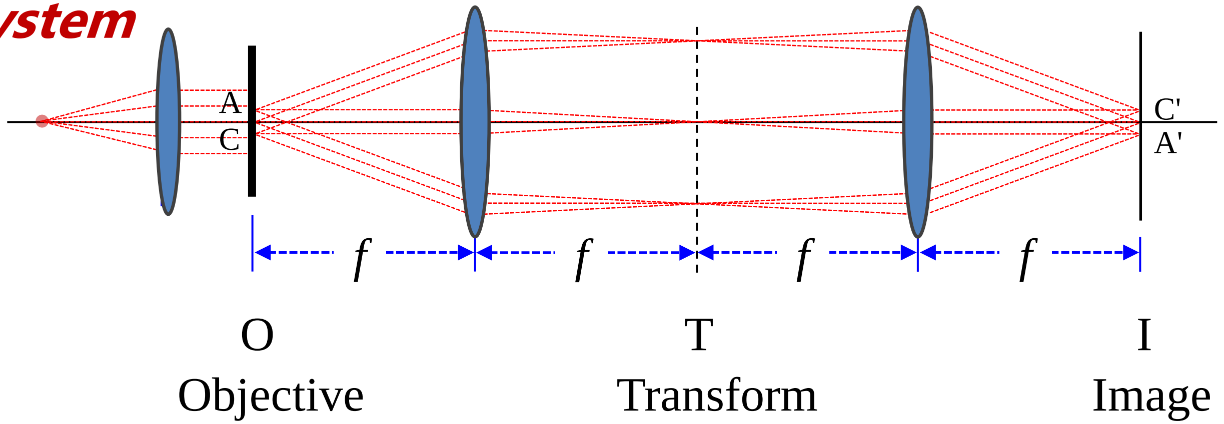
C8 Fourier Optics

- Abbe image theory;
- Understand spatial frequency filtering, 4*f* systems;
- Understand the basic application of FO.

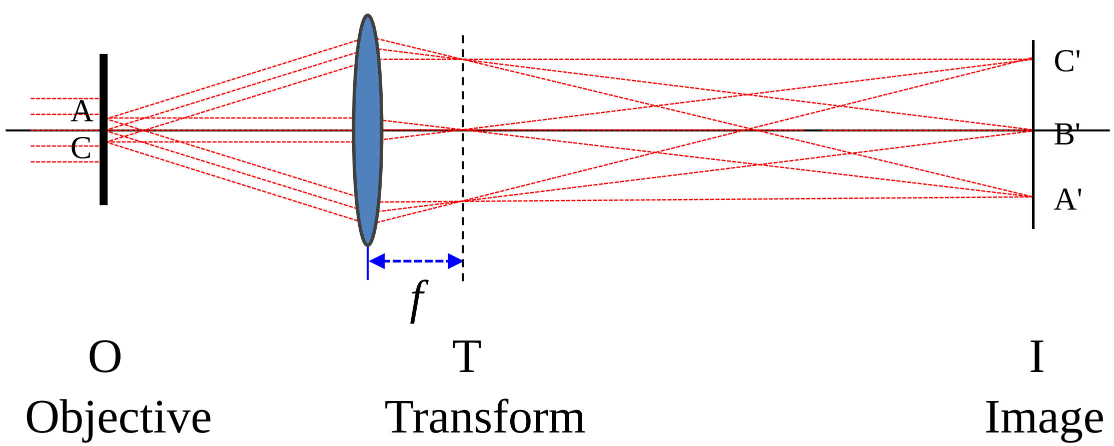
***4f* system**



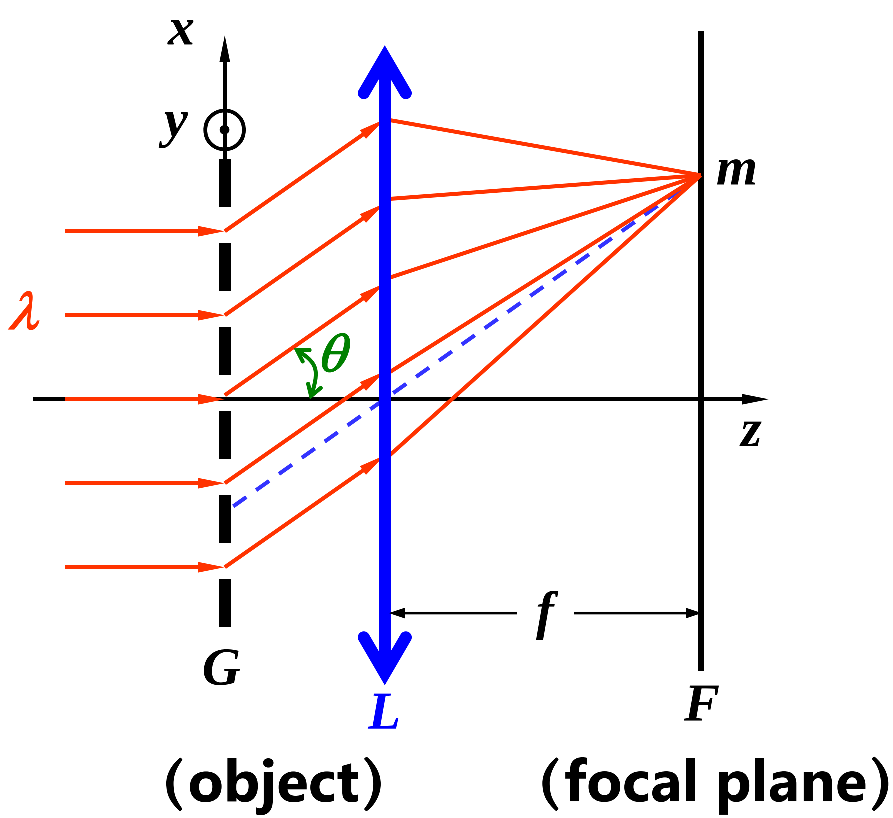
§ 8.1 Abbe's image theory

- E. Abbe (Germany) proposed a two-step diffraction process:
- 1st: form a Fraunhofer diffraction spot on the back focal plane of the lens;
- 2nd: each diffraction spot as new wavelet source emits spherical waves which continue to propagate (diffraction), The images that are superimposed on each other in the plane form the inverse of the spectral function.

- From the wave optics, called the **Abbe's image theory**.



- Fraunhofer diffraction of gratings by using Abbe's image theory.



$$f_x = \frac{k_x}{2\pi} = \frac{\sin\theta}{\lambda}$$

$$f_y = \frac{k_y}{2\pi} = 0$$

$$f_z = \frac{k_z}{2\pi} = \frac{\cos\theta}{\lambda}$$

Grating equation

$$d\sin\theta = m\lambda$$

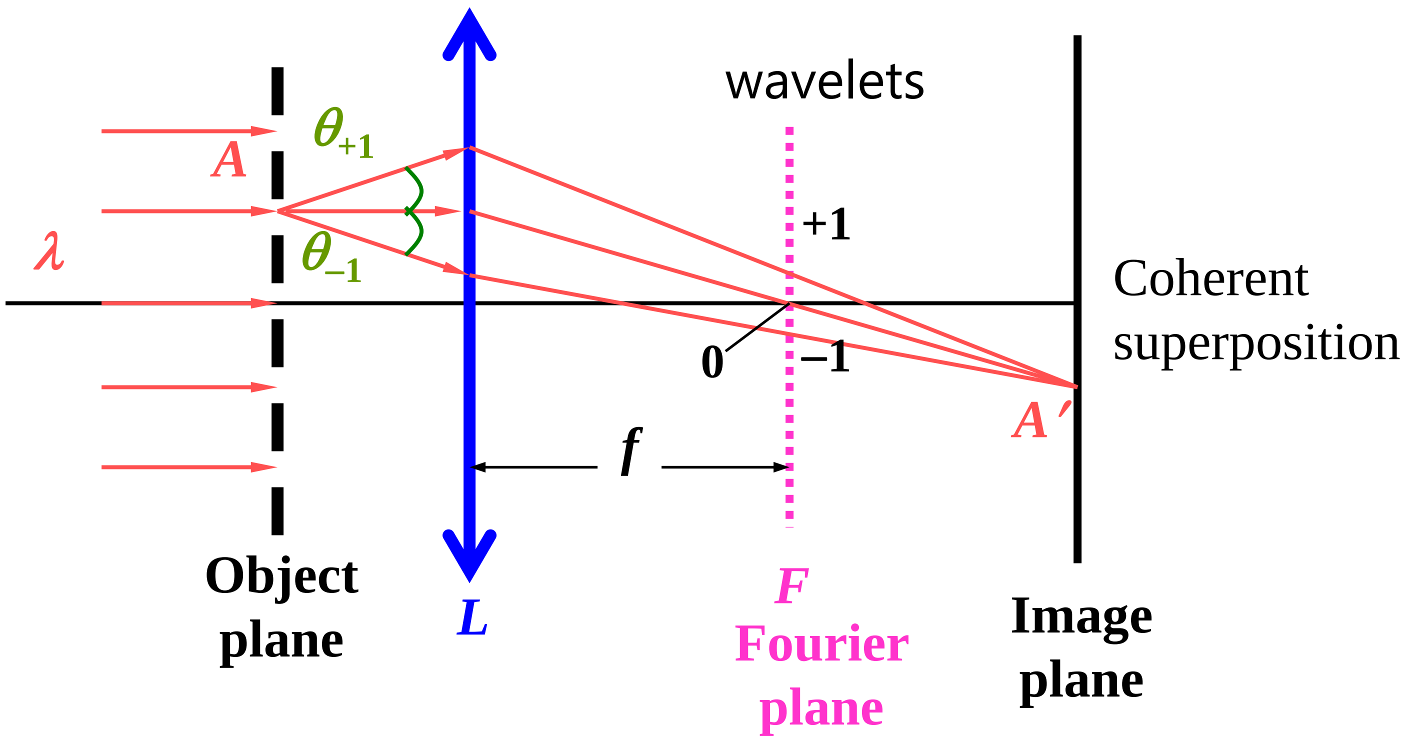
$$m = 0, \pm 1, \pm 2 \cdots$$

$$\boxed{f_x = \frac{\sin\theta}{\lambda} = \frac{m}{d}}$$

Some discussions about $f_x = \dfrac{\sin\theta}{\lambda} = \dfrac{m}{d}$

(1) An object is a collection of information with different spatial frequencies. A certain θ corresponds to a certain f_x, and also corresponds to a certain $\mathbf{k}$.

(2) Unchanging part in the object $d \to \infty$, that is $f_x = 0$, $\theta = 0$. The zero-order bright spot reflects the parts that do not change on the object.



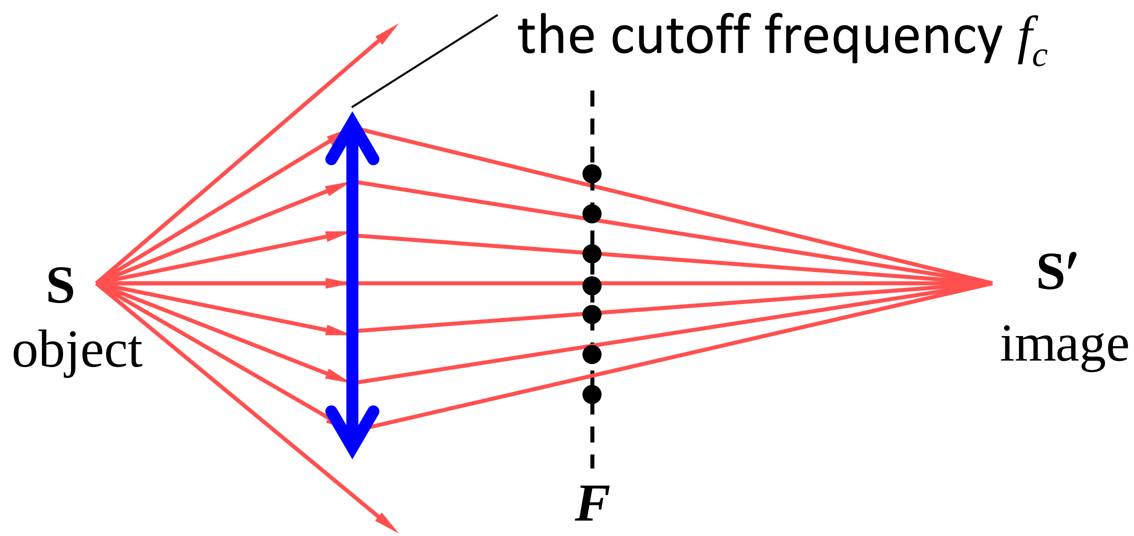
First step: The incident light is diffracted at the object plane, and a series of patterns(spatial spectrum) are formed on the focal plane of L.

Second step: Wavelets emitted by the diffraction fringes coherently superimpose at the image plane and form the image.

Low pass filtering

- The higher the spatial frequency corresponds to the larger the diffraction angle.

$$f_x = \frac{\sin\theta}{\lambda} = \frac{m}{d}$$

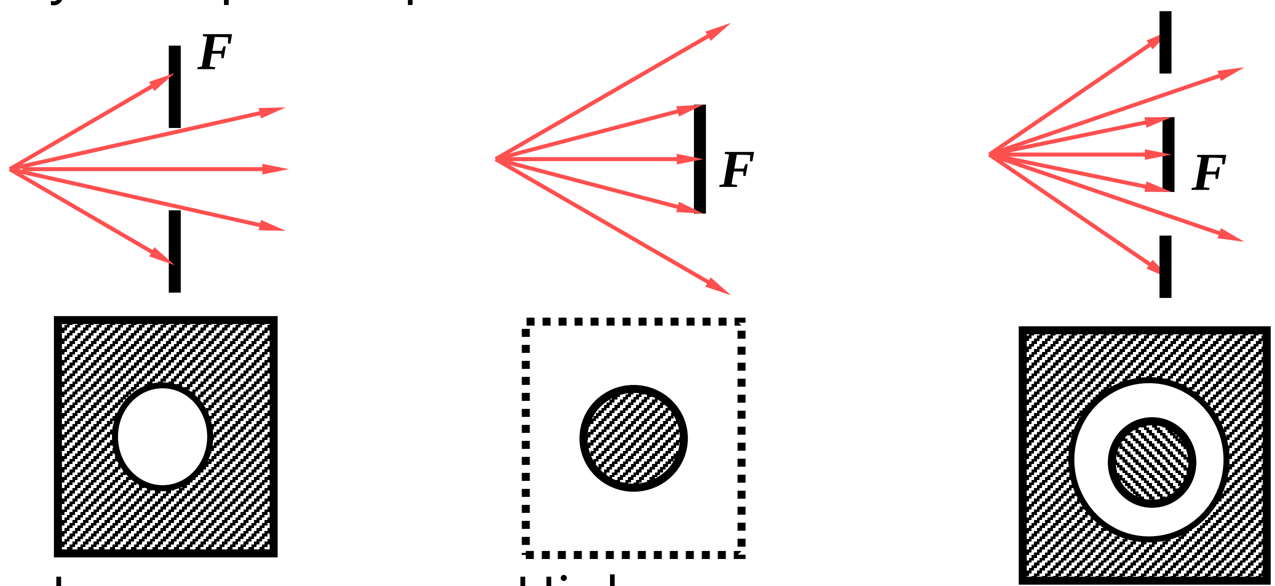


- Aperture of the lens is limited, after the object light passes through the lens, the higher frequency information is always lost, has a loss of detail.
- The lens is a "low-pass filter". To improve the image quality, it is necessary to enlarge the lens.

§ 8.2 Spatial filtering

Lens lose high frequency information, changing the spectrum can change the information of the object light: **spatial filtering**.

A spatial filter can be placed on the spectrum plane to modify the spatial spectrum.



Low pass High pass Band pass

Fourier optics

- After the 1940s, optics has made many major breakthroughs and advances in theoretical methods and practical applications, which gave birth to modern optics. Representative event:

1948s Holographic

1955s Optical transfer function

1960s The birth of the laser

Fourier optics

- One important thing in modern optics is the introduction of the concept of transformation, and thus the formation of a new branch of optics - **Fourier optics**, also known as **Information optics** or **Transformation optics**.

Basic idea

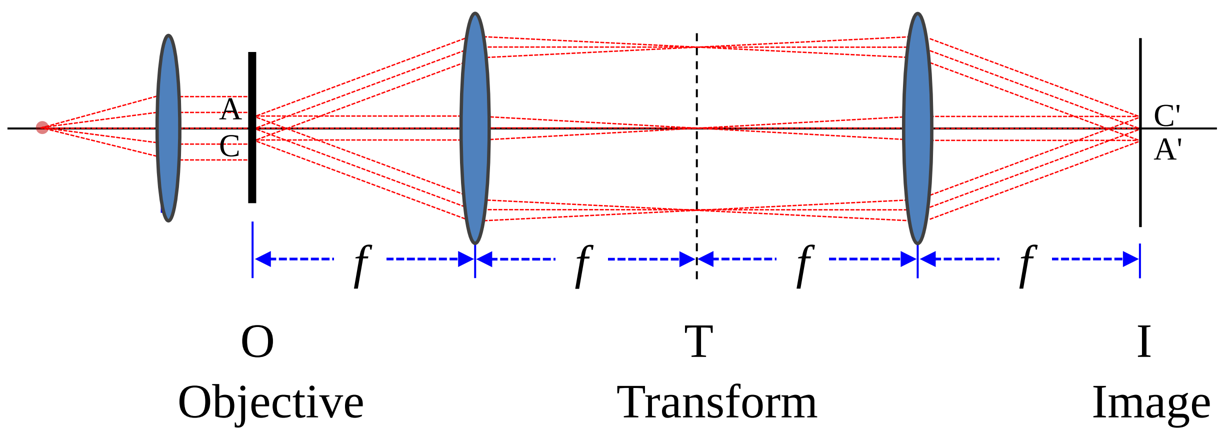
The information of the object plane is analyzed by the language of the spectrum, and the information is processed by means of changing the spectrum.

Object plane → Spatial spectrum analysis system → Spatial spectrum → Processing system → Object image

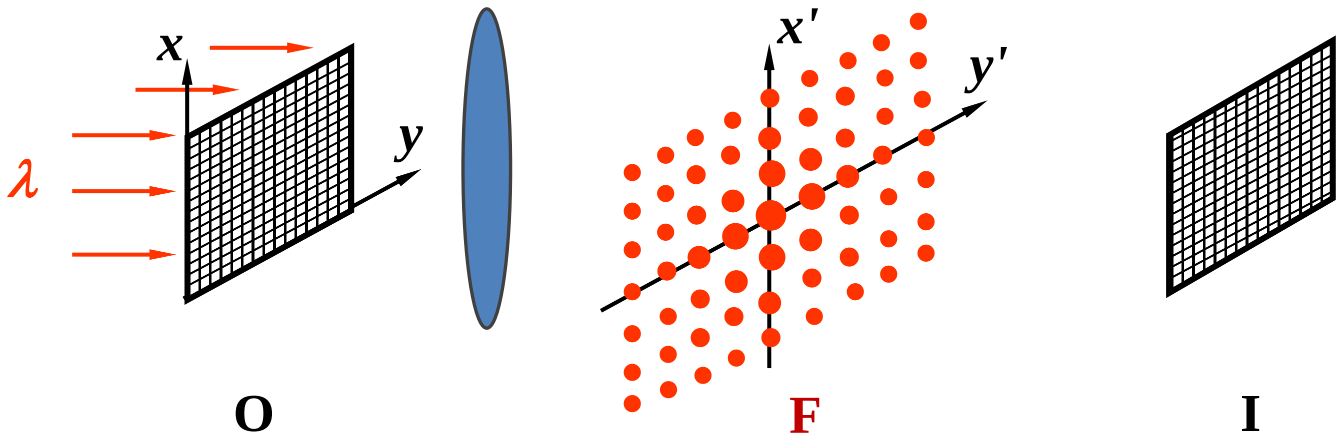
4*f* system

Object plane → Spatial spectrum analysis system → Spatial spectrum → Processing system → Object image

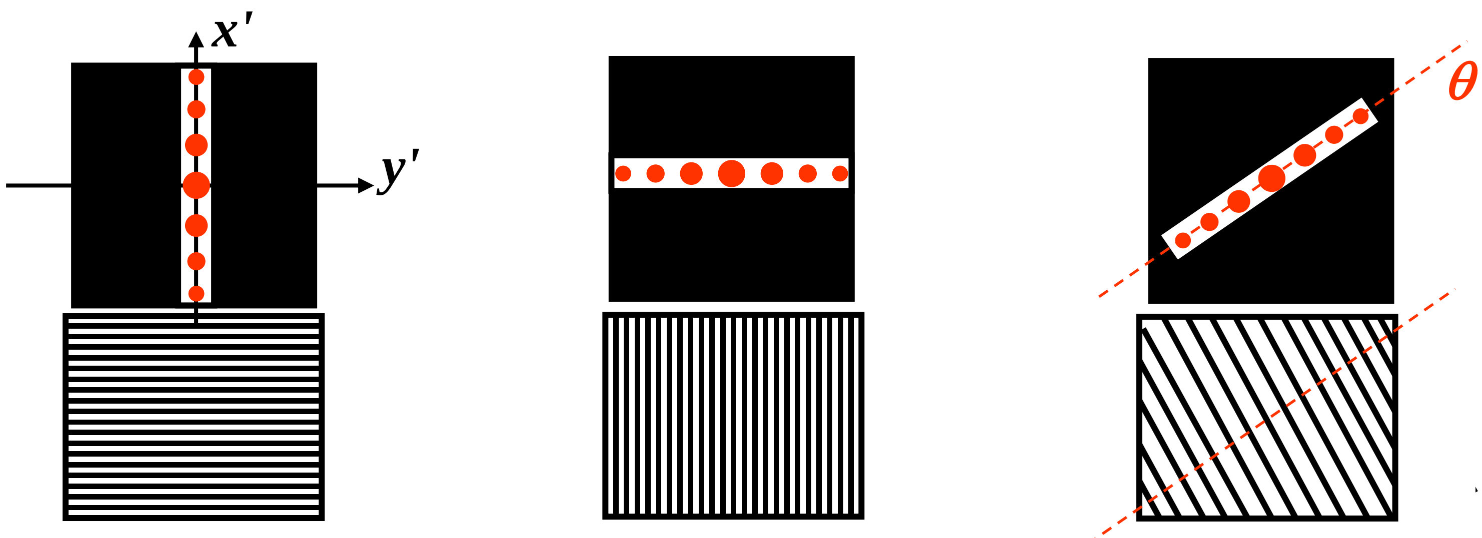
- A system that performs spatial filtering and optical processing using Fourier transform of lightwave propagation is called a **coherent optical processing system**.
- The commonly used system is a 4F system, as shown below.



Abbe-Porter experiment



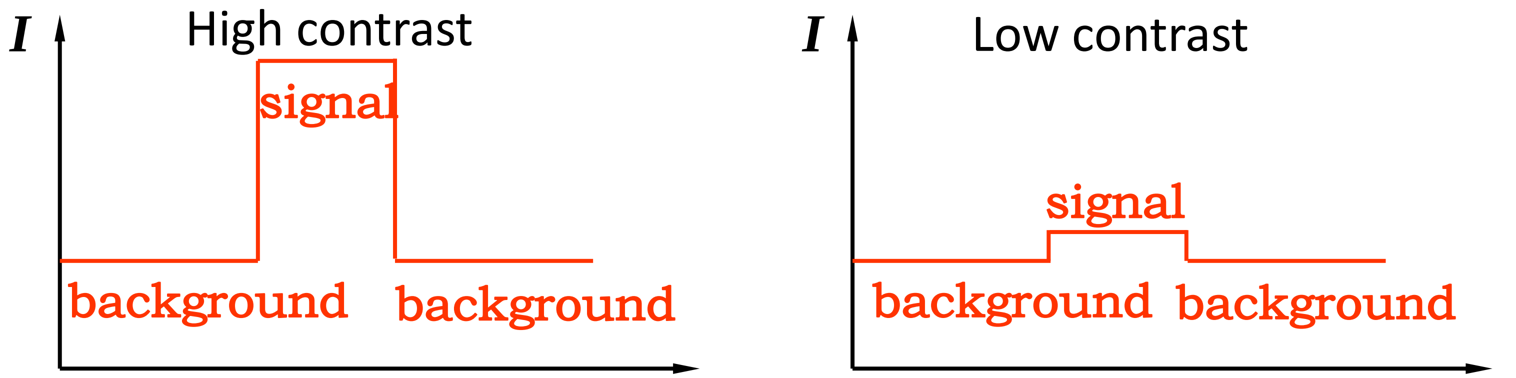
Preserve f_x Preserve f_y Preserve f_θ



Optical information processing

① **Contour highlighting (edge enhancement) and recognition of low contrast graphics**

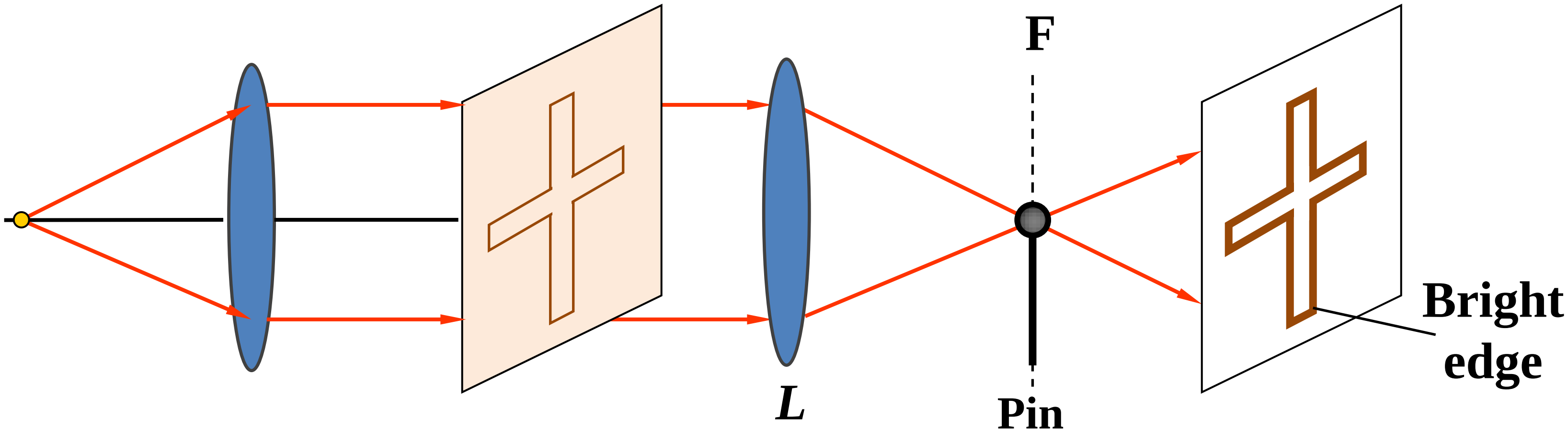
The ratio of the brightness of an object to the brightness of the background is called contrast.



Hooking out the outline of an object is an effective way to identify low-contrast objects. The edge brightness changes drastically and the high frequency components are abundant.

Optical information processing

① **Contour highlighting (edge enhancement) and recognition of low contrast graphics**



Method Place a high-pass filter on the spectral plane F of the object, blocking the 0-order and low-frequency components, thereby highlighting the brightness of the outline - forming a bright edging.

Optical information processing

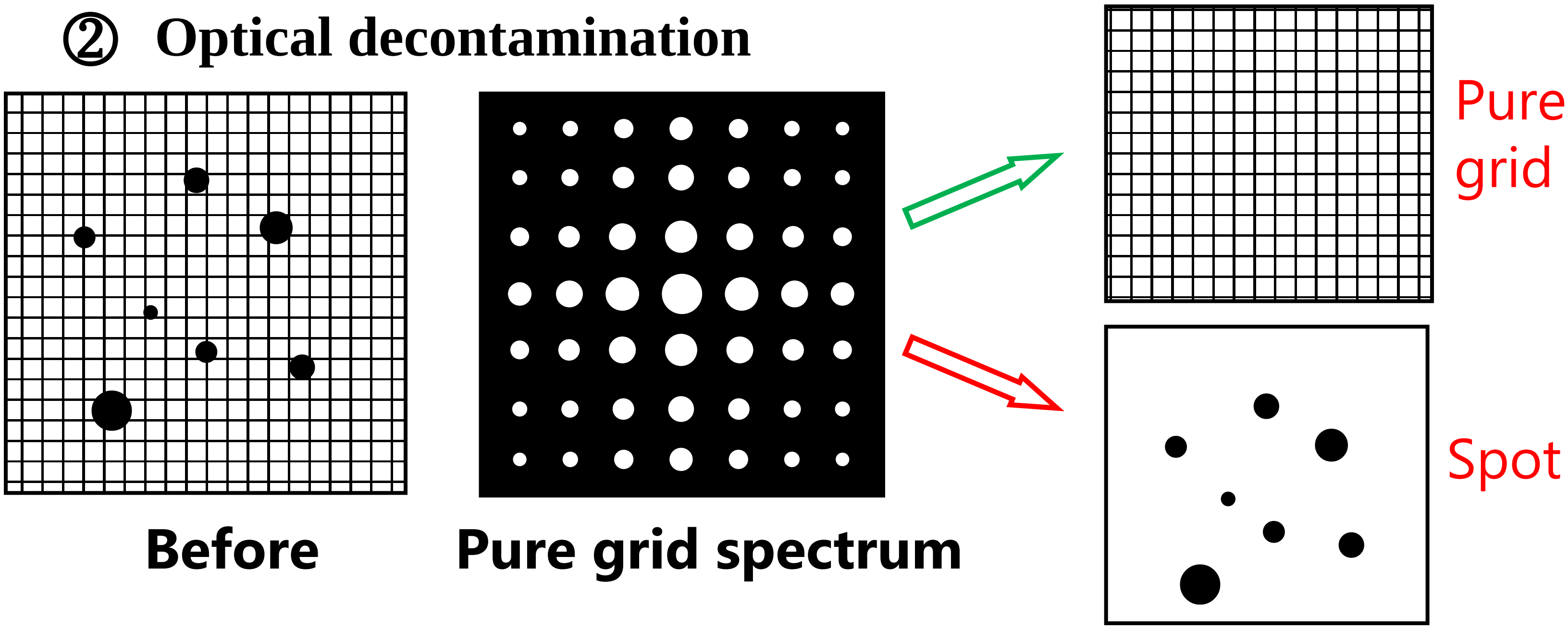
① **Contour highlighting (edge enhancement) and recognition of low contrast graphics**

Before highlighting contour

After highlighting contour

Optical information processing

② **Optical decontamination**



Mesh is obtained by positive filtering of the grid spectrum.

Spot is obtained by negative filtering of the grid spectrum.

Mainly used for industrial plate making (large scale integrated circuits) to check contamination or masks printing.

Optical information processing

③ Optical recognition

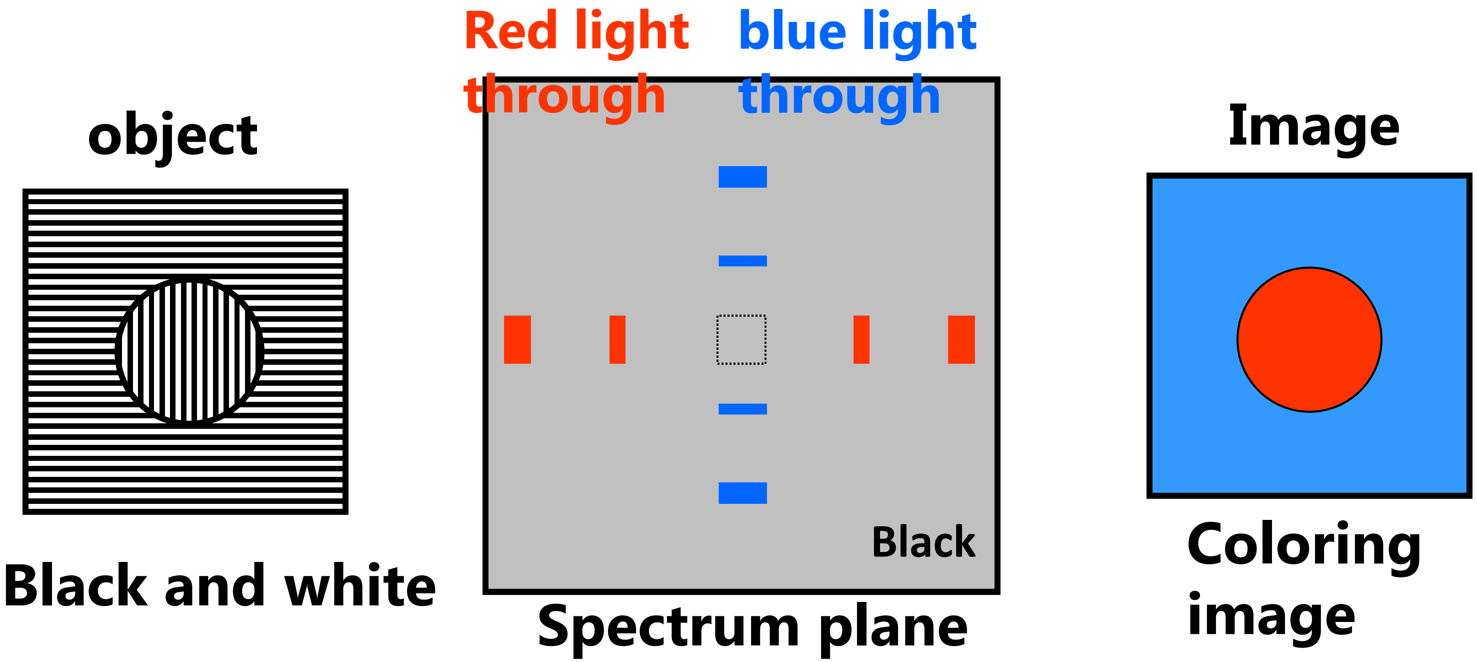
Compare the Fourier spectrum of the known object with the Four's spectrum of the object to be tested to find the target to be tested.

Application：

- Detect military targets from satellite photos
- Finding a word from a file
- Identifying cancer cells
- Conducting aerial surveys
- Optical detection (fingerprint recognition)

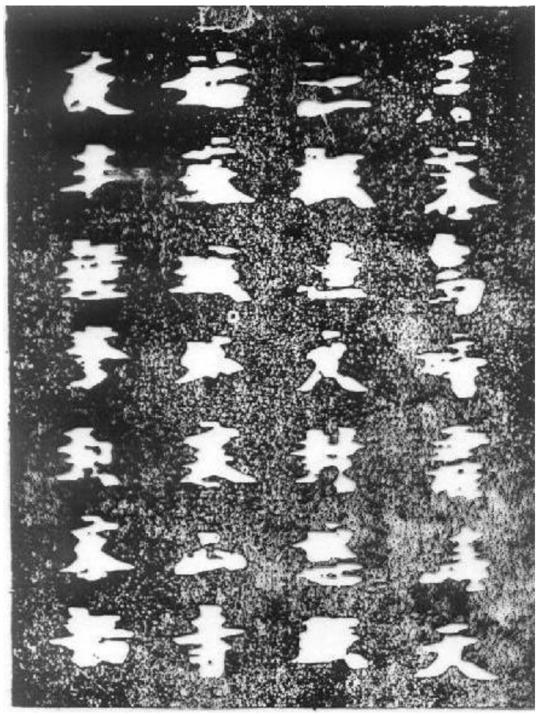
Optical information processing

④ θ modulation (false color coding)



Optical information processing

⑤ Fuzzy image processing (defocus blur, motion blur)



Before processing

After processing

The key is to find a suitable spatial frequency filter function.

Optical information processing

⑥ Phase contrast microscope

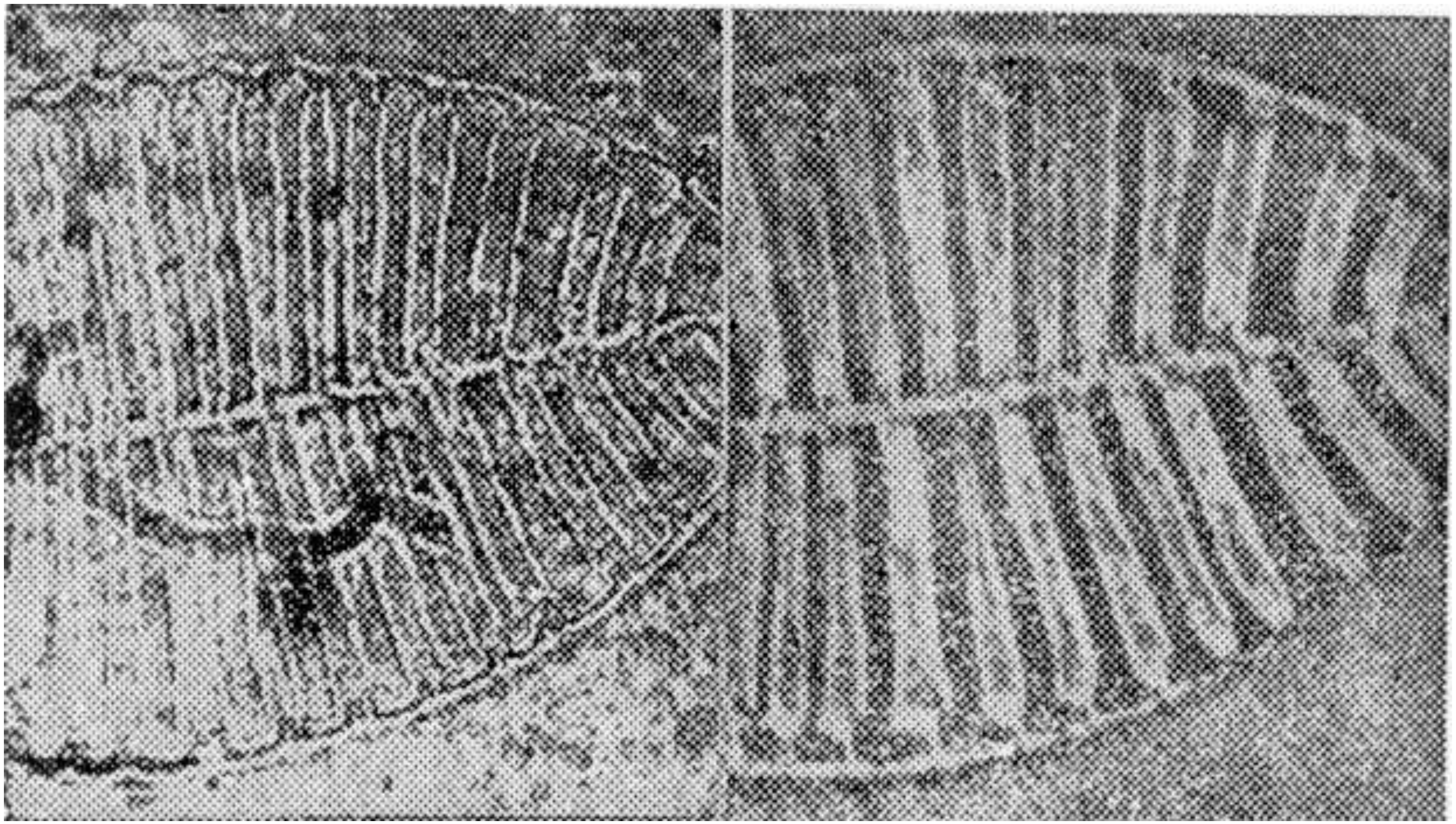
- Zernike phase contrast microscopy observed cells that were not stained but still viable and transparent cytoplasm;

Nobel Prize in Physics, 1953

[Frits (Frederik) Zernike](#)
Netherlands, 1888-1966

The key to information processing is to study the spectral characteristics of the information, and develop a corresponding spatial filter for it, so as to change the spectrum as needed to achieve the purpose of processing the image information.

Optical information processing

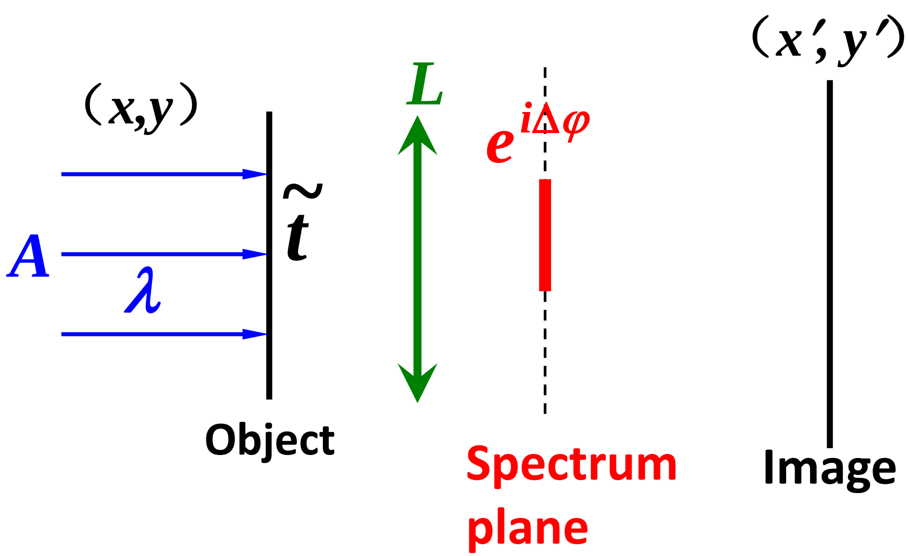


Diatom photos taken with a normal microscope (left) and a phase contrast microscope (right)

Optical information processing

⑥ Phase contrast microscope - increase the contrast

When the sample is colorless and transparent, its transmittance function is phase type. $\tilde{t}(x,y)=e^{i\varphi(x,y)}$, φ is small.



Zernike proposed a phase contrast method: drop a small liquid (thickness *h*) at the center of the glass sheet and put it on the spectrum plane to cause a 0-order phase shift.

$$\Delta\varphi=\frac{2\pi nh}{\lambda}$$

0^{th}-order

$$\tilde{U}_{\text{object}}(x,y)=A\tilde{t}(x,y)=Ae^{i\varphi(x,y)}=A\left[1+i\varphi-\frac{1}{2!}\varphi^2+\cdots\right]$$

Optical information processing

After passing through the phase plate, **0^{th} shift $\Delta\varphi$**. Other changes are not big.

$$\tilde{U}_{\text{image}}(x',y') = A\left[e^{i\Delta\phi} + i\phi - \frac{1}{2!}\phi^2 + \cdots\right]$$

$$= \boldsymbol{A\left[(e^{i\Delta\varphi} - 1) + e^{i\varphi(x,\ y)}\right]}$$

Intensity of image

$$I(x',y') = \tilde{U}_{\text{image}}(x',y') \cdot \tilde{U}^*_{\text{image}}(x',y')$$

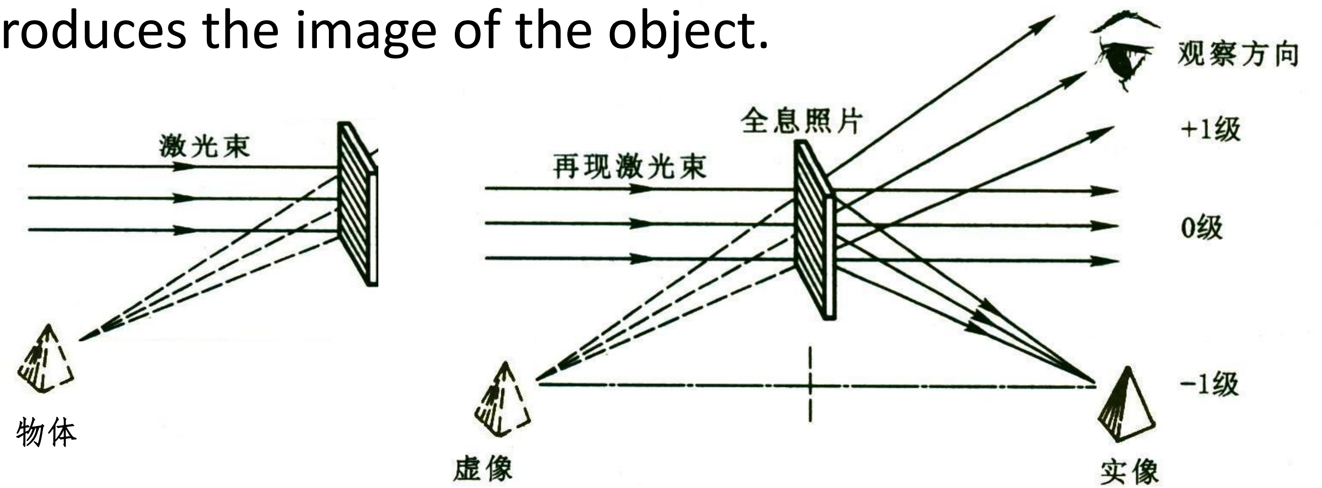
$$\boldsymbol{\approx A^2[1 + 2\sin\Delta\varphi \cdot \varphi(x',y')]} \quad \boldsymbol{(\varphi << 1)}$$

In order to highlight the phase change, usually selected $\boldsymbol{\Delta\varphi = \frac{\pi}{2}}$, that is $h = \frac{\lambda}{4n}$,

So, $$I(x',y') = A^2[1 + 2\varphi(x',y')]$$

§ 8.3 Holography

- In 1948s, Hungarian scientist Dennis Gabor proposed the principle of Holography.
- **Wavefront record**： The coherent object light wave O is superimposed on the reference light wave R, and interference fringes formed on the photosensitive sheet are holograms.
- **Wavefront reproduction:** The hologram is again illuminated with reference light, which diffracts the wavefront and reproduces the image of the object.



Holographic features

- Holography must be illuminated by coherent light;
- Point-to-surface correspondence between the object and the hologram: the information (spatial frequency) of each point is filled with the entire film, and each point contains information about each point on the object. Therefore, the original object can still be reproduced by part of hologram, but resolution is lowered.
- Hologram can be exposed multiple times with different angles of illumination to record multiple images.
- Interferometry: Convert spatial phase information into spatial strength information! ! Achieve observation of relative phase.