

C4 Geometrical Optics

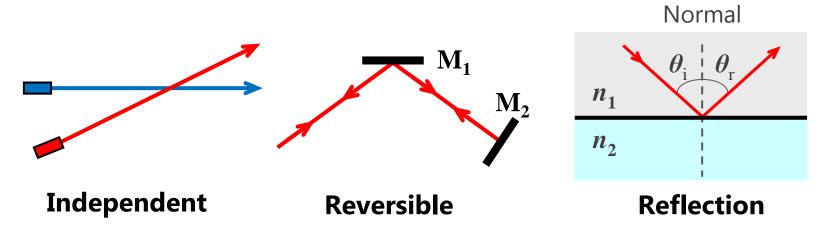
- Geometrical optics, also known as Ray optics, ignores the wave nature of light and uses geometrical methods to study the propagation of light in a homogeneous medium and their applications.
- It applies when the size of the object is much larger than the wavelength of light, e.g., imaging, illumination. $\lambda/D \rightarrow 0$





§ 4.1 Basic laws of ray optics

- ① Rectilinear propagation: In a homogeneous medium, light rays travel in straight lines.
- 2 Independent propagation. When light rays meet in the medium, each one keeps its original propagation direction.
- 3 Law of reflection: The reflected ray lies in the plane of incidence; the angle of reflection equals the angle of incidence.



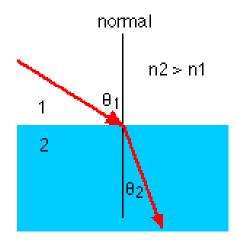


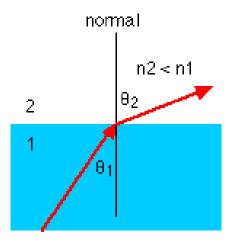
Snell's Law

Law of refraction: The refracted ray lies in the plane of 4 incidence; the angle of refraction θ_2 is related to the angle of incidence θ_1 by :

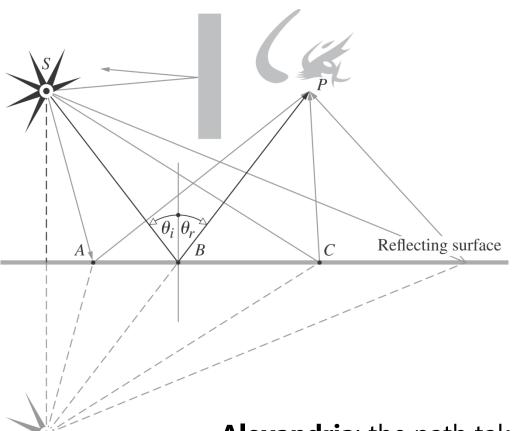
$$\sin \theta_1 / \sin \theta_2 = n_2 / n_1$$

$\sin \theta_1 / \sin \theta_2 = n_2 / n_1$ known as **Snell's law**





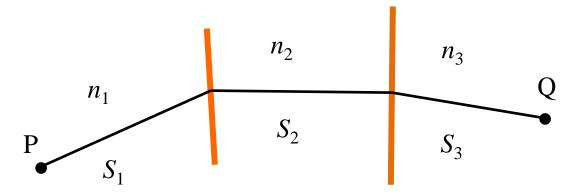
Least time?



Alexandria: the path taken by light in going from some point S to point P via a reflecting surface was the **shortest possible** one.

Optical path length

1657, **Fermat** first introduced the concept of optical path length, which unified the basic laws in geometric optics .



The travelling time for optical rays from P to Q

$$t = S_1/v_1 + S_2/v_2 + S_3/v_3 = (n_1S_1 + n_2S_2 + n_3S_3)/c \qquad v_i = c/n_i$$
$$[l] = n_1S_1 + n_2S_2 + n_3S_3$$

[l] is known as **optical path length (OPL)** from P to Q.



Optical path length

So the equation can be described as: t = [l]/c

The meaning of the t is: whatever the medium is, light spends the same amount of time for travelling the same [l].

- For homogeneous medium: [l] = nS
- For piecewise homogenous medium: $[l] = \sum_{i=1}^{N} n_i S_i$
- For medium with continuously varying refractive index:

$$[l] = \int_{P}^{Q} n dS$$

$$d[l] = dn \cdot S + n \cdot dS = (dn + n) dS \qquad \frac{dn}{n} \to 0 \qquad d[l] = n \cdot dS$$



- Fermat summarized the experimental laws of light propagation:
 - The actual path between two points taken by a beam of light is the one that is traversed in the least time. >> Principle of Least Time.
- Modern version: Optical rays traveling between two points, A and B, follow a path such that the transit time (or the optical path length) is stationary with respect to the variation of that path. That is to say, in the actual path of light, the variation of the optical path is zero,





Fermat, 1601~1665, France

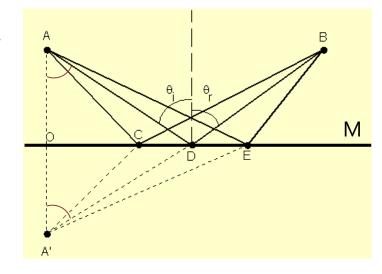


1 The optical path length is a minimum

Law of reflection deduced by Fermat's principle

What is the minimal length from A to B considering reflection?

Supposed C and E is on the mirror M, A' is the image of the A about the M.



$$\therefore$$
 ACB = A'CB (or AEB = A'EB)

So the AD+DB is the smallest *OPL*.

$$\theta_r = \angle DA'A = \angle DAA' = \theta_i$$



Law of refraction deduced by Fermat's principle

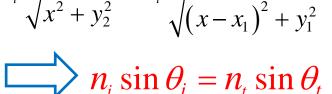
Supposed A(0, y_2), C(x, 0), B(x_1 , y_1), so the OPL from A to B

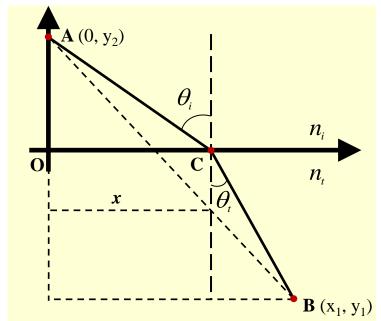
$$[l] = n_i \cdot AC + n_t \cdot CB$$

= $n_i \sqrt{x^2 + y_2^2} + n_t \sqrt{(x - x_1)^2 + y_1^2}$

$$\frac{d[l]}{dx} = n_i \frac{x}{\sqrt{x^2 + y_2^2}} + n_t \frac{(x - x_1)}{\sqrt{(x - x_1)^2 + y_1^2}} = 0$$

$$n_{i} \frac{x}{\sqrt{x^{2} + y_{2}^{2}}} = n_{t} \frac{(x_{1} - x)}{\sqrt{(x - x_{1})^{2} + y_{1}^{2}}}$$





Snell's law

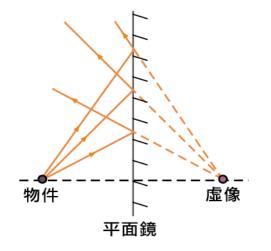


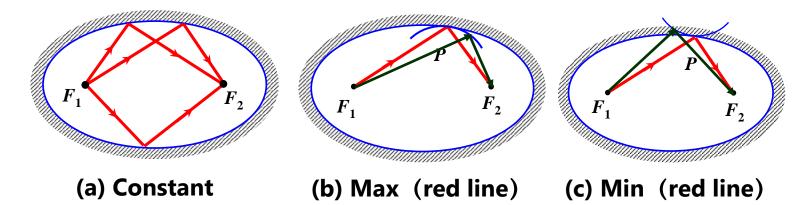
② The OPL is a constant

Example: imaging of a plane mirror

③ The OPL is a maximum

Example: A concave (凹) mirror internally tangent to an ellipse.

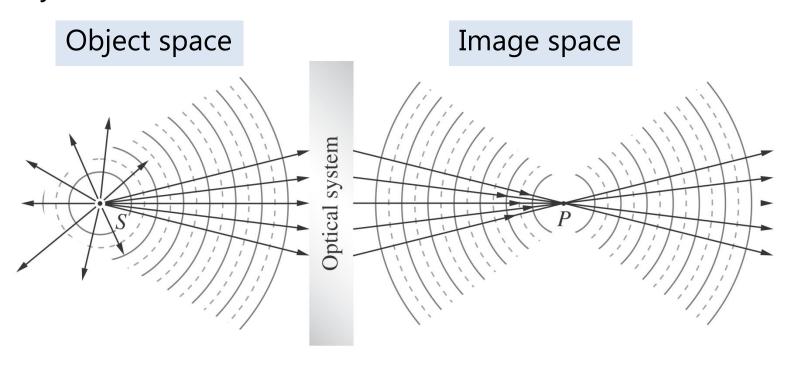






§ 1.2 Imaging

In geometric optics, **imaging** is a major issue for an optical system.



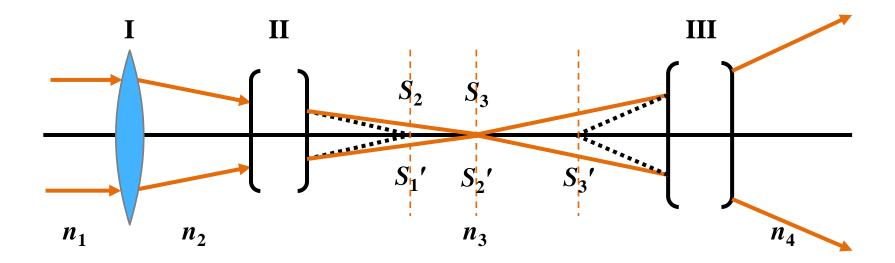
- According to the Principle of Reversibility, a point source at P would be imaged by the same optical system at S.
- S and P are conjugate points.



Optical system: a system consisting of single or multiple optical elements.

Object space: The region where the incident light is

Image space: The region where the emerging light is

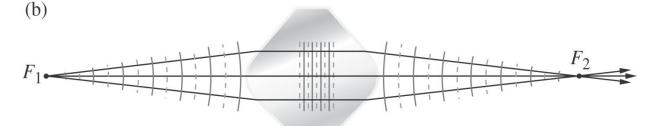


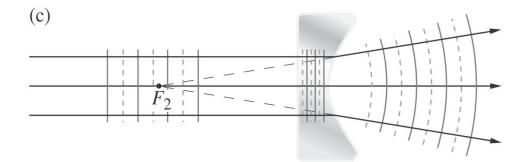


- ☐ Generally, the image of a point source after an optical system is no longer a point (aberration & diffraction), but instead a blur spot.
- ☐ If it is perfectly imaged, >> ideal optical system. A plane mirror is an ideal optical system.
- ☐ Object point and image point are **conjugate points**.
- ☐ For an ideal optical system, the OPL of each light between object point S and image point P is equal. It is an example of a constant OPL in Fermat's principle.

Optical system

☐ The image can be **real** or **virtual**.





- Both virtual and real images can be seen by the human eyes, but only real images can be received by a screen. Why?
- ☐ The object can also be **real** or **virtual** (related to the system)

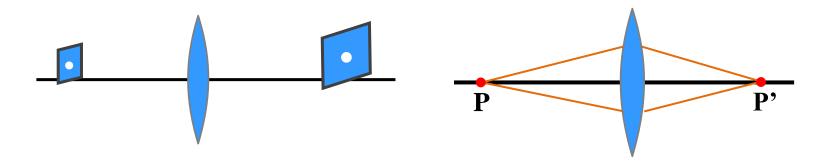


Ideal optical system: the homocentric beam pass through the optical system and is exactly imaged at one point.

① Conjugacy

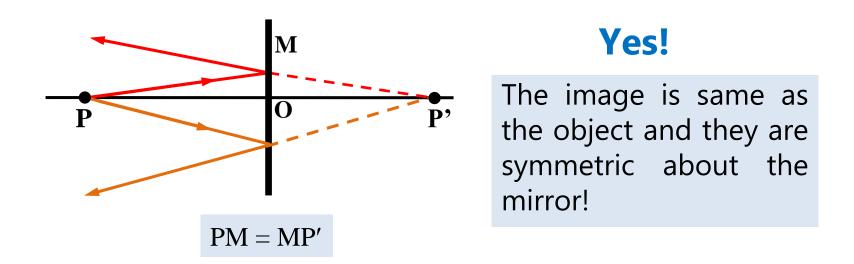
 Each point/line/plane on the object corresponds to a point/line/plane on the image.

2 Equal optical path length





Question 1: Is a plane mirror an ideal optical system?



The homocentric rays is still homocentric after being reflected by the plane mirror.



Question 2: Is a planar refraction system an ideal optical system?

No!

$$n_2$$
 A M i_2
 n_1 S' i_1

$$\overline{AM} = \overline{AS} \cdot \tan i_1 = \overline{AS'} \cdot \tan i_2$$

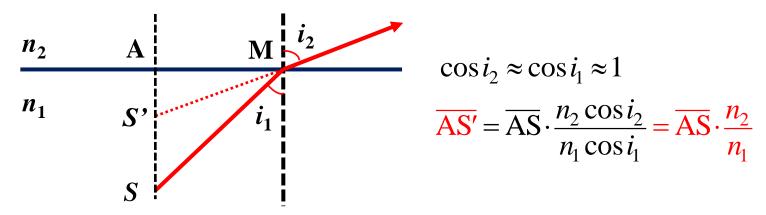
$$\overline{AS'} = \overline{AS} \cdot \sin i_1 \cos i_2$$

$$\overline{AS'} = \overline{AS} \cdot \frac{\sin i_1 \cos i_2}{\sin i_2 \cos i_1}$$
$$= \overline{AS} \cdot \frac{n_2 \cos i_2}{n_1 \cos i_1}$$

: the position of S' is changing with the angel i_1 .

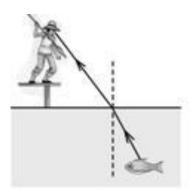


When the i_1 and i_2 is very small



When the incident angle is small, the plane refraction system is approximately an ideal optical system.

Fishing?

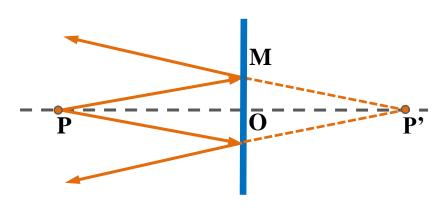




Aplanatic Surface

Aplanatic Surface: A surface that allows all light rays to travel an equal OPL between two points.

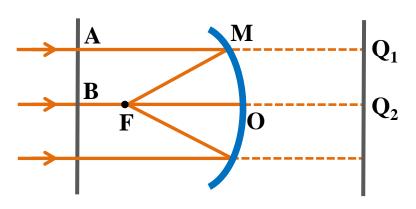
Aplanatic surface using reflection



$$PM - MP' = 0$$

(**P** and **P**')

Aplanatic Surface is a plane



$$AM+MQ_1 = BO+OQ_2$$

(infinity and F)

Aplanatic Surface is a paraboloid

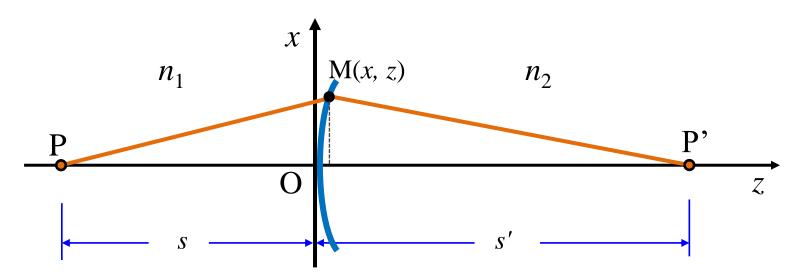
Aplanatic Surface

$$n_1 PM + n_2 MP' = n_1 s + n_2 s'$$

Aplanatic surface using refraction

$$n_1\sqrt{(s+z)^2+x^2}+n_2\sqrt{(s'-z)^2+x^2}=n_1s+n_2s'$$

The equation describes an oval line. The curve rotates around the optical axis to form the Cartesian oval surface, which is the aplanatic surface of points P and P'.

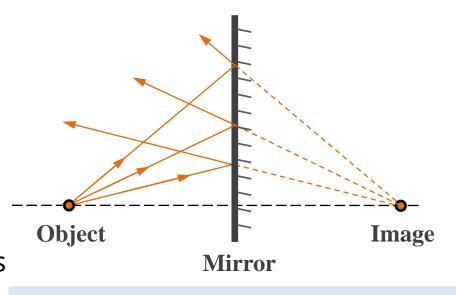




Aplanatic Surface

Some comments

- Object and image is relevance only to one optical system.
- The equal OPL between a object and its image applies to both real/virtual object, and real/virtual image.



The OPL between the object and its virtual image is constant 0.

- For virtual object/image, a virtual OPL (negative value) can be introduced.
- An point object can be imaged perfectly by an aplanatic surface, but it not for objects of finite size.



Homework

Problem 7.36

Homework*

Read about the **Principle of Least time**

(Feynman Lectures on Physics).

Generalized Reflection and Refraction.

Next week

Lens, Mirrors:

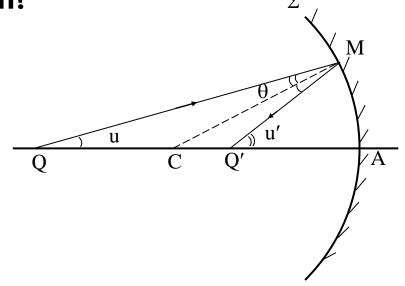
Sections 5.2, 5.4



§ 1.3 Spherical surface imaging

① What is paraxial condition?

• Spherical surface mirror Σ , whose center is C. A is the vertex. CA defines the **optical axis** (main axis).



$$u \leq 5^{\circ}$$

The ray with a small angle with the optical axis is called the paraxial rays. >> paraxial condition

$$\cos i = 1 - \frac{i^2}{2!} + \frac{i^4}{4!} \cdots \approx 1 \qquad \sin i = i - \frac{i^3}{3!} + \frac{i^5}{5!} \cdots \approx i \qquad \tan i \approx \sin i \approx i$$

Sign convention

为了导出物像的一般关系,须对有关参量规定一套符号规则。注意符号规则的定义不是唯一的。

- 基准点: 球面顶点(单球面系统)、焦点
- 基准线: 光轴、各折射点的法线
- 长度量:由指定原点量起顺光线传播方向为正,反之为负。
- 高度量: 垂直向上为正, 反之为负。
- 角度量规定:以锐角衡量,以主光轴顺时针转到光线为正。
- 规定:图上只标绝对值。



Sign Convention II

TABLE 5.1 Sign Convention for Spherical Refracting Surfaces and Thin Lenses* (Light Entering from the Left)

 s_o, f_o

 χ_{o}

 s_i, f_i

 x_i

 \boldsymbol{R}

 y_o, y_i

+ left of V

+ left of F_o

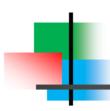
+ right of V

+ right of F_i

+ if C is right of V

+ above optical axis

^{*}This table anticipates the imminent introduction of a few quantities not yet spoken of.



Sign Convention II

TABLE 5.2	Meanings Associated with the Signs of	
Various Thin	Lens and Spherical Interface Parameter	S

Quantity Sign

+

Real object Virtual object

Real image Virtual image

Converging lens Diverging lens

Erect object Inverted object

Erect image Inverted image

Erect image Inverted image

 S_O

 s_i

f

 y_o

 y_i

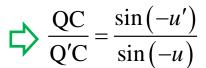
 M_T

Reflection at spherical surfaces

According to Law of Sines, in $\Delta CQ'M$

$$\frac{Q'C}{\sin(-\theta')} = \frac{MC}{\sin(\pi - (-u'))} \implies \frac{Q'C}{\sin\theta} = \frac{MC}{\sin(-u')}$$

$$\ln \Delta CQM \quad \frac{QC}{\sin \theta} = \frac{MC}{\sin(-u)}$$



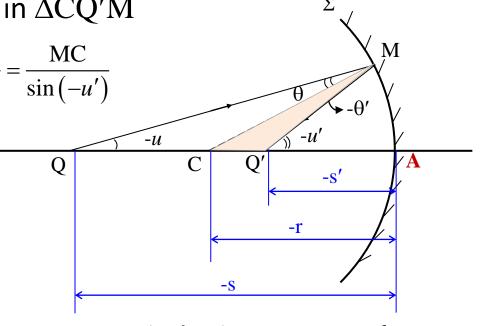
$$QC = -s - (-r) = r - s$$

$$Q'C = -r - (-s') = s' - r$$

Using paraxial condition:

$$\sin(-u) \approx \tan(-u) = MA/QA$$

 $\sin(-u') \approx \tan(-u') = MA/Q'A$



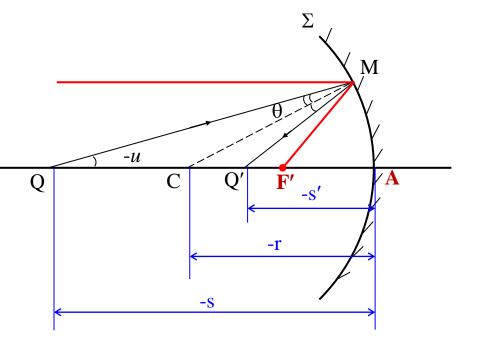
QA: optical axis. A: vertex of Σ .

$$\frac{QC}{Q'C} = \frac{QA}{Q'A} \begin{cases} \frac{r-s}{s'-r} = \frac{-s}{-s'} \\ \frac{1}{-s} + \frac{1}{-s'} = -\frac{2}{r} \end{cases}$$

Reflection at spherical surfaces

$$\frac{1}{-s} + \frac{1}{-s'} = -\frac{2}{r}$$

When the incident light is parallel to the optical axis,
 -s = ∞, then -s' = -r/2.
 This point is called image focus, marked as F'.



• Otherwise, if -s = -r/2, then $-s' = \infty$. The point where -s = -r/2 is also called **object focus**, marked as F.

Reflection at spherical surfaces

The distance from the focus to A is called focal length. By definition,

Object focal length fand image focal **length** f' are:

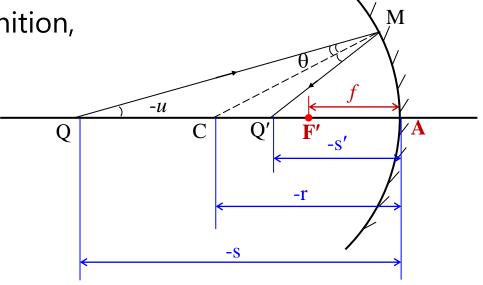
$$f = \lim_{s' \to \infty} s$$
$$f' = \lim_{s \to \infty} s'$$

$$\frac{1}{-s} + \frac{1}{-s'} = -\frac{2}{r}$$
 So, $f = f' = r/2$

So,
$$f = f' = r/2$$

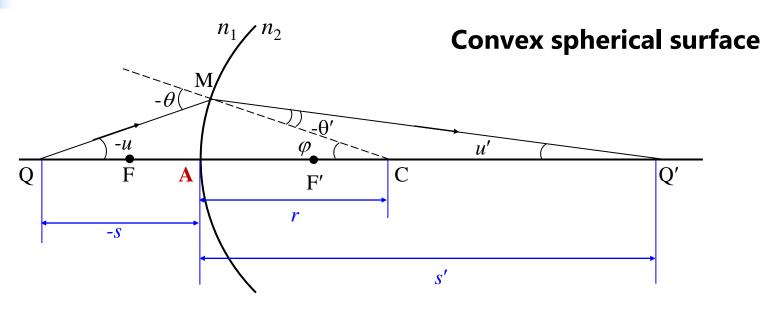
$$\frac{1}{s} + \frac{1}{s'} = \frac{1}{f}$$

If $r \rightarrow \infty$, s = s'. This is the case of plane mirror imaging.





Refraction at spherical surfaces



From the figure, $-\theta = -u + \varphi$, $-\theta' = \varphi - u'$

According to paraxial condition:

$$-u = -\tan u = MA/-s$$

 $u' = \tan u' = MA/s'$
 $\varphi = MA/r$

Snell's Law: $n_1 \sin(-\theta) = n_2 \sin(-\theta')$

$$n_1(-\theta) = n_2(-\theta')$$

$$n_1(-u+\varphi) = n_2(\varphi-u')$$

$$n_1\left(\frac{MA}{-s} + \frac{MA}{r}\right) = n_2\left(\frac{MA}{r} - \frac{MA}{s'}\right)$$

$$\frac{n_1}{-s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$



Refraction at spherical surfaces

$$\frac{n_1}{-s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r} \equiv \Phi$$
 光焦度

$$f = \lim_{s' \to \infty} s = -\frac{n_1 r}{n_2 - n_1} = -\frac{n_1}{\Phi}$$

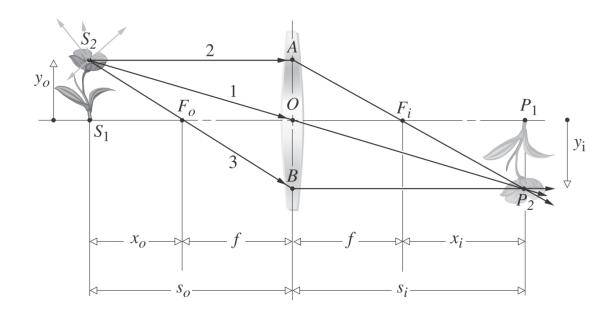
$$f' = \lim_{s \to \infty} s' = \frac{n_2 r}{n_2 - n_1}$$

$$\frac{f}{s} + \frac{f'}{s'} = 1$$

- This formula is called Gaussian Lens Formula.
- Reflection at spherical surface is a special case (f=f')



Newton form of Lens Formula

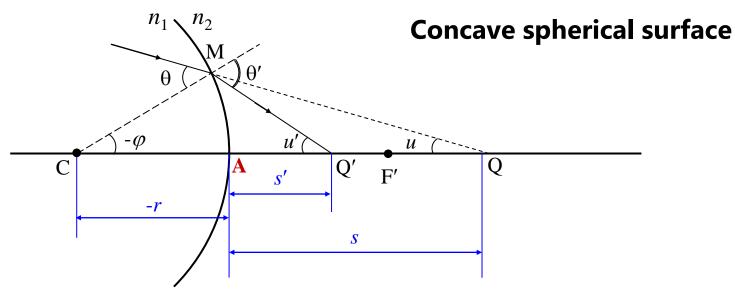


If the object distance and image distance are defined from F and F', represented by $x_{\rm o}$ and $x_{\rm i}$, and the sign convention is the same as before, then the lens formula is:

$$x_o x_i = ff'$$



Refraction at spherical surfaces



From the figure, $\theta = u - \varphi$, $\theta' = -\varphi + u'$

Using paraxial condition:

$$u = \tan u = MA/s$$

 $u' = \tan u' = MA/s'$
 $-\varphi = MA/(-r)$

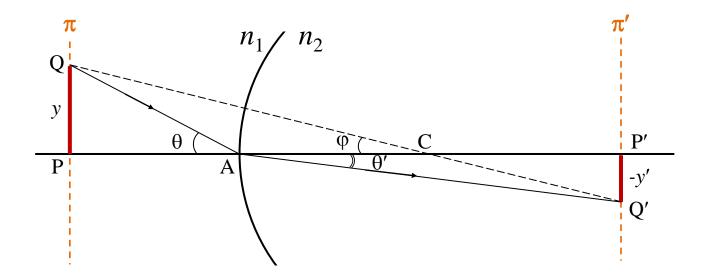
Snell's Law: $n_1 \sin \theta = n_2 \sin \theta'$

$$n_1\theta = n_2\theta'$$

$$\begin{cases}
n_1(u-\varphi) = n_2(-\varphi+u') \\
n_1\left(\frac{MA}{s} + \frac{MA}{-r}\right) = n_2\left(\frac{MA}{-r} + \frac{MA}{s'}\right) \\
n_1 + n_2 - n_2 - n_1
\end{cases}$$

$$\frac{n_1}{-s} + \frac{n_2}{s'} = \frac{n_2 - n_1}{r}$$

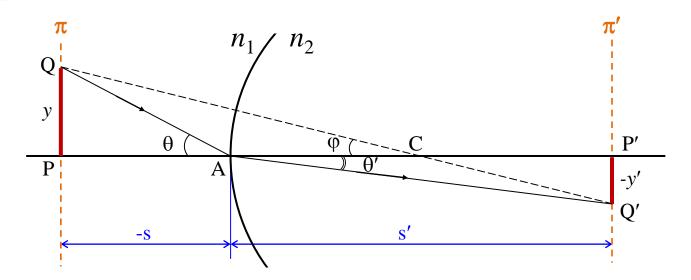
Imaging for off-axis points



From the imaging of an on-axis point P, rotate the PP' around C by φ , P \rightarrow Q, P' \rightarrow Q'. Then, Q and Q' are conjugate points.

Since φ is small, PQ can be seen as a line segment perpendicular to the main axis. The plane is called the **object plane** and represented by π . Similarly, there is a plane π' where P'Q' lies, called the **image plane**. π and π' are conjugate planes.

Imaging for off-axis points



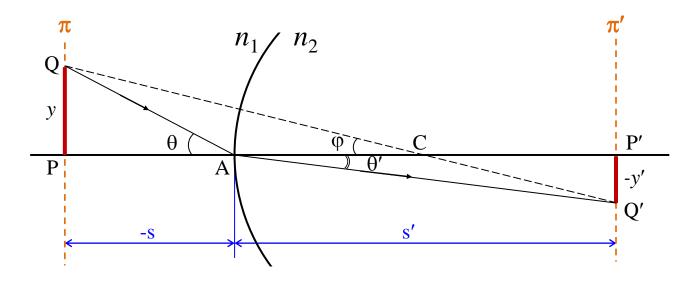
If the length of PQ is y, P'Q' is -y'.

Transverse magnification β : the ratio of the height of image and object: $\beta = y'/y$

 $\beta > 0$, Upright; $\beta < 0$, Inverted.

 $|\beta| > 1$, Magnified; $|\beta| < 1$, Minified.

Imaging for off-axis points



$$\theta = PQ/PA = y/(-s)$$

$$\theta' = P'Q'/Q'A = -y'/s'$$

 \therefore Snell's law: $n_1\theta = n_2\theta'$

$$\therefore -n_1 y/s = -n_2 y'/s'$$

$$\beta = \frac{y'}{y} = \frac{n_1 s'}{n_2 s}$$

$$\beta < 0 \qquad \text{Inverted}$$

$$n_1 < n_2 \quad |\beta| < 1 \quad \text{Minified}$$

$$\beta < 0$$

$$n_1 < n_2 \quad |\beta| < 1 \quad Mini$$

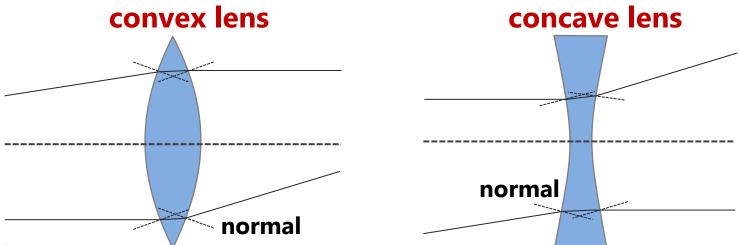
For spherical mirrors, $\beta = -s'/s$



§ 1.4 Imaging by thin lens

- Thin lens: the lens whose central thickness is negligible compared with the radius of curvature.
- Optic axis: the axis that connects two curvature centers.
- Optic center: In a thin lens, the vertices of two refraction spheres can be approximated as coincident, and the point is called optical center.

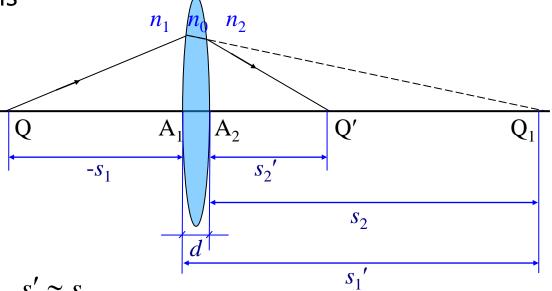
The thin lens is composed of two coaxial refraction spheres:



Lens equation

According to the Lens equation:

$$\begin{cases} \frac{n_1}{-s_1} + \frac{n_0}{s_1'} = \frac{n_0 - n_1}{r_1} \\ -\frac{n_0}{s_2} + \frac{n_2}{s_2'} = \frac{n_2 - n_0}{r_2} \end{cases}$$



For thin lens, $d \approx 0$, $\therefore s_1' \approx s_2$

$$\frac{n_1}{-s_1} + \frac{n_2}{s_2'} = \frac{n_0 - n_1}{r_1} + \frac{n_2 - n_0}{r_2} \equiv \Phi$$

• Similar to single spherical mirror, Φ is the focal power. It is the sum optical power of two refraction surface.



Lens equation

Definition of the thin lens: f, f'

$$f = \lim_{s' \to \infty} s = -\frac{n_1}{\frac{n_0 - n_1}{r_1} + \frac{n_2 - n_0}{r_2}}$$

$$f' = \lim_{s \to \infty} s' = \frac{n_2}{\frac{n_0 - n_1}{r_1} + \frac{n_2 - n_0}{r_2}}$$

$$\frac{n_1}{-s_1} + \frac{n_2}{s_2'} = \frac{n_0 - n_1}{r_1} + \frac{n_2 - n_0}{r_2} \equiv \Phi$$

$$\Rightarrow \frac{f}{f'} = -\frac{n_1}{n_2}$$

: Thin lens equation:

$$\frac{n_1}{-s_1} + \frac{n_2}{s_2'} = \Phi$$



$$\frac{f}{s_1} + \frac{f'}{s_2'} = 1$$

$$n_1 = n_2$$

Inin lens equation:
$$\frac{n_1}{-s_1} + \frac{n_2}{s_2'} = \Phi \qquad \Longrightarrow \qquad \frac{f}{s_1} + \frac{f'}{s_2'} = 1 \qquad \Longrightarrow \qquad \begin{cases} f = -f' \\ -\frac{1}{s} + \frac{1}{s'} = \frac{1}{f'} \end{cases}$$



Lens equation

Transverse magnification of single spherical lens:

$$\beta = \frac{y'}{y} = \frac{n_1 s'}{n_2 s}$$

Transverse magnification of single spherical lens:

$$\beta = \beta_1 \cdot \beta_2 = \frac{n_1 s_1'}{n_0 s_1} \cdot \left(\frac{n_0 s_2'}{n_2 s_1'}\right) = \frac{n_1 s_2'}{n_2 s_1}$$

• When the thin lens is in air, then $n_1 = n_2 = 1$, so

$$\beta = \frac{s_2'}{s_1}$$

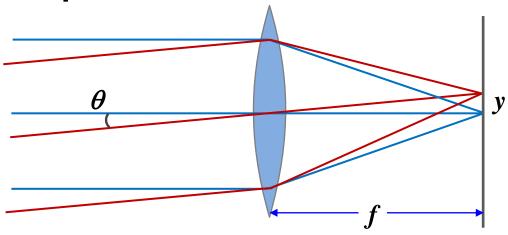


Focal plane: a plane normal to the optical axis and passing though the image focus. First/front focal plane and Second/back focal plane.

- If the light emanate from point P on the front focal plane passes through the thin lens, the outgoing light must be collimated, and along the direction of OP.
- Parallel light incident at any angle will converge at one point on the back focal plane.

Focal plane

A lens makes a correlation between a direction and a location on a plane.

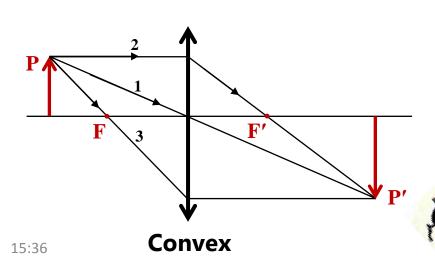


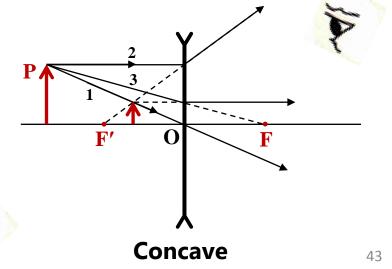
- Two parallel beams incidence with an angle of θ converse on the focal plane with a distance : $y = f \cdot \tan \theta \approx f \cdot \theta$
- Angle (i.e., propagation direction, momentum) v.s. position.
- Imaging by a lens is equivalent to perform twice the Fourier transform. >> Chapter 8



Ray diagram

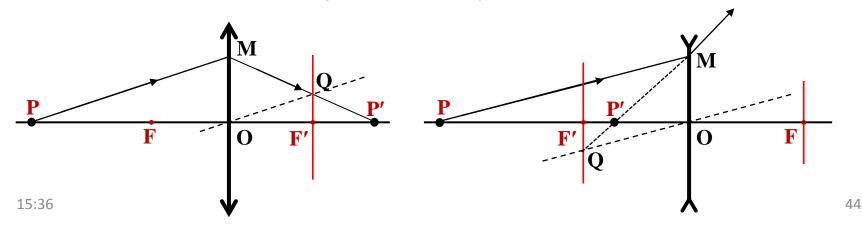
- For off-axis point P, any two of the following three pairs.
 - ① The incident light PO passing through the optical center is still in the original direction when it exits;
 - ② The incident light parallel to the main axis passes through the image focus F';
 - When the incident light passes though the focal point F, it is parallel to the main axis after the lens.





Ray diagram

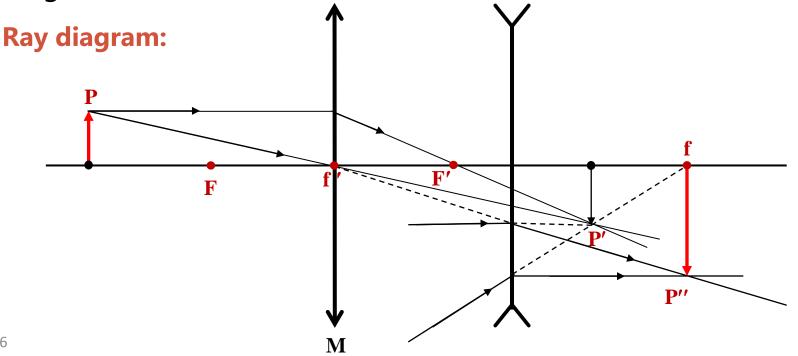
- For on-axis Point P, two ways to get the image point P'.
- (a) Move the P away from the main axis (vertical), find the image point, move the image point back to the main axis, and get P'.
- (b) Using focal plane:
 - ① Plot an arbitrary line PM;
 - ② Plot a parallel line that passes the optical center O with the OQ crossing the focal plane at Q;
 - ③ Connected M and Q, the line MQ intersects the main axis at P'.





Thin-Lens combinations

Exercise: The distance from P to the lens M ($f_1 = 15$ cm) is 30 cm. Place a concave lens 20 cm away from the convex lens on the other side ($f_2 = 20$ cm), where is the P"? And compute the magnification.





Thin-Lens combinations

According to the lens equation, $-1/s + 1/s' = 1/f_1$

then:
$$1/s' = 1/f - 1/s = 1/30$$

$$\therefore$$
 s' = 30 cm

Magnification: $\beta = -s'/s = -1$

For concave lens, object distance: $s_1 = 30 \text{ cm} - 20 \text{ cm} = 10 \text{ cm}$

Similarly, $s'_1 = 20 \text{ cm. } \beta_{\text{total}} = -2$

This system changes the size of the image by slightly changing the position of the concave mirror.



§ 1.5 Stops in optical system

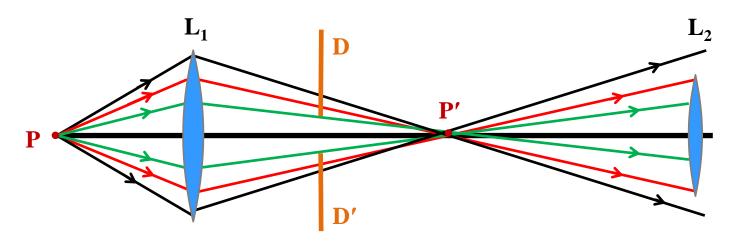
- **Stop**: Optical elements that limit the imaging beam. e.g., a lens, an aperture, the frame of windows.
- The functionality of an optical stop?
 - ① Improve the image quality and reduce some aberrations
 - ② Control light intensity through the system
 - ③ Limit the scope of imaging (field size)
 - 4 Block harmful light
- Aperture stop, Field stop, Glare stop (消杂散光光阑)

-

Aperture stop

 Aperture stop: The aperture that limits the breadth of the beam of light coming from an axial object point as it passes through the system.

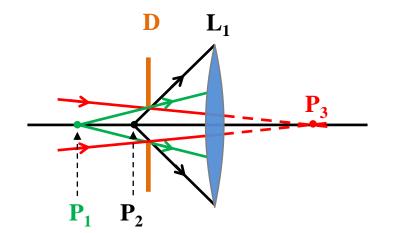
It determines the **cross-section size** of the optical beam and **optical power** through the optical system.



$$L_1 \rightarrow L_2 \rightarrow DD'$$

Aperture stop

 For a fixed point, the aperture stop is determined by the relative position and size of the apertures in the system.



For P_1 , D is aperture stop

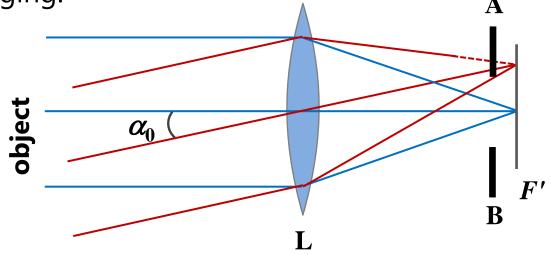
For P₂, L is aperture stop

For P_3 , D is aperture stop

 Comments: The actual aperture stop is related to the position of the object point on the axis.

Field stop

Field stop: A stop that limits the size or angular breadth of imaging.

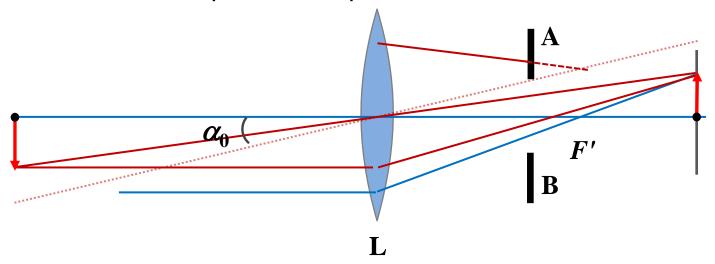


- L: aperture stop. AB, the field stop.
- For an object point on-axis, the presence of AB does not change the brightness of the F'.
- The light beam off the axis is blocked by AB.
- Vignetting: The center of the image is bright and the edges are dark.



Aperture stop and field stop

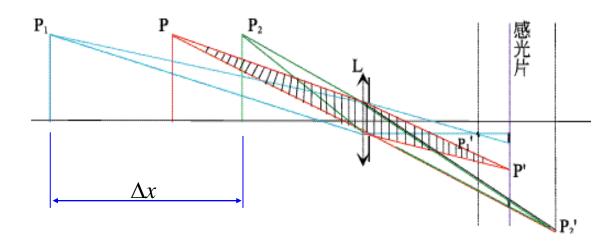
 An aperture can be both an aperture stop (point on the axis) and a field stop (off-axis point)



- When the stop is in the object plane or image plane, it is only the field stop.
- When the stop is in the focal plane, it is only the aperture stop.



Depth of field/focus



For a given aperture, if the position of P varies a distance Δx , the image of it on the photosensitive film can be seen as clear, then the distance within the object space Δx is called the **depth of field**. The corresponding distance in the image space is called the **depth of focus**.

Aperture, fnumber,

- **Relative aperture**: The ratio of the diameter of an aperture stop to the focal length of an objective lens D/f.
- The larger the D/f, the stronger the capability of the system to collect light.
- f number: reciprocal of D/f

E.g. an object lens f = 160 mm, diameter D = 20 mm. f number: 160/20 = 8, marked as f/8. Indicating that the focal length of the system is 8 times that of the diameter.

• Numerical Aperture: $NA = n_0 \sin \alpha$

 n_0 : the refractive index of the surrounding medium α : Incidence angle



Homework

Problem 7.36

Problem 5.5, 5.26, 5.47 and the one in

next slice

Homework*

Read about the **Principle of Least time**

(Feynman Lectures on Physics).

Generalized Reflection and Refraction.

Next week

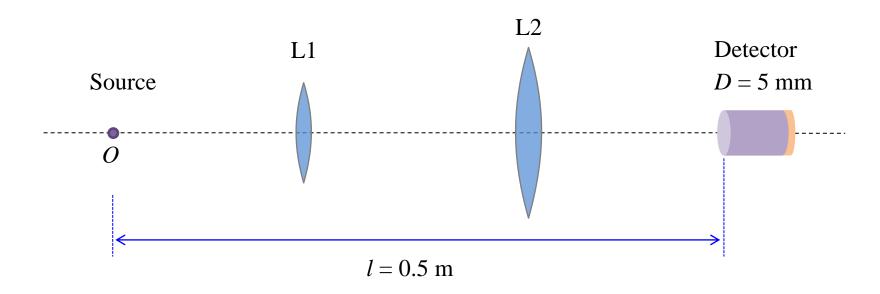
Reflection and Refraction at interfaces,

Fresnel Equations,

Sections 4.2-4.4, 4.6

Homework

已知某光路中,一点光源(光源亮度与方向无关)固定在如图所示位置,两个无像差的薄透镜直径 D_1 = 25 mm、 D_2 = 38 mm ,焦距为 f_1 = 0.1m、 f_2 = 0.2m,与光源相距l = 0.5m处固定一个开口直径5 mm,长度2 cm的探测器,试求如何放置两个透镜才能使收集效率最大?(假设空气折射率为1)



注: L1、L2的先后顺序可调、位置可动,也可只取一个或不用。但Source和 Detector位置固定。