4 Geometric Constructions

Let's study.

- Construction of a triangle similar to the given triangle
  - ✻ To construct a triangle, similar to the given triangle, bearing the given ratio with the sides of the given triangle.
    - (i) When vertices are distinct
    - (ii) When one vertex is common
- Construction of a tangent to a circle.
  - ✻ To construct a tangent at a point on the circle.
    - (i) Using centre of the circle.
    - (ii) Without using the centre of the circle.
  - ✻ To construct tangents to the given circle from a point outside the circle.

Let's recall.

In the previous standard you have learnt the following constructions. Let us recall those constructions.

- To construct a line parallel to a given line and passing through a given point outside the line.
- To construct the perpendicular bisector of a given line segment.
- To construct a triangle whose sides are given.
- To divide a given line segment into given number of equal parts
- To divide a line segment in the given ratio.
- To construct an angle congruent to the given angle.

In the ninth standard you have carried out the activity of preparing a map of surroundings of your school. Before constructing a building we make its plan. The surroundings of a school and its map, the building and its plan are similar to each other. We need to draw similar figures in Geography, architecture, machine drawing etc. A triangle is the simplest closed figure. We shall learn how to construct a triangle similar to the given triangle.

Let's learn.

Construction of Similar Triangle

To construct a triangle similar to the given triangle, satisfying the condition of given ratio of corresponding sides.

The corresponding sides of similar triangles are in the same proportion and the corresponding angles of these triangles are equal. Using this property, a triangle which is similar to the given triangle can be constructed.

Ex. (1) Δ ABC ~ Δ PQR, in Δ ABC, AB = 5.4 cm, BC = 4.2 cm, AC = 6.0 cm. AB: PQ = 3:2. Construct Δ ABC and Δ PQR.

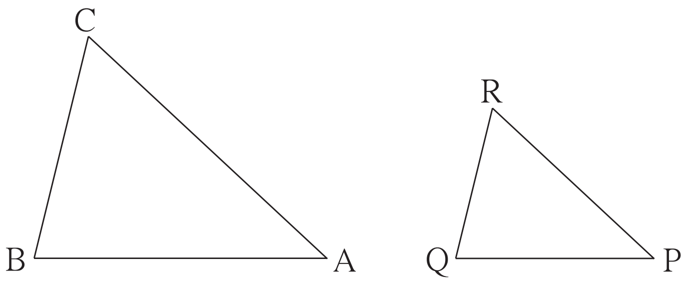

Fig. 4.1
Rough Figure

Construct Δ ABC of given measure.

Δ ABC and Δ PQR are similar.

$\therefore$ their corresponding sides are proportional.

$$\frac{AB}{PQ} = \frac{BC}{QR} = \frac{AC}{PR} = \frac{3}{2} \quad \text{..........} \quad (I)$$

As the sides AB, BC, AC are known, we can find the lengths of sides PQ, QR, PR.

Using equation [I]

$$\frac{5.4}{PQ} = \frac{4.2}{QR} = \frac{6.0}{PR} = \frac{3}{2}$$

$\therefore$ PQ = 3.6 cm, QR = 2.8 cm and PR = 4.0 cm

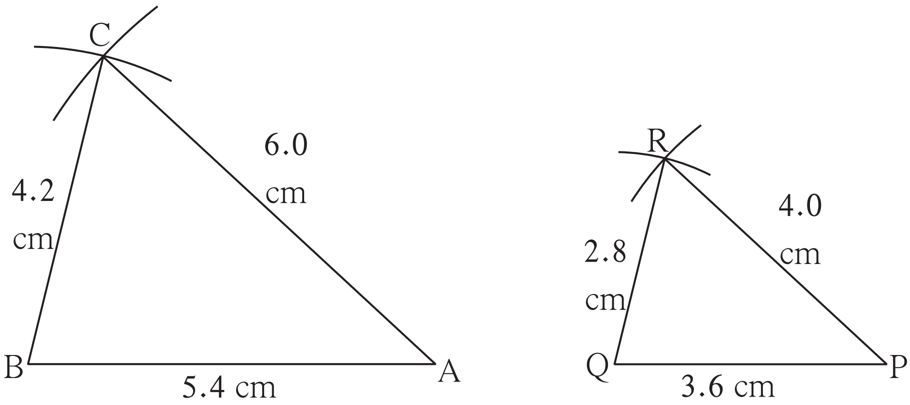

Fig. 4.2

For More Information

While drawing the triangle similar to the given triangle, sometimes the lengths of the sides we obtain by calculation are not easily measureable by a scale. In such a situation we can use the construction 'To divide the given segment in the given number of eqaul parts'.

For example, if length of side AB is $\frac{11.6}{3}$ cm, then by dividing the line segment of length 11.6 cm in three equal parts, we can draw segment AB.

If we know the lengths of all sides of Δ PQR, we can construct Δ PQR.

In the above example (1) there was no common vertex in the given triangle and the triangle to be constructed. If there is a common vertex, it is convenient to follow the method in the following example.

Ex.(2) Construct any Δ ABC. Construct Δ A′BC′ such that AB : A′B = 5:3 and Δ ABC ~ Δ A′BC′

Analysis : As shown in fig 4.3 , let the points B, A, A′ and B, C, C′ be collinear.

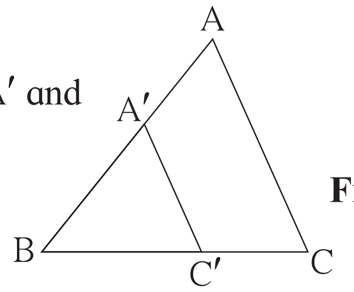

Fig. 4.3

Rough Figure

Δ ABC ~ Δ A′BC′ $\therefore$ $\angle$ ABC = $\angle$A′BC′

$$\frac{AB}{A'B} = \frac{BC}{BC'} = \frac{AC}{A'C'} = \frac{5}{3}$$

$\therefore$ sides of Δ ABC are longer than corresponding sides of Δ A′BC′ .

$\therefore$ the length of side BC′ will be equal to 3 parts out of 5 equal parts of side BC. So if we construct Δ ABC, point C′ will be on the side BC, at a distance equal to 3 parts from B. Now A′ is the point of intersection of AB and a line through C′, parallel to CA.

$$\frac{BA'}{BA} = \frac{BC'}{BC} = \frac{3}{5} \text{ i.e, } \frac{BA}{BA'} = \frac{BC}{BC'} = \frac{5}{3} \text{ ...... Taking inverse}$$

Steps of construction :

(1) Construct any Δ ABC.

(2) Divide segment BC in 5 equal parts.

(3) Name the end point of third part of seg BC as C′ $\therefore$ BC′ = $\frac{3}{5}$ BC

(4) Now draw a line parallel to AC through C′. Name the point where the parallel line intersects AB as A′.

(5) Δ A′BC′ is the required trinangle similar to Δ ABC

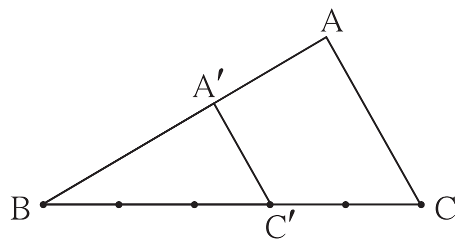

Fig. 4.4

Note : To divide segment BC, in five equal parts, it is convenient to draw a ray from B, on the side of line BC in which point A does not lie.

Take points T_1, T_2, T_3, T_4, T_5 on the ray such that

$BT_1 = T_1T_2 = T_2T_3 = T_3T_4 = T_4T_5$

Join T_5C and draw lines parallel to T_5C through T_1, T_2, T_3, T_4 .

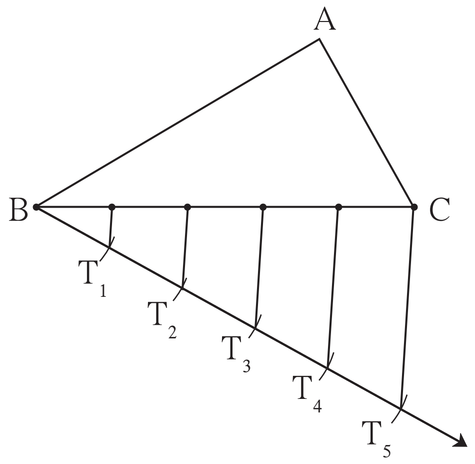

Fig. 4.5

Let's think

Δ A′BC′ can also be constructed as shown in the adjoining figure.

What changes do we have to make in steps of construction in that case ?

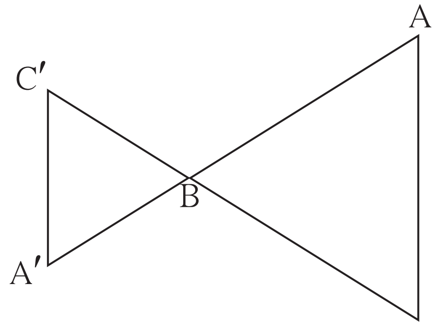

Fig. 4.6

Ex. (3) Construct Δ A′BC′ similar to Δ ABC such that AB:A′B = 5:7

Analysis : Let points B, A, A′ as well as points B, C, C′ be collinear.

Δ ABC ~ Δ A′BC′ and AB : A′B = 5:7

$\therefore$ sides of Δ ABC are smaller than sides of Δ A′BC′

and $\angle$ABC $\cong$ $\angle$A′BC′

Let us draw a rough figure with these considerations. Now $\frac{BC}{BC'} = \frac{5}{7}$

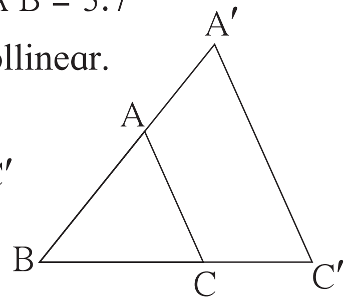

Fig. 4.7
Rough Figure

$\therefore$ If seg BC is divided into 5 equal parts, then seg BC′ will be 7 times each part of seg BC.

$\therefore$ let us divide side BC of Δ ABC in 5 equal parts and locate point C′ at a distance equal to 7 such parts from B on ray BC. A line through point C′ parallel to seg AC is drawn it will intersect ray BA at point A′. According to the basic proportionality theorem we will get Δ A′BC′ as described.

Steps of construction :

(1) Construct any Δ ABC.

(2) Divide segment BC into 5 five equal parts. Fix point C′ on ray BC such that length of BC′ is seven times of each equal part of seg BC

(3) Draw a line parallel to side AC, through C′. Name the point of intersection of the line and ray BA as A′.

We get the required Δ A′BC′ similar to Δ ABC.

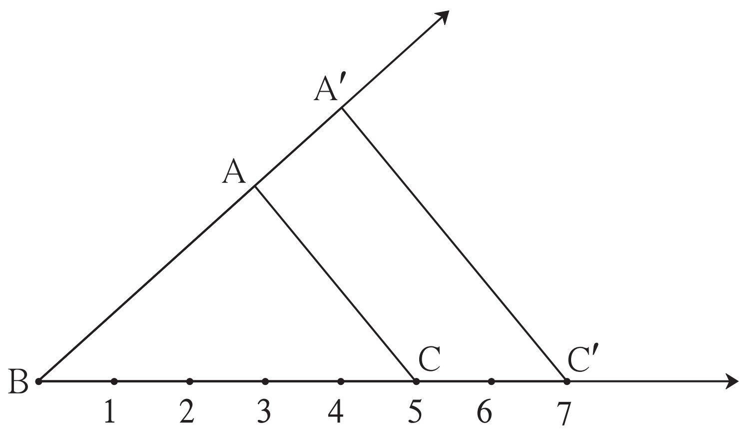

Fig. 4.8

Practice set 4.1

1. Δ ABC ~ Δ LMN. In Δ ABC, AB = 5.5 cm, BC = 6 cm, CA = 4.5 cm. Construct Δ ABC and Δ LMN such that $\frac{BC}{MN} = \frac{5}{4}$.

2. Δ PQR ~ Δ LTR. In Δ PQR, PQ = 4.2 cm, QR = 5.4 cm, PR = 4.8 cm. Construct Δ PQR and Δ LTR, such that $\frac{PQ}{LT} = \frac{3}{4}$.

3. Δ RST ~ ΔXYZ. In Δ RST, RS = 4.5 cm, $\angle$RST = 40°, ST = 5.7 cm Construct Δ RST and Δ XYZ, such that $\frac{RS}{XY} = \frac{3}{5}$.

4. Δ AMT ~ Δ AHE. In Δ AMT, AM = 6.3 cm, $\angle$TAM = 50°, AT = 5.6 cm. $\frac{AM}{AH} = \frac{7}{5}$. Construct Δ AHE.

Construction of a tangent to a circle at a point on the circle

(i) Using the centre of the circle.

Analysis :

Suppose we want to construct a tangent l passing through a point P on the circle with centre C. We shall use the property that a line perpendicular to the radius at its outer end is a tangent to the circle. If CP is a radius with point P on the circle, line l through P and perpendicular to CP is the tangent at P. For this we will use the construction of drawing a perpendicular to a line through a point on it.

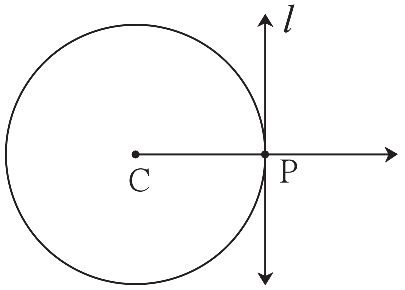

Fig. 4.9

For convenience we shall draw ray CP

Steps of construction

(1) Draw a circle with centre C.
Take any point P on the circle.

(2) Draw ray CP.

(3) Draw line l perpendicular to ray CX through point P.
Line l is the required tangent to the circle at point 'P'.

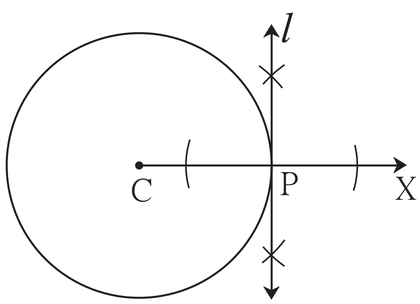

Fig. 4.10

ii) Without using the centre of the circle.

Example: Construct a circle of any radius. Take any point C on it. Construct a tangent to the circle without using centre of the circle.

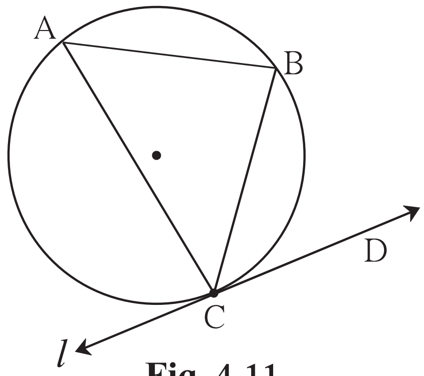

Fig. 4.11

Analysis :

As shown in the figure, let line l be the tangent to the circle at point C. Line CB is a chord and $\angle$ CAB is an inscribed angle. Now by tangent- secant angle theorem, $\angle CAB \cong \angle BCD$.

By converse of tangent- secant theorem, if we draw the line CD such that, $\angle CAB \cong \angle BCD$, then it will be the required tangent.

Steps of Construction :

(1) Draw a circle of a suitable radius. Take any point C on it.

(2) Draw chord CB and an inscribed $\angle CAB$.

(3) With the centre A and any convenient radius draw an arc intersecting the sides of $\angle BAC$ in points M and N.

(4) Using the same radius and centre C, draw an arc intersecting the chord CB at point R.

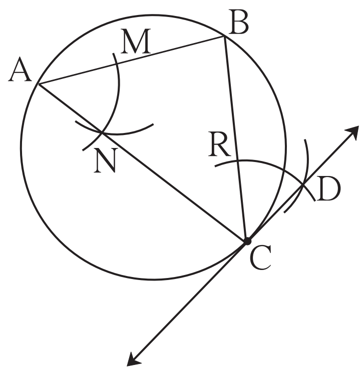

Fig. 4.12

(5) Taking the radius equal to d(MN) and centre R, draw an arc intersecting the arc drawn in the previous step. Let D be the point of intersection of these arcs. Draw line CD. Line CD is the required tangent to the circle.

Note that $\angle$ MAN and $\angle$ BCD in the above figure are congruent. If we draw seg MN and seg RD, then Δ MAN and Δ RCD are congruent by SSS test.

$\therefore \angle MAN \cong \angle BCD$

To construct tangents to a circle from a point outside the circle.

Analysis :

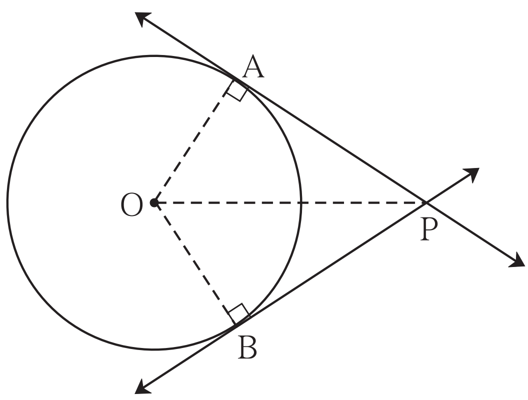

Fig. 4.13

As shown in the figure let P be a point in the exterior of the circle. Let PA and PB be the tangents to the circle with the centre O, touching the circle in points A and B respectively. So if we find points A and B on the circle, we can construct the tangents PA and PB. If OA and OB are the radii of the circle, then OA $\perp$ line PA and seg OB $\perp$ line PB.

Δ OAP and OBP are right angled triangles and seg OP is their common hypotenuse. If we draw a circle with diameter OP, then the points where it intersects the circle with centre O, will be the positions of points A and B respectively, because angle inscribed in a semicircle is a right angle.

Steps of Construction

(1) Construct a circle of any radius with centre O.

(2) Take any point P in the exterior of the circle.

(3) Draw segment OP. Draw perpendicular bisector of seg OP to get its midpoint M.

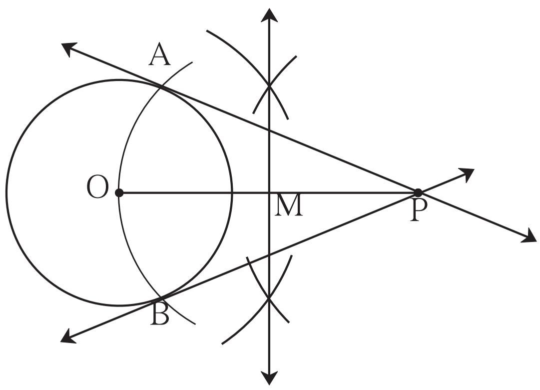

Fig. 4.14

(4) Draw a circle with radius OM and centre M

(5) Name the points of intersection of the two circles as A and B.

(6) Draw line PA and line PB.

Practice set 4.2

1. Construct a tangent to a circle with centre P and radius 3.2 cm at any point M on it.

2. Draw a circle of radius 2.7 cm. Draw a tangent to the circle at any point on it.

3. Draw a circle of radius 3.6 cm. Draw a tangent to the circle at any point on it without using the centre.

4. Draw a circle of radius 3.3 cm Draw a chord PQ of length 6.6 cm. Draw tangents to the circle at points P and Q. Write your obeservation about the tangents.

5. Draw a circle with radius 3.4 cm. Draw a chord MN of length 5.7 cm in it. construct tangents at point M and N to the circle.

6. Draw a circle with centre P and radius 3.4 cm. Take point Q at a distance 5.5 cm from the centre. Construct tangents to the circle from point Q.

7. Draw a circle with radius 4.1 cm. Construct tangents to the circle from a point at a distance 7.3 cm from the centre.

Problem set 4

1. Select the correct alternative for each of the following questions.

(1) The number of tangents that can be drawn to a circle at a point on the circle is ....

(A) 3 (B) 2 (C) 1 (D) 0

(2) The maximun number of tangents that can be drawn to a circle from a point out side it is ....

(A) 2 (B) 1 (C) one and only one (D) 0

(3) If $\Delta ABC \sim \Delta PQR$ and $\frac{AB}{PQ} = \frac{7}{5}$, then ....

(A) Δ ABC is bigger. (B) Δ PQR is bigger.

(C) Both triangles will be equal. (D) Can not be decided.

2. Draw a circle with centre O and radius 3.5 cm. Take point P at a distance 5.7 cm from the centre. Draw tangents to the circle from point P.

3. Draw any circle. Take any point A on it and construct tangent at A without using the centre of the circle.

4. Draw a circle of diameter 6.4 cm. Take a point R at a distance equal to its diameter from the centre. Draw tangents from point R.

5. Draw a circle with centre P. Draw an arc AB of 100° measure. Draw tangents to the circle at point A and point B.

6. Draw a circle of radius 3.4 cm and centre E. Take a point F on the circle. Take another point A such that E-F-A and FA = 4.1 cm. Draw tangents to the circle from point A.

7. Δ ABC $\sim$ ΔLBN. In Δ ABC, AB = 5.1cm, $\angle B = 40^\circ$, BC=4.8 cm, $\frac{AC}{LN} = \frac{4}{7}$. Construct Δ ABC and ΔLBN.

8. Construct Δ PYQ such that, PY = 6.3 cm, YQ = 7.2 cm, PQ = 5.8 cm. If $\frac{YZ}{YQ} = \frac{6}{5}$, then construct Δ XYZ similar to Δ PYQ.

PUN6L7

(3) 90° ; MS : SR = 2 : 1 **9.** $4\sqrt{3}$ cm

13. (1) 180° (2) $\angle AQP \cong \angle ASQ \cong \angle ATQ$

(3) $\angle QTS \cong \angle SQR \cong \angle SAQ$ (4) 65°, 130° (5) 100° **14.**(1) 70°

(2) 130° (3) 210° **15.** (1) 56° (2) 6 (3) 16 or 9 **16.** (1) 15.5°

(2) 3.36 (3) 6 **18.** (1) 68° (2) OR = 16.2, QR = 13 (3) 13 **21.** 13

Chapter 4 Geometric Constructions

Problem set 4

1. (1) C (2) A (3) A

Chapter 5 Co-ordinate Geometry

Practice set 5.1

1. (1) $2\sqrt{2}$ (2) $4\sqrt{2}$ (3) $\frac{11}{2}$ (4) 13 (5) 20 (6) $\frac{29}{2}$

2. (1) are collinear. (2) are not collinear. (3) are not collinear. (4) are collinear.

3. $(-1, 0)$ **7.** 7 or -5

Practice set 5.2

1. $(1, 3)$ **2.** (1) $\left(-\frac{1}{3}, -\frac{1}{3}\right)$ (2) $\left(\frac{4}{7}, -\frac{11}{7}\right)$ (3) $\left(0, \frac{13}{3}\right)$ **3.** 2:7 **4.** $(-6, 3)$

5. 2:5, $k = 6$ **6.** $(11, 18)$ **7.** (1) $(1, 3)$ (2) $(6, -2)$ (3) $\left(\frac{19}{3}, \frac{22}{3}\right)$

8. $(-1, -7)$ **9.** $h = 7$, $k = 18$ **10.** $(0, 2)$; $(-2, -3)$

11. $(-9, -8)$, $(-4, -6)$, $(1, -4)$ **12.** $(16, 12)$, $(12, 14)$, $(8, 16)$, $(4, 18)$

Practice set 5.3

1. (1) 1 (2) $\sqrt{3}$ (3) slope cannot be determined.

2. (1) 2 (2) $-\frac{3}{8}$ (3) $\frac{5}{2}$ (4) $\frac{5}{4}$ (5) $\frac{1}{2}$ (6) slope cannot be determined.

3. (1) are collinear. (2) are collinear. (3) are not collinear. (4) are collinear.

(5) are collinear. (6) are collinear.

4. -5; $\frac{1}{5}$; $-\frac{2}{3}$ 6. $k = 5$ 7. $k = 0$ 8. $k = 5$

Problem set 5

1. (1) D (2) D (3) C (4) C

2. (1) are collinear. (2) are collinear. (3) are not collinear. 3. $(6, 13)$ 4. 3:1