

第六章习题、参考解答

6-1 [解] 设振动方程为 $x = A \cos(\omega t + \varphi_0)$

① 由题给 $\omega = 2\pi\nu = 2 \times \pi \times 0.25 = 0.5\pi = 1.57$ (弧度/秒)

而 $t=0$ 时 $x = 0.37 = A \cos \varphi_0$, $\dot{x} = 0 = -\omega A \sin \varphi_0$

可得 $A = 0.37$ cm, $\varphi_0 = 0$, $T = \frac{1}{\nu} = 4$ (sec)

∴ $x = 0.37 \cos(0.5\pi t)$

② 位移 $x = 0.37 \cos(\frac{\pi}{2}t)$, 速度 $\dot{x} = -\frac{\pi}{2} \times 0.37 \sin(\frac{\pi}{2}t)$

③ 最大速度 $|\dot{x}|_{\max} = \frac{\pi}{2} \times 0.37$ (cm/sec) = 0.58 cm/sec

最大加速度 $\ddot{x}_{\max} = (\frac{\pi}{2})^2 \times 0.37 = 0.91$ cm/sec²

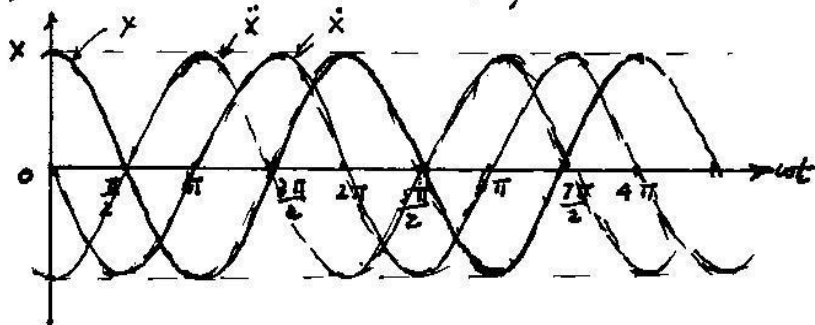
6-2 [解]

$$x = A \cos(\omega t + \alpha) = A \cos(\omega t) \quad (\alpha = 0)$$

$$\dot{x} = -A\omega \sin(\omega t + \alpha) = -A\omega \sin \omega t = A\omega \cos(\omega t + \frac{\pi}{2})$$

$$\ddot{x} = -A\omega^2 \cos(\omega t + \alpha) = -A\omega^2 \cos(\omega t) = A\omega^2 \cos(\omega t + \pi)$$

∴ \dot{x} 超前 x 的相位是 $\frac{\pi}{2}$, \ddot{x} 超前 x 的相位是 π .



6-3 [解] 物体的运动方程为: $x = A \cos(\omega t + \varphi_0) = A \cos(4\pi t + \varphi_0)$

$$\text{∴ 有 } \dot{x} = -4\pi A \sin(4\pi t + \varphi_0), \quad \ddot{x} = -16\pi^2 A \cos(4\pi t + \varphi_0)$$

而物体受力(水平方向)为摩擦力, $f = \mu mg$

$$\text{∴ 有 } f = \mu mg \geq m\ddot{x} = -m16\pi^2 A \cos(4\pi t + \varphi_0)$$

$$\text{∴ 要求: } A \leq \frac{\mu g}{16\pi^2} = \frac{0.5 \times 9.8}{16 \times \pi^2} = 0.031 \text{ m} = 3.1 \text{ cm}$$

6-4 [解] 单摆周期 $T = \sqrt{\frac{l}{g}}$

在准确之处: $T = T_1$, $g = g_1 = 9.8$ $\therefore T_1 = \sqrt{\frac{l}{g_1}}$

减慢处: $T = T_2$, $g = g_2$ $\therefore T_2 = \sqrt{\frac{l}{g_2}}$

设摆钟一天走动 N 次, 应有 $T_1 \times N = 3600 \times 24$

$T_2 \times N = 3600 \times 24 + 10$

\therefore 有 $\frac{T_2}{T_1} = 1 + \frac{10}{3600 \times 24}$

$\therefore \frac{\sqrt{\frac{l}{g_2}}}{\sqrt{\frac{l}{g_1}}} = \sqrt{\frac{g_1}{g_2}} = 1 + \frac{10}{3600 \times 24} = 1 + \frac{1}{360 \times 24} = \frac{360 \times 24 + 1}{360 \times 24}$

$\therefore g_2 = g_1 \times \left(\frac{360 \times 24}{360 \times 24 + 1} \right)^2 = 9.798 \text{ m/s}^2$

6-5 [解] 由 $Y = \frac{F/s}{\Delta l/l}$ \therefore 有 $F = sY \frac{\Delta l}{l} = \frac{sY}{l} \Delta l = k \Delta l$

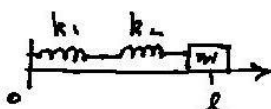
$\therefore k = \frac{sY}{l}$

现在 $l_1 + l_2 = l$, 且 $l_1 = n l_2$, \therefore 有 $l_2(n+1) = l$

$\therefore l_1 = \frac{l}{n+1}$, $l_2 = n l_2 = \frac{n}{n+1} l$

$\therefore k_1 = \frac{sY}{l_1} = \frac{n+1}{n} \frac{sY}{l} = \left(1 + \frac{1}{n}\right) k$

$k_2 = \frac{sY}{l_2} = (n+1) \frac{sY}{l} = (n+1) k$

6-6 [解] (1) $\frac{1}{\omega_1} = \sqrt{\frac{m}{k_1+k_2}}$; (2) $\frac{1}{\omega_2} = \sqrt{\frac{m}{k_1+k_2}}$; (3) 

\therefore 总伸长 Δl , 弹簧1伸长 Δl_1 , 弹簧2伸长

有 $\Delta l = \Delta l_1 + \Delta l_2$, 由每个弹簧的恢复力分别为

$F_1 = -k_1 \Delta l_1$, $F_2 = -k_2 \Delta l_2$, 而 $F_1 = F_2$

$\therefore k_1 \Delta l_1 = k_2 \Delta l_2$ $\therefore \Delta l_1 = \frac{k_2}{k_1} \Delta l_2$

$\therefore \Delta l = \left(\frac{k_2}{k_1} + 1 \right) \Delta l_2 = \frac{k_1+k_2}{k_1} \Delta l_2$ 或 $\Delta l_2 = \frac{k_1}{k_1+k_2} \Delta l$

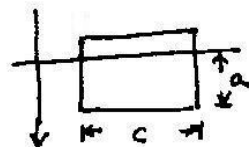
作用于物体上的力 $F_2 = -k_2 \Delta l_2 = -\frac{k_1 \cdot k_2}{k_1+k_2} \Delta l = m \ddot{x} = -k \Delta l = m \ddot{x}$

$\therefore k = \frac{k_1 k_2}{k_1+k_2}$ $\therefore \frac{1}{\omega_3} = \sqrt{\frac{m}{k}} = \sqrt{\frac{(k_1+k_2)m}{k_1 \cdot k_2}}$

或 $\omega_3 = \sqrt{\frac{k_1 k_2}{(k_1+k_2) \cdot m}}$

6-7 [解] 设立方体边长为 c

$$\text{应有 } \rho_{\text{水}} a c^2 = \rho_{\text{木}} c^3$$



$$\text{或 } \rho_{\text{水}} a = \rho_{\text{木}} c \quad (\rho_{\text{水}}, \rho_{\text{木}} \text{ 分别是水和木块的质量密度}) \quad ①$$

设木块浸入水中的增量部分为 x , 则应有

$$[\rho_{\text{木}} c^3 g - \rho_{\text{水}} (a+x) c^2 g] = \rho_{\text{木}} c^3 \ddot{x} \quad ②$$

$$\text{代入①得 } -\rho_{\text{水}} x c^2 g = \rho_{\text{木}} c^3 \ddot{x} \quad \text{即 } \rho_{\text{木}} c \ddot{x} + \rho_{\text{水}} x g = 0$$

$$\text{由①可得 } \rho_{\text{水}} a \ddot{x} + \rho_{\text{水}} x g = 0 \quad \text{即 } \ddot{x} + \frac{x}{a} g = 0 \quad ③$$

$$\text{解得: } x = A \cos(\omega t + \varphi_0), \text{ 其中 } \omega = \sqrt{\frac{g}{a}}, \quad T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{a}{g}} \quad (\text{秒})$$

$$x: \dot{x} = -A\omega \sin(\omega t + \varphi_0) \quad \text{由 } t=0 \text{ 时 } x = b-a, \quad \dot{x} = 0$$

$$\text{可得: } \varphi_0 = 0 \quad A = b-a \quad \therefore x = (b-a) \cos \omega t$$

6-8 [解] 摆至右侧时, 相当于摆长为 $l_1 = 1.05 \text{ m}$ 的单摆, 周期 $T_1 = 2\pi \sqrt{\frac{l_1}{g}} = 2\pi/\omega_1$
摆至左侧时, $\dots \quad l_2 = 1.5 \text{ m} \quad \dots \quad \therefore T_2 = 2\pi \sqrt{\frac{l_2}{g}} = 2\pi/\omega_2$

$$\therefore \text{合系统的周期 } T = \frac{1}{2}(T_1 + T_2) = \pi \left[\sqrt{\frac{l_1}{g}} + \sqrt{\frac{l_2}{g}} \right] \approx 2.26 \text{ (sec)}$$

$$\therefore \text{系统的机械能守恒, 因此有 } \frac{1}{2} m \omega_1^2 A_1^2 = \frac{1}{2} m \omega_2^2 A_2^2$$

$$\therefore \text{振幅比为 } \frac{A_2}{A_1} = \frac{\omega_1}{\omega_2} = \sqrt{\frac{l_2}{l_1}} \approx 1.2$$

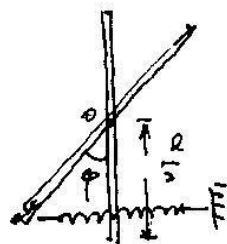
6-10 [解]

杆偏转 φ 角, 所受的弹性力矩为:

$$-k \frac{l}{2} \varphi \cdot \frac{l}{2} = I \ddot{\varphi}$$

$$\because I = \frac{1}{12} m l^2 \quad \therefore -k \frac{l^2}{4} \varphi = \frac{1}{12} m l^2 \ddot{\varphi}$$

$$\ddot{\varphi} + \frac{3k}{m} \varphi = 0 \quad \therefore \text{振动周期 } T = 2\pi \sqrt{\frac{m}{3k}}$$



6-11 [解] 时间平均: $\bar{E}_p = \frac{1}{T} \int_0^T \frac{1}{2} k A^2 \cos^2(\omega t + \alpha) dt$

$$= \frac{1}{T} \int_0^T \frac{1}{2} k A^2 \left[\frac{1}{2} (1 + \cos 2(\omega t + \alpha)) \right] dt$$

$$= \frac{1}{4} k A^2$$

$$\bar{E}_k = \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \varphi) dt$$

$$\bar{E}_k = \frac{1}{T} \int_0^T \frac{1}{2} m \omega^2 A^2 \left[\frac{1}{2} (1 - \cos 2(\omega t + \alpha)) \right] dt = \frac{1}{4} m \omega^2 A^2 \frac{T}{T} = \frac{1}{4} m \omega^2 A^2$$

$$\because \omega^2 = \frac{k}{m} \text{ or } m \omega^2 = k \quad \therefore \bar{E}_k = \frac{1}{4} k A^2 = \bar{E}_p$$

同理: 对空间平均 $\bar{E}_p = \frac{1}{2A} \int_{-A}^A \frac{1}{2} k A^2 \cos^2(\omega t + \alpha) dx$

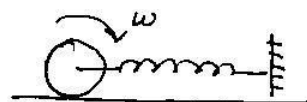
$$\bar{E}_p = \frac{2}{2A} \int_{-A}^A \frac{1}{2} k x^2 dx = \frac{k}{4A} \cdot \frac{4}{3} A^3 = \frac{k}{3} A^2$$

$$\bar{E}_k = \frac{2}{2A} \int_{-A}^A \frac{1}{2} m \omega^2 A^2 \sin^2(\omega t + \varphi) dx = \frac{1}{A} \int_{-A}^A \frac{1}{2} m \omega^2 A^2 [1 - \cos 2(\omega t + \varphi)] dx$$

$$= \frac{1}{A} \int_{-A}^A \frac{1}{2} m \omega^2 A^2 dx - \frac{1}{A} \int_{-A}^A \frac{1}{2} m \omega^2 x^2 dx$$

$$= \frac{1}{A} \left\{ m \omega^2 A^3 - \frac{1}{2} m \omega^2 \frac{2}{3} A^3 \right\} = \frac{2}{3} m \omega^2 A^2 = \frac{2}{3} k A^2$$

6-12 [解]



∵ 机械能守恒

$$\text{有 } \frac{1}{2} M V_c^2 + \frac{1}{2} I_c \omega^2 + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$\text{即 } \frac{1}{2} M \dot{x}^2 + \frac{1}{4} M R^2 \frac{\dot{x}^2}{R^2} + \frac{1}{2} k x^2 = \frac{1}{2} k A^2$$

$$\text{即 } \frac{3}{2} M \dot{x}^2 + k x^2 = \frac{1}{2} k A^2$$

两边求导得 $3M \dot{x} \ddot{x} + 2k x \dot{x} = 0$ 或 $\ddot{x} + \frac{2k}{3M} x = 0$

$$\therefore \omega = \sqrt{\frac{2k}{3M}}, \quad T = 2\pi \sqrt{\frac{3M}{2k}}$$

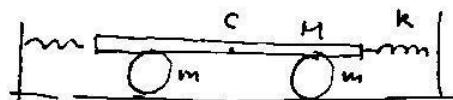
在平衡位置 $x=0$ $\frac{1}{2} M \dot{x}^2 + \frac{1}{4} M \dot{x}^2 = \frac{1}{2} k A^2$

$$\text{即 } E_T = 2 E_{gx} \quad E_T = \frac{2}{3} \left(\frac{1}{2} k A^2 \right) = \frac{1}{3} k A^2 = 0.0625 \text{ N} \cdot \text{m}$$

$$E_{gx} = \frac{1}{3} \left(\frac{1}{2} k A^2 \right) = \frac{1}{6} k A^2 = 0.0312 \text{ N} \cdot \text{m}$$

6-13 [解] 以右端平衡位置为坐标原点,

x 为偏离平衡位置的坐标.



∵ 弹簧力与重力都不做功, ∴ 系统机械能守恒

$$\text{即 } 2 \times \frac{1}{2} k x^2 + \frac{1}{2} M \dot{x}^2 + 2 \left(\frac{1}{2} m V_m^2 + \frac{1}{2} I \omega^2 \right) = C$$

$$\text{又 } \omega = \frac{V_m}{r}, \quad \dot{x} = 2r\omega = 2V_m \quad \text{or} \quad V_m = \frac{1}{2} \dot{x}$$

$$\text{即有 } k x^2 + \frac{1}{2} M \dot{x}^2 + m V_m^2 + I \omega^2 = C$$

$$\text{代入 } I = m r^2 \text{ 得到 } k x^2 + \frac{1}{2} M \dot{x}^2 + m \left(\frac{\dot{x}}{2} \right)^2 + m r^2 \frac{V_m^2}{r^2} = C$$

$$\text{即 } k x^2 + \dot{x}^2 \left[\frac{1}{2} (M+m) \right] = C$$

两边求导得 $2k x \dot{x} + (M+m) \dot{x} \ddot{x} = 0$

$$\text{即 } \ddot{x} + \frac{2k}{M+m} x = 0 \quad \therefore \omega = \sqrt{\frac{2k}{M+m}}, \quad T = 2\pi \sqrt{\frac{M+m}{2k}}$$

6-15 [解] ① 证: $E = E_k + E_p = \frac{1}{2} k x^2 + \frac{1}{2} m \dot{x}^2$

$$\therefore \frac{dE}{dt} = m \dot{x} \ddot{x} + k x \dot{x} = \dot{x} (m \ddot{x} + k x)$$

由无阻尼振动中: $f = -r \dot{x} = m \ddot{x} + k x$

因此: $\frac{dE}{dt} = \dot{x} (-r \dot{x}) = -r \dot{x}^2 = -U f$

② $\because x = A_0 e^{-\beta t} \cos(\omega_0 t + \alpha)$

任意时刻, 系统的能量为 $E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega_0^2 x^2$

($\because E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} k x^2 \quad \omega_0^2 = \frac{k}{m} \quad k = m \omega_0^2$)

$$\therefore E = \frac{1}{2} m \dot{x}^2 + \frac{1}{2} m \omega_0^2 x^2$$

$$\begin{aligned} \therefore E &= \frac{1}{2} m A_0^2 e^{-2\beta t} \{ [\beta \cos(\omega_0 t + \alpha) + \omega_0 \sin(\omega_0 t + \alpha)]^2 + \omega_0^2 \cos^2(\omega_0 t + \alpha) \} \\ &= \frac{1}{2} m A_0^2 e^{-2\beta t} f(t) = E(t) \end{aligned}$$

其中: $f(t) = [\beta \cos(\omega_0 t + \alpha) + \omega_0 \sin(\omega_0 t + \alpha)]^2 + \omega_0^2 \cos^2(\omega_0 t + \alpha)$

显然有: $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}, \quad f(t+T) = f(t)$

$$\begin{aligned} \therefore \text{有} \quad E(t+T) &= \frac{1}{2} m A_0^2 e^{-2\beta(t+T)} f(t+T) \\ &= \frac{1}{2} m A_0^2 e^{-2\beta t} f(t) e^{-2\beta T} = E(t) e^{-2\beta T} \end{aligned}$$

$\therefore E(t+T) = E(t) e^{-2\beta T}$ 得证,

6-16 [解] 阻尼振动的周期 $T = \frac{2\pi}{\omega} = \frac{2\pi}{\sqrt{\omega_0^2 - \beta^2}}$

而 $\omega_0^2 = \frac{k}{m} = \frac{8}{1.5} \quad \beta = \frac{r}{2m} = \frac{0.23}{3}$

代入上式可得 $T = 2.73 \text{ (s)}$

又: $A = A_0 e^{-\beta t} \quad \text{or} \quad \frac{A_0}{A} = e^{\beta t} = 3 \quad \text{解得} \quad t = 14.3 \text{ (s)}$

\therefore 振动次数 $N = \frac{t}{T} \approx 5 \text{ 次}$

6-17 解: - 一个周期内, 第 n 次做功 $dA_n = F_n \dot{x} dt$

$$A_{st} = \int_0^T \frac{1}{2} H A p \cos pt \sin(pt + \phi) dt = \frac{1}{2} p A^2 \pi$$

同理可得

$$A_{re} = \int_0^T -\frac{1}{2} p A^2 p^2 \sin^2(pt + \phi) dt = -\frac{1}{2} p A^2 \pi$$

$$\therefore A_{st} + A_{re} = 0 \quad \Rightarrow A_{re} = -A_{st}$$

$$(dA_{re} = f \dot{x} dt = -f \dot{x}^2 dt)$$

6-18 解: 受迫振动. 稳定时 $A = \frac{h}{\sqrt{(\omega_0^2 - p^2)^2 + 4\beta^2 p^2}}$

$$\frac{1}{2} \quad p \gg \omega_0 \text{ 时 } A = \frac{h}{\sqrt{p^4 + 4\beta^2 p^2}} = \frac{h}{p \sqrt{p^2 + 4\beta^2}}$$

$$\frac{1}{2} \quad \omega_0 \gg p \text{ 时 } A = \frac{h}{\sqrt{\omega_0^4 + 4\beta^2 p^2}} \approx \frac{h}{\omega_0^2}$$

第七章习题参考解答

7-1 解: $x = 6.0 \cos(4\pi t + 0.02\pi y) = A \cos(\omega t - ky + \phi)$

① 振幅: $A = 6.0 \text{ cm}$, $\omega = 2\pi\nu = 4\pi$

② 频率: $\nu = \frac{\omega}{2\pi} = \frac{4\pi}{2\pi} = 2 \text{ Hz}$

③ 波长: $k = \frac{2\pi}{\lambda} = 0.02\pi$; $\lambda = \frac{2\pi}{k} = \frac{2\pi}{0.02\pi} = 100 \text{ cm}$

④ 波速: $\lambda = \nu v = \frac{v}{\nu}$, $v = \lambda\nu = 100 \times 2 = 200 \text{ cm/s}$

⑤ 传播方向: $-y$

⑥ 弦上质点的最大振动速度: $\dot{x} = -A\omega \sin(\omega t - ky + \phi)$

$\dot{x}_{\max} = A\omega = 6.0 \times 4\pi = 75.4 \text{ cm/s}$

7-2 解: $\nu = 500 \text{ Hz}$, $v = 350 \text{ m/s}$

① $\Delta\phi = k\Delta y = \frac{2\pi}{\lambda}\Delta y = \frac{2\pi}{v\nu}\Delta y = 2\pi \frac{\nu}{v}\Delta y = \frac{\pi}{3}$

$\therefore \Delta y = \frac{v}{\nu} \frac{\Delta\phi}{2\pi} = \frac{350}{500 \times 2\pi} \times \frac{\pi}{3} = \frac{7}{60} = 0.117 \text{ (m)}$

② $\Delta\phi = \omega t_2 - ky_0 - \omega t_1 + ky_0 = \omega(t_2 - t_1) = 2\pi\nu \Delta t$
 $= 2\pi \times 500 \times 0.001 = \pi \text{ (360度)}$

7-3 $x = A \cos(\omega t - ky + \phi)$

斜率: $\frac{\partial x}{\partial y} = -kA \sin(\omega t - ky + \phi)$

振动速度: $|\frac{\partial x}{\partial t}| = A\omega \sin(\omega t - ky + \phi)$

$\therefore \frac{\partial x}{\partial y} = -kA \sin(\omega t - ky + \phi) = \frac{k}{\omega} A\omega \sin(\omega t - ky + \phi)$
 $= \frac{k}{\omega} |\frac{\partial x}{\partial t}| = \frac{2\pi}{\lambda\omega} |\frac{\partial x}{\partial t}| = \frac{1}{v} |\frac{\partial x}{\partial t}|$

7-4 解: $x = 0.02 \cos 30(t + \frac{y}{30}) = A \cos(\omega t - ky) = A \cos \omega(t - \frac{y}{v})$

$\therefore \omega = 30$; $v = 30 \text{ m/s}$

又 $\therefore v = \sqrt{\frac{\tau}{\rho}}$ $\therefore \tau = v^2 \rho = 30^2 \times 1.3 \times 10^{-4} \text{ N/m} = 0.117 \text{ N}$

7-6 解: $I = \frac{1.0 \text{ (W)}}{4\pi r^2 \text{ (m}^2\text{)}} = \frac{1}{4\pi} \text{ (W/m}^2\text{)}$

7-7 解: 设两列波为 $x_A = A \cos(\omega t - ky_A)$
 $x_B = A \cos(\omega t - ky_B)$

① 位相差: $\Delta\phi = k(y_A - y_B) = \frac{2\pi}{\lambda}(50 - 45.5) = 9\pi \Rightarrow \pi$

② 要得到最大的合振幅, 则两列波在该点的位相差为 0 或 $2\pi N$

则 $\Delta\phi = \frac{2\pi}{\lambda}(50 - y_B) = 2N\pi \quad (N=0, \pm 1, \pm 2)$

$\therefore 50 - y_B = N$ 或 $y_B = 50 - N$

本题是求在 $y_0 = 45.5$ 附近, 应取 $N_1 = 5$ 或 $N_2 = 4$

当 $N_1 = 5$ 时 $y_B = 45$ 此时 $\Delta\phi = 10\pi \Rightarrow 0$

当 $N_2 = 4$ 时 $y_B = 46$ 此时 $\Delta\phi = 8\pi \Rightarrow 0$

7-8 解: $x_1 = 6.0 \cos \frac{\pi}{2}(8.0t - 0.020y) = 6.0 \cos(4\pi t - 0.01\pi y)$

$x_2 = 6.0 \cos \frac{\pi}{2}(8.0t + 0.020y) = 6.0 \cos(4\pi t + 0.01\pi y)$

$\therefore \omega = 4\pi, k = 0.01\pi, \text{ 而 } k = \frac{2\pi}{\lambda} \therefore \lambda = \frac{2\pi}{k} = \frac{2\pi}{0.01\pi} = 200 \text{ cm}$

合成波: $x = x_1 + x_2 = 12 \cos ky \cos \omega t = 12 \cos 0.01\pi y \cos 4\pi t$
 $= 12 \cos \frac{2\pi}{200} y \cos 4\pi t$

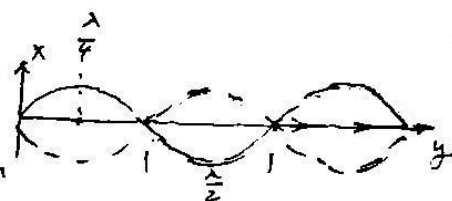
∴ 两端为波节, 端点为波腹

波腹: $y = 0, \frac{\lambda}{2}, \lambda, \frac{3}{2}\lambda, \dots$
 $= 0, 100, 200, 300, \dots$

波节: $y = 50, 150, 250, 350, \dots$

7-9 解: 三个波腹, 波腹间波节距离 $\frac{\lambda}{4}$

∴ 有 $\frac{\lambda}{4} \times 6 = 3 \text{ m}, \therefore \lambda = 2 \text{ m} = 200 \text{ cm}$



$x_1 = A \cos(\omega t - ky), x_2 = A \cos(\omega t + ky)$

$x = 2A \cos ky \cos \omega t$

由 $2A = 1 \text{ cm}$ 得 $A = 0.5 \text{ cm}$; $\therefore \lambda = 200 \text{ cm} = 2 \text{ m}$

$\therefore T = \frac{\lambda}{v} = \frac{200}{100 \times 100} = 0.02 \text{ s}; \therefore k = \frac{2\pi}{200} = \frac{\pi}{100} = \frac{2\pi}{200}$

$\therefore \omega = \frac{2\pi}{T} = \frac{2\pi}{0.02} = 100\pi = 2\pi \times 50$

$\therefore x_1 = 0.5 \cos 2\pi(50t - \frac{y}{200}) = 0.5 \cos 2\pi(50t - 0.005y)$

$x_2 = 0.5 \cos 2\pi(50t + 0.005y)$

$$7-10 \text{ 解: } ① x = 0.5 \sin \frac{\pi}{3} y \cos 40\pi t = 2 \times 0.25 \cos \left(\frac{\pi}{3} y - \frac{\pi}{2} \right) \cos 40\pi t$$

$$= 2A \cos \left(Ky - \frac{\varphi_1 - \varphi_2}{2} \right) \cos \left(\omega t + \frac{\varphi_1 + \varphi_2}{2} \right)$$

$$\therefore A = 0.25 \text{ (m)}, K = \frac{\pi}{3}, \omega = 40\pi$$

$$\text{又 } k = \frac{2\pi}{\lambda} = \frac{\pi}{3}, \lambda = 6 \text{ cm}, T = \frac{2\pi}{\omega} = \frac{2\pi}{40\pi} = 0.05 \text{ s}$$

$$v = \frac{\lambda}{T} = \frac{6}{0.05} = 120 \text{ cm/s}$$

$$② \Delta y = \frac{\lambda}{2} = \frac{6}{2} = 3 \text{ cm}$$

$$③ \text{ 由 } x = 0.5 \cos \left(Ky - \frac{\pi}{2} \right) \cos 40\pi t = 0.5 \cos \left(\frac{\pi}{3} y - \frac{\pi}{2} \right) \cos 40\pi t$$

$$\text{得 } \dot{x} = -0.5 \times 40\pi \cos \left(\frac{\pi}{3} y - \frac{\pi}{2} \right) \sin 40\pi t$$

$$\text{代入 } y = 1.5 \text{ cm}, t = \frac{9}{8} \text{ s}$$

$$\text{得 } \dot{x} = -0.5 \times 40\pi \cos \left(\frac{\pi}{3} \times 1.5 - \frac{\pi}{2} \right) \sin 40\pi \times \frac{9}{8}$$

$$= -20\pi \cos 0 \sin 45\pi = 0$$

$$7-11 \text{ 解: } v = 2 \text{ Hz}, u_{\text{源}} = 60 \text{ km/h}$$

$$\therefore v' = \left(1 + \frac{v_s}{v} \right) v = \left(1 + \frac{60 \times 10^3 / 3600}{340} \right) \times 2 = 2.1 \text{ Hz}$$

$$\therefore N = v' \times 5 \times 60 = 2.1 \times 300 = 630 \text{ (次)}$$

$$7-12 \text{ 解: } (1) \text{ 接收者静止而波源运动 } v'$$

$$v' = \frac{v}{v - v_s} v, \text{ 则 } \Delta v = v' - v = 3 \text{ (Hz)}$$

$$\text{得 } 3 = \left(\frac{v}{v - v_s} - 1 \right) v = \frac{v_s}{v - v_s} v$$

$$\text{整理得 } 3v - 3v_s = v_s v \text{ 或 } v_s(v + 3) = 3v$$

$$\therefore v_s = \frac{3v}{v + 3} = \frac{3 \times 340}{2040 + 3} = \frac{3 \times 340}{2043} = 0.498 \text{ m/s}$$

$$(2) \text{ 反射面运动而波源静止 } v', v' = \frac{v + v_s}{v} v \text{ (相当接收者运动)}$$

$$\text{接收者静止而波源运动 } v'', v'' = \frac{v}{v - v_s} v \text{ (相当波源运动)}$$

$$v'' = \frac{v}{v - v_s} v' = \frac{v + v_s}{v - v_s} v$$

$$\text{则 } \Delta v = v'' - v = 4 = \left(\frac{v + v_s}{v - v_s} - 1 \right) v = \frac{2v_s}{v - v_s} v$$

$$\therefore v = \frac{\Delta v \times (v - v_s)}{2v_s} = \frac{4 \times (340 - 0.2)}{2 \times 0.2} = \frac{340 - 0.2}{0.4} \times 4$$

$$= 3398 \text{ Hz}$$