C6 Interference

- Interference is the **major feature** of waves. The nature of wave.
- Conditions for interference;
- Methods and apparatus for interference;
- Typical interferometers.

linear superposition principle applies?

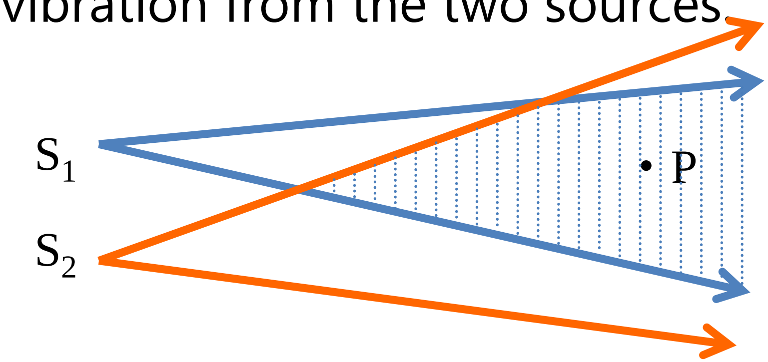
Linear optics ⇔ Nonlinear optics

→

Generally, electric field strength

§ 6.1 Superposition of waves

- **Principle of superposition**: When the simple harmonic wave propagates freely in space, all points in the space vibrates. The total vibration for a point in the overlapping zone is a superposition of the vibration from the two sources.



- Light waves are vector waves. >>Vector superposition; When treated as scalar, it means a special component.
- The principle of superposition for light wave applies when:

1） Strictly OK in the vacuum.
2） When the medium is non-linear and the intensity of the light wave is large ($\sim 10^{13}\,\mathrm{W/cm^2}$), it does not hold.

How to judge whether an interference happen?
Because of the high frequency of light, we cannot follow the temporal oscillation of **E**-field.
>> Look at the light intensity distribution in the overlapping region.

How to analysis?

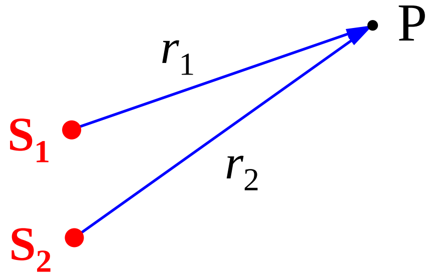
- The amplitude of the combined wave.
- The intensity of light.
- Analyze the light intensity as a function of position.

Coherent superposition

Two light wave have same frequency and the same polarization component. So,

$$E_1 = A_1 e^{i(kr_1 - \omega t + \varphi_{01})}$$

$$E_2 = A_2 e^{i(kr_2 - \omega t + \varphi_{02})}$$



According to **principle of superposition**, at P :

$$E = E_1 + E_2$$

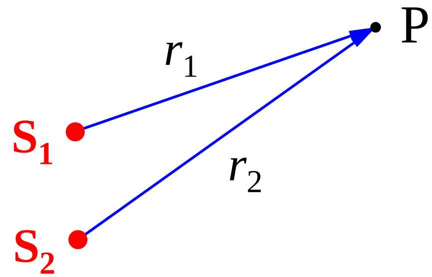
$$= \left[A_1 e^{i(kr_1 + \varphi_{01})} + A_2 e^{i(kr_2 + \varphi_{02})} \right] e^{-i\omega t}$$

So the amplitude is:

$$\tilde{E} = A_1 e^{i(kr_1 + \varphi_{01})} + A_2 e^{i(kr_2 + \varphi_{02})}$$

Coherent superposition

Complex amplitude : $\tilde{E} = A_1 e^{i(kr_1+\varphi_{01})} + A_2 e^{i(kr_2+\varphi_{02})}$



At P :

$$I \propto \tilde{E} \cdot \tilde{E}^*$$

$$= \left[A_1 e^{i(kr_1+\varphi_{01})} + A_2 e^{i(kr_2+\varphi_{02})} \right] \cdot \left[A_1 e^{-i(kr_1+\varphi_{01})} + A_2 e^{-i(kr_2+\varphi_{02})} \right]$$

$$= A_1^2 + A_2^2 + A_1 A_2 \left(e^{i(kr_1-kr_2+\varphi_{01}-\varphi_{02})} + e^{-i(kr_1-kr_2+\varphi_{01}-\varphi_{02})} \right)$$

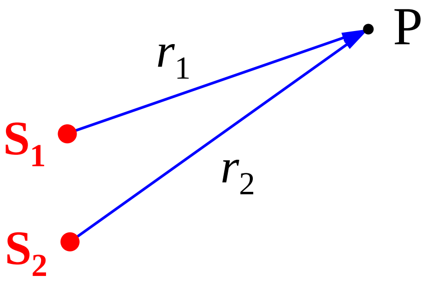
$$= A_1^2 + A_2^2 + 2A_1 A_2 \cos\left(kr_1 - kr_2 + \varphi_{01} - \varphi_{02} \right)$$

$$\propto I_1 + I_2 + \underbrace{2\sqrt{I_1 I_2} \cos\delta}_{\text{interference}}$$

Coherent superposition

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = kr_1 - kr_2 + \varphi_{01} - \varphi_{02}$$



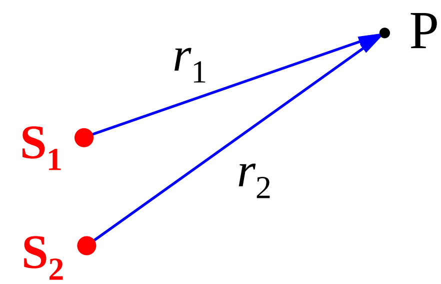
① At P the intensity $\neq I_1 + I_2$, an extra interference term related to the locations.

② For a given point P, d is fixed (r_2 - r_1 is fixed), the light intensity is constant (does not change with time).

Coherent superposition

Analysis: $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$

$$\delta = kr_1 - kr_2 + \varphi_{01} - \varphi_{02}$$



③ For different points, δ changes so that I is different, i.e., a spatial distribution.

\>\> **Interference**

④ When $\delta = 2m\pi$ $\cos 2m\pi = 1$ $I \propto A_1^2 + A_2^2 + 2A_1A_2\cos\delta$

$I = (A_1 + A_2)^2$ (maxima, **total constructive interference**)

When $\delta = (2m+1)\pi$ $\cos(2m+1)\pi = -1$

$I = (A_1 - A_2)^2$ (minima, **total destructive interference**)

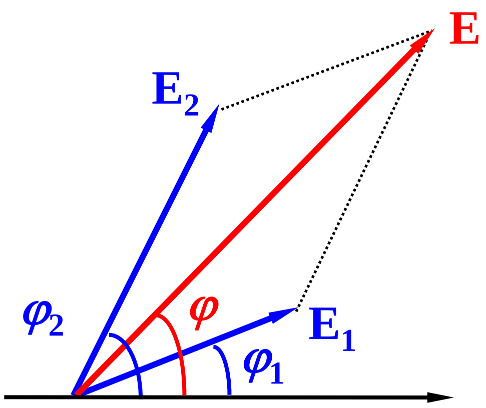
Incoherent superposition

Generally, for two monochromatic sources, the **frequency** of the emitted light is different, and the **polarization** is different.

$$\mathbf{E}_1 = \mathbf{A}_1 e^{i\left(k_1 r_1 - \omega_1 t + \varphi_{01}\right)}$$

$$\mathbf{E}_2 = \mathbf{A}_2 e^{i\left(k_2 r_2 - \omega_2 t + \varphi_{02}\right)}$$

$$\mathbf{E} = \mathbf{E}_1 + \mathbf{E}_2$$



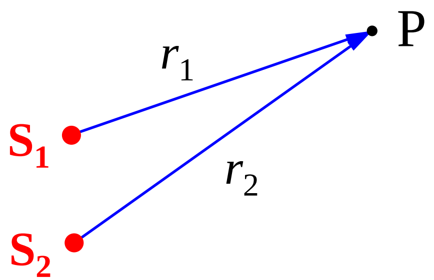
- Time average during observation time

At P: $I \propto \mathbf{E} \cdot \mathbf{E}^*$

$$= \mathbf{E}_1 \cdot \mathbf{E}_1^* + \mathbf{E}_2 \cdot \mathbf{E}_2^* + \mathbf{E}_1 \cdot \mathbf{E}_2^* + \mathbf{E}_1^* \cdot \mathbf{E}_2$$

$$\propto I_1 + I_2 + I_{12}$$

Only S_1 **Only S_2** **Interference**



Incoherent superposition

$$I_{12} \propto \mathbf{E}_1 \cdot \mathbf{E}_2^* + \mathbf{E}_1^* \cdot \mathbf{E}_2$$

$$= \mathbf{A}_1 \cdot \mathbf{A}_2 \left[e^{i\left(k_1 r_1 - k_2 r_2 - \omega_1 t + \omega_2 t + \varphi_{01} - \varphi_{02}\right)} + e^{-i\left(k_1 r_1 - k_2 r_2 - \omega_1 t + \omega_2 t + \varphi_{01} - \varphi_{02}\right)} \right]$$

$$= 2\mathbf{A}_1 \cdot \mathbf{A}_2 \cos\left(k_1 r_1 - k_2 r_2 - \omega_1 t + \omega_2 t + \varphi_{01} - \varphi_{02}\right)$$

$$= 2\mathbf{A}_1 \cdot \mathbf{A}_2 \cos\left[\left(\omega_2 - \omega_1\right)t + \theta_1 - \theta_2\right]$$

$I_{12} = 0$

① When $\omega_1 \neq \omega_2$, $\left\langle \cos\left[\left(\omega_1 - \omega_2\right)t + \theta_1 - \theta_2\right]\right\rangle = 0$

② When $\mathbf{A}_1 \perp \mathbf{A}_2$ $\mathbf{A}_1 \cdot \mathbf{A}_2 = 0$

② When $\delta = \theta_1 - \theta_2$ changes quickly and randomly over time,

Even if $\mathbf{A}_1 // \mathbf{A}_2$ and $\omega_1 = \omega_2$ $\left\langle \cos\left(\theta_1 - \theta_2\right)\right\rangle = 0$

Incoherent superposition

- The **conditions for interference** are:

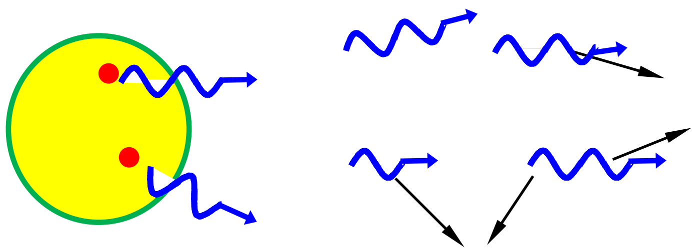
  1. **the same frequency**
  2. **same parallel vibration components**
  3. **fixed phase difference**

- As long as the two light waves do not satisfy any of the coherent conditions, the superposition is incoherent superposition.

So, $\boldsymbol{I} = \boldsymbol{I}_1 + \boldsymbol{I}_2$

Acquisition of coherent light

Ordinary source: The emitting unit is a molecule or an atom. Each luminescence is interstitial; the luminescence of different atoms is completely independent.



independent (from different atom)

independent (from the same atom but at different moment)

Frequency, polarization, phase, direction are different.

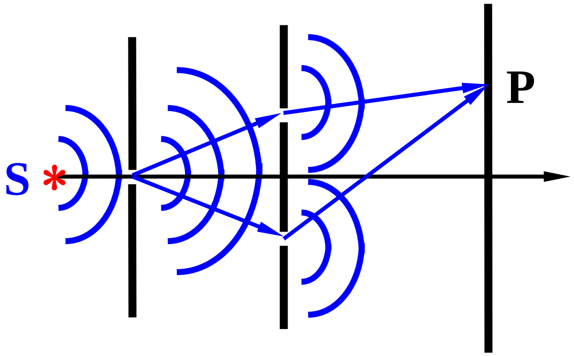
Generally, light emitted by ordinary light sources does not interference. For example: lump, flame, sunlight, etc.

How to get the coherent light? ？

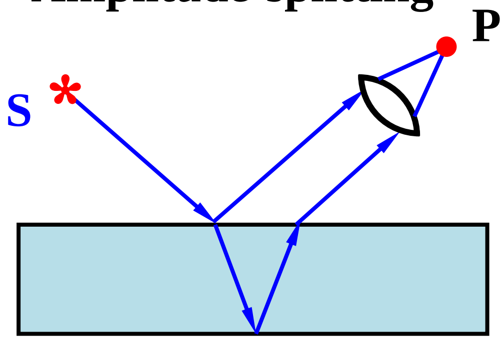
Acquisition of coherent light

- Obtaining coherent light from ordinary light source:

Wavefront -splitting



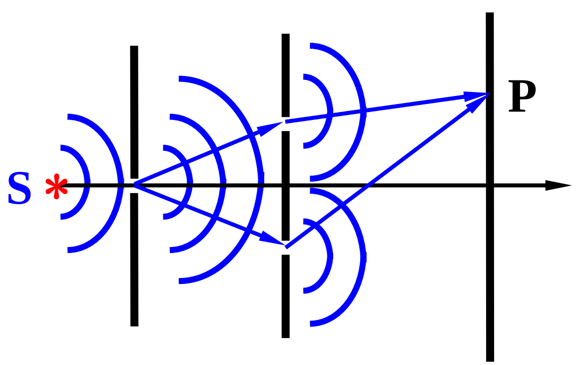
Amplitude-splitting

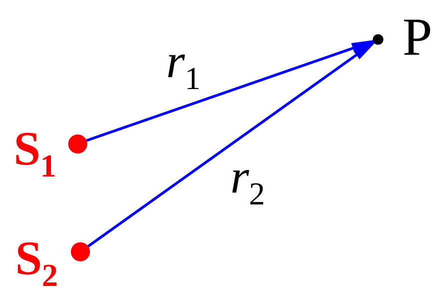


§ 6.2 Young’s Experiment

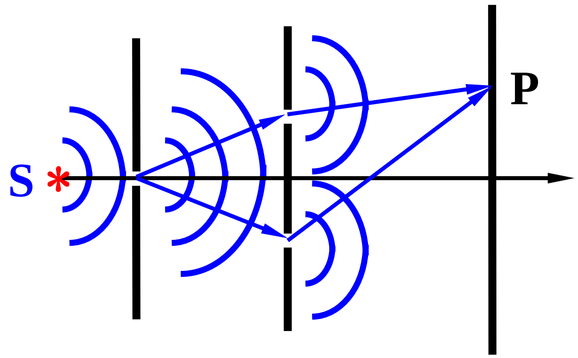
- A method to divide the wavefront of a light source into two (or more) portions as a coherent light source is called **wavefront-splitting interferometers**.
- For example: **Young’s Experiment, Fresnel Double Prism, Lloyd mirror.**

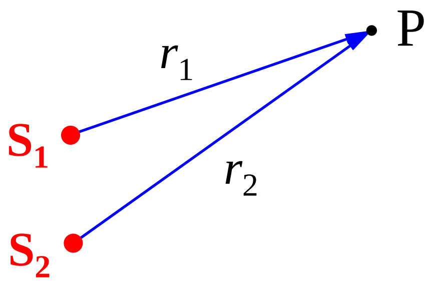
Thomas Young
1773~1829, UK





Young's Experiment





Coherent superposition:

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\delta = kr_1 - kr_2 + \varphi_{01} - \varphi_{02}$$

For Young's Experiment:

$$I_1 = I_2 \qquad \varphi_{01} - \varphi_{02} = 0$$

$$I = 2I_1\left(1+\cos\delta\right) = 4I_1\cos^2\frac{\delta}{2}$$

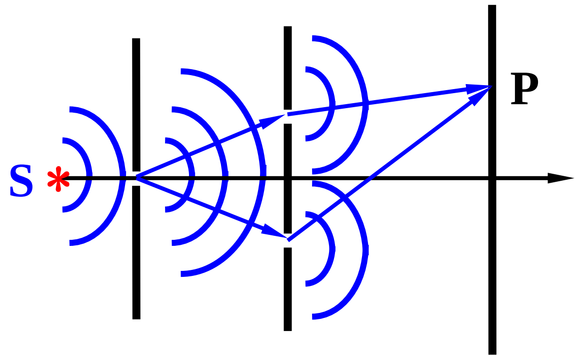
$$= 4I_1\cos^2\frac{k\left(r_1 - r_2\right)}{2}$$

$$= 4I_1\cos^2\frac{\pi\Delta}{\lambda}$$

$$\Delta = n\left(r_1 - r_2\right)$$

The optical pathlength difference

Young's Experiment



$$I = 4I_1 \cos^2 \frac{\pi\Delta}{\lambda} \qquad \Delta = n\left(r_1 - r_2\right)$$

The intensity at P only depends on the Δ, and have a constant intensity distribution.

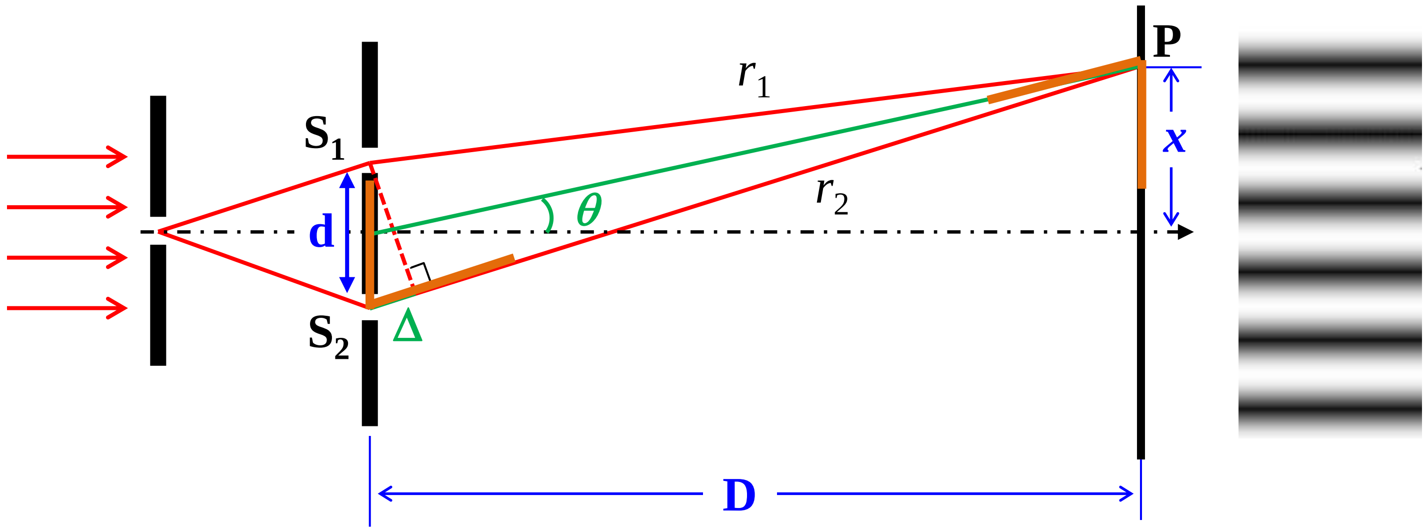
- When $\Delta = m\lambda$ $\quad m = 0, \pm 1, \pm 2 \cdots$

  $I = I_{max} = 4I_1$ For same m, forming the same **bright** fringe.

- when $\Delta = (2m' - 1)\lambda/2$ $\quad m' = \pm 1, \pm 2 \cdots$

  $I = I_{min} = 0$ For same m', forming the same **dark** fringe.

Young's Experiment



$n_{air} = 1$

D ~ 1 m
d ~ 0.1 mm
D >> d
θ → 0

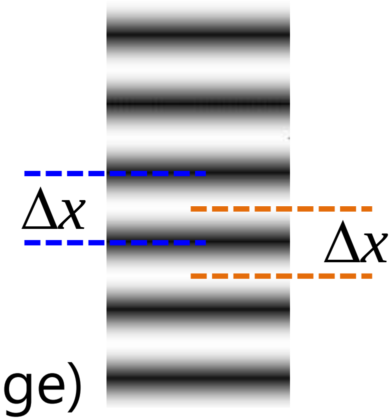
$$\Delta = r_1 - r_2 \approx d\sin\theta \approx d\tan\theta = d\,\frac{x}{D}$$

$$I = 4I_1\cos^2\frac{\pi\Delta}{\lambda} = 4I_1\cos^2\frac{\pi d}{\lambda D}x$$

The intensity of the screen is distributed in the x direction according to the square of the cosine function.
Along y, it is a constant.

Young's Experiment

Determine the position of the m^{th} bright fringe



$$\Delta = \frac{d}{D}x = m\lambda \quad \Rightarrow \quad x = \frac{D}{d}m\lambda \quad \Rightarrow \quad x_m$$

$x = 0$ 0-level bright fringe (central bright fringe)

The difference in the positions of two maxima is

$$\boxed{\Delta x = x_{m+1} - x_m = \frac{D}{d}\lambda}$$

when θ is not large, the fringes are **equally spaced**. Means that the fringes are distributed with x as a period, then Δx is the **spatial period**.

Why the condition that $D/d >> 1$ is important?

Young's Experiment

Summery:

$$\Delta \approx \frac{d}{D}x$$

$$I = 2I_1\left(1+\cos\delta\right) = 4I_1\cos^2\frac{\delta}{2} = 4I_1\cos^2\frac{\pi\Delta}{\lambda} = 4I_1\cos^2\frac{\pi d}{\lambda D}x$$

$$\delta = kr_1 - kr_2 + \varphi_{01} - \varphi_{02}$$
$$= k\Delta$$

Interference max/min

$$\delta = 2m\pi$$
$$\Delta = m\lambda$$

$$+ \quad \Delta \approx \frac{d}{D}x \quad =$$

Bright fringe $x_m = \pm m\dfrac{D}{d}\lambda$

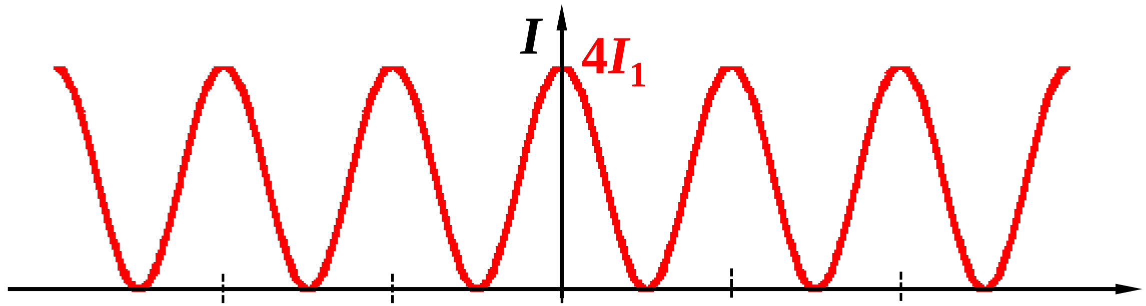
Dark fringe $x_{m'} = \pm\left(2m'-1\right)\dfrac{D}{d}\dfrac{\lambda}{2}$

Spatial period $\Delta x = \dfrac{D}{d}\lambda$

Young's Experiment

$$I = 4I_1\cos^2\frac{\delta}{2} = 4I_1\cos^2\frac{\pi\Delta}{\lambda} = 4I_1\cos^2\frac{\pi d}{\lambda D}x = 2I_1\left(1+\cos\delta\right)$$

Intensity curve:



$\delta = 2m\pi$	-4π	-2π		0		2π		4π	δ
Interference order m	-2	-1		0		1		2	
Pathlength difference	-2λ	$-\lambda$	$-\frac{\lambda}{2}$	0	$\frac{\lambda}{2}$	λ	$-\frac{3\lambda}{2}$	2λ	
	-2nd bright	-1st bright	-1st dark	0th bright	1st dark	1st bright	2nd dark	2nd bright	

$$\Delta = \frac{\delta\lambda}{2\pi} = \begin{cases} 2m\dfrac{\lambda}{2} \\ \left(2m'+1\right)\dfrac{\lambda}{2} \end{cases}$$

Young's Experiment

$$\boxed{\Delta x = \frac{D}{d}\lambda}$$

- When $D/d >> 1$, it can be used to measure the wavelength.

E.g., two narrow slits in a thin metal sheet are 0.6 mm apart center to center. When illuminated directly by Hg light source (in air) a fringe pattern appears on a screen 2500 mm away and the distance between dark fringe center is 2.27 mm. Determine the wavelength.

Solution:

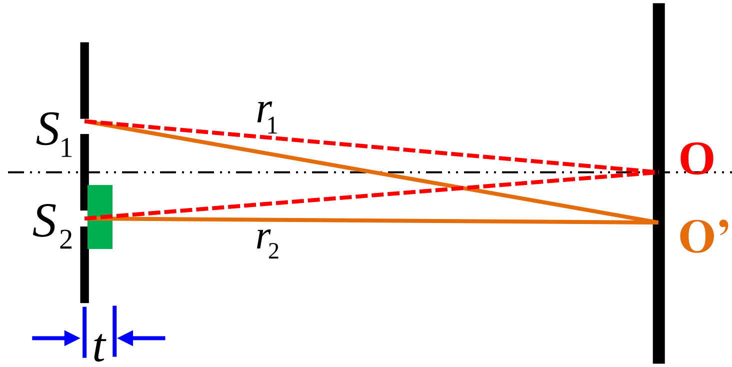
$$\lambda = \frac{d}{D}\Delta x = \frac{0.60 \times 2.27}{2500} = 5.45 \times 10^{-4}\,\text{mm} = 545\,\text{nm}$$

The exact value 546.07 nm

Young's Experiment

A change in optical path causes the movement of the fringes.

Insert a mica sheet in front of one slit of Young's interference, $n = 1.58$, $\lambda = 550$ nm. Interference fringes moved 7. Determine the thickness?



Solution: The 0th fringe will move downward, the difference of OPL at O is:

$$\Delta = (n-1)t$$

If O now becomes the k^{th} bright fringe, then: $k\lambda = (n-1)t$

$$t = \frac{k\lambda}{(n-1)} = \frac{7\times 550\times 10^{-6}\,\text{mm}}{1.58-1} = 6.64\times 10^{-3}\,\text{mm}$$

A method of accurately measuring the thickness. If the thickness is known, the refractive index can be measured.

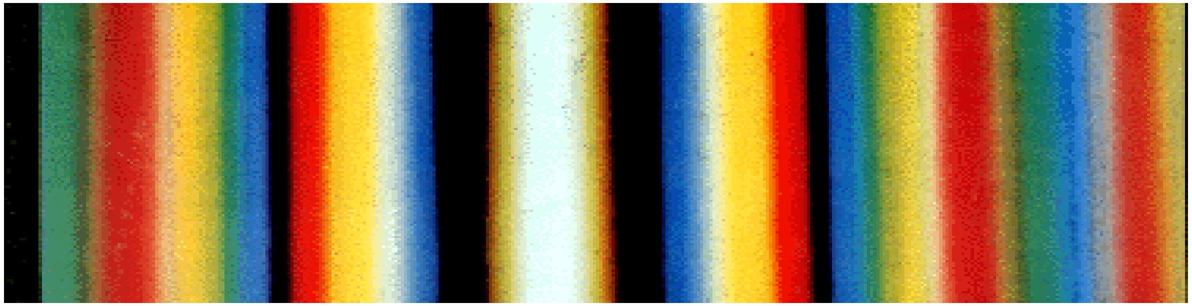
Young's Experiment

$$\boxed{\Delta x = \frac{D}{d}\lambda}$$

- In a same experiment, λ is different, Δx is also different.
- For white light, **0th fringe is white**.
- The rest of the level of the highlights is centered on level 0 to form the ribbon (from blue to red); overlap of the fringe for the second and third order, and higher order (why?)
- A way of dispersing light without dispersion of the medium.

Young's Experiment

Incident light is red light

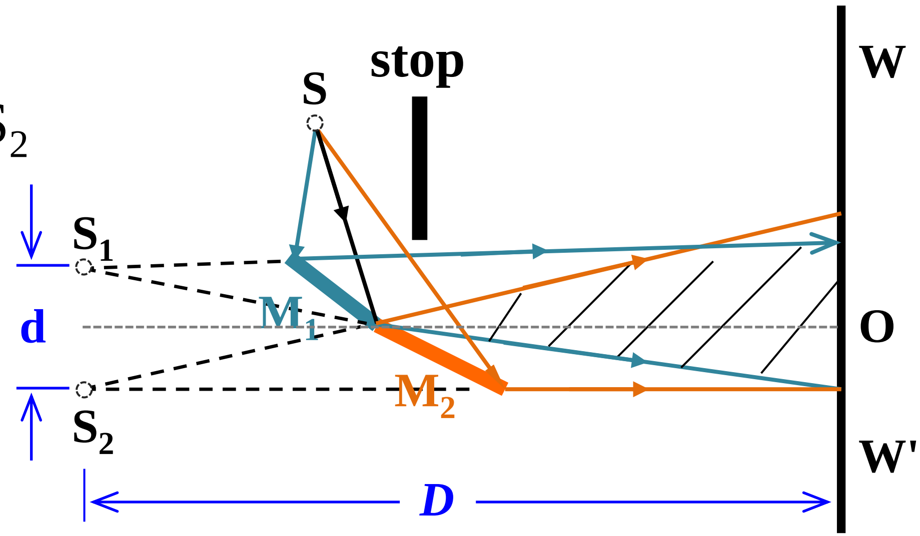


Incident light is white light

More wavefront-splitting

① Fresnel Double Mirror

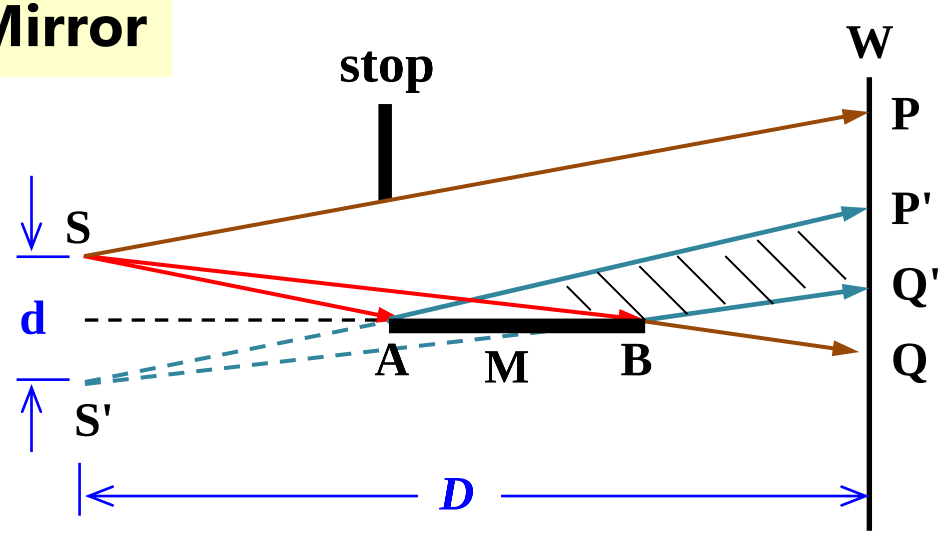
- Imaginary image S_1 and S_2
- $\overline{S_1S_2}$ is parallel to $\overline{WW'}$
- $d << D$



The center O point of the interference fringes on the screen is on the vertical bisector of the line connecting the two imaginary images.

More wavefront-splitting

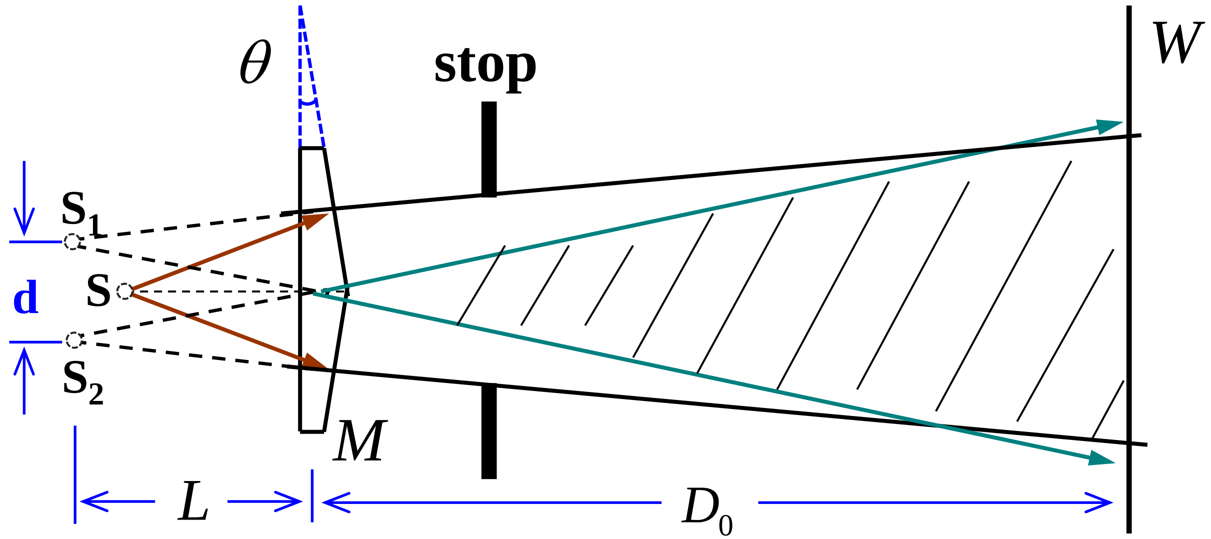
② Lloyd's Mirror



- When the screen W is moved to B, the optical path difference from S and S' to point B is zero, but a dark fringe is observed. This verifies that there is a half-wave loss during reflection .
- The obtained fringe patterns is complementary to that of the Young's double-slit experiment.

More wavefront-splitting

③ Fresnel Double Prism



It can be proved in Geometrical optics that:

$$d = 2\theta L(n-1)$$

Visibility

- **Visibility**

$$\boxed{V = \frac{I_{\max} - I_{\min}}{I_{\max} + I_{\min}}}$$

$I_{\min} = 0$, $V = 1$: clearest fringes and perfect coherent;

$I_{\min} = I_{\max}$, $V = 0$: no fringes and completely incoherent;

Generally, $0 < I_{\min} < I_{\max}$, $0 < V < 1$: partly coherent.

The visibility of interference fringes characterizes the degree of coherence of lightwaves.

Visibility

- **The main reason for the decline of V:**

(1) The amplitude of the two coherent beams have a big difference;

$$V=\frac{I_{\max}-I_{\min}}{I_{\max}+I_{\min}}=\frac{(A_1+A_2)^2-(A_1-A_2)^2}{(A_1+A_2)^2+(A_1-A_2)^2}$$

$$=\frac{2A_1A_2}{A_1^2+A_2^2}=\frac{2\,A_1/A_2}{1+(A_1/A_2)^2}$$

V depends on the amplitude ratio of the two coherent lightwaves.

(2) The light source always has a certain geometrical width, not a perfect point or line source.

(3) The light source always has a certain spectral width, non-monochromatic light source.

Next week

Young's Experiment, Temporal and Spatial Coherence, Equal Inclination:
Sections 9.2, 9.3, 9.4.1

§ 6.3 Spatial/temporal coherence

Assumptions in Young's experiment :

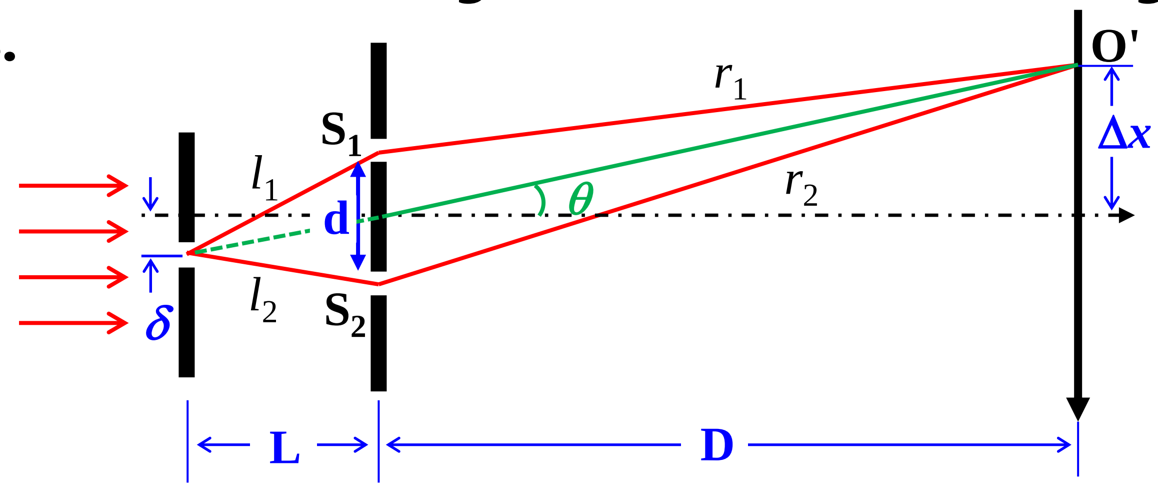
1. **S** is monochromatic light source;
2. The widths of the sources are very small.

Actually, the incident wave always has a certain frequency bandwidth, and $\mathbf{S}$, $\mathbf{S_1}$ and $\mathbf{S_2}$ always have a certain width.

- **Spatial coherence**: The effect of the width of the light source on the visibility of the interference fringes.
- **Temporal coherence**: The effect of the spectral width of light source on the visibility of interference fringes.

Spatial coherence

The movement of the light source causes a change in the fringes.



Look at a particular fringe (e.g. the 0^{th}). If the position of the 0^{th} fringe O move to O', and since its position is determined by the condition

$$\Delta = (l_1 + r_1) - (l_2 + r_2) = 0 \quad \text{that is} \quad l_1 - l_2 = r_2 - r_1$$

Source moves down, $l_1 > l_2$, $r_1 < r_2$, fringes move up.

Source moves up, $l_1 < l_2$, $r_1 > r_2$, fringe move down.

Spatial coherence

Similar to D and x

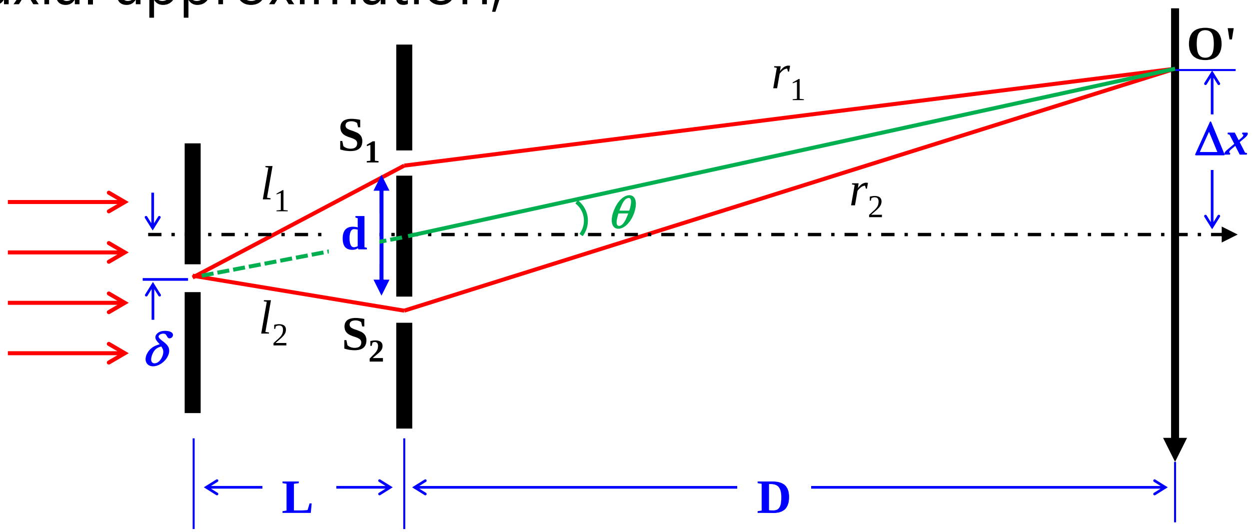
$$\boxed{\Delta = r_1 - r_2 = \frac{d}{D}x}$$

Under the paraxial approximation,

$$r_1 - r_2 = \frac{d}{D}\Delta x'$$

$$l_1 - l_2 = d\,\frac{\delta}{L}$$

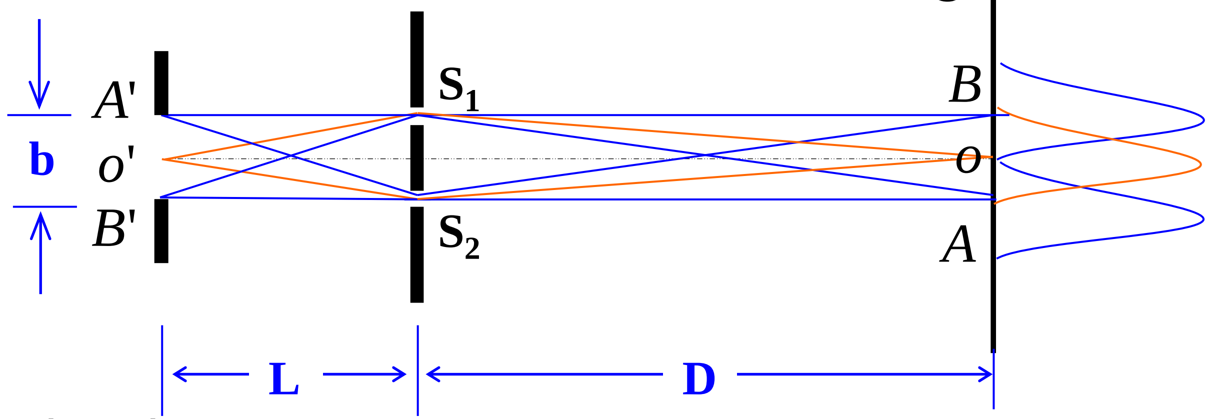
$$\Delta x' = -\frac{D}{L}\delta$$



In summary, the movement of the light source changes the difference of OPL after the two slits, resulting in a movement of the fringes.

Spatial coherence

Each point within the width *b* of the light source can be regarded as a point source and a set of interference fringes are formed on the screen. The overall effect is equivalent to the incoherent superposition of each set of interference fringes.



- Consider the extreme A', B' situation

Fringes of A'(B') move up or down $\frac{D}{L} \cdot \frac{b}{2}$

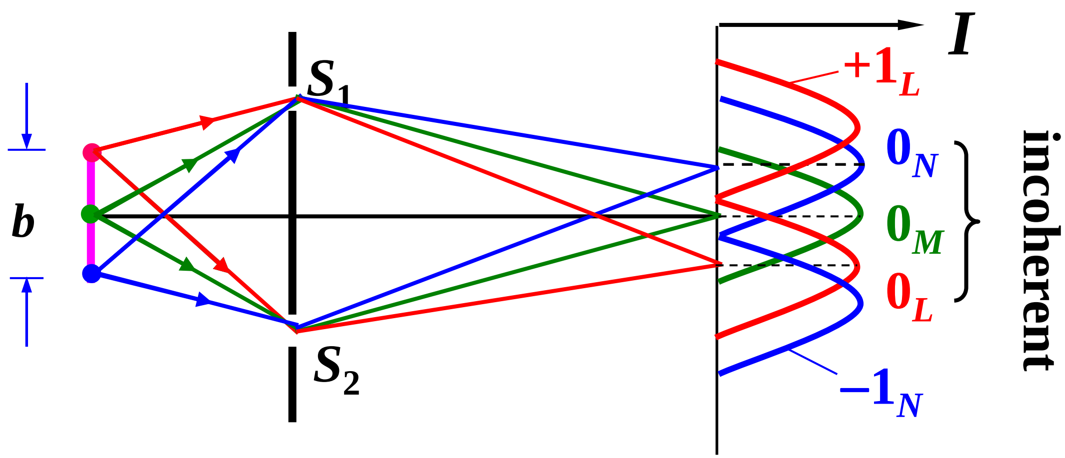
$$\boxed{\Delta x' = -\frac{D}{L}\delta}$$

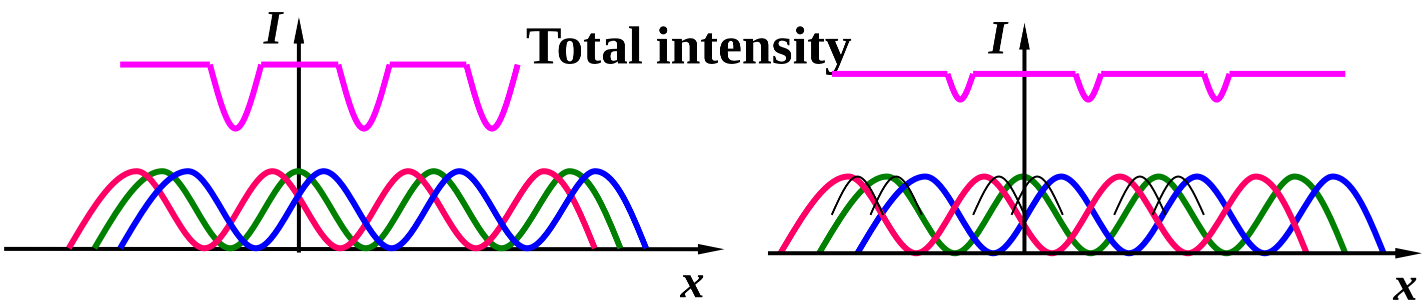
The interference formed by the light source of width *b*, the width of zero-order bright lines is:

$$\boxed{\Delta x' = \frac{D}{L} b}$$

Spatial coherence

- V is mainly determined by the width b of the light source. The smaller b is, the larger the visibility V is, and the clearer the interference fringes are.





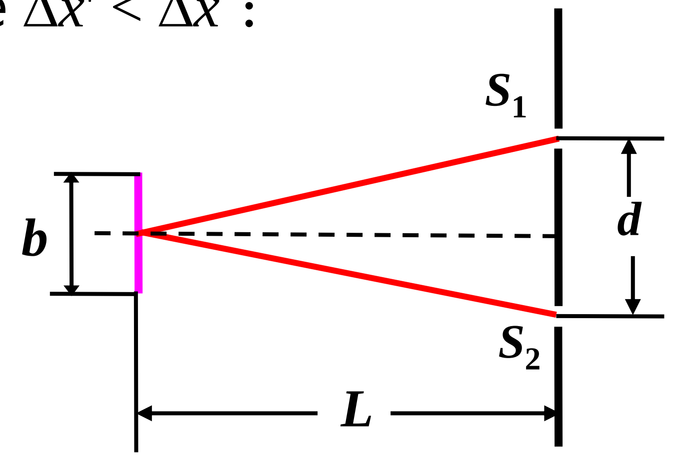
Spatial coherence

- The spacing of the fringe: $\boxed{\Delta x = \frac{D}{d}\lambda}$

When the width of the 0th bright fringe $\Delta x' > \Delta x$. no interference.

To observe fringes, we should have $\Delta x' < \Delta x$:

$$\Delta x' = \frac{D}{L} b < \frac{D}{d}\lambda = \Delta x$$



$$\boxed{b < b_c \equiv \frac{L}{d}\lambda}$$ Critical width

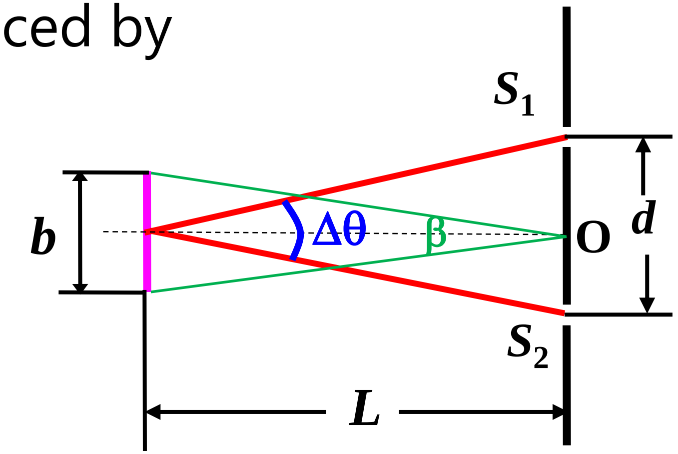
$$d = \frac{L}{b}\lambda$$

Indicates the maximum lateral distance between S_1 and S_2 (a distance L away from the source), can be apart if they are kept coherent.

Spatial coherence

The critical width can also be replaced by the aperture angle

$$\boxed{\Delta\theta \approx \frac{d}{L} = \frac{\lambda}{b}}$$



$\Delta\theta$ becomes larger, the spatial coherence gets better.

The aperture angle can also be expressed by the angle of the light source to the **O** point

$$\beta \approx \frac{b}{L}$$

$$b < b_c \equiv \frac{L}{d}\lambda \quad \Rightarrow \quad \boxed{\beta < \beta_c \equiv \frac{\lambda}{d}}$$

Spatial coherence

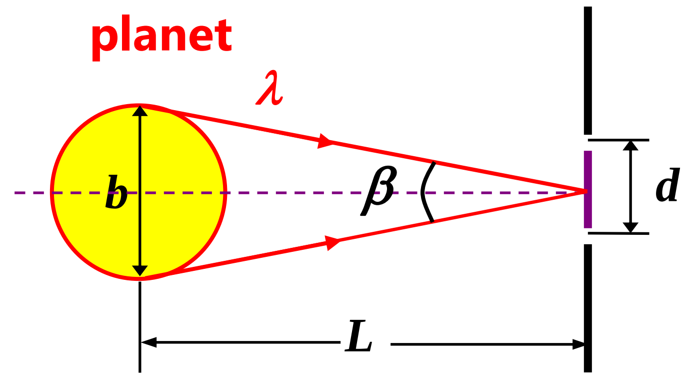
- **Application: Using spatial coherence to measure the angular diameter of a planet.**

$$d_c = \frac{L}{b}\lambda$$

Indicates the maximum lateral distance between S_1 and S_2 (a distance L away from the source), can be apart if they are kept coherent.

Increase d slowly until the fringe disappear, we get d_c.

$$\boxed{\beta < \beta_c \equiv \frac{\lambda}{d_c}}$$

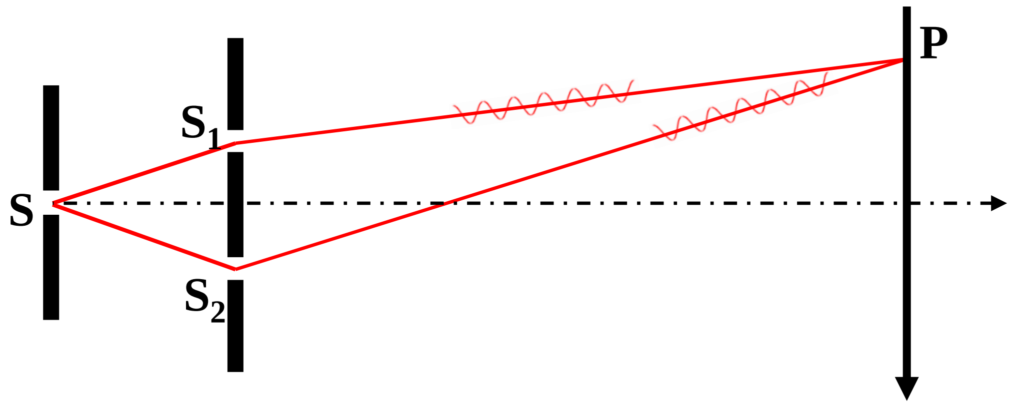


Taking into account the diffraction

$$\boxed{\beta_c \equiv 1.22\frac{\lambda}{d_c}}$$

e.g. planet $d \sim 7$ m

Temporal coherence



$\Delta \geq L$ Completely incoherence

$\Delta = 0$ Completely coherence

$0 \leq \Delta \leq L$ Partly coherence

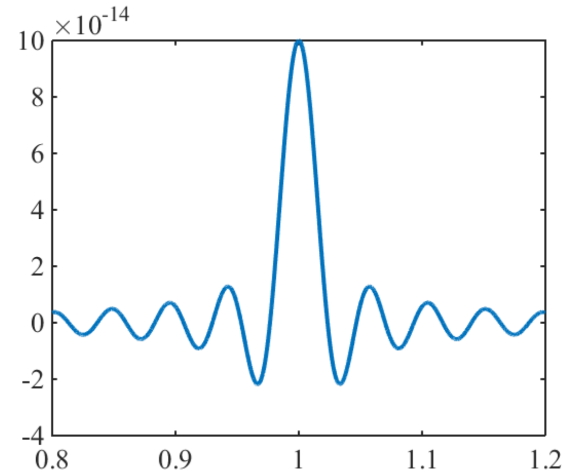
Coherence time: duration of the wave train

Coherence length: the length of the wave train

non-monochromaticity of light influence the visibility of interference fringes - **temporal coherence**.

Temporal coherence

$\Delta\nu$ ⇔ Limited coherence length



When m+1$^{\text{th}}$ bright fringe for λ overlaps with m^{th} bright fringe (λ+$\Delta\lambda$), $V = 0$

$$\Delta = (m+1)\lambda = m(\lambda + \Delta\lambda)$$

$$\Rightarrow \quad m = \frac{\lambda}{\Delta\lambda} \qquad \Rightarrow \quad \Delta = \left(\frac{\lambda}{\Delta\lambda}+1\right)\lambda \approx \frac{\lambda^2}{\Delta\lambda}$$

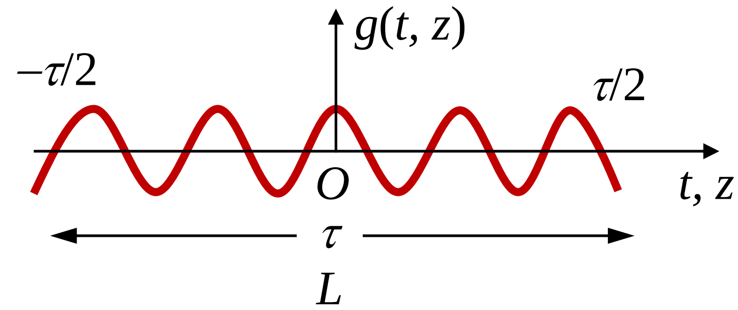
Maximum optical path difference caused by non-monochromatic light interference is the coherence length of the wavetrain.

$$L = c\tau = \frac{\lambda^2}{\Delta\lambda}$$

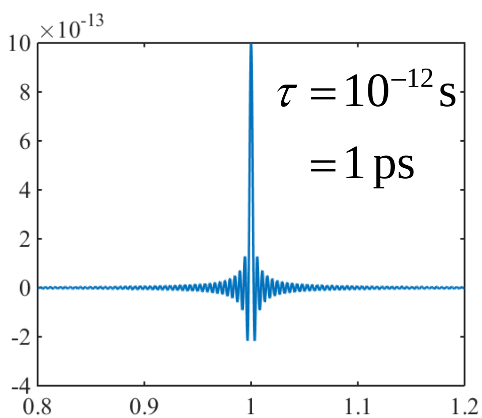
$$\Delta\nu = \Delta\left(\frac{c}{\lambda}\right) = -\frac{c}{\lambda^2}\Delta\lambda$$

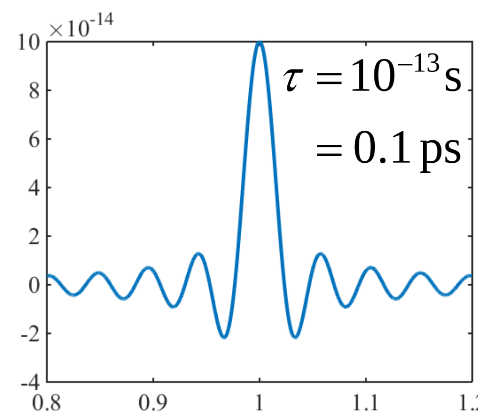
Length of wavetrain

- The relationship between wavetrain（波列）duration and spectrum width:



$$\Delta \nu = 1/\tau$$





- Spatial length of wavetrain:

$$L = c\tau = \frac{\lambda^2}{\Delta\lambda} \qquad \text{and,} \qquad \Delta \nu = \Delta\left(\frac{c}{\lambda}\right) = -\frac{c}{\lambda^2}\Delta\lambda$$

§ 6.4 Thin film interference

- Thin film interference is amplitude-splitting interference. Light is reflected and refracted at the upper and lower surfaces of the film. The reflected or transmitted light is a coherent beam, which is obtained by the splitting the energy of the incident light.

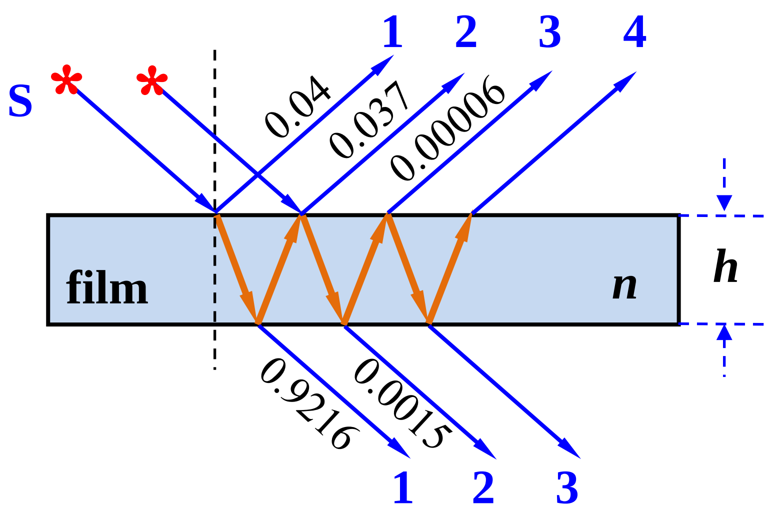
From **Baidu**

Thin film interference in daily life:

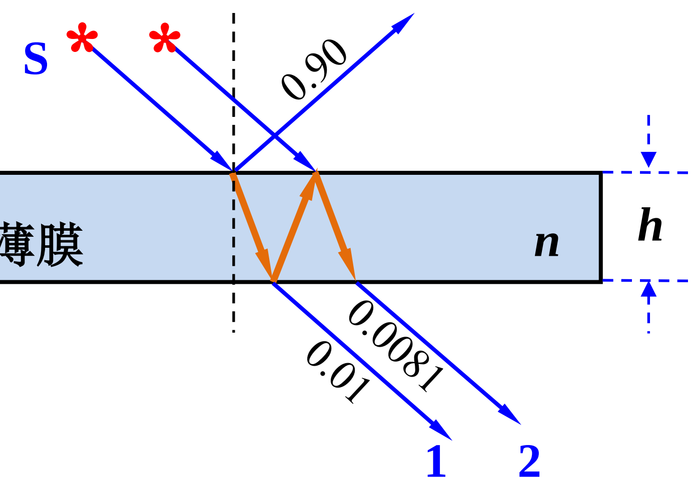
Soap bubble, Oil film, Insect wings…

When ***T*** is high | When ***R*** is high

Reflected light interference



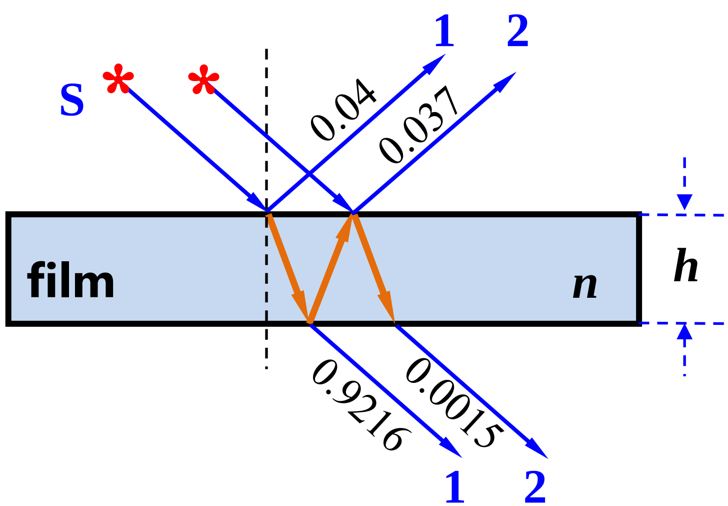
Transmitted light interference

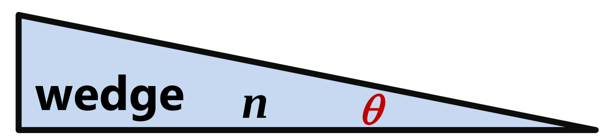


The data is taken from air-glass interface at normal incidence, regardless of external or internal reflection, for simplicity.

- For a general transparent medium, only the interference of the first two reflected beams (of similar amplitude) are considered, and the rest ones are negligible.

Film shape	Interference type
Parallel plate	Equal inclination interference
wedge-shaped film	Equal thickness interference





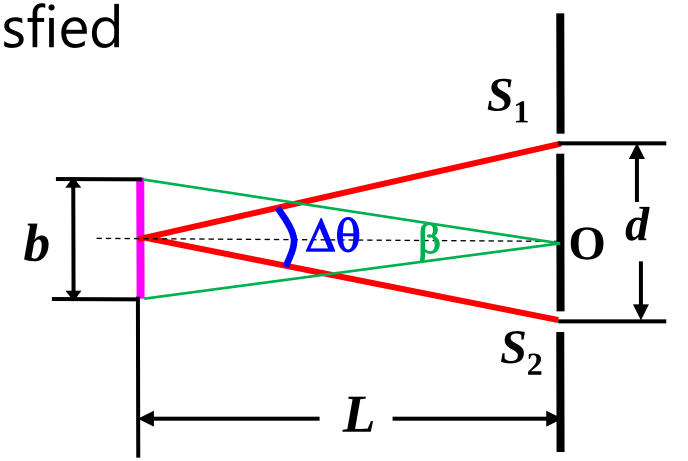
Newton's Ring

Equal inclination interference

- The interference visibility V is determined by the width b of the light source: the smaller b is, the larger V is. **Interference aperture angle** is satisfied

$$\boxed{b\Delta\theta \le \lambda}$$

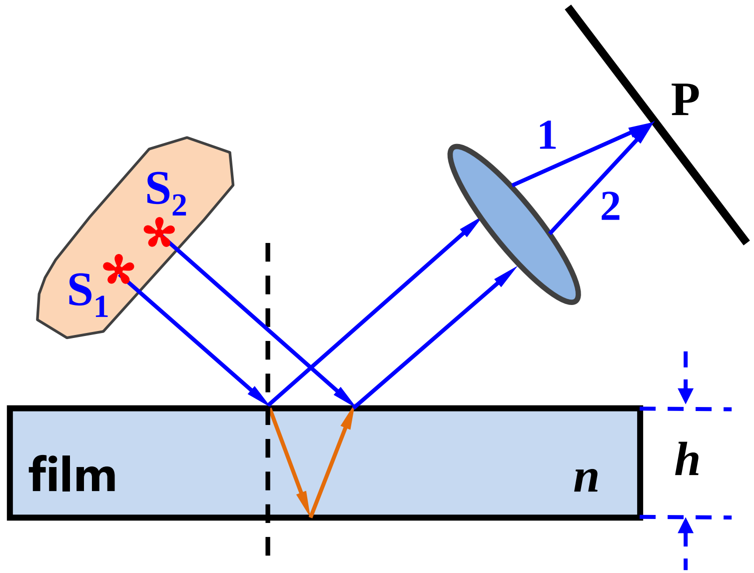
$$\begin{cases} b\Delta\theta = \lambda & V = 0 \quad \text{Critical width} \\ b\Delta\theta = 0 & V = 1 \end{cases}$$



- Young's experiment: $\Delta\theta \neq 0$ $b = 0$, point source
- Thin film interference: $b \neq 0$ $\Delta\theta = 0$, Parallel light

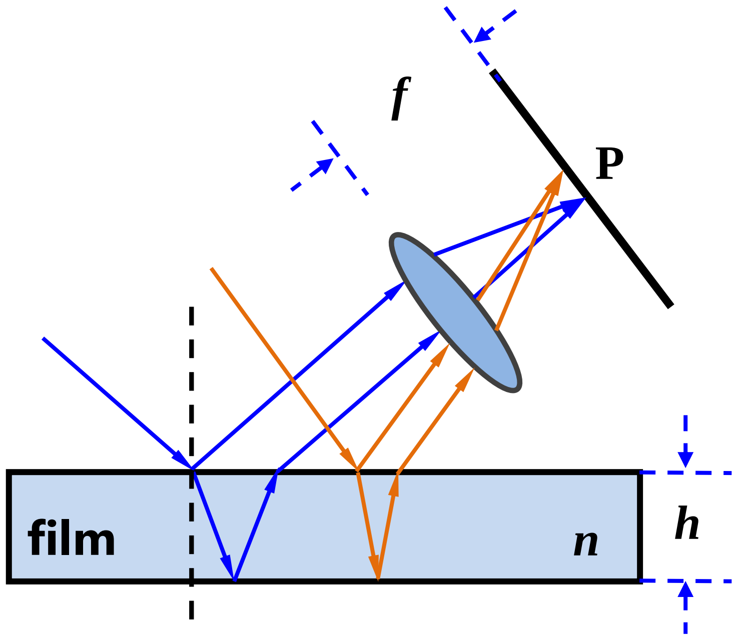
The same incident light is taken and reflected by the upper and lower surfaces of the flat plate into two parallel lights, which intersect at infinity.

Equal inclination interference



- Light of the **same incident angle** from different points on the light source is focused on the **same interference fringe**.
- The total light intensity at point P is the **incoherent** superposition of the interference fringes of the point sources at point P, and the stripe brightness is increased.

Equal inclination interference



- Light with different incident angles is focused at different positions on the focal plane of the lens.
- Interference fringe of equal inclination interference are localized at infinity (or on the focal plane of the lens).

Equal inclination interference

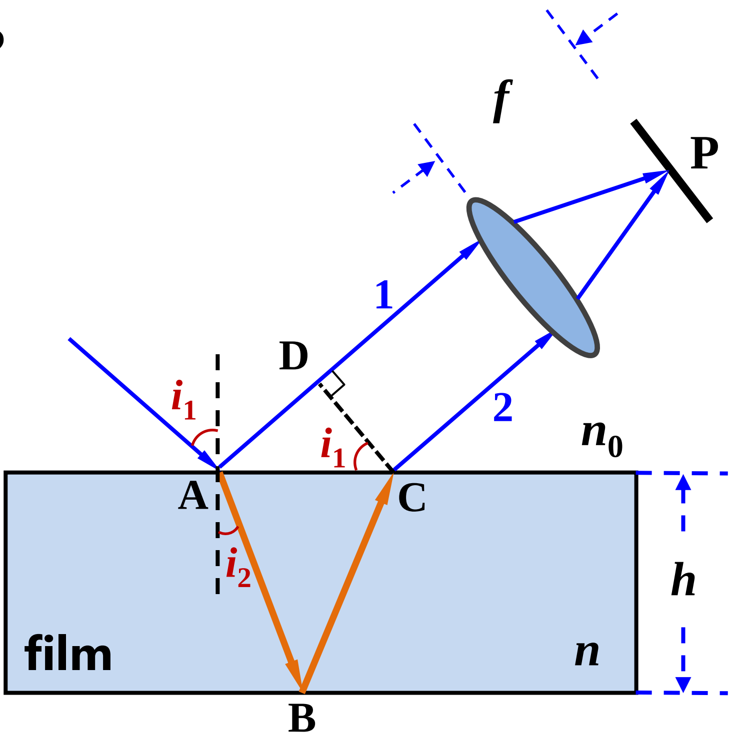
How to get the light intensity?

Assuming $I_1 \approx I_2$

The intensity at arbitrary P

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\delta$$

$$\approx 4I_1\cos^2\frac{\delta}{2} = 4I_1\cos^2\frac{\pi\Delta}{\lambda}$$



The key: find out the OPL difference between the two beams.

Equal inclination interference

（1）Determine the OPL difference

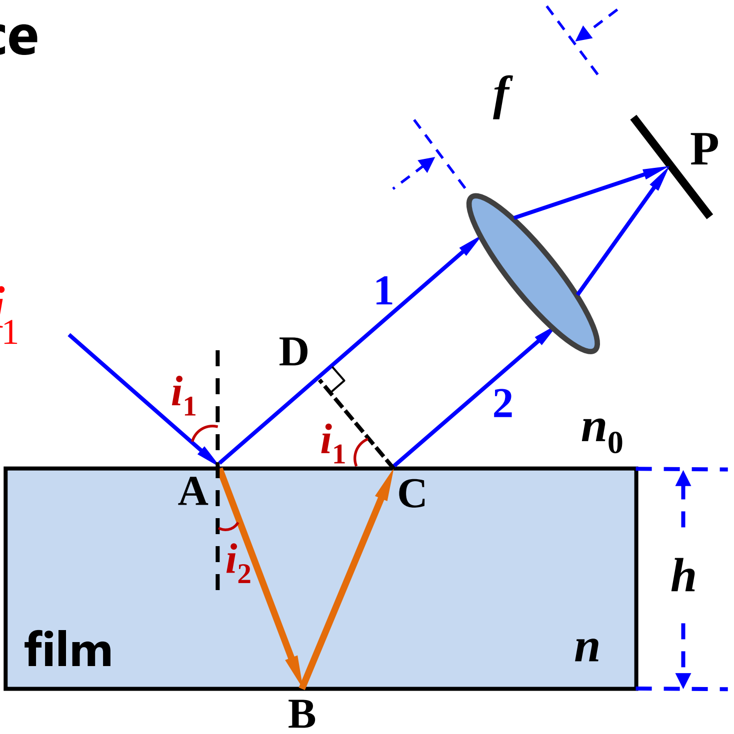
$$\Delta = n(AB + BC) - n_0 AD$$

$$= n\left(\frac{h}{\cos i_2} + \frac{h}{\cos i_2}\right) - n_0 AC \cdot \sin i_1$$

$$\begin{cases} AC = 2h \cdot \tan i_2 \\ n_0 \cdot \sin i_1 = n \cdot \sin i_2 \end{cases}$$

$$\Delta = 2nh\left(\frac{1}{\cos i_2} - \tan i_2 \cdot \sin i_2\right)$$

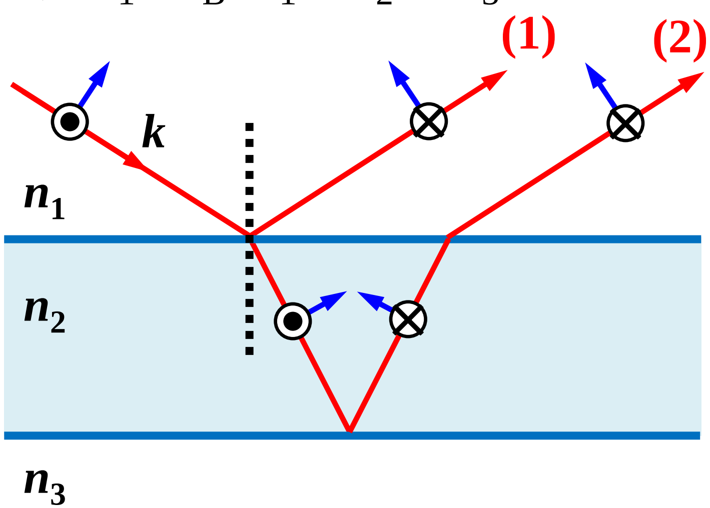
$$= 2nh \cos i_2 = 2h\sqrt{n^2 - n_0^2 \sin^2 i_1}$$



Phase shift

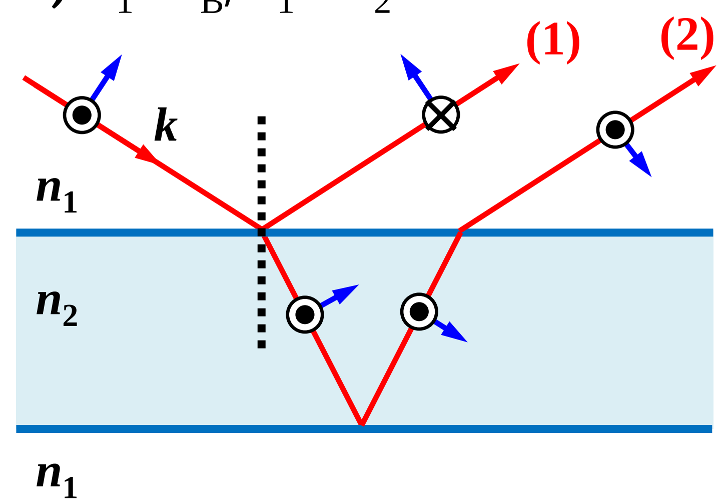
E.g： Reflection of the upper and lower surfaces of the film

1) $i_1 < i_B, n_1 < n_2 < n_3$



Both reflections are external reflections. There is no additional phase difference between the two beams (1)(2), and the vibrations are in phase.

2) $i_1 < i_B, n_1 < n_2$



External reflection at the first interface and internal reflection at the second interface. There is an **additional phase difference** π between the beams (1)(2) , and the vibrations are out-of-phase.

'**additional**' means that it is not the absolute phase difference between the two beams.

Equal inclination interference

(2) the additional phase during reflection π

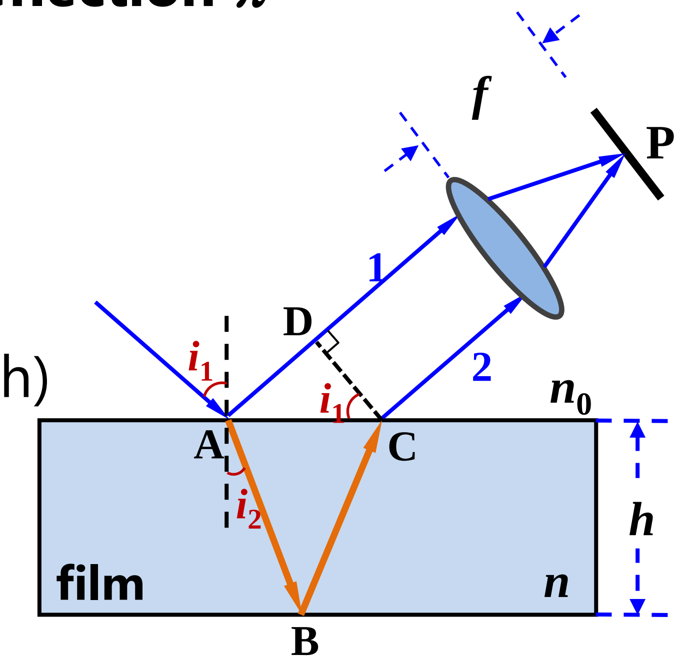
Known from the Fresnel formula

$$n_0 < n \text{ or } n_0 > n$$

There is a π-phase shift between the two reflected beams (half wavelength)

So,

$$\boxed{\Delta = 2h\sqrt{n^2 - n_0^2 \sin^2 i_1} \pm \frac{\lambda}{2}}$$



- For a parallel plane film, the path difference is only determined by the angle of incidence, the interference fringes only change with the incident angle. >> why it is called equal inclination interference.

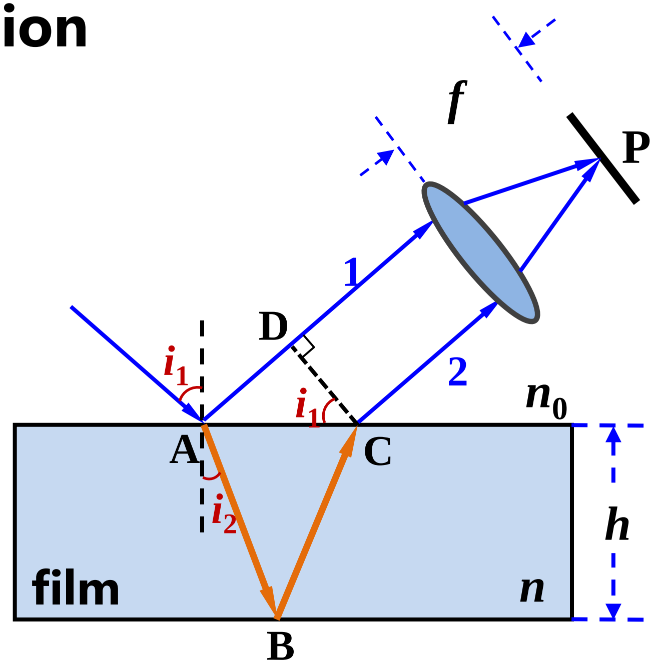
Equal inclination interference

(3) Analyze light intensity expression

$$I = 4I_1 \cos^2 \frac{\pi\Delta}{\lambda}$$

$$\Delta = 2h\sqrt{n^2 - n_0^2 \sin^2 i_1} + \frac{\lambda}{2}$$

$\therefore$ same i_1 $\rightarrow$ same Δ $\rightarrow$ same I $\rightarrow$ same fringe



- The fringe shape is determined by the trajectory of light having the same angle of incidence on the focal plane of the lens.

Equal inclination interference

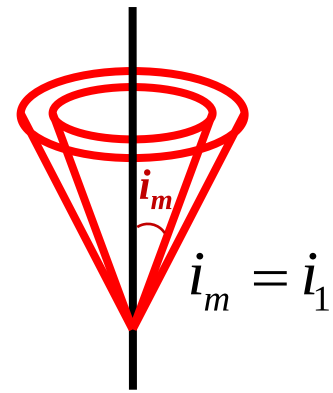
- **Point source**

The trajectory of bright fringes: (the half-wave loss is not considered)

$$\Delta = 2h\sqrt{n^2 - n_0^2 \sin^2 i_1} = m\lambda$$

$$i_1 = \sin^{-1}\frac{1}{n_0}\sqrt{n^2 - \left(\frac{m\lambda}{2h}\right)^2}$$

$m = 0、1、2\ldots$

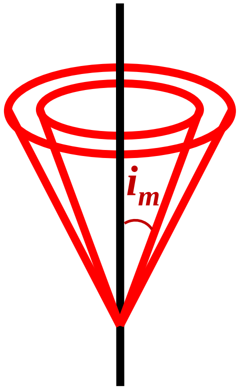


The optical path trajectory is a coaxial conical surface with equal inclination angle, for the same ***m*** value points. Angular radius i_m: the opening angle of the interference ring to the center of the lens, equal to the incident angle corresponding to the $\boldsymbol{m}^{th}$ order fringe.

Equal inclination interference

- **Fringe spacing**

$m = 0、1、2\ldots$



For bright fringes： $\Delta = 2nh\cos i_2 = m\lambda$

$$-2nh\sin i_2 \cdot \Delta i_2 = (m+1)\lambda - m\lambda$$

$$\Delta i_2 = -\frac{\lambda}{2nh\sin i_2}$$

$$\Delta i_m \equiv i_{m+1} - i_m \propto \Delta i_2$$

Negative sign indicates： $i_{m+1} < i_m$

The higher the order, the smaller the corner radius of the ring

The larger the i_2, the smaller the absolute value of Δi_2

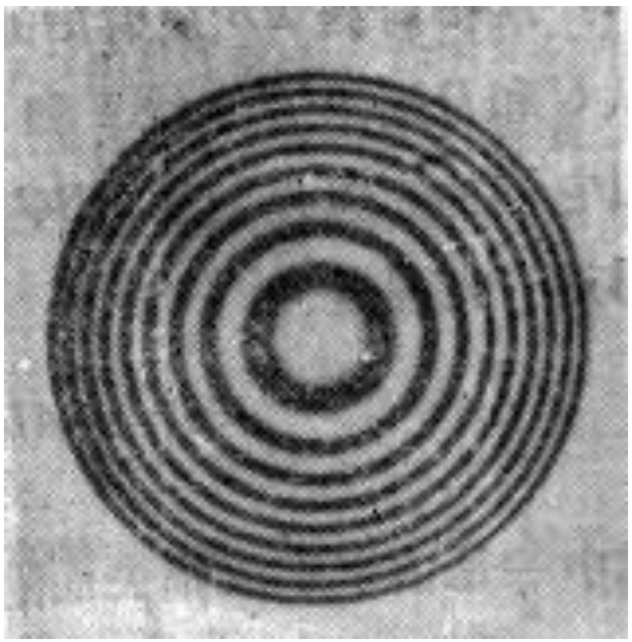
the farther away from the center, the closer the ring

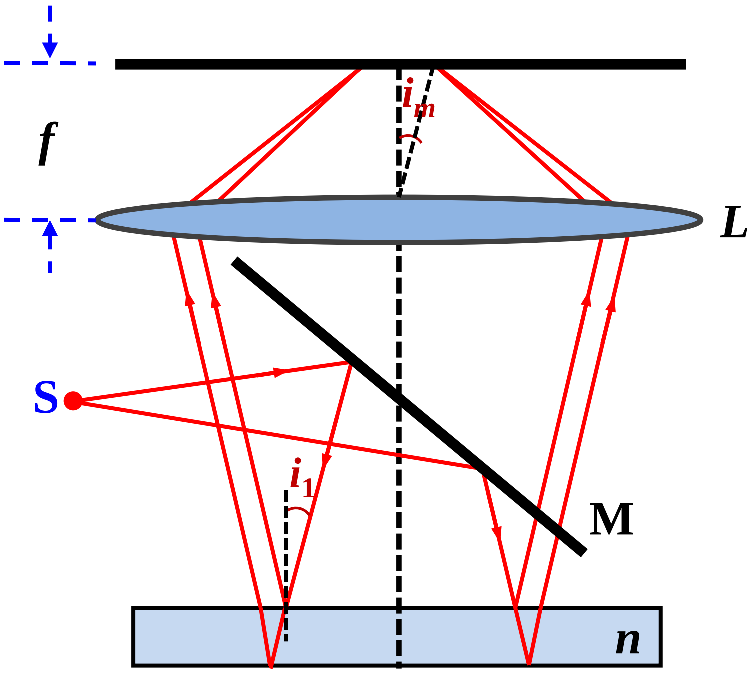
The larger h is, the smaller the absolute value of Δi_2

the thicker the ring is, the denser the ring is

Equal inclination interference

- **Experimental device and fringes**





$$i_m = i_1$$

Equal inclination interference

- **How does the fringes move when the thickness changes?**

$$\Delta = 2nh\cos i_2 + \frac{\lambda}{2}$$

Quantitatively, take the difference between the above equations and see how the points of the optical path difference move:

$$0 = 2n\Delta h\cos i_2 - 2nh\sin i_2 \cdot \Delta i_2 \qquad \Delta h > 0 \quad \Rightarrow \quad \Delta i_2 > 0$$

$$\Delta h = h\tan i_2 \cdot \Delta i_2 \qquad\qquad i_2 \text{ becomes larger}$$

- **When the film is thickened (thinned), the radius of the interference fringes at each order increases (decreases), that is, new fringes are continuously emerging (falling in) from the center.**

Homework

Problem 9.8 and 9.13.

Homework*

Find a text book of Quantum optics, and find out what's the first-order correlation and its relationship with out text.

Next week

Equal thickness interference, Michelson interferometer, thin films

Sections 9.4, 9.6 and 9.7.2