C10 Birefringence

- The propagation of light in an anisotropic medium. It exhibits many unique optical phenomena: (linear) birefringence, circular birefringence (optical rotation), polarization effect, cone (锥形) refraction.
- Application: prism polarizer, polarizer, wave plate, etc.;

Crystal optics ⇔ **Nonlinear optics**

Recommended reference: 石顺祥《光的电磁理论》Chapter 2;

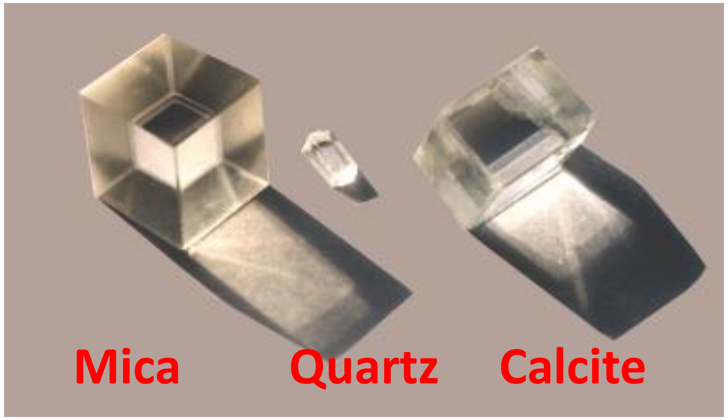
赵建林《高等光学》Chapter 5

§ 10.1 Birefringence

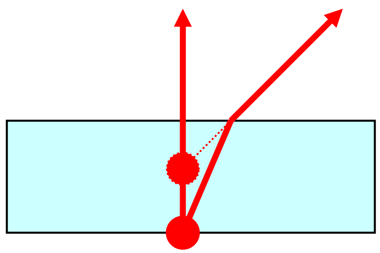
- In some materials, the refraction of light satisfies the law of refraction and is independent of the direction of vibration. Such a medium is referred to as an isotropic medium. Such as water, glass, general polymers, cubic crystals.

- Anisotropic media, such as oxide crystals such as quartz, calcite, mica, organic crystals (e.g. sugars), semiconductors.

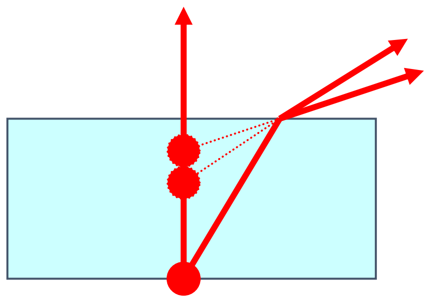
- When natural light is refracted by an anisotropic crystal, two linearly polarized light with different vibration directions are generated. >>> birefringence.



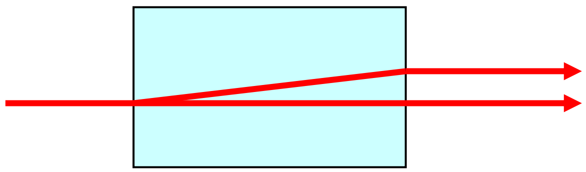
- In 1669s, Bartholinus found that the writing on the paper below the calcite crystal becomes a double line.



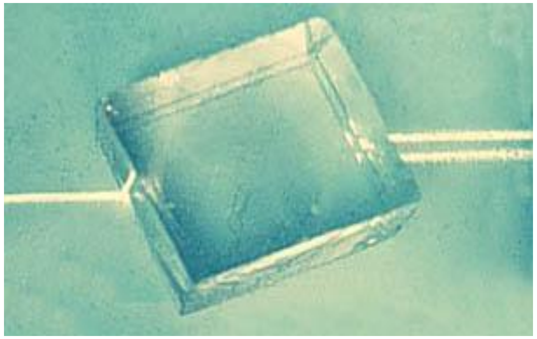
Glass



Calcite crystal



Calcite crystal



Ordinary /extraordinary light

Ordinary (o) light： Compliance with the law of refraction.

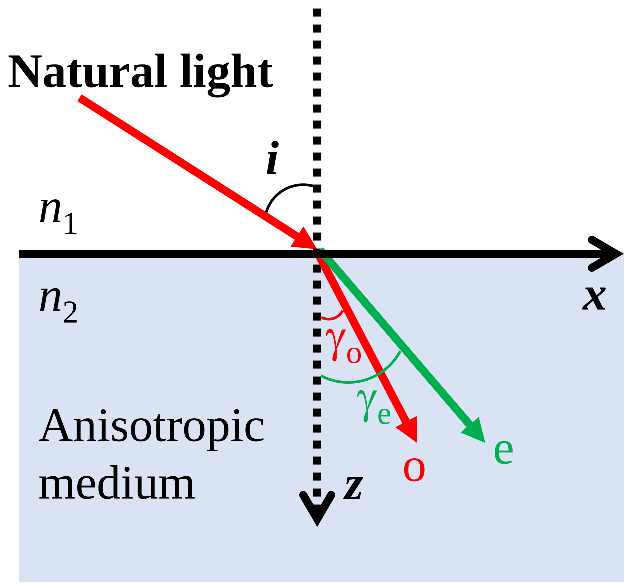
$$n_1 \sin i = n_2 \sin \gamma_o$$

Extraordinary (e) light： Generally it does not follow the law of refraction

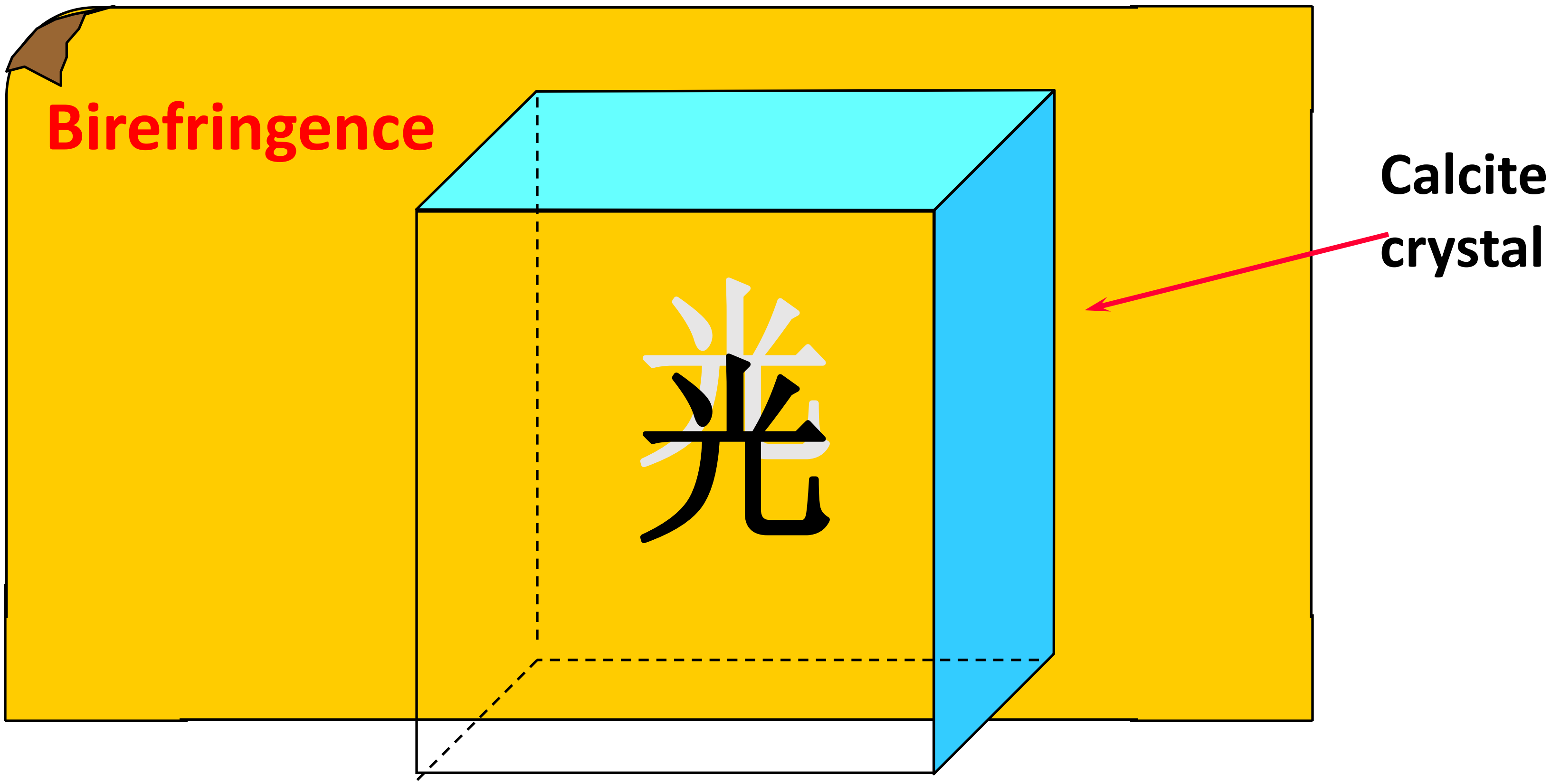
$$\frac{\sin i}{\sin \gamma_e} \neq \text{Const.}$$

The refracted light of the e light is also **not necessarily in the plane of incident**.

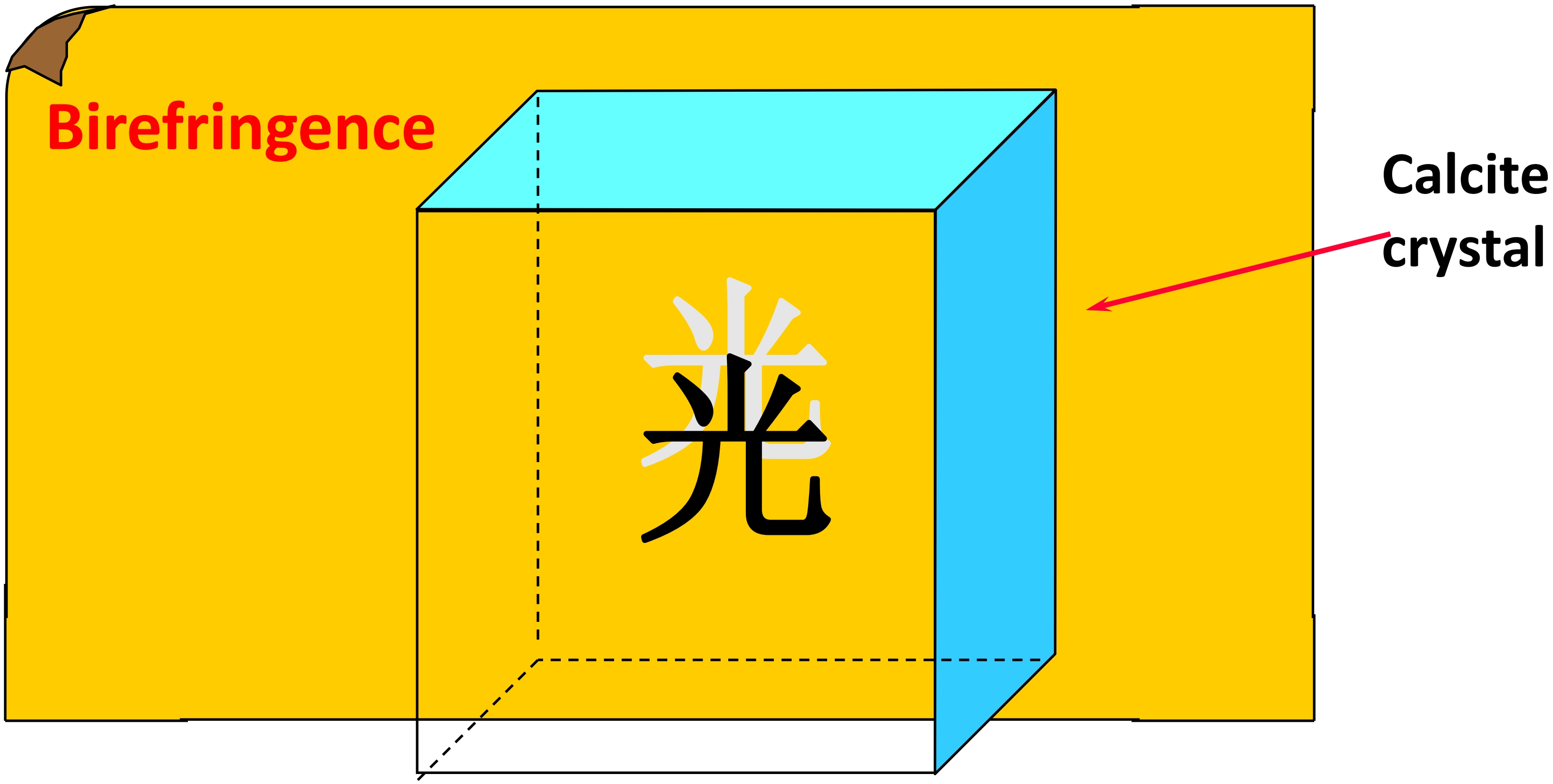
※ o Light and e light only make sense in the birefringent crystal.



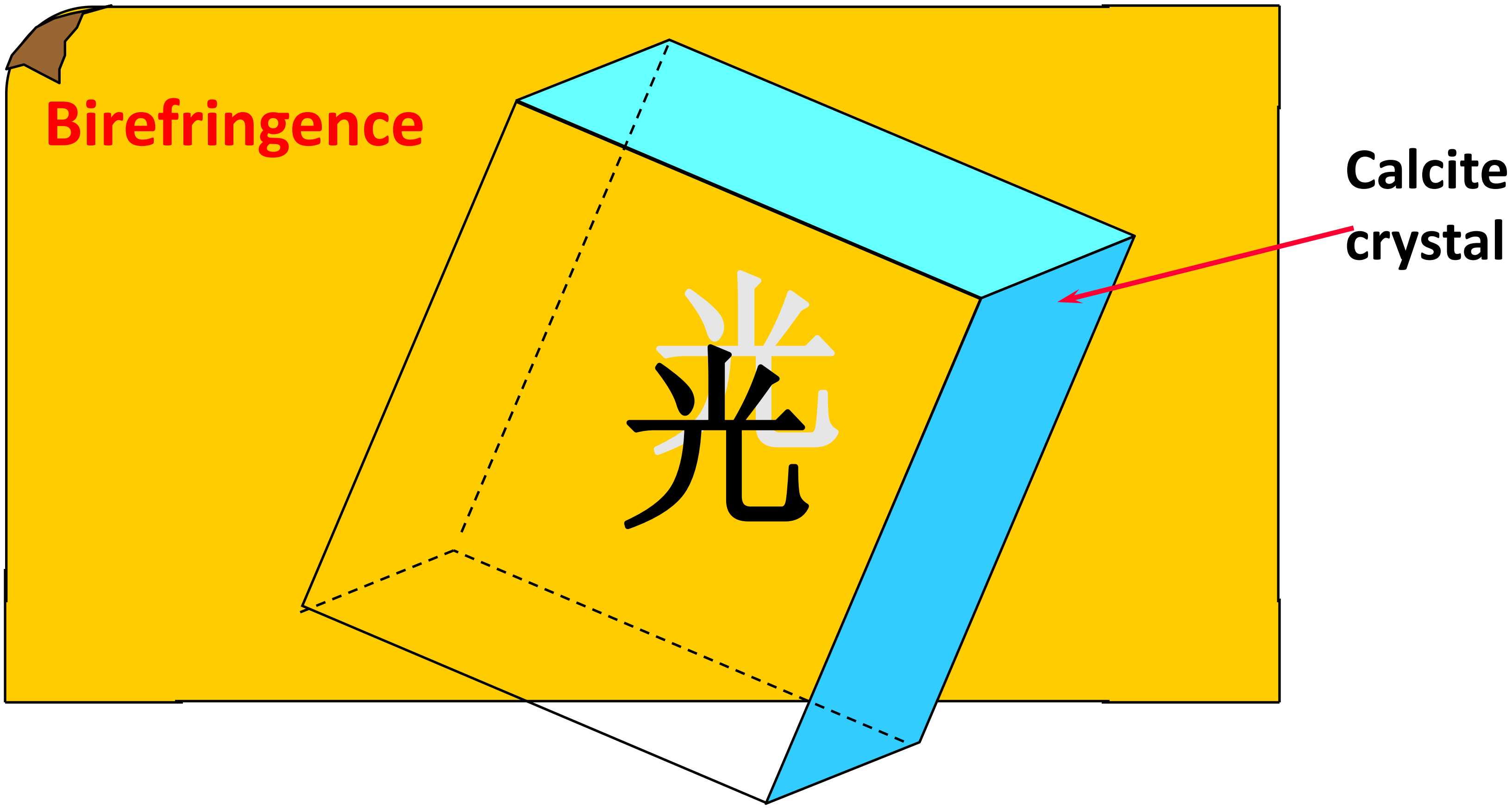
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



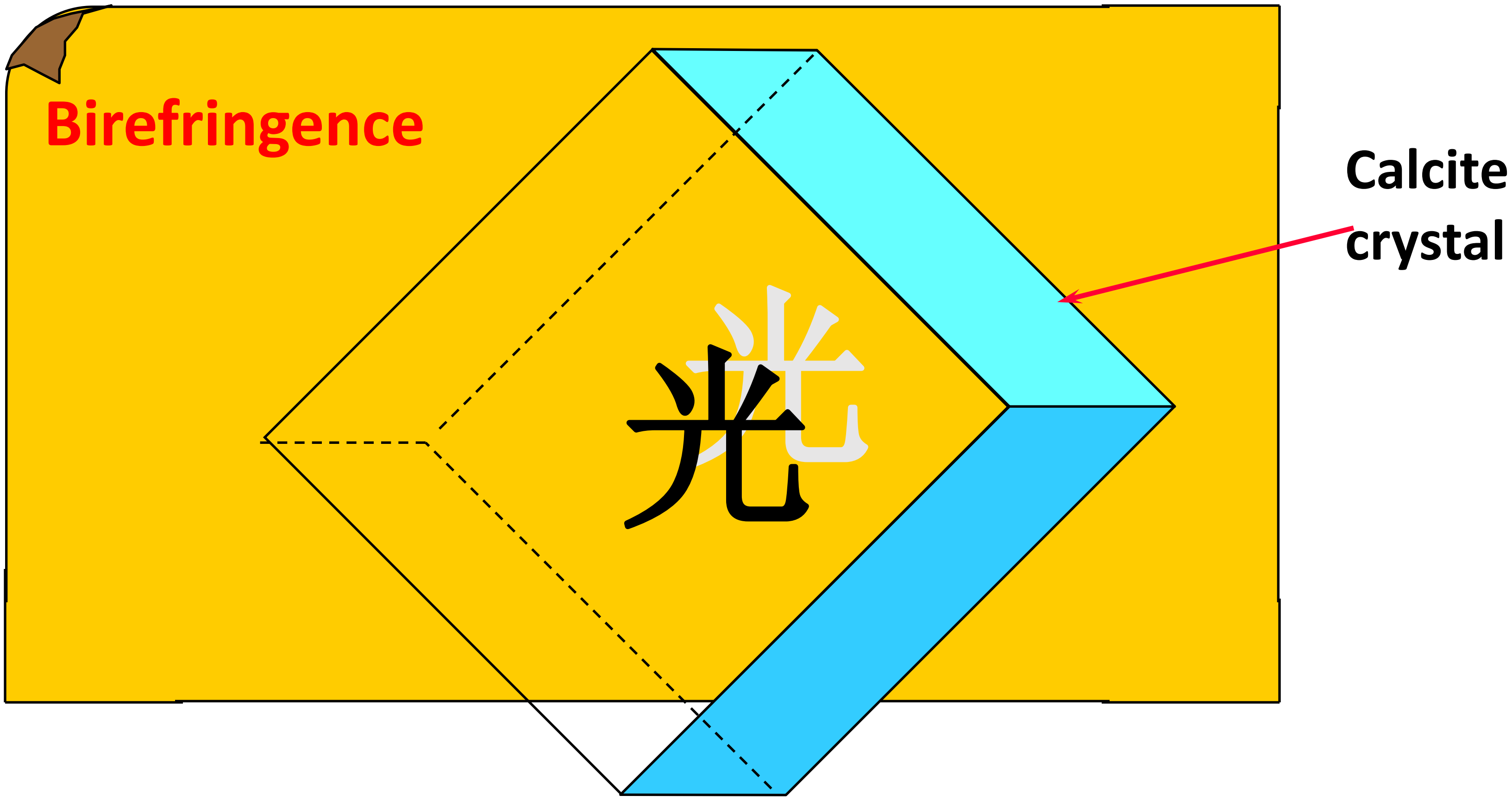
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



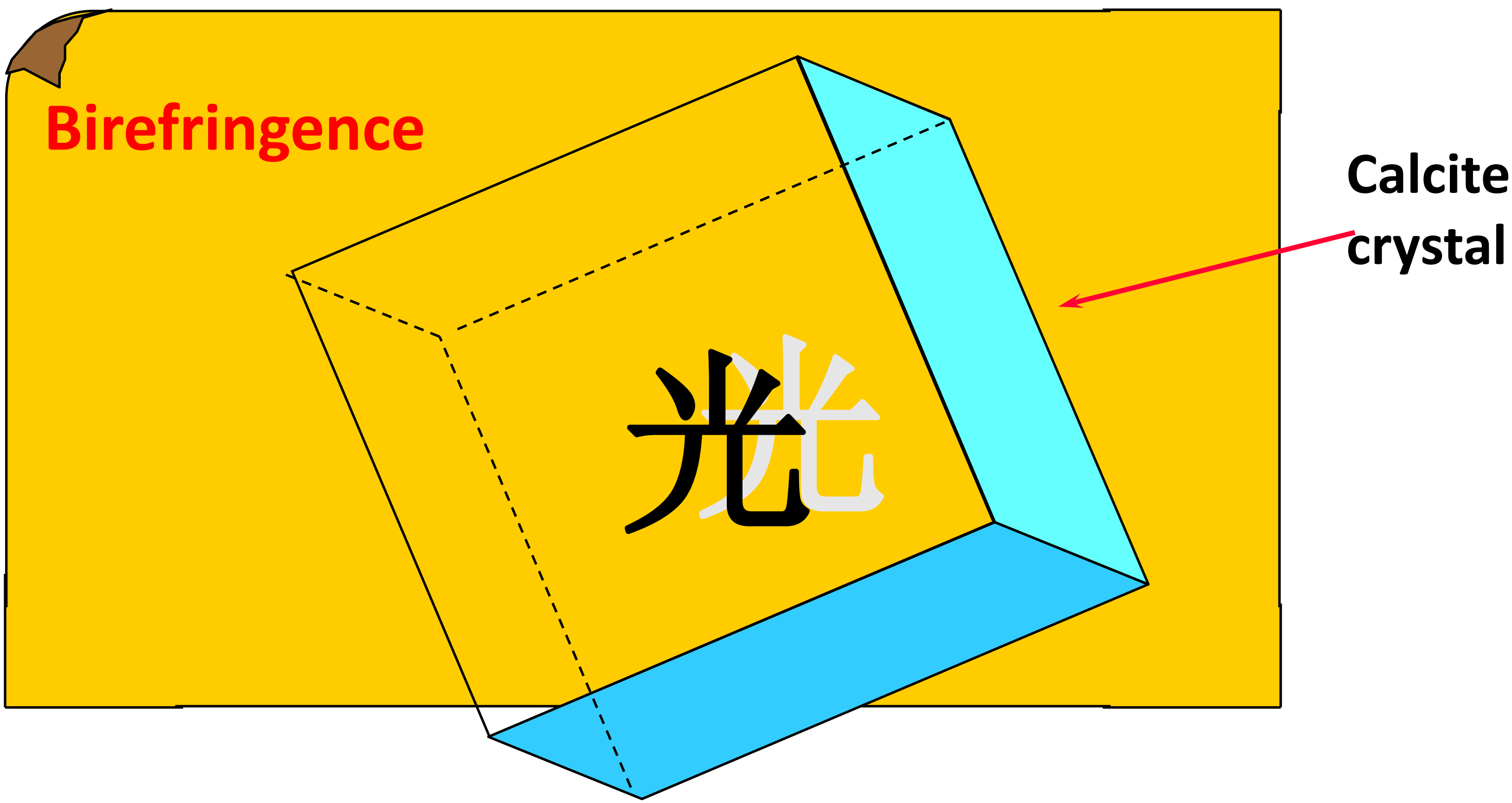
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



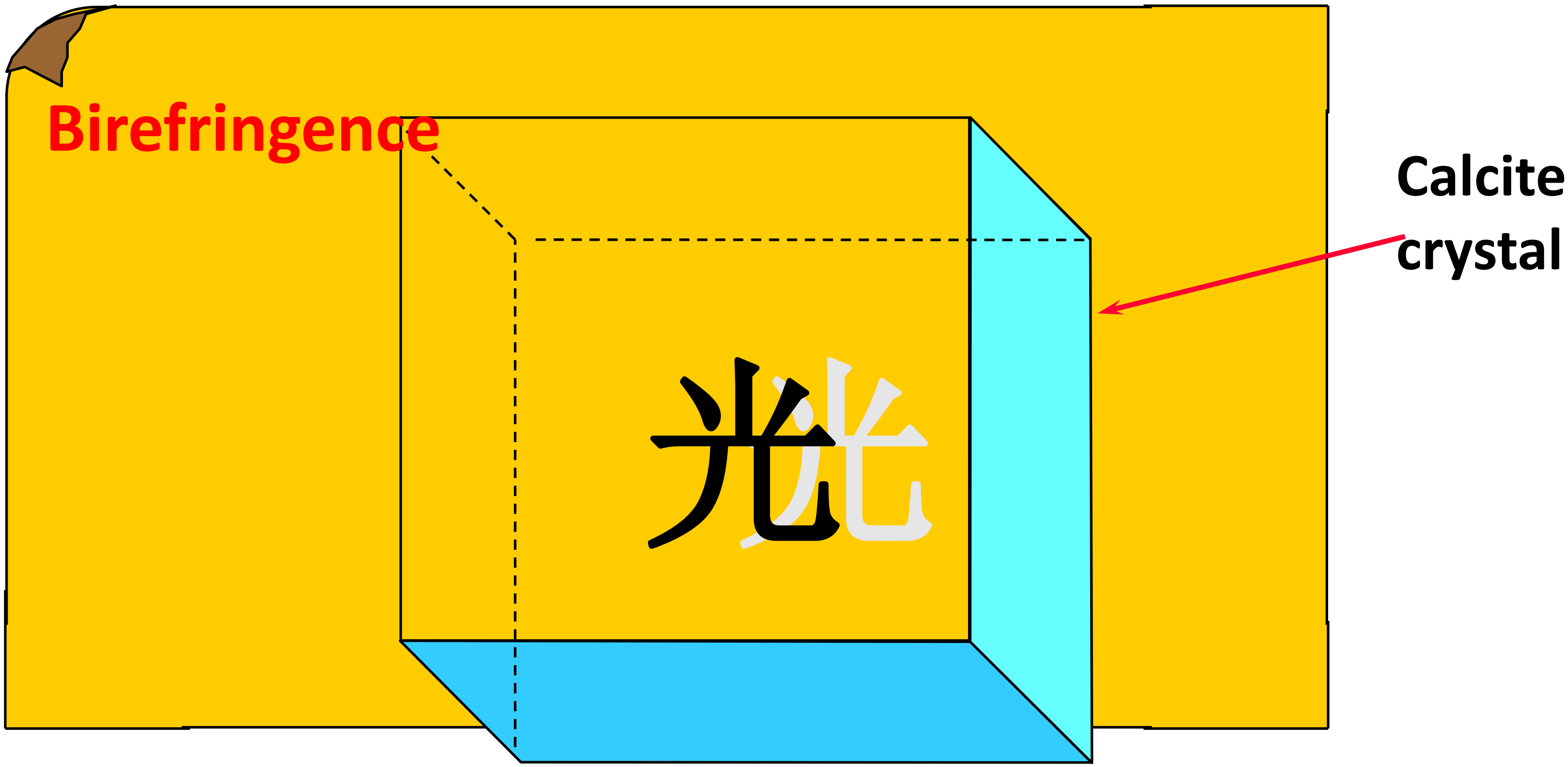
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



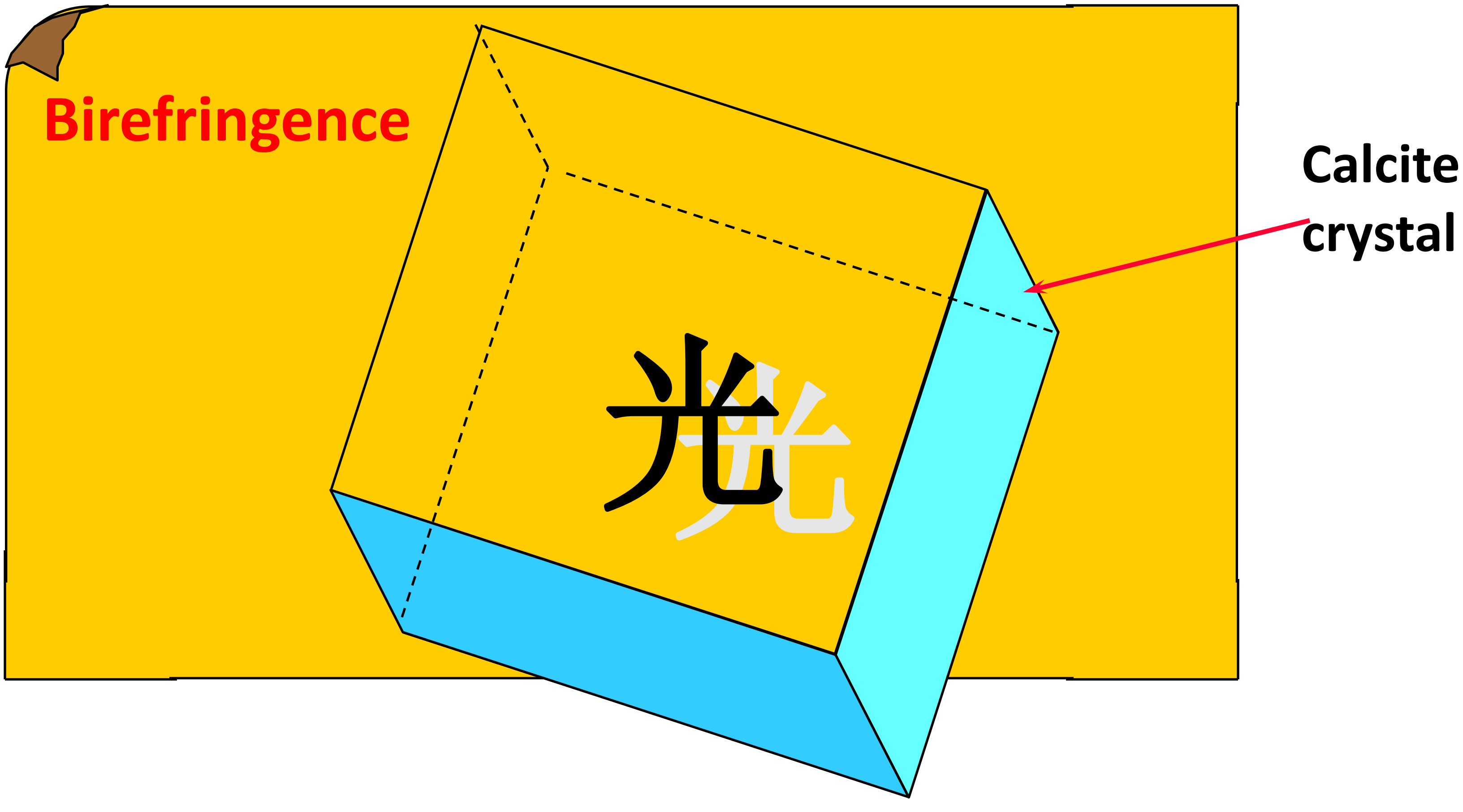
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



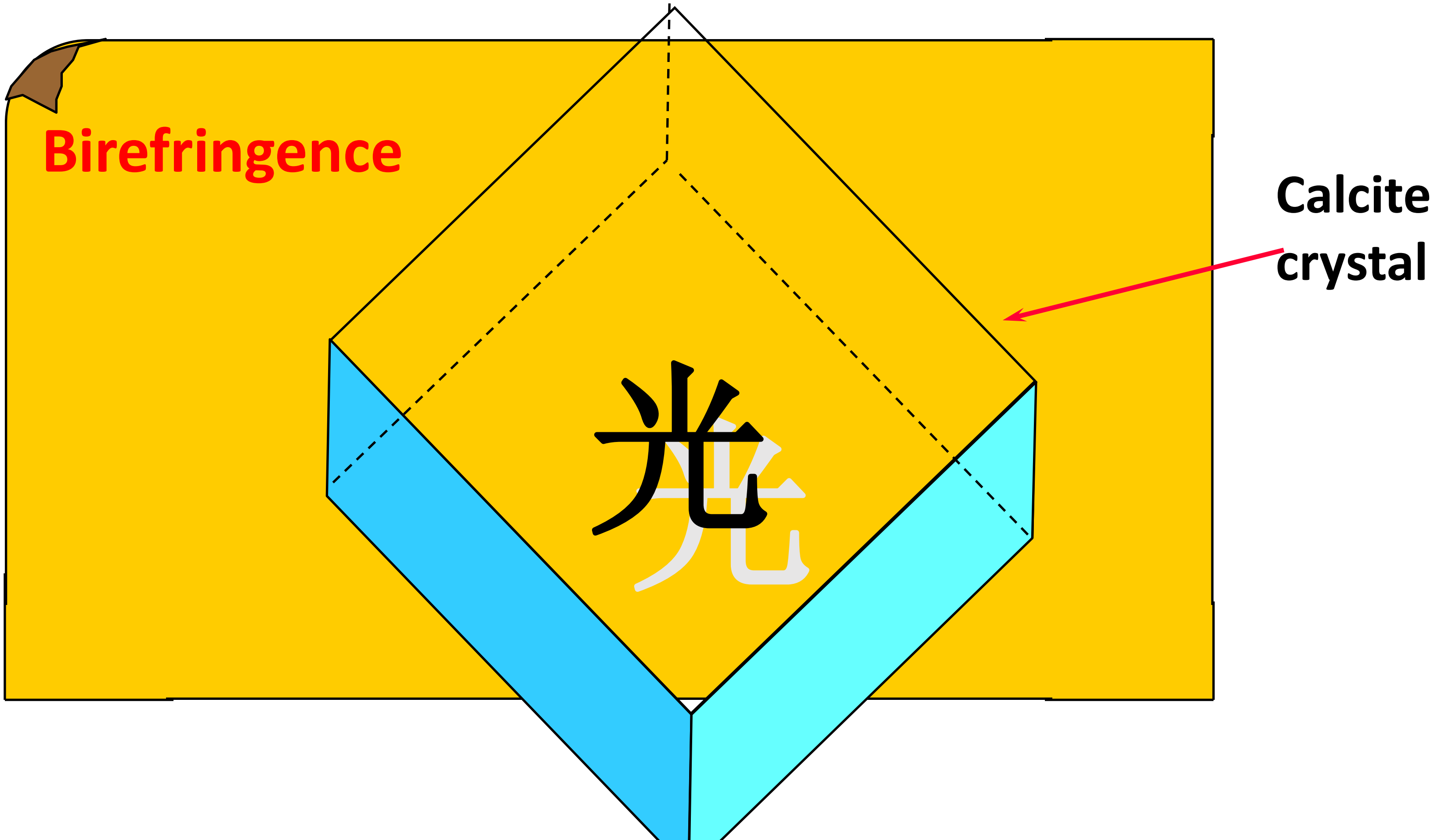
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



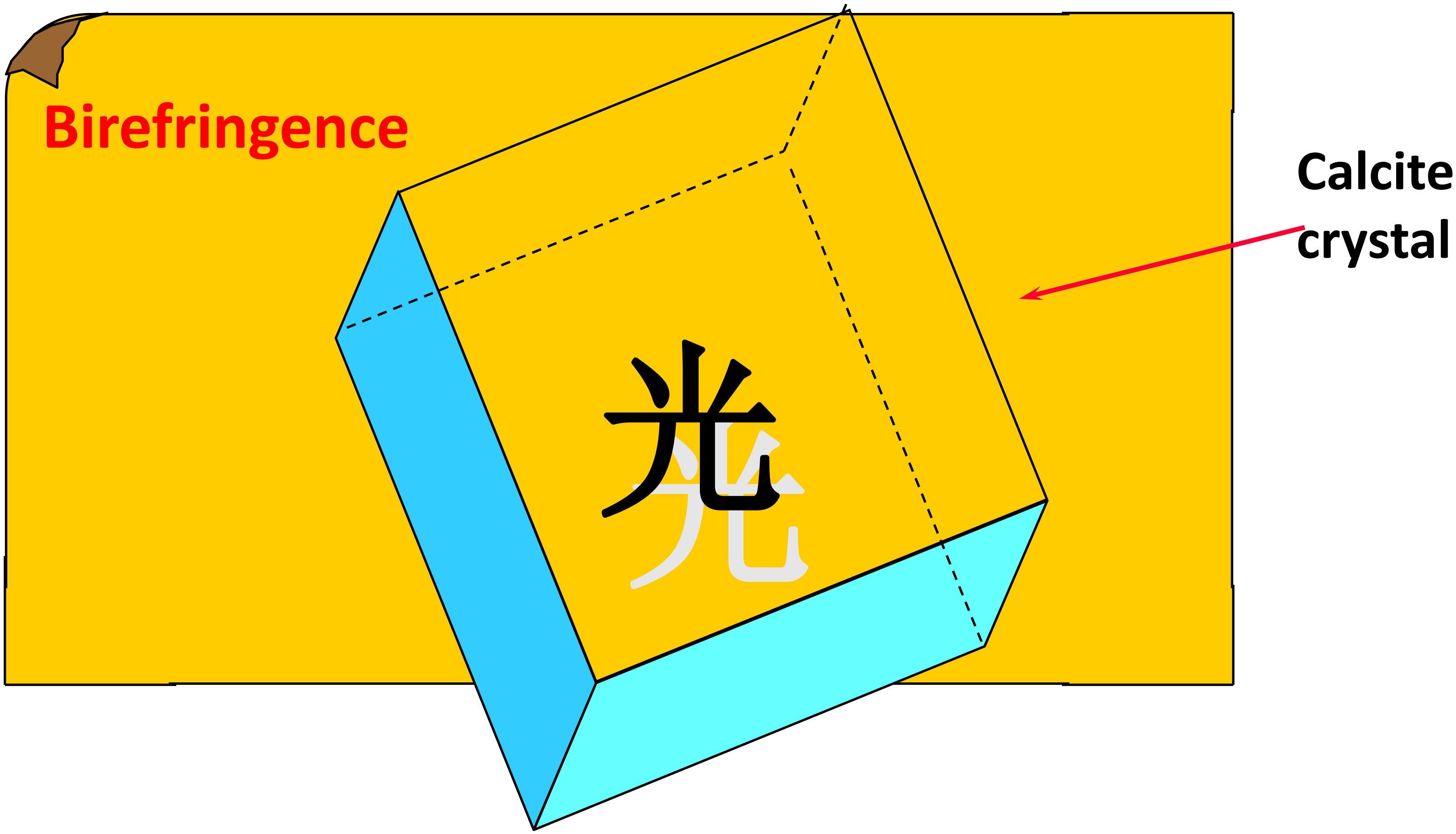
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



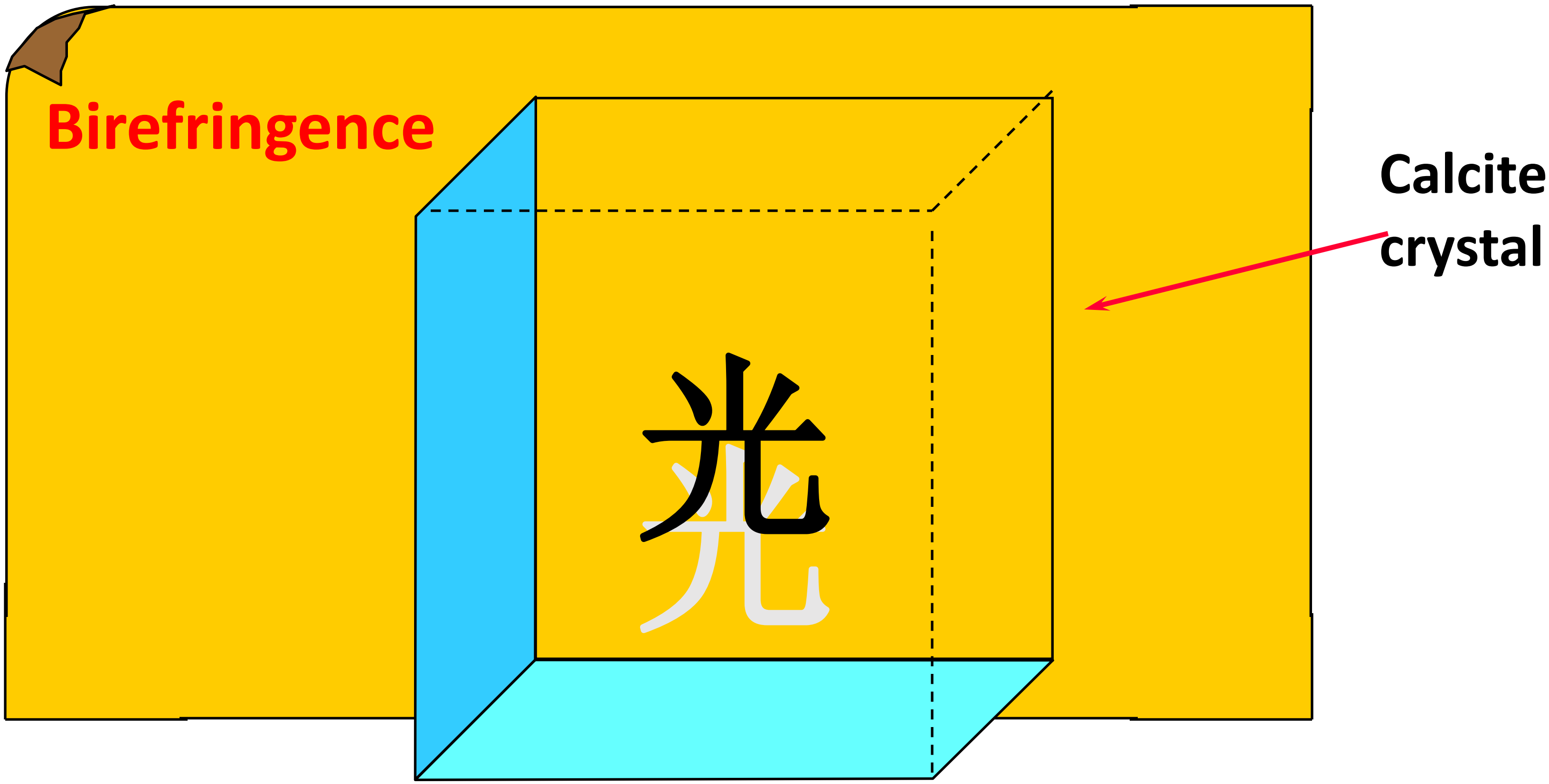
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



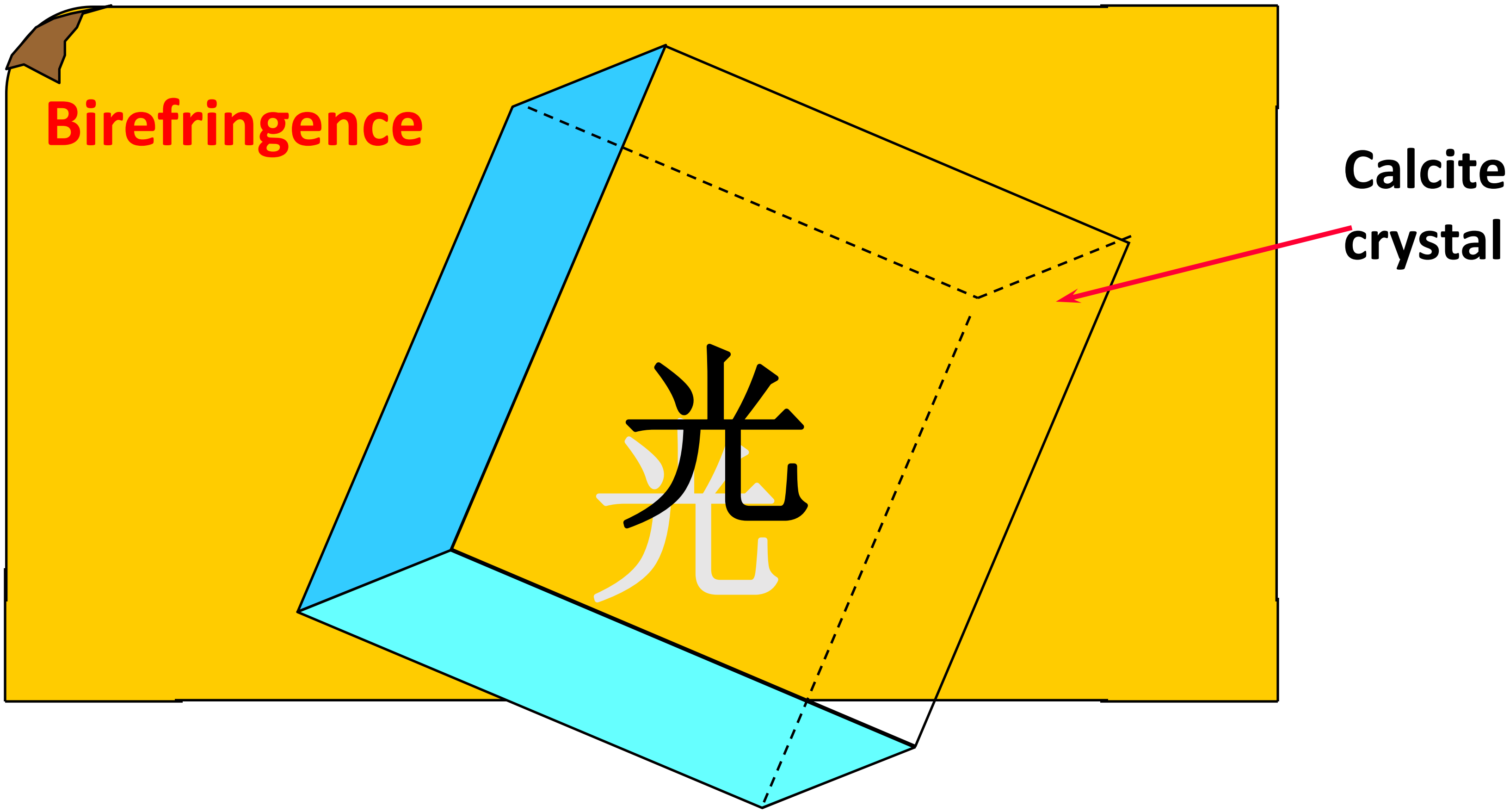
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



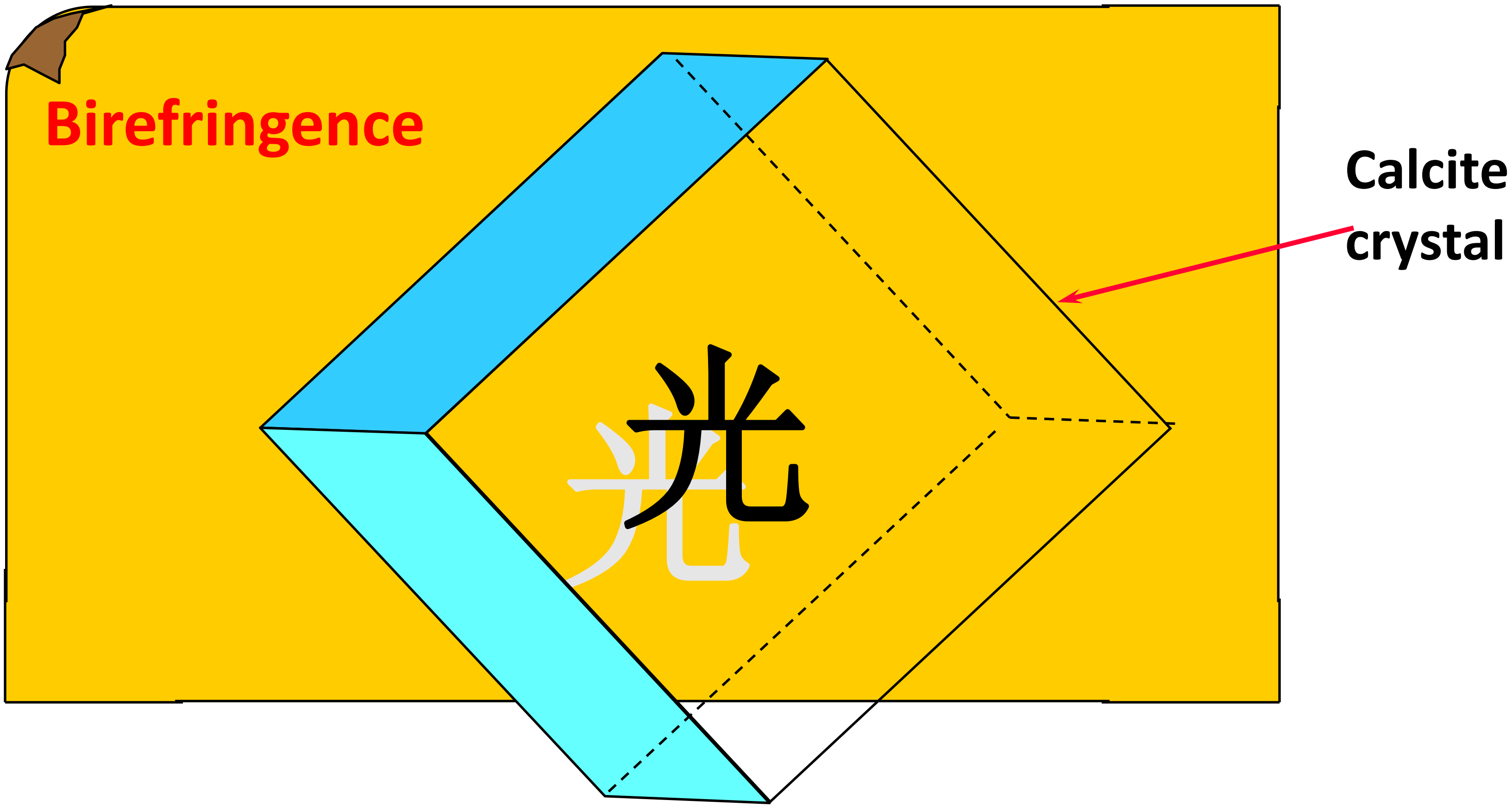
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



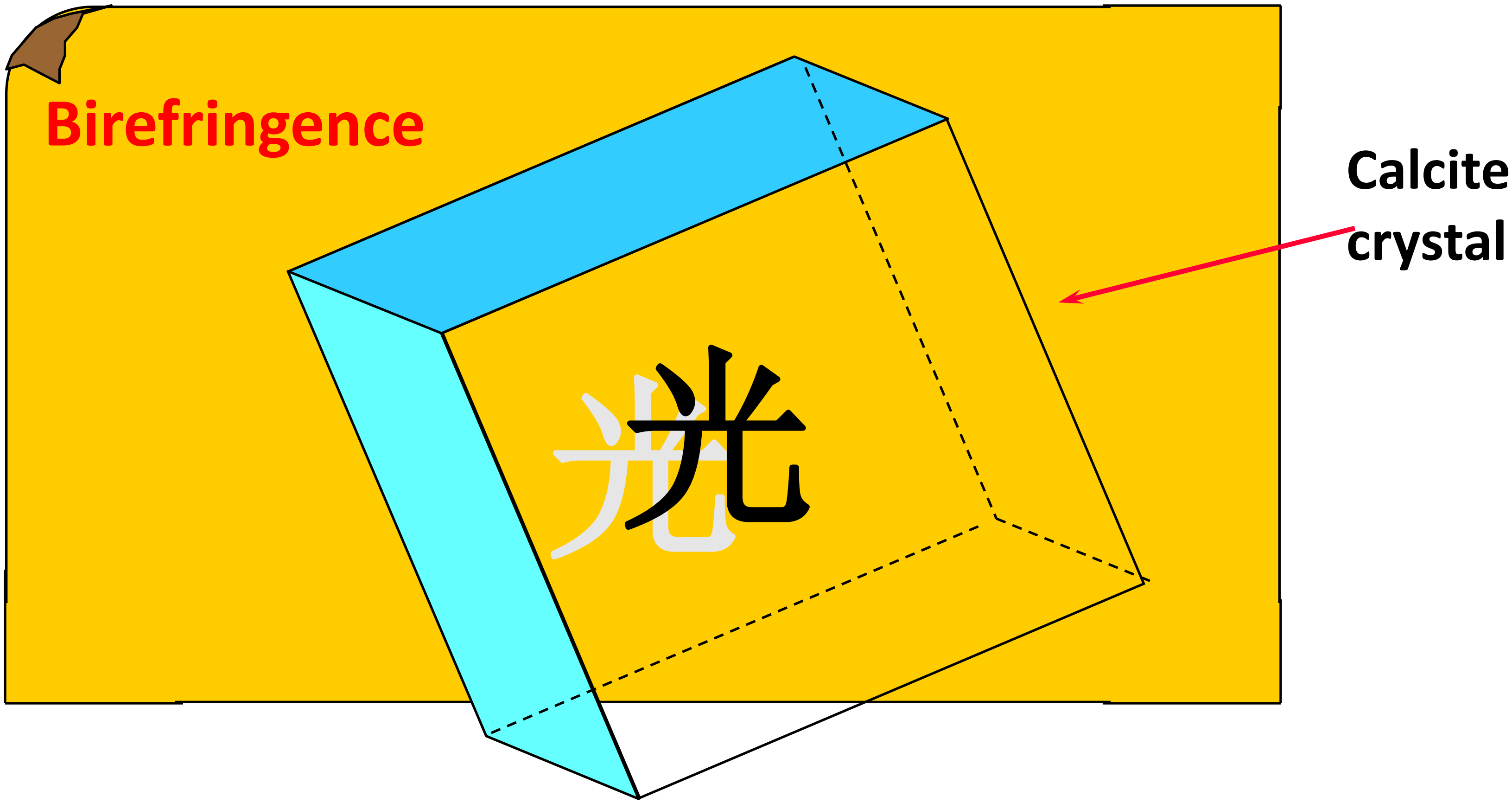
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



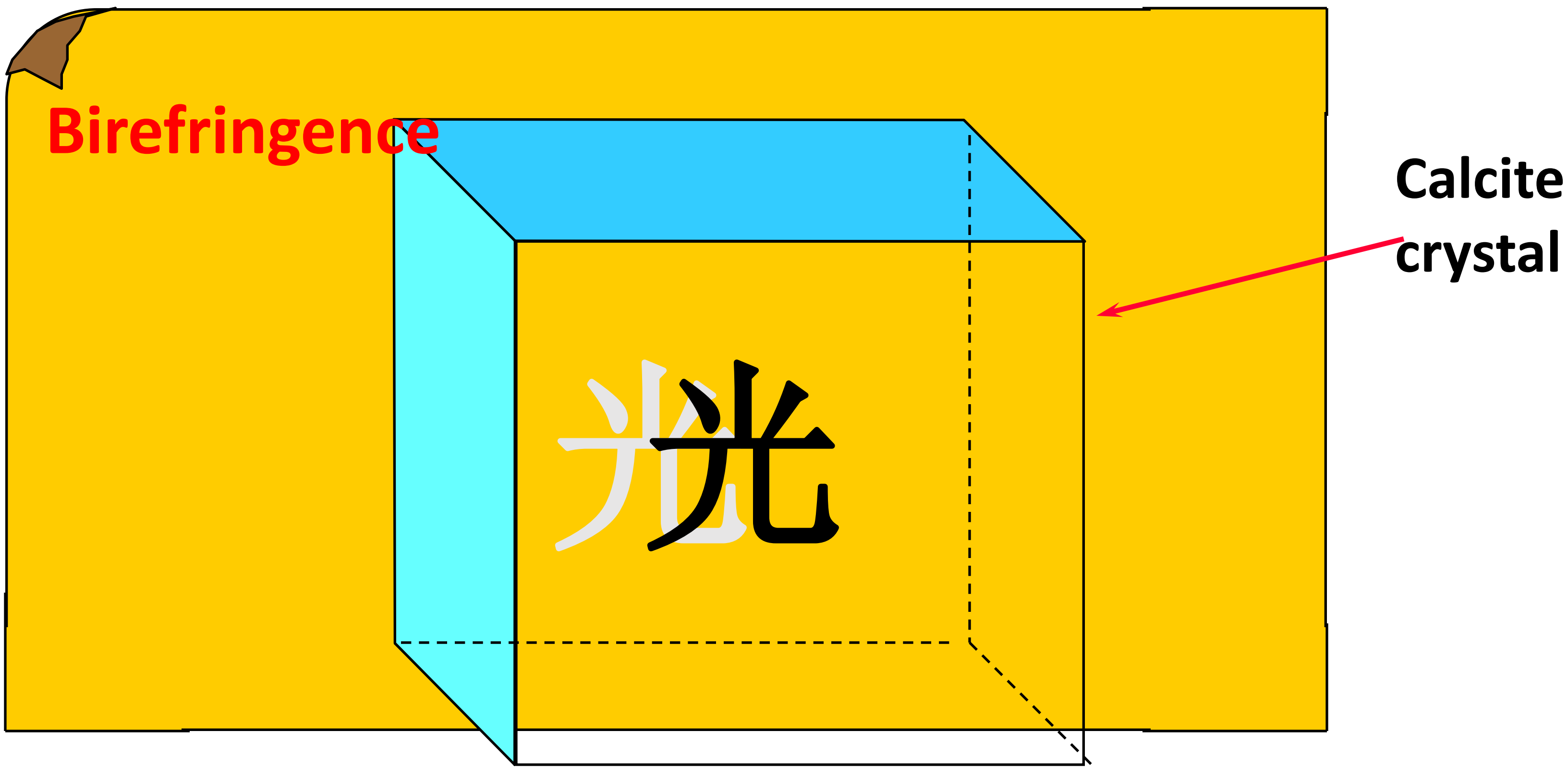
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



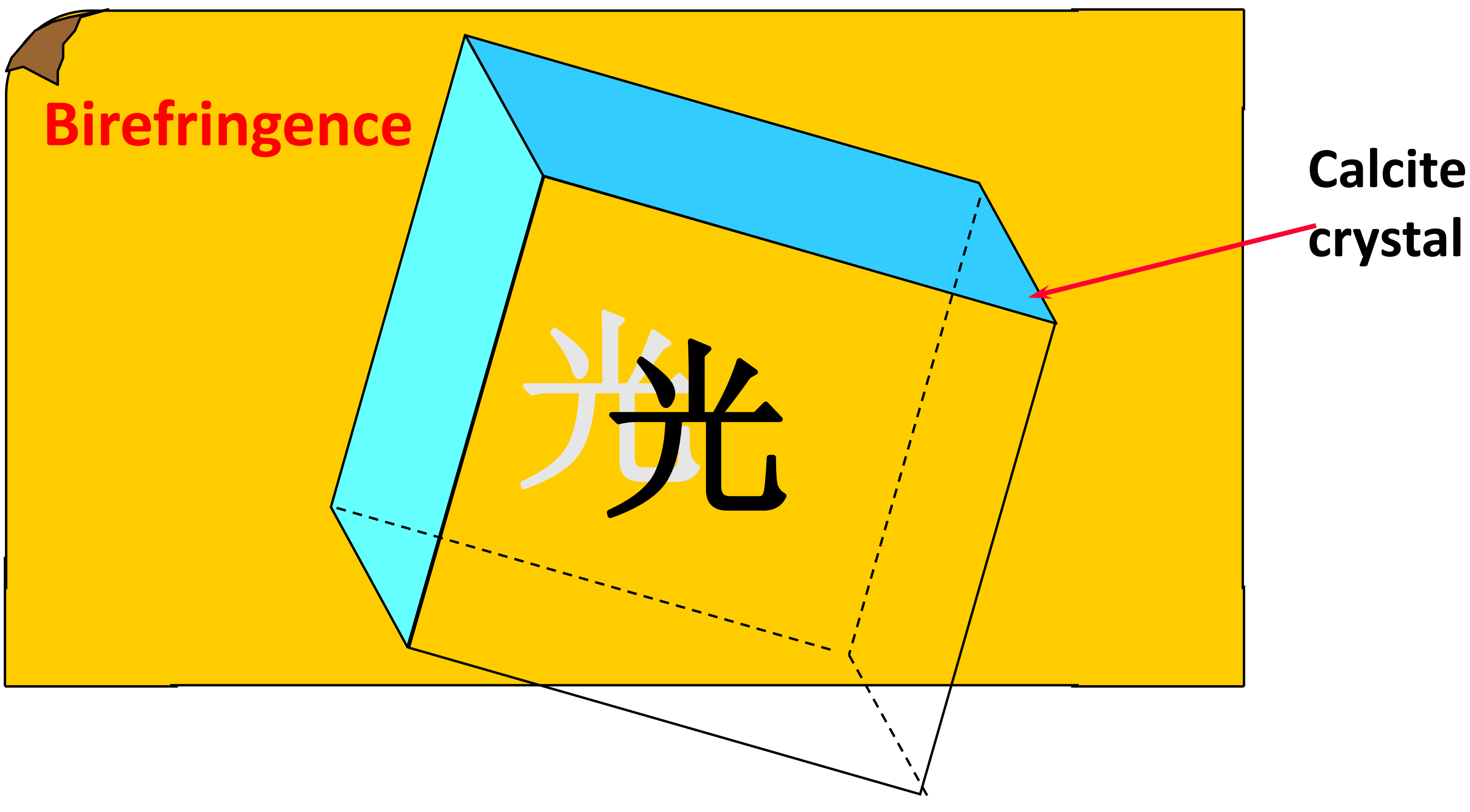
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



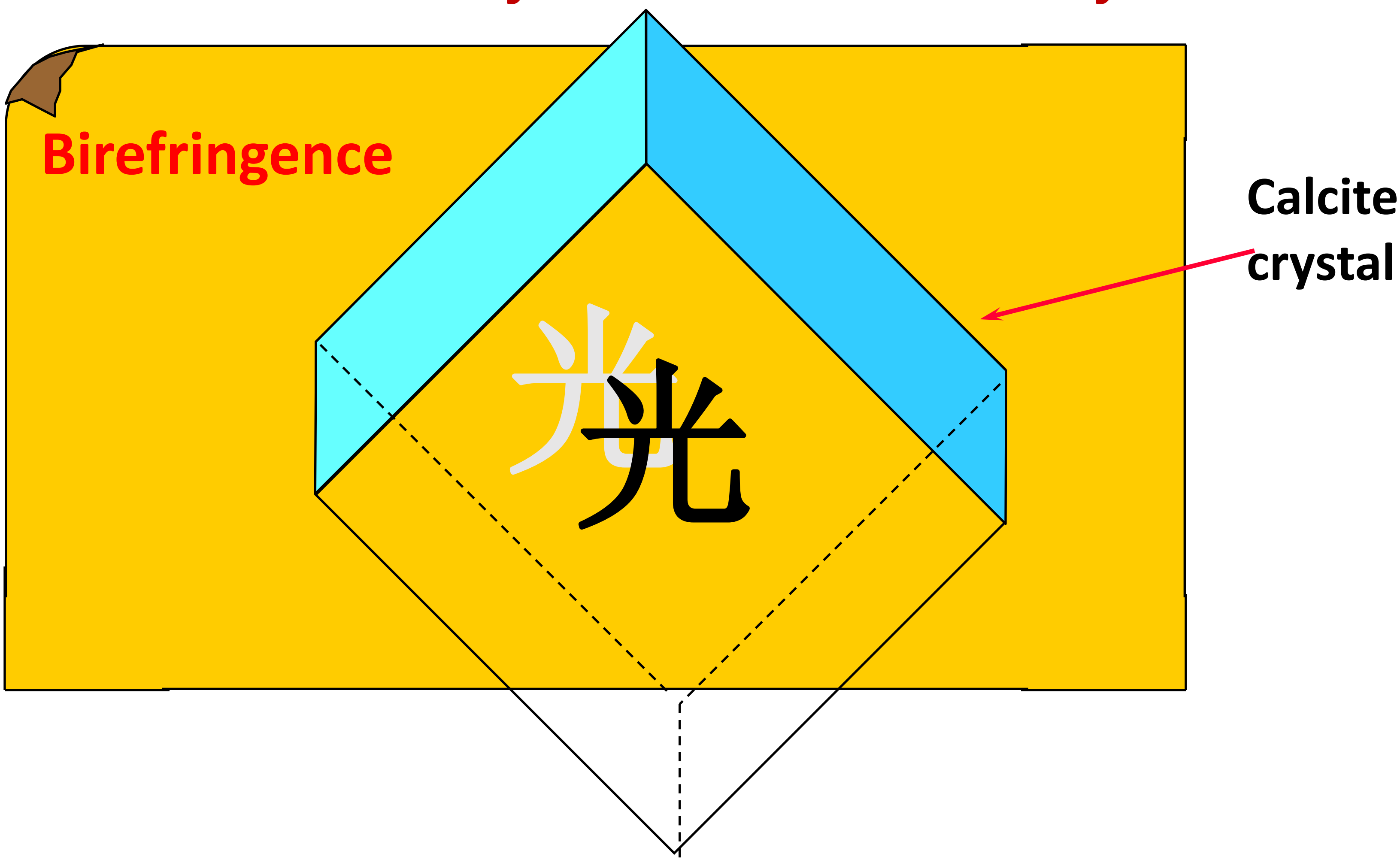
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



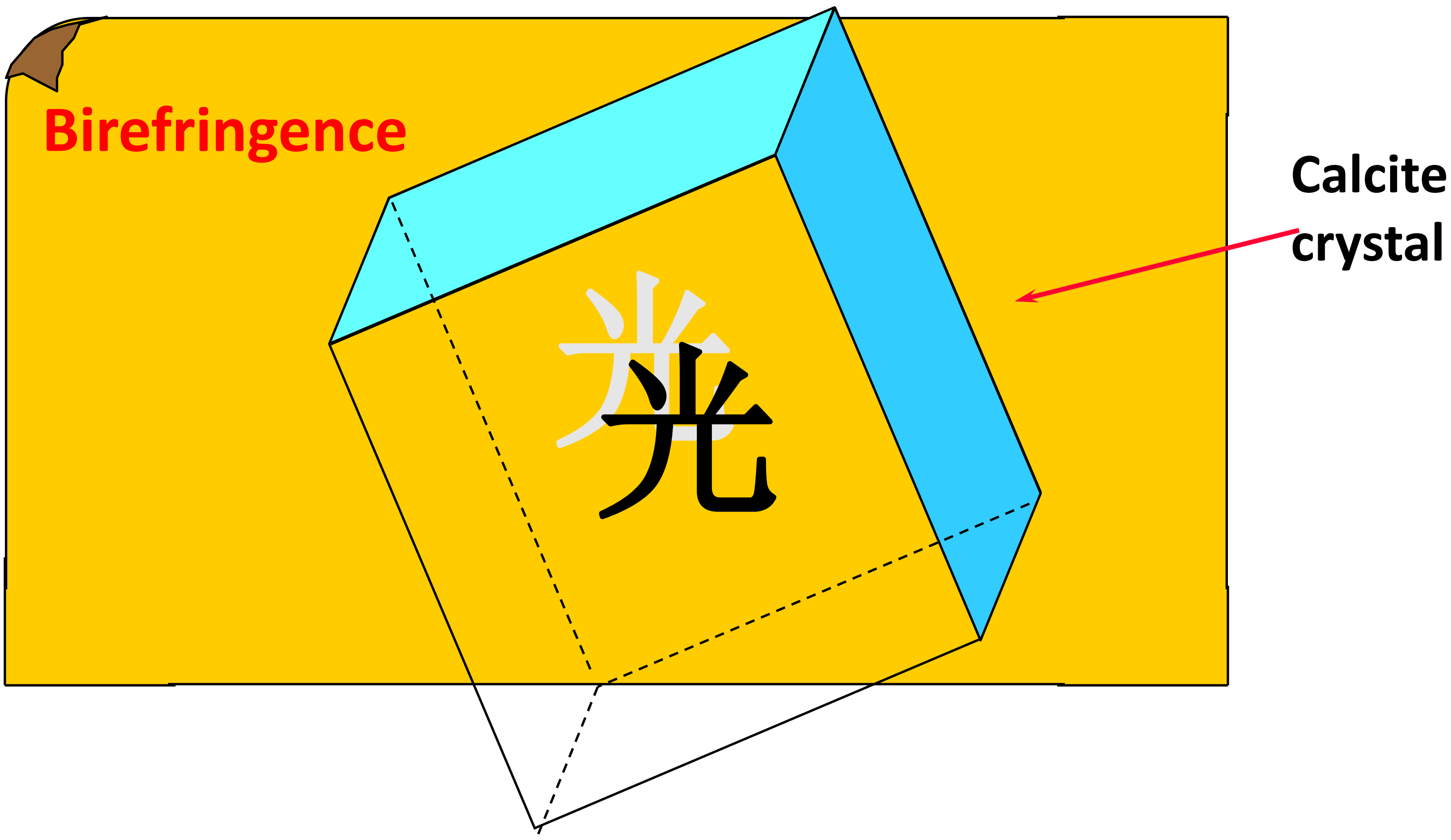
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



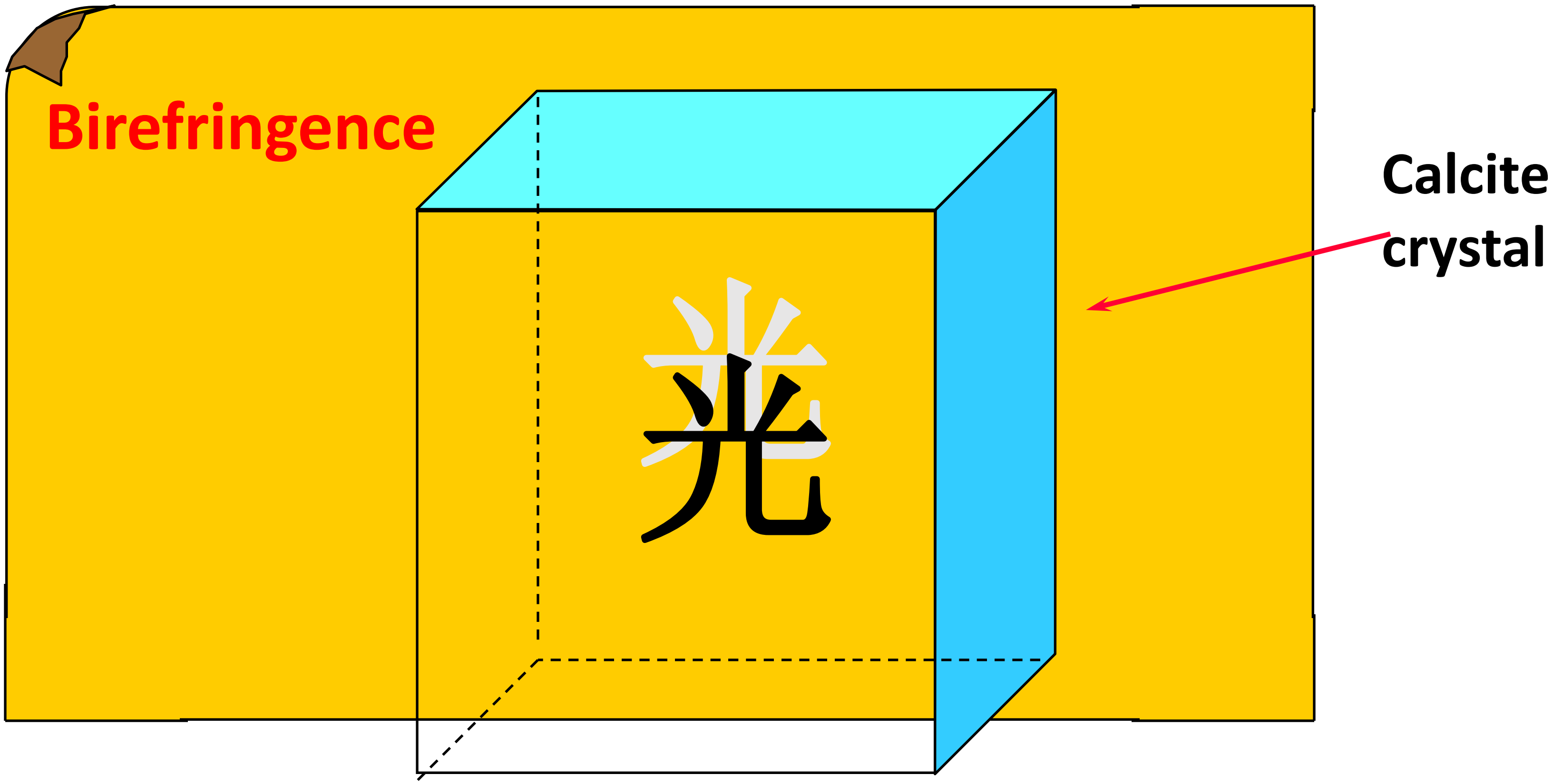
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



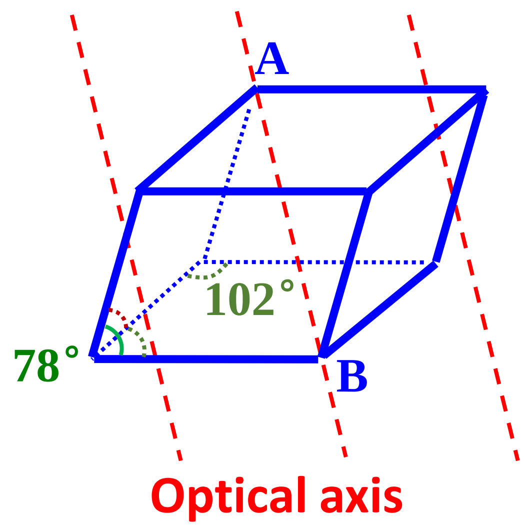
When the calcite crystal rotates, the o ray does not move, and the e ray rotates around the o ray.



Optical axis of crystal

① **Optical axis** of the crystal: along the optical axis, the o-wave and the e-wave coincide. **The birefringence doesn't occur when light propagates in this direction in the crystal** (propagation velocity of the o and e lights are the equal).

The optical axis of Calcite crystal ($CaCO_3$):

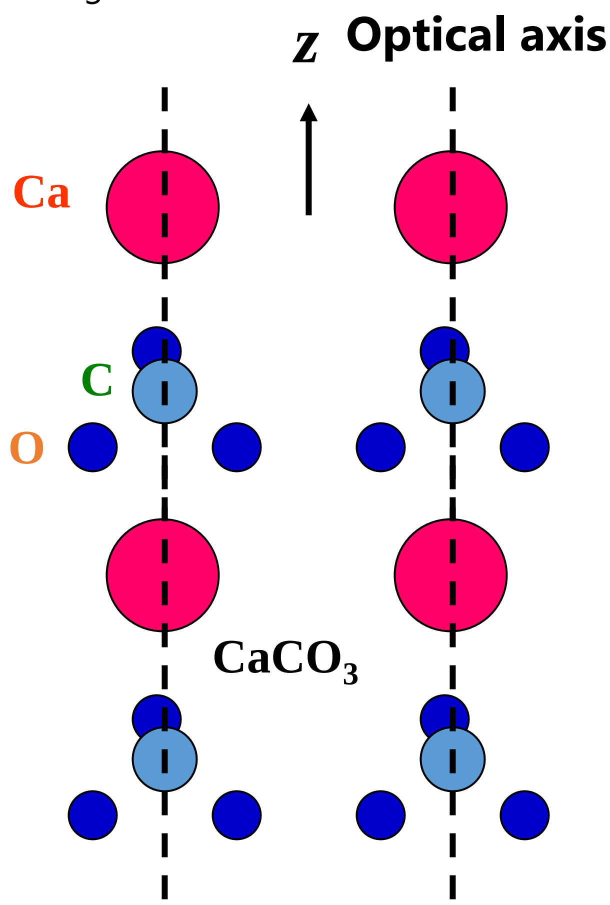


The optical axis is a **special direction** (not one special line). Any line parallel to this direction is the optical axis.

Optical axis of crystal

The optical axis of Calcite crystal ($CaCO_3$):

Origin of crystal anisotropy: the anisotropy from the **crystal structure (unit cell)** that appear **periodically**.



Optical axis of crystal

（uniaxis crystal）

Only one optical axis

方解石 calcite

石英 quartz

冰 ice

红宝石 ruby

Man-made

铌酸锂 $LiNiO_3$

磷酸二氢氨 ADP

（biaxis crystal）

Two optical axis

云母 mica

蓝宝石 sapphire

硫黄 brimstone

黄玉 topaz

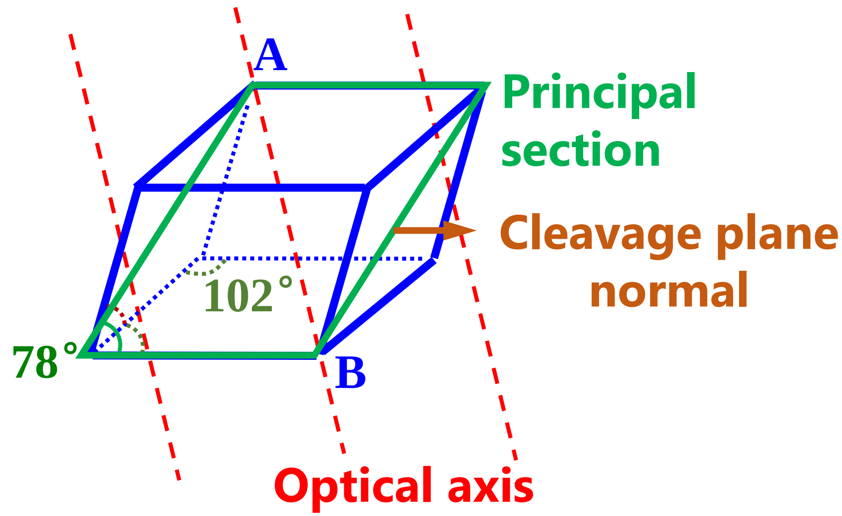
From the Baidu.com

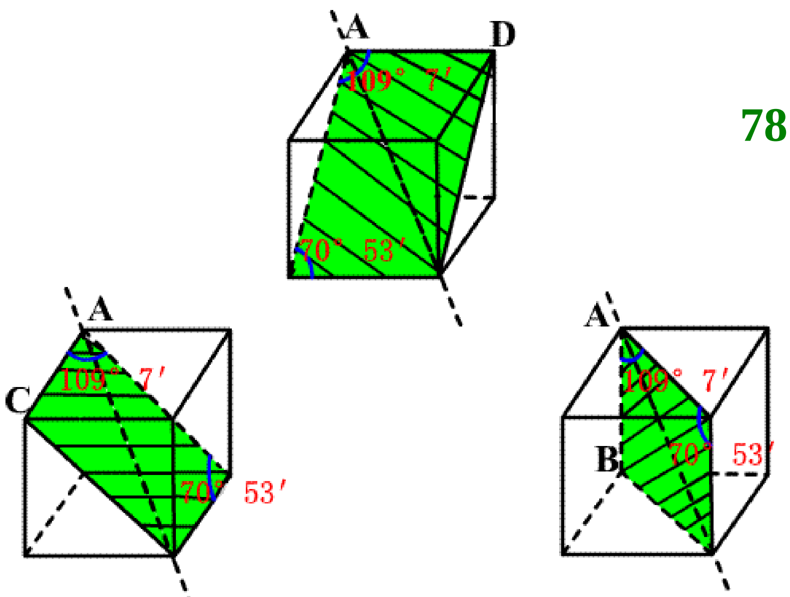
Principal section and plane

② Principal section

A plane containing the optical axis and the surface normal of the **cleavage plane**.

Determined by the crystal structure itself.



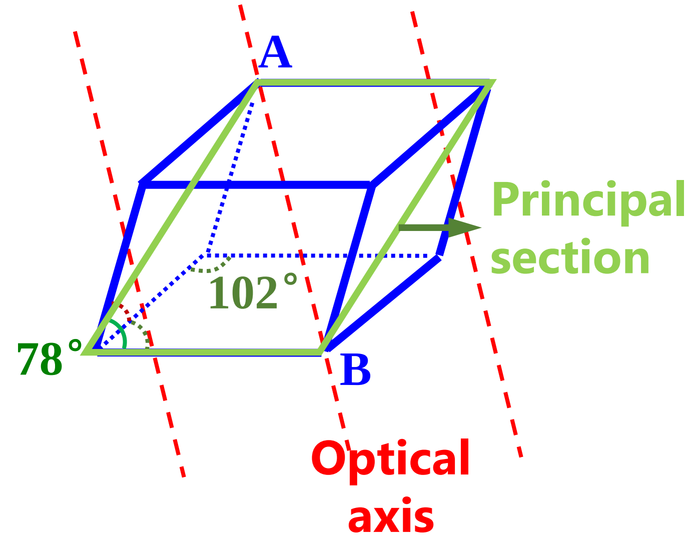


Principal section and plane

③ Principal plane

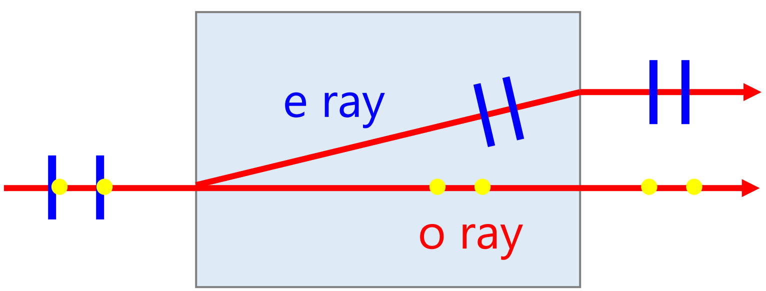
A plane formed by a light and an optical axis in a crystal.

※ Generally, the principal plane of the o-ray and the e-ray do not coincide.



If the incident plane of the light coincides with the principal section, then, the principal planes of the refracted o and e rays coincide with the incident plane.

In practice, the incident plane is intentionally selected to coincide with the principal section.

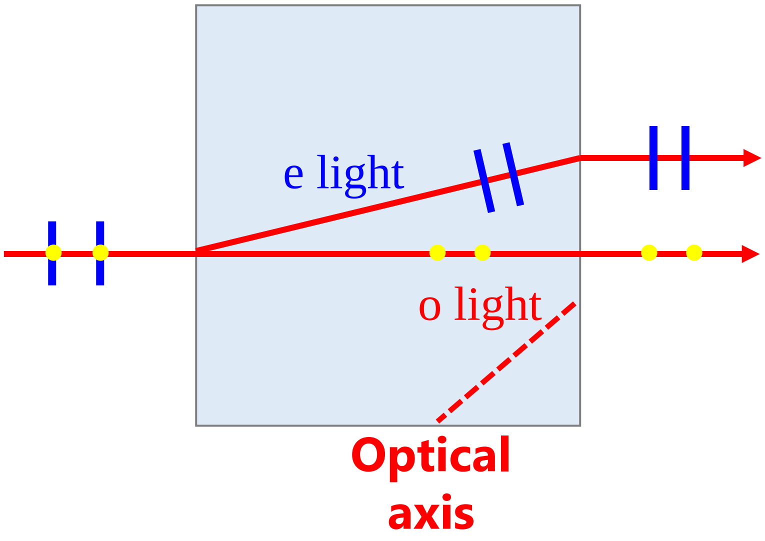


④ Polarization of refracted light

※ Both o and e are linearly polarized.

The vibration of the o wave: **perpendicular** to its principal plane, perpendicular to the optical axis.
The vibration of the e wave: in its principal plane, the angle with the optical axis is variable.

When the light is incident in the principal section of the crystal, the principal planes of the o and e rays coincide; the polarization of the o and the e are perpendicular to each other.



Malus' law

Determine the relative intensity of o and e light? (ignoring the absorption of light by crystals)

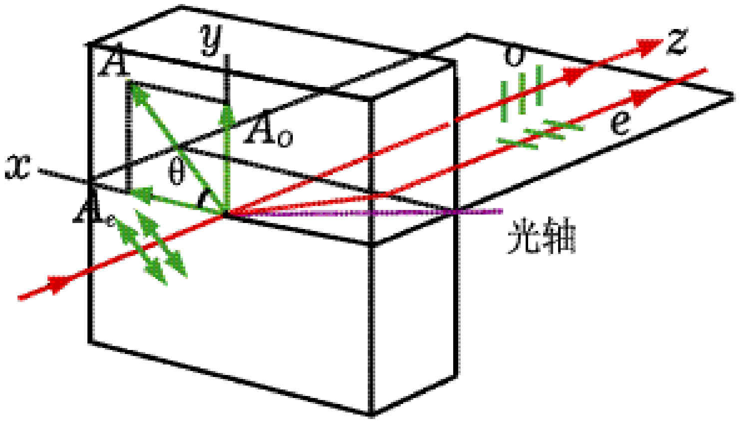
a. Natural light incidence

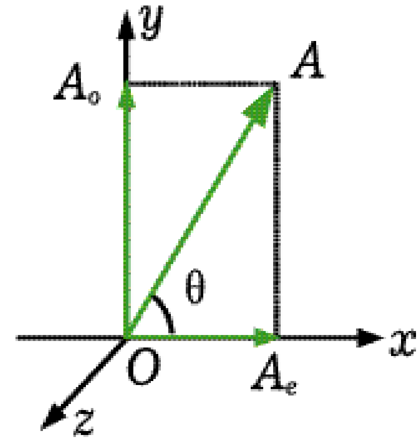
$$I_o = I_e = \frac{I}{2}$$

b. Linearly polarized light incident

o ray, e-ray intensity can be given by Marius's law

Malus' law





Principal plane

If the amplitude of the light is A, the angle between the incident plane of the incident light and the principal section of the crystal is θ, then the amplitudes of the o and e lights are respectively

$$A_o = A\sin\theta \qquad I_o = I\sin^2\theta$$

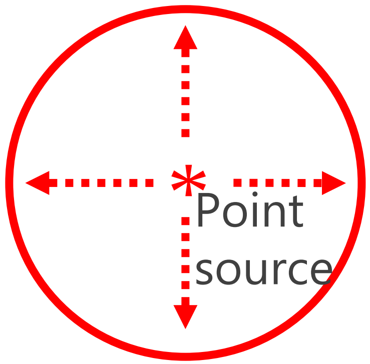
$$A_e = A\cos\theta \qquad I_e = I\cos^2\theta$$

$$\Rightarrow \frac{I_o}{I_e} = \tan^2\theta$$

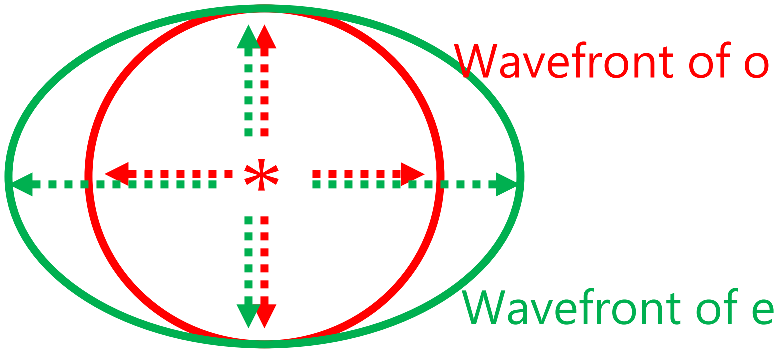
Malus' law

Propagation in a uniaxial crystal

Isotropic medium



Anisotropic medium



In the crystal, the speed of light travel is related to the direction. At time t = 0, light is emitted from the light source, after time t, the wavefront of the light consists of two closed curved surfaces

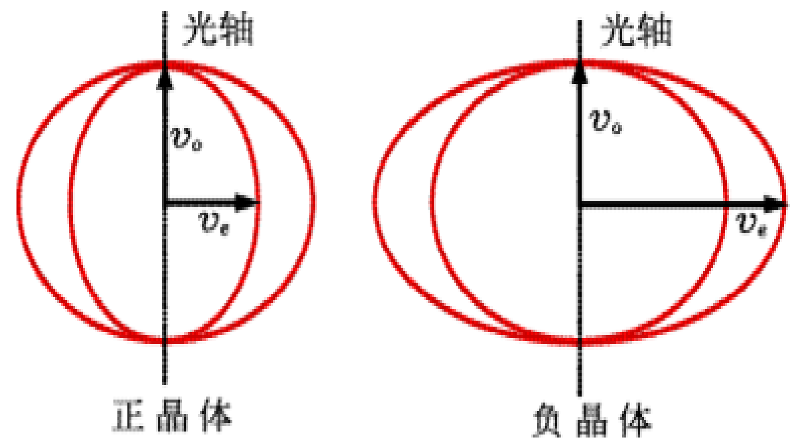
The wavefront of o ray is a spherical surface, and the wavefront of e light is a spheroidal surface (uniaxial crystal. These two surface are tangent in the optical axis direction (the two light speeds are the same along the optical axis direction).

o/e light in uniaxial crystal

According to the difference in the principal refractive index, crystals can be divided into two categories:

Positive crystal $v_o > v_e$

Negative crystal $v_o < v_e$



Principal refractive index of o and e：

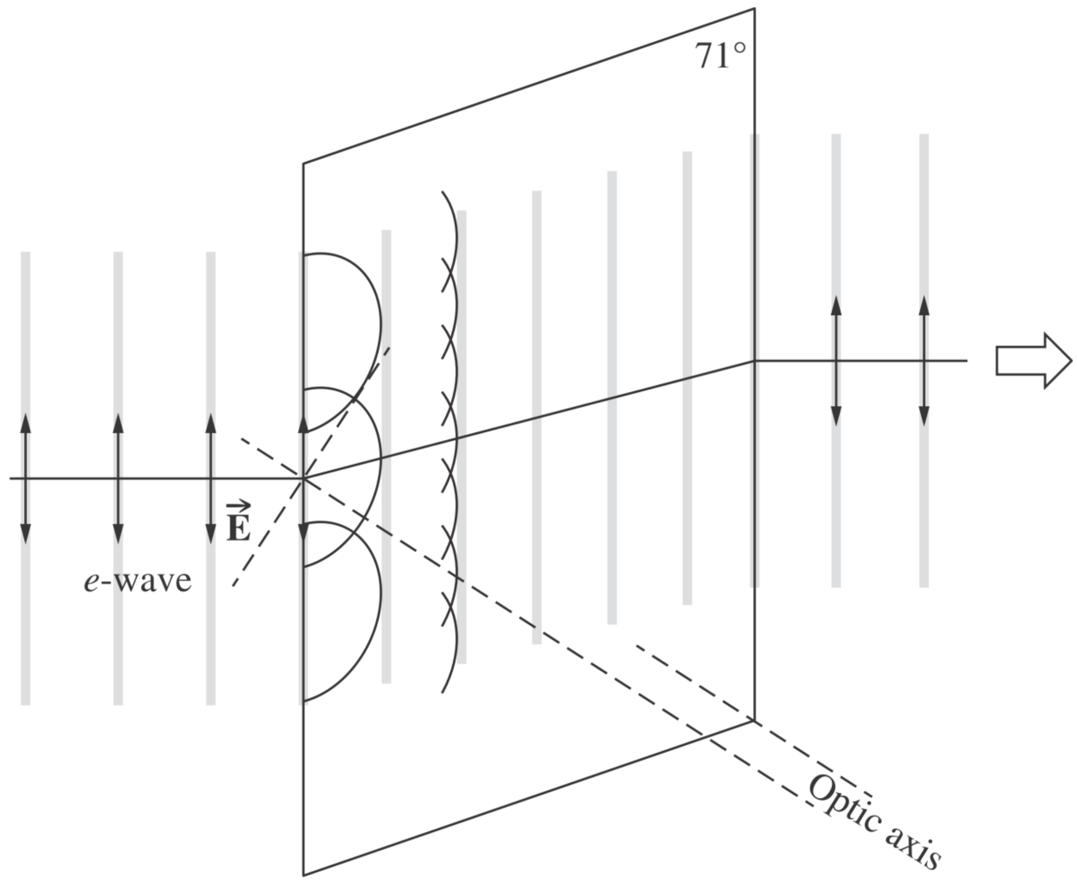
$$n_o = \frac{c}{v_o} \qquad n_e = \frac{c}{v_e}$$

The refractive index of the crystal is a second order tensor (3×3).

e ray satisfies the law of refraction when it propagates in parallel and perpendicular to the optical axis.
At the same speed in all directions as it propagates perpendicular to the optical axis.

Huygens principle

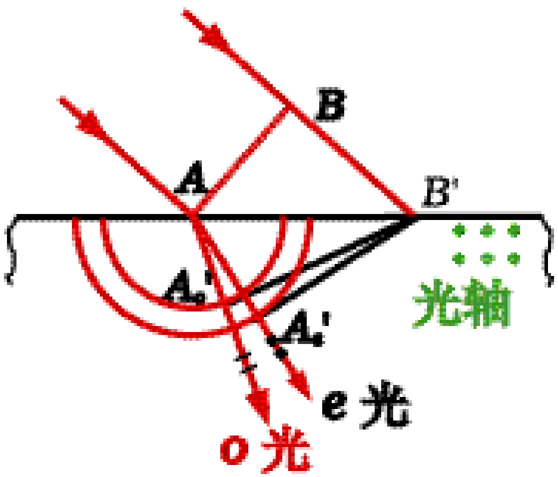
Ray direction vs. the **wavefront**



Huygens principle

Using Huygens principle to determine the ray direction of the o and e light in the crystal.

Condition: The optical axis is in the incident plane or perpendicular to the incident plane.



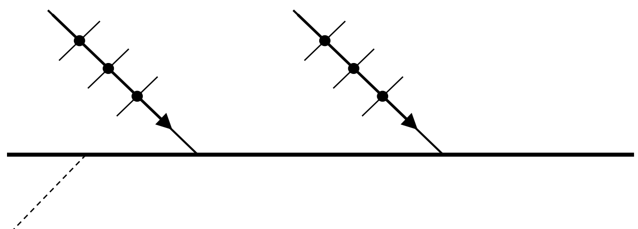
(1) The optical axis is perpendicular to the incident plane, and the plane wave is obliquely incident.

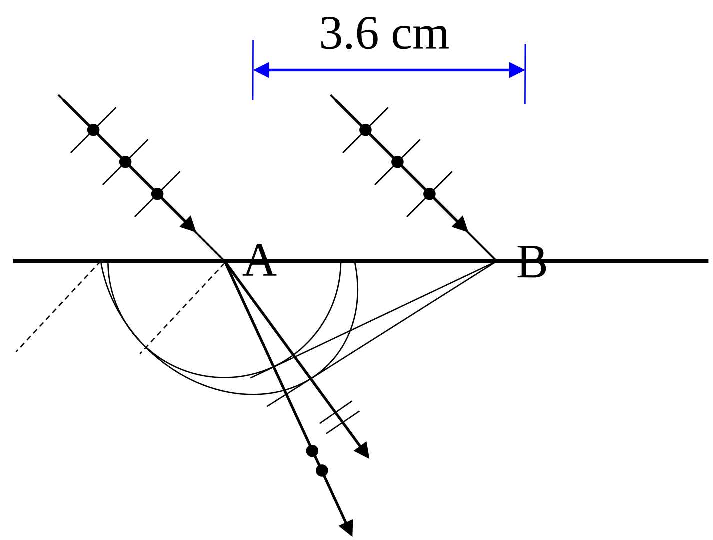
$$\frac{\sin i}{\sin \gamma_e} = \text{Const.}$$

In this case, in the incident plane both the o ray and the e ray satisfy the law of refraction.

Huygens principle

Natural light is incident on the surface of a uniaxial crystal ($n_o \approx 1.6$, $n_e \approx 1.3$) at 45° , and the optical axis (dashed line in the figure) is on the paper surface. Please use Huygens principle to determine the direction and polarization of the refracted light.





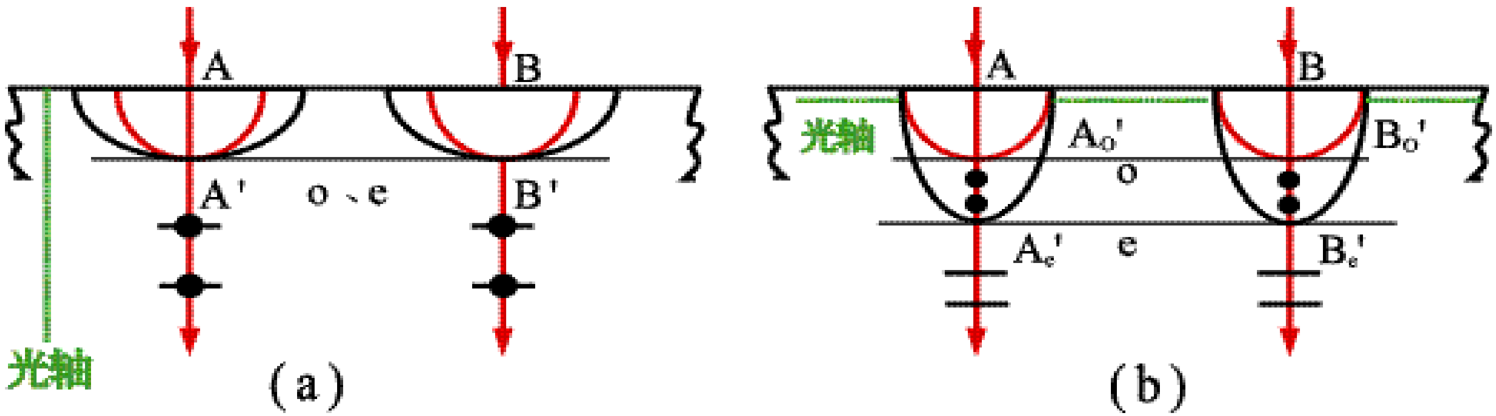
Take the point A as the center and make a semicircle

$$r = 3.6\text{cm} \times \frac{\sin 45^\circ}{1.6} = 1.59\text{cm}$$

Draw a line parallel to optical axis through point A, intersecting the circle at a point. Using this point as a tangent point, draw an ellipse with a long axis of

$$a = 3.6\text{cm} \times \frac{\sin 45^\circ}{1.3} = 1.96\text{cm}$$

Huygens principle



(3) The optical axis is perpendicular to the interface, and the plane wave is incident normally.

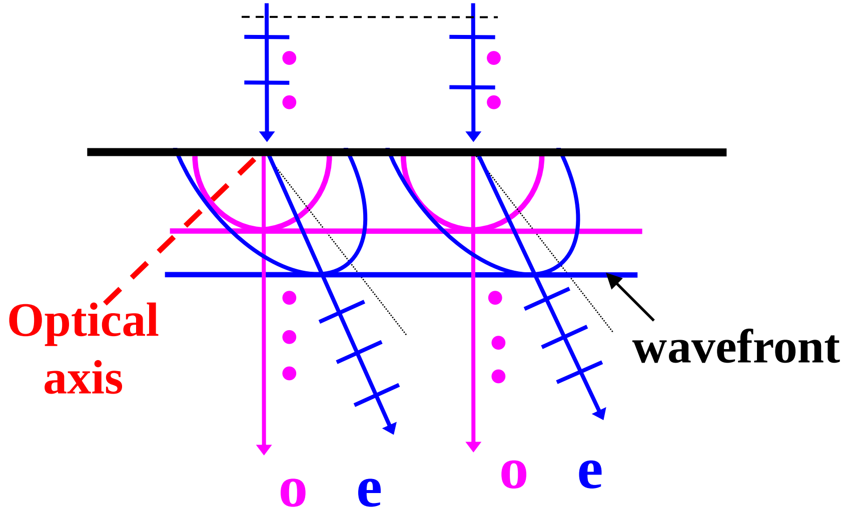
o ray and e ray propagate at the same speed in the same direction, and **no birefringence** occurs.

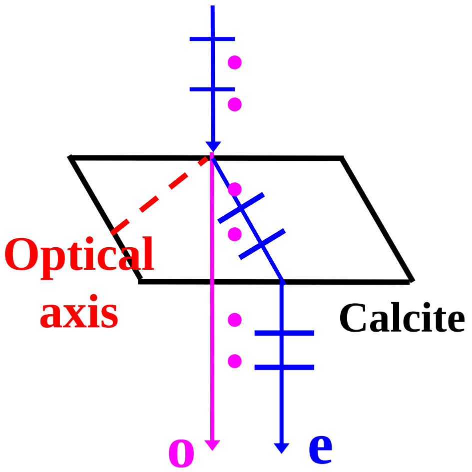
(4) The optical axis is parallel to the interface, and the plane wave is incident normally.

o ray and e ray travel in the same direction, but at different speeds, there is still a birefringence.

Huygens principle

Special situation of case (2): the optical axis is in the incident plane and is at an angle to the interface, and the plane wave is normally incident.





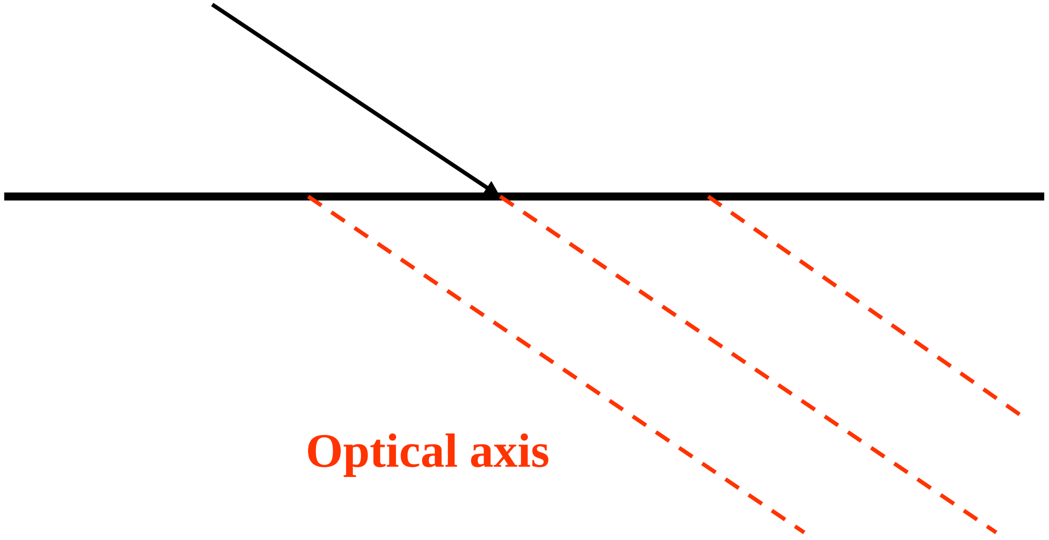
What is the direction of vibration of o wave and e wave?

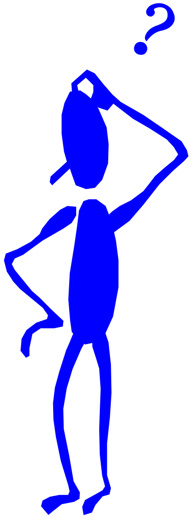
o ----point; e ----line.

Huygens principle

Question：

Someone say: "Because the light does not birefringent when it travels along the optical axis, there is no birefringence as shown in the picture." Right?



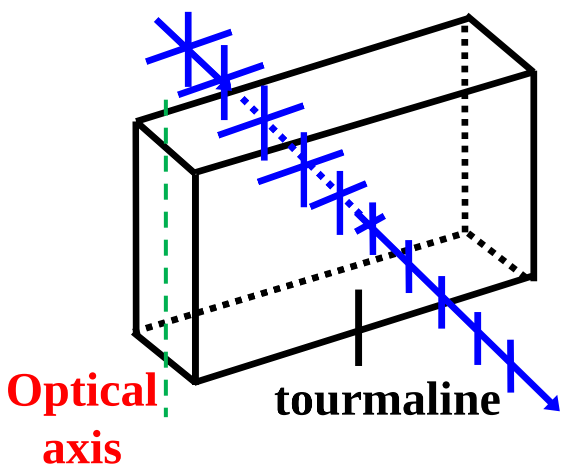


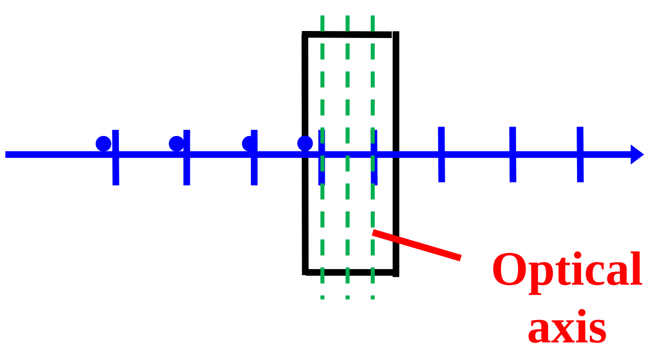
§ 10.2 Polarizers

① Crystal polarizer

Difference in the absorption of o and e ray, which is the **dichroism** (二向色性) of the crystal.

For example, tourmaline (**电气石**) has a strong absorption of o ray but a weak absorption of e ray, which can be used as a crystal polarizer to generate a linearly polarized light.

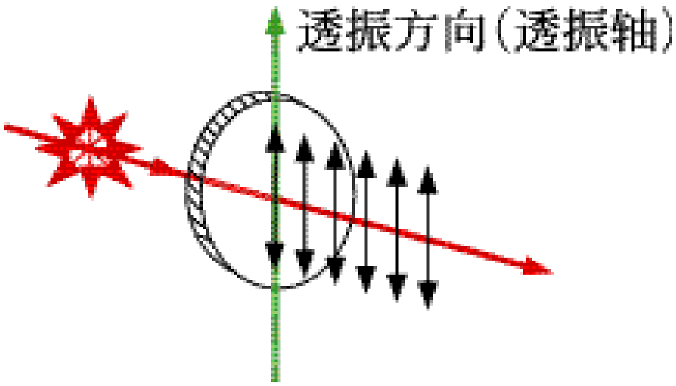




Polarizer

Polarizer: An optical element that produces and examines linearly polarized light.

Polarizer vs. **analyzer**



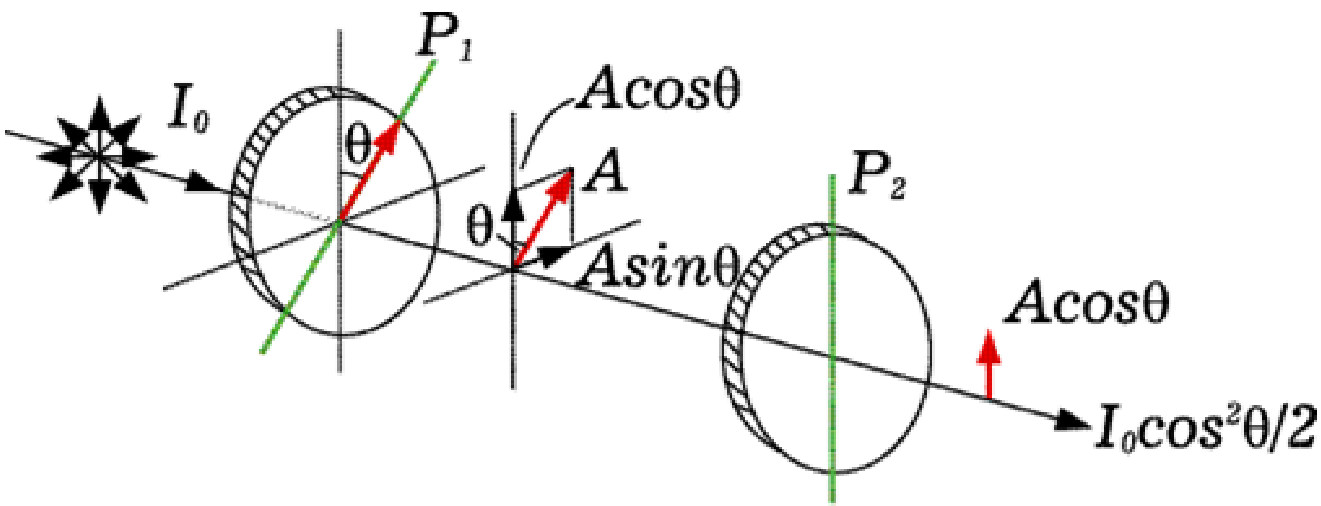
Natural crystal polarizers are small in size and expensive. Artificial polarizers are widely used today (manually aligning the optical axes of fine grains with dichroism on a plastic film)

Disadvantages: The polarized light obtained by the polarizer is not pure enough (the degree of polarization is lower than that of the prism).

Polarizer

Natural light ➡ Linearly polarized light

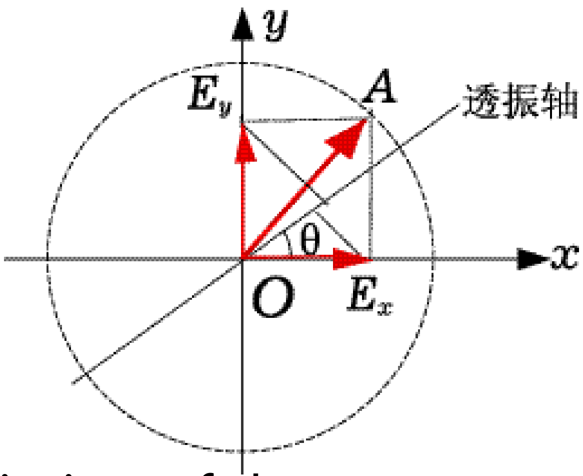
$I = I_0/2$ $I = I_0 \cos^2 \theta/2$



P_2 is fixed, rotating P_1, the transmitted light intensity changes with θ, and when $\theta = 0$, the transmission is the strongest.

Polarizer

Circularly polarized light pass through a polarizer



$$E_x = A\sin\omega t$$

$$E_y = A\cos\omega t$$

$$A = \sqrt{E_x^2 + E_y^2}$$

The angle between the vibration transmission of the polarizer is θ angle to the x-axis. so,

$$E_1 = A\cos\theta\sin\omega t; \quad E_2 = A\sin\theta\cos\omega t$$

$$E = E_1 + E_2 = A\sin(\omega t + \theta)$$

The average is $\langle E^2 \rangle = A^2/2 = I_0/2$

The intensity of circularly polarized light is reduced to a half after passing through a polarizer.

Prism polarizer

② Prism polarizer

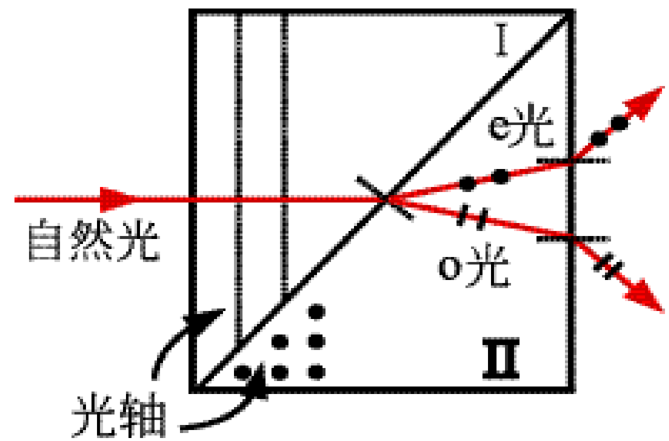
Obtaining linearly polarized light with **high degree of polarization**.

After the natural light is incident on the birefringent crystal, it is divided into o light and e light. By combining the prisms, the two lights are separated widely, and it is easy to obtain single or double beam linearly polarized light.

Wollaston prism

(1) Double beam prism polarizer

① In region I, $n_e < n_o$, $v_e > v_o$
The two light propagation speeds are different, but the directions are the same.



② When the light enters II, the optical axis rotate 90°:

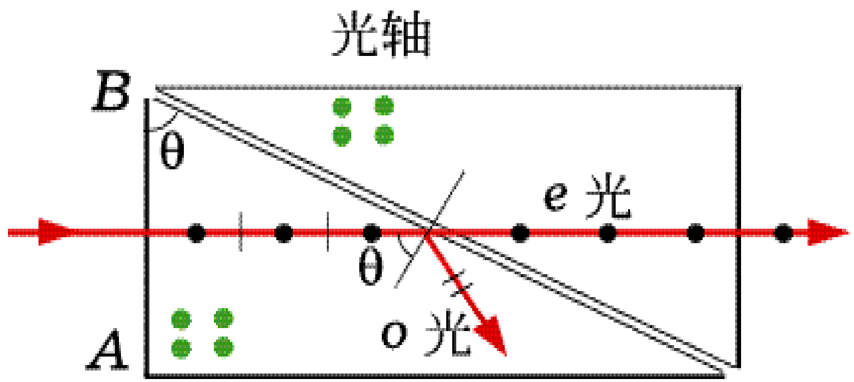
o ray in I → e ray in II	e ray in I → o ray in II
denser medium → tenuous medium	tenuous medium → denser medium
Refraction angle > Incident angle	Refraction angle < Incident angle

③ When the two lights enter the air, they are refracted again, and the angle between the two lights increases.

Glan prism

2．Single beam polarizing prism

The two prisms are glued together. $n_e < n_{glue} < n_o$. It is also possible to replace the glue layer with an air layer.

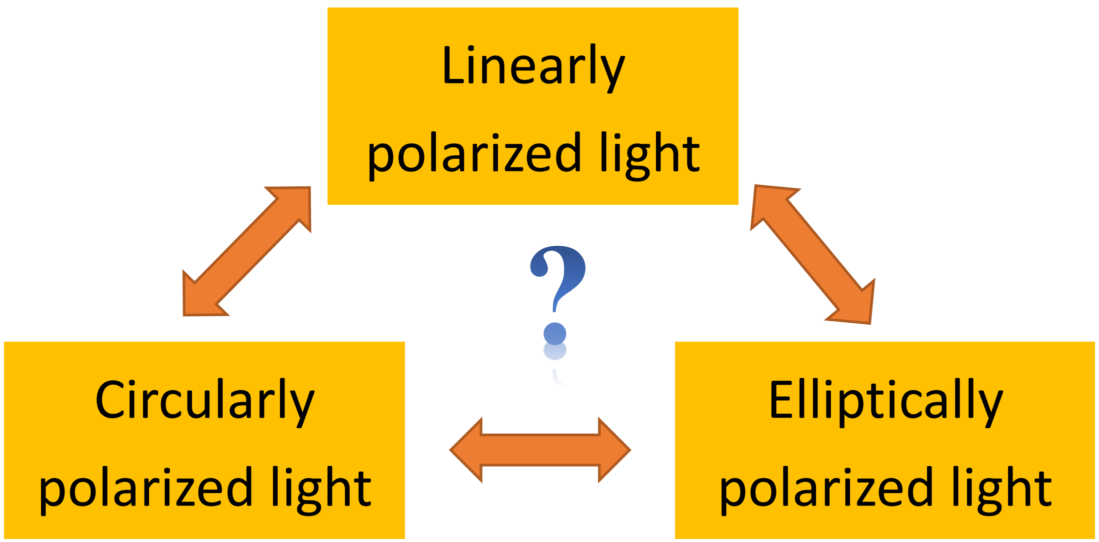


Select the appropriate angle θ so that the o-light is totally reflected at the first interface and absorbed by the side of the prism. At the same time, the e-light does not undergo total reflection at the interface, but is transmitted into II and exits from the prism

§ 10.3 Wave plates

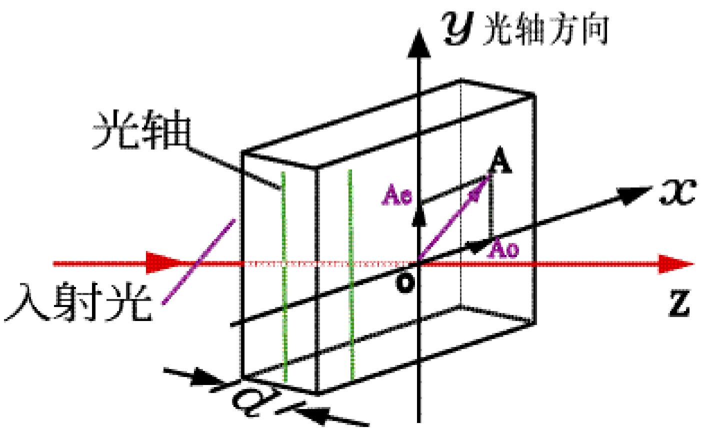
Question: How to convert the polarization state of light?

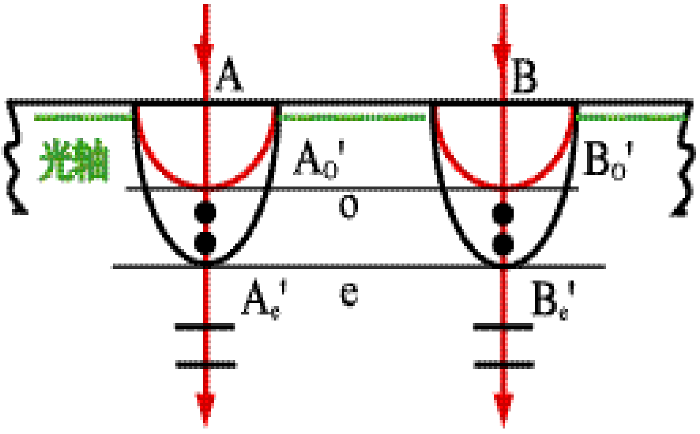
e.g., circularly polarized light → linearly polarized light?



Wave plate: a parallel plate cut from birefringent crystal with its optical axis parallel to the cutting surface.

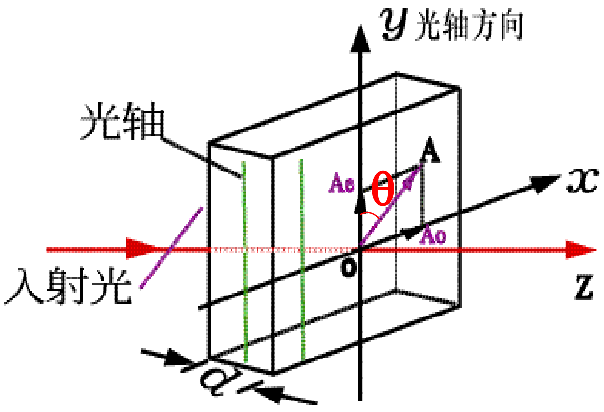
Change the polarization state of the incident light and the phase delay of the o-light e-light.





The refractive index is different, affecting the **phase difference** between the two lights at the exit interface, and the change of the combined light state.

Let the thickness be d, and the monochromatic linearly polarized light is incident normally.



OPL and phase difference is

$$\Delta = (n_o - n_e) d$$

$$\delta = \frac{2\pi}{\lambda}(n_o - n_e) d$$

Positive crystal $n_0 < n_e$, $\delta < 0$,

Negative crystal $n_0 > n_e$, $\delta > 0$,

The light vector after passing through the plate

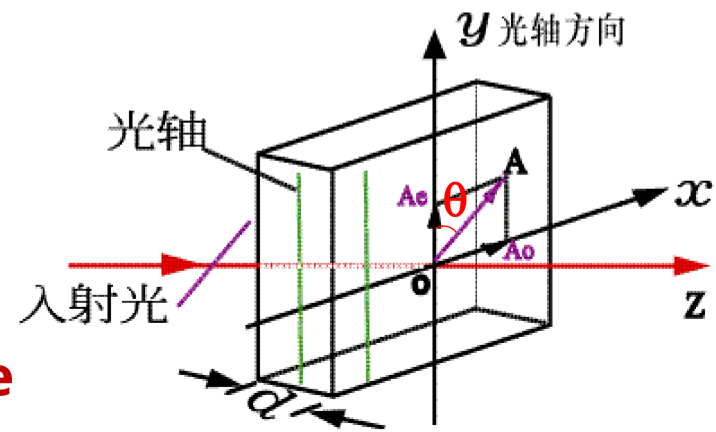
$$\mathbf{E} = \mathbf{E}_o + \mathbf{E}_e$$

$$E_x = E_{0x} \cos(\omega t)$$
$$E_y = E_{0y} \cos(\omega t + \delta_0)$$

$$E_x = E_{0x} \cos(\omega t + \delta)$$
$$E_y = E_{0y} \cos(\omega t + \delta_0)$$

Half wave plate(HWP)

$$\delta = \frac{2\pi}{\lambda}(n_o - n_e)d$$



① When $\delta = 2m\pi \quad \Delta = m\lambda$, Two light waves return to the same phase.

This wave plate is called a **full wave plate**.

② When $\delta = (2m+1)\pi \quad \Delta = (2m+1)\frac{\lambda}{2}$, called **half wave plate**.

Incident		Emerging
LP	➡	LP, The plane-of-vibration is rotated by 2θ.
ECP	➡	L(R)ECP →R(L)ECP. The orientation changes.
CP	➡	L(R) CP →R(L) CP

Quarter wave plate(QWP)

$$\delta = \frac{2\pi}{\lambda}(n_o - n_e)d$$

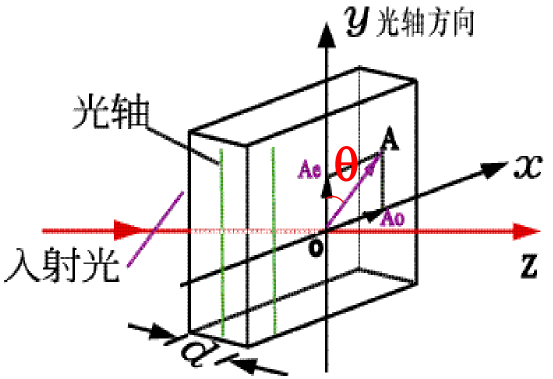
③ When $\delta = (2m+1)\frac{\pi}{2}$

$$\Delta = (2m+1)\frac{\lambda}{4}$$

Quarter wave plate

$$E_x = E_{0x}\cos(\omega t)$$
$$E_y = E_{0y}\cos(\omega t + \delta_0)$$

$$\Rightarrow \quad E_x = E_{0x}\cos(\omega t + \delta)$$
$$E_y = E_{0y}\cos(\omega t + \delta_0)$$

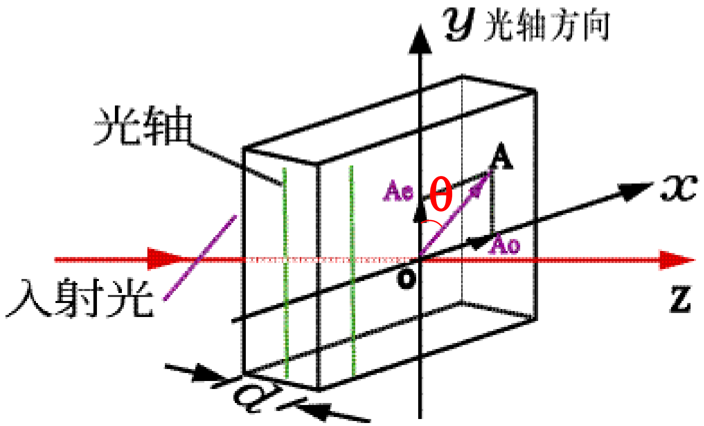


Incident **Emerging**

- LP ➡ ECP $\theta = 0^\circ$ $\theta = 90^\circ$ LP, $\theta = 45^\circ$ CP.
- ECP ➡ LP (Long or short axis along the optical axis) or ECP
- CP ➡ LP

Quarter wave plate(QWP)

$$\delta = \frac{2\pi}{\lambda}\left(n_{o} - n_{e}\right)d$$



Incident light is linearly polarized

④ When $0 < \delta < \pi$， emerging light is RECP

$0 > \delta > \pi$ ， is LECP

The polarization state after **linearly polarized light** passing through the wave plate (normal incident)

d	α	Polarization
Full wave plate	Arbitrarily	Same as the incident light
Arbitrarily	0^0 or 90^0	Same as the incident light
Half wave plate	α	The vibration direction of the emerging and incident light are symmetric to the optical axis. The angle between the two is 2α.
Quarter wave plate	45^0	CP
	0^0 or 90^0	LP
	$\alpha \neq 90^0$ $\alpha \neq 45^0$	Positive elliptical polarization with a ratio of long and short axes to $\tan\alpha$ or $\cot\alpha$

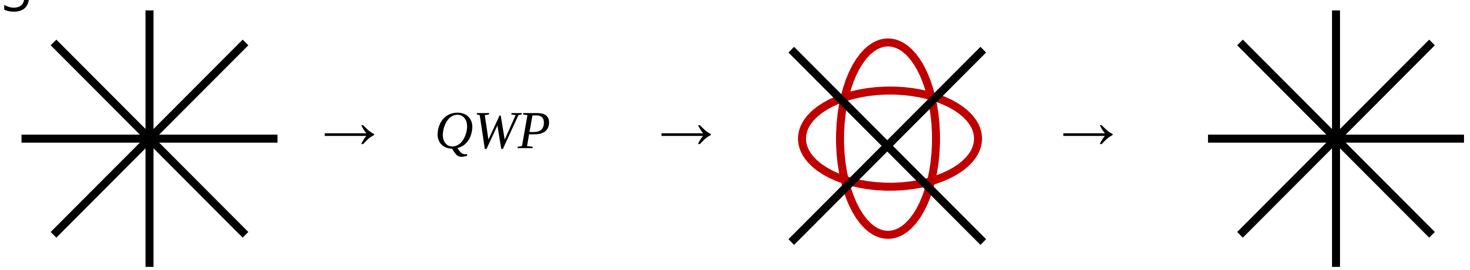
The polarization state after polarized light passing through a $\lambda/4$ plate

Incident light	Position of $\lambda/4$	Emerging light
LP	$\theta = 0^\circ$ or 90°	LP
	$\theta = 45^\circ$	CP
	Other position	ECP
CP	Arbitrarily	LP
ECP	Ellipse length (short) axis parallel to the optical axis (vertical)	LP
	Other position	ECP

Comments:

1. Since $\delta = \frac{2\pi}{\lambda}(n_o - n_e)d$ is related to λ, a wave plate works only for a certain wavelength.

2. Wave plate can't convert unpolarized light into polarized light.



3. When ignoring the absorption of the crystal, the wave wafer only changes the polarization state of the light and does not change its light intensity.

4．椭圆偏振光、圆偏振光的产生；1/2 波片和 1/4 波片的作用

当线偏振光垂直射入一块表面平行于光轴的晶片时，若其振动面与晶片的光轴成α角，该线偏振光将分为 e 光、o 光两部分，它们的传播方向一致，但振动方向平行于光轴的 e 光与振动方向垂直于光轴的 o 光在晶体中传播速度不同，因而产生的光程差为

$$\Delta = d(n_e - n_o)$$

位相差为

$$\delta = \frac{2\pi}{\lambda} d(n_e - n_o) \tag{2}$$

式中 n_e 为 e 光的主折射率，n_0 为 o 光的主折射率（正晶体中，$\delta > 0$，在负晶体中$\delta < 0$）。d 为晶体的厚度，如图 4 所示。当光刚刚穿过晶体时，此两光的振动可分别表示如下：

$$\begin{aligned} E_x &= A_o \cos \omega t \\ E_y &= A_e \cos(\omega t + \delta) \end{aligned} \tag{3}$$

式中 $A_e=A\cos\alpha$，$A_o=A\sin\alpha$， 由（3）中的两式消去 t，得轨迹方程

$$\frac{E_x^2}{A_o^2}+\frac{E_y^2}{A_e^2}-2\frac{E_xE_y}{A_oA_e}\cos\delta=\sin^2\delta \qquad (4)$$

这是个一般的椭圆方程。

当改变厚度 d 时，光程差 Δ 亦改变。

（1）当 $\Delta=k\lambda\ (k=0,1,2,\cdots)$，即 $\delta=0$ 时，由（4）式可得

$$E_y=\frac{A_e}{A_o}E_x \qquad (5)$$

这是直线方程，故出射光为平面偏振光，与原入射光振动方向相同，满足此条件之晶片叫全波片。光通过全波片不发生振动状态的变化。

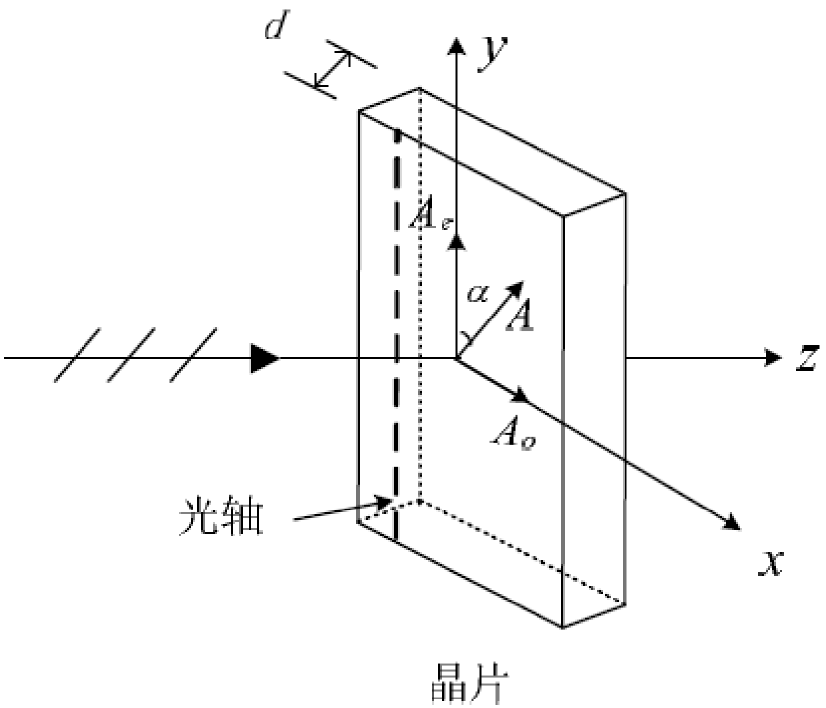


图 4 偏振光通过晶片的情形

（2）当$\Delta=(2k+1)\lambda/2\ (k=0,1,2,\cdots\cdots)$，即$\delta=\pi$时，由（4）式可得

$$E_y=-\frac{A_e}{A_o}E_x \tag{6}$$

出射光也是平面偏振光，但与原入射光夹角为2α，满足此条件的晶片叫1/2波片，或半波片，平面偏振光通过半波片后，振动面转过2α角，若$\alpha=45^\circ$，则出射光的振动面与入射光的振动面垂直。

（3）当$\Delta=(2k+1)\lambda/4\ (k=0,1,2,\cdots\cdots)$，即$\delta=\pm\pi/2$时，由（4）式可得

$$\frac{\boldsymbol{E_x}^2}{\boldsymbol{A_o^2}}+\frac{\boldsymbol{E_y}^2}{\boldsymbol{A_e^2}}=1 \tag{7}$$

出射光为椭圆偏振光，椭圆的两轴分别与晶体的主截面平行及垂直，满足此条件的晶片叫1/4波片。1/4波片是作偏振光实验重要的常用元件。

若$A_e=A_o$，于是$x^2+y^2=A^2$，出射光为圆偏振光。

由于o光和e光的振幅是α的函数，所以通过1/4波片后的合成偏振状态也将随角度α变化而不同。

当$\alpha=0^\circ$时，出射光为振动方向平行1/4波片光轴的平面偏振光。

当$\alpha=\pi/2$时，出射光为振动方向垂直于光轴的平面偏振光。

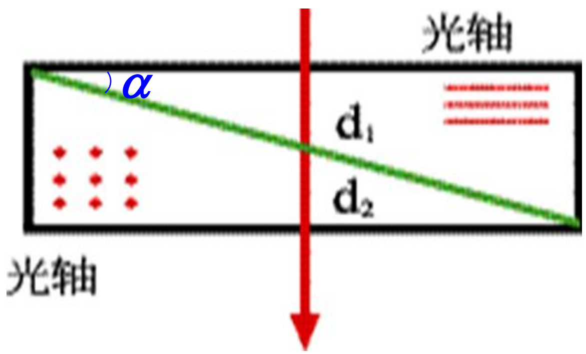
当$\alpha=\pi/4$时，出射光为圆偏振光。

当α为其它值时，出射光为椭圆偏振光。

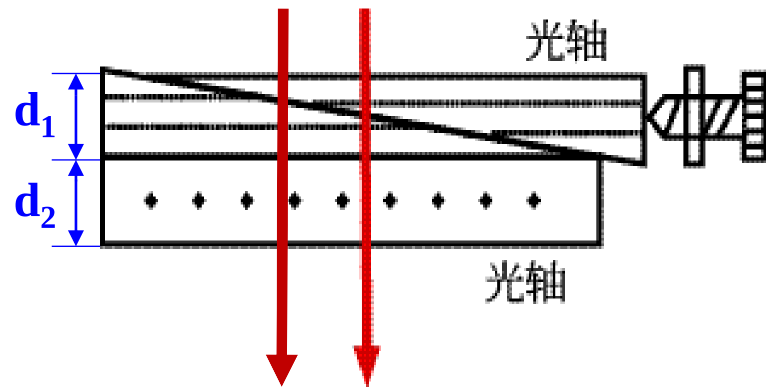
Compensators

Compensator: A waveplate (thickness-adjustable wave plate) with **adjustable phase delay**.

Babinet compensator



Soleil compensator



$$\delta = \frac{2\pi}{\lambda}\left[\left(n_e d_1 + n_o d_2\right) - \left(n_o d_1 + n_e d_2\right)\right] = \frac{2\pi}{\lambda}\left(n_e - n_o\right)\left(d_1 - d_2\right)$$

Working mode: Move the **whole compensator** with the beam

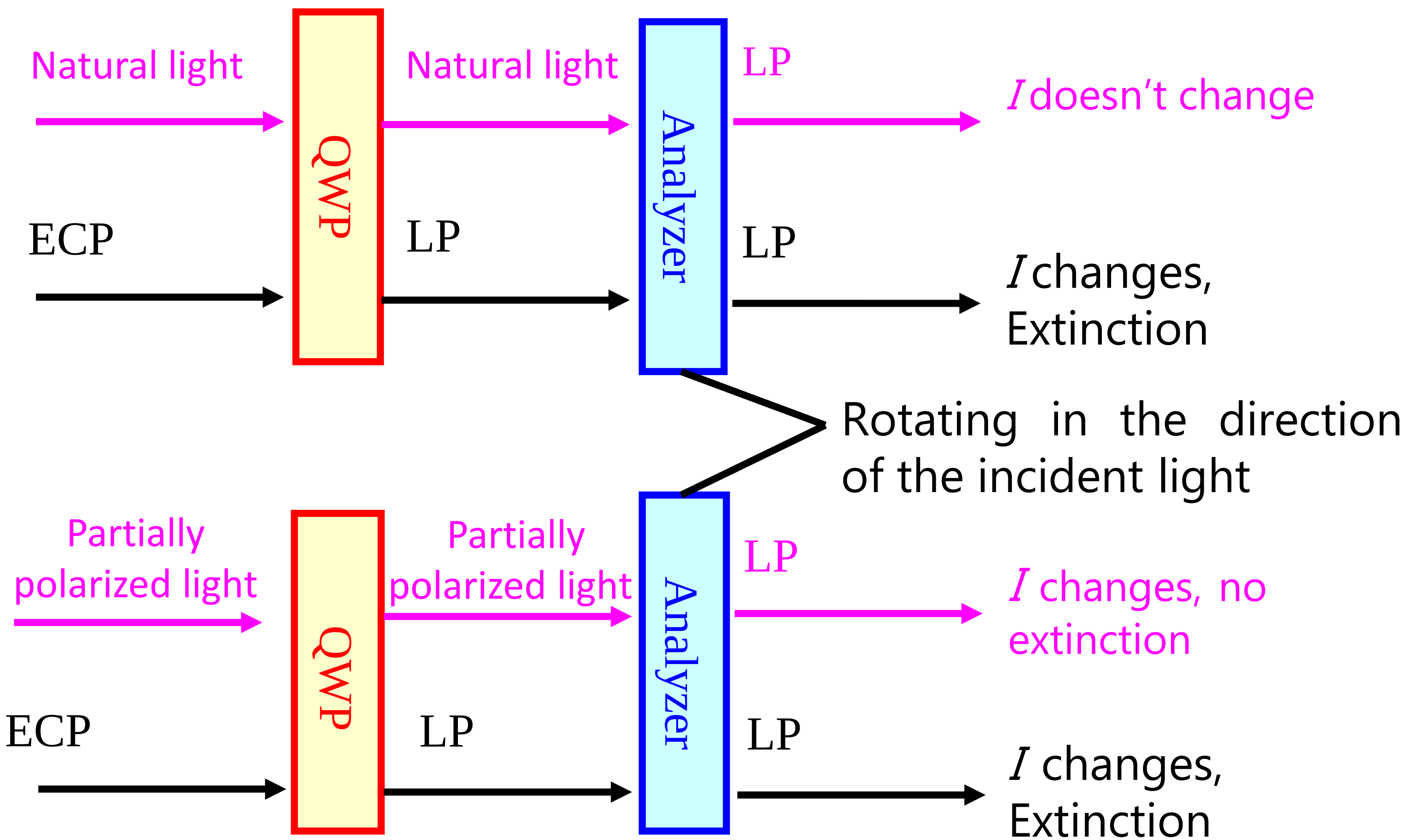
Move the **upper wedge**

§ 10.4 Polarization detection

Question:

- How to distinguish linearly polarized light and circularly polarized light?
- How to distinguish natural light and circularly polarized light?
- How to distinguish partially polarized light and elliptically polarized light?
- How to distinguish left-handed and right-handed polarized light?

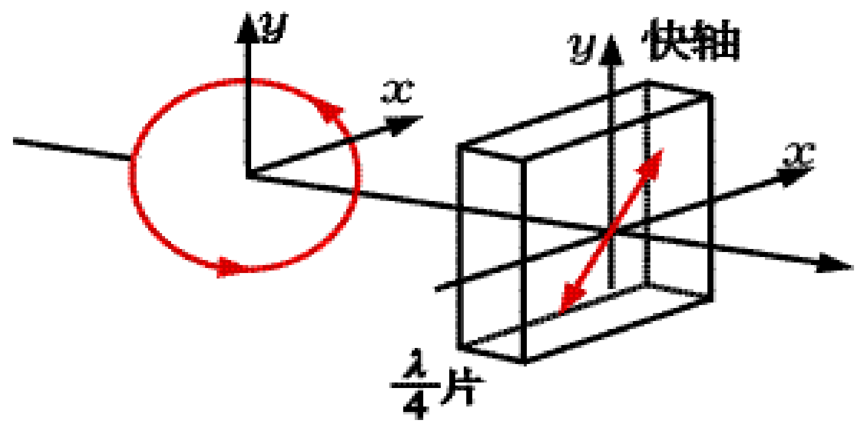
Polarization detection



The optical axis is parallel to the maximum light intensity or the direction of the minimum light intensity

Polarization detection

Distinction between RCP/LCP



Fast axis along the y direction

Incident light is LCP

$$E_x = E_0 \cos(\omega t)$$

$$E_y = E_0 \cos(\omega t - \frac{\pi}{2})$$

Emerging light:

$$E'_x = E_0 \cos(\omega t)$$

$$E'_y = E_0 \cos(\omega t - \frac{\pi}{2} + \frac{\pi}{2}) = E_0 \cos(\omega t)$$ (I、III) Quadrant $y = x$

If incident light is RCP, the emerging light

$$E'_x = E_0 \cos(\omega t)$$

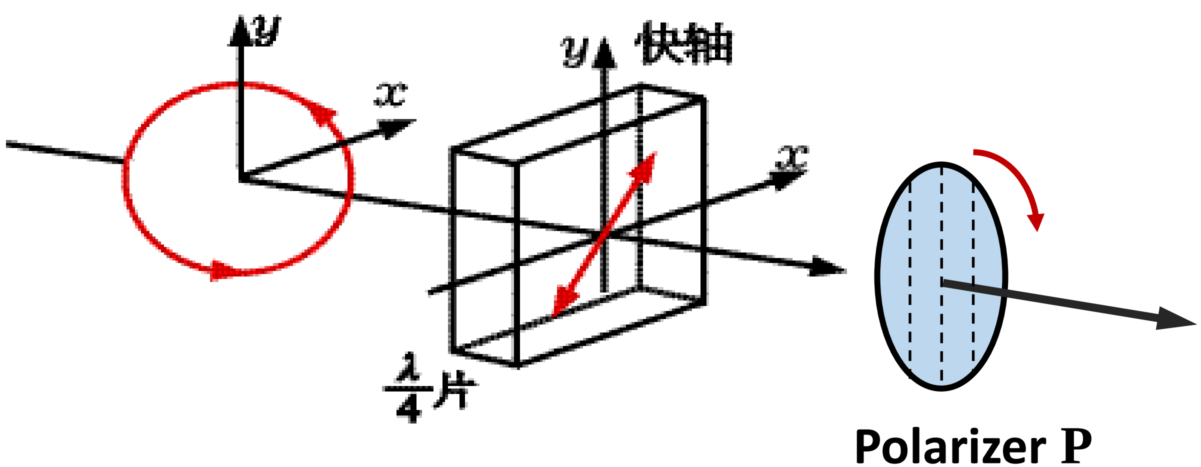
$$E'_y = E_0 \cos(\omega t + \frac{\pi}{2} + \frac{\pi}{2}) = -E_0 \cos(\omega t)$$ (II、IV) Quadrant $y = -x$

Polarization detection

Distinction between RCP/LCP

∵ LCP→*QWP*→LP(Ⅰ、 Ⅲ)

∴ Extinction in the II and IV quadrants after P.



Similarly： RCP→*QWP*→LP(Ⅱ、Ⅳ)，

Extinction in the Ⅰ、Ⅲ quadrants after P.

Homework

Problem 8.57, 8.62, 8.76.

Homework*

^_^

Next week

Blackbody radiation, photons, laser

Sections 4.11, 13.1