# Chapter 5: Part B Simple applications of macroscopic thermodynamics

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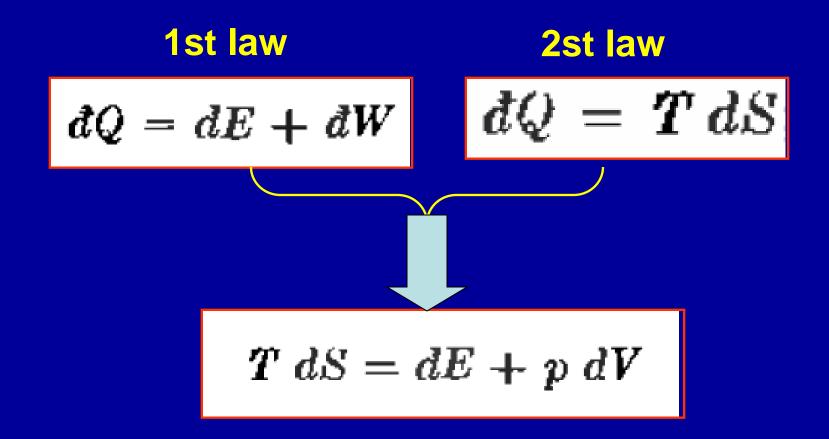
2019 spring semester

- Important relationships between macroscopic quantities
- ➤ Based on the fundamental thermodynamic laws (0th, 1st, 2nd, 3rd)
- Quantities such as p, E always are mean values

#### **Independent variables:**

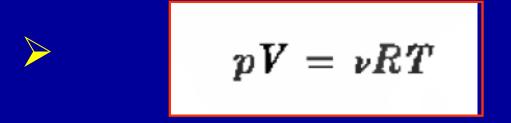
- Most of systems only have the external para V;
- two independent variables can specify the system, such as E and V; or E and P; or T and V... Other variables can be determined by the two independent variables;

#### **Fundamental equations:**



#### **Properties of ideal gas**

#### 5.1 Equation of state and internal energy



$$\Omega \propto V^{N} \chi(E)$$

$$\beta = \frac{\partial \ln \chi(E)}{\partial E}$$

$$\beta = \beta(E)$$



# **5.6 Summary of Maxwell relations and thermodynamic functions**

**Maxwell relations** 

$$dE = T dS - p dY$$

$$\begin{pmatrix} \frac{\partial T}{\partial V} \end{pmatrix}_{S} = -\left(\frac{\partial p}{\partial S}\right)_{V} \\
\begin{pmatrix} \frac{\partial T}{\partial p} \end{pmatrix}_{S} = \left(\frac{\partial V}{\partial S}\right)_{p} \\
\begin{pmatrix} \frac{\partial S}{\partial V} \end{pmatrix}_{T} = \left(\frac{\partial p}{\partial T}\right)_{V} \\
\begin{pmatrix} \frac{\partial S}{\partial p} \end{pmatrix}_{T} = -\left(\frac{\partial V}{\partial T}\right)_{p}
\end{pmatrix}$$

T, p, V, S are not completely in-dependent

One can use Ω to give a complete macroscopic description on system

#### 5.6 Summary of Maxwell relations and thermodynamic functions

Maxwell relations

??? One can use  $\Omega$  to give a complete macroscopic description on system

$$\frac{1}{T} = \left(\frac{\partial S}{\partial E}\right)_{V} \quad \text{and} \quad p = T\left(\frac{\partial S}{\partial V}\right)_{E}$$

$$dS = \left(\frac{\partial S}{\partial E}\right)_{V} dE + \left(\frac{\partial S}{\partial V}\right)_{B} dV$$

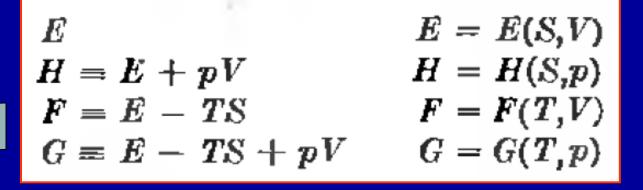
$$= \frac{1}{T} dE + \frac{p}{T} dV \quad \text{by (5.6.6)}$$

$$(T,S)$$
 and  $(p,V)$ 

**Number of states** 

generalized force

# General relations for a homogeneous substance thermodynamic functions



$$dE = T dS - p dV$$

$$dH = T dS + V dp$$

$$dF = -S dT - p dV$$

$$dG = -S dT + V dp$$

#### 5.7 Specific heats

at constant volume

$$C_{V} = \left(\frac{dQ}{dT}\right)_{V} = T\left(\frac{\partial S}{\partial T}\right)_{V}$$

at constant pressure

$$C_{p} = \left(\frac{dQ}{dT}\right)_{p} = T\left(\frac{\partial S}{\partial T}\right)_{p}$$



$$dQ = T dS = T \left[ \left( \frac{\partial S}{\partial T} \right)_{p} dT + \left( \frac{\partial S}{\partial p} \right)_{T} dp \right]$$

$$dQ = T dS = C_p dT + T \left(\frac{\partial S}{\partial p}\right)_T dp$$

#### 5.7 Specific heats

dp

$$dQ = T dS = C_{p} dT + T \left( \frac{\partial S}{\partial p} \right)_{T} \left[ \left( \frac{\partial p}{\partial T} \right)_{V} dT + \left( \frac{\partial p}{\partial V} \right)_{T} dV \right]$$

when V is constant

$$dQ = C_V dT$$

$$C_{V} = T \left( \frac{\partial S}{\partial T} \right)_{V} = C_{P} + T \left( \frac{\partial S}{\partial P} \right)_{T} \left( \frac{\partial P}{\partial T} \right)_{V}$$

$$\left(\frac{\partial S}{\partial p}\right)_T = -\left(\frac{\partial V}{\partial T}\right)_p$$

The quantities in the right of equation is easily to be measured

#### 5.7 Specific heats

$$\alpha \equiv \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p}$$

 $\alpha \equiv \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_n$  volume coefficient of expansion

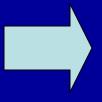
$$\left(\frac{\partial S}{\partial p}\right)_{T} = -V_{\alpha}$$

$$dV = \left(\frac{\partial V}{\partial T}\right)_p dT + \left(\frac{\partial V}{\partial p}\right)_T dp$$

$$dV = \left(\frac{\partial V}{\partial T}\right)_{p} dT + \left(\frac{\partial V}{\partial p}\right)_{T} dp \qquad \left(\frac{\partial P}{\partial T}\right)_{V} = -\frac{\left(\frac{\partial V}{\partial T}\right)_{p}}{\left(\frac{\partial V}{\partial p}\right)_{T}}$$

5.7 Specific heats 
$$C_V = T \left( \frac{\partial S}{\partial T} \right)_V = C_P + T \left( \frac{\partial S}{\partial P} \right)_T \left( \frac{\partial P}{\partial T} \right)_V$$

$$\kappa \equiv -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$
 isothermal compressibility



$$\left(\frac{\partial p}{\partial T}\right)_{V} = \frac{\alpha}{\kappa}$$

$$C_{V} = C_{p} + T(-V\alpha) \left(\frac{\alpha}{\kappa}\right)$$

$$C_{p} - C_{V} = VT^{\frac{\alpha^{2}}{\kappa}}$$

#### 5.7 Specific heats: example---ideal gas

#### For ideal gas:

$$lpha \equiv rac{1}{V} igg( rac{\partial V}{\partial T} igg)_{\!P}$$

$$\frac{p \, dV}{\left(\frac{\partial V}{\partial T}\right)_p} = \frac{\nu R}{p}$$

$$\alpha = \frac{1}{V} \left(\frac{\nu R}{p}\right) = \frac{\nu R}{\nu RT} = \frac{1}{T}$$

$$\kappa \equiv -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

$$p \, dV + V \, dp = 0$$

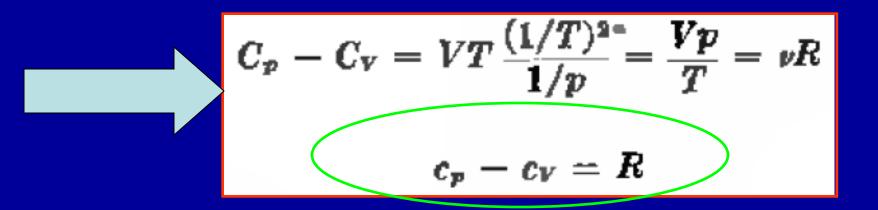
$$\left(\frac{\partial V}{\partial p}\right)_T = -\frac{V}{p}$$

$$\kappa = -\frac{1}{V}\left(-\frac{V}{p}\right) = \frac{1}{p}$$

#### 5.7 Specific heats: example---ideal gas

For ideal gas:

$$C_p - C_V = VT^{\frac{\alpha^2}{\kappa}}$$



#### 5.7 Specific heats

Limiting properties near absolute zero

3rd law

as 
$$T \to 0$$
,  $S \to S_0$ 

$$S \rightarrow S_0$$

 $\partial S/\partial T \approx \text{finite value}$ 

In differential form

as 
$$T \rightarrow 0$$

$$C_V = \left(\frac{\partial Q}{\partial T}\right)_V = T\left(\frac{\partial S}{\partial T}\right)_V \longrightarrow 0$$

$$C_{\mathfrak{p}} = \left(\frac{dQ}{dT}\right)_{\mathfrak{p}} = T\left(\frac{\partial S}{\partial T}\right)_{\mathfrak{p}} \longrightarrow 0$$

#### 5.7 Specific heats

Limiting properties near absolute zero

In integration form

$$S(T) - S(0) = \int_0^T \frac{C_V(T')}{T'} dT'$$

finite value

Guarantee converge

**Maxwell relation** 

as 
$$T \rightarrow 0$$

$$C_{\mathcal{V}}(T) \to 0$$

$$\alpha = \frac{1}{V} \left( \frac{\partial V}{\partial T} \right) = = = -\frac{1}{V} \left( \frac{\partial S}{\partial p} \right) = = 0$$

### 5.7 Specific heats

Limiting properties near absolute zero

$$\kappa \equiv -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$$

 $\kappa = -\frac{1}{V} \left( \frac{\partial V}{\partial p} \right)_T$  Remains finite value in ground state

$$C_p - C_V = V T \frac{\alpha^2}{\kappa}$$
 Decrease to 0 very rapidly

as 
$$T \rightarrow 0$$
,

$$\frac{C_{\mathfrak{p}}-C_{\mathfrak{p}}}{C_{\mathfrak{p}}}\to 0$$

5.7 Specific heats
Limiting properties near absolute zero

How about ideal gas?

$$c_P - c_V = R$$

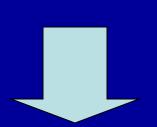
T→0, system approaches the ground state, quantum mechanic effects will become important, and the classic equation of pV=vRT will become invalid.

#### 5.8 Entropy and internal energy

$$C_{\mathbf{v}} = T \left( \frac{\partial S}{\partial T} \right)_{V}$$

$$S = S(T,V)$$

$$dS = \left(\frac{\partial S}{\partial T}\right)_{V} dT + \left(\frac{\partial S}{\partial V}\right)_{T} dV$$



$$\begin{pmatrix} \frac{\partial S}{\partial T} \end{pmatrix}_{v} = \frac{1}{T} C_{1}$$

$$\begin{pmatrix} \frac{\partial S}{\partial V} \end{pmatrix}_{T} = \begin{pmatrix} \frac{\partial P}{\partial T} \end{pmatrix}_{V}$$

$$dS = \frac{C_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_V dV$$

?



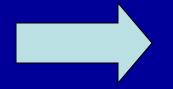
**Equation of state** 

#### **Entropy and internal energy**

$$C_{\mathbf{V}} = T \left( \frac{\partial S}{\partial T} \right)_{V}$$

$$\left(\frac{\partial C_V}{\partial V}\right)_T = \left(\frac{\partial}{\partial V}\right)_T \left[T\left(\frac{\partial S}{\partial T}\right)_V\right]$$

$$= T \frac{\partial^{2} S}{\partial T \partial V} = T \left( \frac{\partial}{\partial T} \right)_{V} \left( \frac{\partial S}{\partial V} \right)_{T}$$
$$= T \left( \frac{\partial}{\partial T} \right)_{V} \left( \frac{\partial P}{\partial T} \right)_{V}$$



$$\left(\frac{\partial C_V}{\partial V}\right)_T = T \left(\frac{\partial^2 p}{\partial T^2}\right)_V$$
 Equation of state



### 5.8 Entropy and internal energy $\left(\frac{\partial C_V}{\partial V}\right)_T = T\left(\frac{\partial^2 p}{\partial T^2}\right)_V$

$$\left(\frac{\partial C_{V}}{\partial V}\right)_{T} = T \left(\frac{\partial^{2} p}{\partial T^{2}}\right)_{V}$$

All the thermodynamic properties can be evaluated by the entropy!

- 1, heat capacity as a function of T at V=V<sub>1</sub>
- 2, equation of state

$$C_V(T,V) = C_V(T,V_1) + \int_{V_1}^V \left(\frac{\partial C_V(T,V')}{\partial V}\right)_T dV'$$

$$S(T,V) - S(T_0,V_0) = [S(T,V) - S(T_0,V)] + [S(T_0,V) - S(T_0,V_0)]$$

$$\int dT$$
  $\int dV$ 

#### 5.8 Entropy and internal energy

$$dS = \frac{C_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_V dV$$

$$S(T,V) - S(T_0,V_0) = \int_{T_0}^T \frac{C_V(T',V)}{T'} dT' + \int_{V_0}^V \left(\frac{\partial p(T_0,V')}{\partial T}\right)_V dV'$$

#### **Entropy and internal energy**

$$dE = T dS - p dV$$

# dE = T dS - p dV Fundamental equation

$$dS = \frac{C_V}{T}dT + \left(\frac{\partial p}{\partial T}\right)_V dV$$

$$dE = C_V dT + \left[ T \left( \frac{\partial p}{\partial T} \right)_V - p \right] dV$$

$$dE = \left(\frac{\partial E}{\partial T}\right)_{V} dT + \left(\frac{\partial E}{\partial V}\right)_{T} dV$$

$$\begin{pmatrix} \frac{\partial E}{\partial T} \rangle_{V} = C_{V} \\
\left( \frac{\partial E}{\partial V} \right)_{T} = T \left( \frac{\partial P}{\partial T} \right)_{V} - P$$

5.8 Entropy and internal energy: example

Van de Waals gas

**Equation of state** 

$$\left(p+\frac{a}{v^2}\right)(v-b)=RT$$

#### Additional positive pressure and occupied volume

$$p = \frac{RT}{v - b} - \frac{a}{v^2}$$

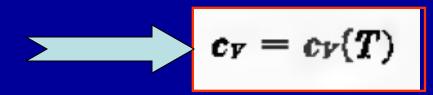
$$\left(\frac{\partial p}{\partial T}\right)_v = \frac{R}{v - b}$$

$$\left(\frac{\partial e}{\partial v}\right)_T = T\left(\frac{\partial p}{\partial T}\right)_v - p = \frac{RT}{v - b} - p$$

$$\left(\frac{\partial \epsilon}{\partial v}\right)_{T} = \frac{a}{v^2}$$

# 5.8 Entropy and internal energy: example Van de Waals gas

$$\left(\frac{\partial c_{v}}{\partial v}\right)_{T} = T\left(\frac{\partial^{2} p}{\partial T^{2}}\right) = T\left(\frac{\partial}{\partial T}\right)_{v}\left(\frac{R}{v-b}\right) = 0$$



$$d\epsilon = c_V(T) dT + \frac{a}{v^2} dv$$

$$\epsilon(T,v) - \epsilon(T_0,v_0) = \int_{T_0}^T c_V(T') dT' - a\left(\frac{1}{v} - \frac{1}{v_0}\right)$$

$$\epsilon(T,v) = \int_{T_0}^T c_V(T') dT' - \frac{a}{v} + \text{constant}$$

# 5.8 Entropy and internal energy: example Van de Waals gas

If  $c_V$  is independent of temperature,

$$\epsilon(T,v) = c_V T - \frac{a}{v} + \text{constant}$$

$$ds = \frac{c_V(T)}{T} dT + \frac{R}{v - b} dv$$

$$s(T,v) - s(T_0,v_0) = \int_{T_0}^T \frac{c_V(T') dT'}{T'} + R \ln \left(\frac{v-b}{v_0-b}\right)$$

If  $c_V$  is independent of temperature,

$$s(T,v) = c_V \ln T + R \ln (v - b) + \text{constant}$$

# 5.8 Entropy and internal energy: example Van de Waals gas

$$ds = \frac{c_V(T)}{T} dT + \frac{R}{v - b} dv$$

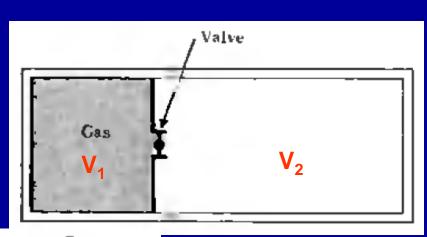
$$s(T,v) - s(T_0,v_0) = \int_{T_0}^T \frac{c_V(T') dT'}{T'} + R \ln \left( \frac{v-b}{v_0-b} \right)$$

If  $c_V$  is independent of temperature,

$$s(T,v) = c_V \ln T + R \ln (v - b) + \text{constant}$$

# Free energy expansion and throttling process 5.9 free expansion

Open valve and the gas is free to expand to fill the volume  $V_2$  from  $V_1$ 



container is adiabatically insulated,

$$Q = 0$$

does no work in the process

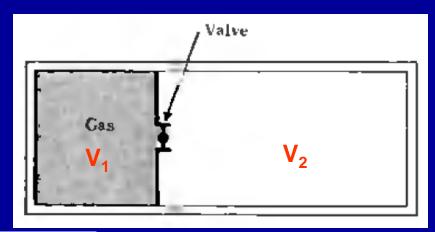
$$W = 0$$

$$\Delta E = 0$$

### Free energy expansion and throttling process 5.9 free expansion

**First law** 

$$\Delta E = 0$$



**Then** 

$$E(T_2,V_2)=E(T_1,V_1)$$

Specially, for ideal gas

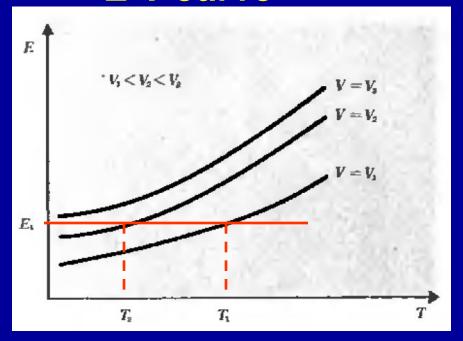
$$E(T_2) = E(T_1)$$

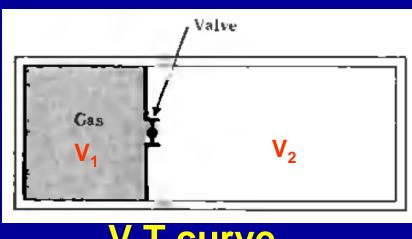
Generally,

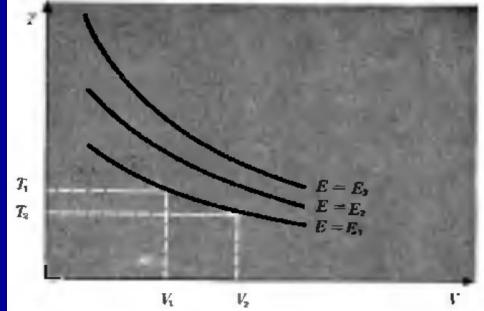
$$E(T_2, V_2) = E(T_1, V_1)$$

#### Free energy expansion and throttling process 5.9 free expansion









## Free energy expansion and throttling process 5.9 free expansion: example

#### Van de Waals gas

$$\epsilon(T_2,v_2) = \epsilon(T_1,v_1)$$

$$\int_{T_1}^{T_2} c_V(T') dT' - \frac{a}{v_2} = \int_{T_2}^{T_2} c_V(T') dT' - \frac{a}{v_1}$$

$$\int_{T_1}^{T_1} c_V(T') dT' - \int_{T_0}^{T_1} c_V(T') dT' = a \left( \frac{1}{v_2} - \frac{1}{v_1} \right)$$

$$\int_{T_1}^{T_2} c_V(T') dT' = a \left( \frac{1}{v_2} - \frac{1}{v_1} \right)$$

#### Free energy expansion and throttling process

5.9 free expansion: example

 $\epsilon(T_2,v_2) = \epsilon(T_1,v_1)$ 

Van de Waals gas

Ignore  $c_V$  change in  $[T_1, T_2]$ 

$$c_{V}(T_{2}-T_{1})=a\left(\frac{1}{v_{2}}-\frac{1}{v_{1}}\right)$$

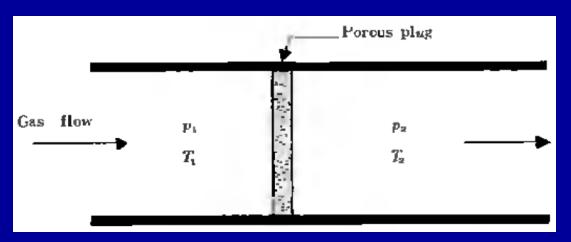
$$T_{2}-T_{1}=-\frac{a}{c_{V}}\left(\frac{1}{v_{1}}-\frac{1}{v_{2}}\right)$$

For an expansion where  $v_2 > v_1$ ,

$$T_2 < T_1$$

## Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

**Steady-state**experiment by J-T



A porous plug provide a constriction to the flow of gas;

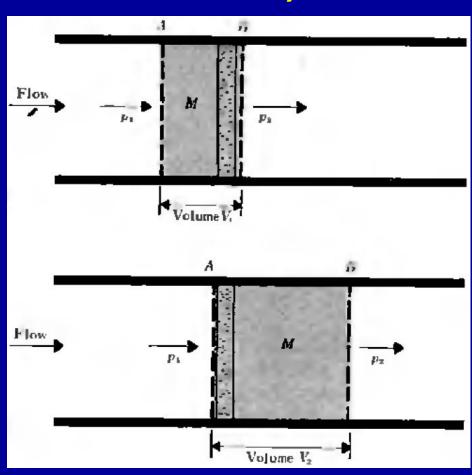
A continuous stream of gas flow from left to right;  $p_1$  in the left  $> p_2$  in the right;

T<sub>1</sub> is the temperature in left, what is T<sub>2</sub> in right?

## Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

Initial: Left, p<sub>1</sub>, V<sub>1</sub>

Final: right, p<sub>2</sub>, V<sub>2</sub>



$$\Delta E = E_2 - E_1 = E(T_2, p_2) - E(T_1, p_1)$$

# Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

$$W = p_2 V_2 - p_1 V_1$$

To external and by external

adiabatically insulated

$$Q = 0$$

Then,

$$\Delta E + W = Q = 0$$

$$(E_2 - E_1) + (p_2V_2 - p_1V_1) = 0$$
  
 $E_2 + p_2V_2 = E_1 + p_1V_1$ 

#### Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

Already define 
$$H \equiv E + pV$$

$$(E_2 - E_1) + (p_2V_2 - p_1V_1) = 0$$

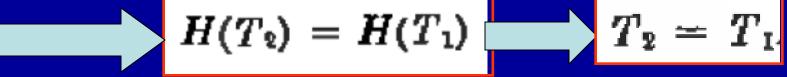
$$E_2 + p_2V_2 = E_1 + p_1V_1$$

$$H(T_2,p_2) = H(T_1,p_1)$$

$$H = H(T)$$

For ideal gas

$$H = E + pV = E(T) + \nu RT$$



in throttling process

### Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

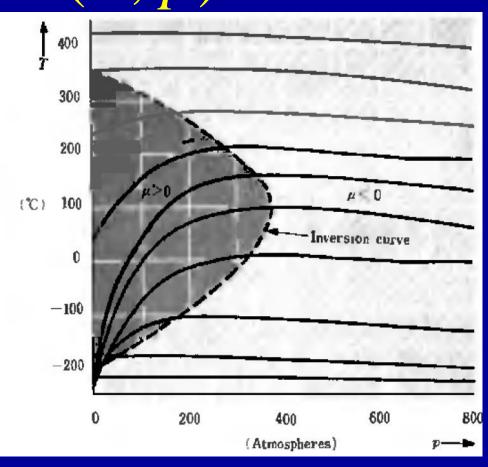
More generally, H = H(T, p)

$$\mu \equiv \left(\!\frac{\partial T}{\partial p}\!\right)_{\! H}$$

μ>0, T increases with p μ<0, T decreases with p

**Inversion curve** 

$$\mu == ??$$



#### Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

$$dE = T dS - p dV$$

$$dH \equiv d(E + pV) = T dS + V dp$$
 $dH = 0.$ 
 $C_p = T(\partial S/\partial T)_p.$ 

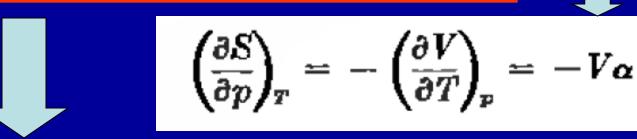
$$0 = T \left[ \left( \frac{\partial S}{\partial T} \right)_{p} dT + \left( \frac{\partial S}{\partial p} \right)_{T} dp \right] + V dp$$

$$C_{p} dT + \left[ T \left( \frac{\partial S}{\partial p} \right)_{T} + V \right] dp = 0$$

#### Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

$$\mu \equiv \left(\!\frac{\partial T}{\partial p}\!\right)_{\! H}$$

$$\mu \equiv \left(\frac{\partial T}{\partial p}\right)_{H} = -\frac{T(\partial S/\partial p)_{T} + V}{C_{p}}$$
 Maxwell



$$\mu = \frac{V}{C_p} \left( T\alpha - 1 \right)$$

For ideal gas,  $\alpha = T^{-1}$ 

$$\alpha = T^{-1}$$



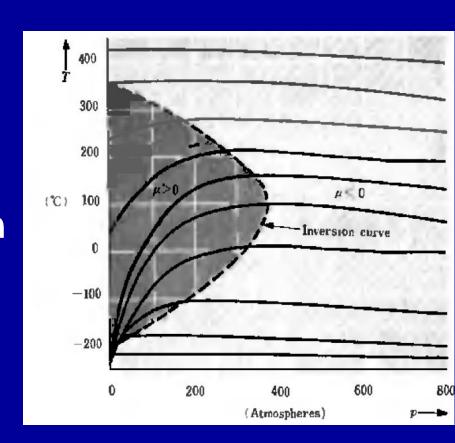
$$\mu = 0$$

# Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..) Application:

J-T effect constitute a practical method for cooling gas.

1, It is necessary to work in the region of pressure and T where  $\mu > 0$ .

2, The initial T < T maximum on the inversion curve



Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..)

Joule-Thomson effect and molecular force

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For ideal gas,

T does not change for free expansion

for throttling process
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These process becomes interesting for realistic gas virial expansion

For any gas,  $n \equiv N/V$ .

$$p = kT[n + B_2(T)n^2 + B_3(T)n^3 \cdot \cdot \cdot]$$

Virial coefficent

$$p = \frac{N}{V} kT \left( 1 + \frac{N}{V} B_2 \right)$$

Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ...)

Joule-Thomson effect and molecular force

$$p = \frac{N}{V} kT \left( 1 + \frac{N}{V} B_2 \right)$$

At low T, attractive force play dom. role,  $B_2<0$ ; At high T, (exclusion) collision play dom. role,  $B_2>0$  $B_2$  increases with T

$$\mu == ??$$

#### Free energy expansion and throttling process 5.10 throttling process (Joule-Thomson ..) Joule-Thomson effect and molecular force

$$p = \frac{N}{V} kT \left( 1 + \frac{N}{V} B_2 \right)$$

$$p = \frac{NkT}{V}\left(1 + \frac{p}{kT}B_2\right) = \frac{N}{V}(kT + pB_2)$$

$$V = N\left(\frac{kT}{p} + B_2\right)$$

$$\mu = \frac{V}{C_p} \left( T\alpha - 1 \right)$$

$$\alpha \equiv \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_p$$

$$\alpha \equiv \frac{1}{V} \left( \frac{\partial V}{\partial T} \right)_{p}$$

$$\mu = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] = \frac{N}{C_p} \left( T \frac{\partial B_2}{\partial T} - B_2 \right)$$

Free energy expansion and throttling process
5.10 throttling process (Joule-Thomson ..)
Joule-Thomson effect & molecular force: discussion

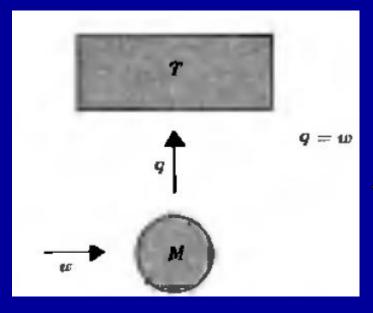
$$\mu = \frac{1}{C_p} \left[ T \left( \frac{\partial V}{\partial T} \right)_p - V \right] = \frac{N}{C_p} \left( T \frac{\partial B_2}{\partial T} - B_2 \right)$$

At low T,  $B_2<0$ ,  $\mu>0$ At high T,  $B_2>0$ ,  $\mu$  can <0

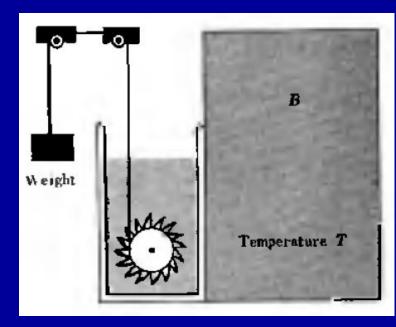
The inversion curve  $(\mu=0)$  indicates the competition between attraction and repulsion.

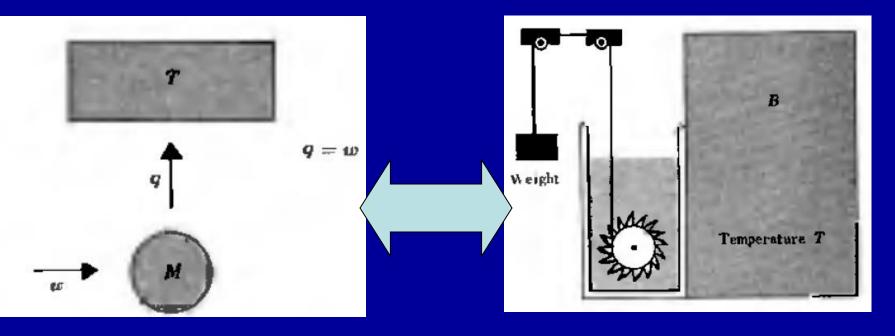
Historically, the subject of thermodynamics began with the study of engines:

- 1, great technological important
- 2, intrinsic physical interests





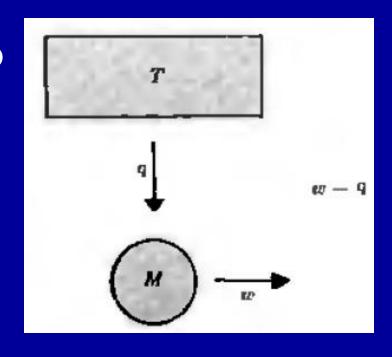




It is easy to do mechanical work w upon a device, and then extract from it heat q (q=w)

To what extent is it possible to proceed in the reversal way?

To build a device to extract internal energy from a heat reservoir in form of heat, and convert it to work?



The device is called heat engine!

#### **Heat engine--- key points:**

1, The work cannot be provided by the engine itself; or the heat-to-work process cannot be continued. Thus one wish the heat engine keeps the same macro-state at the end of process (cycle);

**Heat engine--- Question?** 

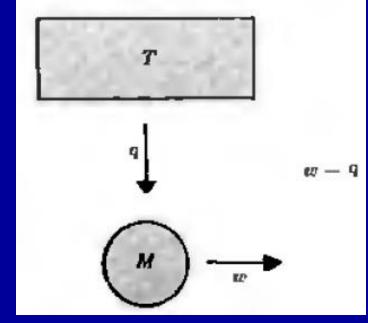
To what extent is it possible to exact a net amount of energy from heat reservoir?

In reservoir, energy is randomly distributed over many degree of freedom.

To energy associated the single freedom connected with the external parameter.

**Heat engine--- Question?** 

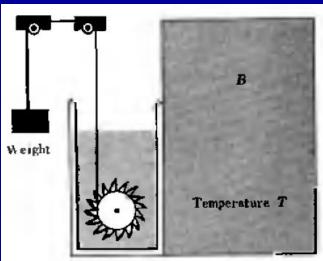
First law since E of M does not change







not realizable.

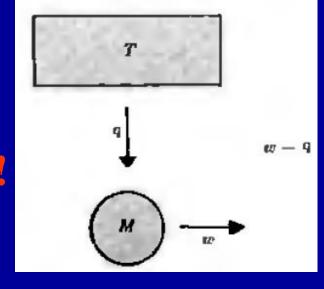


Work→ heat is an irreversible process

Since accessible states more random and entropy increases

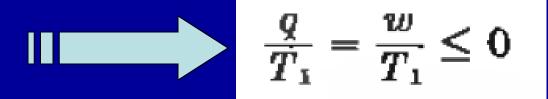
Ideal heat engine violates 2nd law!

$$\Delta S \geq 0$$



Heat reservoir, absorbed heat == (-q)The entropy change  $-q/T_{1}$ 





Wish w>0,
So it cannot be satisfied!!!

$$\frac{q}{T_1} = \frac{w}{T_1} \le 0$$

Wish w>0, So it cannot be satisfied!!!

It is impossible to construct a perfect heat engine.

Kelvin's formulation of the second law

Historically, the subject of thermodynamics began with the study of engines:

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#### Class-work

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#### Homework

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