**PROBLEM 1:** Inference in Propositional Logic (**55 points**)

a/ A propositional 2-CNF expression is a conjunction of clauses, each containing exactly two literals, e.g.:

$(X \lor Y) \land (\neg X \lor Z) \land (\neg Y \lor W) \land (\neg Z \lor G) \land (\neg W \lor G)$

**(15 points)** Prove using resolution that the above sentence entails G.

b/ Given the following Knowledge Base:
1. $X \land B \land C \Rightarrow A$
2. $A \land D \land E \Rightarrow C$
3. $B$
4. $E \land \neg F$
5. $F \lor D$
6. $G \land \neg F \Rightarrow X$
7. $G$

**(15 points)** Use backward-chaining inference to prove the query A.

c/ Use propositional logic inference rules to decide which of the following sentences are entailed by the Sentence 1: $(X \lor Y) \land (\neg Z \lor \neg W \lor Q)$:

Sentence 2: $(X \lor Y)$

Sentence 3: $(X \lor Y \lor Z) \land (Y \land Z \land W \Rightarrow Q)$

Sentence 4: $(X \lor Y) \land (\neg W \lor Q)$

To get full credit you need to write if:

   i.      S1 Entails S2 or not, and if so, which propositional inference rules you have applied to reach the conclusion. Detail the results of each inference rule. **(5 points)**

ii. S1 Entails S3 or not, and if so, which propositional inference rules you have applied to reach the conclusion. Detail the results of each inference rule. **(5 points)**

iii. S1 Entails S4 or not, and if so, which propositional inference rules you have applied to reach the conclusion. Detail the results of each inference rule. **(5 points)**

d/ Demonstrate whether the following sentences are valid, satisfiable or neither. Motivate and detail your demonstrations.

*Sentence 1:* ((Smart ∨ Beautiful) ⇒ (Interesting ∨ Boring)) ⇔ ((Smart ⇒ Interesting) ∨ (Beautiful ⇒ boring)) **(5 points)**

*Sentence 2:* (Tall ∨ Gorgeous) ∨ ¬(Tall ⇒ Gorgeous)) **(5 points)**

**PROBLEM 2:** (**25 points**) Logic Representations

a/ According to political pundits, a person who is a radical ( R) is electable ( E) if he/she is conservative (C ) , but otherwise is not electable. Which of the following are correct representations in propositional logic of this assertion?

i. (R ∧ E) ⇔ C

ii. R ⇒ (E ⇔ C)

iii. R ⇒ ((C ⇒ E) ∨ ¬E)

Explain why. **(15 points)**

b/ Unification: For each pair of literals, find the Most General Unifier and the Most General Common Substitution Instance:

(**2 points**) {P(x), P(A)}

(**4 points**) {P[f(x), y, g(y)], P[f(x), z, g(x)]}

(**4 points**) {P[f(x, g(A,y)), g(A,y)], P[f(x,z),z]}

**PROBLEM 3:** First-Order Logic (FOL) representations (**40 points**)

*Write in FOL the following statements by defining first your vocabulary (i.e. predicates, constants, variables, functions, etc):*

1. (**2 points**) *Some leaves turn red each Fall*.
2. (**3 points**) *Some trees lose all their leaves when winter comes.*
3. (**2 points***) Flowers are always nice and they smell lovely*.
4. (**2 points**) *One flower does not bring Spring.*
5. (**2 points**) *Every flower fades at some point.*
6. (**2 points**) *Only one flower is left in the vase.*
7. (**2 points**) *Every person that buys flowers is sensitive.*
8. (**3 points***) Poets are sensitive but they do not buy flowers, they write beautiful poems.*
9. (**2 points**) *No poet will kill an animal.*
10. (**3 points**) *There is an agent who sells policies only to people who are not insured.*
11. (**2 points**) *There is a barber who shaves all men in town who do not shave themselves.*
12. (**10 points**) *A person born outside the US, one of who has at least one parent who is a US citizen by birth is a US citizen by descent.*
13. (**2 points**) *There is a flower that smells nice in the house.*
14. (**3 points**) *John bought only two flowers.*

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**PROBLEM 4:** Refutation in First-Order Logic (**80 points**)

The purpose of this assignment is to give you experience in proving facts with the resolution method and in exposing you to Prover9, an automatic theorem prover that can help you devise your refutations.
Consider the following helpful pointers for using Prover9:

Installation (https://www.cs.unm.edu/~mccune/mace4/gui/v05.html)
        For linux users, install python-wxtools also.

Help Manual (https://www.cs.unm.edu/~mccune/prover9/manual/2009-02A/)

Simple tutorial (www.cs.utsa.edu/~bylander/cs5233/prover9-intro.pdf)

**You are asked to solve the following puzzle.**
1. Anyone who rides a Harley is a rough character.
2. Every biker rides [something that is] either a Harley or a BMW.
3. Anyone who rides any BMW is a yuppie.
4. Every yuppie is a lawyer.
5. Any nice girl does not date anyone who is a rough character.
6. Mary is a nice girl, and John is a biker.
7. (Conclusion) If John is not a lawyer, then Mary does not date John.

i. (**14 points**) Represent these clauses in first order logic, using only these predicates:

$Harley(x)$ , $Rides(x,y)$ , $Rough(x)$ , $Biker(x)$ , $BMW(x)$ , $Yuppie(x)$ , $Lawyer(x)$ , $Nice(x)$ , $Date(x,y)$
.

ii. (**14 points**) Convert the logic sentences to clause form, skolemizing as necessary.

iii. (**42 points**) Prove <u>by hand</u> whether the conclusion is true by using resolution refutation (i.e. negate the conclusion and show its unsatisfiability with the rest of the knowledge base). Make sure to document the substitutions you use.

iv. (**20 points**) Use Prover9 to perform automatically the refutation. Submit a report with three parts:
   I.    Assumptions and goal;
   II.    The input and output of prover9 (The input of prover 9 should be in plain text)
   III.   Conclusion

*The Pigs and Balloons Puzzle*

(1) All, who neither dance on tight ropes nor eat penny-buns,
   are old.
(2) Pigs, that are liable to giddiness, are treated with respect.
(3) A wise balloonist takes an umbrella with him.
(4) No one ought to lunch in public who looks ridiculous and eats
   penny-buns.
(5) Young creatures, who go up in balloons, are liable to
   giddiness.
(6) Fat creatures, who look ridiculous, may lunch in
   public, provided that they do not dance on tight ropes.
(7) No wise creatures dance on tight ropes, if liable to giddiness.
(8) A pig looks ridiculous, carrying an umbrella.
(9) All, who do not dance on tight ropes, and who are treated
   with respect are fat.
   Show that no wise young pigs go up in balloons.
                    -Lewis Carroll, Symbolic Logic,

Submit a report with three parts:
   I.     Assumptions and goal;
   II.    The input and output of prover9 (The input of prover 9 should be in plain text)
   III.   Conclusion