Problems: Simply Connected Regions

1. Is the paraboloid described by $z = x^2 + y^2$ a simply connected surface? Why or why not?

2. Is the Möbius strip described in lecture a simply connected surface? Why or why not?

Problems: Surface Independence

Suppose that $\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{G}$, where the components of $\mathbf{G}$ have continuous second partial derivatives. Suppose also that S is a closed, positively-oriented surface divided into two parts by a closed curve C. Apply Stokes' theorem to show that $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = 0$.

Answer: We start by drawing a picture.

By Stokes' theorem, $\oint_C \mathbf{G} \cdot d\mathbf{r} = \iint_{S_1} \mathrm{curl}\mathbf{G}\, dA = \iint_{S_1} \mathbf{F} \cdot \mathbf{n}\, dS$. Similarly,

$$\oint_{-C} \mathbf{G} \cdot d\mathbf{r} = \iint_{S_2} \mathbf{F} \cdot \mathbf{n}\, dS.$$

We sum the two results to get:

$$\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = \iint_{S_1} \mathbf{F} \cdot \mathbf{n}\, dS + \iint_{S_2} \mathbf{F} \cdot \mathbf{n}\, dS = \oint_C \mathbf{G} \cdot d\mathbf{r} + \oint_{-C} \mathbf{G} \cdot d\mathbf{r} = 0.$$

18.02SC Multivariable Calculus
Fall 2010

Problems: Surface Independence

Suppose that $\mathbf{F} = \boldsymbol{\nabla} \times \mathbf{G}$, where the components of $\mathbf{G}$ have continuous second partial derivatives. Suppose also that S is a closed, positively-oriented surface divided into two parts by a closed curve C. Apply Stokes' theorem to show that $\iint_S \mathbf{F} \cdot \mathbf{n}\, dS = 0$.