Chapter 6

Magnetic Fields in Matter

- 6.1 Magnetization
- 6.2 The Field of a Magnetized Object
- 6.3 The Auxiliary Field H
- 6.4 Linear and Nonlinear Media

Magnetization

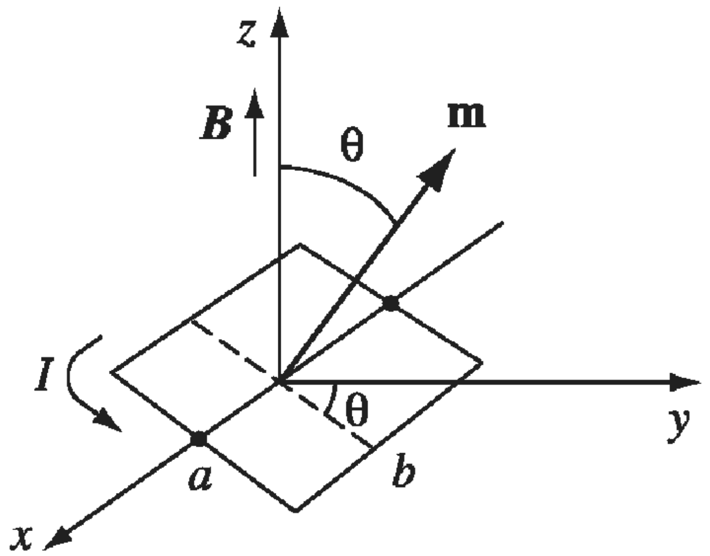
Diamagnets, Paramagnets, Ferromagnets

- All magnetic phenomena are due to electric charges in motion, and in fact, if you could examine a piece of magnetic material on an atomic scale you would find tiny currents: electrons orbiting around nuclei and spinning about their axes.
- For macroscopic purposes, these current loops are so small that we may treat them as magnetic dipoles. Ordinarily, they cancel each other out because of the random orientation of the atoms.
- But when a magnetic field is applied, a net alignment of these magnetic dipoles occurs, and the medium becomes magnetically polarized, or magnetized.

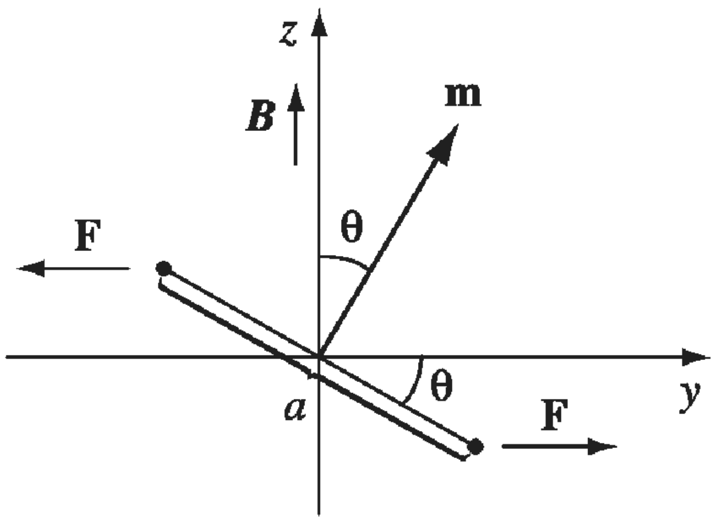
Magnetization parallel to **B** → ***Paramagnets***
Magnetization opposite to **B** → ***Diamagnets***
Retain their magnetization even after the external field has been remove → ***Ferromagnets***

Torques and Forces on Magnetic Dipoles

A magnetic dipole experiences a torque in a magnetic field, just as an electric dipole does in an electric field.



(a)



(b)

The forces on the "horizontal" sides are likewise equal and opposite (so the net force on the loop is zero), but they do generate a torque:

$$N = aF\sin\theta\vec{x}$$

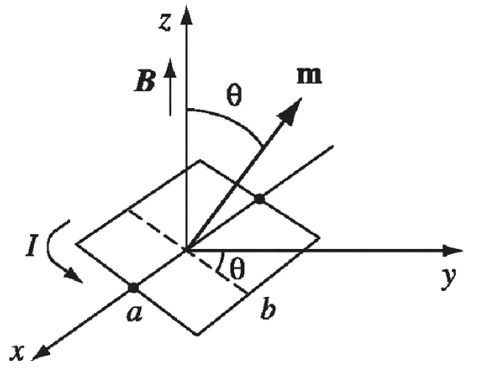
The magnitude of the force on each of these segments is:

$$F = IbB$$

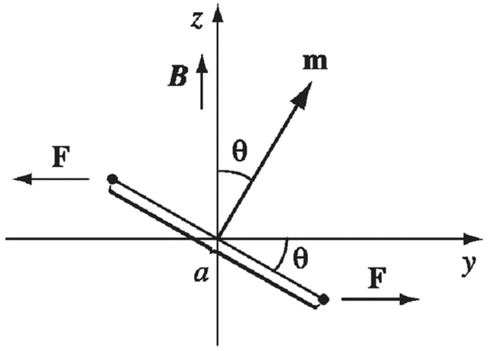
and therefore $$N = IabB\sin\theta\vec{x} = mB\sin\theta\,\vec{x}$$

or $$\boldsymbol{N} = \boldsymbol{m}\times \boldsymbol{B}$$

Where $m = Iab$ is the magnetic dipole moment of the loop



(a)



(b)

Note:
Here $\boldsymbol{N}$ is identical in form to the electrical analog, $\boldsymbol{N} = \boldsymbol{p}\times\mathbf{E}$. In particular, the torque is again in such a direction as to line the dipole up parallel to the field (paramagnetism).

- Since every electron constitutes a magnetic dipole, you might expect paramagnetism to be a universal phenomenon. Actually, quantum mechanics (specifically, the Pauli exclusion principle) tends to lock the electrons within a given atom together in pairs with opposing spins, and this effectively neutralizes the torque on the combination.

- As a result, paramagnetism most often occurs in atoms or molecules with an odd number of electrons, where the "extra" unpaired member is subject to the magnetic torque. Even here, the alignment is far from complete, since random thermal collisions tend to destroy the order.

In a uniform field, the net force on a current loop is zero

$$\mathbf{F} = I\oint(d\mathbf{l}\times\mathbf{B}) = I\left(\oint d\mathbf{l}\right)\times\mathbf{B} = \mathbf{0}$$

In a nonuniform field, the net force on a current loop could be no zero.
For example, suppose a circular wire ring of radius R, carrying a current I, is suspended above a short solenoid in the "fringing" region (Fig. 6.3). Here **B** has a radial component, and there is a net downward force on the loop (Fig. 6.4):

$$F = 2\pi IRB\cos\theta$$

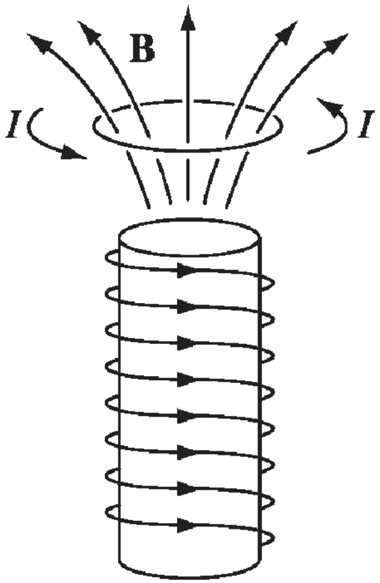


FIGURE 6.3

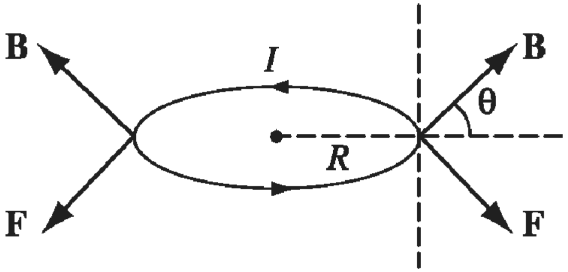


FIGURE 6.4

For an infinitesimal loop, with dipole moment **m**, in a field **B**, the force is

$$F = \nabla(\boldsymbol{m}\cdot\boldsymbol{B}) \quad \text{(Prob. 6.4)}$$

Once again, the magnetic formula is identical to its electrical "twin," if we write the latter in the form

$$F = \nabla(\boldsymbol{p}\cdot\boldsymbol{E})$$

Effect of a Magnetic Field on Atomic Orbits

Electrons not only have spin; they also revolve around the nucleus. For simplicity, let's assume the orbit is a circle of radius R

period $$T = \frac{2\pi R}{v}$$

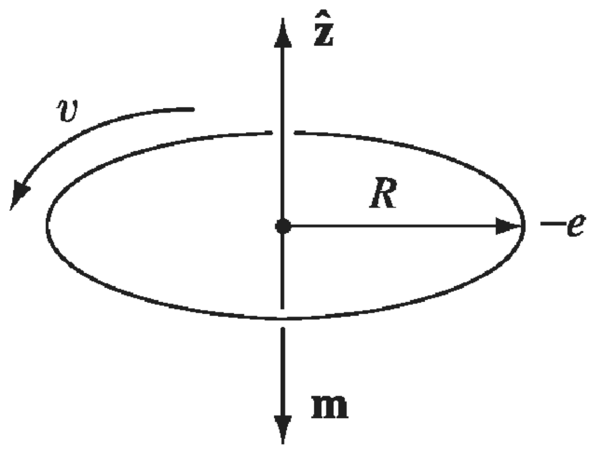


FIGURE 6.9

It's going to look like a steady current:

$$I = \frac{-e}{T} = -\frac{ev}{2\pi R}$$

The orbital dipole moment is

$$\boldsymbol{m} = I\pi R^2 = -\frac{1}{2}evR\vec{z}$$

Note:
(The minus sign accounts for the negative charge of the electron) Like any other magnetic dipole, this one is subject to a torque $(m \times B)$ when you turn on a magnetic field

The Coulomb force satisfies

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} = m_e\frac{v^2}{R}$$

In the presence of a magnetic field there is an additional force, $-e(\boldsymbol{v} \times \boldsymbol{B})$.
For the sake of argument, let's say that **B** is perpendicular to the plane of the orbit:

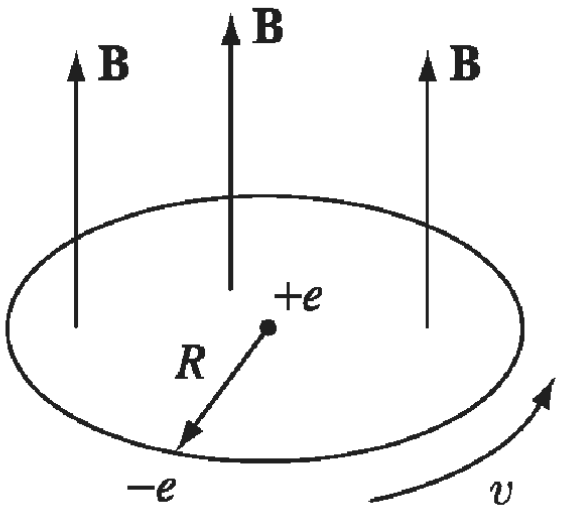


FIGURE 6.10

$$\frac{1}{4\pi\epsilon_0}\frac{e^2}{R^2} + e\bar{v}B = m_e\frac{\bar{v}^2}{R}$$

Under these conditions, the new speed $\vec{v}$ greater than v:

$$e\bar{v}B = \frac{m_e}{R}(\bar{v}^2 - v^2) = \frac{m_e}{R}(\bar{v} + v)(\bar{v} - v)$$

Assuming the change $\Delta v = \vec{v} - v$ is small

$$\Delta v = \frac{eRB}{2m_e}$$

When **B** is turned on, then, the electron speeds up (a changing magnetic field induces an electric field, and it is the latter that accelerates the electrons in this instance)

Note:
The electron speeds up or slows down, depending on the orientation of **B**

A change in orbital speed means a change in the dipole moment

$$\Delta \mathbf{m} = -\frac{1}{2}e(\Delta v)R\,\hat{\mathbf{z}} = -\frac{e^2R^2}{4m_e}\mathbf{B}$$

Note :
The change in m is opposite to the direction of **B**

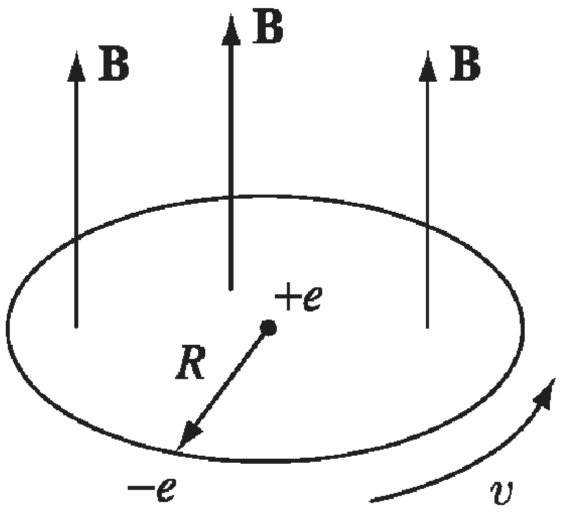


FIGURE 6.10

- Ordinarily, the electron orbits are randomly oriented, and the orbital dipole moments cancel out. But in the presence of a magnetic field, each atom picks up a little "extra" dipole moment, and these increments are all antiparallel to the field. This is the mechanism responsible for **diamagnetism**.

- It is a universal phenomenon, affecting all atoms. However, it is typically much weaker than paramagnetism, and is therefore observed mainly in atoms with even numbers of electrons, where paramagnetism is usually absent.

This classical model is fundamentally flawed (diamagnetism is really a quantum phenomenon), so there's not much point in refining the details. What is important is the empirical fact that in diamagnetic materials the induced dipole moments point opposite to the magnetic field.

Magnetization

- In the presence of a magnetic field, matter becomes magnetized; that is, upon microscopic examination, it will be found to contain many tiny dipoles, with a net alignment along some direction.

- We have discussed two mechanisms that account for this magnetic polarization:
(1) **paramagnetism** (the dipoles associated with the spins of unpaired electrons experience a torque tending to line them up parallel to the field)
(2) **diamagnetism** (the orbital speed of the electrons is altered in such a way as to change the orbital dipole moment in a direction opposite to the field).

- Whatever the cause, we describe the state of magnetic polarization by the vector quantity.

$$M \equiv \textit{magnetic dipole moment per unit volume}$$

M is called the **magnetization**; it plays a role analogous to the polarization **P** in electrostatics.

The Field of a Magnetized Object

Bound Currents

Suppose we have a piece of magnetized material; the magnetic dipole moment per unit volume, **M**, is given. What field does this object produce? Well, the vector potential of a single dipole **m** is given.

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \frac{\mathbf{m} \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$$

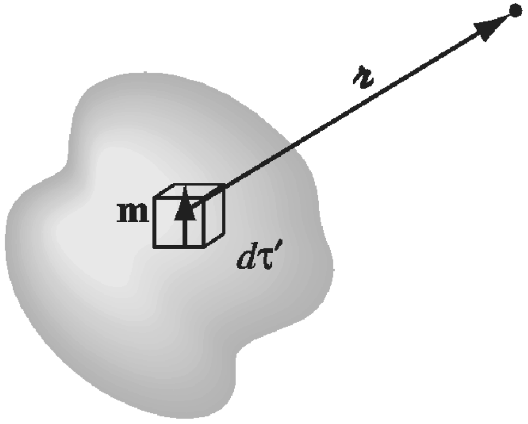


FIGURE 6.11

The total vector potential is

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \frac{\mathbf{M}(\mathbf{r}') \times \hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2} d\tau'$$

With $\nabla' \frac{1}{\mathscr{r}} = \frac{\hat{\boldsymbol{\mathscr{r}}}}{\mathscr{r}^2}$

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi} \int \left[\mathbf{M}(\mathbf{r}') \times \left(\nabla' \frac{1}{\mathscr{r}}\right)\right] d\tau'$$

Integrating by parts, using product rule 7, gives

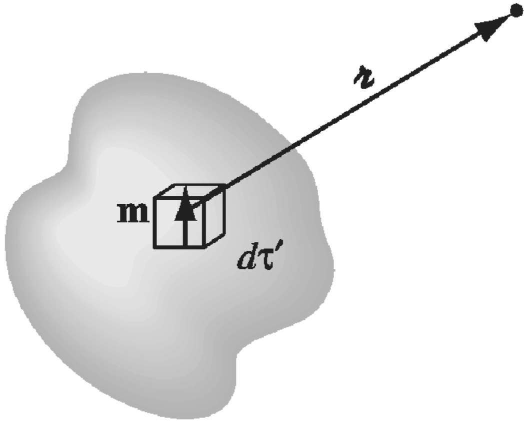


FIGURE 6.11

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\left\{\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\, d\tau' - \int \nabla' \times \left[\frac{\mathbf{M}(\mathbf{r}')}{\mathscr{r}}\right] d\tau'\right\}$$

Express the latter as a surface integral

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int \frac{1}{\mathscr{r}}[\nabla' \times \mathbf{M}(\mathbf{r}')]\, d\tau' + \frac{\mu_0}{4\pi}\oint \frac{1}{\mathscr{r}}[\mathbf{M}(\mathbf{r}') \times d\mathbf{a}']$$

The first term looks just like the potential of a volume current

$$\mathbf{J}_b = \nabla \times \mathbf{M}$$

The second looks like the potential of a surface current

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

where $\vec{n}$ is the normal unit vector.

With these definitions

$$\mathbf{A}(\mathbf{r}) = \frac{\mu_0}{4\pi}\int_{\mathcal{V}} \frac{\mathbf{J}_b(\mathbf{r}')}{\mathscr{r}}\, d\tau' + \frac{\mu_0}{4\pi}\oint_{\mathcal{S}} \frac{\mathbf{K}_b(\mathbf{r}')}{\mathscr{r}}\, da'$$

$$\mathbf{J}_b = \nabla \times \mathbf{M}$$

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$$

- What this means is that the potential (and hence also the field) of a magnetized object is the same as would be produced by a volume current $\boldsymbol{J}_b = \nabla \times \boldsymbol{M}$ throughout the material, plus a surface current $\boldsymbol{K}_b$=$\mathbf{M}\times \vec{\boldsymbol{n}}$, on the boundary.

- Instead of integrating the contributions of all the infinitesimal dipoles, we first determine the bound currents, and then find the field they produce, in the same way we would calculate the field of any other volume and surface currents.

- Notice the striking parallel with the electrical case: there the field of a polarized object was the same as that of a bound volume charge $\rho_b = -\nabla \cdot \boldsymbol{P}$ plus a bound surface charge $\sigma_b = \boldsymbol{P} \cdot \vec{n}$.

Example

Find the magnetic field of a uniformly magnetized sphere

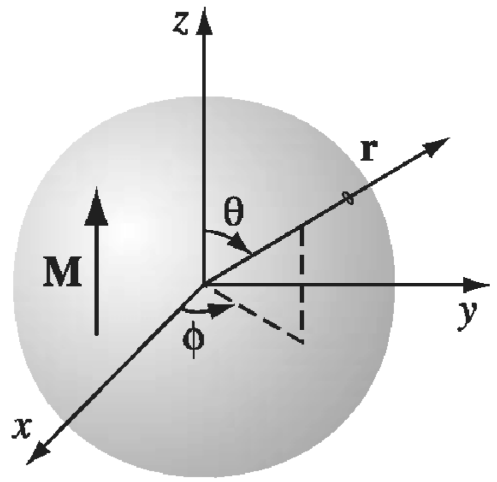


FIGURE 6.12

Solution

Choosing the z axis along the direction of **M**, we have

$$J_b = \nabla \times M = 0 \qquad K_b = \boldsymbol{M} \times \vec{n} = M \sin\theta \vec{\phi}$$

Consider a rotating spherical shell of uniform surface charge, corresponds to a surface current density

$$K = \sigma v = \sigma\omega R \sin\theta \vec{\phi}$$

It follows, therefore, that the field of a uniformly magnetized sphere is identical to the field of a spinning spherical shell, with the identification $\sigma R\boldsymbol{\omega} \to \boldsymbol{M}$.

Referring back to Ex. 5.11, we get

Inside the sphere

$$\boldsymbol{B} = \frac{2}{3}\mu_0 \boldsymbol{M}$$

The field outside is the same as that of a perfect dipole,

$$\boldsymbol{m} = \frac{4}{3}\pi R^3 \boldsymbol{M}$$

Note:
the internal field is uniform, like the electric field inside a uniformly polarized sphere.

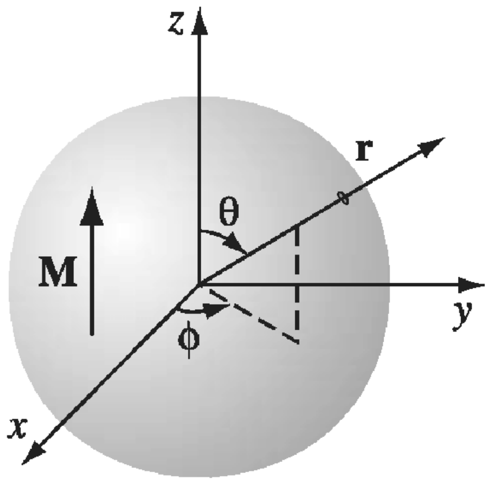


FIGURE 6.12

The Auxiliary Field H

Ampere's Law in Magnetized Materials

In any event, the total current can be written as

$$\mathbf{J} = \mathbf{J}_b + \mathbf{J}_f$$

Here, it is simply a convenience to separate the current into these two parts, because they got there by quite different means: the free current is there because somebody hooked up a wire to a battery-it involves actual transport of charge; the bound current is there because of magnetization-it results from the conspiracy of many aligned atomic dipoles.

Ampere's law can be written

$$\frac{1}{\mu_0}(\nabla \times \mathbf{B}) = \mathbf{J} = \mathbf{J}_f + \mathbf{J}_b = \mathbf{J}_f + (\nabla \times \mathbf{M})$$

collecting together the two curls

$$\nabla \times \left(\frac{1}{\mu_0}\mathbf{B} - \mathbf{M}\right) = \mathbf{J}_f$$

The quantity in parentheses is designated by the letter **H**:

$$\mathbf{H} \equiv \frac{1}{\mu_0}\mathbf{B} - \mathbf{M}$$

In terms of **H**, then, Ampere's law reads

$$\nabla \times \mathbf{H} = \mathbf{J}_f$$

or, in integral form

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}}$$

Note:
I_{fenc} is the total current passing through the Amperian loop

H plays a role in magnetostatics analogous to **D** in electrostatics

Example

A long copper rod of radius R carries a uniformly distributed (free) current I (Fig. 6.19). Find **H** inside and outside the rod.

Solution

Copper is weakly diamagnetic, so the dipoles will line up opposite to the field. This results in a bound current running anti parallel to I, within the wire, and parallel to I along the surface (Fig. 6.20).
In order to calculate H, it is sufficient to realize that all the currents are longitudinal, so **B**, **M**, and therefore also **H**, are circumferential.

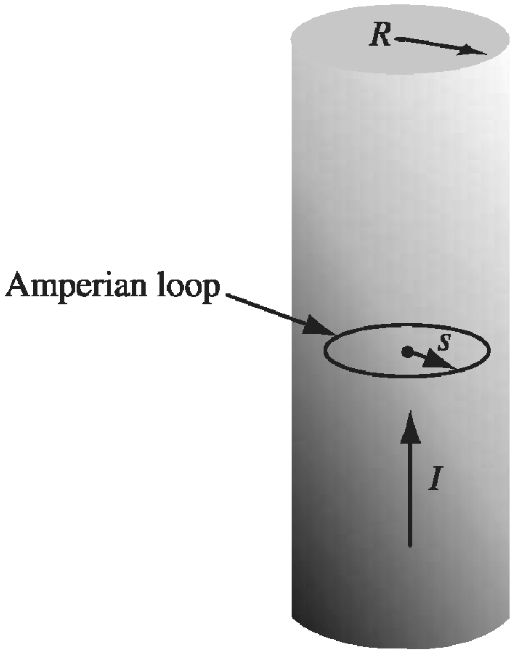


FIGURE 6.19

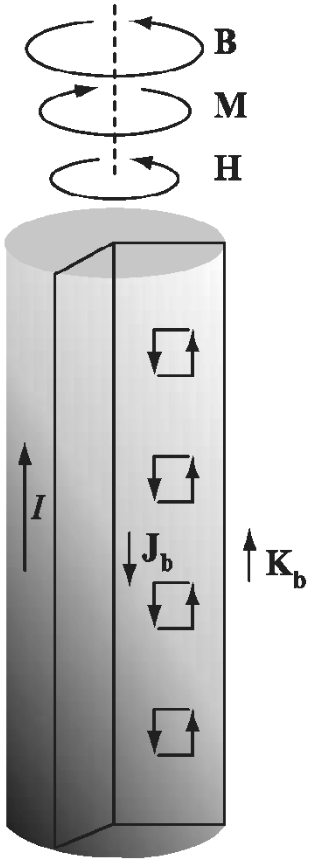


FIGURE 6.20

Applying $\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{enc}}$ to an Amperian loop of radius s < R

$$H(2\pi s) = I_{f_{enc}} = I \frac{\pi s^2}{\pi R^2}$$

so, inside the wire

$$\mathbf{H} = \frac{I}{2\pi R^2} s \,\hat{\boldsymbol{\phi}} \qquad (s \le R)$$

Outside the wire

$$\mathbf{H} = \frac{I}{2\pi s} \hat{\boldsymbol{\phi}} \qquad (s \ge R)$$

In the latter region (as always, in empty space) $M = 0$, so

$$\mathbf{B} = \mu_0 \mathbf{H} = \frac{\mu_0 I}{2\pi s} \hat{\boldsymbol{\phi}} \qquad (s \ge R)$$

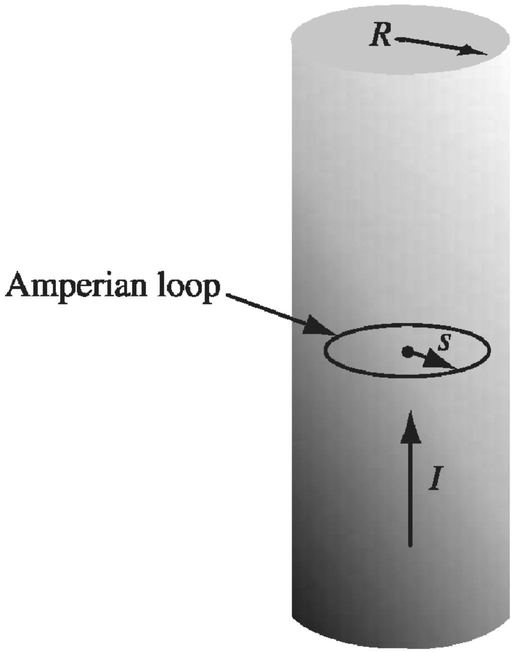


FIGURE 6.19

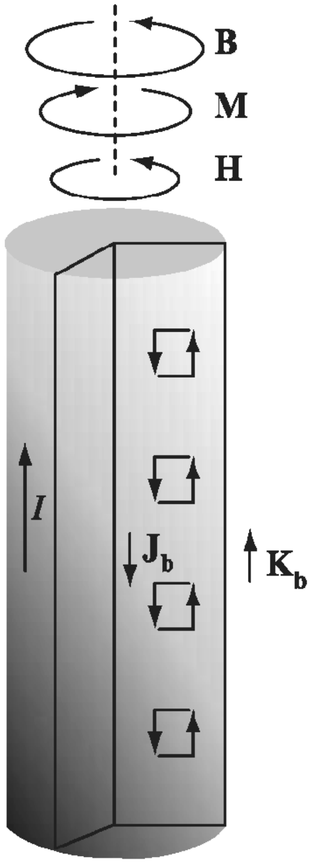


FIGURE 6.20

H: magnetic field
B: "flux density" or the magnetic "induction".

A Deceptive Parallel

- **H** is a more useful quantity than **D**. The reason is this: To build an electromagnet you run a certain (free) current through a coil. The current is the thing you read on the dial, and this determines **H**; But **B** depends on the specific materials you used.

- $\nabla \cdot \boldsymbol{B} = 0$, the divergence of H is not zero. In fact, $\nabla \cdot \boldsymbol{H} = -\nabla \cdot \boldsymbol{M}$

Boundary Conditions

The magnetostatic boundary conditions of Sect. 5.4.2 can be rewritten in terms of the free current.

$$B^{\perp}_{\text{above}} - B^{\perp}_{\text{below}} = 0$$

$$H^{\perp}_{\text{above}} - H^{\perp}_{\text{below}} = -(M^{\perp}_{\text{above}} - M^{\perp}_{\text{below}})$$

$$\mathbf{B}^{\parallel}_{\text{above}} - \mathbf{B}^{\parallel}_{\text{below}} = \mu_0(\mathbf{K} \times \hat{\mathbf{n}})$$

$$\mathbf{H}^{\parallel}_{\text{above}} - \mathbf{H}^{\parallel}_{\text{below}} = \mathbf{K}_f \times \hat{\mathbf{n}}$$

Linear and Nonlinear Media

Magnetic Susceptibility and Permeability

In paramagnetic and diamagnetic materials, the magnetization is sustained by the field; when **B** is removed, **M** disappears. In fact, for most substances the magnetization is proportional to the field, provided the field is not too strong.

$$\mathbf{M} = \chi_m \mathbf{H}$$

Note: $\mathbf{M} = \frac{1}{\mu_0}\chi_m \mathbf{B}$ Incorrect !

Material	Susceptibility	Material	Susceptibility
Diamagnetic:		*Paramagnetic:*	
Bismuth	-1.7×10^{-4}	Oxygen (O_2)	1.7×10^{-6}
Gold	-3.4×10^{-5}	Sodium	8.5×10^{-6}
Silver	-2.4×10^{-5}	Aluminum	2.2×10^{-5}
Copper	-9.7×10^{-6}	Tungsten	7.0×10^{-5}
Water	-9.0×10^{-6}	Platinum	2.7×10^{-4}
Carbon Dioxide	-1.1×10^{-8}	Liquid Oxygen ($-200°$ C)	3.9×10^{-3}
Hydrogen (H_2)	-2.1×10^{-9}	Gadolinium	4.8×10^{-1}

TABLE 6.1 Magnetic Susceptibilities (unless otherwise specified, values are for 1 atm, 20° C). *Data from Handbook of Chemistry and Physics,* 91st ed. (Boca Raton: CRC Press, Inc., 2010) and other references.

The constant of proportionality χ_m is called the **magnetic susceptibility**; it is a dimensionless quantity that varies from one substance to another-positive for paramagnets and negative for diamagnets. Typical values are around 10^{-5}.

Materials that obey $\mathbf{M} = \chi_m \mathbf{H}$ are called linear media:

$$\mathbf{B} = \mu_0(\mathbf{H} + \mathbf{M}) = \mu_0(1 + \chi_m)\mathbf{H}$$

Thus B is also proportional to **H**:

$$\mathbf{B} = \mu \mathbf{H}$$

where $$\mu \equiv \mu_0(1 + \chi_m)$$

μ is called the **permeability** of the material. In a vacuum, where there is no matter to magnetize, the susceptibility χ_m vanishes, and the permeability is μ_0. That's why μ_0 is called the ***permeability of free space***

Example

An infinite solenoid (n turns per unit length, current I) is filled with linear material of susceptibility χ_m. Find the magnetic field inside the solenoid.

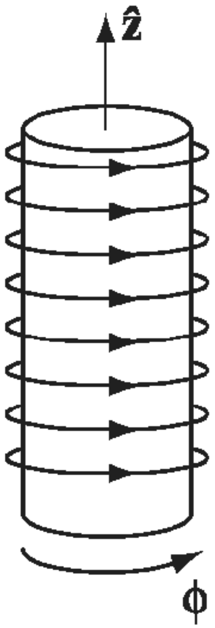


FIGURE 6.22

Solution

Since B is due in part to bound currents (which we don't yet know), we cannot compute it directly. However, this is one of those symmetrical cases in which we can get H from the free current alone, using Ampere's law in H:

$$\oint \mathbf{H} \cdot d\mathbf{l} = I_{f_{\text{enc}}} \quad \longrightarrow \quad \mathbf{H} = nI\,\hat{\mathbf{z}}$$

$$\mathbf{B} = \mu_0(1 + \chi_m)nI\,\hat{\mathbf{z}}.$$

$$\mathbf{B} = \mu_0(1+\chi_m)nI\,\hat{\mathbf{z}}$$

- If the medium is paramagnetic, the field is slightly enhanced;
- If it's diamagnetic, the field is somewhat reduced.

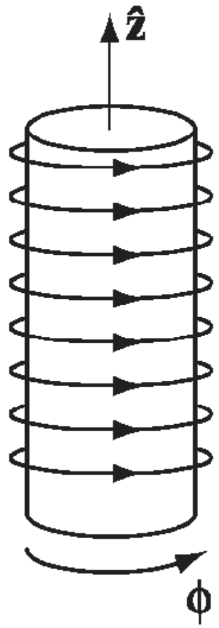


FIGURE 6.22

This reflects the fact that the bound surface current

$$\mathbf{K}_b = \mathbf{M}\times\hat{\mathbf{n}} = \chi_m(\mathbf{H}\times\hat{\mathbf{n}}) = \chi_m nI\,\hat{\boldsymbol{\phi}}$$

$\boldsymbol{K}_b$ is in the same direction as I, in the former case ($\chi_m > 0$), and opposite in the latter ($\chi_m < 0$).

Ferromagnetism

- Ferromagnet require no external fields to sustain the magnetization.
- Ferromagnetism involves the magnetic dipoles associated with the spins of unpaired electrons.
- Each dipole "likes" to point in the same direction as its neighbors. The reason for this preference is essentially quantum mechanical.

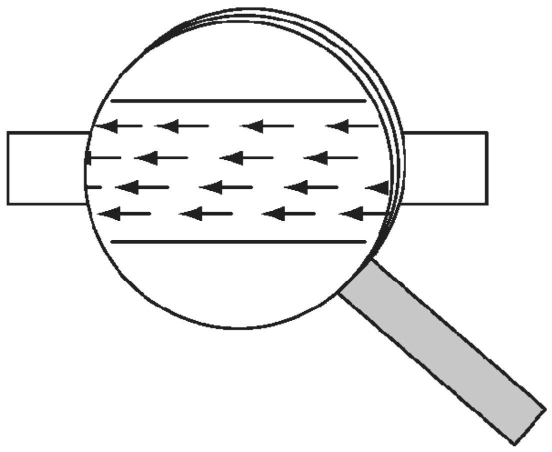
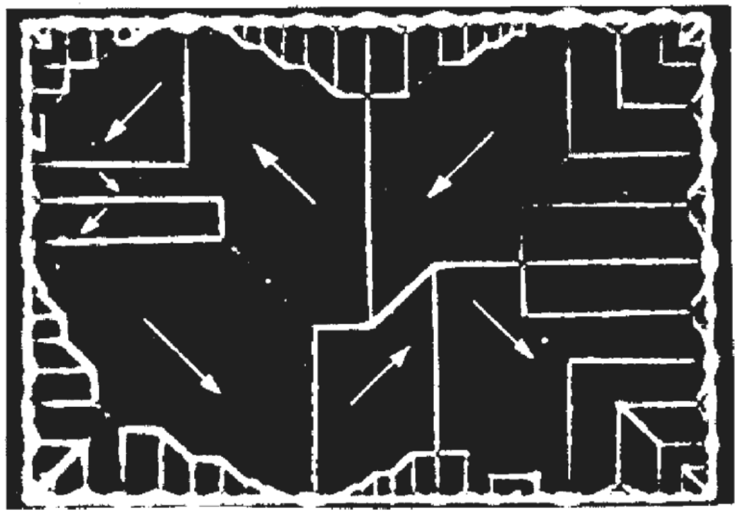


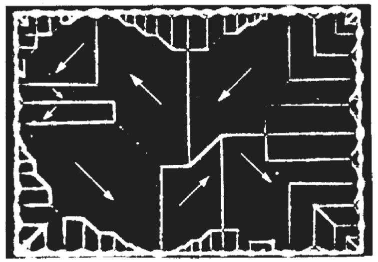
FIGURE 6.25

Domains: Each domain contains billions of dipoles, all lined up. These domains are actually *visible* under a microscope, but are randomly oriented.



Ferromagnetic domains. (Photo courtesy of R. W. DeBlois)

FIGURE 6.26



Ferromagnetic domains. (Photo courtesy of R. W. DeBlois)

FIGURE 6.26

If you put a piece of iron into a strong magnetic field, the torque $\boldsymbol{N} = \boldsymbol{m} \times \boldsymbol{B}$ tends to align the dipoles parallel to the field. Therefore domains parallel to the field grow, and the others shrink. If the field is strong enough, one domain takes over entirely, and the iron is said to be saturated.

The shifting of domain boundaries in response to an external field is not entirely reversible: When the field is switched off, there will be some return to randomly oriented domains, but it is far from complete. There remains a preponderance of domains in the original direction. You now have a permanent magnet.

Wrap a coil of wire around the object to be magnetized. Run a current I through the coil; this provides the external magnetic field (pointing to the left in the diagram)

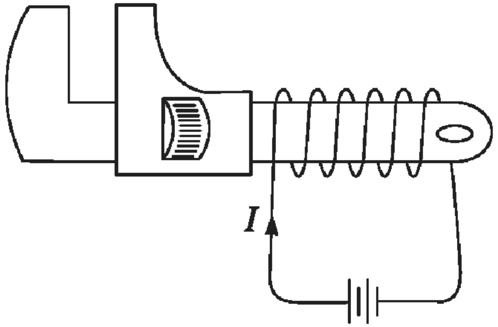


FIGURE 6.27

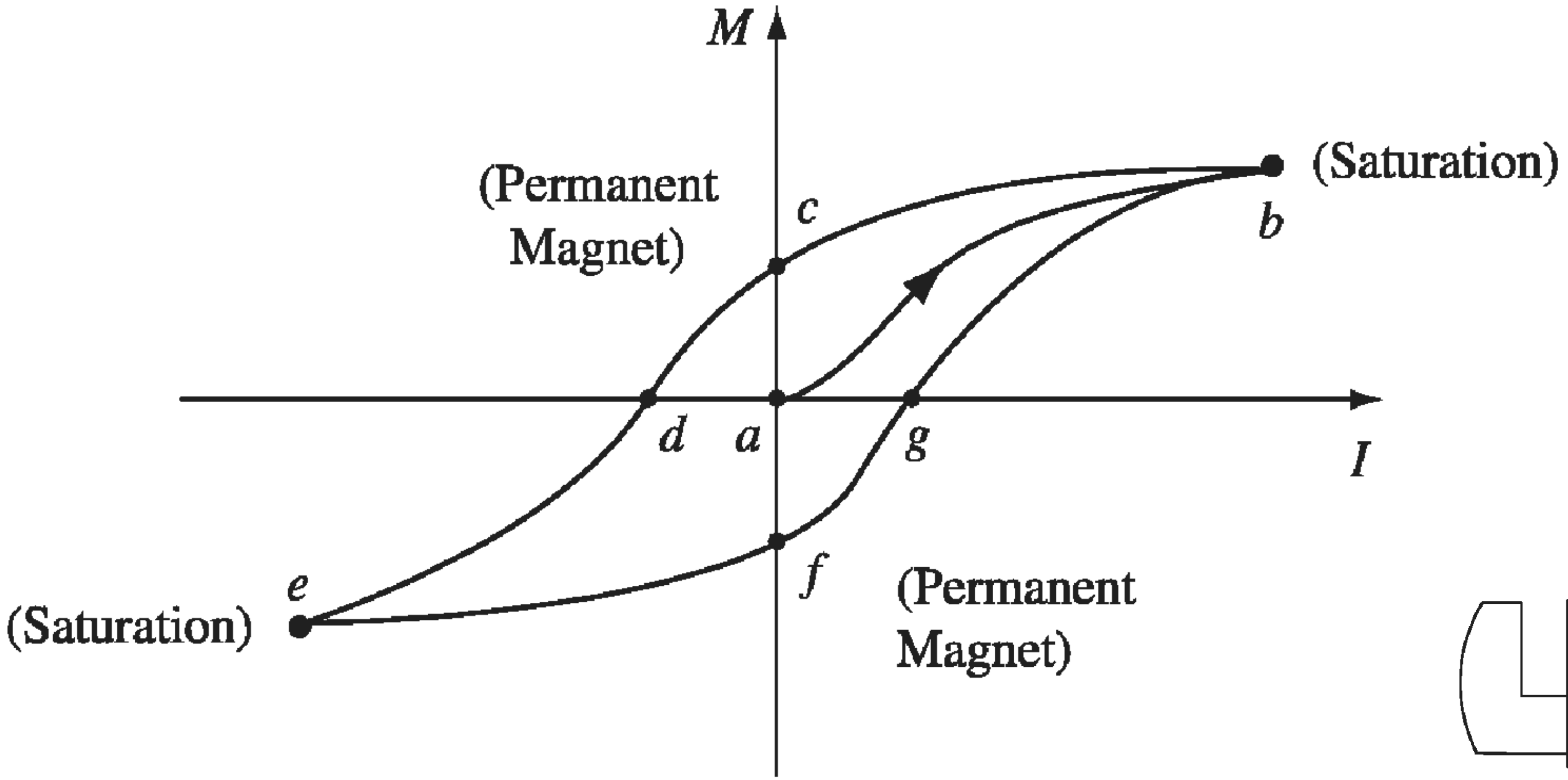


FIGURE 6.28

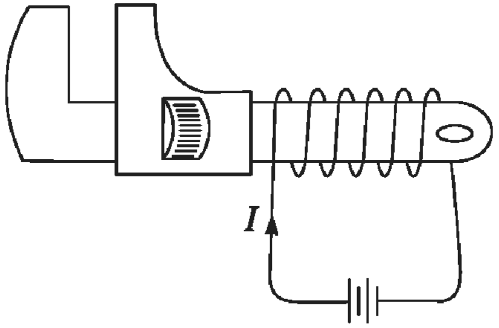


FIGURE 6.27

Note:
the word *hysteresis* derives from a Greek verb meaning "lag behind“.

Hysteresis loop

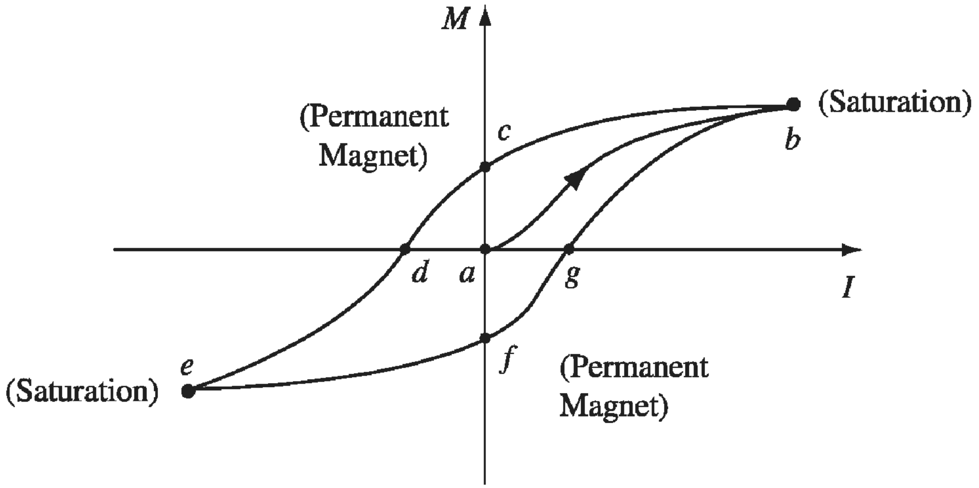


FIGURE 6.28

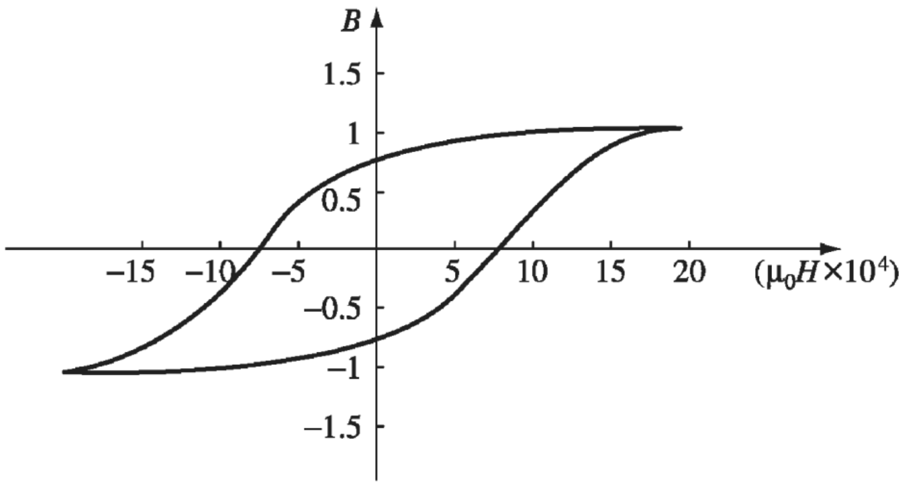


FIGURE 6.29

- The magnetization of the wrench depends not only on the applied field, but also on its previous magnetic "history".

- For instance, at three different times in our experiment the current was zero (a, c, and f), yet the magnetization was different for each of them.

- It is customary to draw hysteresis loops as plots of ***B*** **against** ***H***, rather than ***M*** against *I*. (If our coil is approximated by a long solenoid, with n turns per unit length, then ***H*** = nI, so ***H*** and ***I*** are proportional. $\boldsymbol{B} = \mu_0(\mathbf{H} + \mathbf{M})$, but in practice **M** is huge compared to **H**, so to all intents and purposes **B** is proportional to **M**.

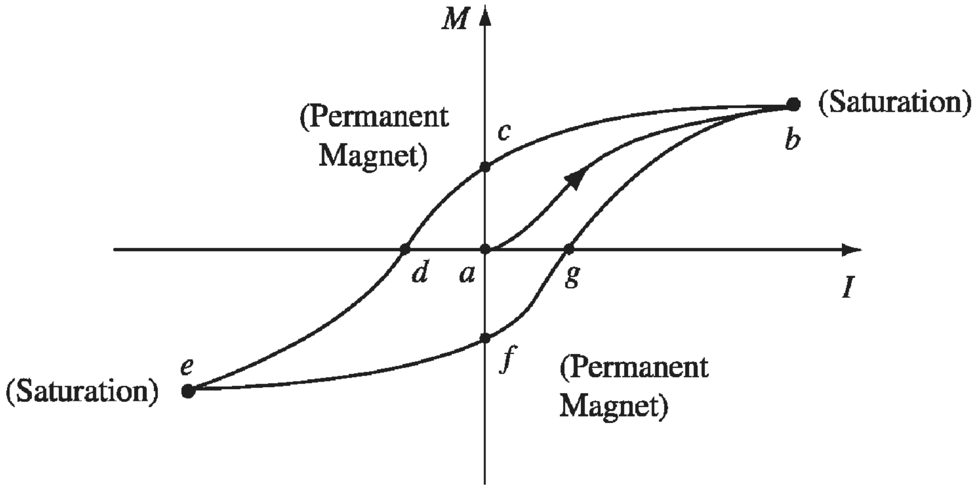


FIGURE 6.28

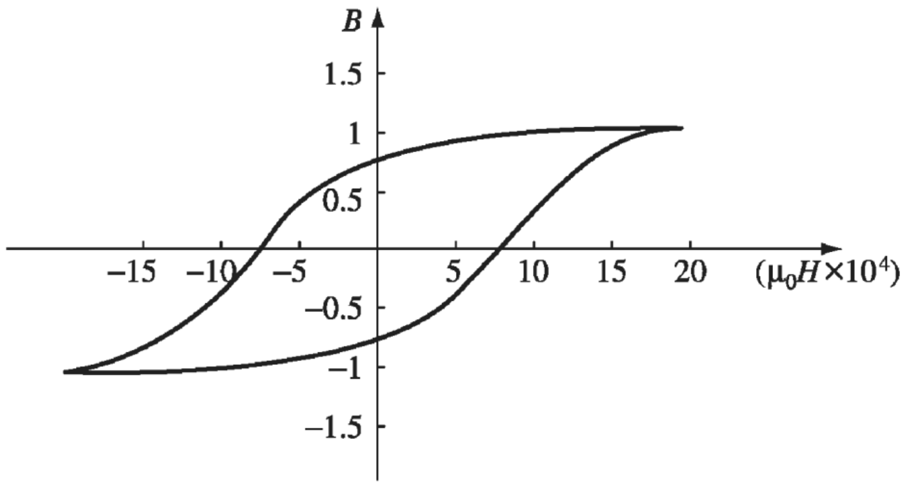
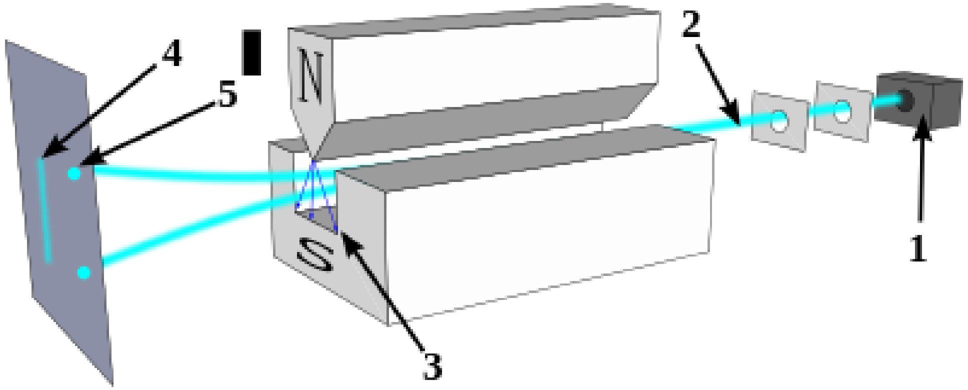


FIGURE 6.29

- $\mu_0 \boldsymbol{H}$ is the field our coil would have produced in the absence of any iron; $\boldsymbol{B}$ is what we actually got, and compared to $\mu_0 \boldsymbol{H}$, it is gigantic.
- The dipoles within a given domain line up parallel to one another. Random thermal motions compete with this ordering, but as long as the temperature doesn't get too high, they cannot budge the dipoles out of line.
- At *very* high temperatures the alignment is destroyed, and this occurs at a precise temperature, called the **Curie point**. Below this temperature, iron is ferromagnetic; above, it is paramagnetic. there is no gradual transition from ferro- to para-magnetic behavior.
- These abrupt changes in the properties of a substance, occurring at sharply defined temperatures, are known in statistical mechanics as **phase transitions**.

Quantum Aspect of Electron's Spin and Angular Momentum



Stern–Gerlach experiment: Silver atoms (containing 47 electrons) travelling through an inhomogeneous magnetic field, and being deflected up or down depending on their spin; (1) furnace, (2) beam of silver atoms, (3) inhomogeneous magnetic field, (4) classically expected result, (5) observed result

Remember that there is no classical analogue of the spin and it must be considered as an intrinsic quantum property of matter!

Electrons are spin 1/2 particles. These have only two possible spin angular momentum values measured along any axis, +1/2 or -1/2 , a purely quantum mechanical phenomenon. Because its value is always the same, it is regarded as an intrinsic property of electrons, and is sometimes known as "intrinsic angular momentum" (to distinguish it from orbital angular momentum, which can vary and depends on the presence of other particles). If one measures the spin along a vertical axis, electrons are described as "spin up" or "spin down", based on the magnetic moment pointing up or down, respectively.

Spin magnetic dipole moment

$$\boldsymbol{\mu}_{\mathrm{s}} = -g_{\mathrm{s}}\,\mu_{\mathrm{B}}\,\frac{\mathbf{S}}{\hbar}$$

μ_B is the Bohr magneton

Here **S** is the electron spin angular momentum. The spin g-factor is approximately two: $g_s \approx 2$. The magnetic moment of an electron is approximately twice what it should be in classical mechanics. The factor of two implies that the electron appears to be twice as effective in producing a magnetic moment as the corresponding classical charged body.

Orbital magnetic dipole moment

$$\boldsymbol{\mu}_{L} = -g_{\mathrm{L}}\,\mu_{\mathrm{B}}\,\frac{\mathbf{L}}{\hbar}$$

Here **L** is the angular momentum for the orbital motion, and $g_L = 1$ is the electron orbital g-factor . The revolution of an electron around an axis through another object, such as the nucleus, gives rise to the orbital magnetic dipole moment.