热力学与统计物理-第四次作业

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Problem 3.1

Answer:

(a)

While in equilibrium state, the densities of the mixed gas on both sides of the box should be the same.

For the larger side:

$$\overline{N_1(Ne)} = 750, \overline{N_1(He)} = 75 \tag{1.1}$$

And for the smaller side:

$$\overline{N_2(Ne)} = 250, \overline{N_2(He)} = 25 \tag{1.2}$$

(b)

$$P = (\frac{3}{4})^{1000}(\frac{1}{4})^{100} \approx 7.16 \times 10^{-176} \tag{1.3}$$

Problem 3.2

Answer:

(a)

From problem 2.4 (b):

$$ln\,\Omega(E) = -\frac{1}{2}(N-\frac{E}{\mu H})\,ln\frac{1}{2}(1-\frac{E}{N\mu H})-\frac{1}{2}(N+\frac{E}{\mu H})ln\frac{1}{2}(1+\frac{E}{N\mu H}) \tag{2.1}$$

Then:

$$\beta = \frac{\partial}{\partial E}ln\,\Omega(E) = \frac{1}{2\mu H}ln\frac{\frac{1}{2}(1-\frac{E}{N\mu H})}{1-\frac{1}{2}(1-\frac{E}{N\mu H})} \tag{2.2}$$

So:

$$E = -N\mu H \, tanh\frac{\mu H}{kT} \tag{2.3}$$

(b)

From equation (2.3), we can find that, while $E > 0$, then $T < 0$

(c)

We have:

$$M = \mu(n_1 - n_2) = \mu(2n_1 - N) \tag{2.4}$$

n_1 is the number of spins aligned parallel to H.

Then:

$$n_1 = \frac{1}{2}(N - \frac{E}{\mu H}) \tag{2.5}$$

Replace n_1 in equation (2.4) with equation (2.5):

$$M = \mu(N - \frac{E}{\mu H} - N) = -\frac{E}{H} \tag{2.6}$$

So:

$$M = N\mu \, tanh\frac{\mu H}{kT} \tag{2.7}$$

Problem3.4

Answer:

For the heat reservoir, the change of entropy is:

$$\Delta S' = -\frac{Q}{T'} \tag{3.1}$$

For the whole system, the change of entropy is:

$$\Delta S + \Delta S' = \Delta S - \frac{Q}{T'} \geq 0 \tag{3.2}$$

So:

$$\Delta S \geq \frac{Q}{T'} \tag{3.3}$$

Problem 3.5

Answer:

(a)

For gas 1 and gas 2, we have:

$$\Omega(E) \propto V^N \chi(E) \tag{4.1}$$

Since gas 1 and gas 2 are noninteracting:

$$\Omega(E) = C\Omega_1(E)\Omega_2(E_0 - E) = CV^{N_1+N_2}\chi_1(E)\chi_2(E) \tag{4.2}$$

(b)

$$\overline{P}V = NkT = (N_1 + N_2)kT \tag{4.3}$$