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Quadratic Equations



Let's study.

- Quadratic equation : Introduction
- Methods of solving quadratic equation
- Nature of roots of quadratic equation
- Relation between roots and coefficients
- Applications of quadratic equations



Let's recall.

You have studied polynomials last year. You know types of polynomials according to their degree. When the degree of polynomial is 1 it is called a linear polynomial and if degree of a polynomial is 2 it is called a quadratic polynomial.

Activity : Classify the following polynomials as linear and quadratic.

$$5x + 9, \quad x^2 + 3x - 5, \quad 3x - 7, \quad 3x^2 - 5x, \quad 5x^2$$

Linear polynomials

Quadratic polynomials

Now equate the quadratic polynomial to 0 and study the equation we get. Such type of equation is known as quadratic equation. In practical life we may use quadratic equations many times.

Ex. Sanket purchased a rectangular plot having area 200 m^2 . Length of the plot was 10 m more than its breadth. Find the length and the breadth of the plot.

Let the breadth of the plot be x metre.

$$\therefore \text{Length} = (x + 10) \text{ metre}$$

Area of rectangle = length \times breadth

$$\therefore 200 = (x + 10) \times x$$

$$\therefore 200 = x^2 + 10x$$

$$\text{That is } x^2 + 10x = 200$$

$$\therefore x^2 + 10x - 200 = 0$$

Now, solving equation $x^2 + 10x - 200 = 0$, we will decide the dimensions of the plot.

Let us study how to solve the quadratic equation.



Let's recall.

Activity : $x^2 + 3x - 5$, $3x^2 - 5x$, $5x^2$; Write the polynomials in the index form.

Observe the coefficients and fill in the boxes.

$$x^2 + 3x - 5, \quad 3x^2 - 5x + 0, \quad 5x^2 + 0x + 0$$

- ◆ Coefficients of x^2 are , and these coefficients are non zero.
- ◆ Coefficients of x are 3, and respectively.
- ◆ Constants terms are , and respectively.

Here constant term of second and third polynomial is zero.



Let's learn.

Standard form of quadratic equation

The equation involving one variable and having 2 as the maximum index of the variable is called the quadratic equation.

General form is $ax^2 + bx + c = 0$

In $ax^2 + bx + c = 0$, a, b, c are real numbers and $a \neq 0$.

$ax^2 + bx + c = 0$ is the general form of quadratic equation.

Activity : Complete the following table

Quadratic Equation	General form	a	b	c
$x^2 - 4 = 0$	$x^2 + 0x - 4 = 0$	1	0	-4
$y^2 = 2y - 7$
$x^2 + 2x = 0$

🔗 Solved Examples 🔗

Ex. (1) Decide which of the following are quadratic equations ?

(1) $3x^2 - 5x + 3 = 0$ (2) $9y^2 + 5 = 0$ (3) $m^3 - 5m^2 + 4 = 0$ (4) $(l + 2)(l - 5) = 0$

Solution : (1) In the equation $3x^2 - 5x + 3 = 0$, x is the only variable and maximum index of the variable is 2

∴ It is a quadratic equation.

(2) In the equation $9y^2 + 5 = 0$, is the only variable and maximum index of the variable is

\therefore It a quadratic equation.

(3) In the equation $m^3 - 5m^2 + 4 = 0$, is the only variable but maximum index of the variable is not 2.

\therefore It a quadratic equation.

(4) $(l + 2)(l - 5) = 0$

$$\therefore l(l - 5) + 2(l - 5) = 0$$

$$\therefore l^2 - 5l + 2l - 10 = 0$$

$\therefore l^2 - 3l - 10 = 0$, In this equation is the only variable and maximum index of the variable is .

\therefore It a quadratic equation.



Let's learn.

Roots of a quadratic equation

In the previous class you have studied that if value of the polynomial is zero for $x = a$ then $(x - a)$ is a factor of that polynomial. That is if $p(x)$ is a polynomial and $p(a) = 0$ then $(x - a)$ is a factor of $p(x)$. In this case 'a' is the root or solution of $p(x) = 0$

For Example ,

Let $x = -6$ in the polynomial $x^2 + 5x - 6$

$$x^2 + 5x - 6 = (-6)^2 + 5 \times (-6) - 6$$

$$= 36 - 30 - 6 = 0$$

$\therefore x = -6$ is a solution of the equation.

Hence -6 is one root of the equation

$$x^2 + 5x - 6 = 0$$

Let $x = 2$ in polynomial $x^2 + 5x - 6$

$$x^2 + 5x - 6 = 2^2 + 5 \times 2 - 6$$

$$= 4 + 10 - 6$$

$$= 8 \neq 0$$

$\therefore x = 2$ is not a solution of the equation $x^2 + 5x - 6 = 0$

Solved Example

Ex. $2x^2 - 7x + 6 = 0$ check whether (i) $x = \frac{3}{2}$, (ii) $x = -2$ are solutions of the equations.

Solution : (i) Put $x = \frac{3}{2}$ in the polynomial $2x^2 - 7x + 6$

$$2x^2 - 7x + 6 = 2\left(\frac{3}{2}\right)^2 - 7\left(\frac{3}{2}\right) + 6$$

$$= 2 \times \frac{9}{4} - \frac{21}{2} + 6$$

$$= \frac{9}{2} - \frac{21}{2} + \frac{12}{2} = 0$$

$\therefore x = \frac{3}{2}$ is a solution of the equation.

(ii) Let $x = -2$ in $2x^2 - 7x + 6$

$$2x^2 - 7x + 6 = 2(-2)^2 - 7(-2) + 6$$

$$= 2 \times 4 + 14 + 6$$

$$= 28 \neq 0$$

$\therefore x = -2$ is not a solution of the equation.

Activity : If $x = 5$ is a root of equation $kx^2 - 14x - 5 = 0$ then find the value of k by completing the following activity.

Solution : One of the roots of equation $kx^2 - 14x - 5 = 0$ is .

\therefore Now Let $x =$ in the equation.

$$k \text{ }^2 - 14 \text{ } - 5 = 0$$

$$\therefore 25k - 70 - 5 = 0$$

$$25k - \text{} = 0$$

$$25k = \text{}$$

$$\therefore k = \frac{\text{}}{\text{}} = 3$$



Let's remember!

- (1) $ax^2 + bx + c = 0$ is the general form of equation where a, b, c are real numbers and ' a ' is non zero.
- (2) The values of variable which satisfy the equation [or the value for which both the sides of equation are equal] are called solutions or roots of the equation.

Practice Set 2.1

- Write any two quadratic equations.
- Decide which of the following are quadratic equations.

(1) $x^2 + 5x - 2 = 0$

(2) $y^2 = 5y - 10$

(3) $y^2 + \frac{1}{y} = 2$

(4) $x + \frac{1}{x} = -2$

(5) $(m + 2)(m - 5) = 0$

(6) $m^3 + 3m^2 - 2 = 3m^3$
- Write the following equations in the form $ax^2 + bx + c = 0$, then write the values of a, b, c for each equation.

(1) $2y = 10 - y^2$

(2) $(x - 1)^2 = 2x + 3$

(3) $x^2 + 5x = -(3 - x)$

(4) $3m^2 = 2m^2 - 9$

(5) $P(3 + 6p) = -5$

(6) $x^2 - 9 = 13$
- Determine whether the values given against each of the quadratic equation are the roots of the equation.

(1) $x^2 + 4x - 5 = 0$, $x = 1, -1$

(2) $2m^2 - 5m = 0$, $m = 2, \frac{5}{2}$
- Find k if $x = 3$ is a root of equation $kx^2 - 10x + 3 = 0$.
- One of the roots of equation $5m^2 + 2m + k = 0$ is $\frac{-7}{5}$. Complete the following activity to find the value of ' k '.

Solution : is a root of quadratic equation $5m^2 + 2m + k = 0$

\therefore Put $m = \text{}$ in the equation.

$$5 \times \text{}^2 + 2 \times \text{} + k = 0$$

$$\text{} + \text{} + k = 0$$

$$\text{} + k = 0$$

$$k = \text{}$$



Let's recall.

Last year you have studied the methods to find the factors of quadratic polynomials like $x^2 - 4x - 5$, $2m^2 - 5m$, $a^2 - 25$. Try the following activity and revise the same.

Activity : Find the factors of the following polynomials.

(1) $x^2 - 4x - 5$

$= \underline{x^2 - 5x} + \underline{1x - 5}$

$= x(\dots) + 1(\dots)$

$= (\dots)(\dots)$

(2) $2m^2 - 5m$

$= \dots \dots$

(3) $a^2 - 25$

$= a^2 - 5^2$

$= (\dots)(\dots)$



Let's learn.

Solutions of a quadratic equation by factorisation

By substituting arbitrary values for the variable and deciding the roots of quadratic equation is a time consuming process. Let us learn to use factorisation method to find the roots of the given quadratic equation.

$$x^2 - 4x - 5 = (x - 5)(x + 1)$$

$(x - 5)$ and $(x + 1)$ are two linear factors of quadratic polynomial $x^2 - 4x - 5$.

So the quadratic equation obtained from $x^2 - 4x - 5$ can be written as

$$(x - 5)(x + 1) = 0$$

If product of two numbers is zero then at least one of them is zero.

$$\therefore x - 5 = 0 \text{ or } x + 1 = 0$$

$$\therefore x = 5 \text{ or } x = -1$$

$\therefore 5$ and the -1 are the roots of the given quadratic equation.

While solving the equation first we obtained the linear factors. So we call this method as 'factorization method' of solving quadratic equation.

🔗 Solved Examples 🔗

Ex. Solve the following quadratic equations by factorisation.

(1) $m^2 - 14m + 13 = 0$

(2) $3x^2 - x - 10 = 0$

(3) $3y^2 = 15y$

(4) $x^2 = 3$

(5) $6\sqrt{3}x^2 + 7x = \sqrt{3}$

(1) $m^2 - 14m + 13 = 0$

$$\therefore m^2 - 13m - 1m + 13 = 0$$

$$\therefore \overline{m(m - 13)} - 1 \overline{(m - 13)} = 0$$

$$\therefore (m - 13)(m - 1) = 0$$

$$\therefore m - 13 = 0 \text{ or } m - 1 = 0$$

$$\therefore m = 13 \text{ or } m = 1$$

$\therefore 13$ and 1 are the roots of the given quadratic equation.

(2) $3x^2 - x - 10 = 0$

$$\therefore \underline{3x^2 - 6x} + \underline{5x - 10} = 0$$

$$\therefore 3x(x - 2) + 5(x - 2) = 0$$

$$\therefore (3x + 5)(x - 2) = 0$$

$$\therefore (3x + 5) = 0 \text{ or } (x - 2) = 0$$

$$\therefore x = -\frac{5}{3} \text{ or } x = 2$$

$\therefore -\frac{5}{3}$, and 2 are the roots of the given quadratic equation.

$$(3) \ 3y^2 = 15y$$

$$\therefore 3y^2 - 15y = 0$$

$$\therefore 3y(y - 5) = 0$$

$$\therefore 3y = 0 \text{ or } (y - 5) = 0$$

$$\therefore y = 0 \text{ or } y = 5$$

\therefore 0 and 5 are the roots of quadratic equation.

$$(4) \ x^2 = 3$$

$$\therefore x^2 - 3 = 0$$

$$\therefore x^2 - (\sqrt{3})^2 = 0$$

$$\therefore (x + \sqrt{3})(x - \sqrt{3}) = 0$$

$$\therefore (x + \sqrt{3}) = 0 \text{ or } (x - \sqrt{3}) = 0$$

$$\therefore x = -\sqrt{3} \text{ or } x = \sqrt{3}$$

$\therefore -\sqrt{3}$ and $\sqrt{3}$ are the roots of given quadratic equation.

$$(5) \ 6\sqrt{3}x^2 + 7x = \sqrt{3}$$

$$\therefore 6\sqrt{3}x^2 + 7x - \sqrt{3} = 0$$

$$\therefore 6\sqrt{3}x^2 + 9x - 2x - \sqrt{3} = 0$$

$$\therefore 3\sqrt{3}x(2x + \sqrt{3}) - 1(2x + \sqrt{3}) = 0$$

$$\therefore (2x + \sqrt{3})(3\sqrt{3}x - 1) = 0$$

$$\therefore 2x + \sqrt{3} = 0 \text{ or } 3\sqrt{3}x - 1 = 0$$

$$\therefore 2x = -\sqrt{3} \text{ or } 3\sqrt{3}x = 1$$

$$\therefore x = -\frac{\sqrt{3}}{2} \text{ or } x = \frac{1}{3\sqrt{3}}$$

$\therefore -\frac{\sqrt{3}}{2}$ and $\frac{1}{3\sqrt{3}}$ are the roots of the given quadratic equation.

$$6\sqrt{3} \times -\sqrt{3} = -18$$

$$\begin{array}{c} -18 \\ \swarrow \quad \searrow \\ 9 \quad -2 \end{array}$$

$$9 = 3\sqrt{3} \times \sqrt{3}$$

Practice Set 2.2

1. Solve the following quadratic equations by factorisation.

$$(1) \ x^2 - 15x + 54 = 0$$

$$(2) \ x^2 + x - 20 = 0$$

$$(3) \ 2y^2 + 27y + 13 = 0$$

$$(4) \ 5m^2 = 22m + 15$$

$$(5) \ 2x^2 - 2x + \frac{1}{2} = 0$$

$$(6) \ 6x - \frac{2}{x} = 1$$

(7) $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$ to solve this quadratic equation by factorisation, complete the following activity.

Solution : $\sqrt{2}x^2 + 7x + 5\sqrt{2} = 0$

$$\sqrt{2}x^2 + \boxed{} + \boxed{} + 5\sqrt{2} = 0$$

$$x(\dots\dots) + \sqrt{2}(\dots\dots) = 0$$

$$(\dots)(x + \sqrt{2}) = 0$$

$$(\dots) = 0 \quad \text{or} \quad (x + \sqrt{2}) = 0$$

$$\therefore x = \boxed{} \quad \text{or} \quad x = -\sqrt{2}$$

$$\therefore \boxed{} \quad \text{and} \quad -\sqrt{2} \quad \text{are roots of the equation.}$$

$$(8)^{\star} 3x^2 - 2\sqrt{6}x + 2 = 0 \quad (9) 2m(m - 24) = 50$$

$$(10) 25m^2 = 9$$

$$(11) 7m^2 = 21m$$

$$(12) m^2 - 11 = 0$$



Let's learn.

Solution of a quadratic equation by completing the square

Teacher : Is $x^2 + 10x + 2 = 0$ a quadratic equation or not ?

Yogesh : Yes Sir, because it is in the form $ax^2 + bx + c = 0$, maximum index of the variable x is 2 and 'a' is non zero.

Teacher : Can you solve this equation ?

Yogesh : No Sir, because it is not possible to find the factors of 2 whose sum is 10.

Teacher : Right, so we have to use another method to solve such equations. Let us learn the method.

Let us add a suitable term to $x^2 + 10x$ so that the new expression would be a complete square.

$$\text{If } x^2 + 10x + k = (x + a)^2$$

$$\text{then } x^2 + 10x + k = x^2 + 2ax + a^2$$

$$\therefore 10 = 2a \text{ and } k = a^2$$

by equating the coefficients for the variable x and constant term

$$\therefore a = 5 \quad \therefore k = a^2 = (5)^2 = 25$$

$$\therefore x^2 + 10x + 2 = (x + 5)^2 - 25 + 2 = (x + 5)^2 - 23$$

Now can you solve the equation $x^2 + 10x + 2 = 0$?

Rehana : Yes Sir, left side of the equation is now difference of two squares and we can factorise it.

$$(x + 5)^2 - (\sqrt{23})^2 = 0$$

$$\therefore (x + 5 + \sqrt{23})(x + 5 - \sqrt{23}) = 0$$

$$\therefore x + 5 + \sqrt{23} = 0 \text{ or } x + 5 - \sqrt{23} = 0$$

$$\therefore x = -5 - \sqrt{23} \text{ or } x = -5 + \sqrt{23}$$

Hameed : Sir, May I suggest another way ?

$$(x + 5)^2 - (\sqrt{23})^2 = 0$$

$$\therefore (x + 5)^2 = (\sqrt{23})^2$$

$$\therefore x + 5 = \sqrt{23} \text{ or } x + 5 = -\sqrt{23}$$

$$\therefore x = -5 + \sqrt{23} \text{ or } x = -5 - \sqrt{23}$$

solved Examples

Ex. (1) Solve : $5x^2 - 4x - 3 = 0$

Solution : It is convenient to make coefficient of x^2 as 1 and then convert the equation as the of difference of two squares, so dividing the equation by 5,

$$\text{we get, } x^2 - \frac{4}{5}x - \frac{3}{5} = 0$$

$$\text{now if } x^2 - \frac{4}{5}x + k = (x - a)^2 \text{ then } x^2 - \frac{4}{5}x + k = x^2 - 2ax + a^2.$$

$$\text{compare the terms in } x^2 - \frac{4}{5}x \text{ and } x^2 - 2ax.$$

$$-2ax = -\frac{4}{5}x \quad \therefore a = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$$

$$\therefore k = a^2 = \left(\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$\text{Now, } x^2 - \frac{4}{5}x - \frac{3}{5} = 0$$

$$\therefore x^2 - \frac{4}{5}x + \frac{4}{25} - \frac{4}{25} - \frac{3}{5} = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{4}{25} + \frac{3}{5}\right) = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 - \left(\frac{19}{25}\right) = 0$$

$$\therefore \left(x - \frac{2}{5}\right)^2 = \left(\frac{19}{25}\right)$$

$$\therefore x - \frac{2}{5} = \frac{\sqrt{19}}{5} \text{ or } x - \frac{2}{5} = -\frac{\sqrt{19}}{5}$$

$$\therefore x = \frac{2}{5} + \frac{\sqrt{19}}{5} \text{ or } x = \frac{2}{5} - \frac{\sqrt{19}}{5}$$

$$\therefore x = \frac{2+\sqrt{19}}{5} \text{ or } x = \frac{2-\sqrt{19}}{5}$$

$$\therefore \frac{2+\sqrt{19}}{5} \text{ and } \frac{2-\sqrt{19}}{5} \text{ are roots of the equation.}$$

When equation is in the form $x^2 + bx + c = 0$, it can be written as

$$x^2 + bx + \left(\frac{b}{2}\right)^2 - \left(\frac{b}{2}\right)^2 + c = 0 \text{ that is,}$$

$$\left(x + \frac{b}{2}\right)^2 = \left(\frac{b}{2}\right)^2 - c$$

Ex. (2) Solve : $x^2 + 8x - 48 = 0$

Method I : Completing the square.

$$\begin{aligned} x^2 + 8x - 48 &= 0 \\ \therefore x^2 + 8x + 16 - 16 - 48 &= 0 \\ \therefore (x + 4)^2 - 64 &= 0 \\ \therefore (x + 4)^2 &= 64 \\ \therefore x + 4 &= 8 \text{ or } x + 4 = -8 \\ \therefore x &= 4 \text{ or } x = -12 \end{aligned}$$

Method II : Factorisation

$$\begin{aligned} x^2 + 8x - 48 &= 0 \\ \therefore x^2 + 12x - 4x - 48 &= 0 \\ \therefore x(x + 12) - 4(x + 12) &= 0 \\ \therefore (x + 12)(x - 4) &= 0 \\ \therefore x + 12 = 0 \text{ or } x - 4 &= 0 \\ \therefore x &= -12 \text{ or } x = 4 \end{aligned}$$

Practice Set 2.3

Solve the following quadratic equations by completing the square method.

- (1) $x^2 + x - 20 = 0$ (2) $x^2 + 2x - 5 = 0$ (3) $m^2 - 5m = -3$
 (4) $9y^2 - 12y + 2 = 0$ (5) $2y^2 + 9y + 10 = 0$ (6) $5x^2 = 4x + 7$



Let's learn.

Formula for solving a quadratic equation

$ax^2 + bx + c$, Divide the polynomial by a ($\because a \neq 0$) to get $x^2 + \frac{b}{a}x + \frac{c}{a}$.

Let us write the polynomial $x^2 + \frac{b}{a}x + \frac{c}{a}$ in the form of difference of two square numbers. Now we can obtain roots or solutions of equation $x^2 + \frac{b}{a}x + \frac{c}{a} = 0$ which is equivalent to $ax^2 + bx + c = 0$.

$$ax^2 + bx + c = 0 \dots (I)$$

$$x^2 + \frac{b}{a}x + \frac{c}{a} = 0 \dots \text{dividing both sides by } a$$

$$\therefore x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2 - \left(\frac{b}{2a}\right)^2 + \frac{c}{a} = 0$$

$$\therefore \left(x + \frac{b}{2a}\right)^2 - \frac{b^2}{4a^2} + \frac{c}{a} = 0$$

$$\therefore \left(x + \frac{b}{2a}\right)^2 - \frac{b^2 - 4ac}{4a^2} = 0 \quad \therefore \left(x + \frac{b}{2a}\right)^2 = \frac{b^2 - 4ac}{4a^2}$$

$$\therefore x + \frac{b}{2a} = \sqrt{\frac{b^2 - 4ac}{4a^2}} \text{ or } x + \frac{b}{2a} = -\sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore x = -\frac{b}{2a} + \sqrt{\frac{b^2 - 4ac}{4a^2}} \text{ or } x = -\frac{b}{2a} - \sqrt{\frac{b^2 - 4ac}{4a^2}}$$

$$\therefore x = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \text{ or } x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$$

In short the solution is written as $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and these values are denoted by α, β .

$$\therefore \alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}, \quad \beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a} \dots \dots \dots (I)$$

The values of a, b, c from equation $ax^2 + bx + c = 0$ are substituted in $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ and further simplified to obtain the roots of the equation. So

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ is the formula used to solve quadratic equation. Out of the two roots any one can be represented by α and the other by β .

$$\text{That is, instead (I) we can write } \alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}, \beta = \frac{-b + \sqrt{b^2 - 4ac}}{2a} \dots \dots (II)$$

Note that : If $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ then $\alpha > \beta$, if $\alpha = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$ then $\alpha < \beta$

Solved Examples

Solve quadratic equations using formula.

Ex. (1) $m^2 - 14m + 13 = 0$

Solution : $m^2 - 14m + 13 = 0$ comparing

with $ax^2 + bx + c = 0$

we get $a = 1, b = -14, c = 13,$

$$\begin{aligned} \therefore b^2 - 4ac &= (-14)^2 - 4 \times 1 \times 13 \\ &= 196 - 52 \\ &= 144 \end{aligned}$$

$$m = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(-14) \pm \sqrt{144}}{2 \times 1}$$

$$= \frac{14 \pm 12}{2}$$

$$\therefore m = \frac{14+12}{2} \text{ or } m = \frac{14-12}{2}$$

$$\therefore m = \frac{26}{2} \text{ or } m = \frac{2}{2}$$

$$\therefore m = 13 \text{ or } m = 1$$

$\therefore 13$ and 1 are roots of the equation.

Ex. (2) : $x^2 + 10x + 2 = 0$

Solution : $x^2 + 10x + 2 = 0$ comparing with $ax^2 + bx + c = 0$

we get $a = 1$, $b = 10$, $c = 2$,

$$\begin{aligned}\therefore b^2 - 4ac &= (10)^2 - 4 \times 1 \times 2 \\ &= 100 - 8 \\ &= 92\end{aligned}$$

$$\begin{aligned}x &= \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-10 \pm \sqrt{92}}{2 \times 1} \\ x &= \frac{-10 \pm \sqrt{4 \times 23}}{2} \\ &= \frac{-10 \pm 2\sqrt{23}}{2} \\ &= \frac{2(-5 \pm \sqrt{23})}{2}\end{aligned}$$

$$\therefore x = -5 \pm \sqrt{23}$$

$$\therefore x = -5 + \sqrt{23} \text{ or } x = -5 - \sqrt{23}$$

\therefore the roots of the given quadratic equation are $-5 + \sqrt{23}$ and $-5 - \sqrt{23}$.

Ex. (3) : $x^2 - 2x - 3 = 0$

Solution : comparing with $ax^2 + bx + c = 0$

we get $a = 1$, $b = -2$, $c = -3$,

$$\therefore b^2 - 4ac = (-2)^2 - 4 \times 1 \times (-3) = 4 + 12 = 16$$

$$\begin{aligned}\therefore x &= \frac{-(-2) \pm \sqrt{16}}{2} \text{ or } x = \frac{-(-2) - \sqrt{16}}{2} \\ &= \frac{2+4}{2} \text{ or } \frac{2-4}{2} \\ &= 3 \text{ or } -1\end{aligned}$$

For more information :

Let us understand the solution of equation $x^2 - 2x - 3 = 0$ when solved graphically.

$x^2 - 2x - 3 = 0 \quad \therefore \quad x^2 = 2x + 3$ The values which satisfy the equation are the roots of the equation.

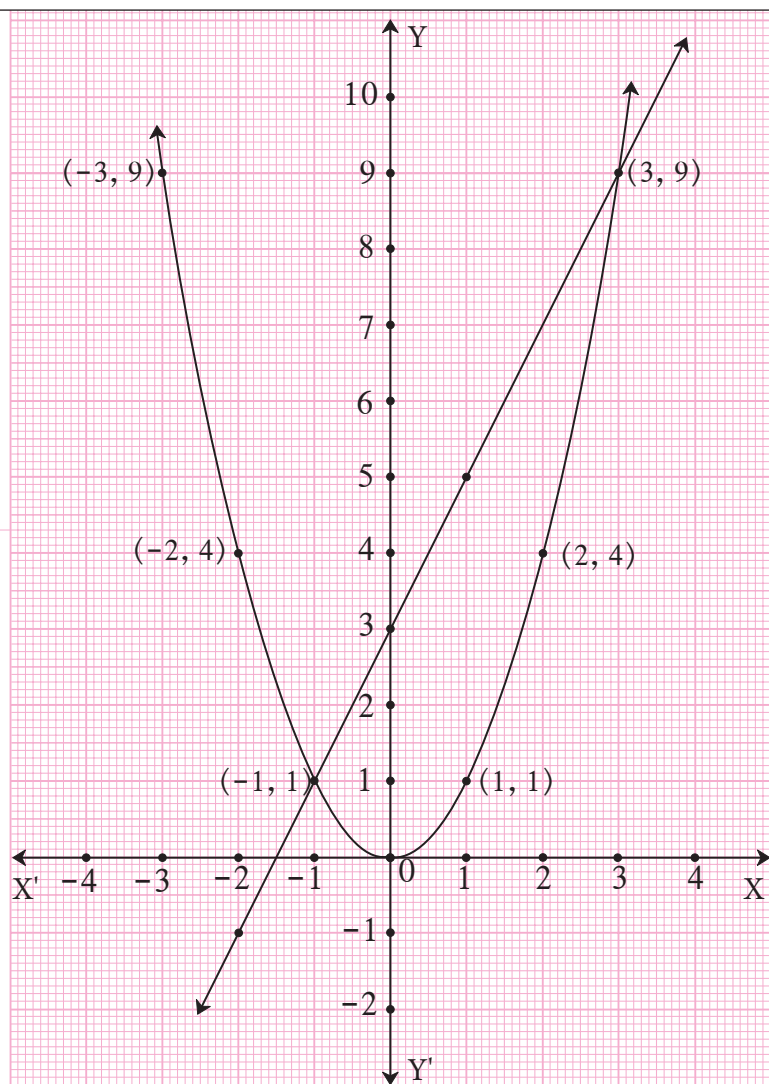
Let $y = x^2 = 2x + 3$. Let us draw graph of $y = x^2$ and $y = 2x + 3$

$$y = x^2$$

x	3	2	1	0	-1	-2	-3
y	9	4	1	0	1	4	9

$$y = 2x + 3$$

x	-1	0	1	-2
y	1	3	5	-1



These graphs intersect each other at $(-1, 1)$ and $(3, 9)$.

\therefore The solutions of $x^2 = 2x + 3$ i.e. $x^2 - 2x - 3 = 0$ are $x = -1$ or $x = 3$.

In the adjacent diagram the graphs of equations $y = x^2$ and $y = 2x + 3$ are given. From their points of intersection, observe and understand how you get the solutions of $x^2 = 2x + 3$ i.e. solutions of $x^2 - 2x - 3 = 0$.

Ex. (4) $25x^2 + 30x + 9 = 0$

Solution : $25x^2 + 30x + 9 = 0$ comparing the equation with $ax^2 + bx + c = 0$ we get $a = 25$, $b = 30$, $c = 9$,

$$\therefore b^2 - 4ac = (30)^2 - 4 \times 25 \times 9 \\ = 900 - 900 = 0$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ = \frac{-30 \pm \sqrt{0}}{2 \times 25}$$

$$\therefore x = \frac{-30+0}{50} \text{ or } x = \frac{-30-0}{50}$$

$$\therefore x = -\frac{30}{50} \text{ or } x = -\frac{30}{50}$$

$$\therefore x = -\frac{3}{5} \text{ or } x = -\frac{3}{5}$$

that is both the roots are equal.

Also note that $25x^2 + 30x + 9 = 0$ means $(5x + 3)^2 = 0$

Ex. (5) $x^2 + x + 5 = 0$

Solution : $x^2 + x + 5 = 0$ comparing with $ax^2 + bx + c = 0$ we get $a = 1$, $b = 1$, $c = 5$,

$$\therefore b^2 - 4ac = (1)^2 - 4 \times 1 \times 5 \\ = 1 - 20 \\ = -19$$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \\ = \frac{-1 \pm \sqrt{-19}}{2 \times 1} \\ = \frac{-1 \pm \sqrt{-19}}{2}$$

But $\sqrt{-19}$ is not a real number. Hence roots of the equation are not real.

Activity : Solve the equation $2x^2 + 13x + 15 = 0$ by factorisation method, by completing the square method and by using the formula. Verify that you will get the same roots every time.

Practice Set 2.4

1. Compare the given quadratic equations to the general form and write values of a , b , c .

(1) $x^2 - 7x + 5 = 0$

(2) $2m^2 = 5m - 5$

(3) $y^2 = 7y$

2. Solve using formula.

(1) $x^2 + 6x + 5 = 0$

(2) $x^2 - 3x - 2 = 0$

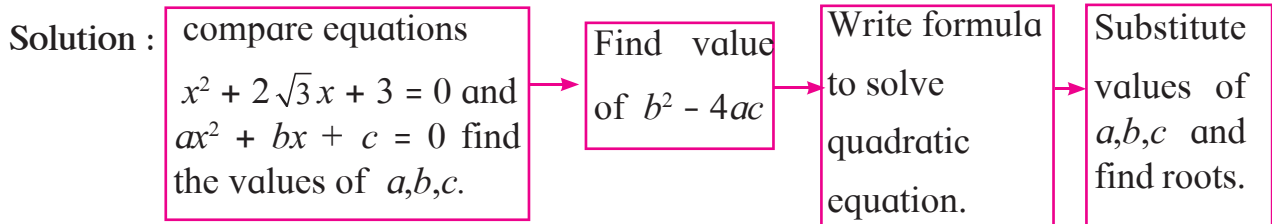
(3) $3m^2 + 2m - 7 = 0$

(4) $5m^2 - 4m - 2 = 0$

(5) $y^2 + \frac{1}{3}y = 2$

(6) $5x^2 + 13x + 8 = 0$

(3) With the help of the flow chart given below solve the equation $x^2 + 2\sqrt{3}x + 3 = 0$ using the formula.



Let's learn.

Nature of roots of a quadratic equation

You know that $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are roots of quadratic equation $ax^2 + bx + c = 0$

(1) If $b^2 - 4ac = 0$ then, $x = \frac{-b \pm \sqrt{0}}{2a} \therefore x = \frac{-b+0}{2a}$ or $x = \frac{-b-0}{2a}$

\therefore the roots of the quadratic equation are real and equal.

(2) If $b^2 - 4ac > 0$, then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

i.e. $x = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $x = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

\therefore roots of the quadratic equation are real and unequal.

(3) If $b^2 - 4ac < 0$ then $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ are not real numbers \therefore the roots of quadratic equations are not real.

Nature of roots of quadratic equation is determined by the value of $b^2 - 4ac$. $b^2 - 4ac$ is called discriminant of a quadratic equation and is denoted by Greek letter Δ (Delta)

Activity - Fill in the blanks.

	Value of discriminant		Nature of roots
(1)	50	→	
(2)	-30	→	
(3)	0	→	

Solved examples

Ex. (1) Find the value of the discriminant of the equation $x^2 + 10x - 7 = 0$

Solution : Comparing $x^2 + 10x - 7 = 0$ with $ax^2 + bx + c = 0$.

$$\begin{aligned} a &= 1, \quad b = 10, \quad c = -7, \\ \therefore b^2 - 4ac &= 10^2 - 4 \times 1 \times -7 \\ &= 100 + 28 \\ &= 128 \end{aligned}$$

Ex. (2) Determine nature of roots of the quadratic equations.

(i) $2x^2 - 5x + 7 = 0$

Solution : Compare $2x^2 - 5x + 7 = 0$ with

$$ax^2 + bx + c = 0$$

$$a = 2, \quad b = -5, \quad c = 7,$$

$$\therefore b^2 - 4ac = (-5)^2 - 4 \times 2 \times 7$$

$$D = 25 - 56$$

$$D = -31$$

$$\therefore b^2 - 4ac < 0$$

\therefore the roots of the equation are not real.

(ii) $x^2 + 2x - 9 = 0$

Solution : Compare $x^2 + 2x - 9 = 0$ with

$$ax^2 + bx + c = 0.$$

$$a = \boxed{}, \quad b = 2, \quad c = \boxed{},$$

$$\therefore b^2 - 4ac = 2^2 - 4 \times \boxed{} \times \boxed{}$$

$$D = 4 - \boxed{}$$

$$D = 40$$

$$\therefore b^2 - 4ac > 0$$

\therefore the roots of the equation are real and unequal.

Ex. (3) $\sqrt{3}x^2 + 2\sqrt{3}x + \sqrt{3} = 0$

Solution : Compare $\sqrt{3}x^2 + 2\sqrt{3}x + \sqrt{3} = 0$ with $ax^2 + bx + c = 0$

$$\text{We get } a = \sqrt{3}, \quad b = 2\sqrt{3}, \quad c = \sqrt{3},$$

$$\therefore b^2 - 4ac = (2\sqrt{3})^2 - 4 \times \sqrt{3} \times \sqrt{3}$$

$$= 4 \times 3 - 4 \times 3$$

$$= 12 - 12$$

$$= 0$$

$$\therefore b^2 - 4ac = 0$$

\therefore Roots of the equation are real and equal.



Let's learn.

The relation between roots of the quadratic equation and coefficients

α and β are the roots of the equation $ax^2 + bx + c = 0$ then,

$$\begin{aligned}\alpha + \beta &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} + \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{-b + \sqrt{b^2 - 4ac} - b - \sqrt{b^2 - 4ac}}{2a} \\ &= -\frac{2b}{2a}\end{aligned}$$

$$\therefore \alpha + \beta = -\frac{b}{a}$$

$$\begin{aligned}\alpha \times \beta &= \frac{-b + \sqrt{b^2 - 4ac}}{2a} \times \frac{-b - \sqrt{b^2 - 4ac}}{2a} \\ &= \frac{(-b + \sqrt{b^2 - 4ac}) \times (-b - \sqrt{b^2 - 4ac})}{4a^2} \\ &= \frac{b^2 - (b^2 - 4ac)}{4a^2} \\ &= \frac{4ac}{4a^2} \\ &= \frac{c}{a}\end{aligned}$$

$$\therefore \alpha \beta = \frac{c}{a}$$

Activity : Fill in the empty boxes below properly

For $10x^2 + 10x + 1 = 0$,

$$\alpha + \beta = \boxed{} \text{ and } \alpha \times \beta = \boxed{}$$

🔗 Solved examples 🔗

Ex. (1) If α and β are the roots of the quadratic equation $2x^2 + 6x - 5 = 0$, then find $(\alpha + \beta)$ and $\alpha \times \beta$.

Solution : Comparing $2x^2 + 6x - 5 = 0$ with $ax^2 + bx + c = 0$.

$$\therefore a = 2, b = 6, c = -5$$

$$\therefore \alpha + \beta = -\frac{b}{a} = -\frac{6}{2} = -3$$

$$\text{and } \alpha \times \beta = \frac{c}{a} = \frac{-5}{2}$$

Ex. (2) The difference between the roots of the equation $x^2 - 13x + k = 0$ is 7 find k.

Solution : Comparing $x^2 - 13x + k = 0$ with $ax^2 + bx + c = 0$

$$a = 1, b = -13, c = k$$

Let α and β be the roots of the equation.

$$\alpha + \beta = -\frac{b}{a} = -\frac{(-13)}{1} = 13 \dots (I)$$

But $\alpha - \beta = 7 \dots \dots \dots$ (given) (II)

$$2\alpha = 20 \dots \text{(adding (I) and (II))}$$

$$\therefore \alpha = 10$$

$$\therefore 10 + \beta = 13 \dots \text{(from (I))}$$

$$\therefore \beta = 13 - 10$$

$$\therefore \beta = 3$$

$$\text{But } \alpha \times \beta = \frac{c}{a}$$

$$\therefore 10 \times 3 = \frac{k}{1}$$

$$\therefore k = 30$$

Ex. (3) If α and β are the roots of $x^2 + 5x - 1 = 0$ then find -

(i) $\alpha^3 + \beta^3$ (ii) $\alpha^2 + \beta^2$.

Solution : $x^2 + 5x - 1 = 0$

$$a = 1, b = 5, c = -1$$

$$\alpha + \beta = -\frac{b}{a} = \frac{-5}{1} = -5$$

$$\alpha \times \beta = \frac{c}{a} = \frac{-1}{1} = -1$$

$$\begin{aligned} \text{(i) } \alpha^3 + \beta^3 &= (\alpha + \beta)^3 - 3\alpha\beta(\alpha + \beta) \\ &= (-5)^3 - 3 \times (-1) \times (-5) \\ &= -125 - 15 \end{aligned}$$

$$\alpha^3 + \beta^3 = -140$$

$$\begin{aligned} \text{(ii) } \alpha^2 + \beta^2 &= (\alpha + \beta)^2 - 2\alpha\beta \\ &= (-5)^2 - 2 \times (-1) \\ &= 25 + 2 \end{aligned}$$

$$\alpha^2 + \beta^2 = 27$$



Let's learn.

To obtain a quadratic equation having given roots

Let α and β be the roots of a quadratic equation in variable x

$$\therefore x = \alpha \text{ or } x = \beta$$

$$\therefore x - \alpha = 0 \text{ or } x - \beta = 0$$

$$\therefore (x - \alpha)(x - \beta) = 0$$

$$\therefore x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

When two roots of equation are given then quadratic equation can be obtained as $x^2 - (\text{addition of roots})x + \text{product of the roots} = 0$.

Activity (I) : Write the quadratic equation if addition of the roots is 10 and product of the roots = 9

$$\therefore \text{Quadratic equation : } x^2 - \boxed{}x + \boxed{} = \boxed{}$$

Activity (II) : What will be the quadratic equation if $\alpha = 2$, $\beta = 5$

$$\text{It can be written as } x^2 - (\boxed{} + \boxed{})x + \boxed{} \times \boxed{} = 0.$$

$$\text{that is } \boxed{}x^2 - \boxed{}x + \boxed{} = 0.$$

Note that, if this equation is multiplied by any non zero number, the roots of the equation are not changed.

🎀🎀🎀 Solved examples 🎀🎀🎀

Ex. Obtain the quadratic equation if roots are -3, -7.

Solution : Let $\alpha = -3$ and $\beta = -7$

$$\therefore \alpha + \beta = (-3) + (-7) = -10 \text{ and } \alpha \times \beta = (-3) \times (-7) = 21$$

$$\therefore \text{and quadratic equation is, } x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

$$\therefore x^2 - (-10)x + 21 = 0$$

$$\therefore x^2 + 10x + 21 = 0$$



Let's remember!

(1) If α and β are roots of quadratic equation $ax^2 + bx + c = 0$,

(i) $\alpha = \frac{-b + \sqrt{b^2 - 4ac}}{2a}$ and $\beta = \frac{-b - \sqrt{b^2 - 4ac}}{2a}$

(ii) $\alpha + \beta = -\frac{b}{a}$ and $\alpha \times \beta = \frac{c}{a}$

(2) Nature of roots of quadratic equation $ax^2 + bx + c = 0$ depends on the value of $b^2 - 4ac$. Hence $b^2 - 4ac$ is called discriminant and is denoted by Greek letter Δ .

(3) If $\Delta = 0$ The roots of quadratic equation are real and equal.

If $\Delta > 0$ then the roots of quadratic equation are real and unequal.

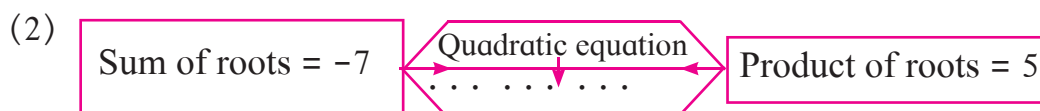
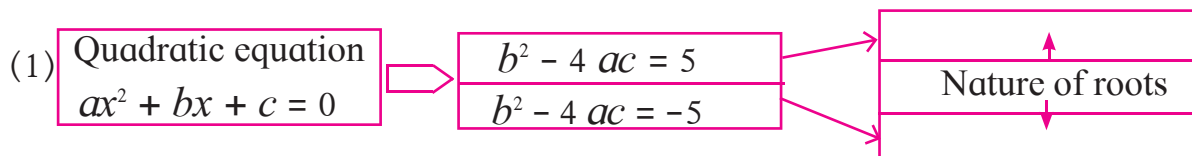
If $\Delta < 0$ then the roots of quadratic equation are not real.

(4) The quadratic equation, whose roots are α and β is

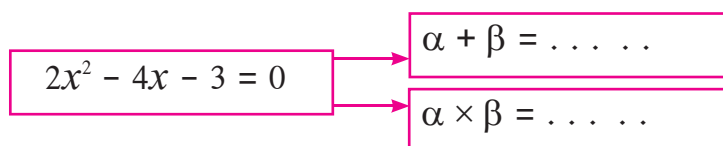
$$x^2 - (\alpha + \beta)x + \alpha\beta = 0$$

Practice Set 2.5

1. **Activity** : Fill in the gaps and complete.



(3) If α, β are roots of quadratic equation,



2. Find the value of discriminant.

(1) $x^2 + 7x - 1 = 0$

(2) $2y^2 - 5y + 10 = 0$

(3) $\sqrt{2}x^2 + 4x + 2\sqrt{2} = 0$

3. Determine the nature of roots of the following quadratic equations.

(1) $x^2 - 4x + 4 = 0$

(2) $2y^2 - 7y + 2 = 0$

(3) $m^2 + 2m + 9 = 0$

4. Form the quadratic equation from the roots given below.

- (1) 0 and 4 (2) 3 and -10 (3) $\frac{1}{2}, -\frac{1}{2}$ (4) $2-\sqrt{5}, 2+\sqrt{5}$

5. Sum of the roots of a quadratic equation is double their product. Find k if equation is $x^2 - 4kx + k + 3 = 0$

6. α, β are roots of $y^2 - 2y - 7 = 0$ find,

- (1) $\alpha^2 + \beta^2$ (2) $\alpha^3 + \beta^3$

7. The roots of each of the following quadratic equations are real and equal, find k .

- (1) $3y^2 + ky + 12 = 0$ (2) $kx(x - 2) + 6 = 0$



Let's learn.

Application of quadratic equation

Quadratic equations are useful in daily life for finding solutions of some practical problems. We are now going to learn the same.

Ex. (1) There is a rectangular onion storehouse in the farm of Mr. Ratnakarrao at Tivasa. The length of rectangular base is more than its breadth by 7 m and diagonal is more than length by 1 m. Find length and breadth of the storehouse.

Solution : Let breadth of the storehouse be x m.

$$\therefore \text{length} = (x + 7) \text{ m, diagonal} = x + 7 + 1 = (x + 8) \text{ m}$$

By Pythagorus theorem

$$x^2 + (x + 7)^2 = (x + 8)^2$$

$$x^2 + x^2 + 14x + 49 = x^2 + 16x + 64$$

$$\therefore x^2 + 14x - 16x + 49 - 64 = 0$$

$$\therefore x^2 - 2x - 15 = 0$$

$$\therefore \underline{x^2 - 5x} + \underline{3x - 15} = 0$$

$$\therefore x(x - 5) + 3(x - 5) = 0$$

$$\therefore (x - 5)(x + 3) = 0$$

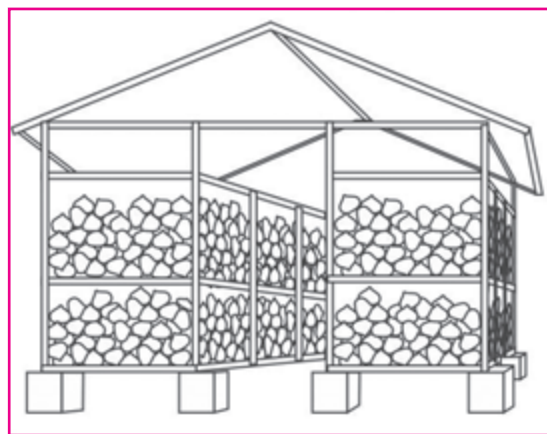
$$\therefore x - 5 = 0 \text{ or } x + 3 = 0$$

$$\therefore x = 5 \text{ or } x = -3$$

But length is never negative $\therefore x \neq -3$

$$\therefore x = 5 \text{ and } x + 7 = 5 + 7 = 12$$

\therefore Length of the base of storehouse is 12m and breadth is 5m.



Onion Storehouse (Kandachal)

Ex. (2) A train travels 360 km with uniform speed. The speed of the train is increased by 5 km/hr, it takes 48 minutes less to cover the same distance. Find the initial speed of the train.

Solution : Let initial speed of the train be x km/hr.

\therefore New speed is $(x + 5)$ km/hr.

time to cover 360 km = $\frac{\text{distance}}{\text{speed}} = \frac{360}{x}$ hours.

New time after increasing speed = $\frac{360}{x+5}$ hours.

from given condition

$$\frac{360}{x+5} = \frac{360}{x} - \frac{48}{60} \quad \text{--- (48 min = } \frac{48}{60} \text{ hrs)}$$

$$\therefore \frac{360}{x} - \frac{360}{x+5} = \frac{48}{60}$$

$$\therefore \frac{1}{x} - \frac{1}{x+5} = \frac{48}{60 \times 360} \quad \text{--- (Dividing both sides by 360)}$$

$$\therefore \frac{x+5-x}{x(x+5)} = \frac{4}{5 \times 360}$$

$$\therefore \frac{5}{x^2+5x} = \frac{1}{5 \times 90}$$

$$\therefore \frac{5}{x^2+5x} = \frac{1}{450}$$

$$\therefore x^2 + 5x = 2250$$

$$\therefore x^2 + 5x - 2250 = 0$$

$$\therefore \underline{x^2 + 50x} - \underline{45x - 2250} = 0$$

$$\therefore \underline{x(x + 50)} - \underline{45(x + 50)} = 0$$

$$\therefore (x + 50)(x - 45) = 0$$

$$\therefore x + 50 = 0 \text{ or } x - 45 = 0$$

$$\therefore x = -50 \text{ or } x = 45$$

But speed is never negative $\therefore x \neq -50$

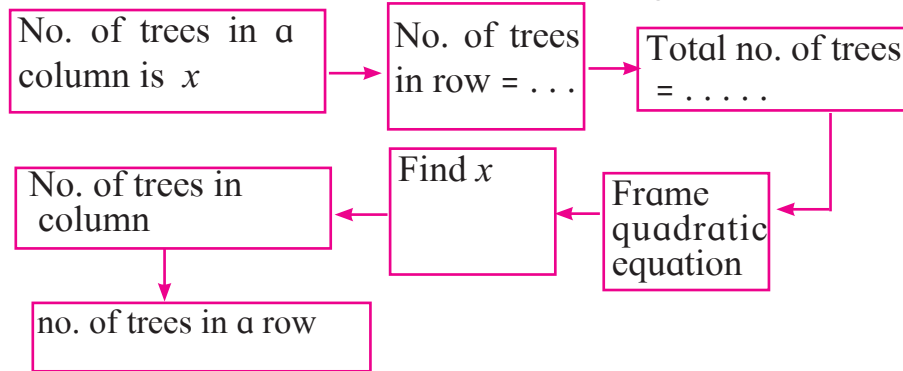
$$\therefore x = 45$$

\therefore Initial speed of the train is 45 km/hr.

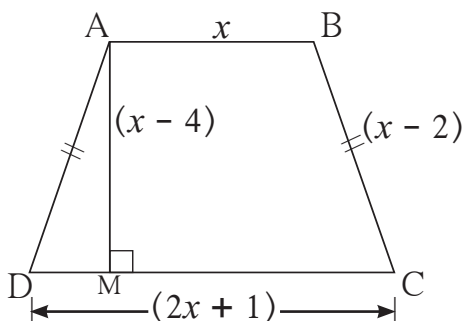
$$\begin{array}{r} -2250 \\ +50 \quad -45 \end{array}$$

Practice Set 2.6

1. Product of Pragati's age 2 years ago and 3 years hence is 84. Find her present age.
2. The sum of squares of two consecutive natural numbers is 244; find the numbers.
3. In the orange garden of Mr. Madhusudan there are 150 orange trees. The number of trees in each row is 5 more than that in each column. Find the number of trees in each row and each column with the help of following flow chart.



4. Vivek is older than Kishor by 5 years. The sum of the reciprocals of their ages is $\frac{1}{6}$. Find their present ages.
5. Suyash scored 10 marks more in second test than that in the first. 5 times the score of the second test is the same as square of the score in the first test. Find his score in the first test.
- 6★ Mr. Kasam runs a small business of making earthen pots. He makes certain number of pots on daily basis. Production cost of each pot is ₹ 40 more than 10 times total number of pots, he makes in one day. If production cost of all pots per day is ₹ 600, find production cost of one pot and number of pots he makes per day.
- 7★ Pratik takes 8 hours to travel 36 km downstream and return to the same spot. The speed of boat in still water is 12 km. per hour. Find the speed of water current.
- 8★ Pintu takes 6 days more than those of Nishu to complete certain work. If they work together they finish it in 4 days. How many days would it take to complete the work if they work alone.
- 9★ If 460 is divided by a natural number, quotient is 6 more than five times the divisor and remainder is 1. Find quotient and divisor.
- 10★



In the adjoining fig. $\square ABCD$ is a trapezium $AB \parallel CD$ and its area is 33 cm^2 . From the information given in the figure find the lengths of all sides of the $\square ABCD$. Fill in the empty boxes to get the solution.

Solution : □ABCD is a trapezium.

$$AB \parallel CD$$

$$A(\square ABCD) = \frac{1}{2}(AB + CD) \times \square$$

$$33 = \frac{1}{2}(x + 2x + 1) \times \square$$

$$\therefore \square = (3x + 1) \times \square$$

$$\therefore 3x^2 + \square - \square = 0$$

$$\therefore 3x(\dots) + 10(\dots) = 0$$

$$\therefore (3x + 10)(\dots) = 0$$

$$\therefore (3x + 10) = 0 \text{ or } \square = 0$$

$$\therefore x = -\frac{10}{3} \text{ or } x = \square$$

But length is never negative.

$$\therefore x \neq -\frac{10}{3} \quad \therefore x = \square$$

$$AB = \dots, CD = \dots, AD = BC = \dots$$

Problem Set - 2

1. Choose the correct answers for the following questions.

(1) Which one is the quadratic equation ?

(A) $\frac{5}{x} - 3 = x^2$ (B) $x(x + 5) = 2$ (C) $n - 1 = 2n$ (D) $\frac{1}{x^2}(x + 2) = x$

(2) Out of the following equations which one is not a quadratic equation ?

(A) $x^2 + 4x = 11 + x^2$ (B) $x^2 = 4x$ (C) $5x^2 = 90$ (D) $2x - x^2 = x^2 + 5$

(3) The roots of $x^2 + kx + k = 0$ are real and equal, find k.

(A) 0 (B) 4 (C) 0 or 4 (D) 2

(4) For $\sqrt{2}x^2 - 5x + \sqrt{2} = 0$ find the value of the discriminant.

(A) -5 (B) 17 (C) $\sqrt{2}$ (D) $2\sqrt{2} - 5$

(5) Which of the following quadratic equations has roots 3, 5 ?

(A) $x^2 - 15x + 8 = 0$ (B) $x^2 - 8x + 15 = 0$

(C) $x^2 + 3x + 5 = 0$ (D) $x^2 + 8x - 15 = 0$

(6) Out of the following equations, find the equation having the sum of its roots -5.

(A) $3x^2 - 15x + 3 = 0$ (B) $x^2 - 5x + 3 = 0$

(C) $x^2 + 3x - 5 = 0$ (D) $3x^2 + 15x + 3 = 0$

(7) $\sqrt{5}m^2 - \sqrt{5}m + \sqrt{5} = 0$ which of the following statement is true for this given equation ?

(A) Real and unequal roots

(B) Real and equal roots

(C) Roots are not real

(D) Three roots.

(8) One of the roots of equation $x^2 + mx - 5 = 0$ is 2; find m.

(A) -2

(B) $-\frac{1}{2}$

(C) $\frac{1}{2}$

(D) 2

2. Which of the following equations is quadratic ?

(1) $x^2 + 2x + 11 = 0$ (2) $x^2 - 2x + 5 = x^2$ (3) $(x + 2)^2 = 2x^2$

3. Find the value of discriminant for each of the following equations.

(1) $2y^2 - y + 2 = 0$ (2) $5m^2 - m = 0$ (3) $\sqrt{5}x^2 - x - \sqrt{5} = 0$

4. One of the roots of quadratic equation $2x^2 + kx - 2 = 0$ is -2, find k.

5. Two roots of quadratic equations are given ; frame the equation.

(1) 10 and -10 (2) $1 - 3\sqrt{5}$ and $1 + 3\sqrt{5}$ (3) 0 and 7

6. Determine the nature of roots for each of the quadratic equation.

(1) $3x^2 - 5x + 7 = 0$ (2) $\sqrt{3}x^2 + \sqrt{2}x - 2\sqrt{3} = 0$ (3) $m^2 - 2m + 1 = 0$

7. Solve the following quadratic equations.

(1) $\frac{1}{x+5} = \frac{1}{x^2}$ (2) $x^2 - \frac{3x}{10} - \frac{1}{10} = 0$ (3) $(2x + 3)^2 = 25$

(4) $m^2 + 5m + 5 = 0$ (5) $5m^2 + 2m + 1 = 0$ (6) $x^2 - 4x - 3 = 0$

8★ Find m if $(m - 12)x^2 + 2(m - 12)x + 2 = 0$ has real and equal roots.

9★ The sum of two roots of a quadratic equation is 5 and sum of their cubes is 35, find the equation.

10. Find quadratic equation such that its roots are square of sum of the roots and square of difference of the roots of equation $2x^2 + 2(p + q)x + p^2 + q^2 = 0$

11★ Mukund possesses ₹ 50 more than what Sagar possesses. The product of the amount they have is 15,000. Find the amount each one has.

12★ The difference between squares of two numbers is 120. The square of smaller number is twice the greater number. Find the numbers.

13★ Ranjana wants to distribute 540 oranges among some students. If 30 students were more each would get 3 oranges less. Find the number of students.

14★ Mr. Dinesh owns an agricultural farm at village Talvel. The length of the farm is 10 meter more than twice the breadth. In order to harvest rain water, he dug a square shaped pond inside the farm. The side of pond is $\frac{1}{3}$ of the breadth of the farm. The area of the farm is 20 times the area of the pond. Find the length and breadth of the farm and of the pond

15★ A tank fills completely in 2 hours if both the taps are open. If only one of the taps is open at the given time, the smaller tap takes 3 hours more than the larger one to fill the tank. How much time does each tap take to fill the tank completely ?



□□□

3. (1) (3, 2) (2) (-2, -1) (3) (0, 5) (4) (2, 4) (5) (3, 1)
4. (1) 22 (2) -1 (3) 13
5. (1) $(-\frac{2}{3}, 2)$ (2) (1, 4) (3) $(\frac{1}{2}, -\frac{1}{2})$ (4) $(\frac{7}{11}, \frac{116}{33})$ (5) (2, 6)
6. (1) (6, -4) (2) $(-\frac{1}{4}, -1)$ (3) (1, 2) (4) (1, 1) (5) (2, 1)
7. (2) Tea; ₹300 per kg.
sugar; ₹ 40 per kg.
(3) ₹100 notes 20
₹50 notes 10
(4) Manisha's age 23 years
Savita's age 8 years.
- (5) Skilled worker's wages ₹ 450.
unskilled worker's wages ₹ 270.
- (6) Hamid's speed 50 km/hr.
Joseph's speed 40 km/hr.

2. Quadratic Equations

Practice Set 2.1

1. Any equations of the type $m^2 + 5m + 3 = 0$, $y^2 - 3 = 0$
2. (1), (2), (4), (5) are quadratic equations.
3. (1) $y^2 + 2y - 10 = 0$, $a = 1, b = 2, c = -10$
(2) $x^2 - 4x - 2 = 0$, $a = 1, b = -4, c = -2$
(3) $x^2 + 4x + 3 = 0$, $a = 1, b = 4, c = 3$
(4) $m^2 + 0m + 9 = 0$, $a = 1, b = 0, c = 9$
(5) $6p^2 + 3p + 5 = 0$, $a = 6, b = 3, c = 5$
(6) $x^2 + 0x - 22 = 0$, $a = 1, b = 0, c = -22$
4. (1) 1 is a root, -1 is not. (2) $\frac{5}{2}$ is a root, 2 is not.
5. $k = 3$ 6. $k = -7$

Practice Set 2.2

1. (1) 9, 6 (2) -5, 4 (3) $-13, -\frac{1}{2}$ (4) $5, -\frac{3}{5}$
- (5) $\frac{1}{2}, \frac{1}{2}$ (6) $\frac{2}{3}, -\frac{1}{2}$ (7) $-\frac{5}{\sqrt{2}}, -\sqrt{2}$ (8) $\frac{\sqrt{2}}{\sqrt{3}}, \frac{\sqrt{2}}{\sqrt{3}}$
- (9) 25, -1 (10) $-\frac{3}{5}, \frac{3}{5}$ (11) 0, 3 (12) $-\sqrt{11}, \sqrt{11}$

Practice Set 2.3

1. (1) $4, -5$ (2) $(\sqrt{6} - 1), (-\sqrt{6} - 1)$ (3) $\frac{\sqrt{13} + 5}{2}, \frac{-\sqrt{13} + 5}{2}$
(4) $\frac{\sqrt{2} + 2}{3}, \frac{-\sqrt{2} + 2}{3}$ (5) $-\frac{5}{4}, -\frac{5}{2}$ (6) $\frac{2 + \sqrt{39}}{5}, \frac{2 - \sqrt{39}}{5}$

Practice Set 2.4

1. (1) $1, -7, 5$ (2) $2, -5, 5$ (3) $1, -7, 0$
2. (1) $-1, -5$ (2) $\frac{3 + \sqrt{17}}{2}, \frac{3 - \sqrt{17}}{2}$ (3) $\frac{-1 + \sqrt{22}}{3}, \frac{-1 - \sqrt{22}}{3}$
(4) $\frac{2 + \sqrt{14}}{5}, \frac{2 - \sqrt{14}}{5}$ (5) $\frac{-1 + \sqrt{73}}{6}, \frac{-1 - \sqrt{73}}{6}$ (6) $-1, -\frac{8}{5}$
3. $-\sqrt{3}, -\sqrt{3}$

Practice Set 2.5

1. (1) Roots are distinct and real when $b^2 - 4ac = 5$, not real when $b^2 - 4ac = -5$.
(2) $x^2 + 7x + 5 = 0$ (3) $\alpha + \beta = 2, \alpha \times \beta = -\frac{3}{2}$
2. (1) 53 (2) -55 (3) 0
3. (1) Real and equal. (2) Real and unequal. (3) Not real.
4. (1) $x^2 - 4x = 0$ (2) $x^2 + 7x - 30 = 0$
(3) $x^2 - \frac{1}{4} = 0$ (4) $x^2 - 4x - 1 = 0$
5. $k = 3$ 6. (1) 18 (2) 50
7. (1) $k = 12$ or $k = -12$ (2) $k = 6$

Practice Set 2.6

1. 9 years 2. 10 and 12 3. In vertical row 10, in horizontal row 15.
4. Kishor's present age is 10 years and Vivek's present age is 15 years
5. 10 marks 6. No. of pots 6, production cost of each is ₹ 100.
7. 6 km/hr 8. For Nishu 6 days, for Pintu 12 days.
9. Divisor = 9, quotient = 51 10. $AB = 7$ cm, $CD = 15$ cm, $AD = BC = 5$ cm.

Problem Set 2

1. (1) B (2) A (3) C (4) B (5) B (6) D (7) C (8) C
2. (1) and (3) are quadratic equations.

3. (1) -15 (2) 1 (3) 21
4. $k = 3$ 5. (1) $x^2 - 100 = 0$ (2) $x^2 - 2x - 44 = 0$ (3) $x^2 - 7x = 0$
6. (1) Not real. (2) Real and unequal (3) Real and equal
7. (1) $\frac{1+\sqrt{21}}{2}, \frac{1-\sqrt{21}}{2}$ (2) $\frac{1}{2}, -\frac{1}{5}$ (3) 1, -4
(4) $\frac{-5+\sqrt{5}}{2}, \frac{-5-\sqrt{5}}{2}$ (5) Roots are not real. (6) $(2 + \sqrt{7}), (2 - \sqrt{7})$
8. $m = 14$ 9. $x^2 - 5x + 6 = 0$ 10. $x^2 - 4pqx - (p^2 - q^2)^2 = 0$
11. ₹ 100 with Sagar, ₹ 150 with Mukund.
12. 12 and $\sqrt{24}$ or 12 and $-\sqrt{24}$ 13. No. of students 60
14. Breadth 45 m. length 100 m, side of the pond 15 m.
15. For larger tap 3 hours and for smaller tap 6 hours.

3. Arithmetic Progression

Practice Set 3.1

1. (1) Yes, $d = 2$ (2) Yes, $d = \frac{1}{2}$ (3) Yes, $d = 4$ (4) No
(5) Yes, $d = -4$ (6) Yes, $d = 0$ (7) Yes, $d = \sqrt{2}$ (8) Yes, $d = 5$
2. (1) 10, 15, 20, 25, ... (2) -3, -3, -3, -3, ... (3) -7, -6.5, -6, -5.5, ...
(4) -1.25, 1.75, 4.75, 7.75, ... (5) 6, 3, 0, -3 ... (6) -19, -23, -27, -31
3. (1) $a = 5, d = -4$ (2) $a = 0.6, d = 0.3$ (3) $a = 127, d = 8$ (4) $a = \frac{1}{4}, d = \frac{1}{2}$

Practice Set 3.2

1. (1) $d = 7$ (2) $d = 3$ (3) $a = -3, d = -5$ (4) $a = 70, d = -10$
2. Yes. 121 3. 104 4. 1115 5. -121 6. 180
7. 55 8. 55th 9. 60 10. 1

Practice Set 3.3

1. 1215 2. 15252 3. 30450 5. 5040
5. 2380 6. 60 7. 4, 9, 14 or 14, 9, 4 8. -3, 1, 5, 9

Practice Set 3.4

1. ₹ 70455 2. First instalment ₹ 1000, last instalment ₹ 560. 3. ₹ 1,92,000
4. 48, 1242 5. $-20^\circ, -25^\circ, -30^\circ, -35^\circ, -40^\circ, -45^\circ$ 6. 325

Problem Set 3

1. (1) B (2) C (3) B (4) D (5) B (6) C (7) C (8) A (9) A (10) B
2. 40 3. 1, 6, 11, ... 4. -195 5. 16, -21 6. -1 7. 6, 10
8. 8 9. 67, 69, 71 10. 3, 7, 11, ..., 147 14. ₹ 2000.