电动力学-第六次作业

吴远清-2018300001031

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Problem 5.10

Answer:

(a)

The forces cancel each other on the two sides.

At the bottom:

$$B = \frac{\mu_0 I}{2\pi s} \Rightarrow F = \left(\frac{\mu_0 I}{2\pi s}\right) Ia = \frac{\mu_0 I^2 a}{2\pi s} \tag{1.1}$$

At the top:

$$B = \frac{\mu_0 I}{2\pi(s+a)} \Rightarrow F = \frac{\mu_0 I^2 a}{2\pi(s+a)} \tag{1.2}$$

The net force is:

$$F_{net} = \frac{\mu_0 I^2 a^2}{2\pi s(s+a)} \tag{1.3}$$

(b)

The force on the bottom is the same as (a), $\frac{\mu_0 I^2 a}{2\pi s}$.

On the left side:

$$\mathbf{B} = \frac{\mu_0 I}{2\pi y}\hat{\mathbf{z}} \tag{1.4}$$

$$d\mathbf{F} = I(d\mathbf{l}\times\mathbf{B}) = I(dx\hat{\mathbf{x}}+dy\hat{\mathbf{y}}+dz\hat{\mathbf{z}})\times\left(\frac{\mu_0 I}{2\pi y}\hat{\mathbf{z}}\right) = \frac{\mu_0 I^2}{2\pi y}(-dx\hat{\mathbf{y}}+dy\hat{\mathbf{x}}) \tag{1.5}$$

But the x component cancels the corresponding term from the right side.

And:

$$F_y = -\frac{\mu_0 I^2}{2\pi}\int_{s/\sqrt{3}}^{(s/\sqrt{3}+a/2)} \frac{1}{y}dx \tag{1.6}$$

So:

$$F_y = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(\frac{s/\sqrt{3}+a/2}{s/\sqrt{3}}\right) = -\frac{\mu_0 I^2}{2\sqrt{3}\pi}\ln\left(1+\frac{\sqrt{3}a}{2s}\right) \tag{1.7}$$

The force on the right side is the same, so the net force on the rectangle is:

$$F = \frac{\mu_0 I^2}{2\pi}\left[1 - \frac{2}{\sqrt{3}}\ln\left(1 + \frac{\sqrt{3}a}{2s}\right)\right] \tag{1.8}$$

Problem 5.11

Answer:

By Eq. 5.38, for a ring of width dz, with $T \to nIdz$:

$$B = \frac{\mu_0 nI}{2}\int \frac{a^2}{\left(a^2+z^2\right)^{3/2}}dz \tag{2.1}$$

And

$$z = a\cot\theta \tag{2.2}$$

So:

$$dz = -\frac{a}{\sin^2\theta}d\theta \tag{2.3}$$

And

$$\frac{1}{\left(a^2+z^2\right)^{3/2}} = \frac{\sin^3\theta}{a^3} \tag{2.4}$$

So:

$$B = \frac{\mu_0 nI}{2}\int \frac{a^2\sin^3\theta}{a^3\sin^2\theta}(-ad\theta) = \frac{\mu_0 nI}{2}\left(\cos\theta_2 - \cos\theta_1\right) \tag{2.5}$$

For and infinite solenoid, $\theta_2 = 0, \theta_1 = \pi$, so $(cos\theta_2 - cos\theta_1) = 2$,and:

$$B = \mu_0 nI \tag{2.6}$$

Problem 5.30

Answer

From Eq. 5.68:

$$\mathbf{A}(\mathbf{r}) = \begin{cases} \frac{\mu_0 R\sigma}{3}(\omega\times\mathbf{r}), & \text{for points inside the sphere} \\ \frac{\mu_0 R^4\sigma}{3r^3}(\omega\times\mathbf{r}), & \text{for points outside the sphere} \end{cases} \tag{3.1}$$

Here, we have $R\ to \bar{r}$ and $\sigma \to \rho d\bar{r}$:

$$\begin{aligned}\mathbf{A} &= \frac{\mu_0\omega\rho}{3}\frac{\sin\theta}{r^2}\hat{\phi}\int_0^r r^{-4}d\bar{r} + \frac{\mu_0\omega\rho}{3}r\sin\theta\hat{\phi}\int_r^R \bar{r}d\bar{r} \\ &= \left(\frac{\mu_0\omega\rho}{3}\right)\sin\theta\left[\frac{1}{r^2}\left(\frac{r^5}{5}\right) + \frac{r}{2}\left(R^2 - r^2\right)\right]\hat{\phi} = \frac{\mu_0\omega\rho}{2}r\sin\theta\left(\frac{R^2}{3} - \frac{r^2}{5}\right)\hat{\phi}\end{aligned} \tag{3.2}$$

And:

$$\mathbf{B} = \nabla \times \mathbf{A} = \frac{\mu_0 \omega \rho}{2} \left\{ \frac{1}{r \sin\theta} \frac{\partial}{\partial \theta} \left[\sin\theta r \sin\theta \left(\frac{R^2}{3} - \frac{r^2}{5} \right) \right] \hat{\mathbf{r}} - \frac{1}{r} \frac{\partial}{\partial r} \left[r^2 \sin\theta \left(\frac{R^2}{3} - \frac{r^2}{5} \right) \right] \hat{\boldsymbol{\theta}} \right\}$$
$$= \mu_0 \omega \rho \left[\left(\frac{R^2}{3} - \frac{r^2}{5} \right) \cos\theta \hat{\mathbf{r}} - \left(\frac{R^2}{3} - \frac{2r^2}{5} \right) \sin\theta \hat{\boldsymbol{\theta}} \right] \tag{3.3}$$

And:

$$\rho = \frac{Q}{(4/3)\pi R^3} \tag{3.4}$$

So:

$$\mathbf{B} = \frac{\mu_0 \omega Q}{4\pi R} \left[\left(1 - \frac{3r^2}{5R^2} \right) \cos\theta \hat{\mathbf{r}} - \left(1 - \frac{6r^2}{5R^2} \right) \sin\theta \hat{\theta} \right] \tag{3.5}$$

Problem 6.12

Answer:

(a)

$$\mathbf{M} = ks\hat{\mathbf{z}} \tag{4.1}$$

$$\mathbf{J}_b = \boldsymbol{\nabla} \times \mathbf{M} = -k\hat{\phi} \tag{4.2}$$

$$\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}} = kR\hat{\boldsymbol{\phi}} \tag{4.3}$$

$\mathbf{B}$ is in the z direction. So $\mathbf{B} = 0$ outside.

Consider a amperian loop with a inner radius s:

$$\oint \mathbf{B} \cdot d\mathbf{l} = Bl = \mu_0 I_{\text{enc}} = \mu_0 \left[\int J_b da + K_b l \right] = \mu_0 [-kl(R-s) + kRl] = \mu_0 kls \tag{4.4}$$

So:

$$\mathbf{B} = \mu_0 ks\hat{\mathbf{z}} \text{ inside.} \tag{4.5}$$