电动力学-第一次作业

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1.11 Find the gradients of the following functions:
(a)$f(x,y,z) = x^2 + y^3 + z^4$
(b)$f(x,y,z) = x^2y^3z^4$
(c)$f(x,y,z) = e^x sin(y)ln(z)$
Answer:
(a):

$$\nabla f = 2x\hat{x} + 3y^2\hat{y} + 4z^3\hat{z} \tag{1.1}$$

(b):

$$\nabla f = 2xy^3z^4\hat{x} + 3x^2y^2z^4\hat{y} + 4x^2y^3z^3\hat{z} \tag{1.2}$$

(c):

$$\nabla f = e^x sin(y)ln(z)\hat{x} + e^x cos(y)ln(z)\hat{y} + \frac{e^x sin(y)}{z}\hat{z} \tag{1.1}$$

1.15 Calculate the divergence of the following vector functions:
(a)$V_a = x^2\hat{x} + 3xz^2\hat{y} - 2xz\hat{z}$
(b)$V_b = xy\hat{x} + 2yz\hat{y} + 3zx\hat{z}$
(c)$V_c = y^2\hat{x} + (2xy + z^2)\hat{y} + 2yz\hat{z}$
Answe:
(a):

$$\nabla \cdot V_a = 2x - 2x = 0 \tag{2.1}$$

(b):

$$\nabla \cdot V_b = y + 2z + 3x \tag{2.2}$$

(c):

$$\nabla \cdot V_c = 2x + 2y \tag{2.3}$$

1.18 Calculate the curls of the vector functions in Prob. 1.15.

Answer:

(a)

$$\nabla \times V_a = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = -6xz\hat{x} + 2z\hat{y} + 3z^2\hat{z} \tag{3.1}$$

(b):

$$\nabla \times V_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ xy & 2yz & 3zx \end{vmatrix} = -2y\hat{x} + -3z\hat{y} + -x\hat{z} \tag{3.1}$$

(c):

$$\nabla \times V_b = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ x^2 & 3xz^2 & -2xz \end{vmatrix} = 0 \tag{3.3}$$

1.36

(a) Show that:

$$\int_S f(\nabla \times A) \cdot da = \int_S [A \times (\nabla f)] \cdot da + \oint_P fA \cdot dl$$

(b) Show that:

$$\int_v B \cdot (\nabla \times A) d\tau = \int_v A \cdot (\nabla \times B) d\tau + \oint_S (A \times B) \cdot da$$

Answer: (a) According to the product rules:

$$f(\nabla \times A) = \nabla \times (fA) + A \times (\nabla f) \tag{4.1.1}$$

So, the integrate transform to:

$$\int_S f(\nabla \times A) \cdot da = \int_S [A \times (\nabla f)] \cdot da + \int_S \nabla \times (fA) \cdot da \tag{4.1.2}$$

And using Stokes theorem:

$$\int_S [\nabla \times (fA)] \cdot da = \oint_P fA \cdot dl \tag{4.1.3}$$

So, we get:

$$\int_S f(\nabla \times A) \cdot da = \int_S [A \times (\nabla f)] \cdot da + \oint_P fA \cdot dl \tag{4.1.4}$$

Q.E.D

(b):

Similarly, according to the product rules:

$$B \cdot (\nabla \times A) = A \cdot (\nabla \times B) + \nabla \cdot (A \times B) \tag{4.2.1}$$

According to the equation (4.2.1), the left side of the equation to be proved could be transform to:

$$\int_v B \cdot (\nabla \times A) \cdot d\tau = \int_v A \cdot (\nabla \times B) \cdot d\tau + \int_v \nabla \cdot (A \times B) \cdot d\tau \tag{4.2.2}$$

Then, using the Green theorem:

$$\int_v \nabla \cdot (A \times B) \cdot d\tau = \int_S (a \times B) \cdot da \tag{4.2.3}$$

So, we get:

$$\int_v B \cdot (\nabla \times A) d\tau = \int_v A \cdot (\nabla \times B) d\tau + \oint_S (A \times B) \cdot da \tag{4.2.4}$$

Q.E.D