物理学中具有里程碑意义实验现代物理前沿

散射

## 1908年 E. Rutherford, 英国物理学家提出了原子的核 式模型获得诺贝尔奖

1927康普顿发现康普顿效应获诺贝尔物理学奖

1935年 J.Chadwick, 英国物理学家, 因发现了中子获得诺贝尔奖。

1938年 E. Fermi, 意大利物理学家发明了热中子链式反应堆, 获得诺贝尔奖。。

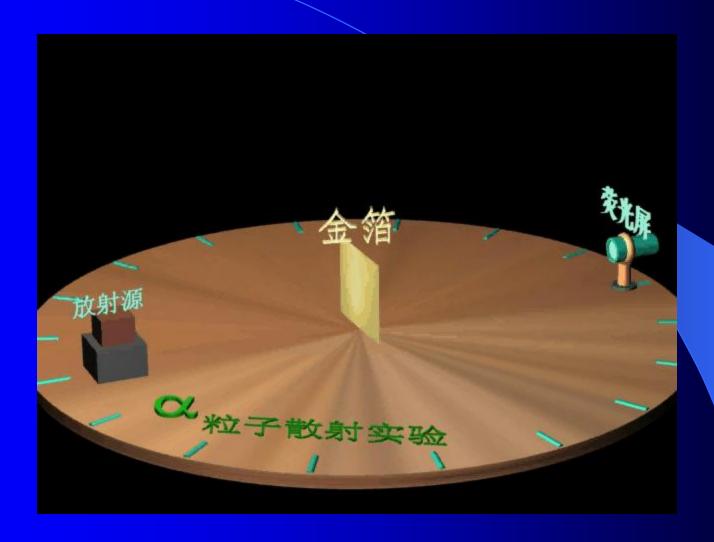
1976丁肇中分别发现J/ψ粒子年获诺贝尔奖。

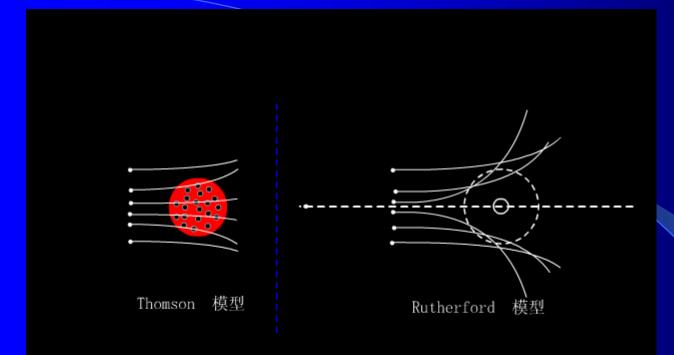
数万亿个质子以每秒1.1245万次的频率急速穿行, 速度接近光速。两束质子束分别以70000亿电子伏 特的相向而行,在功率达到140000亿电子伏特时发 生碰撞。每秒总共能发生大约6亿次撞击。

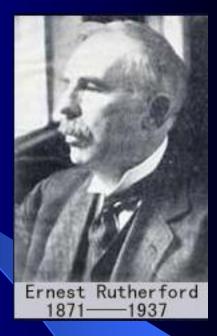
寻找标准模型预言的希格斯粒子

创造夸克-胶子等离子体, 模拟宇宙"大爆炸"

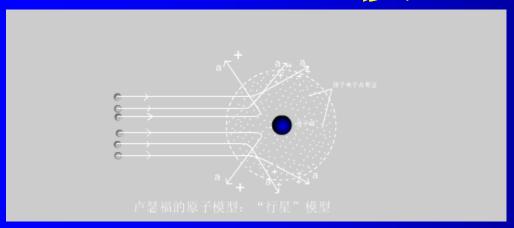
撞击时产生的高温是太阳内部温度的10万倍

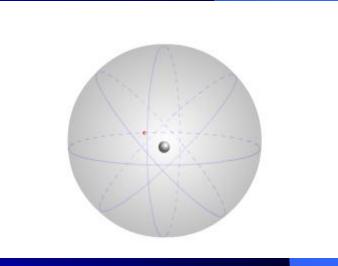






## 原可的核式模型d模型









#### 原子的核式模型:

原子由原子核和核外电子构成,原子核带正电荷,占据整个原子的极小一部分空间,而电子带负电,绕着原子核转动,如同行星绕太阳转动一样。

## 中子发现



查德威克

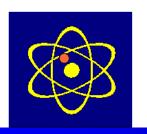
理学奖。

1932年,物理学家查德威克发现了其质量理学奖。

 ${}^{9}_{4}Be + {}^{4}_{2}He \longrightarrow {}^{12}_{6}C + {}^{1}_{0}n$ 

物质	能量密度(焦耳/kg)
TNT	$4.7 \times 10^{6}$
裂变(100%)	7.1 × <b>10</b> <sup>13</sup>
聚变(100%)	7.5 × <b>10</b> <sup>14</sup>
正电子	1.8 × <b>10</b> <sup>17</sup>



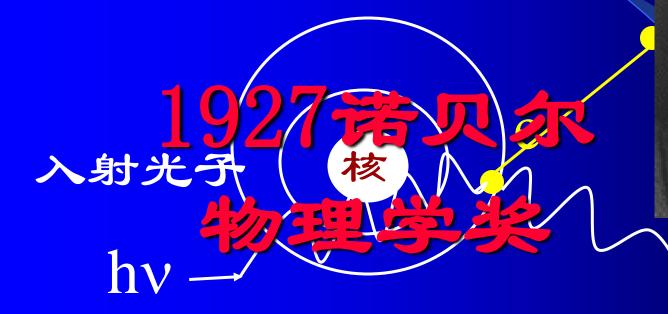




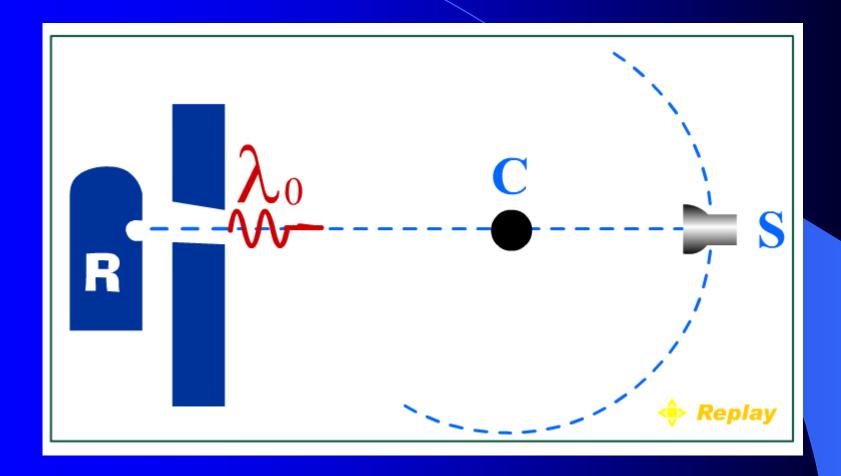


相对论性质能转换过程

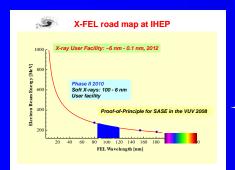
## 康普顿效应



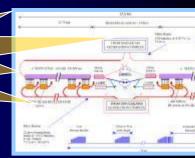
散射光子 hv



## 北京正负电子对撞机



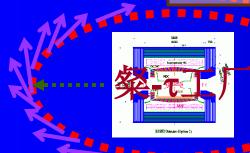
直线对撞机国际合作 <





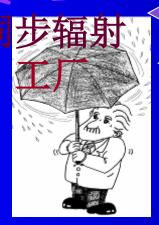
试验束和 慢正电子

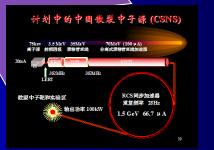
高增益自由电子激光装置



散裂中子源。 和ADS







核技术产业化

## 加速器

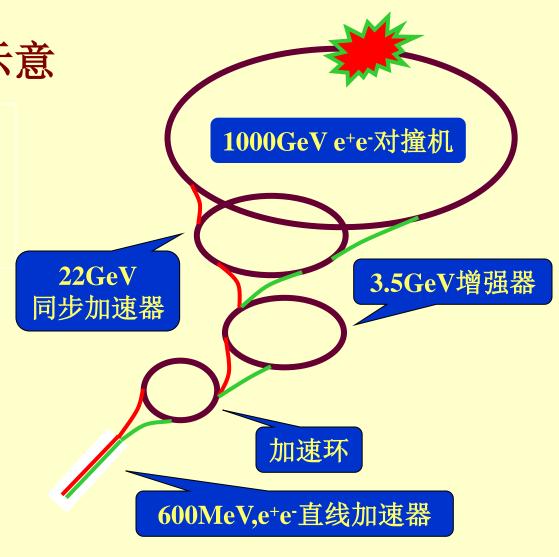
### 三、对撞机

2. 对撞机组成示意

近几十年,加速器的能量每10年提高一个量级,加速器的尺寸由数米增加到数十公里。

美国的LEP加速器是目前能量最高的加速器:

- 最高能量: 1000 GeV
- 周长: 27 km

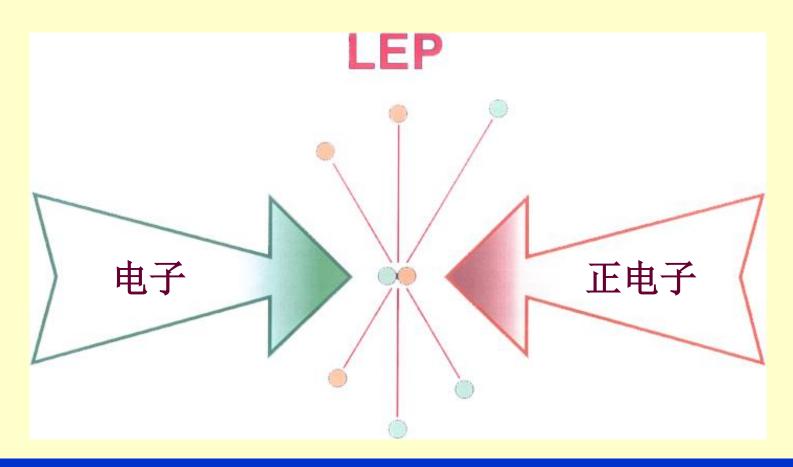


## 北京正负电子对撞机

正负电子在对撞机里相

向高速回旋、对撞,探测 对撞产生的"碎片"——次 级粒子并加以研究,就能 了解物质微观结构的许多 奥秘。

## 对撞机



对撞机模拟环境: 温度是太阳表面温度的4×10<sup>11</sup>倍 宇宙诞生的最初的10<sup>-19</sup>秒

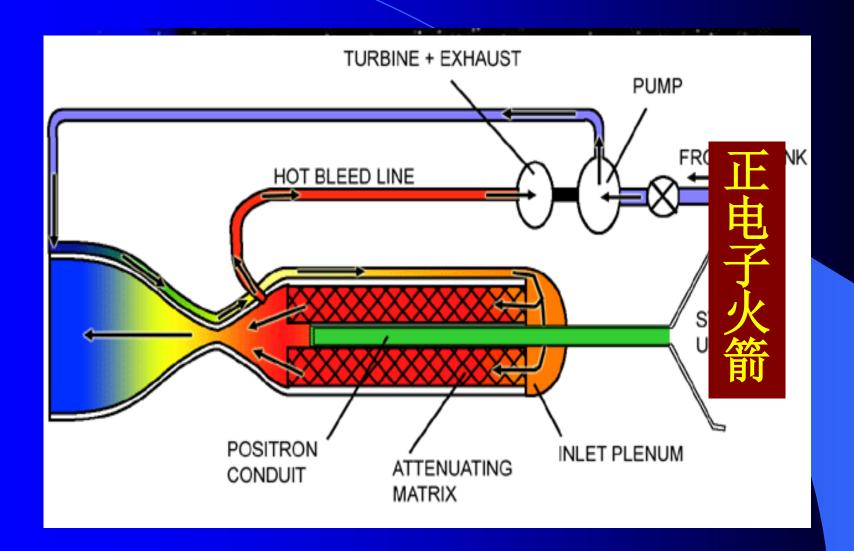
KEK,Japan



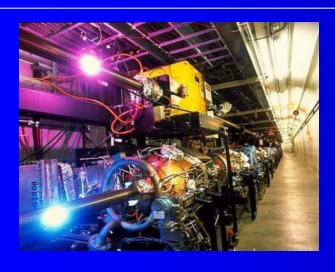


Brookhaven Nat'l Lab相對論性重離子碰撞器RHIC利用金原子核形成的光束,以幾近光速的高速相互碰撞,結果碰撞時產生的高熱火球,行為類似黑洞。

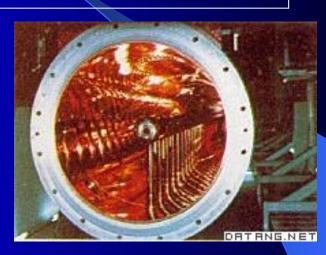
## 未来新能源



美国斯坦福大学的斯坦福直线加速器中心(Stanford Linear Accelerator Center, 简称SLAC)所建造的粒子加速器,用作研究物质与反物质的分别。科学家将电子光束(蓝色)和正电子(粉红色)光束放在不同的圆管,然后观察它们相撞所引致的光束偏差(deflection)和分裂(disruption)。



美国斯坦福大学的斯坦福直线加速器中心(Stanford Linear Accelerator Center, 简称SLAC)所建造的粒子加速器



中国科学院高能物理研究所35 MeV质子直线加速器的加速腔

## 核电起步

#### 核电装机容量及占比趋势



资料来源:中电联、中信建投证券研究所整理

"十五" "适当发展核电"



"十一五" "积极发展核电"

核电战略

#### 我国已建、在建及拟建核电站

核电站	容量MW	并网日期
秦山一期		91. 12
大亚湾-1	900	93.8
大亚湾-2	900	94. 2
秦山二-1	600	02. 2
岭澳-1	984	02. 4
岭澳-2	984	02. 11
秦山三-1	720	02. 12
秦山三-2	720	03.6
秦山二-2	600	04. 3
田湾一1	106	06. 5

规划项目	容量(万千瓦)
浙江三门健挑	
浙江三门扩瑭山	4*100
广东阳江	6*100
广东台山腰古	6*100
大连温垛子	4*100
山东烟台海阳	6*100
山东乳山红石顶	6*100
江苏连云港田湾	8*100
福建惠安	6*100
合计	5200

厂址	容量	预计并网日期
江苏田湾-2		
秦山二期扩建	2台共130万千瓦	2011年
岭澳扩建	2台共200万千瓦	2010年
浙江三门	2台共200万千瓦	2010年后
广东阳江	2台共200万千瓦	2010年后

目前,我国核电装机容量681.4万千瓦,占总 装机容量的1.36%

## 第四章 经典散射

## § 1. 碰撞的特征及分类

动力学特征

●时间极短 不考虑位移

- ●相互作用冲量很大 → 动量守恒
- 碰撞前后总能量守恒但机械能不一定守恒
- ●碰撞前后总质量守恒

## 碰撞的后果

- 弹性碰撞
- ●非弹性碰撞
- ●完全非弹性碰撞

●正碰

相对速度方向与公法线一致

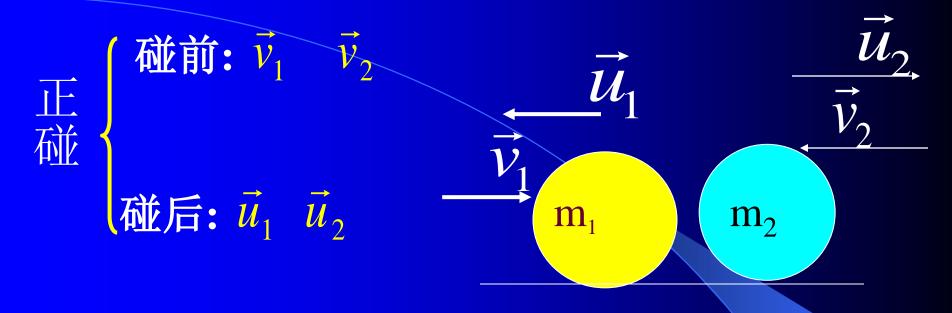
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•斜碰

相对速度方向 与公法线不一致







$$m_1 v_1 + m_2 v_2 = m_1 u_1 + m_2 u_2$$

$$e = \frac{u_2 - u_1}{v_1 - v_2} = \frac{ 恢复沖量}{ 压缩冲量}$$



Attention:

公式中的量均为代数量!!!



推导 
$$e = \frac{u_2 - u_1}{v_1 - v_2}$$

压缩阶段



公共速度V

# $\vec{v}_1$ $\vec{v}_2$ $\vec{v}_2$

设压缩冲量I<sub>1</sub>

$$I_1 = m_1(V - v_1) = -m_2(V - v_2)$$
 ...(1)

$$I_1 = m_1(V - v_1) = -m_2(V - v_2) \dots (1)$$

$$v_1 = V - \frac{I_1}{m_1}$$
  $v_2 = V + \frac{I_1}{m_2}$ 

## 恢复阶段

设恢复冲量I<sub>2</sub>

$$I_2 = m_1(u_1 - V) = -m_2(u_2 - V)$$
 ...(2)

$$u_1 = V + \frac{I_2}{m_1}$$
  $u_2 = V - \frac{I_2}{m_2}$ 

$$v_1 = V - \frac{I_1}{m_1}$$
  $v_2 = V + \frac{I_1}{m_2}$ 

$$v_2 - v_1 = I_1(\frac{1}{m_1} + \frac{1}{m_2})$$

$$u_1 = V + \frac{I_2}{m_1}$$
  $u_2 = V - \frac{I_2}{m_2}$ 

$$(u_2 - u_1 = -I_2(\frac{1}{m_1} + \frac{1}{m_2}))$$

$$e = \frac{I_2}{I_1} = \frac{u_2 - u_1}{v_1 - v_2}$$

斜碰 法线方向 切线方向

## 法线方向

$$m_1 v_{1n} + m_2 v_{2n} = m_1 u_{1n} + m_2 u_{2n}$$

$$e = \frac{u_{2n} - u_{1n}}{v_{1n} - v_{2n}}$$

物体光滑  $v_{1t} = u_{1t}$   $v_{2t} = u_{2t}$ 

切线方向

物体非光滑  $m_1 v_{1t} + m_2 v_{2t} = m_1 u_{1t} + m_2 u_{2t}$ 

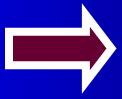
## 解决碰撞问题的注意点

\*恢复系数e正确应用

\* 刚体的碰撞



- 绕质心的转动
- **\*质心的平动** 质心冲量定理



$$\vec{I} = m\Delta \vec{V}_c$$

\* 绕质心的转动

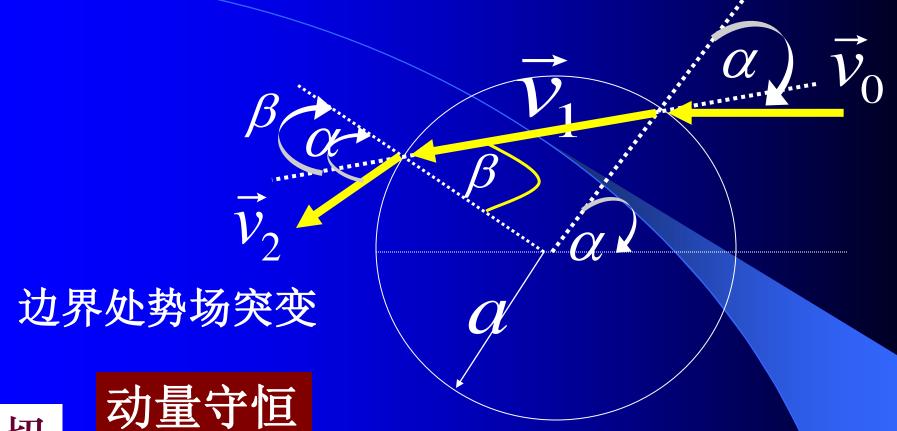
相对质心冲量矩定理

$$\vec{M} = \frac{d\vec{J}}{dt} \longrightarrow Mdt = I\Delta\omega$$



## 例一.一粒子被球形势阱散射,画出粒子出射方向

$$V = \begin{cases} 0 & r > a \\ -V_0 & r \le a \end{cases}$$



切 线方 向

$$m v_0 \sin \alpha = m v_1 \sin \beta$$
  
 $m v_2 \sin \alpha = m v_1 \sin \beta$ 

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_1}{v_2}$$

$$\frac{1}{2}mv_1^2 - V_0 = \frac{1}{2}mv_2^2 = E$$

$$v_1 = \sqrt{\frac{2(E + V_0)}{m}}$$

$$v_2 = \sqrt{\frac{2E}{m}}$$

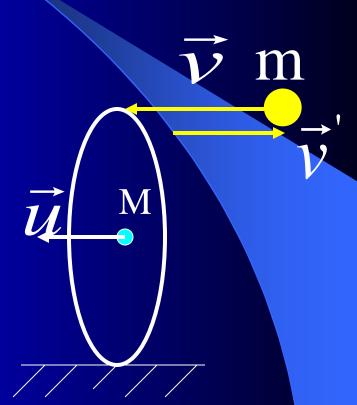
$$\frac{\sin \alpha}{\sin \beta} = \frac{v_1}{v_2} = \sqrt{1 + \frac{V_0}{E}} = n$$
 等效折射率



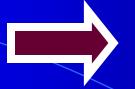
例一. 质量为M, 半径为r的匀质圆盘直立在 光滑的水平面上, 在与环面垂直方向上受一 质量为m速率为v的质点的冲击, 冲击点恰好 在最高点, 证明:

$$v = \frac{\pi (3m+M)\sqrt{2gr}}{8m(1+e)}$$

圆盘恰好呈水平状落地







碰撞过程

动量守恒

$$mv = Mu - mv$$

相对质心冲量矩定理

$$M\Delta \vec{v}_c = \vec{I} \longrightarrow M(u-0) = I$$

$$Ir = \frac{1}{2}Mr^2\dot{\theta} \qquad \dots (2)$$

$$e = \underbrace{(u+r\dot{\theta})-(-v')}_{v} \dots(3)$$

绝对速度 和方向!!!

$$Mur = \frac{1}{2}Mr^2\dot{\theta}$$

$$\dot{\theta} = \frac{2u}{r} \quad ...(4)$$

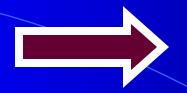
$$m v = M u - m v$$

下一阶段的 初始条件

$$u = \frac{mv(1+e)}{M+3m} ...(5)$$

$$\dot{\theta} = \frac{2mv(1+e)}{(M+3m)r} \quad ...(6)$$

## 第二阶段



圆环质心作平抛运动

圆环绕质心匀速转动

## 落地时间

$$t=rac{\pi/2}{\dot{ heta}}$$



$$t = \frac{\pi/2}{\dot{\theta}}$$
  $\dot{\theta} = \frac{2mv(1+e)}{(M+3m)r}$  ...(6)

$$= \frac{\pi (3m+M)r}{4mv(1+e)} \quad ...(7)$$

$$v_{cy} = gt = \frac{\pi (3m + M)gr}{4mv(1+e)}$$
...

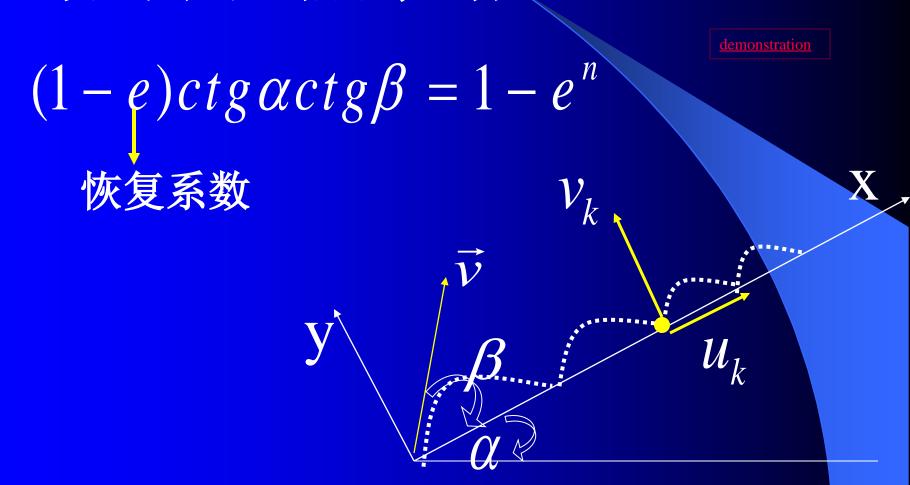
$$\frac{1}{2}Mu^{2} + \frac{1}{2}I_{c}\dot{\theta}^{2} = \frac{1}{2}Mu^{2} + \frac{1}{2}Mv_{cy}^{2} + \frac{1}{2}I_{c}\dot{\theta}^{2} - Mgr$$

$$t = \frac{\pi (3m + M)r}{4mv(1+e)}$$

$$v_{cy} = gt = \frac{\pi (3m + M)gr}{4mv(1+e)}$$

$$v = \frac{\pi (3m+M)\sqrt{2gr}}{8m(1+e)}$$

例三. 质量为m的质点在一倾角为α的斜面底部以一定初速且与斜面成β角方向发射, 如果经n次碰撞后回到原点, 证明:



设第K次碰撞在A点,速度分别为负k,vk)

$$k \Longrightarrow k+1 \Longrightarrow$$
 历经时间为 $t_{k+1}$ 

$$v_k t_{K+1} - \frac{1}{2} g \cos \alpha t_{K+1}^2 = 0$$

$$t_{K+1} = \frac{2v_k}{g\cos\alpha} \qquad (3)$$

从原点0到A<sub>k</sub>所需时间T<sub>k</sub>

$$T_{k} = t_{1} + t_{2} + t_{3} + \dots + t_{k} \qquad \dots (2)$$

$$= \underbrace{2(v_{0} + v_{1} + v_{2} + \dots v_{k-1})}_{= (2)}$$

 $\frac{g\cos\alpha}{}$  .....(2)

$$v_0, v_1, v_2, \dots, v_{k-1}$$

第一次 
$$v_0 = v \sin \beta$$

$$e = \frac{v_1 - 0}{0 - (-v \sin \beta)} \longrightarrow v_1 = ev \sin \beta$$

第二次

$$e = \frac{v_2 - 0}{0 - (-v_1)} \qquad \qquad v_2 = e^2 v \sin \beta$$

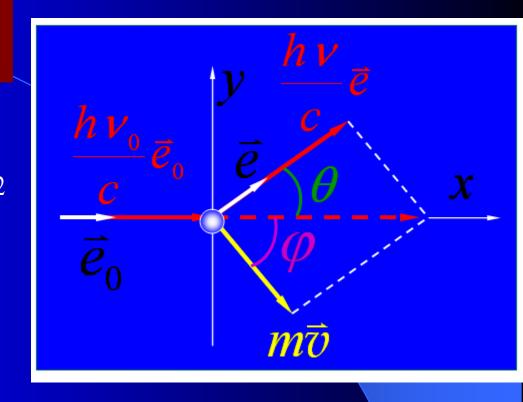
# 康普顿效应

能量守恒

$$hv_0 + m_0 c^2 = hv + mc^2$$

动量守恒

$$\frac{h \, v_0}{c} \, \vec{e}_0 = \frac{h \, v}{c} \, \vec{e} + m \vec{v}$$



$$m^{2}v^{2} = \frac{h^{2}v_{0}^{2}}{c^{2}} + \frac{h^{2}v^{2}}{c^{2}} - 2\frac{h^{2}v_{0}v}{c^{2}}\cos\theta$$

$$m^{2}c^{4}(1-\frac{v^{2}}{c^{2}}) = m_{0}^{2}c^{4} - 2h^{2}v_{0}v(1-\cos\theta) + 2m_{0}c^{2}h(v_{0}-v)$$

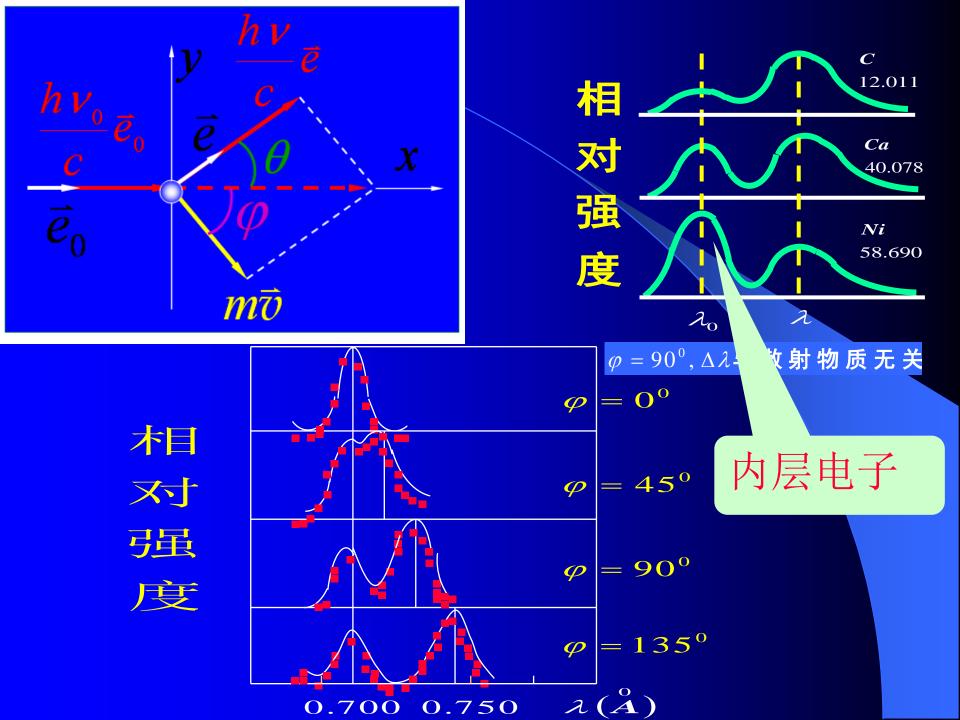
$$m^{2}c^{4}(1-\frac{v^{2}}{c^{2}}) = m_{0}^{2}c^{4} - 2h^{2}v_{0}v(1-\cos\theta) + 2m_{0}c^{2}h(v_{0}-v)$$

$$m = m_0 (1 - v^2 / c^2)^{-1/2}$$

$$\frac{c}{v} - \frac{c}{v_0} = \frac{h}{m_0 c} (1 - \cos \theta) = \lambda - \lambda_0 = \Delta \lambda$$

康普顿公式 
$$\Delta \lambda = \frac{h}{m_0 c} (1 - \cos \theta) = \frac{2h}{m_0 c} \sin^2 \frac{\theta}{2}$$

康普顿波长 
$$\lambda_c = \frac{h}{m_0 C} = 2.41 \times 10^{-12} m = 2.41 \times 10^{-3} nm$$



# Summary:

#### 代数量

● 刚体碰撞点的绝对速度 —  $e = \frac{u_2 - u_1}{v_1 - v_2}$ 

●冲量大小应由质心动量改变量计算

$$\vec{I} = m\Delta \vec{V_c}$$

●冲量矩应对质心而言

$$\vec{M} = \frac{d\vec{J}}{dt}$$
  $\longrightarrow$   $Mdt = I_c \Delta \omega$ 

●冲量对刚体作用的效果

## 定量讨论

# 碰撞前

$$m_1 \Rightarrow \vec{v}_1$$
  $m_2 \Rightarrow \vec{v}_2$  绝对速度  $\vec{v}_c = \frac{m_1 \vec{v}_1 + m_2 \vec{v}_2}{m_1 + m_2}$   $m_1 \to \vec{u}_1$   $m_2 \to \vec{u}_2$  (相对质心速度)  $\vec{u} = \vec{v}_1 - \vec{v}_2 = \vec{u}_1 - \vec{u}_2$  (相对速度)

$$\vec{u}_1 = \vec{v}_1 - \vec{v}_c = \frac{m_2 \vec{u}}{m_1 + m_2}$$

$$\vec{u}_2 = \vec{v}_2 - \vec{v}_c = \frac{-m_1 \vec{u}}{m_1 + m_2}$$

 $m_1 \Rightarrow \vec{v}_1 \qquad m_2 \Rightarrow \vec{v}_2 \qquad$ 绝对速度  $\vec{v}_c = \frac{m_1'\vec{v}_1' + m_2'\vec{v}_2'}{m_1' + m_2'}$ 

碰撞后

$$m_1 \rightarrow \vec{u}_1 \quad m_2 \rightarrow \vec{u}_2$$
 (相对质心速度)

$$\vec{u}' = \vec{v}_1 - \vec{v}_2 = \vec{u}_1 - \vec{u}_2$$
 (相对速度)

$$\vec{u}_{1}' = \vec{v}_{1}' - \vec{v}_{c}' = \frac{m_{2}'\vec{u}'}{m_{1}' + m_{2}'}$$

$$\vec{u}_2' = \vec{v}_2' - \vec{v}_c' = \frac{-m_1 \vec{u}}{m_1' + m_2'}$$

质量守恒 
$$m_1 + m_2 = m_1' + m_2'$$

$$(m_1 + m_2)\vec{v}_c = (m_1' + m_2')\vec{v}_c'$$

碰撞过过程中动能损失



$$\vec{v}_c = \vec{v}_c$$

$$\Delta T = \left[\frac{1}{2}(m_1 + m_2)\vec{v}_c^2 + \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] -$$

$$\left[\frac{1}{2}(m_1 + m_2)\vec{v}_c^2 + \frac{1}{2}m_1u_1^{'2} + \frac{1}{2}m_2u_2^{'2}\right]$$

$$\Delta T = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \left[\frac{1}{2}m_1u_1^{'2} + \frac{1}{2}m_2u_2^{'2}\right]$$

$$\Delta T = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \left[\frac{1}{2}m_1u_1^{'2} + \frac{1}{2}m_2u_2^{'2}\right]$$



$$\Delta T = \left[ \frac{1}{2} \, \mu \, u^2 - \frac{1}{2} \, \mu' u'^2 \right]$$

$$\mu = \frac{m_1 m_2}{m_1 + m_2}$$

$$\mu' = \frac{m_1 m_2}{m_1' + m_2'}$$

(折合质量)

 $m_2u$ 

 $m_1 + m_2$ 

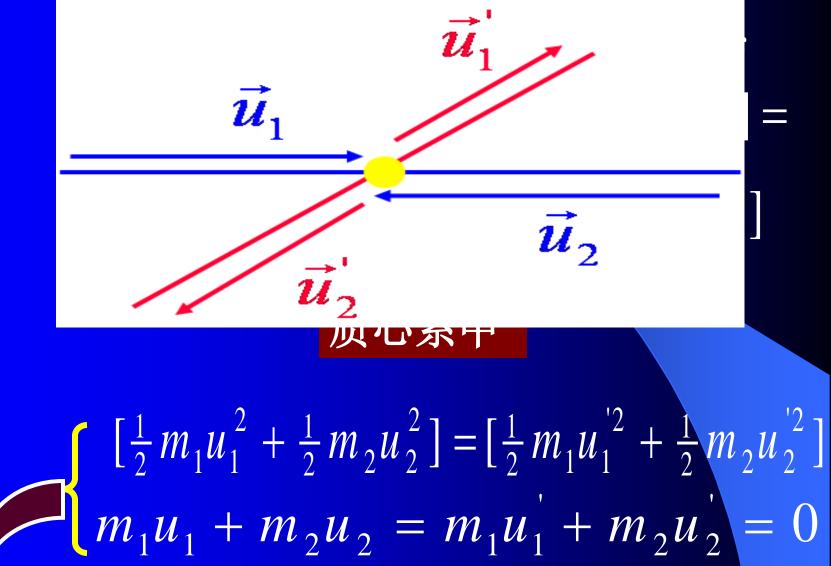
$$\dot{U}_1 = \frac{2}{m_1 + m_2}$$

$$\frac{-\boldsymbol{m_1}\vec{\boldsymbol{u}}}{\boldsymbol{m_1}+\boldsymbol{m_2}}$$

碰 
$$u_1$$

$$\vec{u}_2' = \frac{-m_1'\vec{u}}{m_1' + m_2'}$$





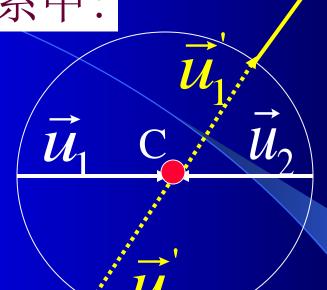
$$u = u'$$
  $u_1 = u'_1$   $u_2 = u'_2$ 

弹性碰撞



$$\vec{u}_1 = \frac{m_2 \vec{u}}{m_1 + m_2}$$

$$\vec{u}_2 = \frac{-m_1 \vec{u}}{m_1 + m_2}$$



n为碰后相对速度方向单位矢量

$$\vec{u}_{1}' = \frac{m_{2}\vec{u}'}{m_{1}' + m_{2}'}$$

$$\vec{u}_{2}' = \frac{-m_{1}'\vec{u}'}{m_{1}' + m_{2}'}$$

$$\vec{u}_{1}' = \frac{m_{2}}{m_{1} + m_{2}} u\vec{n}$$

$$\vec{u}_{2}' = \frac{-m_{1}}{m_{1} + m_{2}} u\vec{n}$$

#### 在L系中:

$$\vec{v}_{1}' = \vec{v}_{c}' + \vec{u}_{1}' = \frac{m_{1}\vec{v}_{1} + m_{2}\vec{v}_{2}}{m_{1} + m_{2}} + \frac{m_{2}u}{m_{1} + m_{2}}\vec{n}$$

$$\vec{v}_{2}' = \vec{v}_{c}' + \vec{u}_{2}' = \frac{m_{1}\vec{v}_{1} + m_{2}\vec{v}_{2}}{m_{1} + m_{2}} - \frac{m_{1}u}{m_{1} + m_{2}}\vec{n}$$

$$\vec{u}_{1}' = \frac{m_{2}}{m_{1} + m_{2}}u\vec{n}$$

$$\vec{u}_{2}' = \frac{-m_{1}}{m_{1} + m_{2}}u\vec{n}$$

$$\vec{u}_{2}' = \frac{-m_{1}}{m_{1} + m_{2}}u\vec{n}$$

$$\vec{u}_{2}' = \frac{m_{2}\vec{v}_{2}'}{m_{1} + m_{2}}(\vec{P}_{1} + \vec{P}_{2}) - \mu u\vec{n} \dots (B)$$

$$\vec{CB} = \mu u \vec{n} \quad \vec{AC} = \frac{m_1}{m_1 + m_2} (\vec{P}_1 + \vec{P}_2)$$

$$\vec{AB} = \vec{P}_1' \quad \vec{CD} = \frac{m_2}{m_1 + m_2} (\vec{P}_1 + \vec{P}_2) \vec{A}$$

$$\vec{BD} = \vec{P}_2' \quad \vec{AD} = \vec{P}_1 + \vec{P}_2$$

A和D两点位置固定

B点可在圆周 上任意变动

# 散射中靶粒子初始静止 $\mathbf{P}_2 = 0$

$$P_2 = 0$$

$$\vec{u} = \vec{v}_1 - \vec{v}_2 = \vec{v}_1$$

$$\vec{CB} = \mu u \vec{n} = \frac{m_1 m_2}{m_1 + m_2} v_1 \vec{n} \qquad \vec{AC} = \frac{m_1 \vec{P}_1}{m_1 + m_2} = \frac{m_1 m_1}{m_1 + m_2} \vec{v}_1$$

$$\vec{CD} = \frac{m_2 \vec{P}_1}{m_1 + m_2} = \frac{m_2 m_1}{m_1 + m_2} \vec{v}_1$$

$$\vec{P}_1' = m_1 \vec{v}_1' = \frac{m_1}{m_1 + m_2} (\vec{P}_1 + \vec{P}_2) + \mu u \vec{n} \quad ...(A)$$

$$\vec{P}_{2}' = m_{2}\vec{v}_{2}' = \frac{m_{2}}{m_{1} + m_{2}}(\vec{P}_{1} + \vec{P}_{2}) - \mu u \vec{n}$$
 ...(B)

## Discussion:

$$\frac{AC}{CD} = \frac{m_1}{m_2}, \quad CB = CD$$

$$m_1 < m_2$$
 AC

φ为c系中入射粒子偏转角

*θ*<sub>1</sub>为入射粒子出射方向对λ射方向偏离

θ<sub>2</sub>为靶粒子在L系中反冲角

$$\theta_2 = \frac{\pi - \varphi}{2}$$

$$0 \le \theta_1 \le \pi$$
 B点变化范围



 $^{\circ}$  m<sub>1</sub> > m<sub>2</sub>

#### A点在圆外

$$0 \le \theta_1 \le \theta_{1 \max}$$

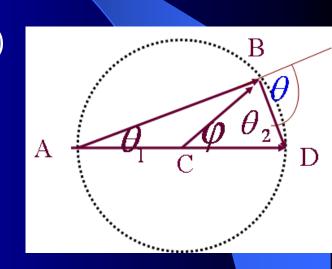
$$\sin \theta_{1\text{max}} = \frac{EC}{AC} = \frac{m_2}{m_1} \quad (\because EC = CD)$$

 $^{\circ}$   $m_1 = m_2$ 

#### A点在圆上

$$\theta_2 = \frac{\pi - \varphi}{2} \qquad \theta_1 = \frac{\varphi}{2}$$

$$\theta = \theta_1 + \theta_2 = \frac{\pi}{2} \qquad 0 \le \theta_1 \le \frac{\pi}{2}$$



## § 2. 有心排斥力场中粒子的散射

#### 一.基本概念

- ●总截面
- ●瞄准距离
- ●入射粒子散射
- ●微分散射截面

#### 一.基本概念

●总截面

入射粒子

靶粒子

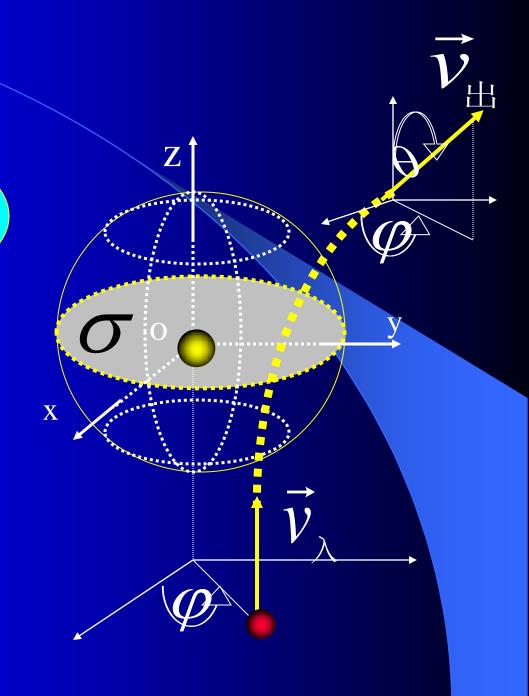


相互作用区域在 xy平面的投影



总截面

O



#### 一基本概念

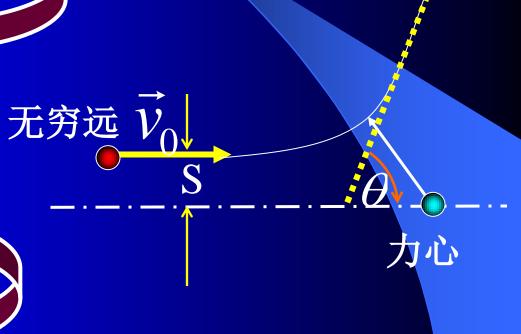
●瞄准距离S

S

相互作用越弱

S

相互作用越强



#### 一基本概念

●入射粒子散射

入射粒子流强度为I:

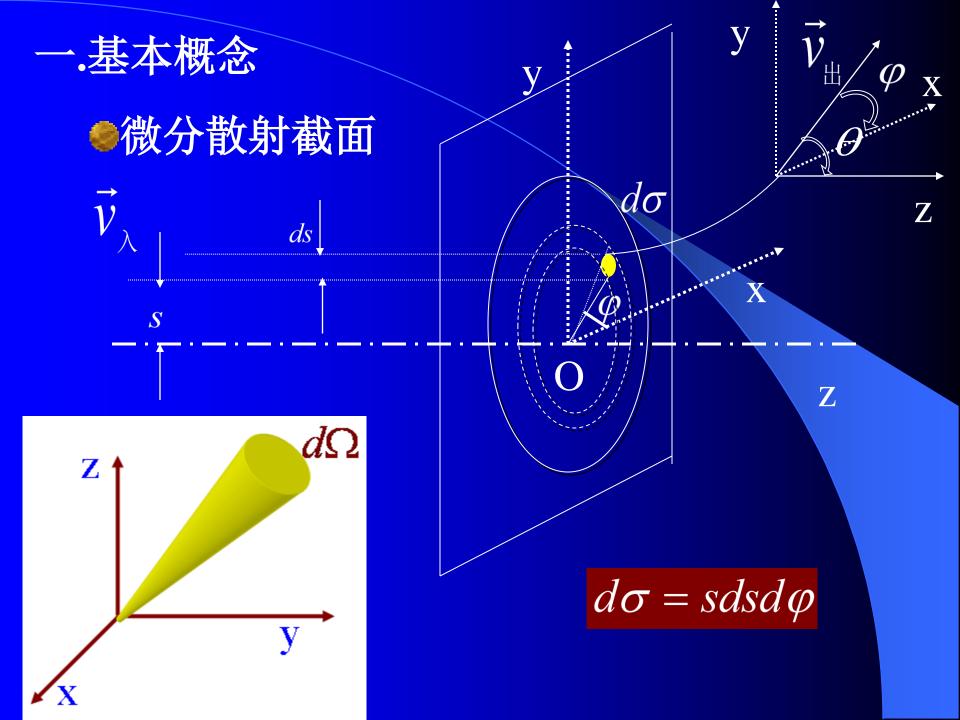
单位时间入射方向垂直的单位面积

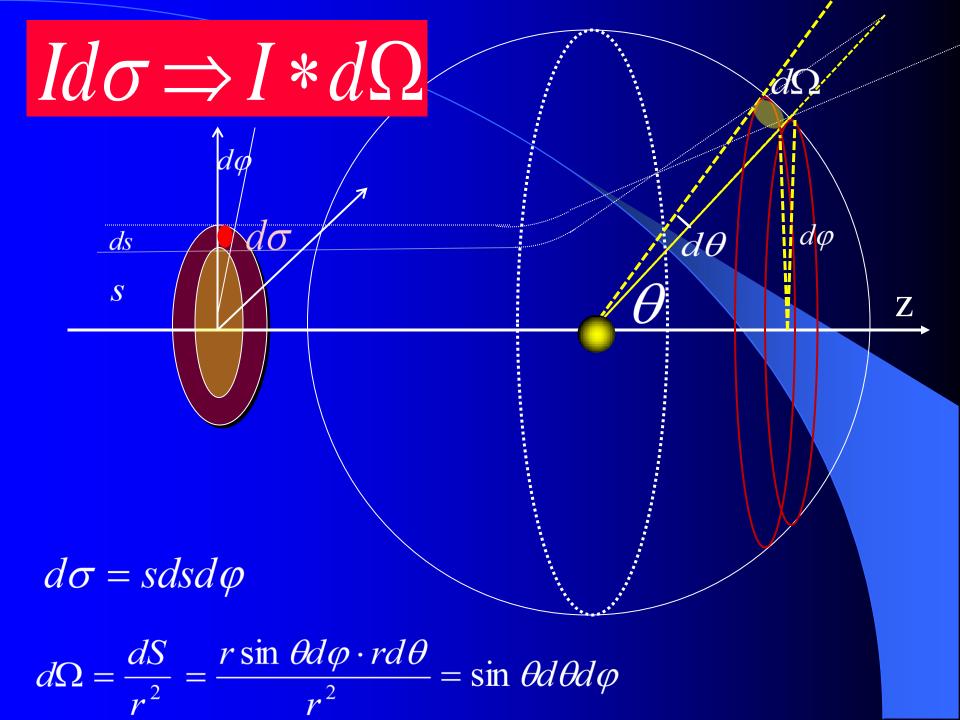
#### 设入射粒子束横截面为A

$$\frac{\sigma I}{AI} = \frac{\sigma}{A}$$



AI = A 入射粒子散射几率





$$d\sigma = sdsd\varphi \qquad d\Omega = \sin\theta d\theta d\varphi$$

$$d\sigma = \sigma(\theta \varphi)d\Omega$$

$$\sigma(\theta \varphi)$$
 微分散射截面

 $d\sigma$ 



部分散射截面

$$\sigma(\theta \varphi) = \sigma(\theta)$$
 相互作用与 $\varphi$ 角无关

# $\sigma(\theta)$ 的物理意义

设单位时间散射到dΩ内的粒子数为dN

$$dN = Id\sigma = I\sigma(\theta)d\Omega$$

$$\sigma(\theta) = \frac{dN}{Id\Omega} = \frac{dN/d\Omega}{I}$$

单位时间散射到单位立体角粒子数 单位时间通过垂直入射方向单位面积粒子数

$$d\sigma = \sigma(\theta \varphi)d\Omega \qquad d\Omega = \sin \theta d\theta d\varphi$$
$$\sigma(\theta) \qquad \qquad \text{量纲为面积}$$



$$dN = Id\sigma = I\sigma(\theta)d\Omega \quad \sigma(\theta) = \frac{dN}{Id\Omega} = \frac{dN}{I}$$

单位时间散射到单位立体角粒子数单位时间通过垂直入射方向单位面积粒子数

粒子被散射到θ方向单位立体角中的几率 (占总入射粒子数)

$$dN = Id\sigma \Longrightarrow I\sigma(\theta)d\Omega$$

几率
$$\rho = \frac{Id\sigma}{IA} = \frac{I\sigma(\theta)d\Omega}{IA} = \frac{\sigma(\theta)d\Omega}{A}$$

 $\sigma(\theta)$ 

量纲为面积

 $d\sigma = \sigma(\theta)d\Omega$   $sdsd\varphi = \sigma(\theta)\sin\theta d\theta d\varphi$ 

$$\sigma(\theta) = \frac{s}{\sin\theta} \left| \frac{ds}{d\theta} \right|$$



微分散射截面计算

# 二. 有心排斥力场中粒子的散射

S P

$$\theta = \pi - 2\psi$$

$$E = \frac{1}{2}mv_0^2$$

$$mr^2\dot{\varphi} = J_0 = mv_0s$$

demonstration

$$m r^2 \dot{\phi} = J_0 = m v_0 s \qquad E = \frac{1}{2} m v_0^2$$

$$J_0^2 = m^2 v_0^2 s^2 = 2m E s^2$$

$$E = \frac{1}{2}(\dot{r}^2 + r^2\dot{\phi}^2) + V(r)$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{J_0^2}{2mr^2} + V(r)$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\varphi} \frac{d\varphi}{dt}$$

$$E = \frac{1}{2}m\dot{r}^2 + \frac{J_0^2}{2mr^2} + V(r)$$

$$\sqrt{\frac{2}{m}} \begin{bmatrix} E - V - \frac{J_0^2}{2mr^2} \end{bmatrix} \qquad mr^2 \dot{\varphi} = J_0 = mv_0 s$$

$$\dot{r} = \frac{dr}{dt} = \frac{dr}{d\varphi} \frac{d\varphi}{dt} \qquad \qquad \dot{\varphi} = \frac{1}{2} mv_0^2$$

$$\dot{\varphi} = \frac{v_0 s}{r^2} = \frac{s}{r^2} \sqrt{\frac{2E}{m}}$$

$$\psi = \int_{0}^{\psi} d\varphi = \int_{r_{\min}}^{\infty} \frac{s}{r^2} \frac{dr}{\sqrt{1 - \frac{V}{E} - \frac{s^2}{r^2}}}$$

# rmin的确定

近日点:

$$\dot{r} = 0$$

$$E = \frac{J_0^2}{2mr_{\min}^2} + V(r_{\min})$$

$$J_0^2 = m^2 v_0^2 s^2 = 2m E s^2$$

$$E = \frac{2mEs^2}{2mr_{\min}^2} + V(r_{\min})$$

$$r_{\min} = s \sqrt{\frac{E}{E - V(r_{\min})}}$$



#### 变量变换

$$z = \frac{S}{r} \begin{cases} r = \infty & z = 0 \\ r = r_{\text{min}} \end{cases} \qquad z = z_0$$

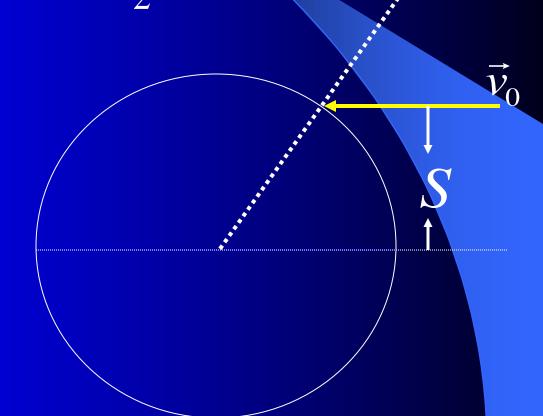
$$dr = -\frac{r^2}{s} dz \qquad 1 - \frac{V(z_0)}{E} - z_0^2 = 0$$

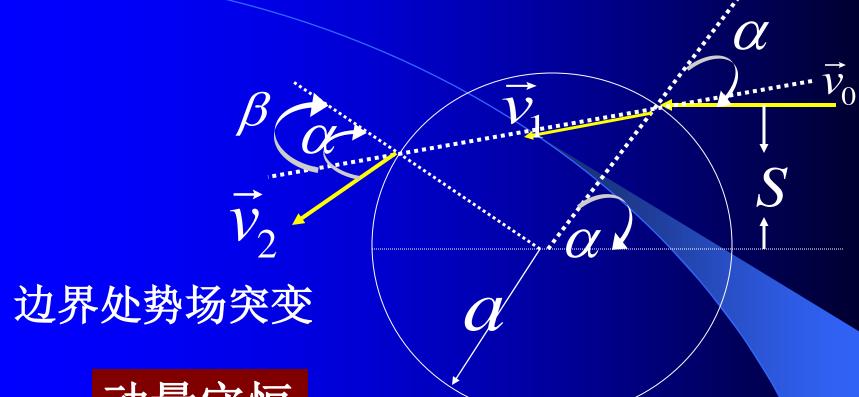
$$\psi = \int_{0}^{\psi} d\varphi = \int_{0}^{z_{0}} \frac{dz}{\sqrt{1 - \frac{V(z)}{E} - z^{2}}}$$

#### 一粒子被球形势阱散射,证明:

$$\sigma(\theta) = \frac{n^2 a^2 (n \cos \frac{\theta}{2} - 1)(n - \cos \frac{\theta}{2})}{4 \cos \frac{\theta}{2} (1 + n^2 - 2n \cos^2 \frac{\theta}{2})}$$

$$\mathbf{V} = \begin{cases} 0 & r > a \\ -\mathbf{V}_0 & r < a \end{cases}$$





## 动量守恒

切线方向

$$m v_0 \sin \alpha = m v_1 \sin \beta$$
  
 $m v_2 \sin \alpha = m v_1 \sin \beta$ 

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_1}{v_2}$$

$$\frac{1}{2}mv_1^2 - V_0 = \frac{1}{2}mv_2^2 = E$$

$$v_1 = \sqrt{\frac{2(E + V_0)}{m}}$$

$$v_2 = \sqrt{\frac{2E}{m}}$$

$$\frac{\sin \alpha}{\sin \beta} = \frac{v_1}{v_2} = \sqrt{1 + \frac{V_0}{E}} = n$$

$$\sigma(\theta) = \frac{s}{\sin \theta} \left| \frac{ds}{d\theta} \right|$$

$$\sigma(\theta) = \frac{a^2 n^2}{4\cos\frac{\theta}{2}} \frac{(n\cos\frac{\theta}{2} - 1)(n - \cos\frac{\theta}{2})}{(1 + n^2 - 2n\cos\frac{\theta}{2})^2}$$

## Discussion:

当S > a散射不能发生

#### 求一质点与半径为a的钢球碰撞的散射截面

$$V(r) = \begin{cases} \infty & r < a \\ 0 & r \ge a \end{cases}$$

$$\theta = \pi - 2 \int_{a}^{\infty} \frac{s}{r^{2}} \frac{1}{\sqrt{1 - \frac{s^{2}}{r^{2}}}} \int_{a}^{\infty} \frac{dx}{\sqrt{1 - x^{2}}} = \sin^{-1} x$$

$$\theta = \pi + 2\sin^{-1}\frac{s}{r} \stackrel{\infty}{a} \qquad \Rightarrow \qquad \theta = \pi - 2\sin^{-1}\frac{s}{a}$$

$$S = a \sin \frac{\pi - \theta}{2} = a \cos \frac{\theta}{2}$$

$$\sigma(\theta) = \frac{s}{\sin \theta} \left| \frac{ds}{d\theta} \right| = \frac{a^2}{4}$$

总截面
$$\sigma = \iint \sigma(\theta) \sin \theta d\theta d\phi$$

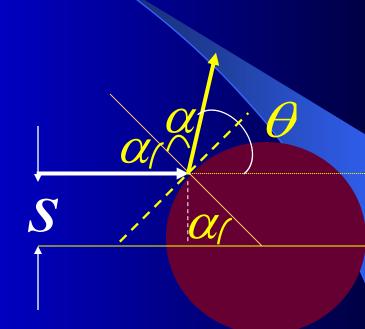
$$= \iint d\phi \int \frac{a^2}{4} \sin \theta d\theta$$

$$\sigma = \pi a^2$$

## Simple Solution:

$$\alpha = \frac{\pi - \theta}{2}$$

$$S = a \sin \frac{\pi - \theta}{2} = a \cos \frac{\theta}{2}$$



$$\sigma(\theta) = \frac{s}{\sin \theta} \left| \frac{ds}{d\theta} \right| = \frac{a^2}{4}$$

# 平方反比有心斥力场 $V = \frac{k}{r}$ 散射的散射截面

$$\psi = \int_{0}^{z_0} \frac{dz}{\sqrt{1 - z^2 - \frac{V(z)}{E}}} \dots (1)$$

# z<sub>0</sub>由下式确定:

$$\frac{1 - Z_0^2 - \frac{V(z_0)}{E} = 0}{E} = 0 \quad (z_0) = \frac{k}{r} \frac{s}{s} = \frac{k}{s} z_0$$

$$z_0 = \frac{-\frac{k}{sE} \pm \sqrt{(\frac{k}{sE})^2 + 4}}{2} \qquad z_0 = \frac{-\frac{k}{sE} + \sqrt{(\frac{k}{sE})^2 + 4}}{2}$$

$$\theta = \pi - 2\int_{0}^{\infty} \frac{dz}{\sqrt{1 - z^2 - \frac{V(z)}{E}}}$$

$$V(z) = \frac{k}{r} \frac{s}{s} = \frac{k}{s} z$$

$$\theta = \pi - 2\int_{0}^{\infty} \frac{dz}{\sqrt{1 + \frac{k^{2}}{4E^{2}s^{2}} - (z + \frac{k}{2Es})^{2}}}$$

$$\theta = \pi - 2 \int_{0}^{z_{0}} \frac{2dz}{\sqrt{4 + \frac{k^{2}}{E^{2}s^{2}}}} \sqrt{1 - (\frac{2z + \frac{k}{SE}}{\sqrt{4 + \frac{K^{2}}{(SE)^{2}}}})}$$

$$\theta = \pi - 2 \int_{0}^{z_{0}} \frac{-1}{\sqrt{1 - \left(\frac{2z + \frac{k}{SE}}{\sqrt{4 + \frac{K^{2}}{(ES)^{2}}}}\right)^{2}}} d\sqrt{\frac{2z + \frac{k}{SE}}{\sqrt{4 + \frac{K^{2}}{(ES)^{2}}}}}$$

$$\theta = \pi - 2 \int_{0}^{z_{0}} \frac{-1}{\sqrt{4 + \frac{K^{2}}{(ES)^{2}}}} d\sqrt{\frac{-2z - \frac{k}{S}}{\sqrt{4 + \frac{K^{2}}{(ES)^{2}}}}}$$

$$\int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1} x$$

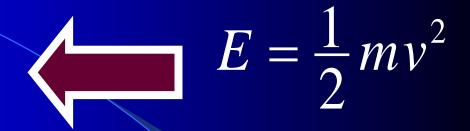
$$\theta = \pi - 2 \left( -\sin^{-1} \frac{-2z - \frac{k}{SE}}{\sqrt{4 + \frac{K^2}{(SE)^2}}} \right)_0^{z_0}$$

$$z_0 = \frac{-\frac{k}{sE} + \sqrt{(\frac{k}{sE})^2 + 4}}{2}$$

$$\theta = 2\sin^{-1}\left[\frac{\frac{k}{SE}}{\sqrt{4 + \frac{k^2}{(SE)^2}}}\right]$$

 $-\sin x = \sin(-x)$ 

$$\theta = 2\sin^{-1}\left(\frac{\frac{k}{SE}}{\sqrt{4 + \frac{k^2}{(SE)^2}}}\right)$$



$$S = \frac{kctg\frac{\theta}{2}}{mv^2}$$

$$\sigma(\theta) = \frac{s}{\sin \theta} \left| \frac{ds}{d\theta} \right|$$

$$\sigma(\theta) = \left[k / 2mv^2 \sin^2 \frac{\theta}{2}\right]^2$$

### 对卢瑟福散射

$$F = \frac{1}{4\pi\varepsilon} \frac{2zq^2}{r^2}$$

$$k = \frac{zq^2}{2\pi\varepsilon}$$

$$\sigma(\theta) = \left(\frac{zq}{4\pi\varepsilon}\right)^2 \csc^4 \frac{\theta}{2}$$



#### § 3. L系C系的关系

一. 散射角的关系

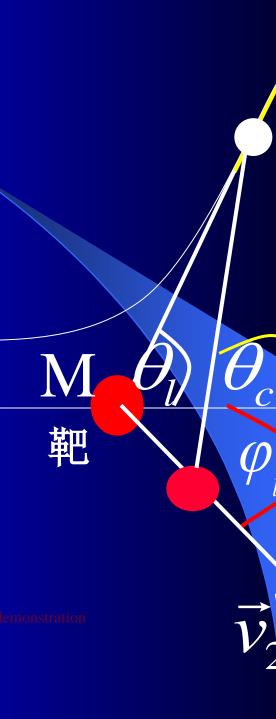
## L系

散射前靶静止m

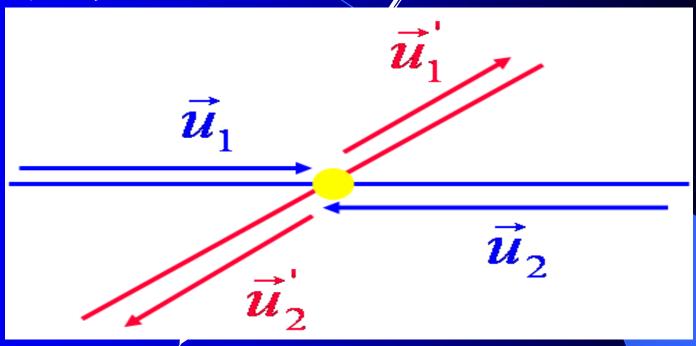
 $\theta_1$  上系中散射角

 $\varphi_l$  L系中靶反冲角

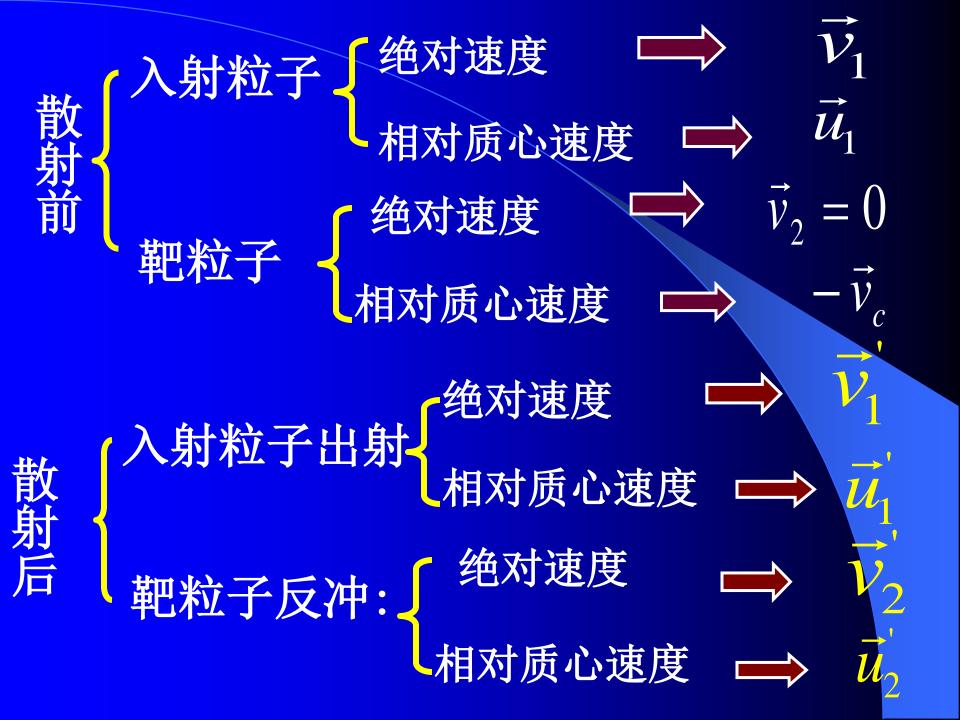
 $\theta_c$  C系中散射角



## C系中的图象



demonstration



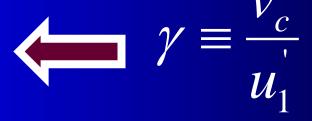
$$\vec{v}_1' = \vec{v}_c + \vec{u}_1$$

$$\vec{v}_1 \cos \theta_l = v_c + u_1 \cos \theta_c$$

$$\vec{v}_1 \sin \theta_l = u_1 \sin \theta_c$$

$$\vec{v}_1' \sin \theta_l = u_1 \sin \theta_c$$

$$tg\theta_{l} = \frac{\sin\theta_{c}}{\frac{v_{c}}{u_{1}} + \cos\theta_{c}}$$



$$\Rightarrow tg\theta_l = \frac{\sin\theta_c}{\gamma + \cos\theta_c}$$

$$\begin{cases} v_1^{'2} = v_c^2 + u_1^{'2} - 2v_c u_1 \cos(\pi - \theta_c) \\ v_1^{'2} = v_c^2 + u_1^{'2} + 2v_c u_1 \cos\theta_c \\ v_1^{'2} \cos\theta_l = v_c + u_1 \cos\theta_c \\ v_1^{'2} \cos^2\theta_l = u_1^{'2} (1 + \gamma \cos\theta_c)^2 \\ v_2^{'2} + u_1^{'2} + 2v_c u_1 \cos\theta_c \\ v_2^{'2} + u_1^{'2} + 2v_c u_1 \cos\theta_c \end{cases}$$

In fact, from 
$$tg\theta_l = \frac{\sin\theta_c}{\gamma + \cos\theta_c}$$
 we can have,

$$\frac{1}{\cos^2 \theta_l} = 1 + tg^2 \theta_l = 1 + \left(\frac{\sin \theta_c}{\gamma + \cos \theta_c}\right)^2$$

$$= \frac{1 + 2\gamma \cos \theta_c + \gamma^2}{(\gamma + \cos \theta_c)^2}$$

i.e.,

$$\cos \theta_l = \frac{\gamma + \cos \theta_c}{\sqrt{1 + \gamma^2 + 2\gamma \cos \theta_c}}$$

#### 对弹性碰撞设靶M散射前静止

$$\vec{v}_{c} = \frac{m\vec{v}_{1}}{m+M}$$

$$\vec{u}_{1} = \vec{v}_{1} - \vec{v}_{c} = \frac{(m+M)\vec{v}_{1} - m\vec{v}_{1}}{M+m} = \frac{M\vec{v}_{1}}{M+m}$$

$$\vec{u}_{1} = \frac{Mv_{1}}{M+m}$$

$$\Rightarrow \gamma \equiv \frac{v_{c}}{u_{1}} = \frac{m}{M}$$

$$tg\theta_{l} = \frac{\sin\theta_{c}}{m+M}$$

 $+\cos\theta_c$ 

$$tg\theta_{l} = \frac{\sin\theta_{c}}{\frac{m}{M} + \cos\theta_{c}}$$

Discussion:

当 
$$m << M$$
,  $\frac{m}{M} = 0$  中  $\theta_l = \theta_d$ 

当 
$$m = M$$
  $tg\theta_l = \frac{\sin \theta_c}{\frac{m}{M} + \cos \theta_c}$ 

$$tg\theta_l = \frac{\sin\theta_c}{1 + \cos\theta_c} = tg\frac{\theta_c}{2} \implies \theta_l = \frac{\theta_c}{2}$$

$$v_2 \cos \varphi_l = v_c + u_2 \cos(\pi - \varphi_c)$$

$$v_2 \sin \varphi_l = u_2 \sin \varphi_c$$

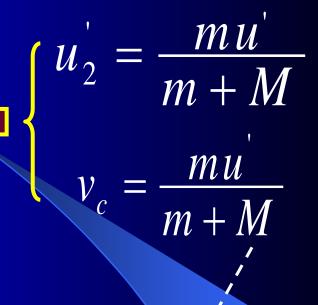
$$tg\varphi_l = \frac{u_2 \sin \varphi_c}{v_c + u_2 \cos \varphi_c}$$

对弹性碰撞设靶M散射前静止

$$u = u' = v_1$$
  $u_2' = \frac{mu'}{m + M}$ 

$$v_c = \frac{mv_1}{m+M} = \frac{mu}{m+M} = \frac{mu}{m+M}$$

$$tg\varphi_l = \frac{u_2' \sin \varphi_c}{v_c + u_2' \cos \varphi_c}$$



$$tg\varphi_l = \frac{\sin\varphi_c}{1 + \cos\varphi_c} = tg\frac{\varphi_c}{2}$$

$$\varphi_l = \frac{\varphi_c}{2} \qquad \varphi_l = \frac{\pi - \theta_c}{2}$$

$$tg\varphi_l = \frac{\sin\varphi_c}{1 + \cos\varphi_c} = tg\frac{\varphi_c}{2}$$

#### 二. 能量关系

$$\varphi_l = \frac{\varphi_c}{2}$$

L系 
$$E_l = \frac{1}{2} m v_1^2$$

C系  $E_c = \frac{1}{2} m u_1^2 + \frac{1}{2} M u_2^2$ 

$$= \frac{1}{2} \frac{mM}{m+M} v_1^2$$

$$\frac{E_l}{E_c} = \frac{m + M}{M}$$

对弹性散射 m=M  $E_l=2E_c$ 

## 入射粒子动能损失

散射前 
$$E_0 = \frac{1}{2} m v_1^2$$
, 靶 粒子静止  
散射后  $E_1 = \frac{1}{2} m (\vec{v}_c + \vec{u}_1')^2$   
 $E_1 = \frac{1}{2} m (v_c^2 + u_1'^2 + 2\vec{v}_c \vec{u}_1')$   
 $E_1 = \frac{1}{2} m u_1'^2 (1 + \gamma^2 + 2\gamma \cos \theta_c)$ 

$$E_1 = \frac{1}{2} m (\frac{Mu}{M+m})^2 (1 + \gamma^2 + 2\gamma \cos \theta_c)$$

$$= \frac{1}{2} m v_1^2 \left(\frac{1}{1+\gamma}\right)^2 (1+\gamma^2+2\gamma) \gamma \equiv \frac{v_c}{u_1} = \frac{m}{M}$$

$$E_0 \quad E_1 = \frac{1}{2} m u_1^2 (1+\gamma^2+2\gamma \cos \theta_c)$$

$$E_0$$
  $E_1 = \frac{1}{2} m u_1^{'2} (1 + \gamma^2 + 2\gamma \cos \theta_c)$ 

$$\frac{E_1}{E_0} = (\frac{1}{1+\gamma})^2 (1 + \gamma^2 + 2\gamma \cos \theta_c)$$

$$\frac{E_1}{E_0} = (\frac{1}{1+\gamma})^2 (1+\gamma^2 + 2\gamma \cos \theta_c)$$

Discussion:

$$m = M \Rightarrow (\gamma = 1)$$

$$\frac{E_1}{E_0} = \frac{2 + 2\cos\theta_c}{4} + \frac{1 + \cos\theta_c}{2}$$

$$\cos \theta_l = \frac{\gamma + \cos \theta_c}{\sqrt{1 + \gamma^2 + 2\gamma \cos \theta_c}} \implies \cos \theta_l = \sqrt{\frac{1 + \cos \theta_c}{2}}$$

$$\frac{E_1}{E_0} = \cos^2 \theta_l$$

 $\cos \frac{\theta_c}{2}$ 

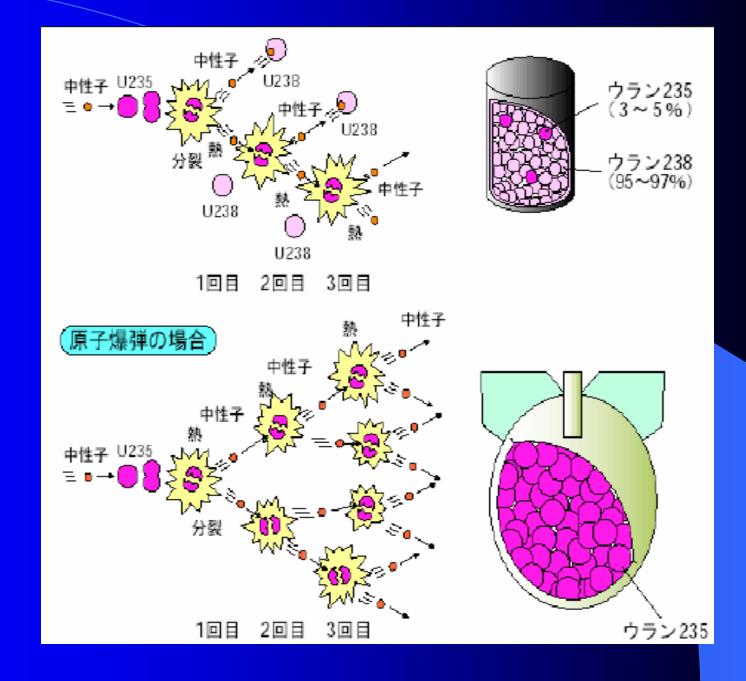
入射粒子能量全部传给靶粒子

235U

裂变能  $n + ^{235}U \rightarrow X + Y + \Delta E$ 

**200** MeV

235**U** 



## 核能从何而来

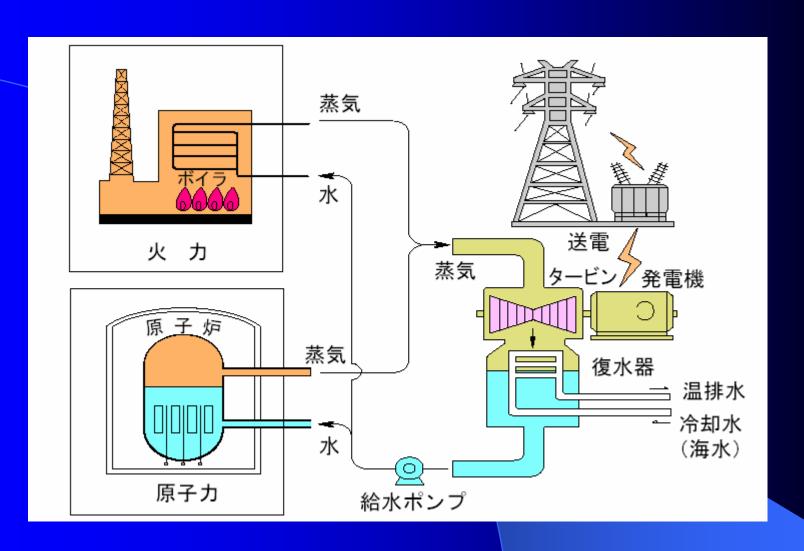
● 爱因斯坦质能关系式: E=mc²

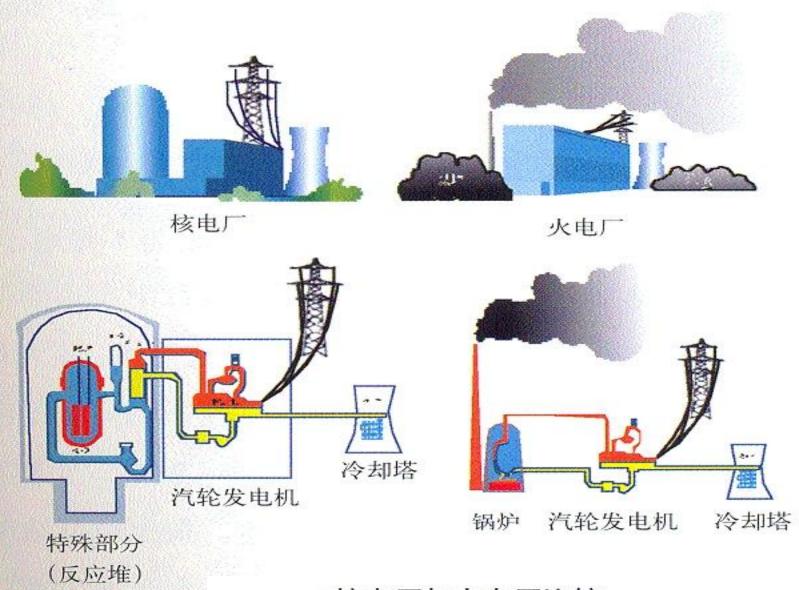
● 裂变反应:

 $^{235}U + n \Rightarrow X + Y + 2.5 n + 200 Mev$ 

燃烧1千克铀-235相当于2700吨标准煤 大270万倍

## 核电站





核电厂与火电厂比较

#### 三. 散射截面关系

总載面 
$$\sigma_l = \sigma_c$$
 与坐标系无关  
 $: \sigma_l = \iint \sigma(\theta_l) \sin \theta_l d\theta_l d\varphi$   
 $= 2\pi \int \sigma(\theta_l) \sin \theta_l d\theta_c$   
 $: \sigma_c = \iint \sigma(\theta_c) \sin \theta_c d\theta_c d\varphi$   
 $= 2\pi \int \sigma(\theta_c) \sin \theta_c d\theta_c$   
 $\int \sigma(\theta_l) \sin \theta_l d\theta_l = \int \sigma(\theta_c) \sin \theta_c d\theta_c$ 

$$d\sigma = sdsd\varphi = \sigma(\theta) \sin \theta d\theta d\varphi$$

$$\int \sigma(\theta_l) \sin \theta_l d\theta_l = \int \sigma(\theta_c) \sin \theta_c d\theta_c$$

$$= \int \sigma(\theta_c) \sin \theta_c \left| \frac{d\theta_c}{d\theta_l} \right| d\theta_l$$

$$\sigma(\theta_l) \sin \theta_l = \sin \theta_c \sigma(\theta_c) \left| \frac{d\theta_c}{d\theta_l} \right|$$

$$\sigma(\theta_l) \sin \theta_l = \sin \theta_c \sigma(\theta_c) \left| \frac{d\theta_c}{d\theta_l} \right|$$

$$\frac{\sigma(\theta_l)}{\sigma(\theta_c)} = \frac{\sin \theta_c}{\sin \theta_l} \left| \frac{d\theta_c}{d\theta_l} \right| \qquad \cos \theta_l = \frac{\gamma + \cos \theta_c}{\sqrt{1 + \gamma^2 + 2\gamma \cos \theta_c}}$$

$$\sigma(\theta_c) = \left| \frac{\gamma + \cos \theta_c}{(1 + \gamma^2 + 2\gamma \cos \theta_c)^{\frac{3}{2}}} \right| \sigma(\theta_l(\theta_c))$$