in \Sigma$, the Turing machine should produce $\widehat{w} = a_1a_2 \dots a_nca_{n+1} \dots a_{2n}$, where $c \in \Gamma - \Sigma$.

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12.47.58 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(a) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary.

$$f(x) = 3x$$

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12.47.59 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(b) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary.

$$\begin{aligned} f(x, y) &= x - y, && x > y, \\ &= 0, && x \leq y. \end{aligned}$$

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12.47.60 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(c) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary

$$f(x, y) = 2x + 3y.$$

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12.47.61 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(d) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary

$$\begin{aligned} f(x) &= \tfrac{x}{2}, && \text{if } x \text{ is even}, \\ &= \tfrac{x+1}{2}, && \text{if } x \text{ is odd}. \end{aligned}$$

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12.47.62 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(e) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary.

$$f(x) = x \bmod 5.$$

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12.47.63 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 11(f) (Page No. 239) top

Design Turing machines to compute the following functions for x and y positive integers represented in unary.

$f(x) = \lfloor \frac{x}{2} \rfloor$, where $\lfloor \frac{x}{2} \rfloor$, denotes the largest integer less than or equal to $\frac{x}{2}$.

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12.47.64 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 12 (Page No. 239) top

Design a Turing machine $\Gamma = \{0, 1, \square\}$ that, when started on any cell containing a blank or a 1, will halt if and only if its tape has a 0 somewhere it.

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12.47.65 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 13 (Page No. 239) top

$\text{Example}:$ Design a Turing machine that accepts

$$L = \{a^n b^n c^n : n \geq 1\}.$$

Write out a complete solution for Example.

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12.47.66 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 14 (Page No. 239 top

$\text{Example}:$ Design a Turing machine that copies strings of $1's$. More precisely, find a machine that performs the computation

$$q_0 w \vdash^* q_f ww,$$

for any $w \in \{1\}^+$.

Give the sequence of instantaneous descriptions that the Turing machine in Example goes through when presented with the input 111. What happens when this machine is started with 110 on its tape?

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12.47.67 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 15 (Page No. 239) top

$\text{Example}:$ Design a Turing machine that copies strings of $1's$. More precisely, find a machine that performs the computation

$$q_0 w \vdash^* q_f ww,$$

for any $w \in \{1\}^+$.

Give convincing arguments that the Turing machine in Example does in fact carry out the indicated computation.

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12.47.68 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 16 (Page No. 239) top

$\text{Example}:$ Let x and y be two positive integers represented in unary notation. Construct a Turing machine that will halt in a final state q_y if $x \geq y$, and that will halt in a nonfinal state q_n if $x < y$. More specifically, the machine is to perform the computation

$$q_0 w(x) 0 w(y) \vdash^* q_y w(x) 0 w(y) \quad \text{if } x \geq y,$$

$$q_0 w(x) 0 w(y) \vdash^* q_n w(x) 0 w(y) \quad \text{if } x < y,$$

Complete all the details in Example

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12.47.69 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 17 (Page No. 239) top

Example : Given two positive integers x and y, design a Turing machine that computes $x + y$.

Suppose that in Example we had decided to represent x and y in binary. Write a Turing machine program for doing the indicated computation in this representation

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12.47.70 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 18 (Page No. 240) top

Example : Given two positive integers x and y, design a Turing machine that computes $x + y$.

Sketch how Example could be solved if x and y were represented in decimal.

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12.47.71 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 19 (Page No. 240) top

You may have noticed that all the examples in these sections had only one final state. Is it generally true that for any Turing machine, there exists another one with only one final state that accepts the same language?

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12.47.72 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 2 (Page No. 238) top

Design a Turing machine with no more than three states that accept the language $L(a(a+b)^*)$. Assume that $\Sigma = \{a, b\}$. Is it possible to do this with a two-state machine ?

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12.47.73 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 3 (Page No. 238) top

Example : For $\Sigma = \{a, b\}$ design a Turing machine that accepts

$$L = \{a^n b^n : n \geq 1\}.$$

Determine what the Turing machine in Example does when presented with the inputs aba and $aaabbbb$.

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12.47.74 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 4 (Page No. 238) top

Example : For $\Sigma = \{a, b\}$ design a Turing machine that accepts

$$L = \{a^n b^n : n \geq 1\}.$$

Is there any input for which the Turing machine in Example goes into an infinite loop?

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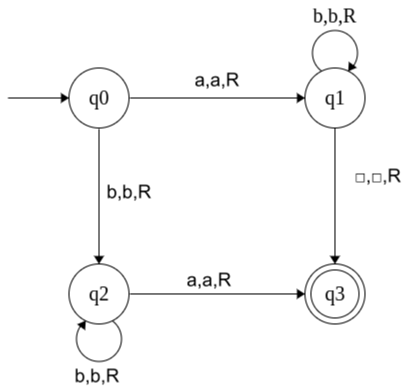
12.47.75 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 5 (Page No. 238) top

What language is accepted by the Turing machine whose transition graph is in the figure below ?



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Answer key

12.47.76 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 6 (Page No. 238) top

Example : Design a Turing machine that copies strings of $1's$. More precisely, find a machine that performs the computation

$$q_0 q \vdash^* q_f ww,$$

for any $w \in \{1\}^+$.

What happens in Example if the string w contains any symbol other than 1?

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12.47.77 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(a) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$.

$$L = L(aba^*b).$$

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12.47.78 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(b) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$

$$L = \{w : |w| \text{ is even }\}.$$

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12.47.79 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(c) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$.

$$L = \{w : |w| \text{ is a multiple of } 3\}.$$

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12.47.80 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(d) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$.

$$L = \{a^n b^m : n \geq 1, n \neq m\}.$$

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12.47.81 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(e) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$

$$L = \{w : n_a(w) = n_b(w)\}.$$

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12.47.82 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(f) (Page No. 238) top

Construct Turing machines that will accept the following languages on $\{a, b\}$.

$$L = \{a^n b^m a^{n+m} : n \geq 0, m \geq 1\}.$$

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12.47.83 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(g) (Page No. 239) top

Construct Turing machines that will accept the following languages on $\{a, b\}$.

$$L = \{a^n b^n a^n b^n : n \geq 0\}.$$

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12.47.84 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 7(h) (Page No. 239) top

Construct Turing machines that will accept the following languages on $\{a, b\}$

$$L = \{a^n b^{2n} : n \geq 0\}.$$

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12.47.85 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 8 (Page No. 239) top

Design a Turing machine that accepts the language.

$$L = \left\{ww : w \in \{a, b\}^+\right\}.$$

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12.47.86 Turing Machine: Peter Linz Edition 5 Exercise 9.1 Question 9 (Page No. 239) top

Construct a Turing machine to compute the function

$$f(w) = w^R,$$

where $w \in \{0, 1\}^+$.

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12.47.87 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 1 (Page No. 244) top

Example : Design a Turing machine that multiples two positive integers in unary notation.

Write out the complete solution to Example.

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12.47.88 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 2 (Page No. 244) top

Establish a convention for representing positive and negative integers in unary notation. With your convention, sketch the construction of a subtracter for computing $x - y$.

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12.47.89 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 3(a) (Page No. 244) top

Using adders, subtracters, comparers, copies or multipliers, draw block diagrams for Turing machines that compute the functions for all positive integers n

$$f(n) = n(n+1),$$

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12.47.90 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 3(b) (Page No. 244) top

Using adders, subtracters, comparers, copies or multipliers, draw block diagrams for Turing machines that compute the functions for all positive integers n

$$f(n) = n^5,$$

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12.47.91 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 3(c) (Page No. 244) top

Using adders, subtracters, comparers, copies or multipliers, draw block diagrams for Turing machines that compute the functions for all positive integers n

$$f(n) = 2^n,$$

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12.47.92 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 3(d) (Page No. 244) top

Using adders, subtracters, comparers, copies or multipliers, draw block diagrams for Turing machines that compute the functions for all positive integers n

$$f(n) = n!,$$

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12.47.93 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 3(e) (Page No. 244) top

Using adders, subtracters, comparers, copiers or multipliers, draw block diagrams for Turing machines that compute the functions for all positive integers n

$$f(n) = n^{n!},$$

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12.47.94 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 4 (Page No. 244) top

Use a block diagram to sketch the implementation of a function f defined for all $w_1, w_2, w_3 \in \{1\}^+$ by

$$f(w_1, w_2, w_3) = i,$$

where i is such that $|w_i| = \max(|w_1|, |w_2|, |w_3|)$ if no two $w's$ have the same length, and $i = 0$ otherwise.

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12.47.95 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 5(a) (Page No. 244) top

Provide a "high-level" description for Turing machines that accept the following languages on $\{a, b\}$. For each problem, define a set of appropriate macroinstructions that you feel are reasonably easy to implement. Then use them for the solution.

$$L = \{ww^R\}.$$

peter-linz peter-linz-edition5 theory-of-computation turing-machine

12.47.96 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 5(b) (Page No. 245) top

Provide a "high-level" description for Turing machines that accept the following languages on $\{a, b\}$. For each problem, define a set of appropriate macroinstructions that you feel are reasonably easy to implement. Then use them for the solution.

$$L = \{w_1 w_2 : w_1 \neq w_2 : |w_1| = |w_2|\}.$$

peter-linz peter-linz-edition5 theory-of-computation turing-machine

12.47.97 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 5(c) (Page No. 245) top

Provide a "high-level" description for Turing machines that accept the following languages on $\{a, b\}$. For each problem, define a set of appropriate macroinstructions that you feel are reasonably easy to implement. Then use them for the solution.

$$\text{The complement of the language in } L = \{ww^R\}.$$

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12.47.98 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 5(d) (Page No. 245) top

Provide a "high-level" description for Turing machines that accept the following languages on $\{a, b\}$. For each problem, define a set of appropriate macroinstructions that you feel are reasonably easy to implement. Then use them for the solution.

$$L = \{a^n b^m : m = n^2, n \geq 1\}.$$

peter-linz peter-linz-edition5 theory-of-computation turing-machine

12.47.99 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 5(e) (Page No. 245) top

Provide a "high-level" description for Turing machines that accept the following languages on $\{a, b\}$. For each problem, define a set of appropriate macroinstructions that you feel are reasonably easy to implement. Then use them for the solution.

$$L = \{a^n : n \text{ is a prime number}\}.$$

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12.47.100 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 6 (Page No. 245) top

Suggest a method for representing rational numbers on a Turing machine, then sketch a method for adding and subtracting such numbers.

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12.47.101 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 7 (Page No. 245) top

Sketch the construction of a Turing machine that can perform the addition and multiplication of positive integers x and y given in the usual decimal notation.

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12.47.102 Turing Machine: Peter Linz Edition 5 Exercise 9.2 Question 8,9,10 (Page No. 245) top

Exercise 8 : Give an implementation of the macroinstruction

$$\text{searchright}(a, q_i, q_j),$$

which indicates that the machine is to search its tape to the right of the current position for the first occurrence of the symbol a. If an a is encountered before a blank, the machine is to go into state q_i, otherwise it is to go into state q_j.

Exercise 9 : Use the macroinstruction in the previous exercise to design a Turing machine on $\Sigma = \{a, b\}$ that accepts the language $L(ab^*ab^*a)$.

Exercise 10 : Use the macroinstructions searchright in Exercise 8 to create a Turing machine program that replaces the symbol immediately to the left of the leftmost a by a blank. If the input contains no a, replace the rightmost nonblank symbol by a b.

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12.47.103 Turing Machine: Peter Linz Edition 5 Exercise 9.3 Question 1 (Page No. 248) top

Consider the set of machine language instructions for a computer of your choice. Sketch how the various instructions in this set could be carried out by a Turing machine.

peter-linz peter-linz-edition5 theory-of-computation turing-machine difficult

Answer Keys

12.0.1	N/A	12.0.2	Q-Q	12.0.3	N/A	12.0.4	N/A	12.0.5	N/A
12.0.6	N/A	12.0.7	Q-Q	12.0.8	N/A	12.0.9	Q-Q	12.0.10	Q-Q
12.1.1	Q-Q	12.2.1	N/A	12.2.2	N/A	12.2.3	N/A	12.2.4	Q-Q
12.2.5	Q-Q	12.2.6	Q-Q	12.2.7	Q-Q	12.2.8	Q-Q	12.2.9	Q-Q
12.2.10	Q-Q	12.2.11	Q-Q	12.2.12	Q-Q	12.2.13	Q-Q	12.2.14	Q-Q
12.2.15	Q-Q	12.2.16	Q-Q	12.2.17	Q-Q	12.2.18	Q-Q	12.2.19	Q-Q
12.2.20	Q-Q	12.2.21	Q-Q	12.2.22	Q-Q	12.2.23	Q-Q	12.2.24	Q-Q
12.2.25	Q-Q	12.2.26	Q-Q	12.2.27	Q-Q	12.2.28	Q-Q	12.2.29	Q-Q
12.2.30	Q-Q	12.2.31	Q-Q	12.3.1	N/A	12.4.1	Q-Q	12.4.2	N/A
12.4.3	N/A	12.5.1	Q-Q	12.5.2	Q-Q	12.5.3	Q-Q	12.5.4	Q-Q
12.5.5	Q-Q	12.5.6	Q-Q	12.5.7	N/A	12.5.8	N/A	12.5.9	N/A
12.5.10	N/A	12.5.11	N/A	12.5.12	N/A	12.5.13	N/A	12.5.14	Q-Q
12.5.15	Q-Q	12.5.16	N/A	12.5.17	N/A	12.5.18	N/A	12.5.19	N/A
12.5.20	N/A	12.5.21	N/A	12.5.22	N/A	12.5.23	N/A	12.5.24	Q-Q
12.5.25	Q-Q	12.5.26	Q-Q	12.5.27	Q-Q	12.5.28	Q-Q	12.5.29	Q-Q
12.5.30	Q-Q	12.5.31	Q-Q	12.5.32	Q-Q	12.5.33	Q-Q	12.5.34	Q-Q
12.5.35	Q-Q	12.5.36	Q-Q	12.5.37	Q-Q	12.5.38	Q-Q	12.5.39	Q-Q
12.5.40	Q-Q	12.5.41	Q-Q	12.5.42	Q-Q	12.5.43	Q-Q	12.5.44	Q-Q
12.5.45	Q-Q	12.5.46	Q-Q	12.5.47	Q-Q	12.5.48	Q-Q	12.5.49	Q-Q
12.5.50	Q-Q	12.5.51	Q-Q	12.5.52	Q-Q	12.5.53	Q-Q	12.5.54	Q-Q
12.5.55	Q-Q	12.5.56	Q-Q	12.5.57	Q-Q	12.5.58	Q-Q	12.5.59	Q-Q
12.5.60	Q-Q	12.5.61	Q-Q	12.5.62	Q-Q	12.5.63	Q-Q	12.5.64	Q-Q
12.5.65	Q-Q	12.5.66	Q-Q	12.5.67	Q-Q	12.5.68	Q-Q	12.5.69	Q-Q
12.5.70	Q-Q	12.5.71	Q-Q	12.5.72	Q-Q	12.5.73	Q-Q	12.5.74	Q-Q
12.5.75	Q-Q	12.5.76	Q-Q	12.5.77	Q-Q	12.5.78	Q-Q	12.5.79	Q-Q
12.5.80	Q-Q	12.5.81	Q-Q	12.5.82	Q-Q	12.5.83	Q-Q	12.5.84	Q-Q
12.5.85	Q-Q	12.5.86	Q-Q	12.5.87	Q-Q	12.5.88	Q-Q	12.5.89	Q-Q
12.5.90	Q-Q	12.5.91	Q-Q	12.5.92	Q-Q	12.5.93	Q-Q	12.5.94	Q-Q
12.5.95	Q-Q	12.5.96	Q-Q	12.5.97	Q-Q	12.5.98	Q-Q	12.5.99	Q-Q
12.5.100	Q-Q	12.6.1	N/A	12.6.2	Q-Q	12.6.3	Q-Q	12.6.4	Q-Q
12.6.5	Q-Q	12.6.6	Q-Q	12.6.7	N/A	12.6.8	N/A	12.6.9	Q-Q
12.6.10	N/A	12.6.11	Q-Q	12.6.12	Q-Q	12.6.13	Q-Q	12.6.14	N/A

12.6.15	N/A	12.6.16	N/A	12.6.17	N/A	12.6.18	N/A	12.6.19	N/A
12.6.20	N/A	12.6.21	Q-Q	12.6.22	Q-Q	12.6.23	Q-Q	12.6.24	Q-Q
12.6.25	Q-Q	12.6.26	Q-Q	12.6.27	Q-Q	12.6.28	Q-Q	12.6.29	Q-Q
12.6.30	Q-Q	12.6.31	Q-Q	12.6.32	Q-Q	12.6.33	Q-Q	12.6.34	Q-Q
12.6.35	Q-Q	12.6.36	Q-Q	12.7.1	N/A	12.7.2	N/A	12.8.1	Q-Q
12.8.2	N/A	12.8.3	N/A	12.8.4	N/A	12.8.5	N/A	12.8.6	N/A
12.8.7	N/A	12.8.8	N/A	12.8.9	N/A	12.8.10	N/A	12.8.11	N/A
12.8.12	N/A	12.8.13	N/A	12.8.14	N/A	12.8.15	N/A	12.8.16	N/A
12.8.17	N/A	12.8.18	N/A	12.8.19	N/A	12.8.20	N/A	12.8.21	N/A
12.8.22	N/A	12.8.23	N/A	12.8.24	N/A	12.8.25	N/A	12.8.26	N/A
12.8.27	N/A	12.8.28	N/A	12.8.29	N/A	12.8.30	N/A	12.8.31	N/A
12.8.32	Q-Q	12.8.33	N/A	12.8.34	Q-Q	12.8.35	N/A	12.8.36	N/A
12.8.37	N/A	12.8.38	N/A	12.8.39	N/A	12.8.40	N/A	12.8.41	N/A
12.8.42	N/A	12.8.43	N/A	12.8.44	N/A	12.8.45	N/A	12.8.46	N/A
12.8.47	N/A	12.8.48	N/A	12.8.49	N/A	12.8.50	N/A	12.8.51	N/A
12.8.52	N/A	12.8.53	N/A	12.8.54	N/A	12.8.55	N/A	12.8.56	N/A
12.8.57	N/A	12.8.58	N/A	12.8.59	N/A	12.8.60	N/A	12.8.61	N/A
12.8.62	N/A	12.8.63	N/A	12.8.64	N/A	12.8.65	N/A	12.8.66	N/A
12.8.67	N/A	12.8.68	N/A	12.8.69	N/A	12.8.70	N/A	12.9.1	N/A
12.10.1	N/A	12.11.1	N/A	12.11.2	N/A	12.11.3	Q-Q	12.11.4	Q-Q
12.11.5	Q-Q	12.11.6	Q-Q	12.11.7	Q-Q	12.11.8	N/A	12.11.9	Q-Q
12.11.10	N/A	12.11.11	N/A	12.11.12	N/A	12.11.13	N/A	12.11.14	N/A
12.11.15	N/A	12.11.16	N/A	12.11.17	N/A	12.11.18	N/A	12.11.19	N/A
12.12.1	Q-Q	12.12.2	N/A	12.13.1	N/A	12.13.2	N/A	12.14.1	Q-Q
12.15.1	Q-Q	12.15.2	Q-Q	12.15.3	Q-Q	12.15.4	Q-Q	12.15.5	Q-Q
12.16.1	Q-Q	12.16.2	Q-Q	12.16.3	Q-Q	12.16.4	Q-Q	12.16.5	Q-Q
12.16.6	Q-Q	12.16.7	Q-Q	12.16.8	Q-Q	12.16.9	Q-Q	12.16.10	Q-Q
12.16.11	Q-Q	12.16.12	Q-Q	12.16.13	Q-Q	12.16.14	Q-Q	12.16.15	Q-Q
12.16.16	Q-Q	12.16.17	Q-Q	12.16.18	Q-Q	12.16.19	Q-Q	12.16.20	Q-Q
12.16.21	Q-Q	12.16.22	Q-Q	12.16.23	Q-Q	12.16.24	Q-Q	12.16.25	Q-Q
12.16.26	Q-Q	12.16.27	Q-Q	12.16.28	Q-Q	12.17.1	N/A	12.18.1	Q-Q
12.18.2	Q-Q	12.18.3	Q-Q	12.18.4	Q-Q	12.19.1	N/A	12.20.1	N/A
12.21.1	Q-Q	12.21.2	Q-Q	12.21.3	Q-Q	12.21.4	Q-Q	12.21.5	Q-Q
12.21.6	Q-Q	12.21.7	Q-Q	12.21.8	Q-Q	12.21.9	Q-Q	12.21.10	Q-Q
12.21.11	Q-Q	12.21.12	Q-Q	12.21.13	Q-Q	12.21.14	Q-Q	12.21.15	Q-Q
12.21.16	Q-Q	12.21.17	Q-Q	12.21.18	Q-Q	12.21.19	Q-Q	12.21.20	Q-Q
12.21.21	Q-Q	12.21.22	Q-Q	12.21.23	Q-Q	12.21.24	Q-Q	12.21.25	Q-Q
12.21.26	Q-Q	12.21.27	Q-Q	12.22.1	Q-Q	12.23.1	Q-Q	12.23.2	Q-Q
12.24.1	N/A	12.24.2	N/A	12.24.3	N/A	12.24.4	N/A	12.24.5	Q-Q
12.24.6	Q-Q	12.24.7	Q-Q	12.24.8	Q-Q	12.24.9	N/A	12.24.10	N/A
12.24.11	Q-Q	12.24.12	Q-Q	12.24.13	Q-Q	12.24.14	Q-Q	12.24.15	Q-Q

12.24.16	Q-Q	12.24.17	Q-Q	12.24.18	Q-Q	12.24.19	Q-Q	12.24.20	Q-Q
12.24.21	Q-Q	12.24.22	Q-Q	12.24.23	Q-Q	12.24.24	Q-Q	12.24.25	Q-Q
12.24.26	Q-Q	12.24.27	Q-Q	12.24.28	Q-Q	12.24.29	Q-Q	12.24.30	Q-Q
12.24.31	Q-Q	12.24.32	Q-Q	12.24.33	Q-Q	12.24.34	Q-Q	12.24.35	Q-Q
12.24.36	Q-Q	12.24.37	Q-Q	12.24.38	Q-Q	12.24.39	Q-Q	12.24.40	Q-Q
12.24.41	Q-Q	12.24.42	Q-Q	12.24.43	Q-Q	12.24.44	Q-Q	12.24.45	Q-Q
12.24.46	Q-Q	12.24.47	Q-Q	12.24.48	Q-Q	12.24.49	Q-Q	12.24.50	Q-Q
12.24.51	Q-Q	12.24.52	Q-Q	12.24.53	Q-Q	12.24.54	Q-Q	12.24.55	Q-Q
12.24.56	Q-Q	12.24.57	Q-Q	12.24.58	Q-Q	12.24.59	Q-Q	12.24.60	Q-Q
12.24.61	Q-Q	12.24.62	Q-Q	12.24.63	N/A	12.24.64	Q-Q	12.24.65	Q-Q
12.24.66	Q-Q	12.24.67	Q-Q	12.24.68	Q-Q	12.24.69	Q-Q	12.24.70	Q-Q
12.24.71	Q-Q	12.24.72	Q-Q	12.24.73	Q-Q	12.24.74	Q-Q	12.24.75	Q-Q
12.24.76	Q-Q	12.24.77	Q-Q	12.24.78	Q-Q	12.24.79	Q-Q	12.24.80	Q-Q
12.24.81	Q-Q	12.24.82	Q-Q	12.24.83	Q-Q	12.24.84	Q-Q	12.24.85	Q-Q
12.24.86	Q-Q	12.24.87	Q-Q	12.24.88	Q-Q	12.24.89	Q-Q	12.24.90	Q-Q
12.24.91	Q-Q	12.24.92	Q-Q	12.24.93	Q-Q	12.24.94	Q-Q	12.24.95	Q-Q
12.24.96	Q-Q	12.24.97	Q-Q	12.24.98	Q-Q	12.24.99	Q-Q	12.25.1	Q-Q
12.26.1	N/A	12.27.1	N/A	12.27.2	N/A	12.27.3	N/A	12.27.4	N/A
12.27.5	N/A	12.27.6	N/A	12.28.1	Q-Q	12.28.2	N/A	12.29.1	Q-Q
12.29.2	N/A	12.29.3	N/A	12.29.4	N/A	12.29.5	Q-Q	12.29.6	N/A
12.29.7	N/A	12.29.8	Q-Q	12.29.9	Q-Q	12.29.10	Q-Q	12.29.11	Q-Q
12.29.12	Q-Q	12.29.13	Q-Q	12.29.14	Q-Q	12.29.15	Q-Q	12.29.16	Q-Q
12.29.17	Q-Q	12.29.18	Q-Q	12.29.19	Q-Q	12.29.20	Q-Q	12.29.21	Q-Q
12.29.22	Q-Q	12.29.23	Q-Q	12.29.24	Q-Q	12.29.25	Q-Q	12.29.26	Q-Q
12.29.27	Q-Q	12.29.28	N/A	12.29.29	N/A	12.29.30	N/A	12.29.31	N/A
12.29.32	N/A	12.29.33	Q-Q	12.29.34	N/A	12.29.35	N/A	12.29.36	N/A
12.29.37	N/A	12.29.38	N/A	12.29.39	N/A	12.29.40	N/A	12.29.41	N/A
12.29.42	N/A	12.29.43	N/A	12.29.44	N/A	12.29.45	N/A	12.30.1	Q-Q
12.30.2	Q-Q	12.30.3	Q-Q	12.30.4	N/A	12.30.5	Q-Q	12.30.6	Q-Q
12.30.7	N/A	12.30.8	N/A	12.30.9	N/A	12.30.10	Q-Q	12.30.11	Q-Q
12.30.12	Q-Q	12.31.1	N/A	12.31.2	N/A	12.31.3	N/A	12.31.4	N/A
12.31.5	Q-Q	12.31.6	N/A	12.31.7	N/A	12.31.8	N/A	12.31.9	N/A
12.31.10	N/A	12.31.11	N/A	12.31.12	N/A	12.31.13	N/A	12.31.14	N/A
12.31.15	N/A	12.31.16	N/A	12.31.17	N/A	12.31.18	N/A	12.31.19	N/A
12.31.20	N/A	12.31.21	N/A	12.31.22	N/A	12.31.23	N/A	12.31.24	N/A
12.31.25	N/A	12.31.26	N/A	12.31.27	N/A	12.31.28	N/A	12.31.29	N/A
12.31.30	N/A	12.31.31	N/A	12.31.32	N/A	12.31.33	N/A	12.31.34	N/A
12.31.35	N/A	12.31.36	N/A	12.31.37	N/A	12.31.38	N/A	12.31.39	N/A
12.31.40	N/A	12.31.41	N/A	12.31.42	N/A	12.31.43	N/A	12.31.44	N/A
12.31.45	Q-Q	12.31.46	Q-Q	12.31.47	N/A	12.31.48	N/A	12.31.49	Q-Q
12.31.50	Q-Q	12.31.51	Q-Q	12.31.52	Q-Q	12.31.53	Q-Q	12.32.1	N/A

12.32.2	N/A	12.32.3	N/A	12.32.4	N/A	12.32.5	N/A	12.32.6	N/A
12.32.7	N/A	12.33.1	Q-Q	12.33.2	Q-Q	12.33.3	Q-Q	12.33.4	Q-Q
12.33.5	Q-Q	12.33.6	Q-Q	12.33.7	Q-Q	12.33.8	Q-Q	12.33.9	Q-Q
12.33.10	Q-Q	12.33.11	Q-Q	12.33.12	Q-Q	12.33.13	Q-Q	12.33.14	Q-Q
12.33.15	Q-Q	12.33.16	Q-Q	12.33.17	Q-Q	12.33.18	Q-Q	12.33.19	Q-Q
12.33.20	Q-Q	12.33.21	Q-Q	12.33.22	Q-Q	12.33.23	Q-Q	12.33.24	Q-Q
12.33.25	Q-Q	12.33.26	Q-Q	12.33.27	Q-Q	12.33.28	Q-Q	12.33.29	Q-Q
12.33.30	Q-Q	12.33.31	Q-Q	12.33.32	Q-Q	12.33.33	Q-Q	12.33.34	Q-Q
12.33.35	Q-Q	12.33.36	Q-Q	12.33.37	Q-Q	12.33.38	Q-Q	12.33.39	Q-Q
12.33.40	Q-Q	12.33.41	Q-Q	12.33.42	Q-Q	12.33.43	Q-Q	12.33.44	Q-Q
12.33.45	Q-Q	12.33.46	Q-Q	12.33.47	Q-Q	12.33.48	Q-Q	12.33.49	Q-Q
12.34.1	Q-Q	12.34.2	Q-Q	12.34.3	Q-Q	12.34.4	Q-Q	12.34.5	Q-Q
12.34.6	Q-Q	12.34.7	Q-Q	12.34.8	Q-Q	12.34.9	Q-Q	12.34.10	Q-Q
12.34.11	Q-Q	12.34.12	Q-Q	12.34.13	Q-Q	12.34.14	Q-Q	12.34.15	Q-Q
12.34.16	Q-Q	12.34.17	Q-Q	12.35.1	Q-Q	12.35.2	N/A	12.35.3	Q-Q
12.35.4	Q-Q	12.35.5	Q-Q	12.35.6	Q-Q	12.35.7	Q-Q	12.35.8	Q-Q
12.35.9	Q-Q	12.35.10	Q-Q	12.35.11	Q-Q	12.35.12	N/A	12.35.13	N/A
12.35.14	N/A	12.35.15	N/A	12.35.16	N/A	12.35.17	N/A	12.35.18	N/A
12.35.19	N/A	12.35.20	N/A	12.35.21	N/A	12.35.22	N/A	12.35.23	N/A
12.35.24	N/A	12.35.25	N/A	12.35.26	N/A	12.35.27	N/A	12.36.1	Q-Q
12.37.1	N/A	12.37.2	N/A	12.37.3	N/A	12.38.1	N/A	12.39.1	N/A
12.40.1	Q-Q	12.40.2	Q-Q	12.40.3	Q-Q	12.41.1	Q-Q	12.42.1	Q-Q
12.43.1	Q-Q	12.43.2	N/A	12.43.3	N/A	12.43.4	N/A	12.43.5	N/A
12.43.6	N/A	12.43.7	Q-Q	12.44.1	N/A	12.45.1	N/A	12.46.1	Q-Q
12.47.1	N/A	12.47.2	N/A	12.47.3	N/A	12.47.4	N/A	12.47.5	N/A
12.47.6	N/A	12.47.7	N/A	12.47.8	Q-Q	12.47.9	N/A	12.47.10	Q-Q
12.47.11	N/A	12.47.12	Q-Q	12.47.13	Q-Q	12.47.14	N/A	12.47.15	Q-Q
12.47.16	N/A	12.47.17	N/A	12.47.18	Q-Q	12.47.19	Q-Q	12.47.20	N/A
12.47.21	Q-Q	12.47.22	Q-Q	12.47.23	N/A	12.47.24	N/A	12.47.25	Q-Q
12.47.26	Q-Q	12.47.27	N/A	12.47.28	N/A	12.47.29	Q-Q	12.47.30	N/A
12.47.31	Q-Q	12.47.32	Q-Q	12.47.33	Q-Q	12.47.34	Q-Q	12.47.35	N/A
12.47.36	Q-Q	12.47.37	Q-Q	12.47.38	Q-Q	12.47.39	Q-Q	12.47.40	Q-Q
12.47.41	N/A	12.47.42	N/A	12.47.43	N/A	12.47.44	Q-Q	12.47.45	Q-Q
12.47.46	N/A	12.47.47	Q-Q	12.47.48	Q-Q	12.47.49	N/A	12.47.50	Q-Q
12.47.51	Q-Q	12.47.52	Q-Q	12.47.53	Q-Q	12.47.54	N/A	12.47.55	N/A
12.47.56	Q-Q	12.47.57	Q-Q	12.47.58	Q-Q	12.47.59	Q-Q	12.47.60	Q-Q
12.47.61	Q-Q	12.47.62	Q-Q	12.47.63	Q-Q	12.47.64	Q-Q	12.47.65	Q-Q
12.47.66	Q-Q	12.47.67	Q-Q	12.47.68	Q-Q	12.47.69	Q-Q	12.47.70	Q-Q
12.47.71	N/A	12.47.72	Q-Q	12.47.73	Q-Q	12.47.74	Q-Q	12.47.75	Q-Q
12.47.76	Q-Q	12.47.77	Q-Q	12.47.78	Q-Q	12.47.79	Q-Q	12.47.80	Q-Q
12.47.81	Q-Q	12.47.82	Q-Q	12.47.83	Q-Q	12.47.84	Q-Q	12.47.85	Q-Q

12.47.86	Q-Q
12.47.91	Q-Q
12.47.96	Q-Q
12.47.101	Q-Q

12.47.87	Q-Q
12.47.92	Q-Q
12.47.97	Q-Q
12.47.102	Q-Q

12.47.88	Q-Q
12.47.93	Q-Q
12.47.98	Q-Q
12.47.103	Q-Q

12.47.89	Q-Q
12.47.94	Q-Q
12.47.99	Q-Q

12.47.90	Q-Q
12.47.95	Q-Q
12.47.100	Q-Q