热力学与统计物理-第六次作业

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Problem 5.1

Answer:

(a):

$$pV^{\gamma} = constant \tag{1.1}$$

And:

$$pV = \mu RT \tag{1.2}$$

Then:

$$\frac{\mu RT_i V_i^{\gamma}}{V_i} = \frac{\mu RT_f V_f^{\gamma}}{V_f} \tag{1.3}$$

So:

$$T_f = T_i(\frac{V_f}{V_i})^{1-\gamma} \tag{1.4}$$

(b):

As mentioned before, the entropy change of an ideal gas can be written as:

$$\Delta S = \mu C_V ln\frac{T_f}{T_i} + \mu Rln\frac{V_f}{V_i} \tag{1.5}$$

And the ehtropy change is 0:

$$ln\frac{T_f}{T_i}(\frac{V_f}{V_i})^{\frac{R}{C_V}} = 0 \tag{1.6}$$

So:

$$T_f = T_i(\frac{V_f}{V_i})^{1-\gamma} \tag{1.7}$$

Problem 5.2

Answer:

(a):

$$W = \int PdV = 100\pi \approx 314J \tag{2.1}$$

(b):
We have:

$$PV = RT \tag{2.2}$$

So:

$$\Delta E = C_V \Delta T = \frac{3}{2}R(\frac{P_C V_C}{R} - \frac{P_A V_A}{R}) = 600J \tag{2.3}$$

(c):

$$Q = \Delta E + W \tag{2.4}$$

$$Q = 600 + (400 + \frac{100\pi}{2}) \approx 1157J \tag{2.5}$$

Problem 5.3

Answer:
(a):

$$C_V = \frac{\partial E}{\partial T} = \frac{5}{2}R \tag{3.1}$$

(b):

$$W = \int PdV = 1300J \tag{3.2}$$

(c):

$$\Delta E = C_V \Delta T = \frac{5}{2}R(T_C - T_A) = \frac{5}{2}(P_C V_C - P_A V_A) = 1500J \tag{3.3}$$

We have:

$$Q = \Delta E + W = 1500 + 1300 = 2800J \tag{3.4}$$

(d):
Same as (5.2):

$$\Delta S = C_V ln\frac{T_C}{T_A} + Eln\frac{V_C}{V_A} \approx 23.6J/K \tag{3.5}$$

Problem 5.5

(a):

The temperature decreases. Beacause the gas does work which makes its internal energy decreases.

(b):

The entropy increases. Because this process is irreversible.

(c):

The system is isolated, so $Q = 0$, Then from the first law:

$$\Delta E = -W = -\frac{mg}{A}(V_f - V_o) \tag{4.1}$$

And:

$$\Delta = \mu C_V (T_f - T_o) \tag{4.2}$$

So:

$$-\frac{mg}{A}(V_f - V_o) = \mu C_V (T_f - T_o) \tag{4.3}$$

Then let's determine V_f.

We have:

$$p = \frac{mg}{A} \tag{4.4}$$

$$pV_f = \mu R T_f \tag{4.5}$$

So:

$$V_f = \frac{\mu A R T_f}{mg} \tag{4.6}$$

So:

$$T_f = \frac{1}{1 + \frac{R}{C_V}}(T_o + \frac{mgV_o}{\mu C_V A}) \tag{4.7}$$

Problem 5.6

Answer:

From Newton second law:

$$m\ddot{x} = pA - mg - p_o A \tag{5.1}$$

And:

$$pV^\gamma = constant = (p_o + \frac{mg}{A})V_o^\gamma \tag{5.2}$$

So:

$$m\ddot{x} = (p_o + \frac{mg}{A})\frac{AV_o^\gamma}{(Ax)^\gamma} - mg - p_o A \tag{5.3}$$

The displacement from the equilibrium position is $\frac{V_o}{A}$, Let's $x = \frac{V_o}{A} + \eta$, and expand about $\frac{V_o}{A}$:

$$\frac{1}{x^gamma} = \frac{1}{(\frac{V_o}{A} + \eta)^\gamma} = (\frac{A}{V_o})^gamma(1 - \frac{\gamma A\eta}{V_o} + ...) \tag{5.4}$$

We only keep the first and second terms in (5.4):

$$m\ddot{\eta} = -(p_o + \frac{mg}{A})\frac{A^2\gamma}{V_o}\eta \tag{5.5}$$

We can find that (5.5) is the motion equation for harmonic motion.
The frequency is:

$$\mu = \frac{1}{2\pi}[(p_o + \frac{mg}{A})\frac{A^2\gamma}{V_o m}]^{\frac{1}{2}} \tag{5.6}$$

So:

$$\gamma = \frac{4\pi^2\mu^2 m V_o}{p_o A^2 + mgA} \tag{5.7}$$

partial 5.7

(a):

Let's consider a thin unit of atmosphere at height z to z+dz

$$p(z+dz)A - p(z)A = -n(Adz)mg \tag{6.1}$$

Where n is the number of particles per unit volume, m is the mass of each particle.

$$\frac{dp}{dz} = -nmg = -\frac{n\mu g}{N_A} \tag{6.2}$$

Since $p = nkT$:

$$\frac{dp}{p} = -\frac{\mu g}{RT}dz \tag{6.3}$$

(b):

Since it's adiabatic:

$$pV^\gamma = constant \tag{6.4}$$

With $pV = \mu RT$:

$$T^\gamma p^{1-\gamma} = constant \tag{6.5}$$

Then:

$$\gamma T^{\gamma-1}p^{1-\gamma}dT + (1-\gamma)T^\gamma p^{-\gamma}dp = 0 \tag{6.6}$$

So:

$$\frac{dp}{p} = \frac{\gamma}{\gamma - 1}\frac{dT}{T} \tag{6.7}$$

(c):

From (6.3) and (6.7):

$$\frac{dT}{dz} = \frac{1-\gamma}{\gamma R}\mu g \tag{6.8}$$

For N_2, $\mu = 28$ and $\gamma = 1.4$, Then:

$$\frac{dT}{dz} = -9.4 \tag{6.9}$$

(d):

Integrating (6.3):

$$p = p_o e^{-\mu g z/RT} \tag{6.10}$$

(e):

From (6.8):

$$T = T_o - \frac{\gamma - 1}{\gamma R}\mu g z \tag{6.11}$$

Then:

$$\int_{p_o}^{p} \frac{dp'}{p'} = \int_{o}^{z} \frac{-\mu g dz'}{R(T_o - \frac{\gamma-1}{\gamma}\frac{\mu g z'}{R})} \tag{6.12}$$

So:

$$p = p_o(1 - \frac{(\gamma - 1)\mu g z}{\gamma R T_o})^{\frac{\gamma}{\gamma-1}} \tag{6.13}$$