热力学与统计物理-第八次作业

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Problem 6.1

Answer:

The probablilty that the system is in a state with energy E is proportional to the Boltzman factor $e^{-E/kT}$.

Then the ratio of the probablilty of being in the first excited state to the probablilty of being in the ground state is:

$$\frac{e^{-3\hbar w/2kT}}{e^{-\hbar\omega/2kT}} = e^{-\hbar\omega/kT} \tag{1.1}$$

(b)

By the definition of mean value:

$$\bar{E} = \frac{\hbar\omega}{2} \quad \frac{1+3\mathrm{e}^{-\hbar\omega/\mathrm{kT}}}{1+\mathrm{e}^{-\hbar\omega/\mathrm{kT}}} \tag{1.2}$$

Problem 6.2

Answer:

The mean energy per particle is:

$$\bar{\epsilon} = \frac{\mu\mathrm{He}^{-\mu\mathrm{H/kT}} - \mu\mathrm{He}^{\mu\mathrm{H/kT}}}{\mathrm{e}^{-\mu\mathrm{H/kT}} + \mathrm{e}^{\mu\mathrm{H}/kT}} = -\mu\mathrm{H}\tanh\frac{\mu\mathrm{H}}{\mathrm{k}T} \tag{2.1}$$

So:

$$E = -N\mu H\tanh\frac{\mu H}{kT} \tag{2.2}$$

Problem 6.4

Answer:

The power absorbed is proportional to the difference in the nuber of nuclei

in the two levels.
This is:

$$n_+ - n_- = \frac{Ne^{\mu H/KT}}{e^{\mu H/KT} + e^{-\mu H/KT}} - \frac{Ne^{-\mu H/KT}}{e^{\mu H/KT} + e^{-\mu H/KT}} \tag{3.1}$$

Since $\mu H << kT$:

$$n_+ - n_- \approx N\frac{\left(1 + \frac{\mu H}{kT} - 1 + \frac{\mu H}{kT}\right)}{1 + \frac{\mu H}{kT} + 1 - \frac{\mu H}{kT}} = \frac{N\mu H}{kT} \tag{3.2}$$

Problem 6.5

Answer:
The volume element of atmosphere shown must be in equilibrium under the forces of the pressure and gravity. Then if m is the mass per particle, A the area, and g the acceleration of grevity, we have:

$$P(z + dz)A - p(z)A = \frac{dp}{dz}dzA = -mn(z)Adzg \tag{4.1}$$

With $p = nKT$:

$$\frac{dn(z)}{dz} = -\frac{mg}{kT}n(z) \tag{4.2}$$

So:

$$n(z) = n(0)e^{-mgz/kT} \tag{4.3}$$

Problem 6.6

Answer:
(a)
As T approaches 0 the system tends to the low energy state, while in the

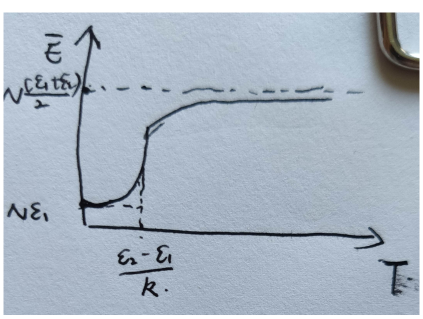

图 1: (a)

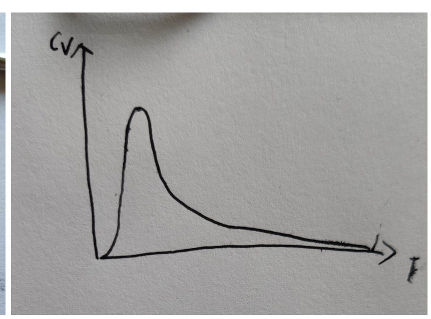

图 2: (b)

limit of high temperatures all states become equally probable. The energy goes from the low to the high temperature Iimit when $\epsilon_2 - \epsilon_1 \approx KT$

The specific heat is, $C_V = \left(\frac{\partial \bar{E}}{\partial T}\right)_V$, i.e., the slope of $\overline{E}$

(c)

$$\bar{E} = N\left[\frac{\epsilon_1 e^{-\epsilon_1/kT} + \epsilon_2 e^{-\epsilon_2/kT}}{e^{-\epsilon_1/kT} + e^{-\epsilon_2/kT}}\right] = N\left[\frac{\epsilon_1 + \epsilon_2 e^{-(\epsilon_2-\epsilon_1)/KT}}{1 + e^{-(\epsilon_2-\epsilon_1)/KT}}\right] \tag{5.1}$$

$$T \to 0, \bar{E} \to N\epsilon_1 \tag{5.2}$$

$$T \to \infty, \bar{E} \to N(\frac{\epsilon_1 + \epsilon_2}{2}) \tag{5.3}$$

So:

$$\frac{\epsilon_2 - \epsilon_1}{\mathrm{k}T} = \ln 3 \approx 1 \quad \text{or} \quad (\epsilon_2 - \epsilon_1) \approx \mathrm{k}T \tag{5.4}$$

Then the heat capacity is:

$$c_V = \frac{\partial \bar{E}}{\partial T} = \frac{N\left(\epsilon_2 - \varepsilon_1\right)^2 e^{-(\varepsilon_2-\epsilon_1)/K^2}}{KT^2\left[1 + e^{-(\epsilon_2-\varepsilon_1)/KT}\right]^2} \tag{5.5}$$

$C_V \to 0$ as $T \to 0, T \to \infty$.

Problem 6.10

Answer:

(a)

The centrifugal force $m\omega^2 r$ yields the potential energy $-\frac{m\omega^2 r^2}{2}$. Since the probablilty that a molecule is at r is proportional to the Boltzman factor, the density is:

$$p(r) = p(0)e^{m\omega^2 r^2/2kT} \tag{6.1}$$

(b)

Substituting $\mu_= N_A m$, the molecular weight, into (6.1) and evaluating this expression at r_1 and r_2 we have:

$$\mu = \frac{2\mathrm{N_A kT}}{\omega^2\left(\mathrm{r}_1^2 - \mathrm{r}_2\right)^2} \ln \frac{\rho\left(\mathrm{r}_1\right)}{\rho\left(\mathrm{r}_2\right)} \tag{6.2}$$