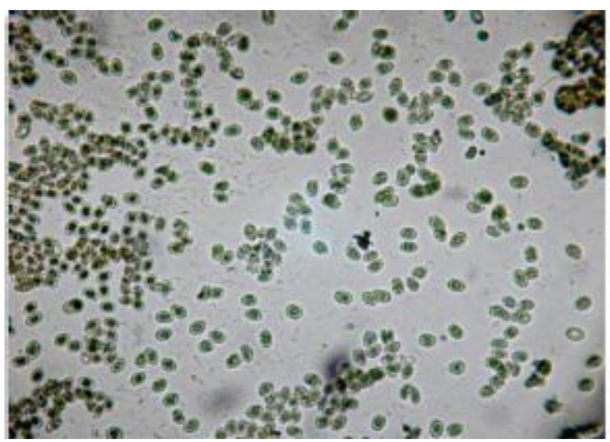
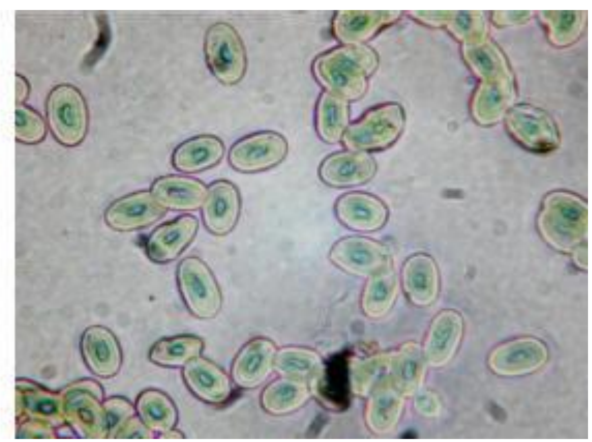
# C9 Optical instruments

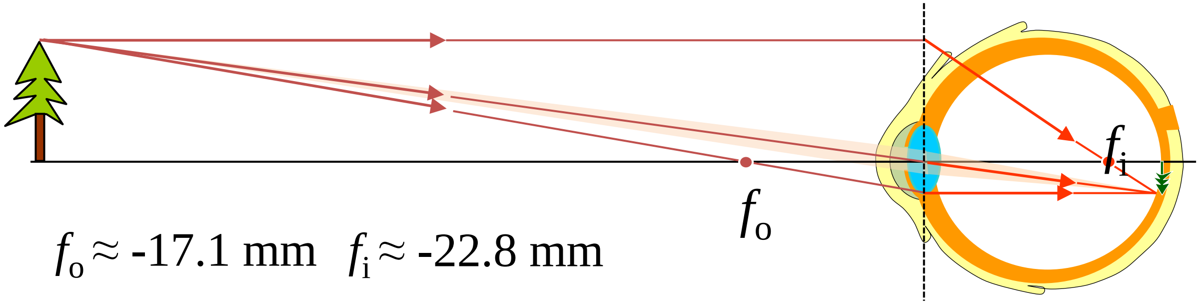
- Imaging instruments, magnification lens, minimum resolution distance/angle;
- Near-field optical microscopy;
- Spectrometer, FP interferometer.





# Eyes

① **Human eye: a complex and efficient imaging system**




$f_o \approx -17.1$ mm $f_i \approx -22.8$ mm

Cornea（角膜）, anterior eye（前眼房）, pupil（瞳孔）, lens（晶状体）, vitreous（玻璃体）, retina（视网膜）

When the object is between the near point and the far point of the eye, the focal length of the lens is changed by muscle expansion and contraction, so that the object becomes a real image on the retina.

>>Why can the human eye see the virtual image?

# § 9.1 Imaging instrument

- **Viewing angle** $\omega$

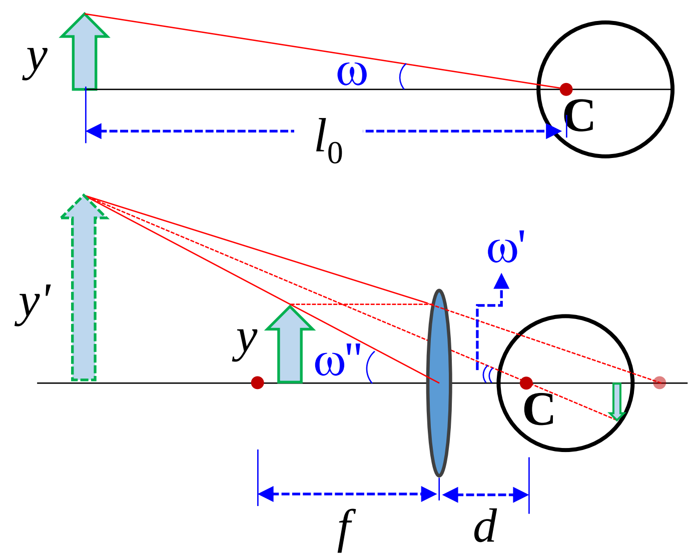
**magnification**: $$M = \frac{\tan\omega'}{\tan\omega}$$

$\omega'$: Viewing angle when using the instrument

$\omega$: Viewing angle without using the instrument

- Magnification: $d$ is small, $\omega' \approx \omega''$,

The object is placed within a focal length, close to the focus:




$$M = \frac{\tan\omega'}{\tan\omega} = \frac{y'}{y} \approx \frac{y/f}{y/l_0} = \frac{l_0}{f}$$

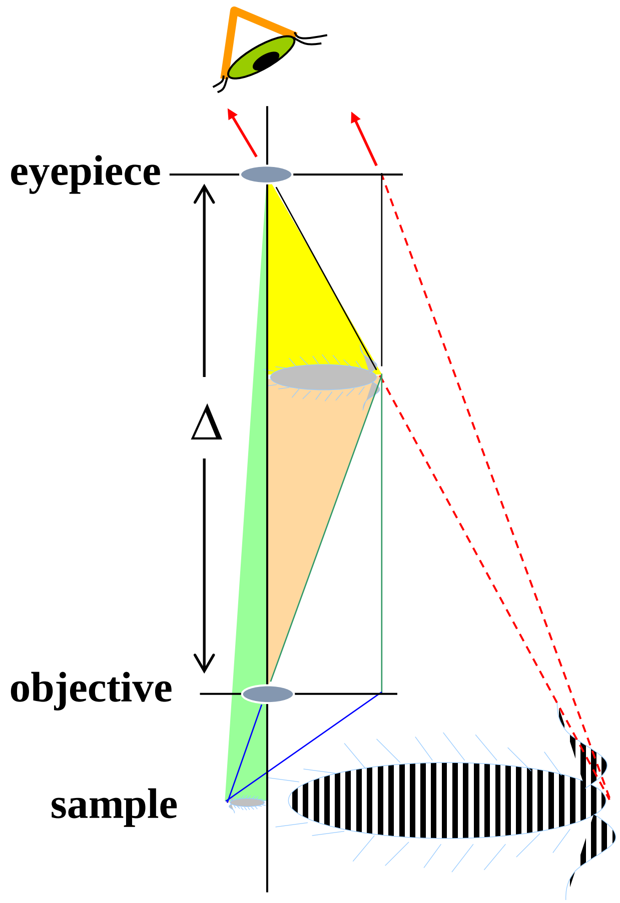
$l_0 \approx 25\ \text{cm} \quad f = 10\ \text{cm} \qquad M \approx 2.5$

# Microscope

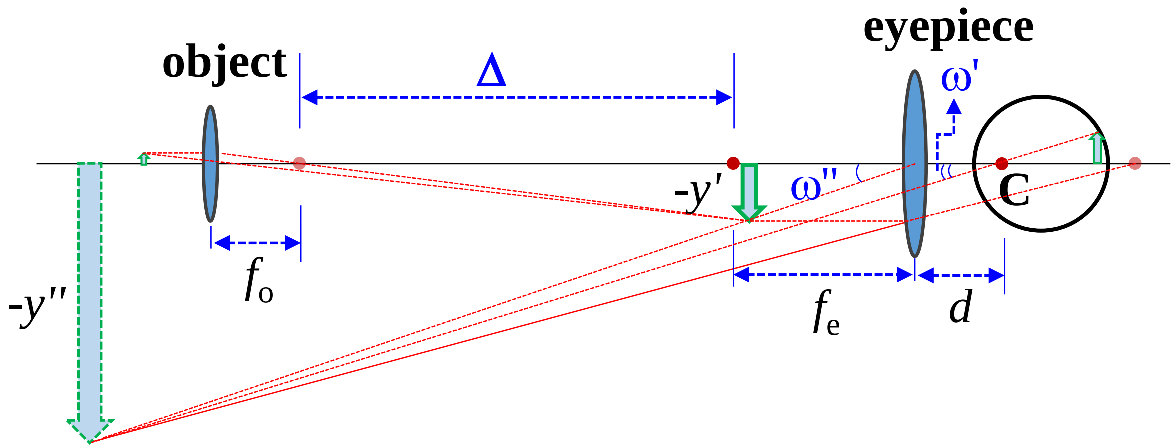
- The imaging system of a microscope consists of an objective (very short focal length) and an eyepiece; the distance $\Delta$ between them is much larger than their focal lengths.

# Microscope

- The objective images the object at a plane less than the focal length of the eyepiece (real image);
- This real image is magnified by the eyepiece into an enlarged virtual image.



# Principle of microscope




Magnification of objective

$$\beta = \frac{-y'}{y} \approx \frac{-\Delta}{f_o'}$$

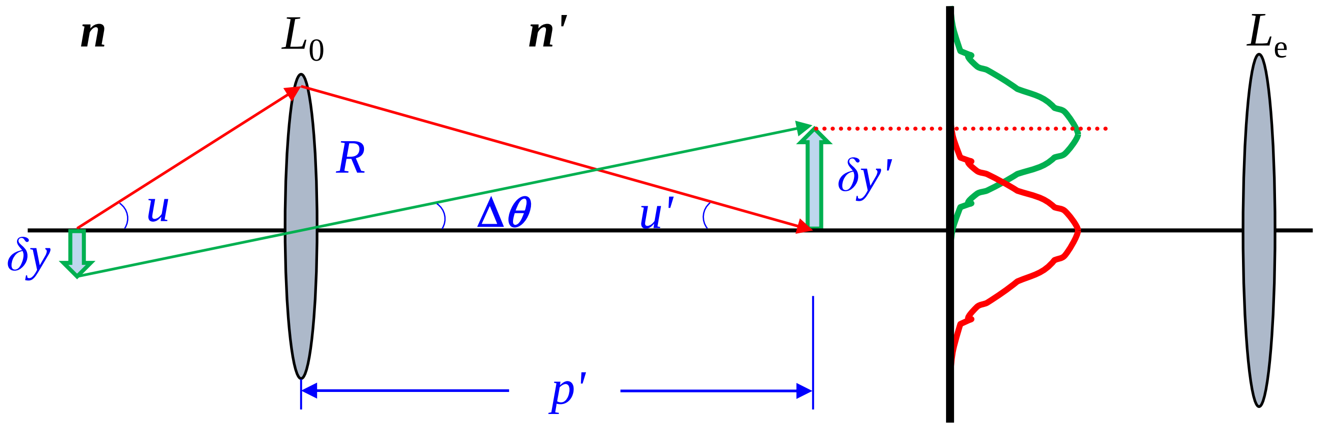
Magnification of eyepiece

$$M_e \approx \frac{25}{f_e'} \qquad l_0 \approx 25\ \text{cm}$$

- The magnification of a microscope is equal to the product of the magnification of the objective and the eyepiece:

$$M = \frac{\tan\omega'}{\tan\omega} \approx \frac{y'/f_e'}{y/l_0} = \frac{25\beta y}{f_e' y} = \beta\frac{25}{f_e'} = \beta M_e$$

# Resolving power



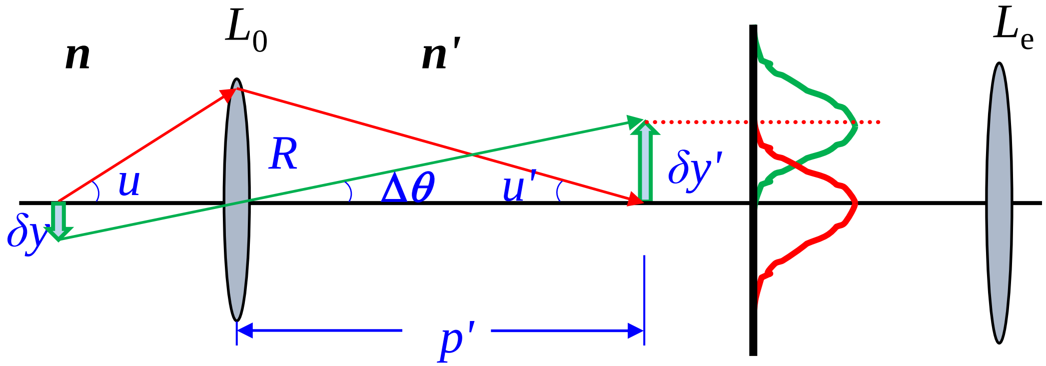

∵ An point object is diffracted by the objective (aperture stop, radius $R$) and forms a Fraunhofer diffraction pattern on the image plane; the diffraction spot is the Airy disk.

Angular radius: $\Delta\theta = 1.22\dfrac{\lambda_{n'}}{D} = 0.61\dfrac{\lambda}{n'R}$

When the distance of the P' Q' point is the radius of the Airy disk, the Rayleigh criterion is satisfied. The two points are just distinguishable.

$$\delta y' = p'\Delta\theta = 0.61\frac{\lambda p'}{n'R}$$

# Resolving power




$$\delta y' = 0.61\frac{\lambda p'}{n'R} \qquad \sin u' \approx \tan u' = \frac{R}{p'} \qquad \delta y' = \frac{0.61\lambda}{n'\sin u'}$$

In order to correct the aberration, the objective lens satisfies the Abbe sine condition

$$\delta y' \cdot n' \sin u' = \delta y \cdot n \sin u$$

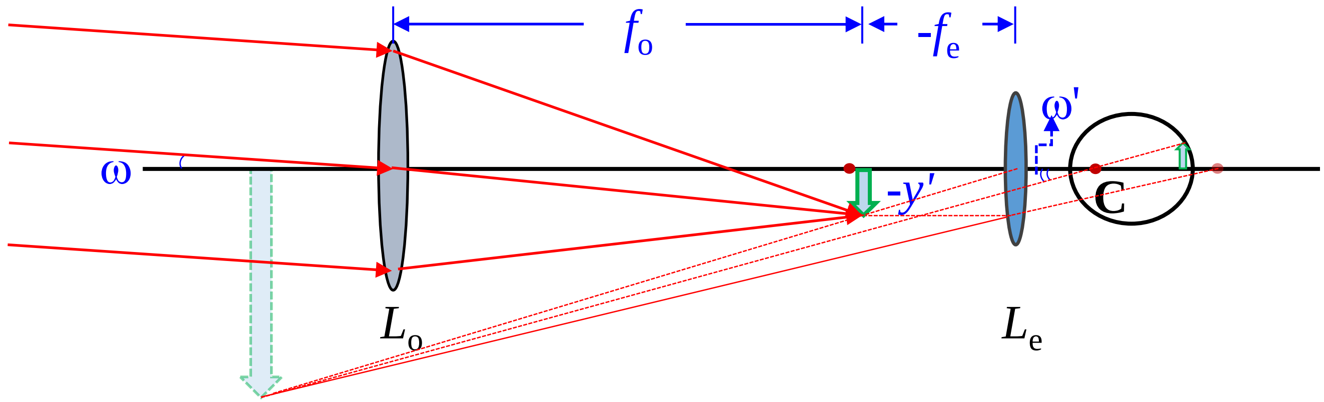
Minimum resolution distance

$$\boxed{\delta y = \frac{0.61\lambda}{n \sin u}}$$

**>>Diffraction limit**

$NA \equiv n \sin u$ is the **numerical aperture** of the objective lens, the larger the NA, the higher the resolution.

# Kepler telescope




Since the object is at infinity, the image focus of the objective lens coincides with the object focus of the eyepiece.

$$M = \frac{\tan\omega'}{\tan\omega} \approx \frac{-y'/f_e}{y'/f_o} = -\frac{f_o}{f_e}$$

The minimum resolution angle is determined by the diameter $D$ of the objective lens of the telescope:

$$\Delta\theta = 1.22\frac{\lambda}{D}$$

# § 9.2 Super-resolution imaging

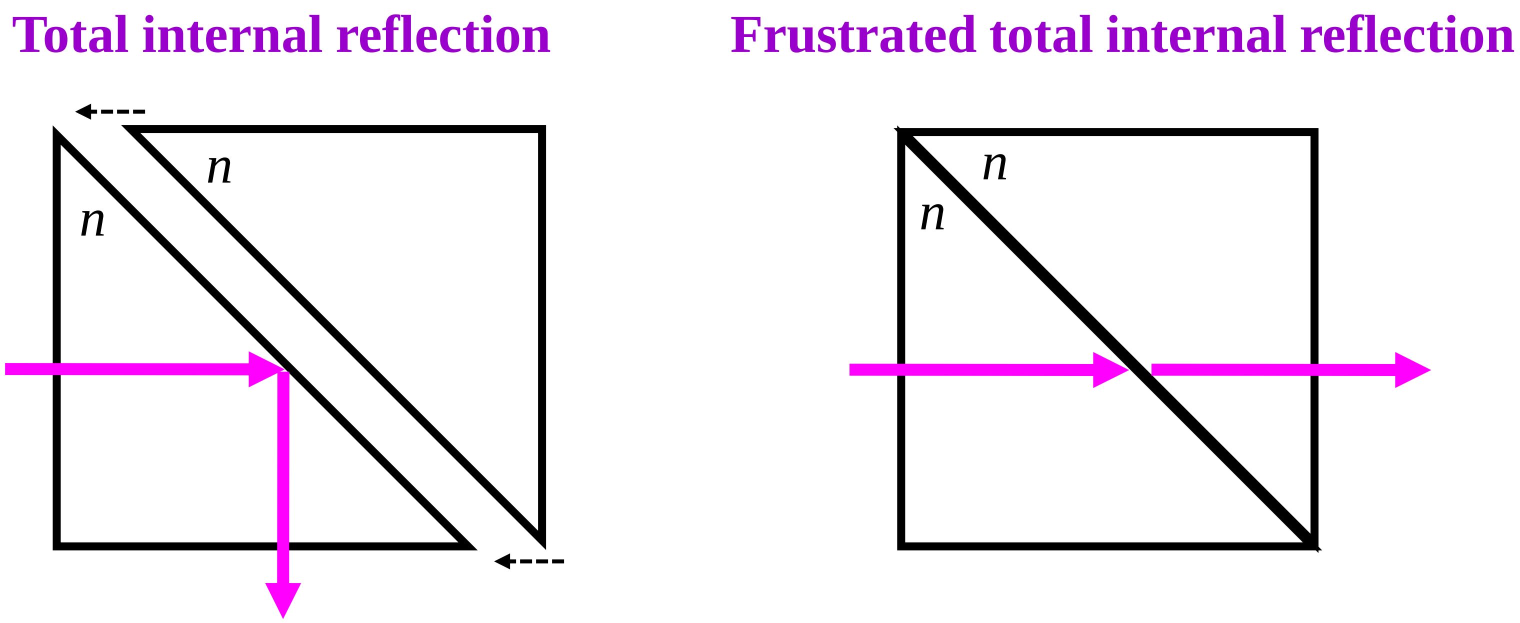
I. $\lambda$ is smaller, X-ray/ electron

II. Scanning near-field optical microscope (SNOM)

III. Super-resolution fluorescence microscope

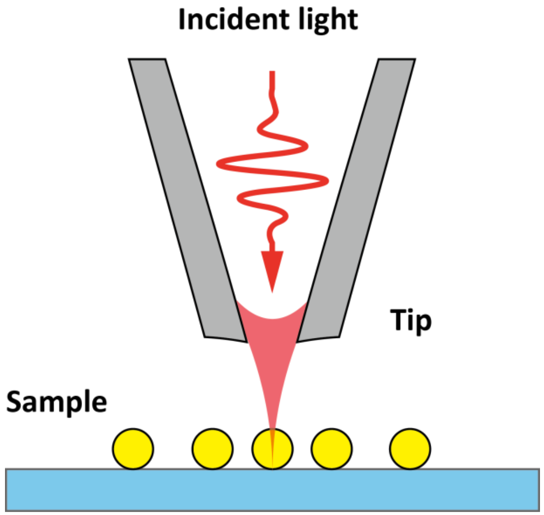
# Frustrated TIR

Taking a surface closer to an interface where total reflection has occurred, total reflection will be gradually frustrated.

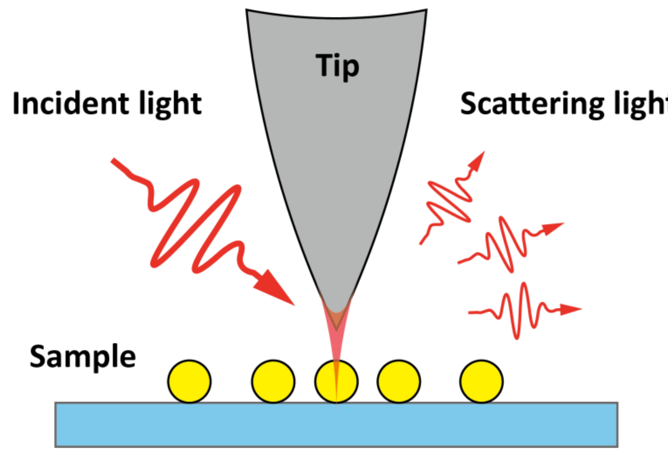



- Photon tunneling effect (analogous to tunneling effects in quantum mechanics)
- Scanning near-field microscopy, prism coupling (coupling light into a waveguide or coupling out)

# Aperture/aperture-less probe




**resolution ~ 50 nm**




**resolution ~ 10 nm**

- Working modes:
  - –Near/far field excitation
  - –Near/far field collection

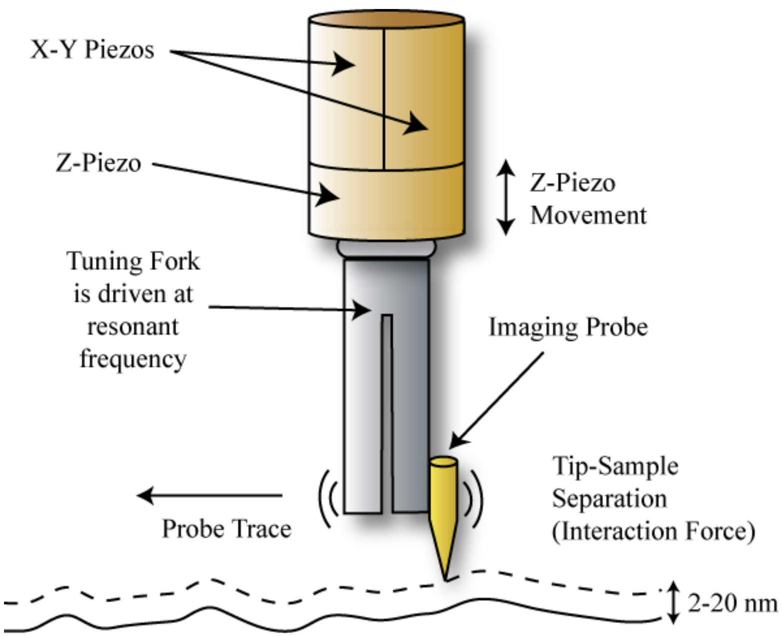
**Limitations: Tapered fiber, weak signal**

Far field excitation/collection
High imaging resolution

**Limitations: background stray light intensity, signal is difficult to extract**

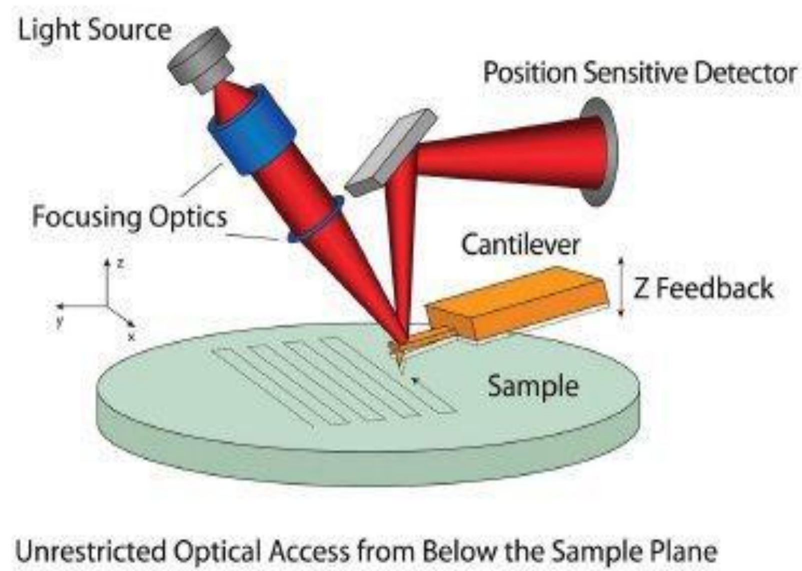
# Scan/feedback system

Tuning fork
+ piezoelectric ceramic




**Horizontal scanning**

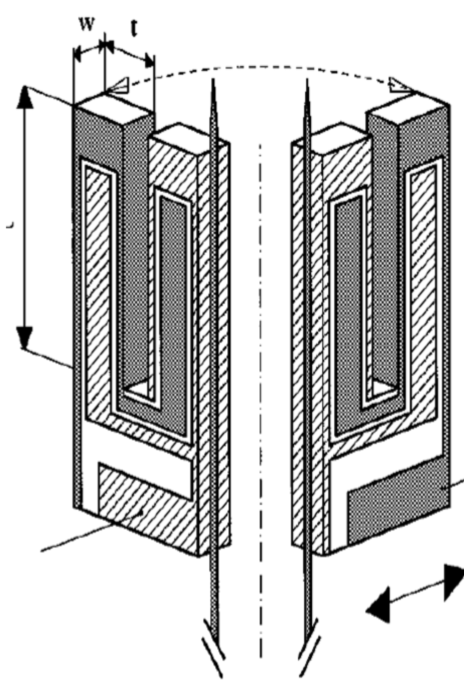
Micro cantilever
+ piezoelectric ceramic

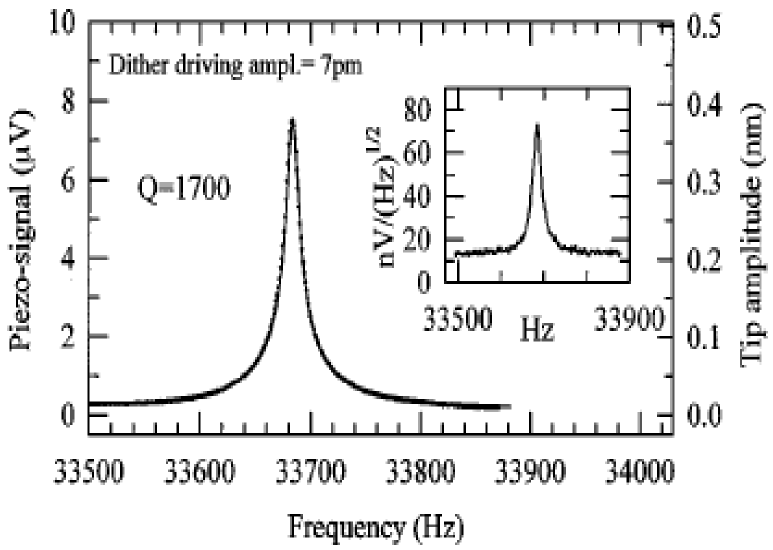



**Vertical feedback**

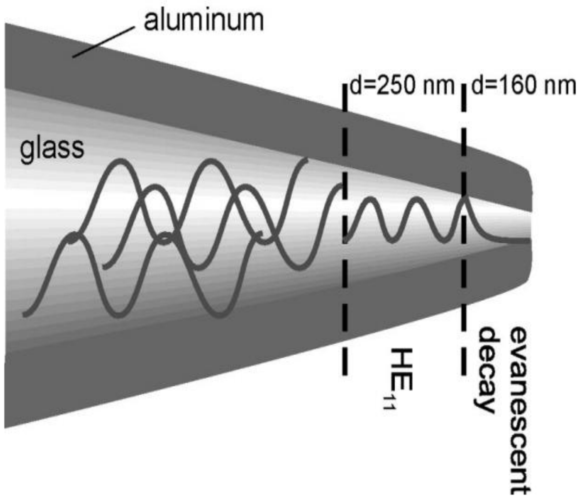
# Scan/feedback system

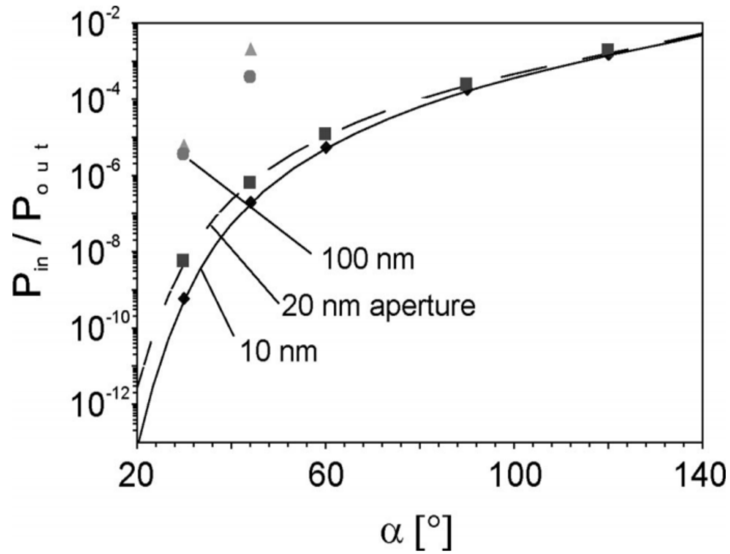
The resonant frequency of the tuning fork moves with the distance between the tip and the surface. Monitoring the resonant frequency knows the distance between the tip and the surface.



# Limitations of perforated SNOM



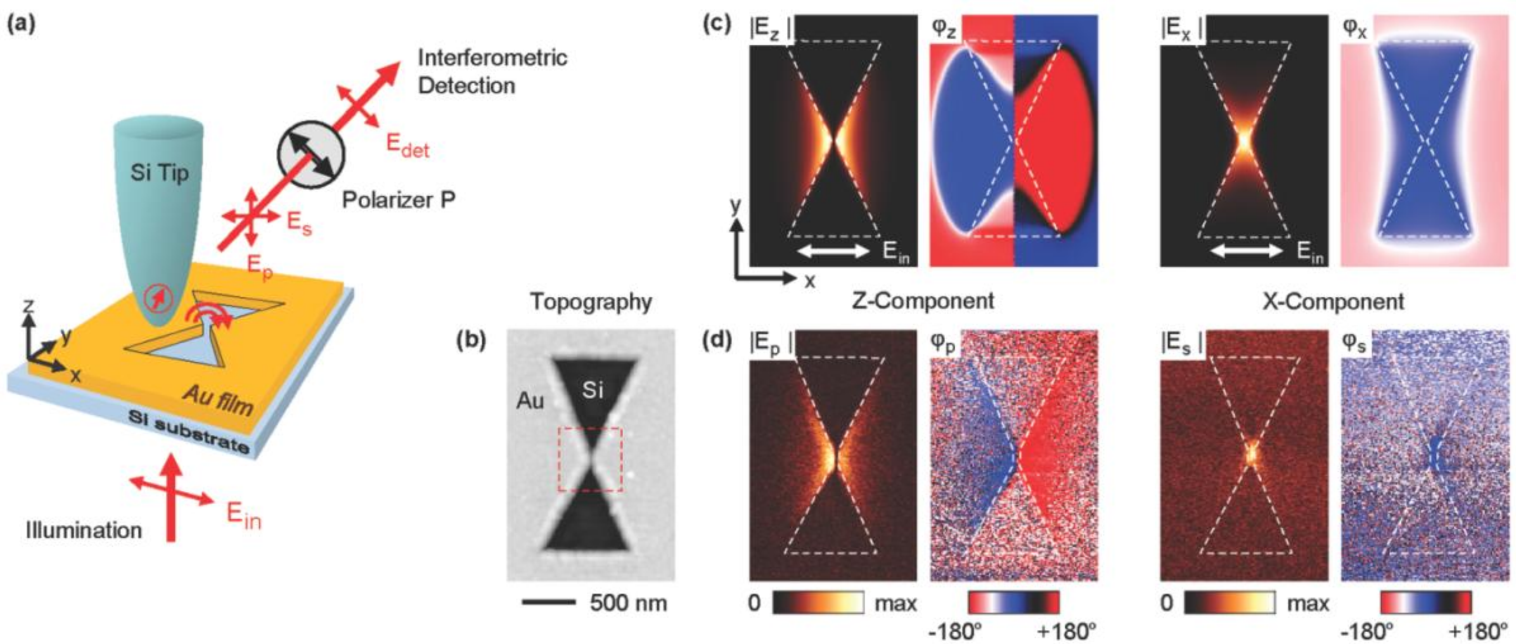




J. Chem. Phys., 112, 18, 2000

- **The sharper the tip, the smaller the cone angle a, the higher the SNOM resolution.**
- **the transmittance decreases sharply as the cone angle decreases.**

**This is why the apertured SNOM resolution is basically only ~50 nm!**

# Scattering type SNOM

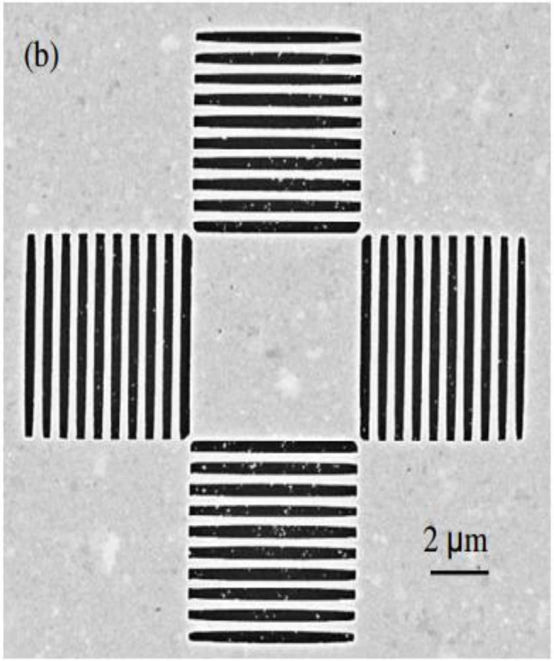


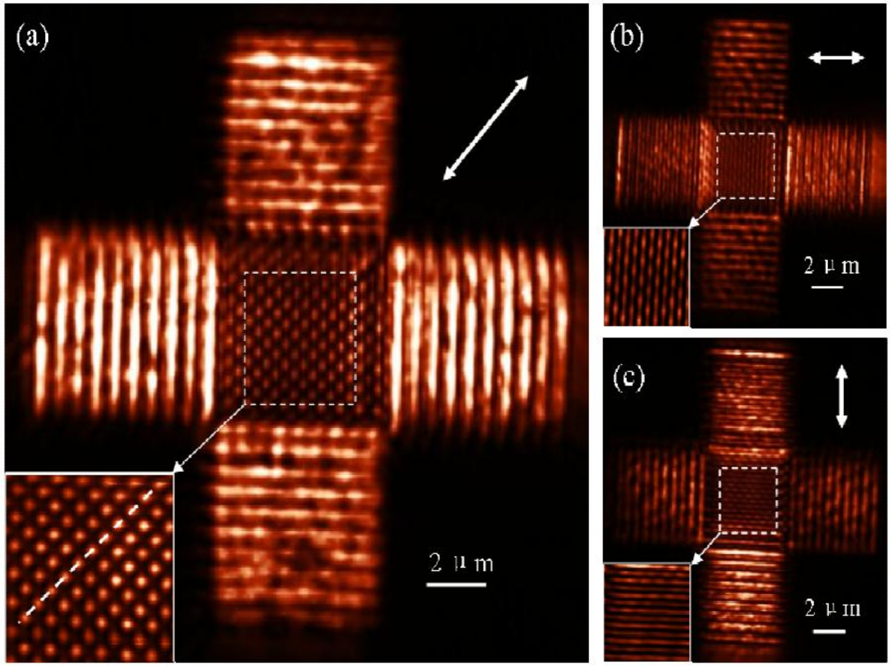

- Interferometry + phase-locked amplification (extracting weak signals)

Nano Lett. 2010, 10, 3524–3528

The resolution depends on the radius of curvature of the tip of the needle, up to ~ 10 nm, about 1000 times smaller than the wavelength of light (mid-infrared 9.3 μm)

# Application of SNOM




**Sample scanning electron microscope**

**Field strength SNOM diagram excited by different polarized lights**

OPTICS EXPRESS Vol. 18, 2662 (2010)

# § 9.3 Spectrometers

**Spectrometer**: instrument that measures the relative intensities of different spectral components of light.

According to the working principle :

**Interference order**

- Prism spectrometer Dielectric dispersion
- Grating spectrometer $m = 1, 2, 3$
- Interference spectrometer（F-P and so on）$m = 10^3, 10^4$

According to the usage can be divided into:

- Spectrometer Look with your eyes
- Spectrograph Photographing with a plate
- Monochromator PMT、CCD, Rotating grating

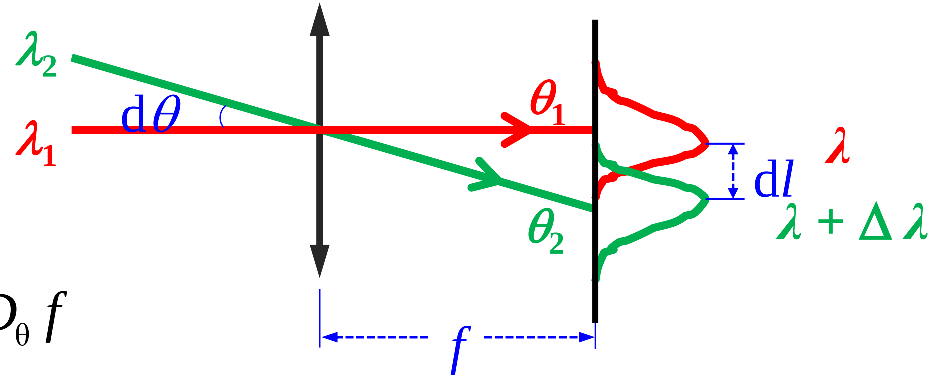
# Basic performance parameters

## ① Dispersion power

**Angular dispersion**: $D_\theta = \dfrac{d\theta}{d\lambda}$

**Linear dispersion**: $D_l = \dfrac{dl}{d\lambda} \approx D_\theta f$

paraxial approximation




$D_\theta$ or $D_l$: An angular distance or distance the principal maximum per unit wavelength difference.

## ② Chromatic resolving power

$$A = \lambda / \Delta\lambda_{\min}$$

$\Delta\lambda_{\min}$: The minimum wavelength difference that can be resolved by the spectrometer near $\lambda$

③ **Free spectral region:** The $m$-order spectrum in the continuous band of $\lambda \sim \lambda + \Delta\lambda$ that does not overlap, and the corresponding $\Delta\lambda$ is called the free spectral region.

Overlapping: $(m + 1)\lambda = m(\lambda + \Delta\lambda)$

According to the definition: $\boxed{\Delta\lambda = \lambda / m}$

$\lambda$: Lower limit of incident wavelength

$\Delta\lambda$: Maximum incident wavelength range that does not produce overlapping

# Grating spectrometer

Dispersion power: Characterizes the extent to which the grating separates the principal maxima of different wavelengths in space.

① **Angular dispersion**:

$$\boxed{D_\theta = \frac{\mathrm{d}\theta}{\mathrm{d}\lambda} = \frac{m}{d\cos\theta}}$$

Grating equation:

$$d\sin\theta = m\lambda$$

When $\theta$ is small, $\cos\theta_m \approx 1$

(1) $D_\theta = m/d$, is determined by $m/d$, so $\theta \sim \lambda$ change **linearly**, Uniform line spectrum. $D_\theta$ is independent of $N$.

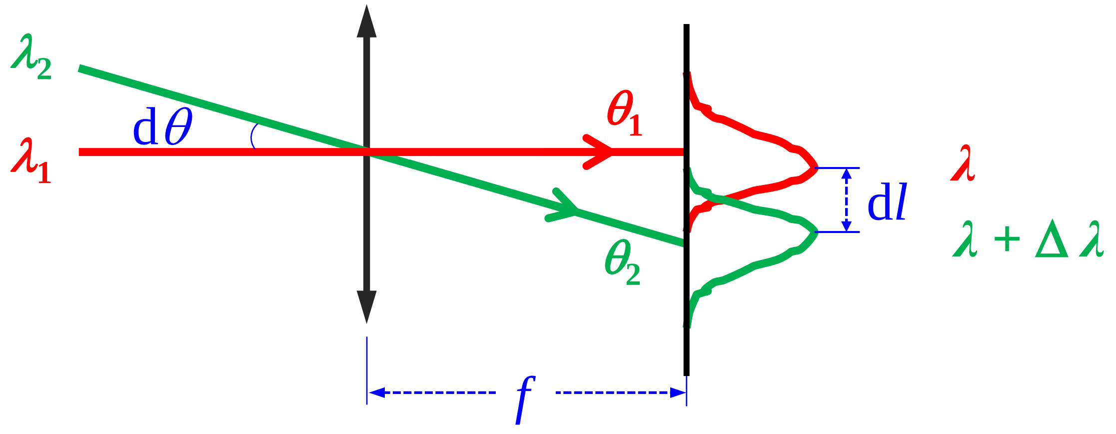
(2) When $m = 0$, no dispersion; $m\uparrow \rightarrow D_\theta\uparrow$.

(3) $d\downarrow \rightarrow D_\theta\uparrow$ (The generally $d$ size is $10^{-2} \sim 10^{-3}$ mm)

# Grating spectrometer

② **Linear dispersion**： ***θ* is small.**

$$\boxed{D_l = \frac{\mathrm{d}l}{\mathrm{d}\lambda} \approx D_\theta f = \frac{mf}{d\cos\theta}}$$



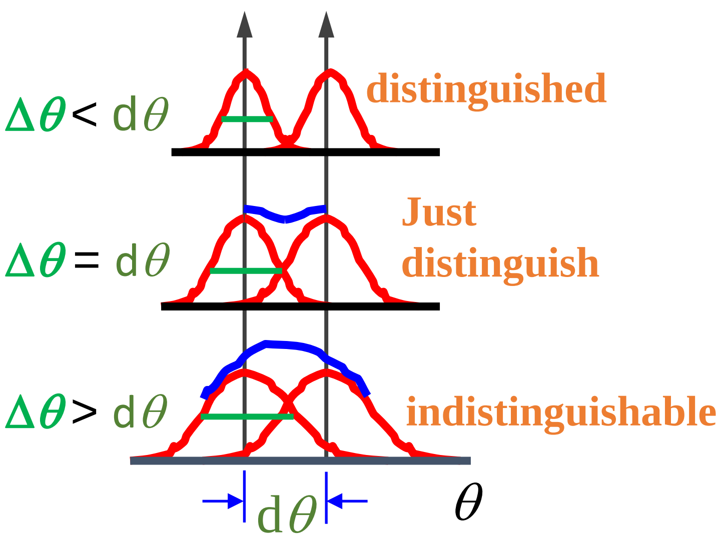
# Grating spectrometer

## ③ Chromatic resolving power

$\mathrm{d}\theta$ is the angular distance of two same order line of $\lambda_1$, $\lambda_2$.

$\Delta\theta$: The half-width of the spectral line.

Rayleigh criterion




$$\Delta\theta = \frac{\lambda}{Nd\cos\theta} \qquad D_\theta = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta} \qquad d\lambda = \frac{\mathrm{d}\theta}{D_\theta} = \frac{\Delta\theta}{D_\theta} \equiv \Delta\lambda_{\min}$$

The minimum wavelength difference corresponding to the spectral line

# Grating spectrometer

$$\Delta\theta = \frac{\lambda}{Nd\cos\theta}$$

$$\Delta\lambda_{\min} = \frac{\Delta\theta}{D_\theta} \equiv \frac{\lambda}{Nd\cos\theta}\cdot\frac{d\cos\theta}{m} = \frac{\lambda}{mN}$$

$$D_\theta = \frac{d\theta}{d\lambda} = \frac{m}{d\cos\theta}$$

**Chromatic resolving power**:

$$\boxed{A = \frac{\lambda}{\Delta\lambda_{\min}} = mN = \frac{Nd\sin\theta}{\lambda}}$$

$\because$ $m\uparrow \rightarrow D_\theta\uparrow$, The center of the two lines is separated more.

$N\uparrow \rightarrow \Delta\theta\downarrow$, The sharper the line is.

$\therefore$ $m\uparrow, N\uparrow \rightarrow A\uparrow$。

# Grating spectrometer

For a grating, if the $d = 1200/\mathrm{mm}$, $L = 5$ cm, then

$$A = mN$$

$$N = 6 \times 10^4$$

$$\Delta\lambda_{\min} = \frac{\lambda}{mN}$$

When $m = 1$, $A = 6 \times 10^4$

If $\lambda = 600$ nm, then $\Delta\lambda = 0.01$ nm

Q: Is it possible to increase $m$ indefinitely and get a big $A$?

a. Limited by single slit diffraction;

b. Limited by the free spectral region.

④ **Free spectral region** $\Delta\lambda = \lambda / m$

Where $\lambda$ is the lower limit of the incident wavelength.

$$\Delta\lambda \sim 400 - 500 \text{ nm}$$

# F-P Interferometer

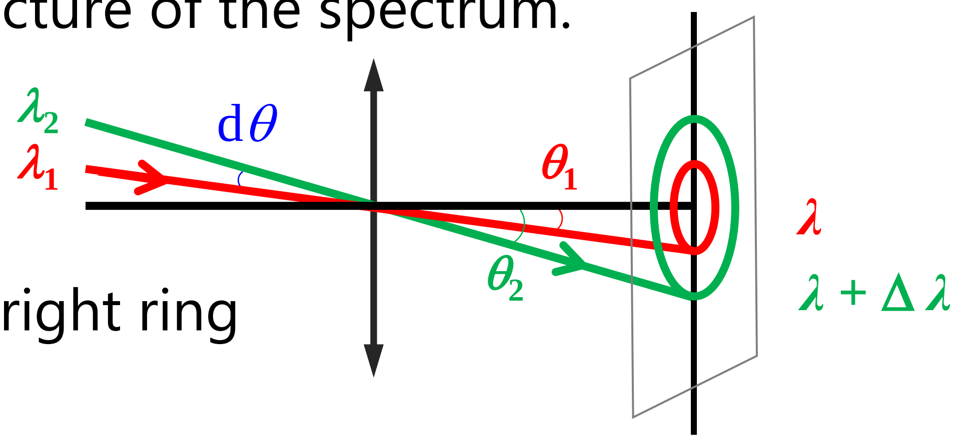
According to the bright-fringes conditions, the ring fringes formed by different wavelengths of light have different sizes (interference splitting), and the fringes are sharp.

$$\Delta = 2h\cos i_2 = m\lambda$$

∴F-P interferometers are high-resolution spectroscopic instruments that are commonly used to study the fine structure and hyperfine structure of the spectrum.

Rewritten as：

$$2nh\cos\theta = m\lambda$$




$\theta$： The angle of the $m^{\text{th}}$ bright ring to the center of the lens.

# F-P Interferometer

## ① Dispersion power

$2nh\cos\theta = m\lambda$

$$D_\theta = \frac{\mathrm{d}\theta}{d\lambda} = -\frac{m}{2nh\sin\theta}$$

$$h = \frac{m\lambda}{2n\cos\theta} \Rightarrow D_\theta = -\frac{m}{2nh\sin\theta}\cdot\frac{2n\cos\theta}{m\lambda} = -\frac{1}{\lambda\tan\theta}$$

Discussion:

(1) "-" indicate $\mathrm{d}\lambda\uparrow \rightarrow \mathrm{d}\theta\downarrow$.

(2) $D \propto 1/\tan\theta$, $\theta\downarrow\rightarrow \tan\theta\downarrow\rightarrow m\uparrow\rightarrow D\uparrow$.

Ring center $\theta\approx 0$, $D\rightarrow\infty$. higher order at the center

(3) $D \propto 1/\lambda$, $\lambda\downarrow\rightarrow D\uparrow$.

The shorter the wavelength, the larger the dispersion power.

# F-P Interferometer

## ② Chromatic resolving power

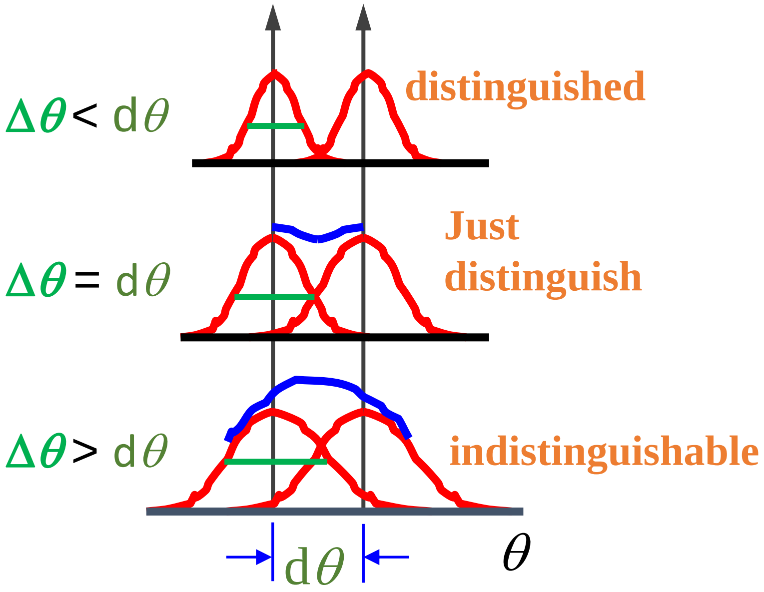
Assume： $I_{\lambda 1} = I_{\lambda 2}$

(1) Angular distance between two main maxima

$$\mathrm{d}\theta = \theta_{m\lambda 1} - \theta_{m\lambda 2}$$

(2) angular width of half max

$$\Delta\theta = \Delta\theta_{m\lambda 1} = \Delta\theta_{m\lambda 2}$$




$\Delta\lambda_{\min}$： is resolvable wavelength difference when $\mathrm{d}\theta = \Delta\theta$.

# F-P Interferometer

Determine the chromatic resolving power ***A*** of F-P interferometer ?

Since $2nh\cos\theta = m\lambda$

$$-2nh\sin\theta \cdot \mathrm{d}\theta = m \cdot \mathrm{d}\lambda$$

When $\mathrm{d}\theta = \Delta\theta$, assume that $\mathrm{d}\lambda = \Delta\lambda_{\min}$, so

$$\Delta\lambda_{\min} = \frac{-2nh\sin\theta \cdot \Delta\theta}{m}$$

So, what is the angular width of the peak of the F-P interferometer?

# F-P Interferometer

The half angular width is the angular distance between the angle and $m^{\text{th}}$ central angle ($\theta_m = 0$) when transmission intensity is half of the peak.

Rewritten as $\delta = \dfrac{4\pi}{\lambda} nh\cos\theta$

$$\delta = \frac{2\pi}{\lambda}\Delta = \frac{2\pi}{\lambda} 2h\cos i_2$$

$\delta_m = 2m\pi$ Maximum transmission

$$I_{T\max} = I_0$$

$$I_T = I_0 \frac{(1-R)^2}{(1-R)^2 + 4R\sin^2\frac{\delta}{2}} = \frac{I_0}{2} \Rightarrow (1-R)^2 = 4R\sin^2\frac{\delta'}{2}$$

$$\sin\frac{\delta'}{2} = \frac{1-R}{2\sqrt{R}}$$

$$\sin\frac{\delta_m}{2} = 0$$

$$\Rightarrow \sin\frac{\delta'}{2} - \sin\frac{\delta_m}{2} = \frac{1-R}{2\sqrt{R}} = \cos\frac{\delta_m}{2}\cdot\frac{\Delta\delta}{2} = 1\cdot\frac{\Delta\delta}{2}$$

since $\Delta\delta = -\dfrac{4\pi}{\lambda} nh\sin\theta\cdot\Delta\theta'$

Half angular width

$$\boxed{\Delta\theta' = -\frac{\lambda}{4\pi nh\sin\theta}\cdot\frac{1-R}{\sqrt{R}}}$$

# F-P Interferometer

So, full angular breadth $\Delta\theta = -\dfrac{\lambda}{2\pi nh\sin\theta}\cdot\dfrac{1-R}{\sqrt{R}}$

$$\Delta\lambda_{\min} = \frac{-2nh\sin\theta\cdot\Delta\theta}{m}$$

$$\frac{\lambda}{\Delta\lambda_{\min}} = \frac{m\lambda}{-2nh\sin\theta\cdot\Delta\theta}$$

$$= -\frac{m\lambda}{2nh\sin\theta}\cdot -\frac{2\pi nh\sin\theta}{\lambda}\frac{\sqrt{R}}{1-R}$$

$$= m\frac{\pi\sqrt{R}}{1-R}$$

Chromatic resolving power:

$$\boxed{A = \frac{\lambda}{\Delta\lambda_{\min}} = m\frac{\pi\sqrt{R}}{1-R} \equiv m\mathcal{F}}$$

$\boxed{\mathcal{F} \equiv \dfrac{\pi\sqrt{R}}{1-R}}$ is called Fineness of the fringes

$\boxed{\dfrac{\mathcal{F}}{\pi}}$ is the number of round-trips photons undergo

$$F \equiv \frac{4R}{(1-R)^2}$$

Coefficient of finesse

larger $\mathcal{F}$ is, the sharper the fringes and the higher the resolution.

# F-P Interferometer

For example: for a F-P interferometer

$$h = 10\text{ mm},\ R = 0.98\ (\mathcal{F} = 150),\ \lambda = 632.8\text{ nm}.$$

Ring center:

$m = 2h/\lambda \approx 3\times 10^4$ Min resolution $\Delta\lambda_{\min}=1.4\times 10^{-4}$ nm

$A = m\mathcal{F} = 4.5\times 10^6$ Free spectral range $\Delta\lambda = \lambda^2/2h \approx 0.02$ nm

Comparison:

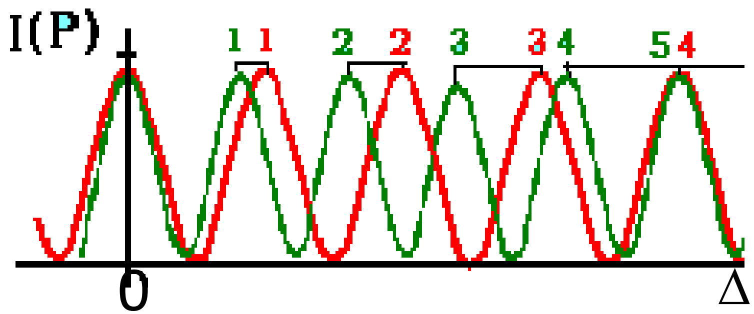
Diffraction grating $N = 60000$, $\Delta\lambda_{\min} = 0.01$ nm

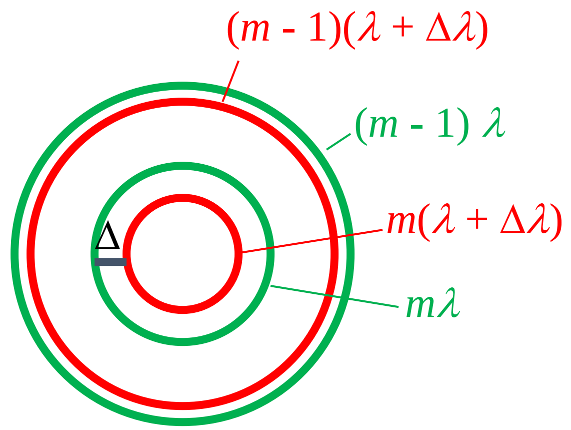
Prism spectrometer (5 cm): $\Delta\lambda_{\min} = 0.1$ nm

F-P interferometer: $A$ is extremely high, and $\Delta\lambda$ is small.

# F-P Interferometer

## ③ Free spectral region $\Delta\lambda$




$m\uparrow$, The easier it will overlap.
Instrument has a certain limit on the spectral range of the incident light.

$\Delta\lambda$: the maximum wavelength range that doesn't overlap.

# F-P Interferometer

$$2nh\cos\theta_m = m\lambda = (m-1)(\lambda+\Delta\lambda)$$

Ring center: $m_{\max}$. So if $\lambda$ and $(\lambda + \Delta\lambda)$ has no overlap in the center, other levels will not overlap. $\theta_m = 0$, so

$$2nh = m\lambda = (m-1)(\lambda+\Delta\lambda)$$

$$\Delta\lambda = \frac{\lambda}{m-1} \approx \frac{\lambda}{m}$$

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2nh}}$$

so:

$$\boxed{A = m\frac{\pi\sqrt{R}}{1-R}}$$

1) $m\uparrow$ (Near the center or $h$ is large) or $R\uparrow \rightarrow A\uparrow$

$$\boxed{D_\theta = -\frac{1}{\lambda\tan\theta}}$$

2) $m\uparrow$ (Near the center or $h$ is large) $\rightarrow D\uparrow$

Combination (1) (2), can make $\boldsymbol{A}$ very high

3) $m\uparrow \rightarrow \Delta\lambda = \lambda / m\downarrow$, Easily overlap.

# F-P Interferometer

$$\boxed{\Delta\lambda = \frac{\lambda^2}{2nh}}$$

So： $h\uparrow \rightarrow \Delta\lambda\downarrow$

For example: $h = 10$ cm; $\lambda_0 = 600$ nm; $n = 1$, so $\Delta\lambda = 0.018$ nm

Upper and lower wavelength limits

$$\lambda_{1,\,2} = 600 \text{ nm} \pm 0.009 \text{ nm}$$

If the incident light wave range is greater than 0.018 nm, the spectrum will overlap. The instrument is not working properly

- F-P interferometer has a large dispersion power, but not suitable for a wide range of spectra;
- Suitable for analyzing high resolution situations near a line.

| | |
|---|---|
| **Homework** | Problem 10.11, 10.43, 10.52, 10.63, 10.65. |
| **Homework*** | ^_^ |

| | |
|---|---|
| **Next week** | Birefringence<br>Sections 8.4 |