电动力学-第五次作业

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Problem 4.10

Answer:

(a)

$$\sigma_b = P \cdot \hat{n} = kR \tag{1.1}$$

$$\rho_b = -\nabla \cdot P = -\frac{1}{r^3}\frac{\partial}{\partial r}(r^2 kr) = -3k \tag{1.2}$$

(b)

For $r < R$:

$$E = \frac{1}{3\epsilon_0}\rho r\hat{r} = -\frac{k}{\epsilon_0}r \tag{1.3}$$

For $r > R$, we can treat it as all charge at center:

$$Q_t = (kR)(4\pi R^2) + (-3k)(\frac{4}{3}\pi R^3) = 0 \tag{1.4}$$

Then:

$$E = 0 \tag{1.5}$$

Problem 4.18

Answer:

(a):

For the surface, we have:

$$\int D \cdot da = Q \tag{2.1}$$

Then:

$$DA = \sigma A \tag{2.2}$$

Which means:

$$D = \sigma \tag{2.3}$$

(b)

$$D = \epsilon E$$

Combine with (2.3), we can determine the E:

$$E = \frac{\sigma}{\epsilon_1}, in\, slab\, 1 \tag{2.4}$$

$$E = \frac{\sigma}{\epsilon 2}, in\, slab\, 2 \tag{2.5}$$

And in total, we have:

$$\epsilon = \epsilon_0 \epsilon_r \tag{2.6}$$

So:

$$E_1 = \frac{\sigma}{2\epsilon_0} \tag{2.7}$$

$$E_2 = \frac{2\sigma}{3\epsilon_0} \tag{2.8}$$

(c)

$$P = \epsilon_0 \chi_e E \tag{2.9}$$

So:

$$P = \frac{\epsilon_0 \chi_e d}{\epsilon_0 \epsilon_r} = \frac{\chi_e}{\epsilon_r} \sigma \tag{2.10}$$

We have:

$$\chi_e = \epsilon_r - 1 \tag{2.11}$$

Then:

$$P_1 = \frac{\sigma}{2} \tag{2.12}$$

$$P_2 = \frac{\sigma}{3} \tag{2.13}$$

(d)

$$V = E_1 a + E_2 a = \frac{\sigma a}{6\epsilon_0} \times (3 + 4) = \frac{7\sigma a}{6\epsilon_0} \tag{2.14}$$

(e)

$$\rho_b = 0$$

Then:

$$\begin{cases} \sigma_b = \dfrac{\sigma}{2} & bottom\,of\,slab\,1 \\ \sigma_b = -\dfrac{\sigma}{2} & top\,of\,slab\,1 \\ \sigma_b = \dfrac{\sigma}{3} & bottom\,of\,slab\,2 \\ \sigma_b = -\dfrac{\sigma}{3} & top\,of\,slab\,2 \end{cases} \tag{2.15}$$

(f)

For slab 1, surface charge above is:$\frac{\sigma}{2}$, surface charge below is:$-\frac{\sigma}{2}$

$$E = \frac{\sigma}{2\epsilon_0} \tag{2.16}$$

For slab 2, surface charge above is:$\frac{\sigma}{3}$, surface charge below is:$-\frac{\sigma}{3}$

$$E = \frac{2\sigma}{3\epsilon_0} \tag{2.17}$$

This result is same as the result in (b)

Problem 4.22

Answer

We have the boundary condition:

$$\begin{cases} V_{in} = V_{out},\ where\,s = a \\ \epsilon_0 \dfrac{\partial V_{in}}{\partial s} = \epsilon_0 \dfrac{\partial V_{out}}{\partial s} & where\,s = a \\ \lim\limits_{\frac{s}{a} \to \infty} V_{out} = -E_0 s\,cos\phi \end{cases} \tag{3.1}$$

From Problem 3.24:

$$V_{in}(s,\phi) = \sum_{k=1}^{\infty} s^k (a_k cos\,k\phi + b_k sin\,k\phi) \tag{3.2}$$

And

$$V_{out}(s,\phi) = -E_0 s\,cos\phi + \sum_{k=1}^{\infty} s^{-k} (c_k cos\,k\phi + d_k sin\,k\phi) \tag{3.3}$$

From (3.1):

$$\sum_{k=1}^{\infty} a^k(a_k cos\,k\phi + b_k sin\,k\phi) = -E_0 s\,cos\phi + \sum_{k=1}^{\infty} a^{-k}(c_k cos\,k\phi + d_k sin\,k\phi) \tag{3.4}$$

$$\epsilon_r \sum_{k=1}^{\infty} k a^{k-1}(a_k cos\,k\phi + b_k sin\,k\phi) = -E_0 cos\phi - \sum_{k=1}^{\infty} k a^{-k-1}(c_k cos\,k\phi + d_k sin\,k\phi) \tag{3.5}$$

So:

$$b_k = d_k = 0 \tag{3.6}$$

$$a_k = c_k = 0 \qquad if\, k \neq 1 \tag{3.8}$$

And when $k = 1$:

$$a_1 = -\frac{E_0}{(1 + \frac{\chi_e}{2})} \tag{3.8}$$

So:

$$V_{in}(s, \phi) = -\frac{E_0}{1 + \frac{\chi_e}{2}} s\,cos\phi \tag{3.9}$$

Problem 4.26

Answer:

From Example 4.5:

$$D = \begin{cases} 0, & (r < a) \\ \dfrac{Q}{4\pi r^2}\hat{r}, & (r > a) \end{cases} \tag{4.1}$$

$$E = \begin{cases} 0, & (r < a) \\ \dfrac{Q}{4\pi \epsilon r^2}\hat{r}, & (a < r < b) \\ \dfrac{Q}{4\pi \epsilon_0 r^2}\hat{r}, & (r > b) \end{cases} \tag{4.2}$$

So:

$$\begin{aligned} W &= \frac{1}{2}\int D \cdot E dr = \frac{1}{2}\frac{Q}{(4\pi)^2}4\pi\{\frac{1}{\epsilon}\int_a^b \frac{1}{r^2}\frac{1}{r^2}r^2 dr + \frac{1}{\epsilon_0}\int_b^{\infty}\frac{1}{r^2}dr\} \\ &= \frac{Q^2}{8\pi^2}\frac{1}{\epsilon}(-\frac{1}{r})|_a^b + \frac{1}{\epsilon_0}(-\frac{1}{r})|_b^{\infty} \\ &= \frac{Q^2}{8\pi\epsilon_0(1+\chi_e)}(\frac{1}{a} + \frac{\chi_e}{b}) \end{aligned} \tag{4.3}$$