

Let's label our solution, from the greedy algorithm: **G**:  $\{(a_1, b_1), (a_2, b_2) \dots (a_n, b_n)\}$  where  $n$  is the total number of pairs.

Let's label our solution, from the optimal algorithm: **O**:  $\{(a_1, b_1), (a_2, b_2) \dots (a_n, b_n)\}$  where  $n$  is the total number of pairs.

In both the cases, all the elements will exist in both sets as the both sticks are of the same number,  $n$ .

However, the ordering can be different in both the sets.

Let's take an example:

G:  $\{(2, 5), (5, 10)\} \Rightarrow \text{cost} = (3+5)/2 \Rightarrow 4$

O:  $\{(5, 5), (2, 10)\} \Rightarrow \text{cost} = (0+8)/2 \Rightarrow 4$

As we can see, both algorithms generate same cost hence the greedy algorithm does as good as the optimal one.

If we exchange pairs between G and O, we can continue doing so without worsening the quality of the algorithm, hence the greedy solution is the optimal one.