热力学与统计物理-第三次作业

吴远清-2018300001031

2020 年 3 月 17 日

1.17

We can treat this as biominal distribution: Each molecules only have two states: In the volume or not. The probability of the molecules in the volume is:

$$p = \frac{V}{V_0} \tag{1.1}$$

Then, use Gaussian approximation:

$$P(N) = (2\pi N_0 pq)^{-\frac{1}{2}} exp[-\frac{(N-N_0p)^2}{2N_0pq}] \tag{1.2}$$

So, we can get the result:

$$\begin{aligned} P(N, N+dN) &= \int_N^{N+dN} (2\pi N_0 pq)^{-\frac{1}{2}} exp[-\frac{(N-N_0p)^2}{2N_0pq}]dN \\ &= (2\pi N_0 pq)^{-\frac{1}{2}} exp[-\frac{(N-N_0p)^2}{2N_0pq}]dN \\ &= (2\pi N_0 \frac{V(V_0-V)}{V_0^2})^{-\frac{1}{2}} exp[-\frac{(V_0N-N_0V)^2}{2N_0V(V_0-V)}]dN \end{aligned} \tag{1.3}$$

1.18

Answer:

This question equal to the off-latice 3D random walk.

For each step, we have the constant length l, and a random direction that can be described by θ, ϕ, θ is the angle between the projection of direction vector on XY plane and the X positive semi-axis, and ϕ is the angle between the direction vector and XY plane. For N steps:

$$\{\theta_1, \theta_2, ..., \theta_N, \phi_1, \phi_2, ..., \phi_N\} \tag{2.1}$$

Then, we have:

$$\begin{cases} \overline{r_x^2} = l^2 \times \overline{\sum_{i=1}^{N}(cos^2\phi_i cos^2\theta_i)} = l^2 \times N \times \overline{cos^2\phi cos^2\theta} \\ \overline{r_x^2} = l^2 \times \overline{\sum_{i=1}^{N}(cos^2\phi_i sin^2\theta_i)} = l^2 \times N \times \overline{cos^2\phi sin^2\theta} \\ \overline{r_z^2} = l^2 \times \overline{\sum_{i=1}^{N}(sin^2\phi_i)} = l^2 \times N \times \overline{sin^2\phi} \end{cases} \tag{2.2}$$

Using the periodicity of triangle function:

$$\begin{cases} \overline{cos^2\theta} \ = \ \int_0^{2\pi} cos^2\theta d\theta \ = \ \frac{1}{2} + \frac{1}{2}\int_0^{2\pi} cos(2\theta)d\theta \ = \ \frac{1}{2} \\ \overline{sin^2\theta} \ = \ \int_0^{2\pi} sin^2\theta d\theta \ = \ \frac{1}{2} - \frac{1}{2}\int_0^{2\pi} cos(2\theta)d\theta \ = \ \frac{1}{2} \end{cases} \tag{2.3}$$

Same for ϕ, ϕ and θ are independent variable, so:

$$\begin{cases} \overline{cos^2\phi cos^2\theta} = \overline{cos^2\phi} \times \overline{cos^2\theta} \\ \overline{cos^2\phi sin^2\theta} = \overline{cos^2\phi} \times \overline{sin^2\theta} \end{cases} \tag{2.4}$$

Substitute (2.4), (2.3) into (2.2):

$$\begin{cases} \overline{r_x^2} = \frac{N}{4}l^2 \\ \overline{r_y^2} = \frac{N}{4}l^2 \\ \overline{r_z^2} = \frac{N}{2}l^2 \end{cases} \tag{2.5}$$

Finally, we get:

$$\overline{r^2} \ = \ \overline{r_x^2} + \overline{r_y^2} + \overline{r_z^2} = N \times l^2 \tag{2.6}$$

2.1

Answer:

For this particle, all of its energy is kinetic energy, so we have:

$$P = \sqrt{2mE} \tag{3.1}$$

And the classical phase space is:

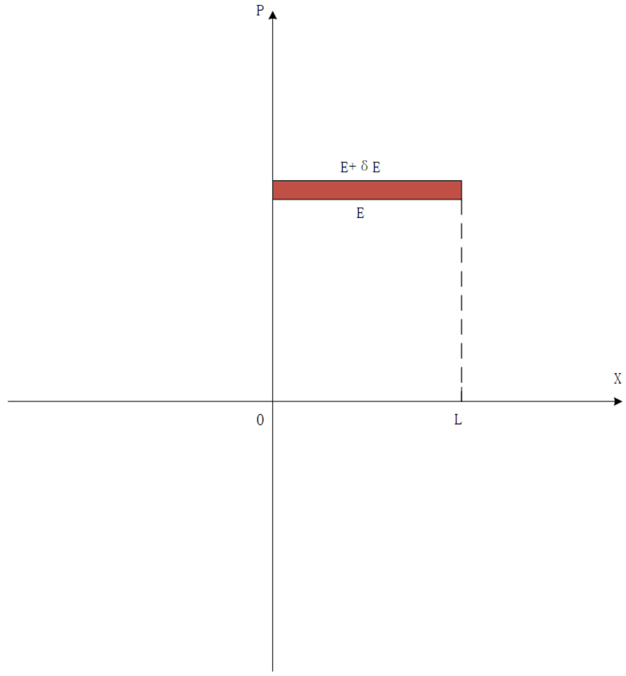

图 1: Classical phase space

2.4

Answer:

(a). The energy of the system is:

$$E = -(n_1 - n_2)\mu H = -(2n_1 - N)\mu H \tag{4.1}$$

So:

$$n_1 = \frac{N}{2} - \frac{E}{2\mu H} \tag{4.2}$$

Since δE is very small compared to E:

$$\Omega(E) = \int_E^{E+\delta E} \Omega(E')dE' \approx \Omega(E)\delta E = \Omega(n_1)\delta n_1 \tag{4.3}$$

From (4.2):

$$\delta n_1 = \frac{\delta E}{2\mu H} \tag{4.4}$$

Then:

$$\Omega(E) = \frac{N!}{n_1!(N-n_1)!}\frac{\delta E}{2\mu H} = \frac{N!}{(\frac{N}{2} - \frac{E}{2\mu H})!(\frac{N}{2} + \frac{E}{2\mu H})!}\frac{\delta E}{2\mu H} \tag{4.5}$$

(b) From (4.5):

$$ln\,\Omega(E) = ln\,N! - ln\,(\frac{N}{2} - \frac{E}{2\mu H})! - ln\,(\frac{N}{2} + \frac{E}{2\mu H})! + ln\,\frac{\delta E}{2\mu H} \tag{4.6}$$

With Stirling's formula:

$$ln\,n! \approx n\,ln\,n - n \tag{4.7}$$

(4.6) aprroximate to:

$$ln\,\Omega(E) = N\,ln\,N - (\frac{N}{2} - \frac{E}{2\mu H})\,ln\,(\frac{N}{2} - \frac{E}{2\mu H})$$

$$-(\frac{N}{2} + \frac{E}{2\mu H})\,ln\,(\frac{N}{2} + \frac{E}{2\mu H}) + ln\,\frac{\delta E}{2\mu H} \tag{4.8}$$

(c) For this system, the spin is parallel or antiparallel have an equal probability. So:

$$P(n_1) = (\frac{1}{2}\pi N)^{-\frac{1}{2}}\,exp[-\frac{(n_1 - \frac{N}{2})^2}{\frac{N}{2}}] \tag{4.9}$$

Consider (4.2):

$$P(E) = (\frac{1}{2}\pi N)^{-\frac{1}{2}}\,exp(-\frac{E^2}{2N\mu^2 H^2}) \tag{4.10}$$

So, the $\Omega(E)$ is:

$$\Omega(E) = P(E) \times 2^N = 2^N(\frac{1}{2}\pi N)^{-\frac{1}{2}}\,exp(-\frac{E^2}{2N\mu^2 H^2}) \tag{4.11}$$

2.7

Answer:

(a). The energy's change is:

$$\Delta E = E(L_x + dL_x) - E(L_x) \tag{5.1}$$

The change of system energy is equal to the negative value of system work to the outside:

$$W = -\Delta E \tag{5.2}$$

The system's work to outside given by:

$$W = \int_{L_x}^{L_x+dL_x} F_x(x')dx' \approx F_x dL_x \tag{5.3}$$

Since dL_x is a small amount, The approximation in the above formula can be satisfied. Then, from (5.1) to (5.3), we can get:

$$F = -\frac{E(L_x + dL_x) - E(L_x)}{dL_x} = -\frac{\partial E}{\partial L_x} \tag{5.4}$$

Q.E.D (b). The energy of this particle is given by:

$$E = \frac{\hbar^2}{2m}\pi^2(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_z^2}{L_z^2}) \tag{5.5}$$

Then we can calculate the force that particle exert on the wall:

$$\begin{cases} F_x = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_x^2}{L_x^3} \\ F_y = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_y^2}{L_y^3} \\ F_z = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_z^2}{L_z^3} \end{cases} \tag{5.6}$$

Then we can calculate the pressure on three direction:

$$\begin{cases} P_x = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_x^2}{L_x^2 V} \\ P_y = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_y^2}{L_y^2 V} \\ P_z = 2\dfrac{\hbar^2}{2m^2}\pi^2\dfrac{n_z^2}{L_z^2 V} \end{cases} \tag{5.6}$$

The total pressure is:

$$P = \frac{P_x + P_y + P_z}{3} = \frac{2}{3V}\frac{\hbar^2}{2m^2}\pi^2(\frac{n_x^2}{L_x^2} + \frac{n_y^2}{L_y^2} + \frac{n_y^2}{L_y^2}) \tag{5.7}$$

The average is:

$$\bar{P} = \frac{2}{3}\frac{\bar{E}}{V} \tag{5.8}$$

2.10

Answer:

We can treat $\bar{P}$ as a function of V:

$$\bar{P} = KV^{-\gamma} \tag{6.1}$$

Since the wrok is done through a quasistatic process, we can divided the change of volume into three dimension's length change, here we only consider X direction, the other two directions will be same and easy to prove:

$$F_x = \bar{P}_x S_x \tag{6.2}$$

$$W = \int_{X_s}^{X_f} F_x dx = \int_{X_s}^{X_f} K(S_x X)^{-\gamma} S_x dx = K S_x^{-\gamma+1} \int_{X_s}^{X_f} X^{-\gamma} dx \tag{6.3}$$

Then:

$$W = -K(\gamma - 1)(V_f^{-\gamma+1} - V_s^{-\gamma+1}) \tag{6.4}$$

Replace K with $\bar{P}_s V_s^{\gamma}$:

$$W = -\bar{P}_s V_s^{\gamma}(\gamma - 1)(V_f^{-\gamma+1} - V_s^{-\gamma+1}) \tag{6.5}$$

2.11.

Answer:

First, we'll calculate the difference of internal energy between state A and B.

$$\Delta E = -W = \int_{V_A}^{V_B} \bar{P} dV = \int_{V_A}^{V_B} \alpha V^(- 5/3) dV = -3600J \tag{7.1}$$

(a).

$$W = \int_{V_A}^{V_{B'}} \bar{P} dV = 22400J \tag{7.2}$$

With the first law:

$$\Delta E = Q - W = -3600J \tag{7.3}$$

$$Q = 18800J \tag{7.4}$$

(b)

$$W = \int_{V_A}^{V_B} \bar{P} dV = 11500J \tag{7.5}$$

From (7.3):

$$Q = 7950J \tag{7.6}$$

(c)

$$W = \int_{V_A'}^{V_B} \bar{P} dV = 700J \tag{7.7}$$

From (7.3):

$$Q = -2900J \tag{7.8}$$