热力学与统计物理-第七次作业

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Problem 5.11

Answer:

Let's the solid expands an increment of dV, we have:

$$V + dV = (x + dx)(y + dy)(z + dz) \tag{1.1}$$

Ignore the orders that higher than 1:

$$V + dV = V + yzdx + xzdy + xydz \tag{1.2}$$

So:

$$\alpha = \frac{1}{V}\frac{dV}{dT} = \frac{1}{x}\frac{dx}{dT} + \frac{1}{y}\frac{dy}{dT} + \frac{1}{z}\frac{dz}{dT} = 3\alpha_L \tag{1.3}$$

Problem 5.13

Answer:

From the first law:

$$C_p = T(\frac{\partial s}{\partial T})_p \tag{2.1}$$

$$(\frac{\partial C_p}{\partial p})_T = T(\frac{\partial}{\partial p})_T(\frac{\partial s}{\partial T})_p = T(\frac{\partial}{\partial T})_p(\frac{\partial s}{\partial p})_T \tag{2.2}$$

From the Maxwell relation and the definition of α:

$$(\frac{\partial C_p}{\partial p})_T = \alpha^2 vT - vT\frac{d\alpha}{dT} \tag{2.3}$$

Problem 5.14

Answer:

(a):

$$TdS = dE - FdL \tag{3.1}$$

(b):

From (3.1) we may read off the Maxwell relation:

$$\left(\frac{\partial S}{\partial L}\right)_T = -\left(\frac{\partial R}{\partial T}\right)_L \tag{3.2}$$

Since:

$$F = aT^2(L - L_o) \tag{3.3}$$

So:

$$\left(\frac{\partial S}{\partial L}\right)_T = -2aT\left(L - L_O\right) \tag{3.4}$$

(c):

$$s\left(L_0, T\right) - S\left(L_0, T_0\right) = \int_{T_0}^{T} \frac{C_L}{T'} dT' = \int_{T_0}^{T} \frac{bT'}{T'} dT' = b\left(T - T_0\right) \tag{3.5}$$

$$S(L,T) - S\left(L_0, T\right) = \int_{I_0}^{L} \left(\frac{\partial S}{\partial L}\right)_T dL = \int_{L_0}^{L} -2aT\left(L' - L_0\right) dL' = -aT\left(L - L_0\right)^2 \tag{3.6}$$

So:

$$S(L,T) = S\left(L_0, T_0\right) + b\left(T - T_0\right) - aT\left(L - L_0\right)^2 \tag{3.7}$$

(d):

In this process which $\Delta S = 0$

$$S\left(T_0, L_0\right) + b\left(T_f - T_0\right) - aT_i\left(L_f - L_0\right)^2 = S\left(T_0, L_0\right) + b\left(T_i - I_0\right) - aT_f\left(L_f - L_0\right)^2 \tag{3.8}$$

So:

$$T_f = T_i \frac{b - a\left(L_1 - L_0\right)^2}{b - a\left(L_f - L_0\right)^2} \tag{3.9}$$

(e):

From the first law:

$$C_L = T(\frac{\partial S}{\partial T})_l \tag{3.10}$$

Then:

$$\left(\frac{\partial C_L}{\partial L}\right)_T = T\left(\frac{\partial}{\partial L}\right)_L \left(\frac{\partial S}{\partial T}\right)_L = T\left(\frac{\partial}{\partial T}\right)_L \left(\frac{\partial S}{\partial L}\right)_T \tag{3.11}$$

From (3.2) and (3.3) we can get:

$$\left(\frac{\partial C_I}{\partial L}\right)_T = -2aT\left(L - L_O\right) \tag{3.11}$$

So:

$$C_L(L,T) = C(L_0,T) + \int_{L_0}^{L} (\frac{\partial C_L}{\partial L})_T dL = bT - aT(L - L_0) \tag{3.12}$$

(f):

$$S\left(L, T_0\right) - S\left(L_0, T_0\right) = \int_{L_0}^{L} \left(\frac{\partial S}{\partial L}\right)_T dL' = \int_{L_0}^{L} -2aT_0\left(L' - L_0\right) dL' = -aT_0\left(L - L_0\right)^2 \tag{3.13}$$

$$\begin{aligned} S(L,T) - S(L,T_0) &= \int_{T_0}^{T} \frac{C_L dT'}{T'} = \int_{T_0}^{T} \frac{bT' - aT'(L-L_0)^2}{T'} dT' \\ &= b(T - T_0) - a(L - L_0)^2 (T - T_0) \end{aligned} \tag{3.14}$$

So:

$$S(L,T) = S\left(T_0, L_0\right) + b\left(T - T_0\right) - aT\left(L - L_0\right)^2 \tag{3.15}$$

Problem 5.15

Answer:

(a):

$$dQ = TdS = dE - 2\sigma\ell dx \tag{4.1}$$

(b):

From (4.1):

$$\mathrm{dS} = \frac{\mathrm{dE}}{\mathrm{T}} - \frac{2\sigma\ell}{\mathrm{T}}\mathrm{dx} \tag{4.2}$$

So:

$$\left(\frac{\partial S}{\partial x}\right)_T dx + \left(\frac{\partial S}{\partial T}\right)_X dT = \frac{1}{T}\left(\frac{\partial E}{\partial x}\right)_T dx + \frac{1}{T}\left(\frac{\partial E}{\partial T}\right)_x dT - \frac{2\sigma\ell}{T} dx \tag{4.3}$$

Then:

$$(\frac{\partial E}{\partial x})_T = T(\frac{\partial S}{\partial x})_T + 2\sigma\ell \tag{4.4}$$

From (4.1):

$$(\frac{\partial S}{\partial x})_T = (\frac{\partial(-2\sigma\ell)}{\partial T})_x = -2\ell\frac{d\sigma}{dT} \tag{4.5}$$

Since $\sigma = \sigma T$

$$\left(\frac{\partial E}{\partial x}\right)_T = 2l\alpha T + 2\ell\sigma_0 - 2\ell\alpha T = 2l\sigma_0 \tag{4.6}$$

If the film is stretched at constant temperature:

$$E(x) - E(0) = 2\ell\sigma_0 x \tag{4.7}$$

(c):

$$\mathrm{W(O \to x)} = -\int \mathrm{Fdx} = -\int_0^{\mathrm{x}} 2d\ell \mathrm{dx}' = -2\sigma\ell x \tag{4.8}$$

Problem 5.17

Answer:

By equation (5.8.12)

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p \tag{5.1}$$

Substituting $p = nkT[1 + B_2(T)n]$we find.

$$\left(\frac{\partial E}{\partial V}\right)_T = p + n^2 KT\frac{dB_2}{dT} - p = n^2 KT\frac{dB_2}{dT} > 0 \tag{5.2}$$

So, it's positive.

Problem 5.18

Answer:

(a).

$$dE = \left(\frac{\partial E}{\partial V}\right)_T dV + \left(\frac{\partial E}{\partial T}\right)_V dT \tag{6.1}$$

Since:

$$\left(\frac{\partial E}{\partial T}\right)_V = C_V \tag{6.2}$$

$$\left(\frac{\partial E}{\partial V}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_V - p \tag{6.3}$$

So:

$$\left(\frac{\partial T}{\partial V}\right)_\Sigma = -\frac{T\left(\frac{\partial p}{\partial T}\right) - P}{C_V} \tag{6.4}$$

(b).

From the first law, $dE = Tds - PdV$, then:

$$O = T\left(\frac{\partial S}{\partial V}\right)_B - P \tag{6.5}$$

$$\left(\frac{\partial S}{\partial V}\right)_E = \frac{P}{T} \tag{6.6}$$

(c).

For Van der Waals gas:

$$p = \frac{\nu RT}{V - \nu b} - \frac{\nu^2 a}{V^2} \tag{6.7}$$

Then:

$$\left(\frac{\partial T}{\partial V}\right)_E = \frac{\nu^2 a}{C_V V^2} \tag{6.8}$$

$$T_2 - T_1 = \int_{V_1}^{V_2} \left(\frac{\partial T}{\partial V}\right)_E dV = \frac{a\nu^2}{C_V} \int_{V_1}^{V_2} \frac{dV}{V^2} = -\frac{a\nu^2}{c_V} \left(\frac{1}{V_2} - \frac{1}{V_1}\right) \tag{6.9}$$

Problem 5.20

Answer:

To fint the inversion curve, we must have:

$$\left(\frac{\partial T}{\partial p}\right)_H = \frac{V}{C_p} \left(\frac{T}{V} \left(\frac{\partial V}{\partial T}\right)_p - 1\right) = 0 \tag{7.1}$$

or:

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{V}{T} \tag{7.2}$$

From the Van der Waals equation:

$$\mathrm{dp} = \frac{\mathrm{RdT}}{\mathrm{v - b}} + \left(-\frac{\mathrm{RT}}{(\mathrm{v - b})^2} - \frac{2\mathrm{a}}{\mathrm{v}^3}\right) \mathrm{dv} \tag{7.3}$$

$$\left(\frac{\partial V}{\partial T}\right)_p = \frac{V - b}{T - \frac{2a}{RV}\left(\frac{V-b}{V}\right)^2} = \frac{V}{T} \tag{7.4}$$

Then:

$$\frac{2a}{\mathrm{RT}} \left(\frac{V - b}{V}\right)^2 = b \tag{7.4}$$

On eliminating V and putting the equation in terms of the dimensionless variables of problem 5.19, it follows that

$$p' = 9 - 12(\sqrt{T'} - \sqrt{3})^2$$

Problem 5.23

Answer:

(a)

$$W = q_1 - q_2 = c\,(T_1 - T_f) + c\,(T_2 - T_f) = c\,(T_1 + T_2 - 2T_f) \tag{8.1}$$

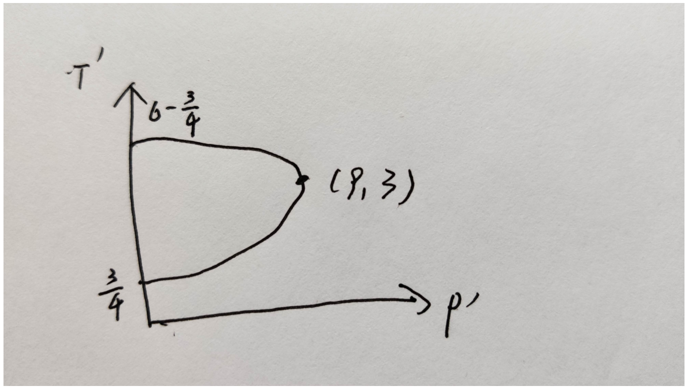

图 1: 5.20 Figure 1

(b)

By the second law:

$$\Delta S \geq 0 \tag{8.2}$$

Then:

$$\int_{T_1}^{T_f} \frac{CdT}{T} + \int_{T_2}^{T_f} \frac{CdT}{T} = C\ln\frac{T_f^2}{T_1T_2} \geq 0 \tag{8.3}$$

So:

$$T_I = \sqrt{T_1T_2} \tag{8.4}$$

(c)

The maximum amount of work vill be obtained when $T_f = \sqrt{T_1T_2}$ From (8.1):

$$W = c\left(T_1 + T_2 - 2T_f\right) = C\left(\sqrt{T_1} - \sqrt{T_2}\right)^2 \tag{8.5}$$

Problem 5.24

Answer:

To freeze an additional mass m of water at T_0, heat mL must be removed from the ice-water mixture resulting in an entrogy change $\Delta S_1 = -\frac{mL}{\mathrm{T}_0}$.The heat rejected to the body of heat capacity C increases its temperature to T_f with an entropy change $\Delta S_2 = C\int_{T_0}^{T_f}\frac{dT}{T} = C\,ln\frac{T_f}{T_0}$. By the second law $\Delta S_1 + \Delta S_2 = -\frac{m_L}{T_0} + C\ln\frac{T_f}{T_0} \geq 0$. For minimun temperature increase and thus ninimm heat rejection the equality holds and it follows that:

$$T_f = T_0 e^{\frac{mL}{T_0C}} \tag{9.1}$$

The heat rejected is $Q = C\left(T_f - T_0\right) = CT_0\left(e^{mL/T_0C} - 1\right)$

Problem 5.26

Answer:

In the processes $a \to b$ and $c \to d$ no heat is absorbed, so by the first law $W = -\Delta E$, and since $\Delta E = C\Delta T$, where C is the heat capacity, we have:

$$W_{a\to b} = -\left(E_b - E_a\right) = -\nu c_V\left(T_b - T_a\right) \tag{10.1}$$

$$W_{c\to d} = -\left(E_d - E_c\right) = -\nu C_V\left(T_d - T_c\right) \tag{10.2}$$

Fovever, in an adiabatic expension $\mathrm{TV}^{\gamma-1} = \mathrm{const}$.

Then:

$$W_{a\to b} = -\upsilon c_V T_b\left(l\frac{T_a}{m_b}\right) = -\upsilon C_V T_b\left(1 - \left(\frac{\upsilon_2}{\upsilon_1}\right)^{\gamma-1}\right) \tag{10.3}$$

$$W_{c\to d} = -\upsilon c_V T_d\left(l\frac{T_d}{m_c}\right) = -\upsilon C_V T_c\left(1 - \left(\frac{\upsilon_2}{\upsilon_1}\right)^{\gamma-1}\right) \tag{10.4}$$

The volume is constant in process $b \to c$ so no work is performed.

Then:

$$Q_1 = \left(E_c - E_b\right) = \nu C_V\left(T_c - T_b\right) \tag{10.5}$$

$$\eta = \frac{W_{a\to b} + W_{c\to d}}{Q_1} = l - \left(\frac{V_2}{V_1}\right)^{\gamma-1} \tag{10.6}$$