THE DOT PRODUCT. (A way to multiply vectors)

For a vector $a = \langle a_1, a_2, a_3 \rangle$ & $b = \langle b_1, b_2, b_3 \rangle$

A dot product is $a \cdot b = a_1 b_1 + a_2 b_2 + a_3 b_3$.

* A dot product is a scalar quantity.

eg:- $a = \langle 1, 2, 3 \rangle$ $b = \langle 4, 5, 6 \rangle$

$$a \cdot b = 1 \cdot 4 + 2 \cdot 5 + 3 \cdot 6$$

$$= 4 + 10 + 18$$

$$= \underline{\underline{32}} \leftarrow \text{scalar quantity}$$

Some Laws.

$(a \cdot b) = (b \cdot a)$

$\text{constant} \cdot (a \cdot b) = (\text{constant} \cdot a) \cdot b) = (\text{Constant} \cdot b) \cdot a)$

$a \cdot a = |a|^2$

length of a ^ squared.

$a \cdot 0 = 0$

A dot product has another formula that is:-

$$\underline{a \cdot b = |a||b| \cos\theta}$$

*(if dot product of two vectors is zero then the vectors are perpendicular as $\cos 90 = 0$)

The above formula is useful and is commonly used as

$$\theta = \cos^{-1}\left(\frac{a \cdot b}{|a||b|} \right)$$

← without using a lot of trigo we can find angle between two vectors, using dot product.

eg:- what is the angle between the following vectors $a = \langle 2, 2, -1 \rangle$ $b = \langle 5, -3, 2 \rangle$

$$|a| = \sqrt{(2)^2 + (2)^2 + (-1)^2}$$

$$= \sqrt{9} = 3$$

$$|b| = \sqrt{(5)^2 + (-3)^2 + (2)^2}$$

$$= \sqrt{25 + 9 + 4} = \sqrt{38}$$

$a \cdot b = (2 \times 5) + (2 \times -3) + (-1 \times 2)$

$= 10 - 6 - 2$

$= \underline{\underline{2}}$

$$\therefore \theta = \cos^{-1}\left(\frac{2}{(3)(38)} \right) = \underline{\underline{84^\circ}}$$