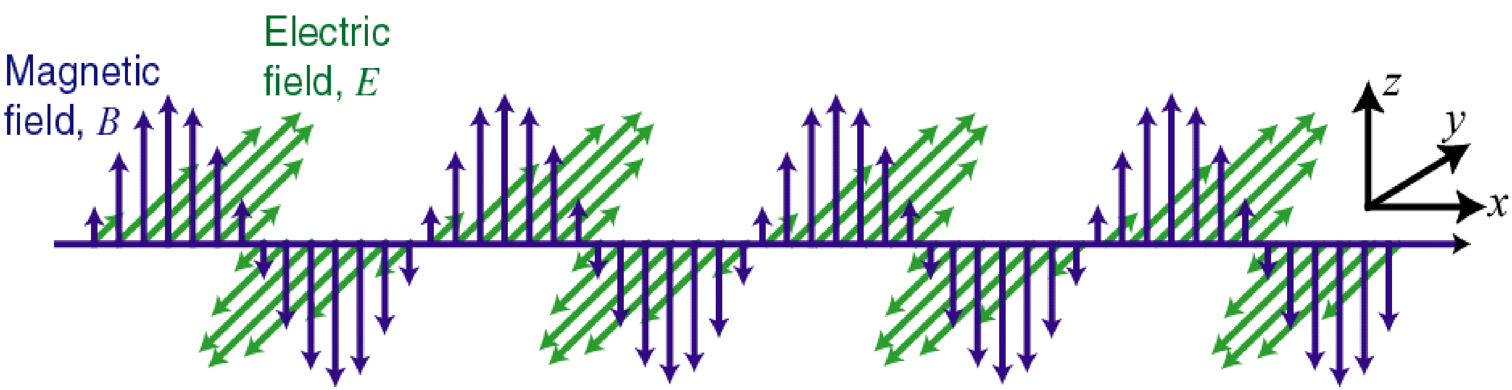
C3 Light as EM Waves

- Transverse wave nature of light, intensity, energy flow.
- Polarization states, Jones matrix/vector.
- Absorption, dispersion, refractive index.
- Scattering of light by small particles.



Vector wave： the wavefunction is a vector. (e.g. EM wave)

Scalar wave： the wavefunction is a scalar. (e.g. Acoustic wave)

- Lightwaves are vector waves, but sometimes they are also treated as scalar waves. In these cases, it should be understood to be one specific component of the EM waves.
- For simplicity, we treat lightwaves as scalar waves when discussing **interference** and **diffraction**. Only when we are talking about phenomenon associated with polarized light, light is treated as a vector wave.

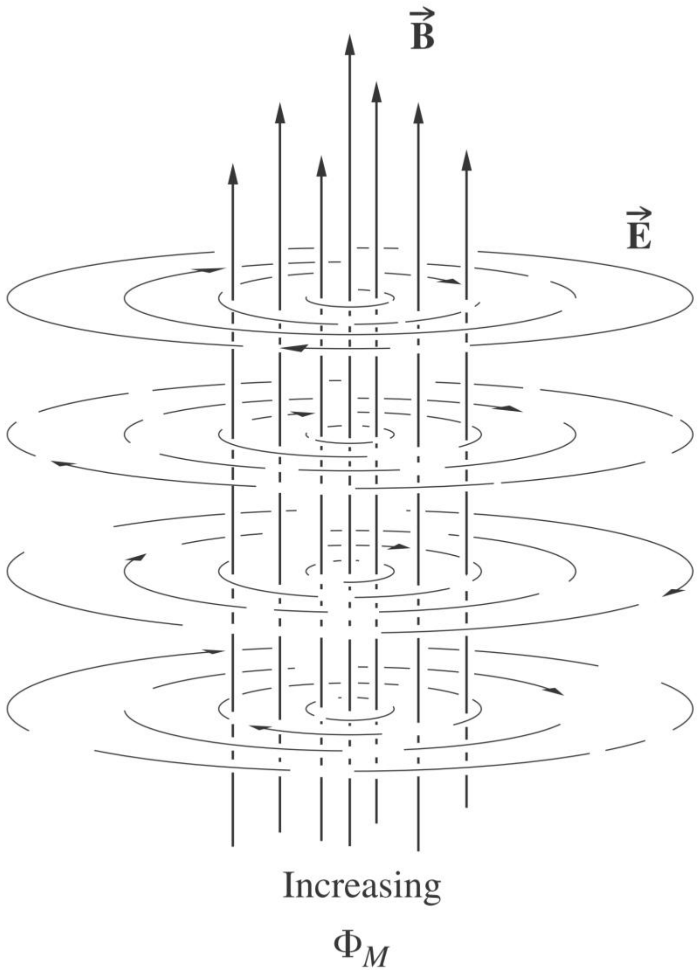
- In the light-matter interaction, the electric field has a dominant effect. Therefore, a lightwave is represented by its $\mathbf{E}$-field. The $\mathbf{E}$-vector is called the **light vector**.

  Interaction strength $H' = \hbar g = \mu \cdot \mathbf{E}$ (dipole approximation)

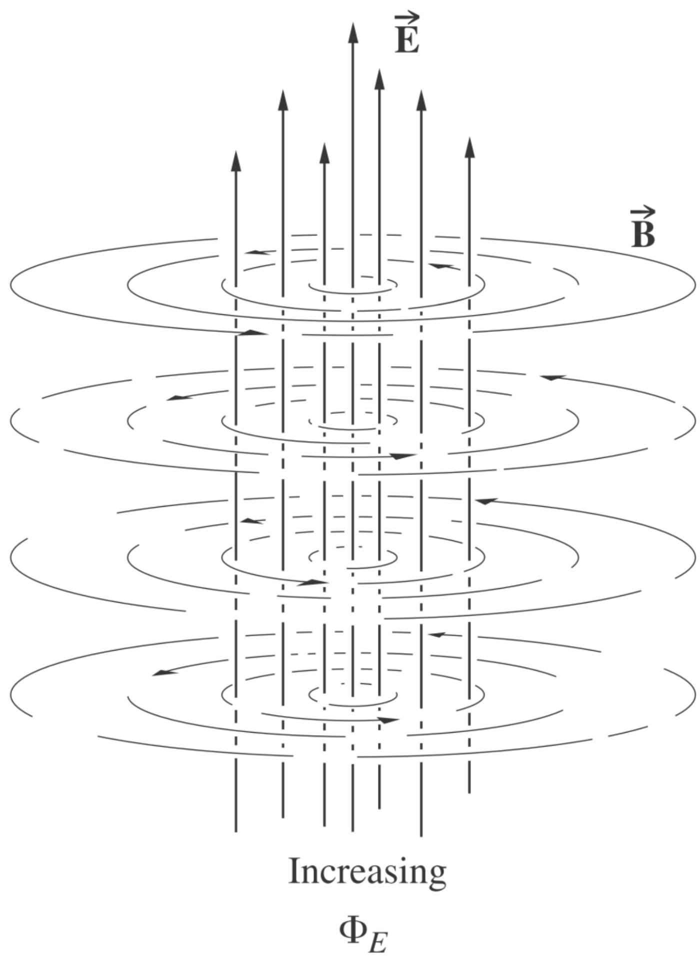
- Experiments have proven that for most light detection components such as photocells, photomultiplier tubes, photographic film, and photosynthesis, eye vision, etc., the response to light is mainly caused by the electric field in the electromagnetic wave.
- Only when a **strong magnetic material** (e.g., a ferromagnetic material) is studied, we have to consider the interaction of the magnetic component $\mathbf{H}$ with the material.

§ 3.1 Electromagnetic wave

Faraday's Induction Law



Ampère's Circuital Law



Maxwell's equations

Maxwell's equations in vacuum (differential version)

$\nabla \times \mathbf{E} = -\dfrac{\partial \mathbf{B}}{\partial t}$	**Faraday's Law**
$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \dfrac{\partial \mathbf{E}}{\partial t}$	**Ampère's Law**
$\nabla \cdot \mathbf{E} = 0$	**Guass's Law for electric field**
$\nabla \cdot \mathbf{B} = 0$	**Guass's Law for magnetic field**

Except for a multiplicative scalar, the electric and magnetic fields appears in the equations with a **remarkable symmetry**. They **inseparably coupled** and **mutually sustaining**, propagate out into space as a single entity.

Wave equations

In vacuum (free of charges and currents)

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \quad \Rightarrow \quad \nabla \times \nabla \times \mathbf{E} = -\frac{\partial (\nabla \times \mathbf{B})}{\partial t} = -\mu_0 \varepsilon_0 \frac{\partial^2 \mathbf{E}}{\partial t^2}$$

$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t} \qquad \nabla \times \nabla \times \mathbf{E} = \nabla (\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\nabla^2 \mathbf{E}$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \cdot \mathbf{B} = 0 \quad \Rightarrow \quad \begin{cases} \nabla^2 \mathbf{E} = \mu_0 \varepsilon_0 \dfrac{\partial^2 \mathbf{E}}{\partial t^2} \\ \nabla^2 \mathbf{B} = \mu_0 \varepsilon_0 \dfrac{\partial^2 \mathbf{B}}{\partial t^2} \end{cases}$$

This is the **wave equations** in free space, again, **symmetric**.

Properties of Plane Waves

For a plane light wave, if we choose the coordinate such that

$$\mathbf{E} = \mathbf{E}_0 e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})} = \mathbf{E}_0 e^{i\left(k_x x + k_y y + k_z z\right)} e^{-i\omega t}$$

According to the Maxwell equation: $\nabla \cdot \mathbf{E} = 0$

$$\because \quad \nabla \cdot \mathbf{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$$

$$\frac{\partial E_x}{\partial x} = ik_x E_x, \quad \frac{\partial E_y}{\partial y} = ik_y E_y, \quad \frac{\partial E_z}{\partial z} = ik_z E_z$$

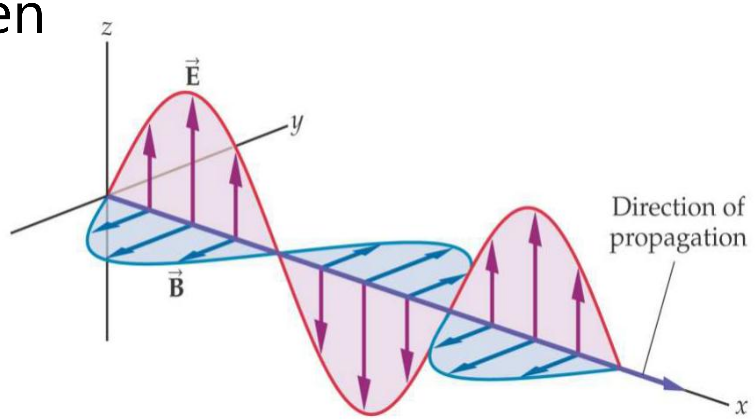
$$\Rightarrow \quad i\left(k_x E_x + k_y E_y + k_z E_z\right) = i\mathbf{k} \cdot \mathbf{E} = 0$$

Similarly, we have $\mathbf{k} \cdot \mathbf{H} = 0$

Properties of Plane Waves

In **isotropic** media, since $\mathbf{D}//\mathbf{E}$, then

$$\mathbf{k}\cdot\mathbf{E}=0 \quad \Rightarrow \quad \mathbf{k}\cdot\mathbf{D}=0$$



In isotropic non-ferromagnetic media, $\mathbf{B}//\mathbf{H}$, then

$$\mathbf{k}\cdot\mathbf{H}=0 \quad \Rightarrow \quad \mathbf{k}\cdot\mathbf{B}=0$$

- These relations show that the $\mathbf{E}$-field vector and the $\mathbf{B}$-field vector of the plane wave are **perpendicular** to the wave vector (the wavefront normal).
- So, plane waves are **transverse electromagnetic waves**.

Properties of Plane Waves

- Substituting plane wave expressions into Maxwell's Equations,

$$\left.\begin{aligned}\nabla\times\mathbf{E}&=-\frac{\partial\mathbf{B}}{\partial t}\\ \mathbf{B}&=\mu\mathbf{H}\end{aligned}\right\}\Rightarrow\quad\left.\begin{aligned}\nabla\times\mathbf{E}&=-\mu\frac{\partial\mathbf{H}}{\partial t}\\ \mathbf{H}&=\mathbf{H}_0\mathrm{e}^{-\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\end{aligned}\right\}\Rightarrow$$

$$\boxed{\nabla\times\mathbf{E}=\mathrm{i}\omega\mu\mathbf{H}_0\mathrm{e}^{-\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}=\mathrm{i}\omega\mu\mathbf{H}}$$

$$\because\quad\nabla\times(\varphi\mathbf{a})=(\nabla\varphi)\times\mathbf{a}+\varphi\,\nabla\times\mathbf{a}$$

$$\therefore\quad\nabla\times\mathbf{E}=\nabla\times\left[\mathbf{E}_0\mathrm{e}^{-\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\right]$$

$$=\left[\nabla\mathrm{e}^{-\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\right]\times\mathbf{E}_0+\cancel{\mathrm{e}^{-\mathrm{i}(\omega t-\mathbf{k}\cdot\mathbf{r})}\nabla\times\mathbf{E}_0}$$

For plane monochromatic light waves: $\nabla\times\mathbf{E}_0=0$

Properties of Plane Waves

In addition: $\nabla\varphi = \frac{\partial\varphi}{\partial x}\hat{\mathbf{x}} + \frac{\partial\varphi}{\partial y}\hat{\mathbf{y}} + \frac{\partial\varphi}{\partial z}\hat{\mathbf{z}}$

$$\Rightarrow \quad \nabla e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})} = \nabla e^{-i(\omega t-k_x x-k_y y-k_z z)}$$

$$= ik_x e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}\hat{\mathbf{x}} + ik_y e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}\hat{\mathbf{y}} + ik_z e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}\hat{\mathbf{z}}$$

$$= i\left(k_x\hat{\mathbf{x}} + k_y\hat{\mathbf{y}} + k_z\hat{\mathbf{z}}\right)e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})} = i\mathbf{k}e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}$$

$$\boxed{\nabla\times\mathbf{E} = \left[\nabla e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}\right]\times\mathbf{E}_0 = i\mathbf{k}e^{-i(\omega t-\mathbf{k}\cdot\mathbf{r})}\times\mathbf{E}_0 = i\mathbf{k}\times\mathbf{E}}$$

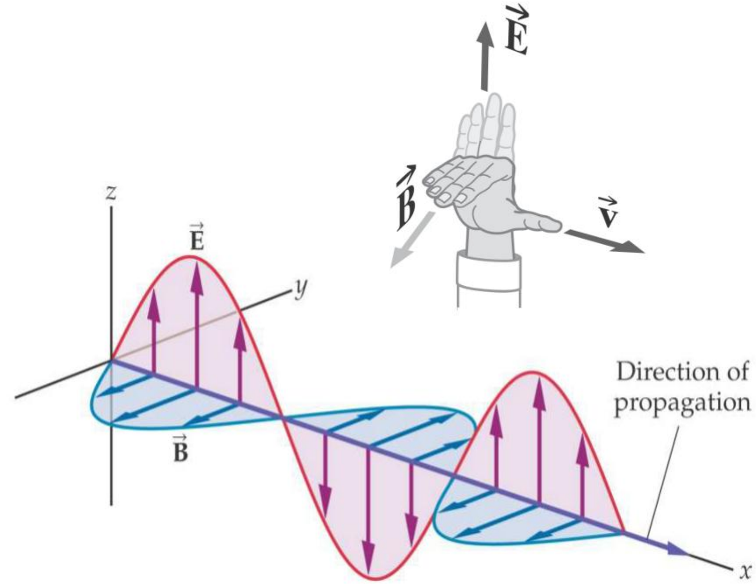
Properties of Plane Waves

So, $\nabla \times \mathbf{E} = i\omega\mu\mathbf{H}$

$\nabla \times \mathbf{E} = i\mathbf{k} \times \mathbf{E}$

$\Rightarrow$ $$\boxed{\mathbf{H} = \frac{1}{\omega\mu}\mathbf{k} \times \mathbf{E}}$$

$\Rightarrow$ $\mathbf{H} \perp \mathbf{k},\ \mathbf{E}$



- Apparently, $\mathbf{E}$ and $\mathbf{B}$ ($\mathbf{H}$) are perpendicular to each other. The vectors $\mathbf{E}$, $\mathbf{B}$, and $\mathbf{k}$ obey the right-hand rule.

Properties of Plane Waves

- Use the following relation, we have

$$k=\frac{2\pi}{\lambda}=\frac{2\pi}{T}\cdot\frac{1}{v}=\frac{\omega}{c}n=\frac{\omega}{c}\sqrt{\mu_r\varepsilon_r}=\omega\sqrt{\mu\varepsilon}$$

$$\mathbf{H}=\frac{1}{\omega\mu}\mathbf{k}\times\mathbf{E}=\sqrt{\frac{\varepsilon}{\mu}}\hat{\mathbf{k}}\times\mathbf{E}$$

$\hat{\mathbf{k}}$ is the unit vector.

- The ratio between $\mathbf{E}$ and $\mathbf{H}$ is **real** and **positive**. So $\mathbf{E}$ and $\mathbf{H}$ are **in phase**.

$$\frac{|\mathbf{E}|}{|\mathbf{H}|}=\sqrt{\frac{\mu}{\varepsilon}}$$ or $$\sqrt{\varepsilon_0\varepsilon_r}\,|\mathbf{E}|=\sqrt{\mu_0\mu_r}\,|\mathbf{H}|$$

$$Z_0=\sqrt{\frac{\mu_0}{\varepsilon_0}}=376.73\Omega$$

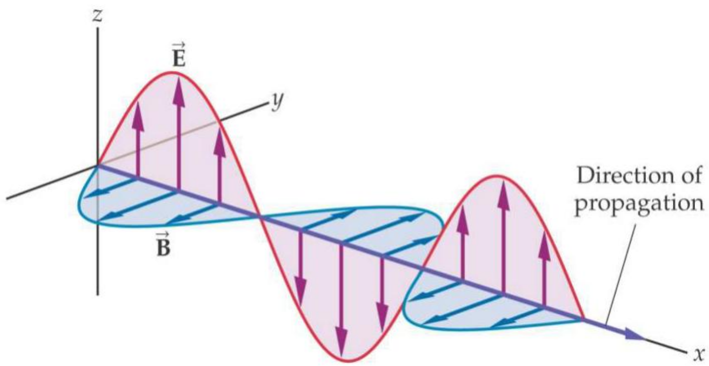
Impedance of vacuum

$$\boxed{\frac{|\mathbf{E}|}{|\mathbf{B}|}=\sqrt{\frac{1}{\mu\varepsilon}}=v}$$

The ratio of the $\mathbf{E}$-field and the $\mathbf{B}$-field is the speed of light.

Properties of Plane Waves

Summary of the properties of a plane EM wave



- **Transverse waves**: $\mathbf{E} \perp \mathbf{k},\ \mathbf{H} \perp \mathbf{k}$

$$\mathbf{E} \perp \mathbf{H}$$

- $\mathbf{E}$ and $\mathbf{H}$ are in phase: $\mathbf{E} \times \mathbf{H} // \mathbf{k}$
- The $\mathbf{E}$ is proportional to $\mathbf{H}$:

$$\sqrt{\varepsilon_0 \varepsilon_r}\,|\mathbf{E}| = \sqrt{\mu_0 \mu_r}\,|\mathbf{H}|$$

- Velocity :

$$\begin{cases} c = \dfrac{1}{\sqrt{\varepsilon_0 \mu_0}} = 2.997\,924\,58 \times 10^8 \text{ m/s} \quad \text{(vacuum)} \\ v = \dfrac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}} = \dfrac{c}{n} \quad \text{(medium)} \quad n = \sqrt{\varepsilon_r \mu_r} \end{cases}$$

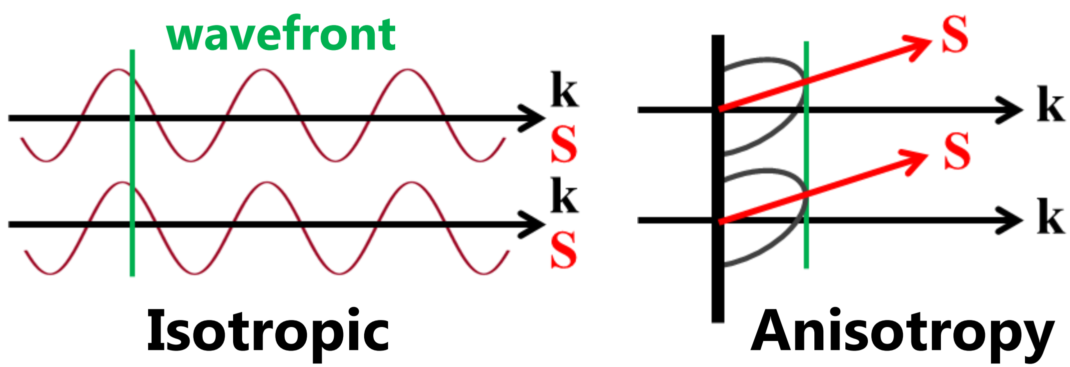
For non-ferromagnetic material, $\mu_r = 1$, so, $n = \sqrt{\varepsilon_r}$

Poynting Vector

- The propagation of wave accompanies the transport of energy, descried by the **Poynting vector** (energy flux density):

$$\mathbf{S} = \mathbf{E} \times \mathbf{H}$$

- $|\mathbf{S}|$: The power per unit area passing through an enclose surface in the direction perpendicular to the direction of energy flow.



- Direction of energy flow (the propagation direction of light):
  In isotropic media, it is the same with the wavevector.
  In anisotropic media, it is usually different from the wavevector.

Poynting Vector

- **Energy flux density** $\mathbf{S} = \mathbf{E} \times \mathbf{H}$

$\because$ $\mathbf{E} \perp \mathbf{H}$ and $\sqrt{\varepsilon_0 \varepsilon_r}\,|\mathbf{E}| = \sqrt{\mu_0 \mu_r}\,|\mathbf{H}|$

So $S = EH = \sqrt{\dfrac{\varepsilon_0 \varepsilon_r}{\mu_0 \mu_r}} E^2 \xrightarrow[\text{non-ferromagnetic}]{\mu_r = 1} S = n\sqrt{\dfrac{\varepsilon_0}{\mu_0}} E^2$

- For monochromatic plane wave

$$\mathbf{E} = \mathbf{E}_0 \cos\left(\omega t - \mathbf{k} \cdot \mathbf{r}\right)$$

$$S = n\sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \cos^2\left(\omega t - \mathbf{k} \cdot \mathbf{r}\right) \qquad \text{(Instantaneous)}$$

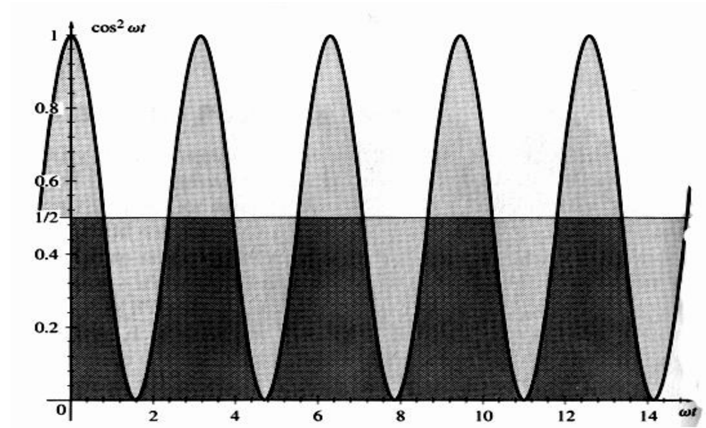
$\because$ $\nu > 10^{14}$ Hz, instruments cannot measure instantaneous values.

$\therefore$ The actual measured intensity is the time-averaged energy flow density within the instrument's response time τ.

Intensity

- **Irradiance** (辐照度), i.e., the time-averaged of the magnitude of the Poynting vector)

$$I = \langle S \rangle \quad \text{unit: W/m}^2$$



$\langle \cos^2 \omega t \rangle = 1/2$

$$\langle S \rangle = \frac{1}{\tau}\int_0^{\tau} S dt \qquad \text{periodic function}$$

$$= \frac{1}{T}\int_0^{T} S dt \equiv \langle S \rangle_T \qquad \tau >> T$$

$$\because \begin{cases} S = n\sqrt{\dfrac{\varepsilon_0}{\mu_0}} E_0^2 \cos^2(\omega t - \mathbf{k}\cdot\mathbf{r}) \\ \langle \cos^2(\omega t - \mathbf{k}\cdot\mathbf{r}) \rangle_T = \dfrac{1}{2} \end{cases} \Rightarrow \begin{cases} I = \dfrac{1}{2} n\sqrt{\dfrac{\varepsilon_0}{\mu_0}} E_0^2 \\ or \quad I = \dfrac{nc\varepsilon_0}{2} E_0^2 \end{cases}$$

- The irradiance is proportional to the **square of the amplitude** of the $\mathbf{E}$-field. Also called **Intensity**.

Intensity

$$\left\langle \cos^2(\omega t - \mathbf{k}\cdot\mathbf{r}) \right\rangle_T = \frac{1}{2}$$

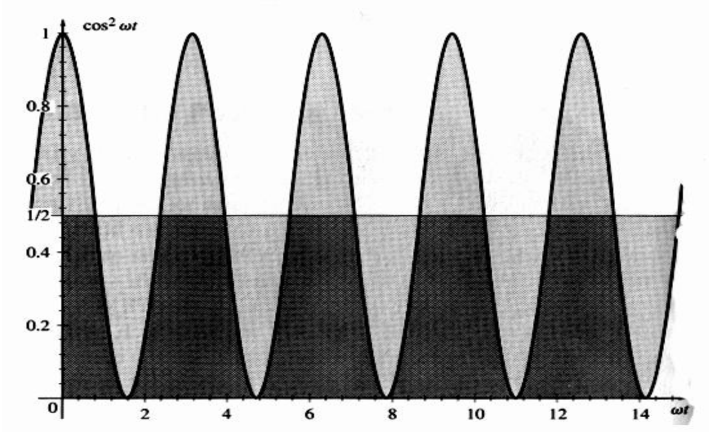
$$\frac{1}{T}\int_0^T \cos^2(\omega t - \mathbf{k}\cdot\mathbf{r})\,dt$$

$$= \frac{1}{2T}\int_0^T \left[1 - \cos 2(\omega t - \mathbf{k}\cdot\mathbf{r})\right]dt$$

$$= \frac{1}{2T}\left[T - \frac{1}{2\omega}\sin 2\left(\frac{2\pi}{T}t - \mathbf{k}\cdot\mathbf{r}\right)\Bigg|_0^T\right]$$

$$= \frac{1}{2T}\left[T - \frac{1}{2\omega}\left[\sin(4\pi - 2\mathbf{k}\cdot\mathbf{r}) - \sin(-2\mathbf{k}\cdot\mathbf{r})\right]\right]$$

$$= \frac{1}{2}$$



$\langle \cos^2 \omega t \rangle = 1/2$

Intensity

- If the wave function is given in the complex representation

$$\mathbf{E} = \mathbf{E}_0 e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}, \quad \mathbf{H} = \mathbf{H}_0 e^{-i(\omega t - \mathbf{k}\cdot\mathbf{r})}$$

Then $\mathbf{S} = \mathrm{Re}(\mathbf{E}) \times \mathrm{Re}(\mathbf{H}) \neq \mathrm{Re}(\mathbf{E} \times \mathbf{H})$

So, $\langle \mathbf{S} \rangle = \frac{1}{2}\mathrm{Re}(\mathbf{E}^* \times \mathbf{H}) = \frac{1}{2} n \sqrt{\frac{\varepsilon_0}{\mu_0}} E_0^2 \mathbf{n} = \frac{1}{2}\sqrt{\frac{\varepsilon}{\mu}} E_0^2 \mathbf{n}$ **Homework**

- The response of the photodetector is proportional to the incident light intensity, so it can only get the amplitude. **The information of the phase is lost**.
- Using the interference, the phase information can be converted into light intensity. >>**Holography**

§ 3.2 The polarization states

- **Polarization**: The vibrational state of the light vector in a 2D plane perpendicular to the propagation direction.
- **Unpolarized light**: Natural light
- **Completely polarized light**
- **Partially polarized light**: Mixture of polarized light and natural light.

According to the trajectory of the endpoint of the vector $\mathbf{E}$ at any point in space at different time, completely polarized light can be divided into:

(1) **Linearly** polarized light;
(2) **Circularly** polarized light;
(3) **Elliptically** polarized light.

If a lightwave propagates in the z direction, since $\mathbf{E}$ is a vector :

$$\boldsymbol{E} = \hat{\mathbf{x}}E_x + \hat{\mathbf{y}}E_y$$

and
$$E_x = E_{0x}\cos(\underline{\omega t - kz + \varphi_x}) = E_{0x}\cos(\varphi)$$
$$E_y = E_{0y}\cos(\omega t - kz + \varphi_y) = E_{0y}\cos(\varphi + \delta)$$

$\boxed{\delta = \varphi_y - \varphi_x}$ Initial phase difference for two vibration directions

To eliminate φ, we have

$$\cos(\varphi + \delta) = \cos\varphi\cos\delta - \sin\varphi\sin\delta$$
$$= \frac{E_x}{E_{0x}}\cos\delta - \sqrt{1 - \left(\frac{E_x}{E_{0x}}\right)^2}\sin\delta = \frac{E_y}{E_{0y}}$$

Rewrite

$$\frac{E_x}{E_{0x}}\cos\delta - \sqrt{1-\left(\frac{E_x}{E_{0x}}\right)^2}\,\sin\delta = \frac{E_y}{E_{0y}}$$

into

$$\boxed{\left(\frac{E_x}{E_{0x}}\right)^2 + \left(\frac{E_y}{E_{0y}}\right)^2 - 2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta = \sin^2\delta}$$

In general, this binary quadratic equation represents an **ellipse**.

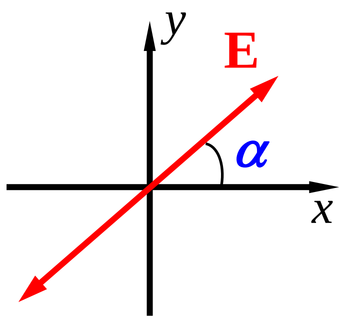
The **phase difference** δ and the **amplitude ratio** E_{0y}/E_{0x} determines the **orientation** and the **ellipticity** of the ellipse, and thus determines the polarization states of light.

Linearly polarized light

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2-2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta=\sin^2\delta$$

① When $\delta=2m\pi\ (m=0,\pm1,\pm2,\cdots)$

$$\left(\frac{E_x}{E_{0x}}-\frac{E_y}{E_{0y}}\right)^2=0 \quad\Rightarrow\quad \frac{E_x}{E_y}=\frac{E_{0x}}{E_{0y}}\equiv\cot\alpha$$



The elliptic equation degenerates into a linear equation. The light is called **linearly polarized light**. Electric field vector vibrates in the I and III quadrants.

Linearly polarized light

$$\left(\frac{E_x}{E_{0x}}\right)^2+\left(\frac{E_y}{E_{0y}}\right)^2-2\left(\frac{E_x}{E_{0x}}\right)\left(\frac{E_y}{E_{0y}}\right)\cos\delta=\sin^2\delta$$

② When $\delta=(2m+1)\pi$, it vibrations in the II, IV quadrants.

$$\frac{E_x}{E_y}=-\frac{E_{0x}}{E_{0y}}$$

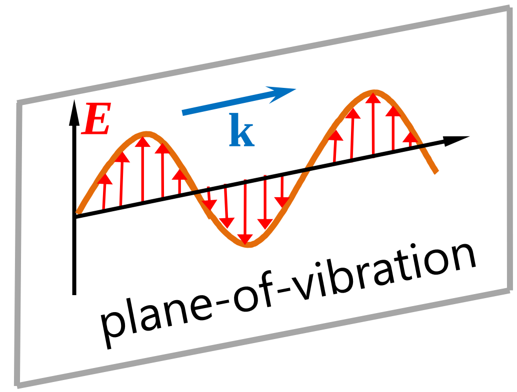
- The above two equations can be combined into

$$\frac{E_x}{E_y}=\frac{E_{0x}}{E_{0y}}\mathrm{e}^{\mathrm{i}m'\pi}$$

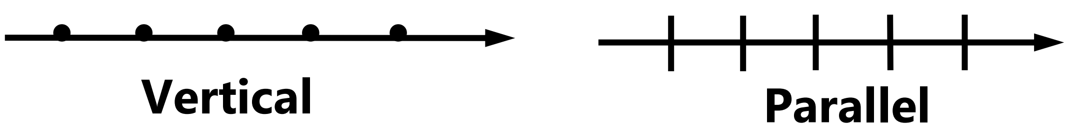
When m' is **0 or even**, the direction of vibration is within quadrants I and III. When m' is **odd**, it is within quadrants II and IV.

Linearly polarized light

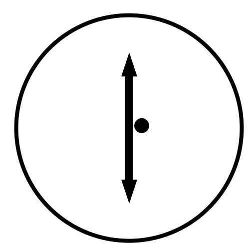
- The electric field $\mathbf{E}$ of a LP light at each point along the propagation direction resides in the same plane, known as the **plane-of-vibration**.
- The linearly polarized light is also called **plane-polarized light**.



LP light can be denoted by (propagating within the plane):



Propagating perpendicular to the plane:



Elliptically polarized light

③ When $\delta = (2m+1)\dfrac{\pi}{2} \quad (m = 0, \pm 1, \pm 2, \cdots)$

$$\frac{E_x^{\,2}}{E_{0x}^2} + \frac{E_y^{\,2}}{E_{0y}^2} = 1$$

Shows that the endpoint of the $\mathbf{E}$-vector will rotate along an ellipse, known as **elliptically polarized light.**

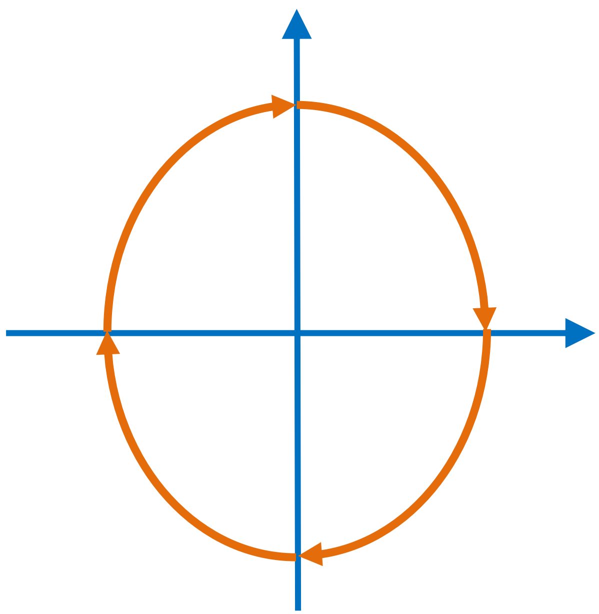
- If E_x is equal to E_y

$$E_{0x} = E_{0y} = E_0$$

The ellipse degenerates into a circle.

$$E_x^2 + E_y^2 = E_0^2$$

>> **circularly polarized light**



Circularly polarized light

③ When $\delta = \pi/2$

$$E_x = E_{0x}\cos(\omega t)$$

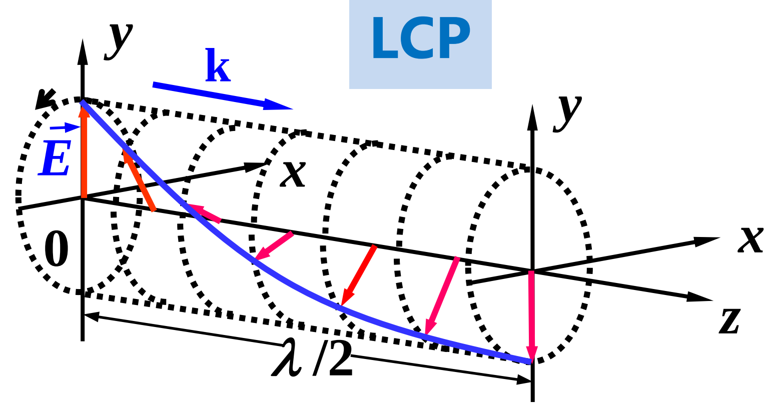
$$E_y = E_{0y}\cos(\omega t + \frac{\pi}{2})$$

The movement in the y-axis leads that in x-axis, and the trajectory of the movement is clockwise. >>**RCP**

- The elliptically polarized light: the rotating light vector **E**, the direction of which is rotated at an angular velocity ω, and its instantaneous value also changes regularly, and the trajectory of its end is an ellipse.

- When $\delta = -\frac{\pi}{2}$, it is called **left-handed**.

LCP

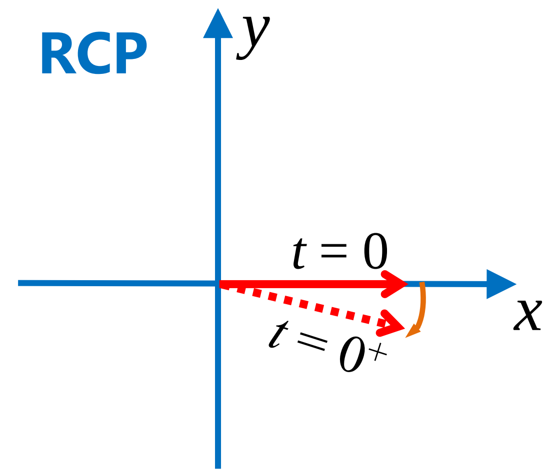


Circularly polarized light

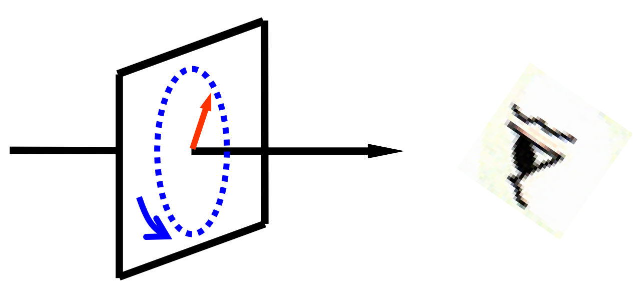
③ When $\delta = \pi/2$

$$E_x = E_{0x}\cos(\omega t)$$

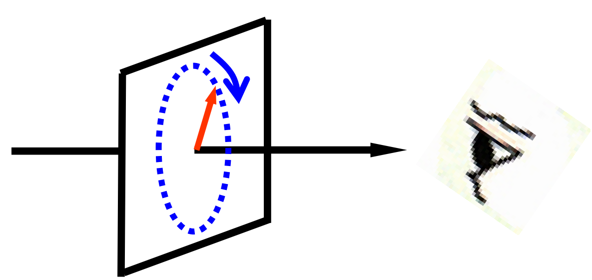
$$E_y = E_{0y}\cos(\omega t + \frac{\pi}{2})$$



propagating in z

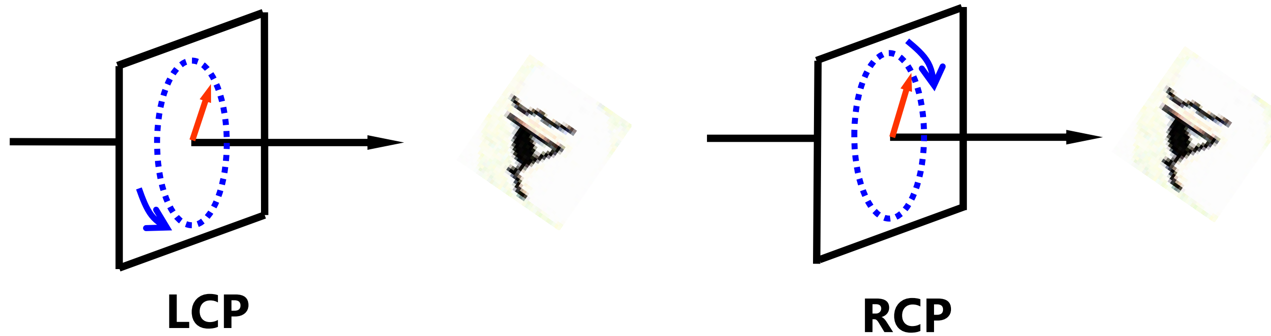


LCP



RCP

Circularly polarized light

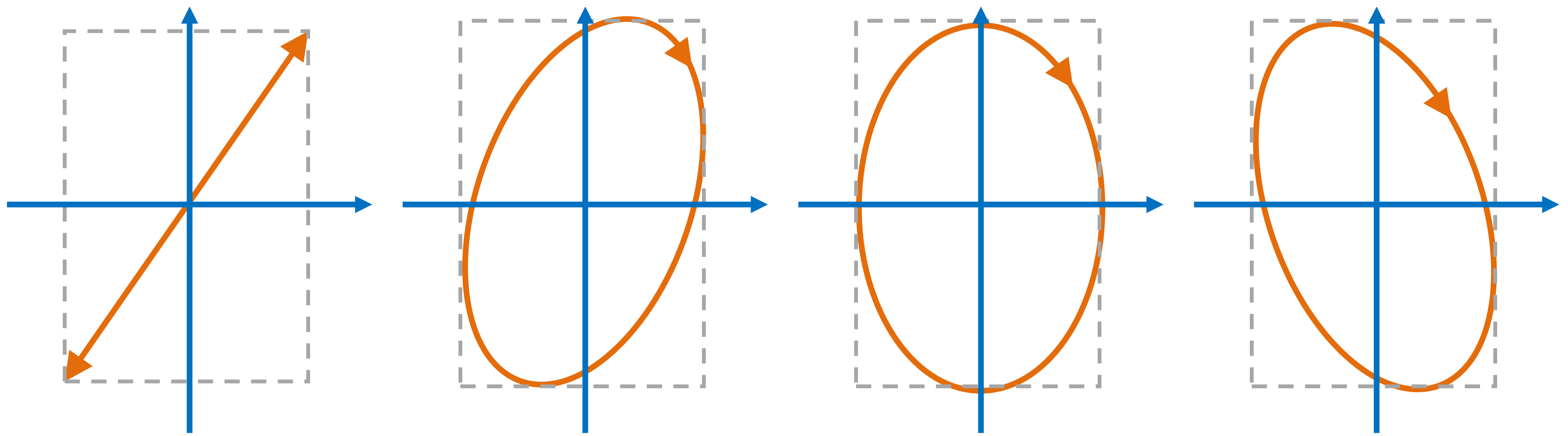


Looking down the propagation direction **toward** the origin.

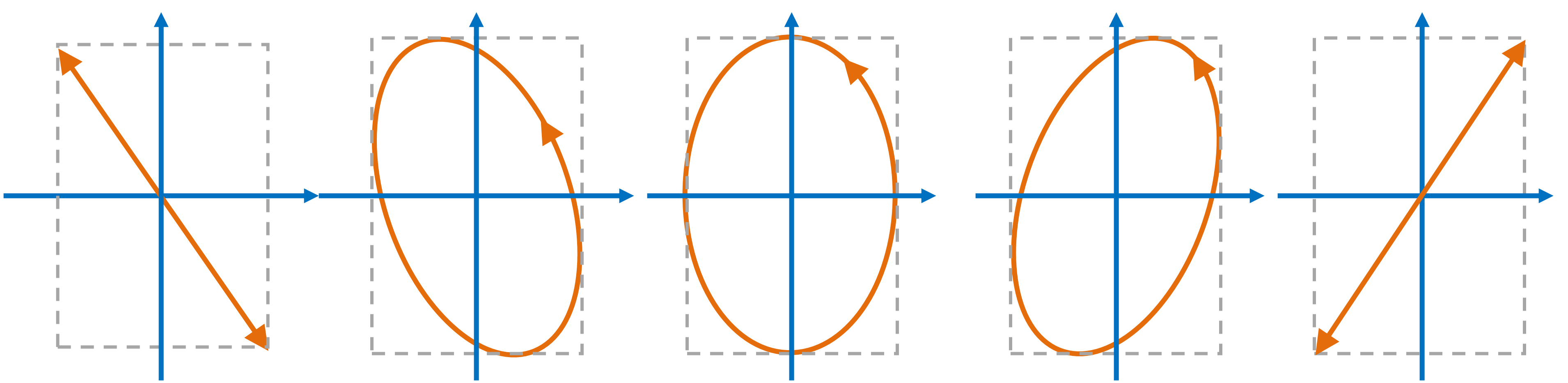
- R-handed polarization:
  Light vector rotates **clockwise** (**left-hand spiral**)
- L-handed polarization
  Light vector rotates **counterclockwise** (**right hand spiral**)

In the field of chemistry, people usually use the opposite definition, since it is convenient when looking at the chiral molecules.

Complete polarized light



$\delta = 0$ $\delta = \pi / 4$ $\delta = \pi / 2$ $\delta = 3\pi / 4$



$\delta = \pi$ $\delta = 5\pi / 4$ $\delta = 3\pi / 2$ $\delta = 7\pi / 4$ $\delta = 2\pi$

Complete polarized light

In complex representation:

Linearly Polarized: $$\frac{E_x}{E_y}=\frac{E_{0x}}{E_{0y}}\mathrm{e}^{\mathrm{i}m'\pi}=\pm\frac{E_{0x}}{E_{0y}}\qquad m'=0,\pm1,\pm2,\cdots$$

Elliptically polarized light: $$\frac{E_x}{E_y}=\frac{E_{0x}}{E_{0y}}\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{2}}$$

Circularly polarized light: $$\frac{E_x}{E_y}=\mathrm{e}^{\pm\mathrm{i}\frac{\pi}{2}}=\pm\mathrm{i}$$

In the formula, the ± corresponds to **right-handed (+)** and **left-handed (-)** circularly polarized light, respectively.

Jones Vectors

Write the two components of polarized light in a vector:

$$\mathbf{E} = \begin{bmatrix} E_x(t) \\ E_y(t) \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ E_{0y} e^{i\varphi_y} \end{bmatrix}$$

φ_i $(i = x, y)$ are the appropriate phases.

So, the horizontal and vertical P-state are given by

$$\mathbf{E}_\mathrm{h} = \begin{bmatrix} E_x(t) \\ 0 \end{bmatrix} = \begin{bmatrix} E_{0x} e^{i\varphi_x} \\ 0 \end{bmatrix}, \mathbf{E}_\mathrm{v} = \begin{bmatrix} 0 \\ E_y(t) \end{bmatrix} = \begin{bmatrix} 0 \\ E_{0y} e^{i\varphi_y} \end{bmatrix}$$

Neglect the term about the amplitude and absolute phase, and normalized the vectors.

$$\mathbf{E}_{45^\circ} = \mathbf{E}_\mathrm{h} + \mathbf{E}_\mathrm{v} = \frac{1}{\sqrt{2}} \left(\begin{bmatrix} 1 \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right) = \frac{1}{\sqrt{2}} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

Jones Vectors

Jones Vectors for some polarization states

State of Polarization	Jones Vectors		
Horizontal P-state	$\begin{bmatrix}1\\0\end{bmatrix}$	P-state at -45°	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-1\end{bmatrix}$
Vertical P-state	$\begin{bmatrix}0\\1\end{bmatrix}$	R-state	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\-i\end{bmatrix}$
P-state at 45°	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\1\end{bmatrix}$	L-state	$\frac{1}{\sqrt{2}}\begin{bmatrix}1\\i\end{bmatrix}$
		P-state, θ to x-axis	$\begin{bmatrix}\cos\theta\\\sin\theta\end{bmatrix}$

Do not confused $\frac{E_x}{E_y} = \mathrm{e}^{\pm \mathrm{i}\frac{\pi}{2}} = \pm \mathrm{i}$

Jones Vectors

- If two polarized lightwaves are **orthogonal**, their Jones vectors satisfy

$$E_1 \cdot E_2^* = \begin{bmatrix} E_{1x} & E_{1y} \end{bmatrix} \begin{bmatrix} E_{2x}^* \\ E_{2y}^* \end{bmatrix} = 0$$

- E.g., linearly polarized light along x-axis and y-axis. Left circularly light and right circularly light.

$$[1,\ 0]\begin{bmatrix} 0 \\ 1 \end{bmatrix} = 0 \qquad\qquad [1,\ \mathrm{i}]\begin{bmatrix} 1 \\ \mathrm{i} \end{bmatrix} = 0$$

Jones Vectors

- Using Jones vectors, the **superposition** of polarized light can be simply described by

$$\begin{bmatrix} E_x \\ E_y \end{bmatrix} = \begin{bmatrix} E_{1x} \\ E_{1y} \end{bmatrix} + \begin{bmatrix} E_{2x} \\ E_{2y} \end{bmatrix} = \begin{bmatrix} E_{1x} + E_{2x} \\ E_{1y} + E_{2y} \end{bmatrix}$$

- When light passes though different optical elements, the final polarization state is given by

$$\begin{bmatrix} E_{tx} \\ E_{ty} \end{bmatrix} = \begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix} \cdots \begin{bmatrix} a_2 & b_2 \\ c_2 & d_2 \end{bmatrix} \begin{bmatrix} a_1 & b_1 \\ c_1 & d_1 \end{bmatrix} \begin{bmatrix} E_{ix} \\ E_{iy} \end{bmatrix}$$

where $\begin{bmatrix} a_n & b_n \\ c_n & d_n \end{bmatrix}$ is called **Jones Matrices**.

Natural light

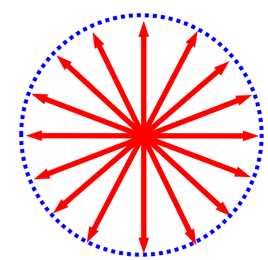
- Ordinary light sources: constantly emitting and random orientation of the emitters.

  Vibration direction

  Wavetrain of different lengths

  Initial phase

- The statistics over a large number of atomic luminescent events results in the natural light. Due to the randomness, there's **no prefer orientation** of the polarization. Then, the vibration direction and amplitude of the light vector show an **axial symmetry** in the plane normal to the propagation direction.



random

Natural light

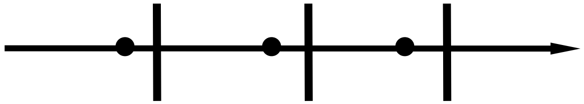
- The natural light can be considered as the superposition of two arbitrary, **incoherent**, orthogonal, linearly polarized light with equal amplitude.

$$E_x = \sum_i E_{ix} \qquad I_x = \sum_i E_{ix}{}^2$$

$$E_y = \sum_i E_{iy} \qquad I_y = \sum_i E_{iy}{}^2$$

$$I_x = I_y = \frac{1}{2} I_{\text{total}}$$

The representation of a natural light

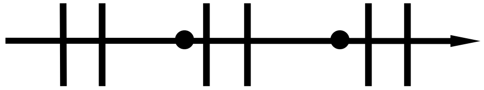


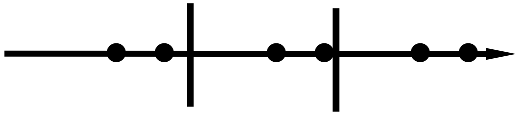
Partially polarized light

- Completely polarized light and natural light are two extremes. The general situation between the two is **partially polarized light**.

- A partially polarized light can be regarded as a mixture of natural light and linearly polarized light.
- E.g., the scattered light of the sky and the reflected light from the a glass slice.

Representation of a partially polarized light





Degree of polarization

- **Degree of polarization**（偏振度）: Used to measure the degree of polarization of partially polarized light.

$$P = \frac{I_p}{I_t} = \frac{I_p}{I_p + I_n}$$

I_t: The total intensity of light

I_n: The component of the natural light

I_p: The component of the complete polarized light

Polarization state
$$\begin{cases} P = 1 & \text{complete polarized light} \\ P = 0 & \text{natural light} \\ 0 < P < 1 & \text{partially polarized light} \end{cases}$$

$$\boxed{P = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}}$$

I_{max}, and I_{min}, the intensity of the light after passing through polarizer.

Homework	Problem 3.32 Problems 8.4, 8.5 and 8.6
Homework*	Read about the **spin** of photons, the **orbital** angular momentum of photons

Next week	Absorption, Dispersion, Scattering: Sections 3.5, 4.2, 7.2.2 Fermat’s Principle, Imaging: Sections 4.5, 5.1-5.2