热力学与统计物理-第十次作业

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Problem 7.14

Answer:

The energy of the megnetic moment in the field H is:

$$E = -\mu \cdot H = -\mu H \cos\theta \tag{1.1}$$

where θ is the angle betwen the magnetic moment and the direction of the field or z-axis. Then the probability that the magnetic moment lies in the range θ to $\theta + d\theta$ is proportional to the Boltzmann factor and the solid angle $2\pi \sin\theta d\theta$

Thus:

$$P(\theta)d\theta \propto e^{\beta\mu H\cos\theta}\sin\theta d\theta \tag{1.2}$$

And:

$$\mathrm{\overline{M}_Z = N_O \bar{\mu}_Z} = \frac{\mathrm{N_O}\int_0^\pi \mathrm{e}^{\beta\mu\mathrm{E}}\cos\theta sin\theta d\theta(\mu cos\theta)}{\int_0^\pi \mathrm{e}^{\beta\mu\mathrm{H}\cos\theta}\sin\theta\mathrm{d}\theta} \tag{1.3}$$

$$\begin{aligned}\overline{M}_Z &= \frac{N_0}{H}\frac{\partial}{\partial\beta}\ln\int_0^\pi e^{\beta\mu H\cos\theta}\sin\theta d\theta = \frac{N_0}{H}\frac{\partial}{\partial\beta}\ln\frac{e^{\beta\mu H}-e^{-\beta\mu H}}{\beta\mu H}\\ &= \frac{N_0}{H}\frac{\partial}{\partial\beta}\ln\frac{2\sinh\beta\mu H}{\beta\mu H} = \frac{N_0}{H\beta\sinh\beta\mu H}\left[\mu H\beta\cosh\beta\mu H - \sinh\beta\mu H\right]\end{aligned} \tag{1.4}$$

So:

$$\overline{M}_Z = N_0\mu\left[\coth\beta\mu H - \frac{1}{\beta\mu H}\right] \tag{1.5}$$

Problem 7.15

Answer:

We have:

$$\overline{M}_Z = N_0 g\mu_0 J B_J(\eta) \tag{2.1}$$

Then:

$$B_J(\eta) = \frac{1}{J}\left[\left(J+\frac{1}{2}\right)\coth\left(J+\frac{1}{2}\right)\eta - \frac{1}{2}\coth\frac{1}{2}\eta\right] \tag{2.2}$$

If $\eta \ll 1$ and $J \gg 1$ in such a way that $J\eta \gg 1, B_J(\eta)$ becomes:

$$B_J(\eta) = \frac{1}{J}\quad\left[J\coth J\eta - \frac{1}{2}\left(\frac{2}{\eta}\right)\right] = \coth J\eta - \frac{1}{J\eta} \tag{2.3}$$

Let $\mu \mathrm{H}\beta = J\eta = Jg\mu_0\mathrm{H}\beta$ where $\mu = \mathrm{g}\mu_{\mathrm{O}}\mathrm{J}$ is by (7.8.2) the classical magnetic moment. , then (2.1) becomes:

$$\overline{M}_Z = N_0\mu\left[\coth\beta\mu H - \frac{1}{\beta\mu H}\right] \tag{2.4}$$

Problem 7.17

Answer:

To find the fraction, ξ, of molecules with x component of velocity between - $\tilde{v}$ and $\tilde{v}$, we must integrate the distribution between these limits, i.e.:

$$\xi = \frac{1}{n}\int_{-\tilde{v}}^{\tilde{v}} g\left(v_x\right)dv_x = \int_{-\sqrt{\frac{2kT}{m}}}^{\sqrt{\frac{2kT}{m}}}\left(\frac{m}{2\pi kT}\right)^{\frac{1}{2}} e^{-\left(mv_x^2/2kT\right)}dv_x \tag{3.1}$$

Making the change of variable, $y = \sqrt{\frac{m}{kT}}v_x$, we have:

$$\xi = \frac{1}{\sqrt{2\pi}}\int_{-\sqrt{2}}^{\sqrt{2}} \mathrm{e}^{-\left(y^2/2\right)}\mathrm{dy} = \frac{2}{\sqrt{2\pi}}\int_0^2 \mathrm{e}^{-\left(\mathrm{y}^2/2\right)} = 2\ \mathrm{erf}\ \sqrt{2} \tag{3.2}$$

Problem 7.18

Answer:

In problem 5.9 we found that the velocity of sound is:

$$u = \left(\frac{\gamma RT}{\mu}\right)^{\frac{1}{2}} \tag{4.1}$$

where $\gamma = \mathrm{C_p}/\mathrm{C_V}$ and μ is the atomic weight. Since $\mu = \mathrm{N_A m}$, we have:

$$u = \left(\frac{\gamma RT}{N_A m}\right)^{\frac{1}{2}} = \left(\frac{\gamma KT}{m}\right)^{\frac{1}{2}} \tag{4.2}$$

The most probable speed is $\tilde{v} = (2kT/m)^{\frac{1}{2}}$. Thus:

$$u = \left(\frac{\gamma}{2}\right)^{\frac{1}{2}}\tilde{v} \tag{4.3}$$

For helium, $\gamma = 1.66$ so that $u = 0.91\tilde{v}$, and the fraction of molecules with speeds less than u is:

$$\xi = \frac{1}{n}\int_0^{0.91\tilde{v}} F(v)dv = 4\pi\int_0^{0.91(2kT/m)^{\frac{1}{2}}} \left(\frac{m}{2\pi k\pi}\right)^{\frac{3}{2}} v^2 e^{-\frac{mv^2}{2kT}} dv \qquad (4.4)$$

Making the change of variable $y = (m/kT)^{\frac{1}{2}}v$, we have:

$$4\xi = \frac{4\pi}{(2\pi)^{\frac{3}{2}}}\int_0^{0.91\sqrt{2}} \mathrm{y}^2\mathrm{e}^{-\mathrm{y}^2/2}\mathrm{dy} \qquad (4.5)$$

This integral may be evaluated by noticing that integration by parts of $\int_0^{\mathrm{a}} \mathrm{e}^{-\mathrm{y}^2/2}\ \mathrm{dy}$ yields:

$$\int_0^a e^{-y^2/2}dy = ye^{-y^2/2}\Big|_0^a + \int_0^a y^2e^{-y^2/2}dy \qquad (5.6)$$

Then we find:

$$\xi = 4\pi^{-\frac{1}{2}}(2)^{-\frac{3}{2}}\int_0^{0.91\sqrt{2}} y^2\mathrm{e}^{-y^2/2}dy = 4\pi^{-\frac{1}{2}}(2)^{-\frac{3}{2}}\int_0^{0.91\sqrt{2}} \mathrm{e}^{-y^2/2}dy - 0.91\sqrt{2}\exp\left[-(0.912)^2/2\right] \approx 0.37 \qquad (5.7)$$

Problem 7.19

Answer:

$$(a)\bar{v}_x = 0$$
$$(b)\overline{v_x^2} = \frac{\mathrm{kT}}{\mathrm{m}}$$
$$(c)\overline{(v^2v_x)} = \left(\overline{v_x^3} + \overline{v_y^2}\bar{v}_x + \overline{v_z^2}\bar{v}_x\right) = 0$$
$$(d)\overline{(v_x^3v_y)} = \overline{v_x^3}\bar{v}_y = 0$$
$$(e)\overline{(v_x + bv_y)^2} = \overline{v_x^2} + 2b\bar{v}_x\bar{v}_y + b^2\overline{v_y^2} = (1+b)^2\frac{kT}{m}$$
$$(f)\overline{v_x^2v_y^2} = \left(\frac{kT}{m}\right)^2$$

Problem 7.21

Answer:

The most probable energy is given by the condition $\frac{dF(\epsilon)}{d\epsilon} = 0$:

$$\frac{1}{2}\epsilon^{-\frac{1}{2}}e^{-\frac{\epsilon}{kT}} - \frac{\epsilon^{\frac{1}{2}}}{\mathrm{kT}}e^{-\frac{\epsilon}{\mathrm{kT}}} = 0 \qquad (7.1)$$

Then:

$$\tilde{\epsilon} = \frac{1}{2}\text{kT} \tag{7.2}$$

The most probable speed is:

$$\tilde{v} = (2kT/m)^{\frac{1}{2}} \tag{7.3}$$

So:

$$\frac{1}{2}mv^2 = \text{kT} \tag{7.4}$$

Problem 7.23

Answer:

(a)

The number of molecules which leave the source slit per second is:

$$\Phi_0 A = \frac{1}{4}n\bar{v}A = \frac{\bar{p}_s A}{\sqrt{2\pi mkT}} = 1.1 \times 10^{18} \text{ molecules/sec} \tag{8.1}$$

where A is the area of slit.

(b)

Approximating the slit as a point source, we have by (7.11.7), the number of molecules with speed in the range between v and v + dv which emerge into solid angle $d\Omega$ is:

$$A\Phi(v)d^3v = A[f(v)v\cos\theta]\left[v^2 dv d\Omega\right] \tag{8.2}$$

$\cos\theta \approx 1$ for molecules arriving at the detector slit; hence the total number which reach the detector is

$$A\int_{\Omega_d} d\Omega \int_0^\infty n\left(\frac{m}{2\pi kT}\right)^{\frac{3}{2}} v^3 e^{-\frac{\beta mv^2}{2}} dv = \frac{A\Omega_a \bar{p}_s}{\pi\sqrt{2\pi mkT}} \tag{8.2}$$

From (8.2), we have:

$$\frac{A^2\overline{p}_S}{\pi L^2\sqrt{2\pi mkT}} = \phi_d A = \frac{A\overline{p}_d}{\sqrt{2\pi mkT}} \tag{8.3}$$

Or:

$$\bar{p}_d = \bar{p}_S\frac{A}{\pi L^2} = 2.4 \times 10^{-8}\text{mm of Hg.} \tag{8.4}$$

Problem 7.27

Answer:

The rate of change of the mumber of perticles inside the container is

$$\frac{d\mathrm{N}}{\mathrm{d}t} = -\frac{1}{4}\mathrm{n}\overline{\mathrm{v}}\mathrm{A} = -\frac{\mathrm{NA}}{4\mathrm{V}}\sqrt{\frac{8}{\mathrm{T}}\frac{\mathrm{kT}}{\mathrm{m}}} = -\frac{\lambda \mathrm{N}}{\sqrt{\mathrm{m}}} \tag{9.1}$$

thus defining λ. since pressure is proportional to the number of particles, we find after integrating:

$$\mathrm{p}/\mathrm{p}_0 = \exp\left[-\frac{\lambda t}{\sqrt{\mathrm{m}}}\right] \tag{9.2}$$

For Helium gas $\mathrm{p}/\mathrm{p}_\mathrm{o} = 1/2$ at $\mathrm{t} = 1$ hour. Substituting we have:

$$\lambda = \sqrt{\frac{m}{\mathrm{He}}}\ln 2 \tag{9.3}$$

So:

$$\mathrm{n}_{\mathrm{Ne}}/\mathrm{n}_{\mathrm{He}} = 2^{(1-\sqrt{\mathrm{m}_{\mathrm{He}}/\mathrm{m}_{\mathrm{Ne}}})} = 2^{(1-\sqrt{\mu_{\mathrm{He}}/\mu_{\mathrm{Ne}}})} \tag{9.4}$$

Problem 7.29

Answer:

(a)

Inside the container $\bar{v}_z = 0$ by symmetry.

(b)

The velocity distribution, $\phi(\overline{v})$, of the molecules which bave effused into the vacuum is:

$$\Phi(\overline{v})\mathrm{d}^3\overline{v} = f(\tilde{v})\mathrm{v}_\mathrm{z}\mathrm{d}^3\overline{\mathrm{v}} \tag{10.1}$$

where $f(\tilde{v})$ is Maxwell distribution.

So:

$$\bar{v}_z = \frac{\frac{\sqrt{\pi}}{4}\left(\frac{2k\pi}{m}\right)^{\frac{3}{2}}}{\frac{1}{2}\left(\frac{2k\pi}{m}\right)} = \sqrt{\frac{\pi}{2}\frac{kT}{m}} \tag{10.2}$$