热力学与统计物理-第五次作业

吴远清-2018300001031

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Problem 4.1

Answer:

(a)

We can calculate the change in entropy of water directly:

$$\Delta S_1 = mC\int_{273}^{373}\frac{1}{T}dT \approx 1310J/K \tag{1.1}$$

With the thermal first law:

$$\Delta Q_{res} + \Delta Q_{water} = 0 \tag{1.2}$$

Then:

$$\Delta S_2 = -\frac{Q_{water}}{T} = -\frac{mC}{T}(373-273) \approx -1120J/K \tag{1.3}$$

The total change in entropy is:

$$\Delta S = \Delta S_1 \Delta S_2 = 190J/K \tag{1.4}$$

(b)

The change in entropy of water still be the same as (a):

$$\Delta S = \Delta S_1 - \frac{mC\Delta T}{323} - \frac{mC\Delta T_2}{373} = 102J/K \tag{1.5}$$

(c)

Through a quasi-static process.

In detail, the system brought to its final temperature by interaction with a succession of heat reserviors differing infinitesimally in temperature.

Problem 4.3

Answer:

$$dQ = c\,dT + \bar{p}\,dV = c\,dT + \frac{RT}{V}\,dV \tag{2.1}$$

So:

$$\Delta S = \int_i^f \frac{dQ}{T} = \int_{T_i}^{T_f} \frac{c}{T}\,dT + \int_{V_i}^{V_f} \frac{R}{V}\,dv \tag{2.2}$$

Result:

$$\Delta S = c\,ln\frac{T_f}{T_i} + R\,ln\frac{V_f}{V_i} \tag{2.3}$$

This result is independent of process.

Problem 4.4

Answer:

Since the spins of atoms is completely random at high temperature:

$$\lim_{T\to\infty} S(T) = k\,ln\,\Omega = k\,ln\,2^N = Nk\,ln2 \tag{3.1}$$

The change in entropy of this system from 0K to T:

$$\Delta S = \int_0^T \frac{C(T)}{T} dT \tag{3.2}$$

Since $C(T) = 0$ while $T \notin (\frac{1}{2}T_1, T_1)$:

$$\Delta S = \int_{\frac{1}{2}T_1}^{T_1} C_1(2\frac{1}{T_1} - \frac{1}{T})dT = C_1(1 - ln2) \tag{3.3}$$

With the third law, we set $S(0) = 0$,Then, from (3.1) and (3.3):

$$Nk\,ln2 = C_1(1 - ln2) \tag{3.4}$$

Then:

$$C_1 = \frac{Nk\,ln2}{1 - ln2} \tag{3.5}$$

Problem 4.5

Answer:

Let's u denote the undiluted system, and d denote the diluted syste.

Similarly with Problem 4.4:

$$S_u(T_1) - S_u(0) = \int_{\frac{1}{2}T_1}^{T_1} C_1(2\frac{1}{T_1} - \frac{1}{T_1})dT = C_1(1 - ln2) \tag{4.1}$$

$$S_d(T_1) - S_d(0) = \int_{\frac{1}{2}T_1}^{T_2} C_2 \frac{1}{T_2} dT = C_2 \tag{4.2}$$

By the third law, let's $S_u(0) = S_d(0) = 0$.
Obviously:

$$S_d(T_1) = \frac{7}{10} S_u(T_1) \tag{4.3}$$

So:

$$\frac{C_2}{C_1} = \frac{7}{10}(1 - ln2) \tag{4.4}$$