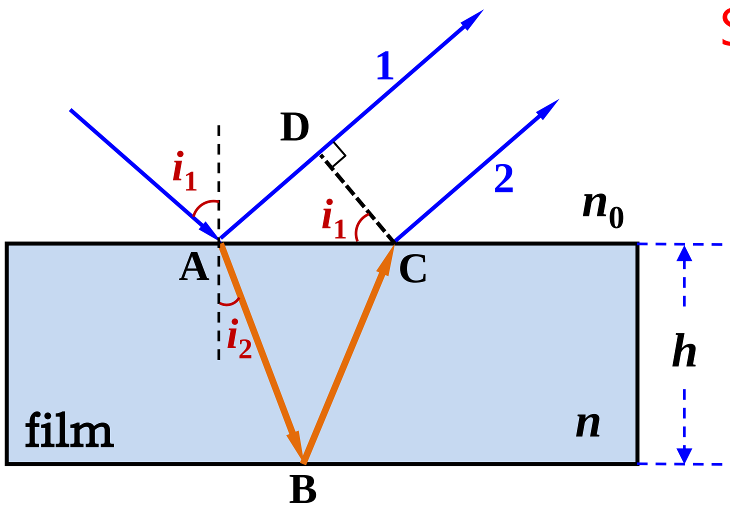
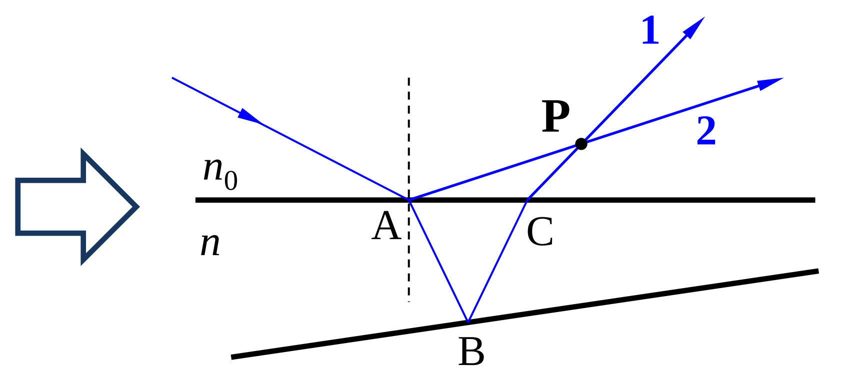
Equal thickness interference

- For a film with unequal thickness, the reflected (or refracted) light will interfere near the surface. The OPL difference is determined mainly by the thickness and is called **equal thickness interference**.



Same incident angel different thickness



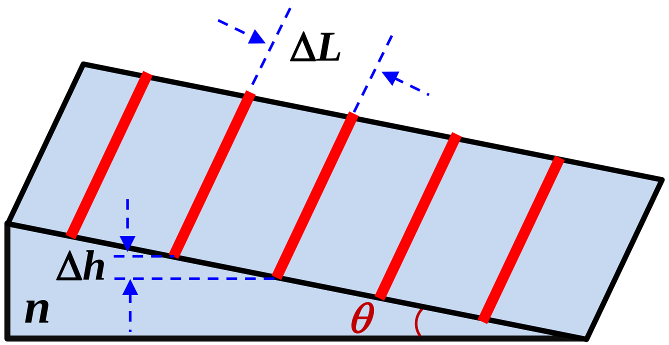
$$\Delta = 2h\sqrt{n^2 - n_0^2 \sin^2 i_1} + \frac{\lambda}{2}$$

Determined by h

Equal thickness interference

- When A is near to C, we can think that $h_A \approx h_C$, so

$$\Delta = 2nh\cos i_2 + \frac{\lambda}{2}$$



- When is normally incident：

$$i_1 \approx i_2 \approx 0$$

$$\Delta \approx 2nh + \frac{\lambda}{2}$$

- m^{th} order bright fringes

$$2nh_m + \frac{\lambda}{2} = m\lambda \quad \Rightarrow \quad \boxed{h_m = (m - 1/2)\frac{\lambda}{2n} = \text{Const}}$$

- The trajectory equation is a straight line parallel to the edge

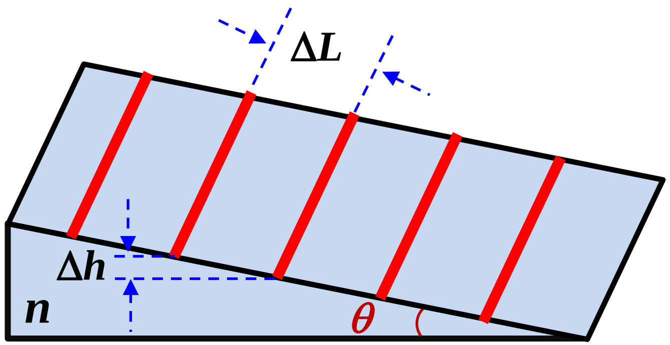
Equal thickness interference

- m+1$^{\text{th}}$ order bright fringes and m^{th} order

$$h_m = (m - 1/2)\frac{\lambda}{2n}$$

$$h_{m+1} = (m + 1\text{-}1/2)\frac{\lambda}{2n}$$

$$\Rightarrow \quad \Delta h = \frac{\lambda}{2n}$$



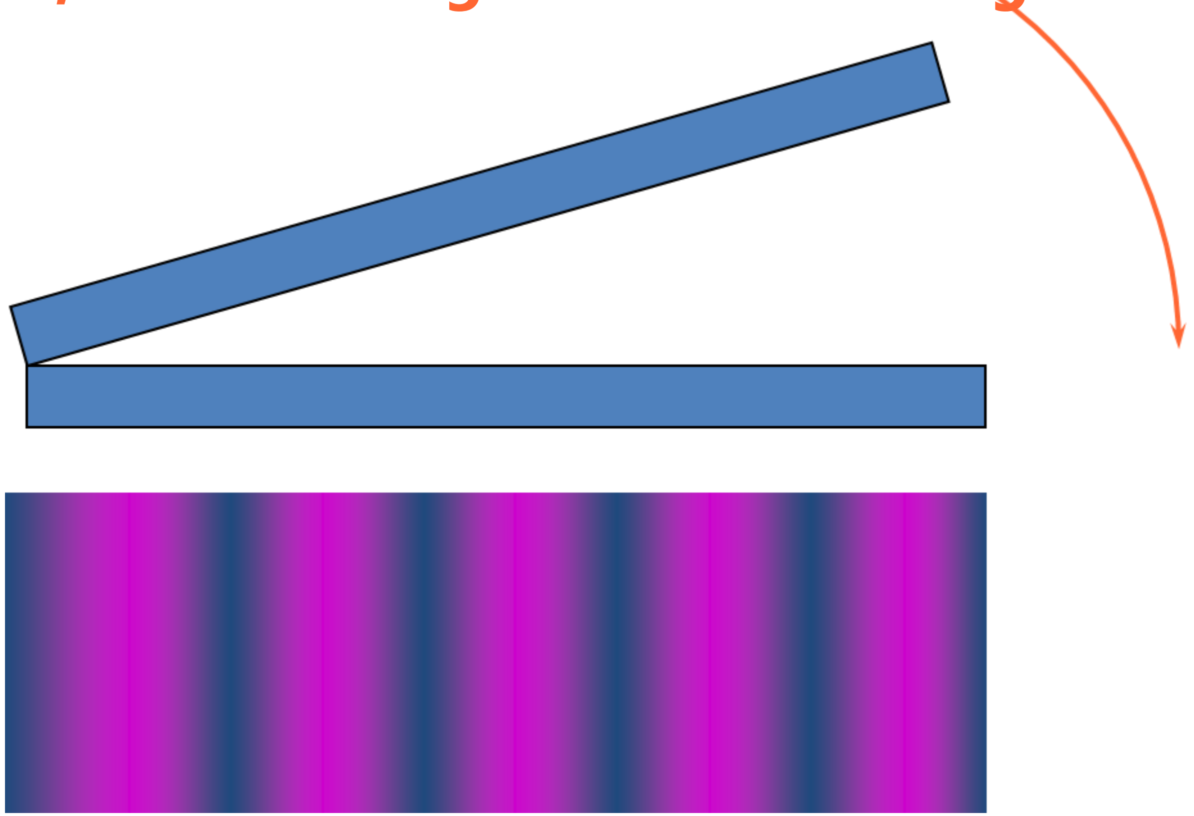
- Fringes spacing

$$\Delta L = \frac{\Delta h}{\tan\theta} \approx \frac{\Delta h}{\theta} = \frac{\lambda}{2n\theta}$$

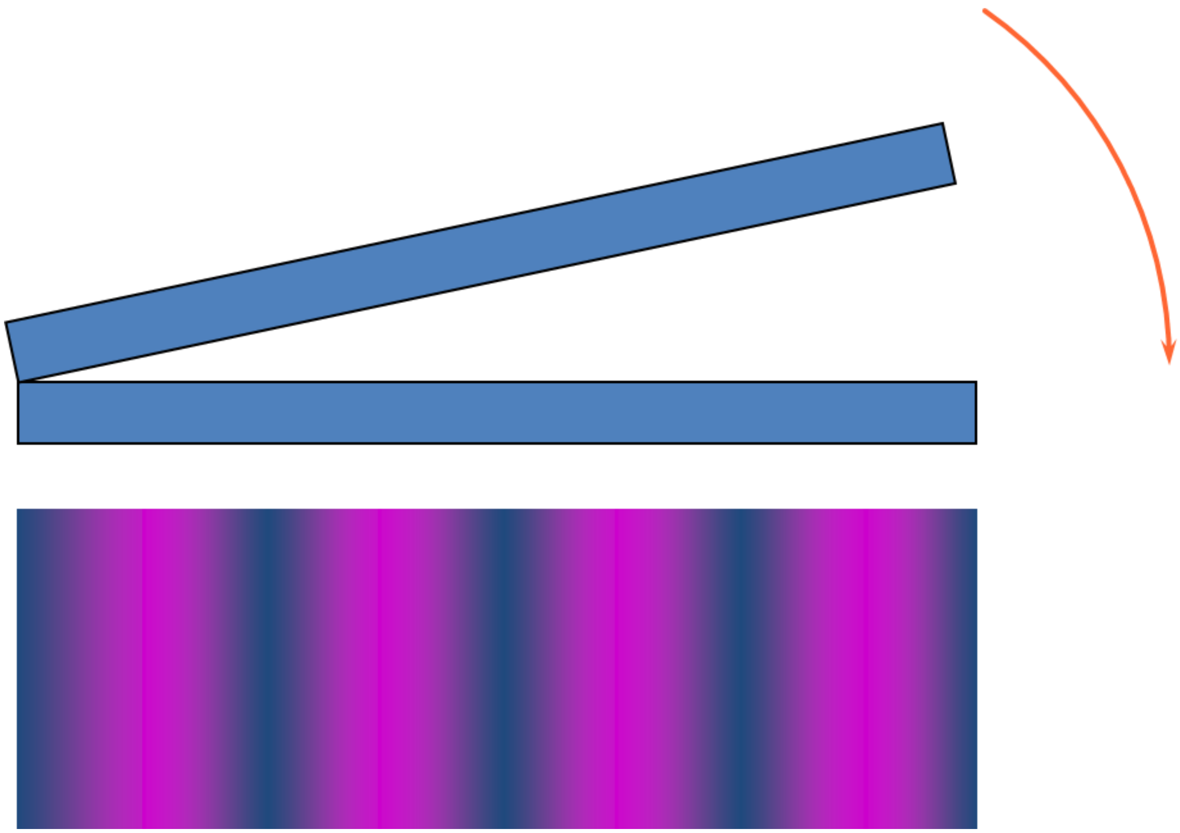
- The change in the density (spacing) of the fringes reflects the change in the wedge angle.
- It can be used to measure changes in small angles.

Equal thickness interference

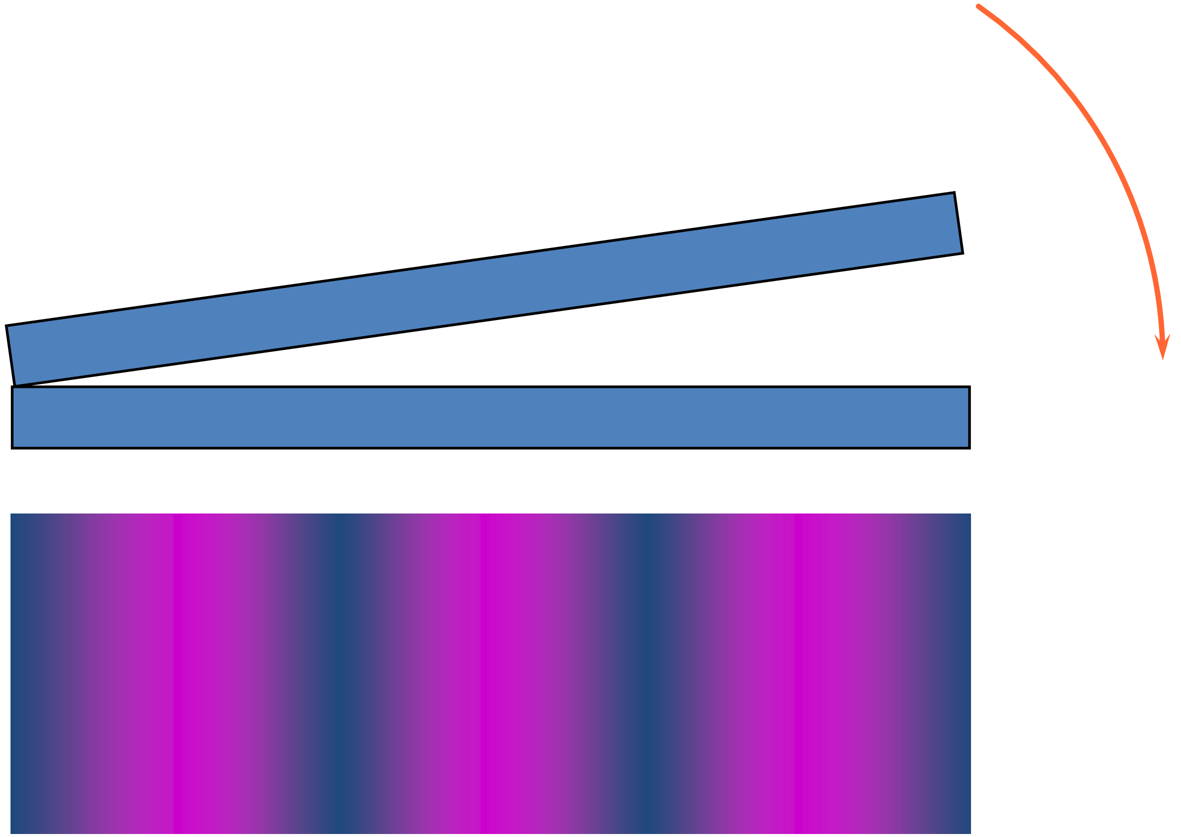
The angle becomes smaller, the fringes become wider, and the fringes move to the right



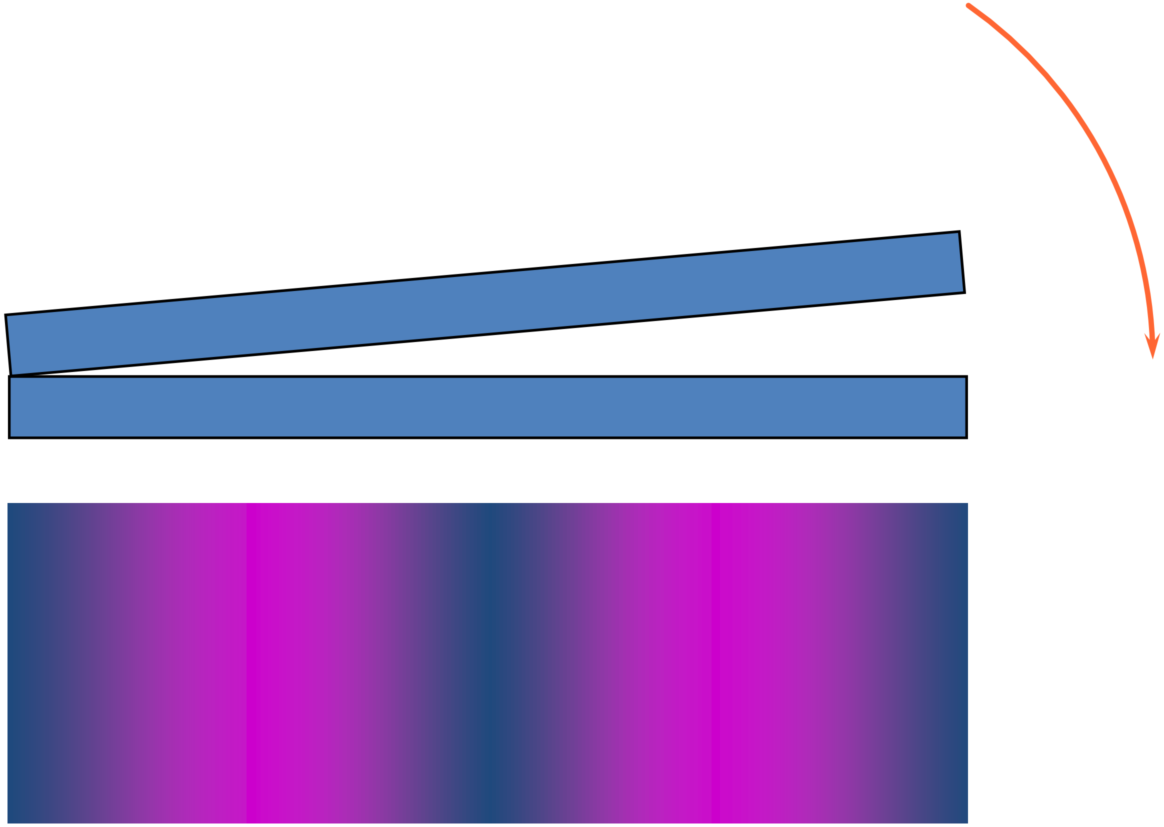
Equal thickness interference



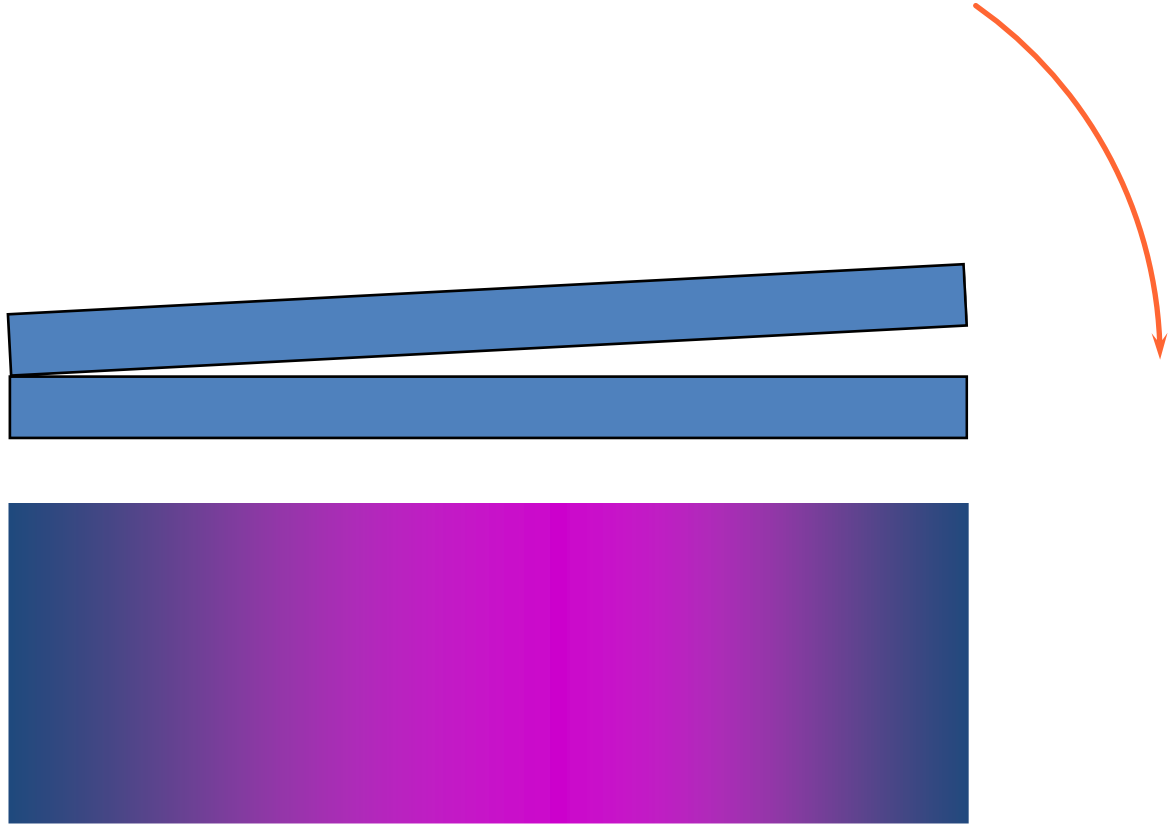
Equal thickness interference



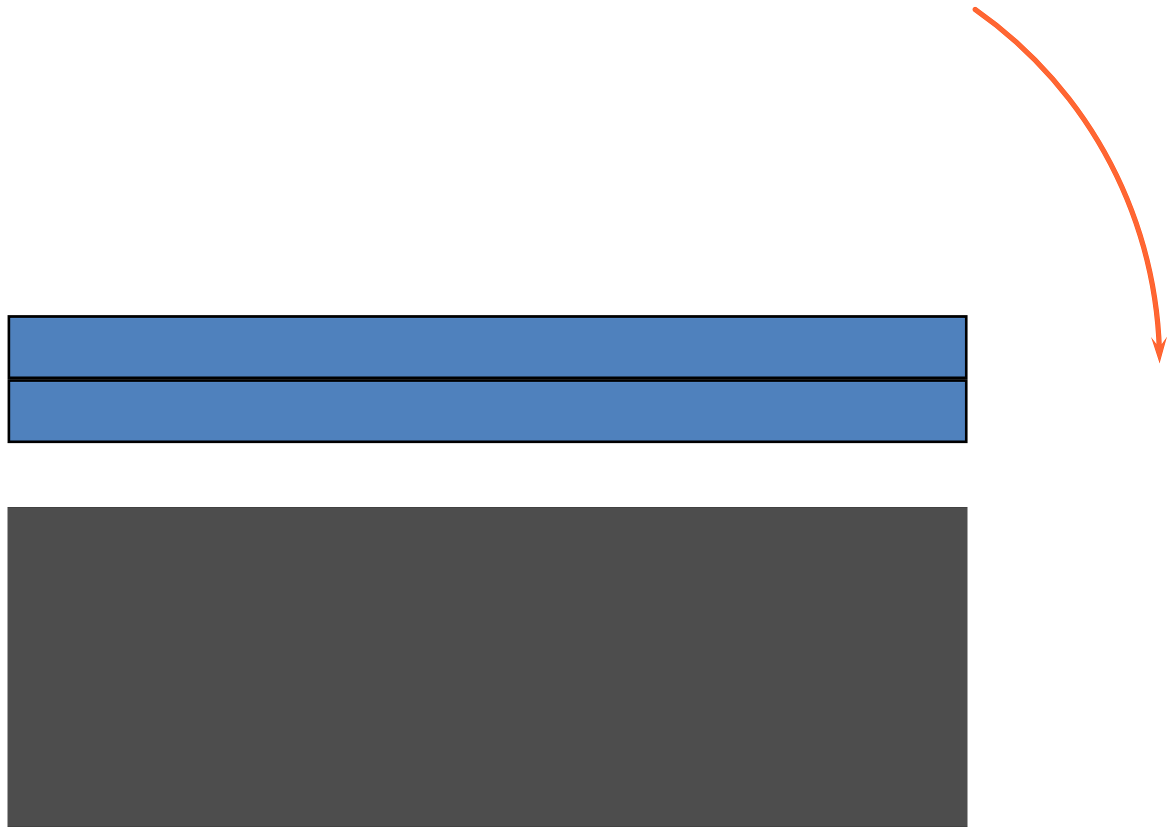
Equal thickness interference



Equal thickness interference



Equal thickness interference

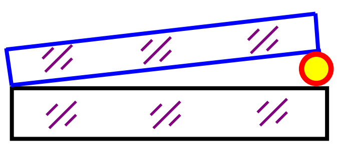


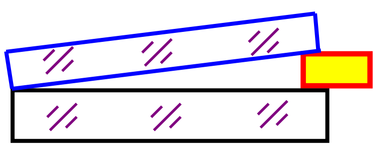
Equal thickness interference

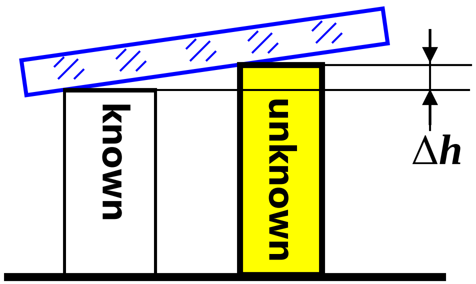
- **Applications:**

$$\Delta L \approx \frac{\lambda}{2n\theta}$$

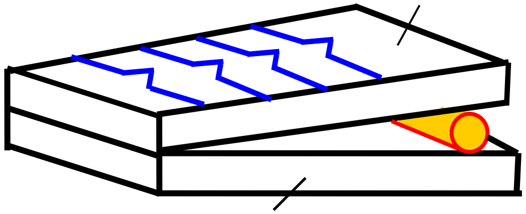
- ☐ Measure wavelength
- ☐ Measure refractive index
- ☐ Measuring small changes







- ☐ Measuring surface irregularity



Equal thickness interference

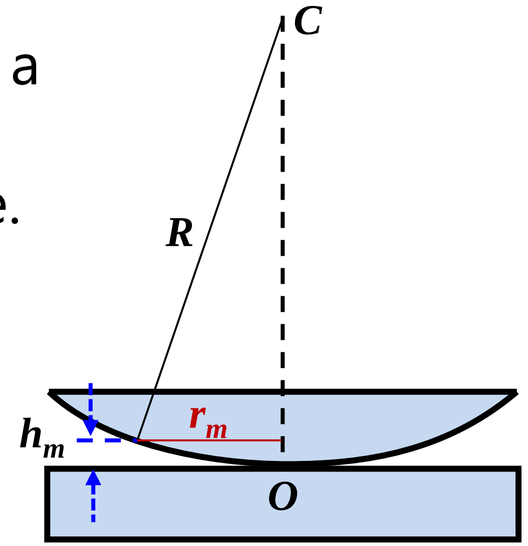
- **Interference fringes**
  - When white light is illuminated,
    >>Colorful fringes.

$$\Delta L \approx \frac{\lambda}{2n\theta}$$

- For same region, uniform thickness, different colors when viewed from different angles: equal inclination interference;
- Unequal thickness: equal thickness interference.

Newton's ring

If a large R plane-convex lens is placed on a flat plate, the reflected light from the two interfaces of an air wedge can interference. The fringes are concentric rings called **Newton's rings.**



Newton's ring

- Normal incidence： $i_1 \approx i_2 \approx 0$

$$\Delta \approx 2nh + \frac{\lambda}{2}$$

- m^{th} order dark fringes （$n_{air} = 1$）：

$$2h_m + \frac{\lambda}{2} = (2m+1)\frac{\lambda}{2} \Rightarrow \boxed{h_m = \frac{m\lambda}{2}}$$

□ The fringe trajectory equation is a concentric ring.

□ At center point, $h = 0$ ($m = 0$), $\Delta = \frac{\lambda}{2} \Rightarrow$ dark

Interference of transmitted light

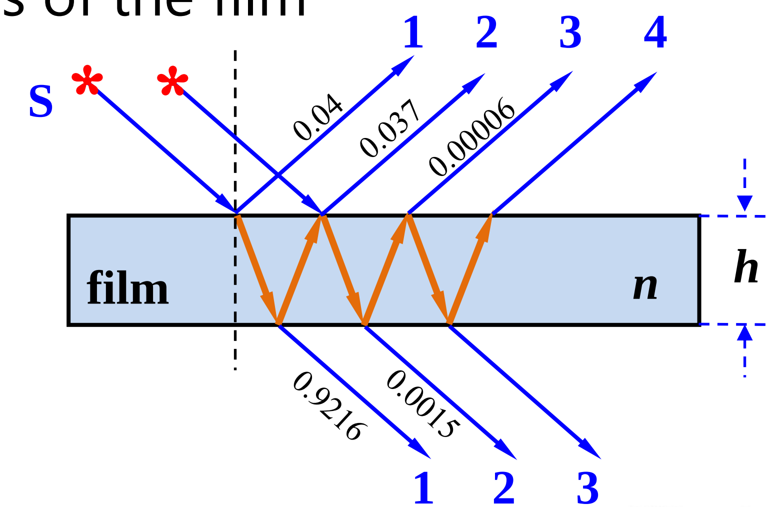
① There is no phase change in transmitted light. If the media on both sides of the film are the same, then

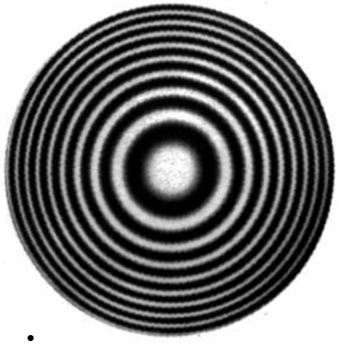
$$\Delta = 2nh\cos i_2$$

② The interference pattern of transmitted and reflected is complementary.

③ Low visibility. ???

The amplitude of the two beams of coherent light differs too much





- The interference fringes are observed from the transmitted light, and the center of the Newton's ring is a bright spot, as shown in the right figure.

Localized or Nonlocalized

- **Localization of interference fringes**: Studying where in the two lightwaves overlapping areas, clear interference fringes can be produced.
- **Nonlocalized interference fringes**: Clear interference fringes can be formed anywhere in the overlapped area;

E.g.: Young's Experiment, Fresnel Double Mirror, Lloyd Mirror

- **Localized interference fringes**: The interference fringes are only visible at certain positions in the overlapped region.

E.g: thin film interference
—Extended light source: a collection of many point sources at different locations

§ 6.5 Michelson interferometer

- In 1887, Michelson-Morey experiment;
- Spectral fine structure analysis by visibility curve;
- The Cd red line spectrum is 643.84696 nm to represent international unit of meter, which becomes the international standard during 1927-1960.

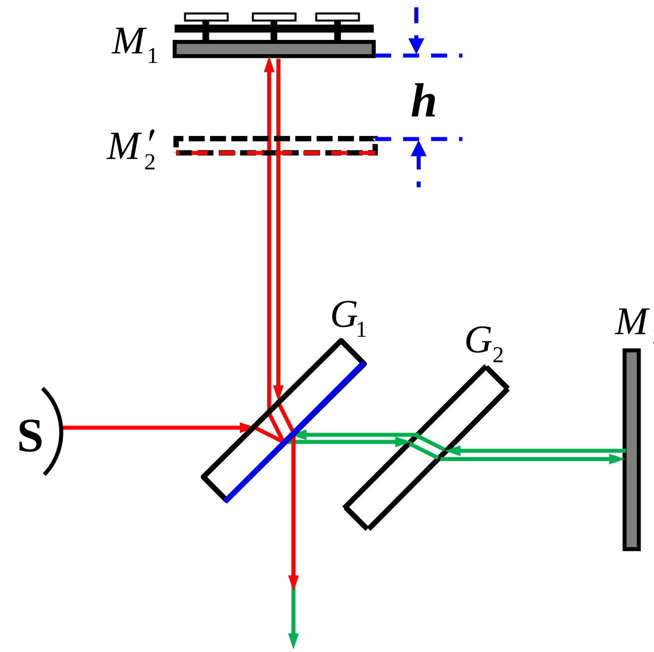
Albert A. Michelson
1852-1931, Poland

- The first American to receive a Nobel Prize in physics, 1907. Because of precise optic instruments and spectroscopy and metrology studies with these instruments.

Michelson interferometer

G_1 and G_2 are optical flat plate with the same thickness and uniform geometry and identical geometry.

The G_1 side is coated with a thin silver layer that is transflective. Placed at an angle of 45° with respect to the horizontal direction; G_2 is called a compensation plate.



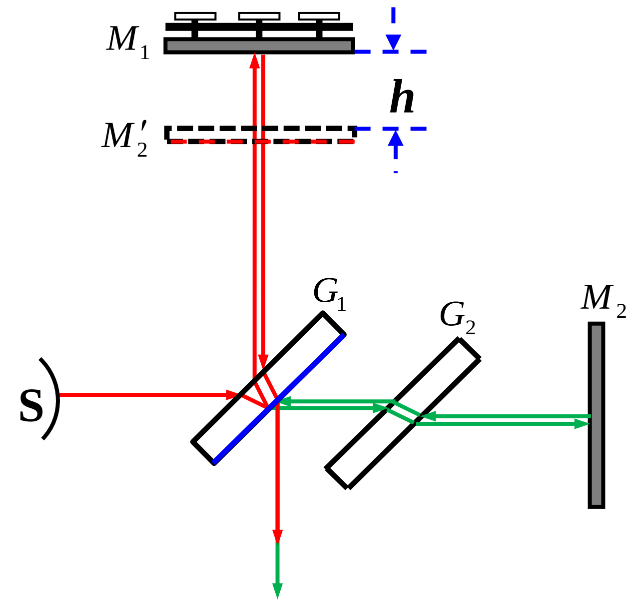
M_1 can move back and forth, M'_2 is fixed, fringe changes.

The interference is equivalently from the virtual image M'_2 and M_1.

- **Interference fringes**

The OPL difference between the two beams is

$$\Delta = 2h \qquad \Delta = 2\underline{n}h\,\underline{\cos i_2} + \cancel{\frac{\lambda}{2}}$$



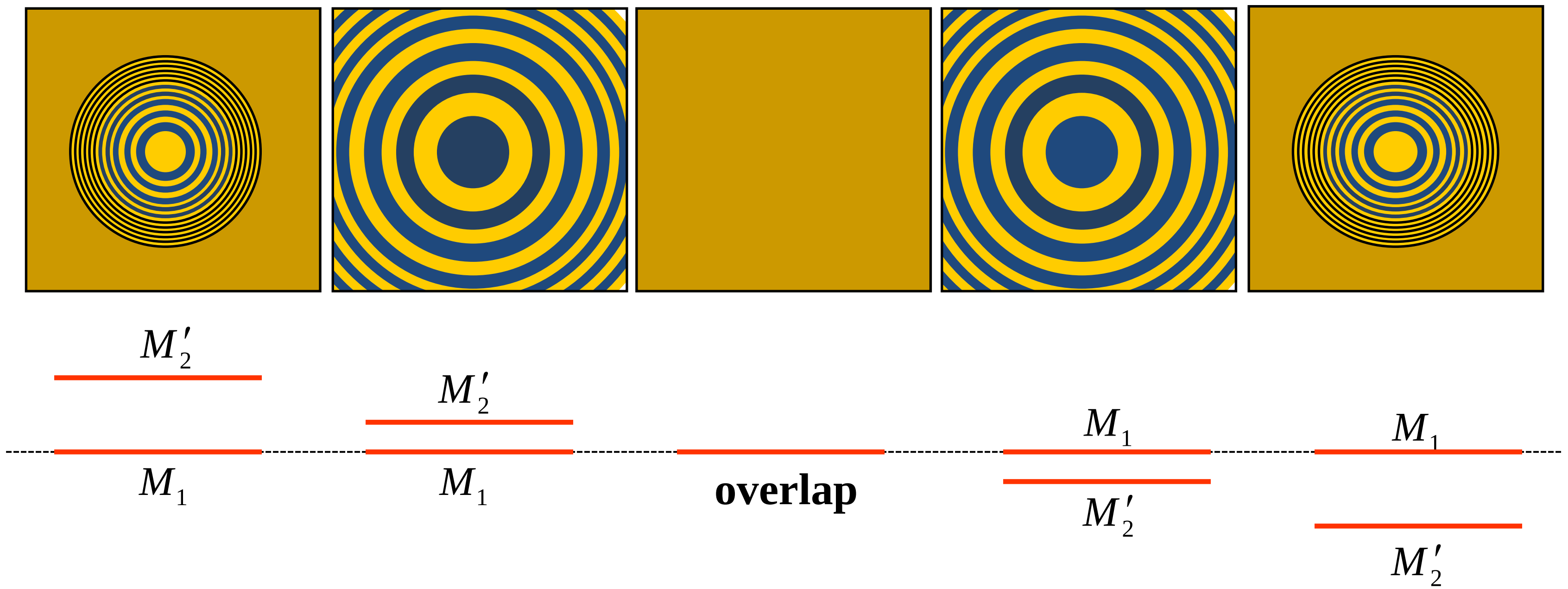
Change the angle of M_2 we can get h = 0, ***h* = constants** (**Equal inclination interference**), ***h* ≠ constants** (**Equal thickness interference**) (wedge-shaped film) .

If M_1 is shifted back and forth by Δh, the interference fringes move N, then:

$$\Delta h = N\frac{\lambda}{2}$$

Interference fringes

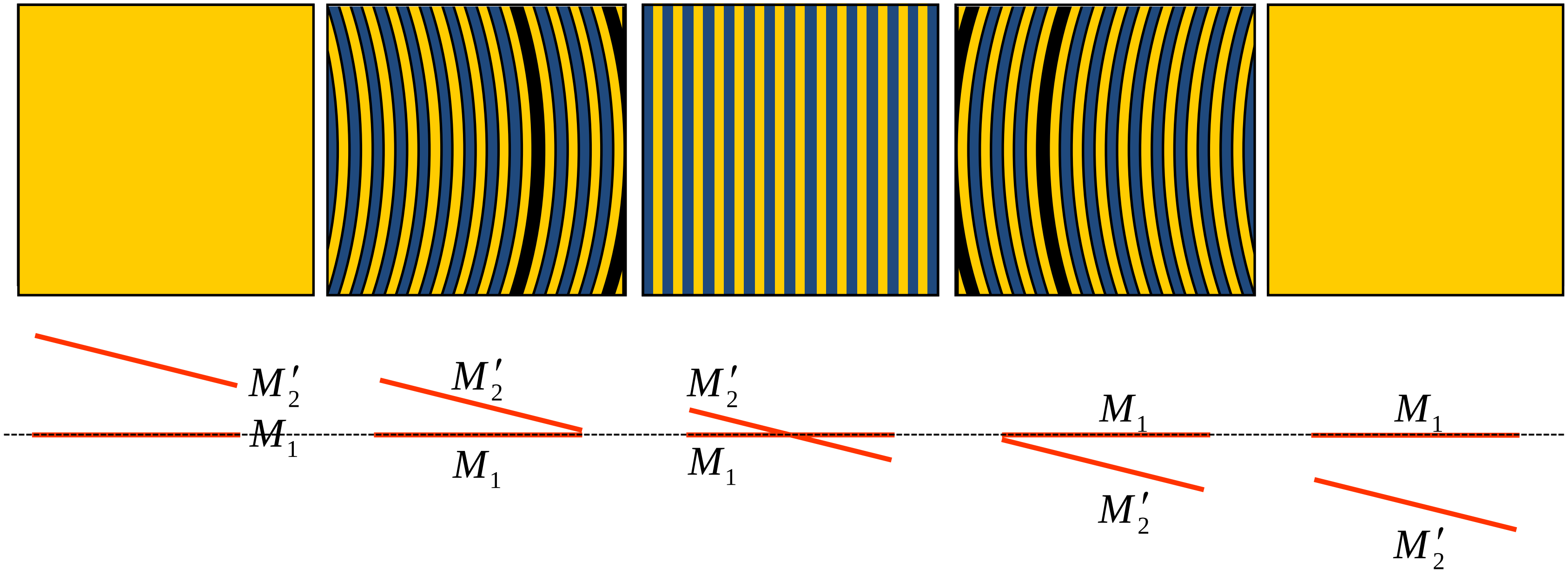
Equal inclination interference fringes



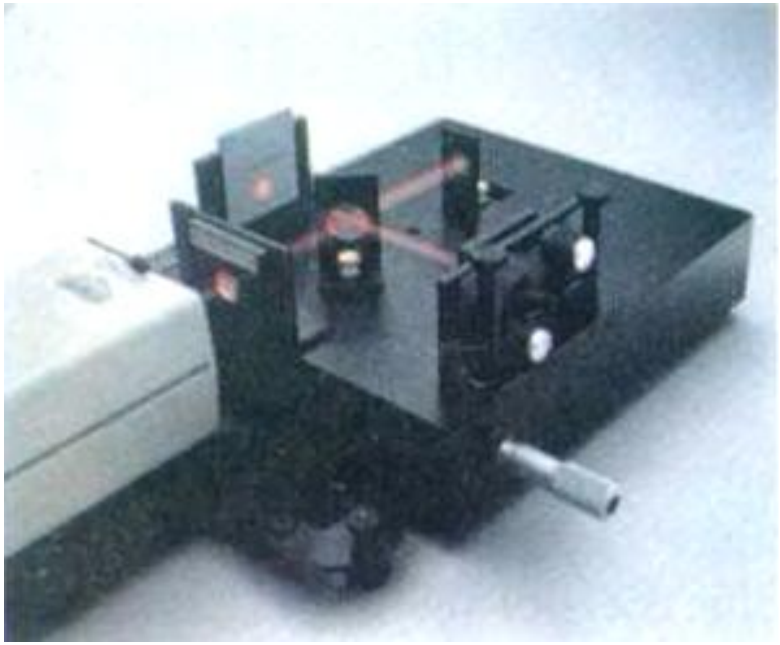
If M_1 moves to M_2, the air film thickness decreases, the interference fringes shrink toward the center and disappear, and when M_1 moves over $\lambda/2$, a fringe disappears.
If move in the opposite direction, a fringe emerges;

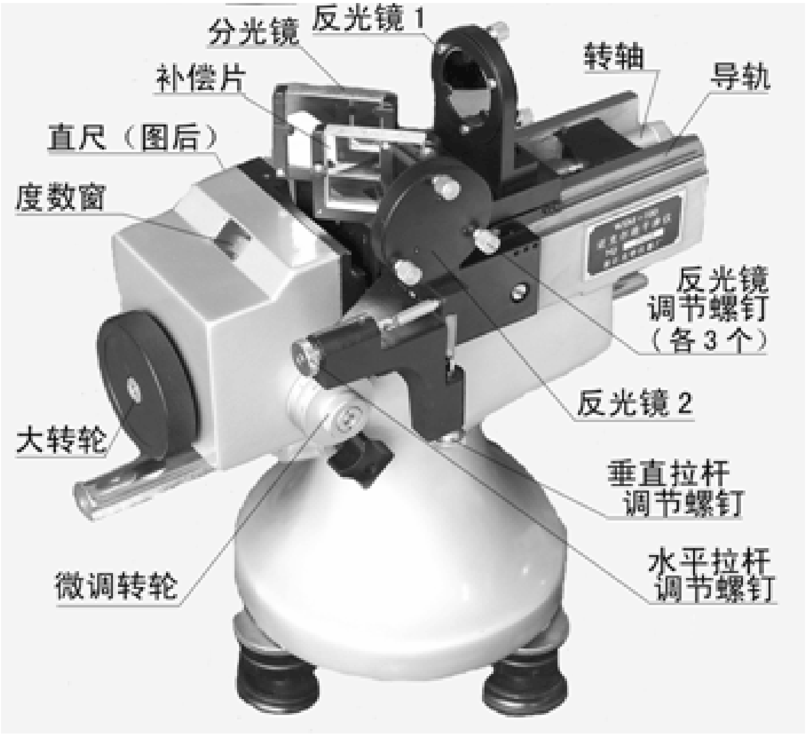
- **Interference fringes**

Equal thickness interference fringes



- **Michelson interferometer**



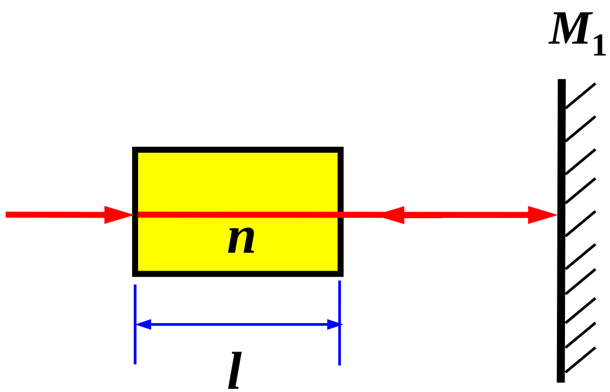


- **Application**

  - **Measuring small displacements, measuring wavelengths** （$\lambda/20$）

$$\boxed{\Delta h = N\frac{\lambda}{2}}$$

  - **Measured refractive index**：

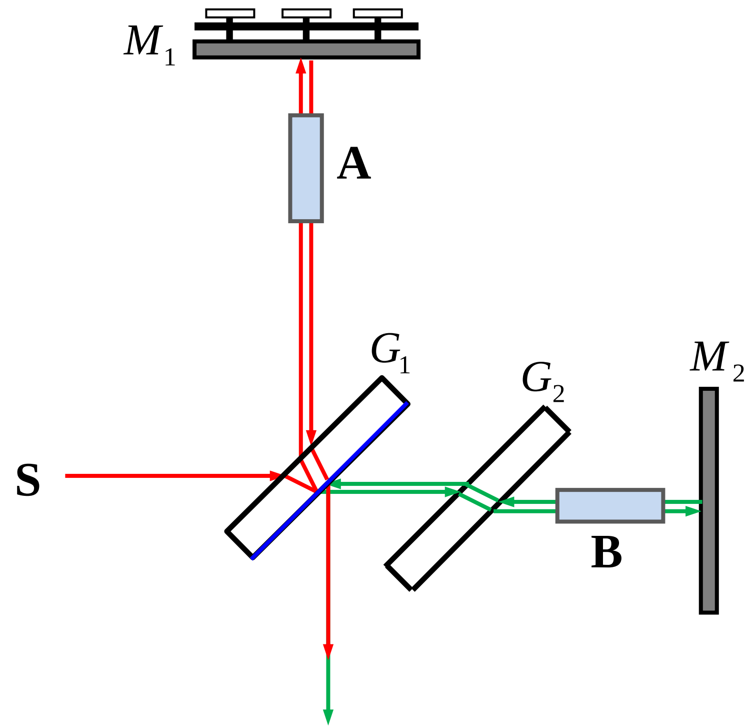


The medium to be tested is inserted into the optical path 1 to generate an additional OPL difference:

$$\delta = 2(n-1)l$$

  - **Measuring coherence length:** $\Delta_{\max} = L_0 = \dfrac{\lambda^2}{\Delta\lambda}$

Example: In the arms of the Michelson interferometer, insert two 10 cm long glass tubes A and B. Both of the tubes are vacuumed. It is observed that 107.2 fringes move during the filling of tube A with air. λ = 546 nm. Determine the refractive index of air.



Assume that the refractive index of air is *n:*

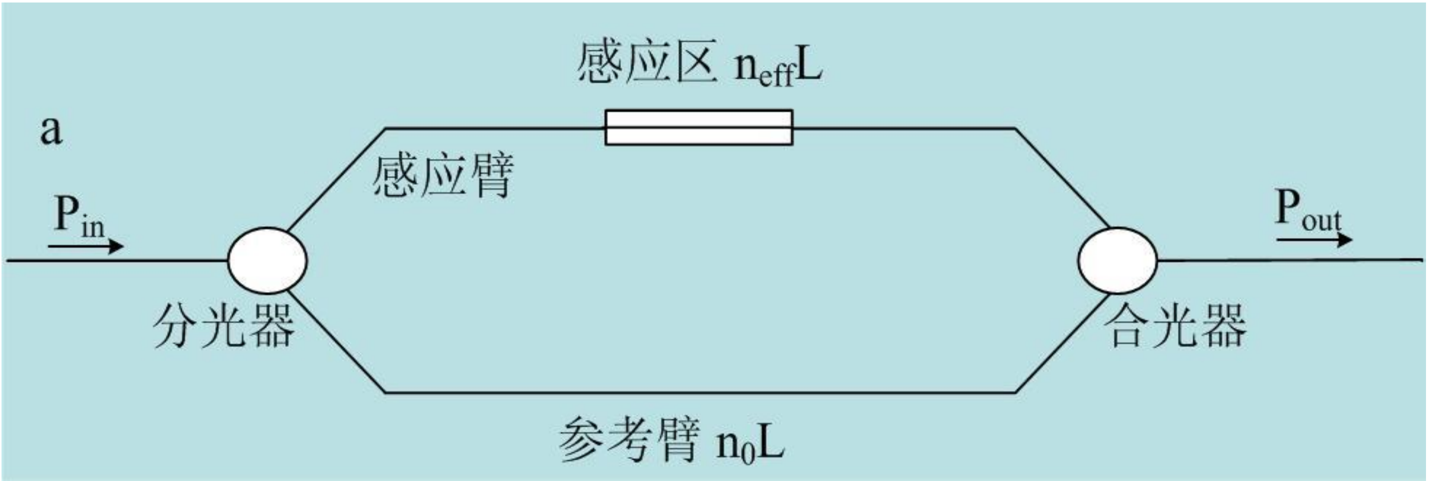
$$\delta = 2(n-1)l = 107.2\lambda$$

$$n = \frac{107.2\times\lambda}{2l} + 1 = 1.0002927$$

High precision

Mach-Zehnder interferometer

- **Mach-Zehnder interferometer**



Output intensity: $I_{out} = I_S + I_R + 2\sqrt{I_S I_R}\cos\delta$

Phase difference between the two arms

$$\delta = \frac{2\pi}{\lambda}\left[n(t)L(t) - n_0 L_0\right]$$

Mach-Zehnder interferometer

$$\delta = \frac{2\pi}{\boxed{\lambda}}\left[n(t)L(t) - n_0 L_0\right]$$

Splitter
Isolator
filter

Electric, thermal, magnetic, acoustic, etc.

Δn

sensor
modulator…

Displacement, force, electric field, etc.

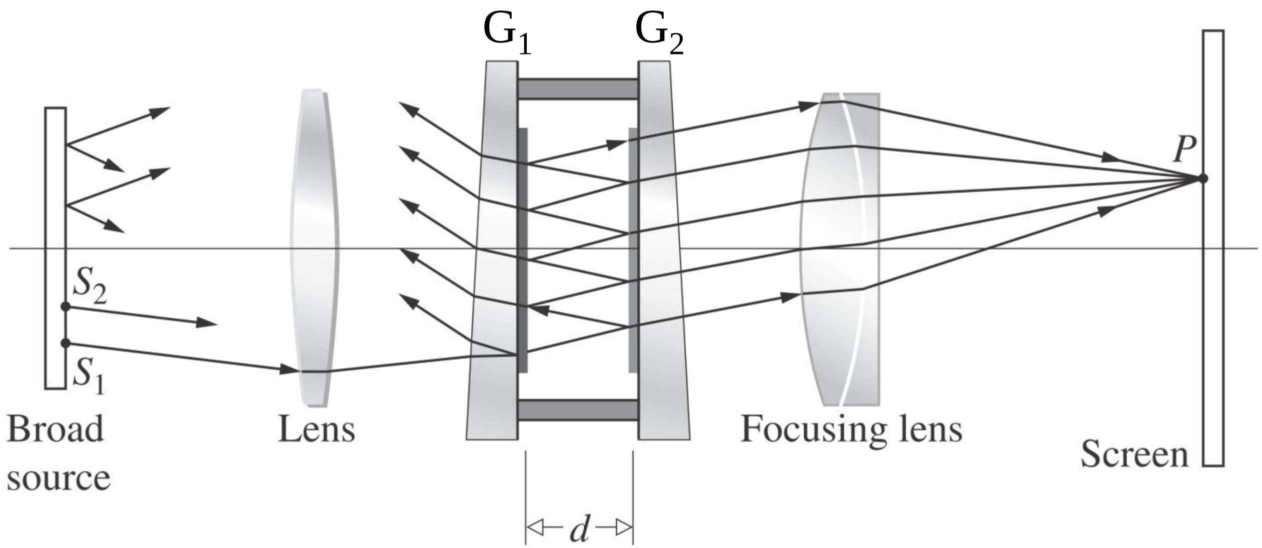
ΔL

sensor

MZ Interferometers are widely used in the fields of modulation and sensing.

§ 6.6 Fabry-Pèrot Interferometer

- Fabry-Pèrot interferometer：



Equal inclination ring fringe on the focal plane of the focusing lens.

High reflection at inner surfaces

The distance d between G_1 and G_2 is adjustable

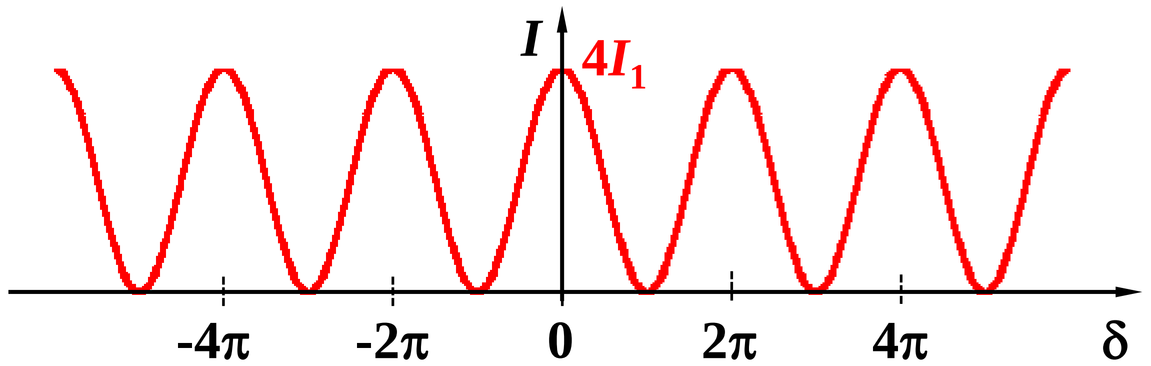
—Fabry-Pèrot interferometer

The distance d between G_1 and G_2 is fixed

—Fabry-Pèrot etalon

Multi-beam interference

Intensity of double beam interference: $I = 4I_1 \cos^2 \frac{\delta}{2} = 2I_1 \left(1 + \cos\delta\right)$



$I \sim \delta$ change slowly>>fringe is not sharp>> low resolution.

In multi-beam interference, $I \sim \delta$ change quickly >> fringe is sharp >> high resolution.

Multi-beam interference
- Amplitude splitting — **F-P interference**
- Wavefront splitting — **Grating diffraction**

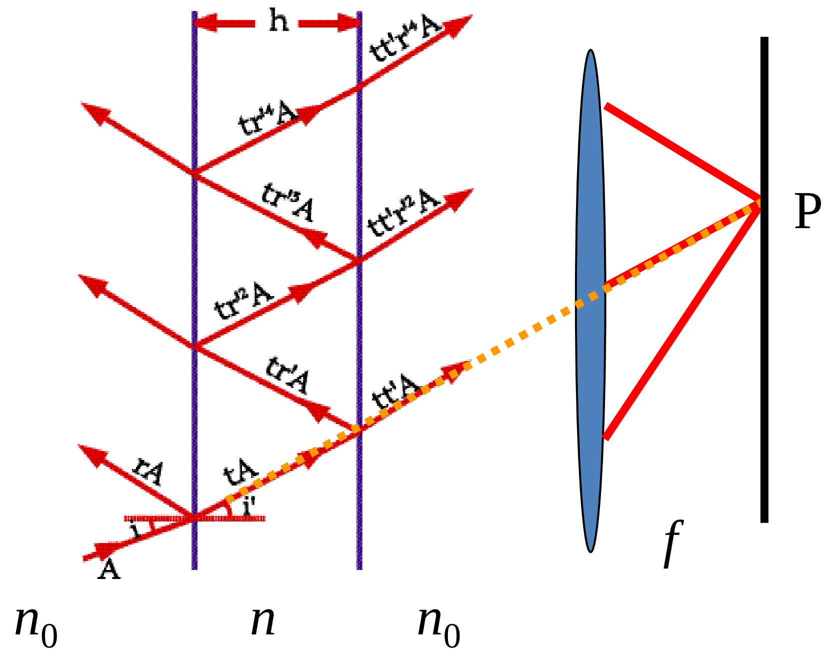
Multi-beam interference

- **Determine multi-beam interference intensity**

Denote

$n_0 \rightarrow n$: amplitude-reflection coefficient r, amplitude-transmission coefficient t.

$n \rightarrow n_0$: amplitude-reflection coefficient r', amplitude-transmission coefficient t'.



Phase shift caused by reflection is included in $r' = -r$.

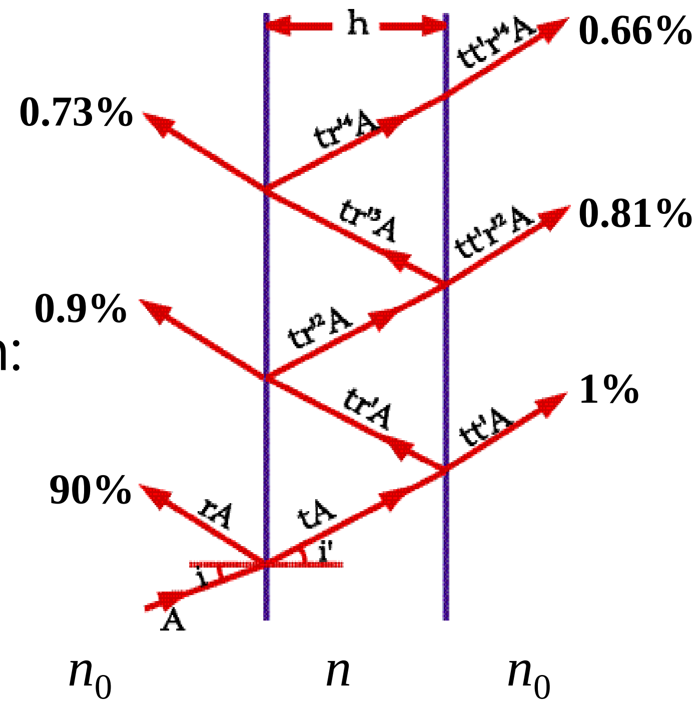
Multi-beam interference requires (i) the film to be long enough, (ii) the incident angle i is not too large.

Multi-beam interference

If the incident light amplitude A, wavelength λ, first beam transmitted light $\varphi_0 = 0$.

Phase difference between adjacent two transmitted light:

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} 2h\cos i_2$$



Amplitude of each transmitted beam:

$$\tilde{E}_1 = tt'Ae^{i0}$$

$$\tilde{E}_2 = tt'r'^2 Ae^{i\delta}$$

$$\tilde{E}_3 = tt'r'^4 Ae^{i2\delta}$$

$$\tilde{E}_4 = tt'r'^6 Ae^{i3\delta}$$

As can be seen from the above expressions, they are infinitely series with a common ratio of $r'^2 e^{i\delta}$.

Multi-beam interference

Complex amplitude of transmitted beam：

$$\tilde{E}_T = \tilde{E}_1 + \tilde{E}_2 + \cdots = \sum_{j=1}^{\infty} \tilde{E}_j = Att'\left(1 + r'^2 e^{i\delta} + r'^4 e^{i2\delta} + \cdots\right)$$

$$= A\frac{tt'}{1 - r'^2 e^{i\delta}} = A\frac{1 - r^2}{1 - r^2 e^{i\delta}}$$

The last step uses the Fresnel reflection/refraction amplitude coefficients:

$$r = -r',\ 1 - r^2 = tt'$$ (You&Yu's Book: Page 54)

Transmission intensity:

$$I_T = \tilde{E}_T \cdot \tilde{E}_T^* = A^2 \frac{\left(1 - r^2\right)^2}{\left(1 - r^2 e^{i\delta}\right)\left(1 - r^2 e^{-i\delta}\right)} = A^2 \frac{\left(1 - r^2\right)^2}{1 - 2r^2 \cos\delta + r^4}$$

Fabry-Pèrot Interferometer

It is known that $A^2 = I_0$ and $r^2 = R$, so

$$I_T = A^2 \frac{\left(1-r^2\right)^2}{1-2r^2\cos\delta + r^4} = I_0 \frac{\left(1-R\right)^2}{1-2R\cos\delta + R^2}$$

Use the equation: $\cos\delta = 1 - 2\sin^2\frac{\delta}{2}$

$$I_T = I_0 \frac{\left(1-R\right)^2}{\left(1-R\right)^2 + 4R\sin^2\frac{\delta}{2}} = \frac{I_0}{1+F\sin^2\frac{\delta}{2}}$$

$F \equiv \dfrac{4R}{\left(1-R\right)^2}$ is called **coefficient of finesse**.

Fabry-Pèrot Interferometer

- **Characteristics of interference fringes**

$$I_T = I_0 \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}}$$

(1) The intensity extremum is determined by ***δ***, not ***R.***

① When $\delta = 2m\pi$, $\sin\delta/2 = 0$;

$$I_{T\max} = I_0 \qquad I_{R\min} = 0$$

② When $\delta = (2m+1)\pi$, $\sin\delta/2 = 1$

$$I_{T\min} = I_0 \frac{(1-R)^2}{(1+R)^2} \qquad I_{R\max} = I_0 - I_{T\min} = I_0 \frac{4R}{(1+R)^2}$$

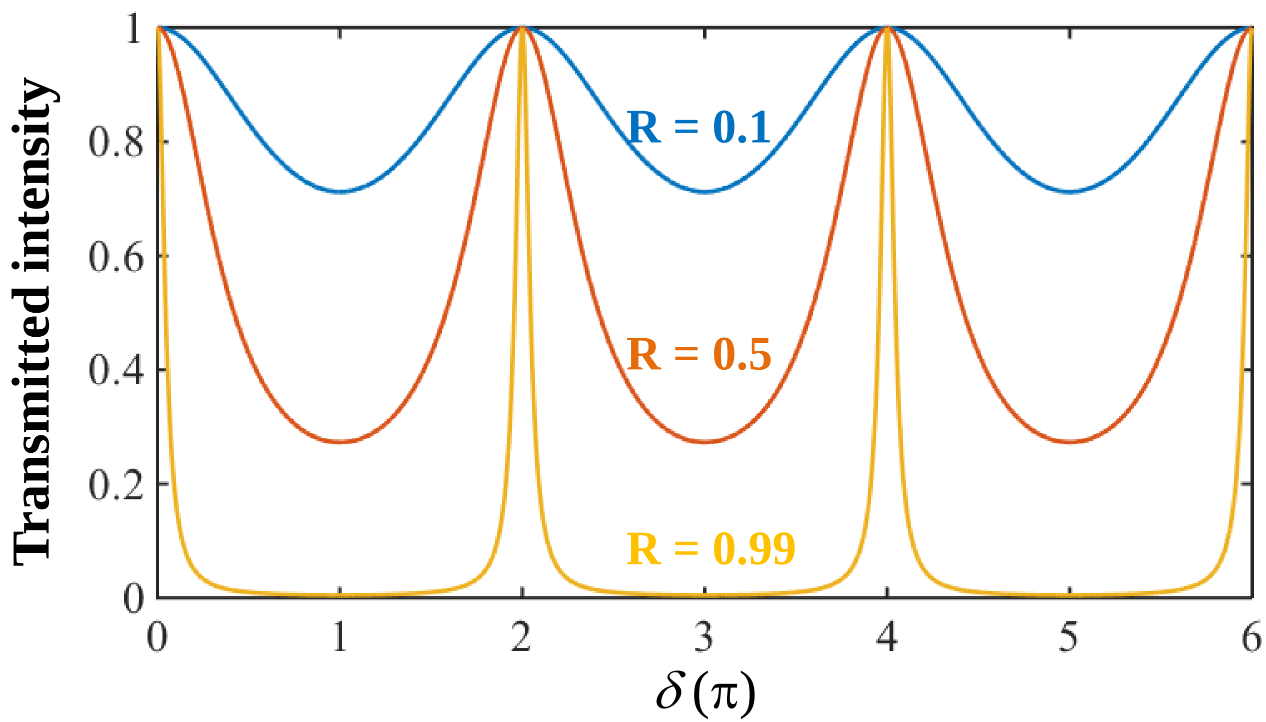
$$R\uparrow \quad \Rightarrow \quad I_{T\min}\downarrow \quad I_{R\max}\uparrow$$

Fabry-Pèrot Interferometer

(2) **Coefficient of finesse is determined by *R*.**

$$\frac{I_T}{I_0} = \frac{1}{1 + F\sin^2\frac{\delta}{2}} \qquad F \equiv \frac{4R}{(1-R)^2}$$

***R* ↑, sharpness ↑**



Fabry-Pèrot Interferometer

（二）参数估算：透过峰的半峰宽（FWHM）。

习题4.23 设法布里-珀罗干涉仪腔长3 cm，反射率R = 0.98，采用扩展光源照明。求λ = 0.5 μm的谱线宽度?

$$\frac{I_T}{I_0}=\frac{1}{1+F\sin^2\frac{\delta}{2}}$$

$$F\equiv\frac{4R}{(1-R)^2}$$

$$\frac{I_T}{I_0}=\frac{1}{2} \Rightarrow F\sin^2\frac{\delta'}{2}=1 \Rightarrow \frac{1}{2}\left(1-\cos\delta'\right)=\frac{1}{F}$$

$$\cos\delta'=\frac{F-2}{F}=0.9798$$

$$\arccos 0.9798=\frac{4\pi h}{\lambda'}-\frac{4\pi h}{\lambda}$$

$$\Rightarrow \Delta\lambda=2\left|\lambda'-\lambda\right|=2.6\times10^{-4}\,\text{nm}$$

Fabry-Pèrot Interferometer

(3) The shape of the interference fringes: the interference pattern is an equal inclination fringes.

- Reflected light interference pattern: very thin dark lines on a bright background.
- Transmitted light interference pattern: very fine bright lines appear on a wide dark background.

※ choose the transmitted light interference usually!

※ The bright fringes of the F-P interferometer are sharper and brighter than the rings of the Michelson interferometer.

Fabry-Pèrot Interferometer

The ring fringes formed by different wavelengths of light have different sizes (interference splitting), and the fringes are sharp.

$$\Delta = 2h\cos i_2 = m\lambda$$

The F-P interferometer is a high-resolution spectroscopic instrument that is commonly used to study the (hyper)fine structure of the spectrum.

F-P interferometer is very important in modern optics:

(1) **Spectrometer**

(2) **Resonant cavity**

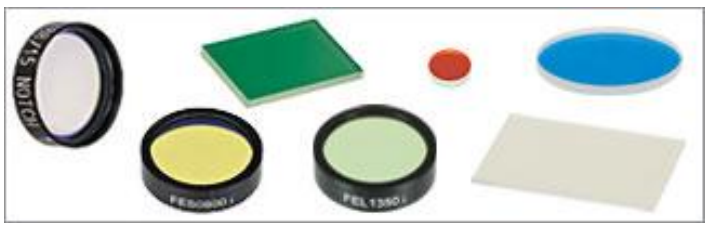
(3) **Wavelength division multiplexing components**

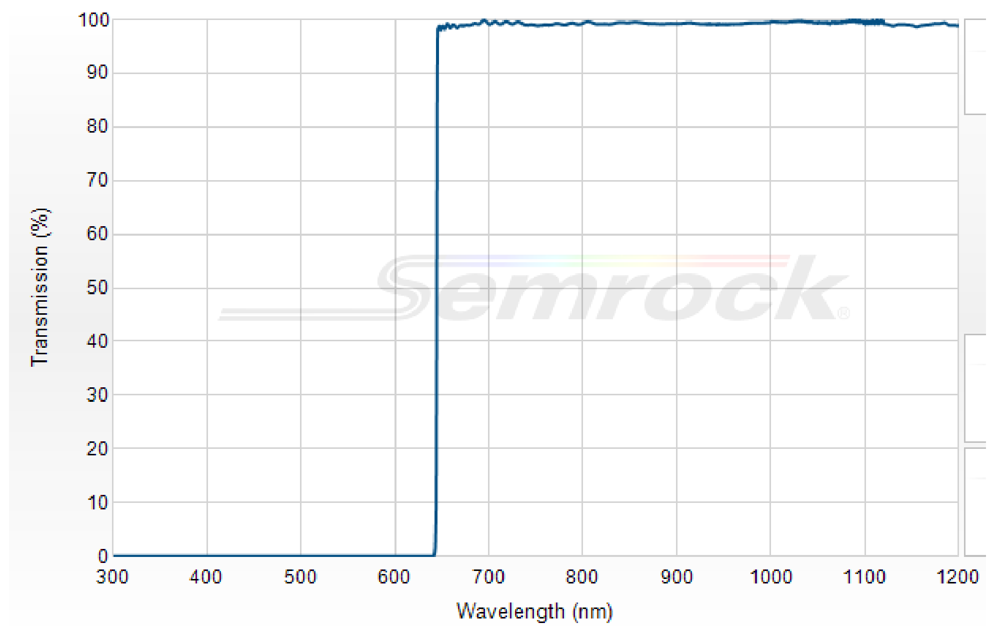
(4) **comparison method measures the wavelength of light**

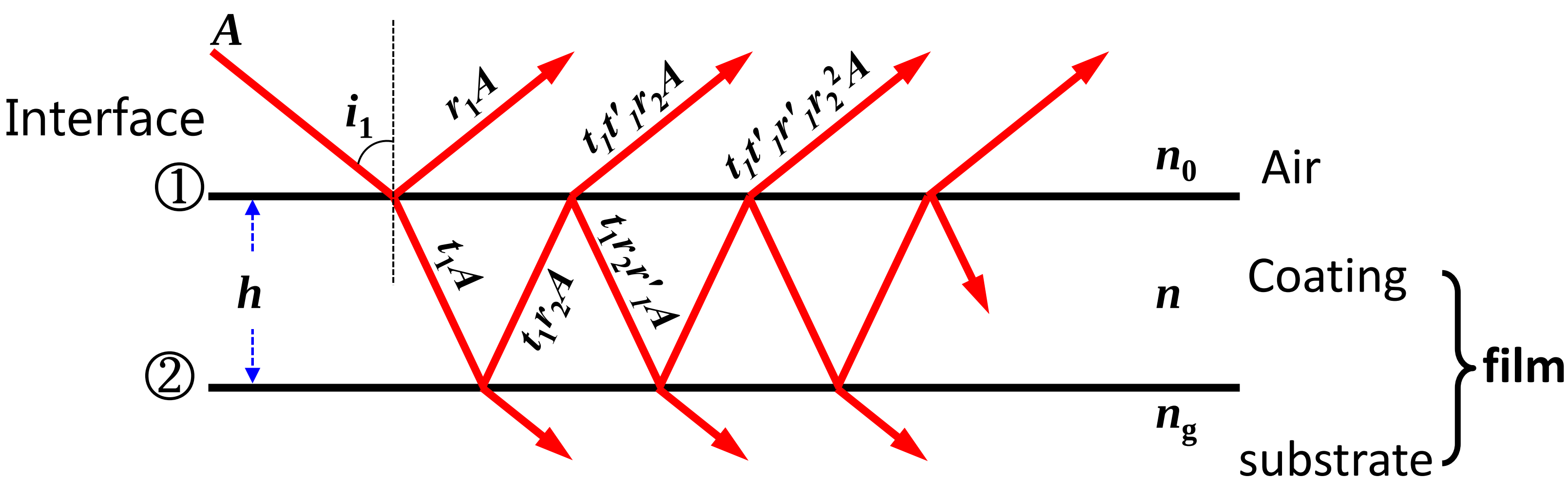
§ 6.7 Optical thin film

Coating technology: A thin layer of transparent or metallic film is coated to the glass or smooth metal surface by evaporation, deposition or spin-coating.

Single / multilayer films







Interface ①: $n_0 \rightarrow n$, reflectivity r_1、 transmittance t_1 ;

$n \rightarrow n_0$, r'_1、 t'_1

Interface ②: $n \rightarrow n_g$, r_2、 t_2 ;

Appropriate choice of n and h causes multi-beam interference of light that is sequentially reflected and transmitted on interface ① ②, so that the reflected light is strengthened or attenuated.

Single layer film

For the sake of simplicity, only the normal incidence of light waves is discussed.

The multi-beam interference problem is fixed at (n_0、n、n_g、h) $i = 0$, determine the R of the single-layer film.

The solution is the same as the reflectivity calculation of the F-P interferometer, except that the amplitude-reflection and amplitude-transmission coefficient of the **upper and lower interfaces are different**.

①Amplitude and intensity expression of the reflected light：

$$\tilde{E}_R = \sum_{j=1}^{\infty} \tilde{E}_j \qquad I_T = \tilde{E}_R \cdot \tilde{E}_R^*$$

②Write a single layer film reflectance expression：

$$R = \frac{I_R}{I_0}$$

Single layer film

Amplitude of reflected wave: $\tilde{E}_R = A\dfrac{r_1 + r_2 e^{i\delta}}{1 + r_1 r_2 e^{i\delta}}$

Membrane reflectivity: $R = \dfrac{\tilde{E}_R \tilde{E}_R^*}{A^2} = \dfrac{r_1^2 + 2r_1 r_2 \cos\delta + r_2^2}{1 + 2r_1 r_2 \cos\delta + r_1^2 r_2^2}$

when $i = 0$ $\quad \delta = \dfrac{2\pi}{\lambda}\Delta = \dfrac{4\pi}{\lambda}nh \qquad r_1 = \dfrac{n_0 - n}{n_0 + n} \qquad r_2 = \dfrac{n - n_g}{n + n_g}$

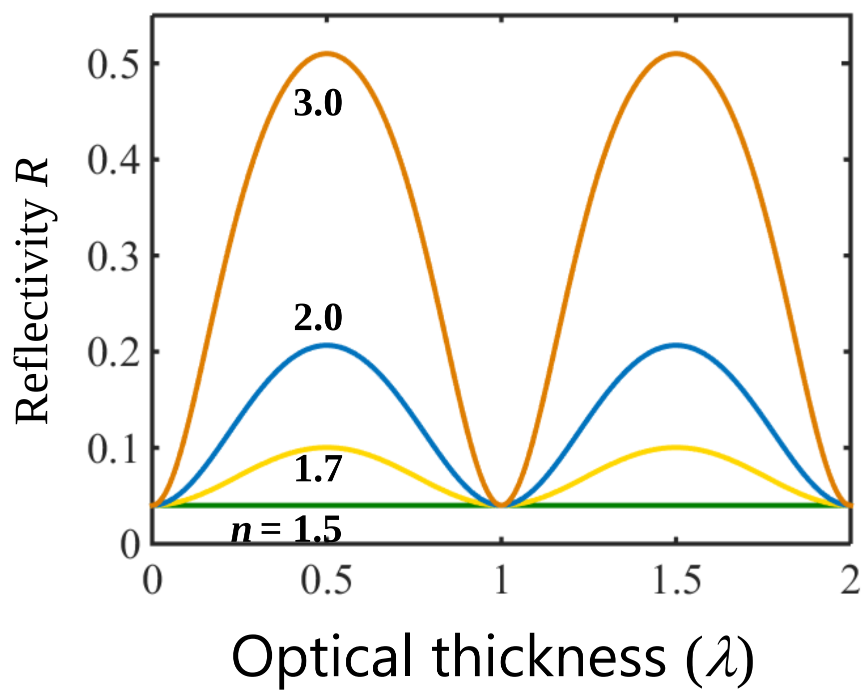
$$\Rightarrow \boxed{R = \frac{(n_0 - n_g)^2 \cos^2\dfrac{\delta}{2} + (\dfrac{n_0 n_g}{n} - n)^2 \sin^2\dfrac{\delta}{2}}{(n_0 + n_g)^2 \cos^2\dfrac{\delta}{2} + (\dfrac{n_0 n_g}{n} + n)^2 \sin^2\dfrac{\delta}{2}}}$$

Single layer film

If: $n_0 = 1.0$, $n_g = 1.5$, $i = 0$

$R \sim nh$ curve is shown:

$$R = \frac{(n_0 - n_g)^2 \cos^2\frac{\delta}{2} + (\frac{n_0 n_g}{n} - n)^2 \sin^2\frac{\delta}{2}}{(n_0 + n_g)^2 \cos^2\frac{\delta}{2} + (\frac{n_0 n_g}{n} + n)^2 \sin^2\frac{\delta}{2}}$$



Single layer film

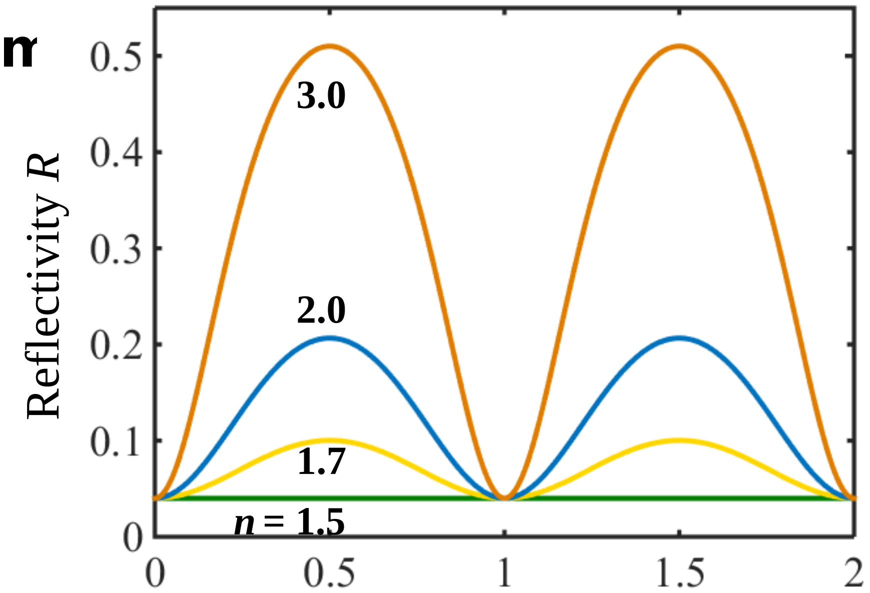
(1) reflection increasing film

Requirement: $n > n_g$

When $\delta = (2m+1)\pi$, R is max.

$$\delta = \frac{4\pi}{\lambda} nh$$

$$\Rightarrow \quad nh = (2m+1)\lambda/4$$



Optical thickness (λ)

When the optical thickness is the odd times of $\lambda/4$, R is the max, **reflection increasing film**.

$$R = R_{\max} = \frac{\left(n_0 n_g - n^2\right)^2}{\left(n_0 n_g + n^2\right)^2}$$

$$\boxed{R = \frac{(n_0 - n_g)^2 \cos^2\frac{\delta}{2} + (\frac{n_0 n_g}{n} - n)^2 \sin^2\frac{\delta}{2}}{(n_0 + n_g)^2 \cos^2\frac{\delta}{2} + (\frac{n_0 n_g}{n} + n)^2 \sin^2\frac{\delta}{2}}}$$

Single layer film

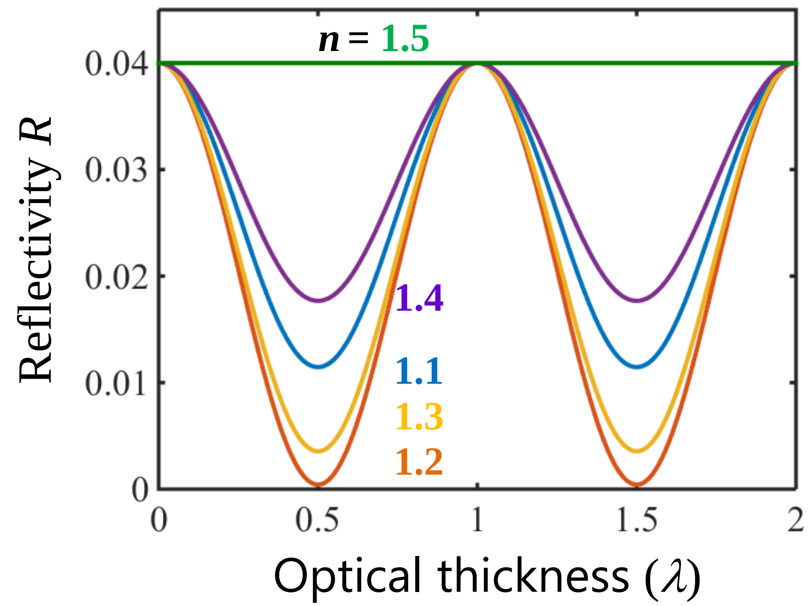
(2) Antireflection film

Requirement: $n < n_g$

When $\delta = (2m+1)\pi$, R is min.

$$\delta = \frac{4\pi}{\lambda} nh$$

$$\Rightarrow \quad nh = (2m+1)\lambda/4$$



When the optical thickness is the odd times of $\lambda/4$, R is the min, **antireflection**.

$$R = R_{\min} = \frac{\left(n_0 n_g - n^2\right)^2}{\left(n_0 n_g + n^2\right)^2}$$

$$\boxed{R = \frac{(n_0 - n_g)^2 \cos^2\frac{\delta}{2} + \left(\frac{n_0 n_g}{n} - n\right)^2 \sin^2\frac{\delta}{2}}{(n_0 + n_g)^2 \cos^2\frac{\delta}{2} + \left(\frac{n_0 n_g}{n} + n\right)^2 \sin^2\frac{\delta}{2}}}$$

When $n = \sqrt{n_0 n_g}$ $\quad R_{\min} = 0$

Single layer film

(3) when $\delta = 2m\pi$ ， $nh = m\lambda/2$。

Whatever $n < n_g$, or $n > n_g$ the r is same as un-coating.

$$R = \frac{\left(n_0 - n_g\right)^2}{\left(n_0 + n_g\right)^2}$$

$$R = \frac{(n_0 - n_g)^2 \cancel{\cos^2\frac{\delta}{2}} + \cancel{(\frac{n_0 n_g}{n} - n)^2 \sin^2\frac{\delta}{2}}}{(n_0 + n_g)^2 \cancel{\cos^2\frac{\delta}{2}} + \cancel{(\frac{n_0 n_g}{n} + n)^2 \sin^2\frac{\delta}{2}}}$$

Single layer film

When $nh = (2m+1)\lambda/4$, R is extremum.

$n > n_g$ reflection increasing film

$n < n_g$ Antireflection film

How do we understand these films in physics? ?

$$\Delta = 2nh = (2m+1)\frac{\lambda}{2}$$

	Interface	Incidence	Reflection	Phase change	film
$n > n_g$	①	$n_0 \to n$	External	Y	Reflection increasing
	②	$n \to n_g$	Internal	N	
$n < n_g$	①	$n_0 \to n$	External	Y	Antireflection
	②	$n \to n_g$	External	Y	

Multilayer film

Reflection increasing film： $R\uparrow$
Antireflection film： $T\uparrow$
Interference filter： Extracting light of a certain wavelength or band from complex light.

Multi-layer medium high-reflection film: The film system is composed of H and L films alternately, and the total number of layers N is odd.

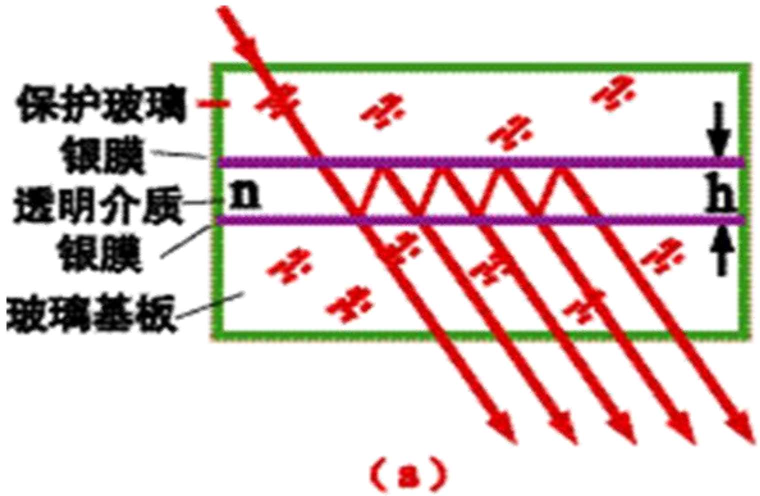
Each layer $nh = \lambda/4$ The number of layer $\uparrow\rightarrow R\uparrow$。

For example: Full reflection mirror of He-Ne laser cavity
$N = 15 \sim 19$, n(ZnS – MgF), for $\lambda = 632.8$ nm, $R \sim 99.6\%$.

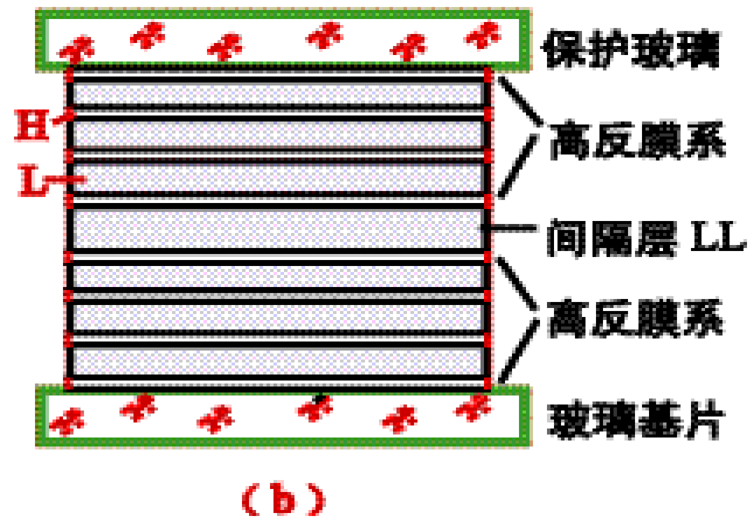
Thin Film Optics: Studying the relationship between n, h and R, T

Interference filter

Structure: Minimal F-P etalon



Metal-coated interference filter



All-dielectric interference filter

Interference filter: The narrowband filter is formed by the frequency selection principle of the F-P cavity.

Homework

Problem 9.8, 9.13, 9.45 and 9.51.

Homework*

Find a text book of Quantum optics, and find out what's the first-order correlation and its relationship with out text.

Next week

Huygen's Principle, Fraunhofer and Fresnel Diffraction

Sections 4.4.2, 10.1 and 10.2