



### Can you recall?

1. Temperature of a body determines its hotness while heat energy is its heat content.
2. Pressure is the force exerted per unit area normally on the walls of a container by the gas molecules due to collisions.
3. Solids, liquids and gases expand on heating.
4. Substances change their state from solid to liquid or liquid to gas on heating up to specific temperature.

### 7.1 Introduction:

In previous lessons, while describing the equilibrium states of a mechanical system or while studying the motion of bodies, only three fundamental physical quantities namely length, mass and time were required. All other physical quantities in mechanics or related to mechanical properties can be expressed in terms of these three fundamental quantities. In this chapter, we will discuss properties or phenomena related to heat. These require a fourth fundamental quantity, the temperature, as mentioned in Chapter 1.

The sensation of hot or cold is a matter of daily experience. A mother feels the temperature of her child by touching its forehead. A cook throws few drops of water on a frying pan to know if it is hot enough to spread the *dosa* batter. Although not advisable, in our daily lives, we feel hotness or coldness of a body by touching or we dip our fingers in water to check if it is hot enough for taking bath. When we say a body or water is hot, we actually mean that its temperature is more than our hand. However, in this way, we can only compare the hotness or coldness of two objects **qualitatively**. Hot and cold are relative terms. You might recall the example given in your science textbook of VIII<sup>th</sup> standard. Lukewarm water seems colder than hot water but hotter than cold water to our hands. We ascribe a property 'temperature' to an object to determine its degree of hotness. The higher the temperature, the hotter is the body. However, the precise temperature of a body can be known only when we have an accurate and easily reproducible way to

quantitatively measure it. Scientific precision requires measurement of a physical quantity in numerical terms. A thermometer is the device to measure the temperature.

In this chapter, we will learn properties of matter and various phenomena that are related to heat. Phenomena or properties having to do with temperature changes and heat exchanges are termed as thermal phenomena or thermal properties. You will understand why the direction of wind near a sea shore changes during day and night, why the metal lid of a glass bottle comes out easily on heating and why two metal vessels locked together can be separated by providing heat to the outer vessel.

### 7.2 Temperature and Heat:

Heat is energy in transit. When two bodies at different temperatures are brought in contact, they exchange heat. After some time, the heat transfer stops and we say the two bodies are in thermal equilibrium. The property or the deciding factor to determine the state of thermal equilibrium is the temperature of the two bodies. Temperature is a physical quantity that defines the thermodynamic state of a system.

You might have experienced that a glass of ice-cold water when left on a table eventually warms up whereas a cup of hot tea on the same table cools down. It means that when the temperature of a body, ice-cold water or hot tea in the above examples, is different from its surrounding medium, heat transfer takes place between the body and the surrounding medium until the body and the surrounding medium are at the same temperature. We then say that the body and its surroundings have reached

a state of thermal equilibrium and there is no net transfer of heat from one to the other. In fact, whenever two bodies are in contact, there is a transfer of heat owing to their temperature difference.

Matter in any state-solid, liquid or gas-consists of particles (ions, atoms or molecules). In solids, these particles are vibrating about their fixed equilibrium positions and possess kinetic energy due to motion at the given temperature. The particles possess potential energy due to the interatomic forces that hold the particles together at some mean fixed positions. Solids therefore have definite volume and shape. When we heat a solid, we provide energy to the solid. The particles then vibrate with higher energy and we can see that the temperature of the solid increases (except near its melting point). Thus the energy supplied to the solid (does not disappear!) becomes the internal energy in the form of increased kinetic energy of atoms/molecules and raises the temperature of the solid. The temperature is therefore a measure of the average kinetic energy of the atoms/ molecules of the body. The greater the kinetic energy is, the faster the molecules will move and higher will be the temperature of the body. If we continue heating till the solid starts to melt, the heat supplied is used to weaken the bonds between the constituent particles. The average kinetic energy of the constituent particles does not change further. The order of magnitude of the average distance between the molecules of the melt remains almost the same as that of solid. Due to weakened bonds liquids do not possess definite shape but have definite volume. The mean distance between the particles and hence the density of liquid is more or less the same as that of the solid. On heating further, the atoms/molecules in liquid gain kinetic energy and temperature of the liquid increases. If we continue heating the liquid further, at the boiling point, the constituents can move freely overcoming the interatomic/molecular forces and the mean distance between the constituents increases so that the particles are farther apart.

As per kinetic theory of gases, for an ideal

gas, there are no forces between the molecules of a gas. Hence gases neither have a definite volume nor shape. Interatomic spacing in solids is  $\sim 10^{-10}$  m while that in liquid is almost twice. The average inter molecular spacing in gases at NTP is  $\sim 10^{-9}$  m.

From the above discussion, we understand that **heat supplied to the substance increases the kinetic energy of molecules or atoms of the substance. The average kinetic energy per particle of a substance defines the temperature.** Temperature measures the degree of hotness of an object and not the amount of its thermal energy.

A glass of water, a gas enclosed in a container, a block of copper metal are all examples of a 'system'. We can say that heat in the form of energy is transferred between two (or more) systems or a system and its surroundings by virtue of their temperature difference. SI unit of heat energy is joule (J) and that of temperature is kelvin (K) or celcius ( $^{\circ}\text{C}$ ). The CGS unit of heat energy is erg and one J =  $10^7$  erg. The other unit of heat energy, that you have learnt in VIII<sup>th</sup> standard, is calorie (cal) and the relation with J is 1 cal = 4.184 J. Heat being energy has dimension  $[\text{L}^2\text{M}^1\text{T}^2\text{K}^0]$  while dimension of temperature is  $[\text{L}^0\text{M}^0\text{T}^0\text{K}^1]$ .

### 7.3 Measurement of Temperature:

In order to isolate two liquids or gases from each other and from the surroundings, we use containers and partitions made of materials like wood, plastic, glass wool, etc. An ideal wall or partition (not available in practice) separating two systems is one that does not allow any flow or exchange of heat energy from one system to the other. Such a perfect thermal insulator is called an **adiabatic wall** and is generally shown as a **thick cross-shaded** (slanting lines) region. When we wish to allow exchange of heat energy between two systems, we use a partition like a thin sheet of copper. It is termed as a **diathermic** wall and is represented as a thin dark region.

Let us consider two sections of a container separated by an adiabatic wall. Let them contain two different gases. Let us call them system A

and system B. We independently bring systems A and B in thermal equilibrium with a system C. Now if we remove the adiabatic wall separating systems A and B, there will be no transfer of heat from system A to system B or vice versa. This indicates that systems A and B are also in thermal equilibrium. Overall conclusion of this activity can be summarized as follows: If systems A and B are separately in thermal equilibrium with a system C, then A and B are also mutually in thermal equilibrium. When two or more systems/ bodies are in thermal equilibrium, their temperatures are same. This principle is used to measure the temperature of a system by using a thermometer.



### Do you know ?

If  $T_A = T_B$  and  $T_B = T_C$ , then  $T_A = T_C$  is not a mathematical statement, if  $T_X$  represents the temperature of system X. It is the zero<sup>th</sup> law of thermodynamics and makes the science of Thermometry possible.

Do you remember that to know the temperature of our body, doctor brings the mercury in the thermometer down to indicate some low temperature. We are then asked to keep the thermometer in our mouth. We have to wait for some time before the thermometer is taken out to know the temperature of our body. There is transfer of heat energy from our body to the thermometer since initially our body is at a higher temperature. When the temperature on the thermometer is same as that of our body, thermal equilibrium is said to be attained and heat transfer stops.

As mentioned above, to precisely know the thermodynamic state of any system, we need to know its temperature. The device used to measure temperature is a thermometer. Thermometry is the science of temperature and its measurement. For measurement of temperature, we need to establish a temperature scale and adopt a set of rules for assigning numbers (with corresponding units).

For the calibration of a thermometer, a standard temperature interval is selected between two easily reproducible fixed

temperatures just as we select the standard of length (metre) to be the distance between two fixed marks. The fact that substances change state from solid to liquid to gas at fixed temperatures is used to define reference temperature called fixed point. The two fixed temperatures selected for this purpose are the melting point of ice or freezing point of water and the boiling point of water. The next step is to sub-divide this standard temperature interval into sub-intervals by utilizing some physical property that changes with temperature and call each sub-interval a degree of temperature. This procedure sets up an empirical scale for temperature.

- \* The temperature at which pure water freezes at one standard atmospheric pressure is called **ice point**/ freezing point of water. This is also the melting point of ice.
- \* The temperature at which pure water boils and vaporizes into steam at one standard atmospheric pressure is called **steam point**/ boiling point. This is also the temperature at which steam changes to liquid water.

Having decided the fixed point phenomena, it remains to assign numerical values to these fixed points and the number of divisions between them. In 1750, conventions were adopted to assign (i) a temperature at which pure ice melts at one atmosphere pressure (the ice point) to be 0° and (ii) a temperature at which pure water boils at one atmosphere (the steam point) to be 100° so that there are 100 degrees between the fixed points. This was the centigrade scale (*centi* meaning hundred in Latin). This was redefined as celcius scale after the Swedish scientist Anders Celcius (1701-1744). It is a convention to express temperature as degree celcius (°C).

To measure temperature quantitatively, generally two different scales of temperature are used. They are describe below.

- 1) **Celsius scale:-** On this scale, the ice point is marked as 0 and the steam point is marked as 100, both taken at normal atmospheric pressure ( $10^5$  Pa or N/m<sup>2</sup>). The interval between these points is divided into 100

equal parts. Each of these is known as degree Celsius and is written as °C.

- 2) **Fahrenheit scale** :- On this scale, the ice point is marked as 32 and the steam point is marked as 212, both taken at normal atmospheric pressure. The interval between these points is divided into 180 equal parts. Each division is known as degree Fahrenheit and is written as °F.

A relationship for conversion between the two scales may be obtained from a graph of fahrenheit temperature ( $T_F$ ) versus celsius temperature ( $T_C$ ). The graph is a straight line (Fig. 7.1) whose equation is

$$\frac{T_F - 32}{180} = \frac{T_C}{100} \quad \text{--- (7.1)}$$

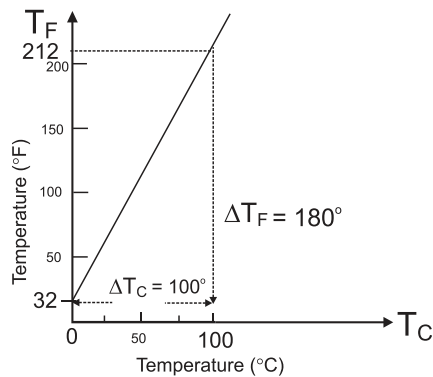


Fig. 7.1: A plot of fahrenheit temperature ( $T_F$ ) versus celsius temperature ( $T_C$ ).

**Example 7.1:** Average room temperature on a normal day is 27 °C. What is the room temperature in °F?

**Solution:** We have

$$\frac{T_F - 32}{180} = \frac{T_C}{100}$$

$$\therefore T_F = \frac{180}{100} T_C + 32$$

Given  $T_C = 27$  °C,

$$\begin{aligned} T_F &= \frac{180}{100} \times 27 + 32 \\ &= 48.6 + 32 \\ &= 80.6 \text{ °F} \end{aligned}$$

**Example 7.2:** Normal human body temperature in feherenheit is 98.4 °F. What is the body temperature in °C?

**Solution:** We have

$$\frac{T_C}{100} = \frac{T_F - 32}{180}$$

$$\therefore T_C = \frac{100}{180} (T_F - 32)$$

Given  $T_F = 98.4$  °F,

$$\begin{aligned} T_C &= \frac{100}{180} (98.4 - 32) \\ &= \frac{100}{180} (66.4) \\ &= 36.89 \text{ °C} \end{aligned}$$

A device used to measure temperature, is based on the principle of thermal equilibrium. To measure the temperature, we use different measurable properties of materials which change with temperature. Some of them are length of a rod, volume of a liquid, electrical resistance of a metal wire, pressure of a gas at constant volume etc. Such changes in physical properties with temperature are used to design a thermometer. Physical property that is used in the thermometer for measuring the temperature is called the thermometric property and the material employed for the purpose is termed as the thermometric substance. Temperature is measured by exploiting the continuous monotonic variation of the chosen property with temperature. A calibration, however, is required to define the temperature scale.

There are different kinds of thermometers each type being more suitable than others for a certain job. In each type, the physical property used to measure the temperature must vary continuously over a wide range of temperature. It must be accurately measurable with simple apparatus.

An important characteristic of a thermometer is its *sensitivity*, i.e., a change in the thermometric property for a very small change in temperature. Two other characteristics are *accuracy* and *reproducibility*. Also it is important that the system attains thermal equilibrium with the thermometer quickly.

If the values of a thermometric property are  $P_1$  and  $P_2$  at the ice point (0 °C) and steam point (100 °C) respectively and the value of this property is  $P_T$  at unknown temperature  $T$ , then  $T$  is given by the following equation

$$T = \frac{100(P_T - P_1)}{(P_2 - P_1)} \quad \text{--- (7.2)}$$

Ideally, there should be no difference



in temperatures recorded on two different thermometers. This is seen for thermometers based on gases as thermometric substances. In a constant volume gas thermometer, the pressure of a fixed volume of gas (measured by the difference in height) is used as the thermometric property. It is an accurate but bulky instrument.

Liquid-in-glass thermometer depends on the change in volume of the liquid with temperature. The liquid in a glass bulb expands up a capillary tube when the bulb is heated. The liquid must be easily seen and must expand (or contract) rapidly and by a large amount over a wide range of temperature. Most commonly used liquids are mercury and alcohol as they remain in liquid state over a wide range. Mercury freezes at  $-39^{\circ}\text{C}$  and boils at  $357^{\circ}\text{C}$ ; alcohol freezes at  $-115^{\circ}\text{C}$  and boils at  $78^{\circ}\text{C}$ . Thermochromic liquids are ones which change colour with temperature but have a limited range around room temperatures. For example, titanium dioxide and zinc oxide are white at room temperature but when heated change to yellow.

**Example 7.3:** The length of a mercury column in a mercury-in-glass thermometer is 25 mm at the ice point and 180 mm at the steam point. What is the temperature when the length is 60 mm?

**Solution:** Here the thermometric property  $P$  is the length of the mercury column. Using Eq. (7.2), we get

$$T = \frac{100(60 - 25)}{(180 - 25)} = 22.58^{\circ}\text{C}$$

Resistance thermometer uses the change of electrical resistance of a metal wire with temperature. It measures temperature accurately in the range  $-2000^{\circ}\text{C}$  to  $1200^{\circ}\text{C}$  but it is bulky and is best for steady temperatures.

**Example 7.4:** A resistance thermometer has resistance  $95.2\ \Omega$  at the ice point and  $138.6\ \Omega$  at the steam point. What resistance would be obtained if the actual temperature is  $27^{\circ}\text{C}$ ?

**Solution:** Here the thermometric property  $P$  is the resistance. Using Eq. (7.2), if  $R$  is the resistance at  $27^{\circ}\text{C}$ , we have

$$\begin{aligned} 27 &= \frac{100(R - 95.2)}{(138.6 - 95.2)}, \\ \therefore R &= \frac{27 \times (138.6 - 95.2)}{100} + 95.2 \\ &= 11.72 + 95.2 = 106.92\ \Omega \end{aligned}$$

Normally in research laboratories, a thermocouple is used to measure the temperature. A thermocouple is a junction of two different metals or alloys e.g., copper and iron joined together. When two such junctions at the two ends of two dissimilar metal rods are kept at two different temperatures, an electromotive force is generated that can be calibrated to measure the temperature.

Thermistor is another device used to measure temperature based on the change in resistance of a semiconductor materials i.e., the resistance is the thermometric property. You will learn more about this device in Chapter 14 on Semiconductors.

## 7.4 Absolute Temperature and Ideal Gas Equation:

### 7.4.1 Absolute zero and absolute temperature

Experiments carried out with gases at low densities indicate that while pressure is held constant, the volume of a given quantity of gas is directly proportional to temperature (measured in  $^{\circ}\text{C}$ ). Similarly, if the volume of a given quantity of gas is held constant, the pressure of the gas is directly proportional to temperature (measured in  $^{\circ}\text{C}$ ). These relations are graphically shown in Fig. 7.2 (a) and (b). Mathematically, this relationship can be written as  $PV \propto T_c$ . Thus the volume-temperature or pressure-temperature graphs for a gas are straight lines. They show that gases expand linearly with temperature on a mercury thermometer i.e., equal temperature increase causes equal volume or pressure increase. The similar thermal behavior of all gases suggests that this relationship of gases can be used to measure temperature in a constant-volume gas thermometer in terms of pressure of the gas.

Although actual experimental measurements might differ a little from the ideal linear relationship, the linear relationship

holds over a wide temperature range.

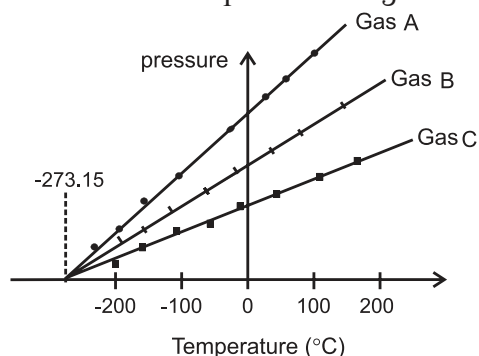


Fig. 7.2 (a): Graph of pressure versus temperature (in °C) at constant volume.

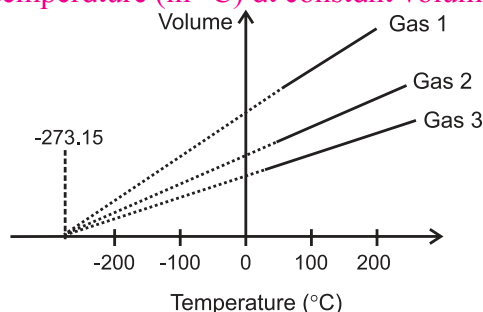


Fig. 7.2 (b): Graph of volume versus temperature (in °C) at constant pressure.

It may be noted that the lines do not pass through the origin i.e., have non-zero intercept along the y-axis. The straight lines have different slopes for different gases. If we assume that the gases do not liquefy even if we lower the temperature, we can extend the straight lines backwards for low temperatures. Is it possible to reach a temperature where the gases stop exerting any pressure i.e., pressure is zero? In a constant pressure thermometer, as the temperature is lowered, the volume decreases. Suppose the gas does not liquefy even at very low temperature, at what temperature, will the volume become zero? Practically it is not possible to keep the material in gaseous state for very low temperature and without exerting any pressure. If we extrapolate the graph of pressure  $P$  versus temperature  $T_c$  (in °C), the temperature at which the pressure of a gas would be zero is  $-273.15^\circ\text{C}$ . It is seen that all the lines for different gases cut the temperature axis at the same point at  $-273.15^\circ\text{C}$ . This point is termed as the **absolute zero of temperature**. It is not possible to attain a temperature lower than this value. Even to achieve absolute zero

temperature is not possible in practice. It may be noted that the point of zero pressure or zero volume does not depend on any specific gas.

The two fixed point scale, described in section 7.3, had a practical shortcoming for calibrating the scale. It was difficult to precisely control the pressure and identify the fixed points, especially for the boiling point as the boiling temperature is very sensitive to changes in pressure. Hence, a one fixed point scale was adopted in 1954 to define a temperature scale. This scale is called the **absolute scale** or thermodynamic scale. It is named as the kelvin scale after Lord Kelvin (1824-1907).

It is possible for all the three phases - solid, liquid and gas/vapour - to coexist in equilibrium. This is known as the *triple point*. To know the triple point one has to see that three phases coexist in equilibrium and no one phase is dominating. This occurs for each substance at a single unique combination of temperature and pressure. Thus if three phases of water - solid ice, liquid water and water vapour- coexist, the pressure and temperature are automatically fixed. This is termed as the triple point of water and is a single fixed point to define a temperature scale.

The absolute scale of temperature, is so termed since it is based on the properties of an ideal gas and does not depend on the property of any particular substance. The zero of this scale is ideally the lowest temperature possible although it has not been achieved in practice. It is termed as Kelvin scale with its zero at  $-273.15^\circ\text{C}$  and temperature intervals same as that on the celsius scale. It is written as K (without °). Internationally, triple point of water has been assigned as  $273.16\text{ K}$  at pressure equal to  $6.11 \times 10^2\text{ Pa}$  or  $6.11 \times 10^{-3}$  atmosphere, as the standard fixed point for calibration of thermometers. Size of one kelvin is thus  $1/273.16$  of the difference between the absolute zero and triple point of water. It is same as one celcius. On celcius scale, the triple point of water is  $0.01^\circ\text{C}$  and not zero.

Three identical thermometers, marked in kelvin, celcius and fahrenheit, placed in a fixed

temperature bath, each thermometer showing the same rise in the level of mercury for human body temperature, are depicted in Fig. 7.3.

The relation between the three scales of temperature is as given in Eq. (7.3) .

$$\frac{T_C}{100} = \frac{T_F - 32}{180} = \frac{T_K - 273.15}{100} \quad \text{--- (7.3)}$$

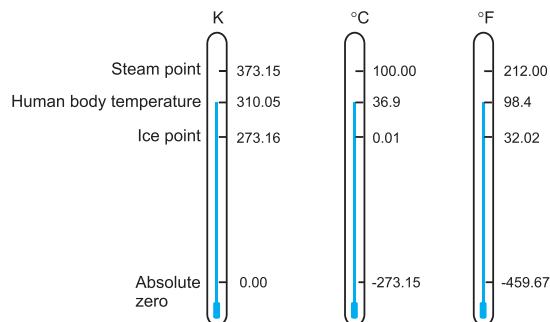


Fig. 7.3: Comparison of the kelvin, celsius and fahrenheit temperature scales.

**Example 7.5:** Express  $T = 24.57$  K in celsius and fahrenheit.

**Solution:** We have

$$\frac{T_F - 32}{180} = \frac{T_C}{100} = \frac{T_K - 273.15}{100}$$

$$\begin{aligned} \therefore T_C &= T_K - 273.15 \\ &= 24.57 - 273.15 \\ &= -248.58^\circ\text{C} \end{aligned}$$

$$\frac{T_F - 32}{180} = \frac{T_K - 273.15}{100}$$

$$\begin{aligned} \therefore T_F &= \frac{180}{100} (T_K - 273.15) + 32 \\ &= \frac{9}{5} (24.57 - 273.15) + 32 \\ &= -447.44 + 32 \\ &= -415.44^\circ\text{F} \end{aligned}$$

**Example 7.6:** Calculate the temperature which has the same value on fahrenheit scale and kelvin scale.

**Solution:** Let the required temperature be  $y$ .

i.e.,  $T_F = T_K = y$  then we have

$$\frac{y - 32}{180} = \frac{y - 273.15}{100}$$

$$\text{or, } 5y - 160 = 9y - 2458.35$$

$$\text{or, } 4y = -160 + 2458.35$$

$$\therefore y = 574.59$$

Thus  $574.59^\circ\text{F}$  and  $574.59$  K are equivalent temperatures.

## 7.4.2 Ideal Gas Equation:

The relation between three properties of a gas i.e., pressure, volume and temperature is called ideal gas equation. You will learn more about the properties of gases in chemistry.

Using absolute temperatures, the gas laws can be stated as given below.

- 1) **Charles' law-** In Fig. 7.2 (b), the volume-temperature graph passes through the origin if temperatures are measured on the kelvin scale, that is if we take 0 K as the origin. In that case the volume  $V$  is directly proportional to the absolute temperature  $T$ .

Thus  $V \propto T$

$$\text{or, } \frac{V}{T} = \text{constant} \quad \text{--- (7.4)}$$

Thus Charles' law can be stated as, the volume of a fixed mass of gas is directly proportional to its absolute temperature if the pressure is kept constant.

- 2) **Pressure (Gay Lussac's) law-** From Fig.7.2, it can be seen that the pressure-temperature graph is similar to the volume-temperature graph.

Thus  $P \propto T$

$$\text{or, } \frac{P}{T} = \text{constant} \quad \text{--- (7.5)}$$

Pressure law can be stated as the pressure of a fixed mass of gas is directly proportional to its absolute temperature if the volume is kept constant.

- 3) **Boyle's law-** For fixed mass of gas at constant temperature, pressure is inversely proportional to volume.

$$\text{Thus } P \propto \frac{1}{V}$$

$$PV = \text{constant} \quad \text{--- (7.6)}$$

Combining above three equations, we get

$$\frac{PV}{T} = \text{constant} \quad \text{--- (7.7)}$$

For one mole of a gas, the constant of proportionality is written as  $R$

$$\therefore \frac{PV}{T} = R \quad \text{or} \quad PV = RT \quad \text{--- (7.8)}$$

If given mass of a gas consists of  $n$  moles, then Eq. (7.8) can be written as

$$PV = nRT \quad \text{--- (7.9)}$$

This relation is called ideal gas equation. The value of constant  $R$  is same for all gases. Therefore, it is known as universal gas constant. Its numerical value is  $8.31 \text{ J K}^{-1} \text{ mol}^{-1}$ .

**Example 7.7:** The pressure reading in a thermometer at steam point is  $1.367 \times 10^3 \text{ Pa}$ . What is pressure reading at triple point knowing the linear relationship between temperature and pressure?

**Solution:** We have  $P_{\text{triple}} = 273.16 \times \left(\frac{P}{T}\right)$  where  $P_{\text{triple}}$  and  $P$  are the pressures at temperature of triple point ( $273.16 \text{ K}$ ) and  $T$  respectively. We are given that  $P = 1.367 \times 10^3 \text{ Pa}$  at steam point i.e., at  $273.15 + 100 = 373.15 \text{ K}$ .

$$\therefore P_{\text{triple}} = 273.16 \times \left(\frac{1.367 \times 10^3}{373.15}\right) \\ = 1.000 \times 10^3 \text{ Pa}$$

## 7.5 Thermal Expansion:

When matter is heated, it normally expands and when cooled, it normally contracts. The atoms in a solid vibrate about their mean positions. When heated, they vibrate faster and force each other to move a little farther apart. This results into expansion. The molecules in a liquid or gas move with certain speed. When heated, they move faster and force each other to move a little farther apart. This results in expansion of liquids and gases on heating. The expansion is more in liquids than in solids; gases expand even more.

A change in the temperature of a body causes change in its dimensions. The increase in the dimensions of a body due to an increase in its temperature is called thermal expansion. There are three types of thermal expansion: 1) Linear expansion, 2) Areal expansion, 3) Volume expansion.

### 7.5.1 Linear Expansion:

The expansion in length due to thermal energy is called linear expansion.

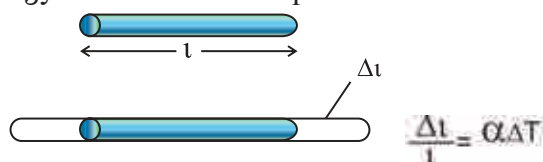


Fig. 7.4: Linear expansion  $\Delta l$  is exaggerated for explanation.

If the substance is in the form of a long rod of length  $l$ , then for small change  $\Delta T$ , in temperature, the fractional change  $\Delta l/l$ , in length (shown in Fig.7.4), is directly proportional to  $\Delta T$ .

$$\frac{\Delta l}{l} \propto \Delta T \\ \text{or } \frac{\Delta l}{l} = \alpha \Delta T \quad \text{--- (7.10)}$$

where  $\alpha$  is called the coefficient of linear expansion of solid. Its value depends upon nature of the material. Rearranging Eq. (7.10), we get

$$\alpha = \frac{\Delta l}{l \Delta T} \\ = \frac{l_T - l_0}{l_0 (T - T_0)} \quad \text{--- (7.11)}$$

where  $l_0$  = length of rod at  $0^\circ \text{C}$

$l_T$  = length of rod when heated to  $T^\circ \text{C}$

$T_0 = 0^\circ \text{C}$  is initial temperature

$T$  = final temperature

$\Delta l = l_T - l_0$  = change in length

$\Delta T = T - T_0$  = rise in temperature

Referring to Eq. (7.11), if  $l_0 = 1$  and  $T - T_0 = 1^\circ \text{C}$ , then

$$\alpha = l_T - l_0 \text{ (numerically).}$$

**Coefficient of linear expansion of a solid is thus defined as increase in the length per unit original length at  $0^\circ \text{C}$  for one degree centigrade rise in temperature.**

The unit of coefficient of linear expansion is per degree celcius or per kelvin. The magnitude of  $\alpha$  is very small and it varies only a little with temperature. For most practical purposes,  $\alpha$  can be assumed to be constant for a particular material. Therefore, it is not necessary that initial temperature be taken as  $0^\circ \text{C}$ . Equation (7.11) can be rewritten as

$$\alpha = \frac{l_2 - l_1}{l_1 (T_2 - T_1)} \quad \text{--- (7.12)}$$

where  $l_1$  = initial length at temperature  $T_1^\circ \text{C}$

$l_2$  = final length at temperature  $T_2^\circ \text{C}$ .

Table 7.1 lists average values of coefficient of linear expansion for some materials in the



temperature range 0°C to 100 °C.

**Table 7.1: Values of coefficient of linear expansion for some common materials.**

Materials	$\alpha$ (K <sup>-1</sup> )
Carbon (diamond)	$0.1 \times 10^{-5}$
Glass	$0.85 \times 10^{-5}$
Iron	$1.2 \times 10^{-5}$
Steel	$1.3 \times 10^{-5}$
Gold	$1.4 \times 10^{-5}$
Copper	$1.7 \times 10^{-5}$
Silver	$1.9 \times 10^{-5}$
Aluminium	$2.5 \times 10^{-5}$
Sulphur	$6.1 \times 10^{-5}$
Mercury	$6.1 \times 10^{-5}$
Water	$6.9 \times 10^{-5}$
Carbon (graphite)	$8.8 \times 10^{-5}$

**Example 7.8:** The length of a metal rod at 27 °C is 4 cm. The length increases to 4.02 cm when the metal rod is heated upto 387 °C. Determine the coefficient of linear expansion of the metal rod.

**Solution:** Given

$$T_1 = 27^\circ\text{C}$$

$$T_2 = 387^\circ\text{C}$$

$$l_1 = 4\text{ cm} = 4 \times 10^{-2}\text{ m}$$

$$l_2 = 4.02\text{ cm} = 4.02 \times 10^{-2}\text{ m}$$

We have

$$\begin{aligned}\alpha &= \frac{l_2 - l_1}{l_1(T_2 - T_1)} \\ &= \frac{(4.02 - 4.0) \times 10^{-2}}{4 \times 10^{-2}(387 - 27)} \\ &= \frac{0.02 \times 10^{-2}}{4 \times 10^{-2} \times 360} \\ &= 1.39 \times 10^{-5} / ^\circ\text{C}\end{aligned}$$

**Example 7.9:** Length of a metal rod at temperature 27 °C is 4.256m. Find the temperature at which the length of the same rod increases to 4.268m. ( $\alpha$  for iron =  $1.2 \times 10^{-5} \text{ K}^{-1}$ )

**Solution:** Given

$$T_1 = 27^\circ\text{C}, l_1 = 4.256\text{ m},$$

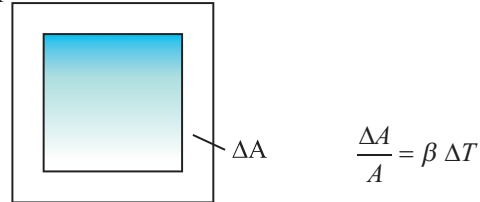
$$l_2 = 4.268\text{ m}, \alpha = 1.2 \times 10^{-5} \text{ K}^{-1}$$

We have

$$\begin{aligned}\alpha &= \frac{l_2 - l_1}{l_1(T_2 - T_1)} = \frac{l_2 - l_1}{l_1 T_2 - l_1 T_1} \\ \therefore l_1 T_2 &= l_1 T_1 + \frac{l_2 - l_1}{\alpha} \\ T_2 &= \frac{1}{l_1} \left[ l_1 T_1 + \frac{l_2 - l_1}{\alpha} \right] \\ &= \frac{1}{4.256} \left[ (4.256 \times 27) + \frac{4.268 - 4.256}{1.2 \times 10^{-5}} \right] \\ &= \frac{1}{4.256} \left[ 114.912 + \frac{0.012}{1.2 \times 10^{-5}} \right] \\ &= \frac{1}{4.256} [114.912 + 1000] \\ &= 261.96^\circ\text{C}\end{aligned}$$

### 7.5.2 Areal Expansion:

The increase  $\Delta A$ , in the surface area, on heating is called areal expansion or superficial expansion.



**Fig. 7.5: Areal expansion  $\Delta A$  is exaggerated for explanation.**

If a substance is in the form of a plate of area  $A$ , then for small change  $\Delta T$  in temperature, the fractional change in area,  $\Delta A/A$  (as shown in Fig. 7.5), is directly proportional to  $\Delta T$ .

$$\begin{aligned}\frac{\Delta A}{A} &\propto \Delta T \\ \text{or } \frac{\Delta A}{A} &= \beta \Delta T \quad \text{--- (7.13)}\end{aligned}$$

where  $\beta$  is called the coefficient of areal expansion of solid. It depends on the material of the solid. Rearranging Eq. (7.13), we get

$$\beta = \frac{\Delta A}{A \Delta T} = \frac{A_T - A_0}{A_0(T - T_0)} \quad \text{--- (7.14)}$$

where  $A_0$  = area of plate at 0 °C

$A_T$  = area of plate when heated to  $T^\circ\text{C}$

$T_0 = 0^\circ\text{C}$  is initial temperature

$T$  = final temperature

$\Delta A = A_T - A_0$  = change in area

$\Delta T = T - T_0$  = rise in temperature.

If  $A_0 = 1 \text{ m}^2$  and  $T - T_0 = 1^\circ\text{C}$ , then

$\beta = A_T - A_0$  (numerically).

Therefore, **coefficient of areal expansion of a solid is defined as the increase in the area per unit original area at 0°C for one degree rise in temperature.**

The unit of  $\beta$  is per degree celcius or per kelvin.

As in the case of  $\alpha$ ,  $\beta$  also does not vary much with temperature. Hence, if  $A_1$  is the area of a metal plate at  $T_1$  °C and  $A_2$  is the area at higher temperature  $T_2$  °C, then

$$\beta = \frac{A_2 - A_1}{A_1(T_2 - T_1)} \quad \text{--- (7.15)}$$

**Example 7.10:** A thin aluminium plate has an area 286 cm<sup>2</sup> at 20 °C. Find its area when it is heated to 180 °C.

( $\beta$  for aluminium =  $4.9 \times 10^{-5}$  /°C)

**Solution:** Given

$$T_1 = 20 \text{ °C}$$

$$T_2 = 180 \text{ °C}$$

$$A_1 = 286 \text{ cm}^2$$

$$\beta = 4.9 \times 10^{-5} \text{ /°C}$$

We have

$$\beta = \frac{A_2 - A_1}{A_1(T_2 - T_1)}$$

$$\begin{aligned} \therefore A_2 &= A_1 [1 + \beta (T_2 - T_1)] \\ &= 286 [1 + 4.9 \times 10^{-5} (180 - 20)] \\ &= 286 [1 + 4.9 \times 10^{-5} \times 160] \\ &= 286 [1 + 784.0 \times 10^{-5}] \\ &= 286 [1 + 0.00784] \\ &= 286 [1.00784] \\ \therefore A_2 &= 288.24 \text{ cm}^2 \end{aligned}$$

### 7.5.3 Volume expansion

The increase in volume due to heating is called volume expansion or cubical expansion.

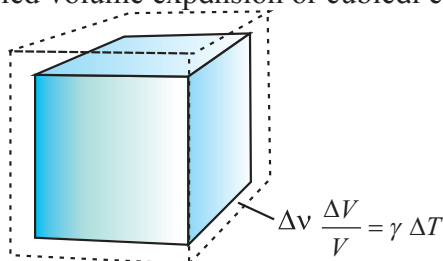


Fig. 7.6: Volume expansion  $\Delta V$  is exaggerated for explanation.

If the substance is in the form of a cube of volume  $V$ , then for small change  $\Delta T$  in temperature, the fractional change,  $\Delta V/V$  (as shown in Fig.7.6), in volume is directly proportional to  $\Delta T$ .

$$\frac{\Delta V}{V} \propto \Delta T$$

$$\text{or } \frac{\Delta V}{V} = \gamma \Delta T \quad \text{--- (7.16)}$$

where  $\gamma$  is called coefficient of cubical or volume expansion. It depends upon the nature of the material. Its unit is per degree celcius or per kelvin. From Eq.(7.16), we can write

$$\gamma = \frac{\Delta V}{V \Delta T} = \frac{V_T - V_0}{V_0(T - T_0)} \quad \text{--- (7.17)}$$

where  $V_0$  = volume at 0 °C

$V_T$  = volume when heated to  $T$  °C

$T_0 = 0$  °C is initial temperature

$T$  = final temperature

$\Delta V = V_T - V_0$  = change in volume

$\Delta T = T - T_0$  = rise in temperature.

If  $V_0 = 1 \text{ m}^3$ ,  $T - T_0 = 1$  °C, then

$\gamma = V_T - V_0$  (numerically).

**The coefficient of cubical expansion of a solid is therefore defined as increase in volume per unit original volume at 0°C for one degree rise in the temperature.**

If  $V_1$  is the volume of a body at  $T_1$  °C and  $V_2$  is the volume at higher temperature  $T_2$  °C, then

$$\gamma_1 = \frac{V_2 - V_1}{V_1(T_2 - T_1)} \quad \text{--- (7.18)}$$

$\gamma_1$  is the coefficient of volume expansion at temperature  $T_1$  °C.

Since fluids possess definite volume and take the shape of the container, only change in volume is significant. Equations (7.17) and (7.18) are valid for cubical or volume expansion of fluids. It is to be noted that since fluids are kept in containers, when one deals with the volume expansion of fluids, expansion of the container is also to be considered. If expansion of fluid results in a volume greater than the volume of the container, the fluid will overflow if the container is open. If the container is closed, volume expansion of fluid will cause additional

pressure on the walls of the container. Can you now tell why the balloon bursts sometimes when you try to fill air in it?

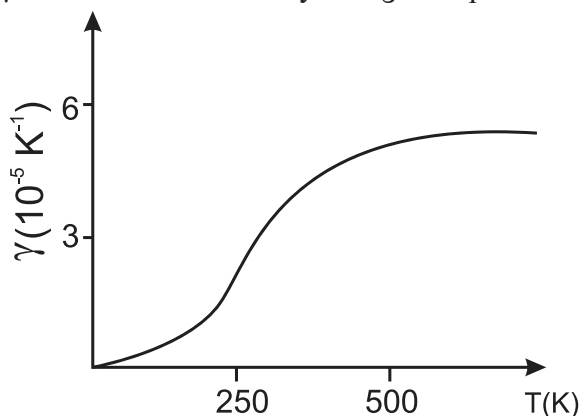
Normally solids and liquids expand on heating. Hence their volume increases on heating. Since the mass is constant, it results in a decrease in the density on heating. You have learnt about the anomalous behaviour of water. Water expand on cooling from 4 °C to 0 °C. Hence its density decreases on cooling in this temperature range.

In Table 7.2 are given typical average values of the coefficient of volume expansion  $\gamma$  for some materials in the temperature range 0°C to 100°C.

**Table 7.2: Values of coefficient of volume expansion for some common materials.**

Materials	$\gamma$ (K <sup>-1</sup> )
Invar	$2 \times 10^{-6}$
Glass (ordinary)	$2.5 \times 10^{-5}$
Steel	$(3.3-3.9) \times 10^{-5}$
Iron	$3.55 \times 10^{-5}$
Gold	$4.2 \times 10^{-5}$
Brass	$5.7 \times 10^{-5}$
Aluminium	$6.9 \times 10^{-5}$
Mercury	$18.2 \times 10^{-5}$
Water	$20.7 \times 10^{-5}$
Paraffin	$58.8 \times 10^{-5}$
Gasoline	$95.0 \times 10^{-5}$
Alcohol (ethyl)	$110 \times 10^{-5}$

$\gamma$  is also characteristic of the substance but is not strictly a constant. It depends in general on temperature as shown in Fig.7.7. It is seen that  $\gamma$  becomes constant only at high temperatures.



**Fig. 7.7: Coefficient of volume expansion of copper as a function of temperature.**

**Example 7.11 :** A liquid at 0 °C is poured in a glass beaker of volume 600 cm<sup>3</sup> to fill it completely. The beaker is then heated to 90 °C. How much liquid will overflow?

$$(\gamma_{\text{liquid}} = 1.75 \times 10^{-4} / ^\circ\text{C}, \gamma_{\text{glass}} = 2.75 \times 10^{-5} / ^\circ\text{C})$$

**Solution:** Given

$$V_1 = 600 \text{ cm}^3$$

$$T_1 = 0 ^\circ\text{C}$$

$$T_2 = 90 ^\circ\text{C}$$

$$\text{We have } \gamma = \frac{V_2 - V_1}{V_1(T_2 - T_1)}$$

$$\therefore \text{ increase in volume} = V_2 - V_1 = \gamma V_1 (T_2 - T_1)$$

Increase in volume of beaker

$$\begin{aligned} &= \gamma_{\text{glass}} \times V_1 (T_2 - T_1) \\ &= 2.75 \times 10^{-5} \times 600 \times (90 - 0) \\ &= 2.75 \times 10^{-5} \times 600 \times 90 \\ &= 148500 \times 10^{-5} \text{ cm}^3 \end{aligned}$$

$$\therefore \text{ increase in volume of beaker} = 1.485 \text{ cm}^3$$

Increase in volume of liquid

$$\begin{aligned} &= \gamma_{\text{liquid}} \times V_1 (T_2 - T_1) \\ &= 1.75 \times 10^{-4} \times 600 \times (90 - 0) \\ &= 1.75 \times 10^{-4} \times 600 \times 90 \\ &= 94500 \times 10^{-4} \text{ cm}^3 \end{aligned}$$

$$\therefore \text{ increase in volume of liquid} = 9.45 \text{ cm}^3$$

$\therefore$  volume of liquid which overflows

$$\begin{aligned} &= (9.45 - 1.485) \text{ cm}^3 \\ &= 7.965 \text{ cm}^3 \end{aligned}$$

#### 7.5.4 Relation between Coefficients of Expansion:

##### i) Relation between $\beta$ and $\alpha$ :

Consider a square plate of side  $l_0$  at 0 °C and  $l_T$  at  $T$  °C.

$$\therefore l_T = l_0 (1 + \alpha T) \text{ from Eq. (7.11).}$$

If area of plate at 0 °C is  $A_0$ ,  $A_0 = l_0^2$ .

If area of plate at  $T$  °C is  $A_T$ ,

$$A_T = l_T^2 = l_0^2 (1 + \alpha T)^2$$

$$\text{or } A_T = A_0 (1 + \alpha T)^2 \quad \text{--- (7.19)}$$

Also from Eq. (7.14),

$$A_T = A_0 (1 + \beta T) \quad \text{--- (7.20)}$$

Using Eqs. (7.19) and (7.20), we get

$$A_0 (1+\alpha T)^2 = A_0 (1+\beta T)$$

$$\text{or } 1 + 2\alpha T + \alpha^2 T^2 = 1 + \beta T$$

Since the values of  $\alpha$  are very small, the term  $\alpha^2 T^2$  is very small and may be neglected.

$$\therefore \beta = 2\alpha \quad \text{--- (7.21)}$$



### Can you tell?

1. Why the metal wires for electrical transmission lines sag?
2. Why a railway track is not a continuous piece but is made up of segments separated by gaps?
3. How a steel wheel is mounted on an axle to fit exactly?
4. Why lakes freeze first at the surface?

The result is general because any solid can be regarded as a collection of small squares.

### ii) Relation between $\gamma$ and $\alpha$ :

Consider a cube of side  $l_0$  at  $0^\circ\text{C}$  and  $l_T$  at  $T^\circ\text{C}$ .

$$\therefore l_T = l_0 (1 + \alpha T) \text{ from Eq. (7.11).}$$

If volume of the cube at  $0^\circ\text{C}$  is  $V_0$ ,  $V_0 = l_0^3$ .

If volume of the cube at  $T^\circ\text{C}$  is

$$V_T, V_T = l_T^3 = l_0^3 (1 + \alpha T)^3$$

$$\text{or } V_T = V_0 (1 + \alpha T)^3 \quad \text{--- (7.22)}$$

Also from Eq. (7.17),

$$V_T = V_0 (1 + \gamma T) \quad \text{--- (7.23)}$$

Using Eqs. (7.22) and (7.23), we get

$$V_0 (1 + \alpha T)^3 = V_0 (1 + \gamma T)$$

$$\text{or } 1 + 3\alpha T + 3\alpha^2 T^2 + \alpha^3 T^3 = 1 + \gamma T$$

Since the values of  $\alpha$  are very small, the terms with higher powers of  $\alpha$  may be neglected.

$$\therefore \gamma = 3\alpha \quad \text{--- (7.24)}$$

Again the result is general because any solid can be regarded as a collection of small cubes.

Finally, the relation between  $\alpha$ ,  $\beta$  and  $\gamma$  is

$$\alpha = \frac{\beta}{2} = \frac{\gamma}{3} \quad \text{--- (7.25)}$$

**Example 7.12:** A sheet of brass is 50 cm long and 8 cm broad at  $0^\circ\text{C}$ . If the surface area at  $100^\circ\text{C}$  is  $401.57\text{cm}^2$ , find the coefficient of linear expansion of brass.

**Solution:** Given

$$T_1 = 0^\circ\text{C}$$

$$T_2 = 100^\circ\text{C}$$

$$A_1 = 50 \times 8 = 400\text{ cm}^2$$

$$A_2 = 401.57\text{ cm}^2$$

We have

$$\begin{aligned} \beta = 2\alpha &= \frac{A_2 - A_1}{A_1(T_2 - T_1)} \\ &= \frac{(401.57 - 400)\text{ cm}^2}{400\text{ cm}^2 \times (100 - 0)^\circ\text{C}} \\ &= \frac{1.57}{400 \times 100} = 0.3925 \times 10^{-4}^\circ\text{C}^{-1} \\ \therefore \alpha &= 0.1962 \times 10^{-4}^\circ\text{C}^{-1} \\ &= 1.962 \times 10^{-5}^\circ\text{C}^{-1} \end{aligned}$$

$\therefore$  Coefficient of linear expansion of brass is  $1.962 \times 10^{-5} / ^\circ\text{C}$ .



### Do you know ?

- \* When pressure is held constant, due to change in temperature, the volume of a liquid or solid changes very little in comparison to the volume of a gas.
- \* The coefficient of volume expansion,  $\gamma$ , is generally an order of magnitude larger for liquids than for solids.
- \* Metals have high values for the coefficient for linear expansion,  $\alpha$ , than non-metals.
- \*  $\gamma$  changes more with temperature than  $\alpha$  and  $\beta$ .
- \* We know that water expands on freezing from  $4^\circ\text{C}$  to  $0^\circ\text{C}$ . Other two substances, metals bismuth (Bi) and antimony (Sb), that expand on freezing. Thus the density of liquid is more than corresponding solid and hence solid Bi or Sb float on their liquids like ice floats on water.

## 7.6 Specific Heat Capacity:

### 7.6.1 Specific Heat Capacity of Solids and Liquids

If 1 kg of water and 1 kg of paraffin are heated in turn for the same time by the same heater, the temperature rise of paraffin is about twice that of water. Since the heater gives equal amounts of heat energy to each liquid, it seems that different substances require different



amounts of heat to cause the same temperature rise of  $1^{\circ}\text{C}$  in the same mass of 1 kg.

If  $\Delta Q$  stands for the amount of heat absorbed or given out by a substance of mass  $m$  when it undergoes a temperature change  $\Delta T$ , then the specific heat capacity of that substance is given by

$$s = \frac{\Delta Q}{m\Delta T} \quad \text{--- (7.26)}$$

If  $m = 1 \text{ kg}$  and  $\Delta T = 1^{\circ}\text{C}$  then  $s = \Delta Q$ .

**Thus specific heat capacity is defined as the amount of heat per unit mass absorbed or given out by the substance to change its temperature by one unit (one degree)  $1^{\circ}\text{C}$  or  $1\text{K}$ .**

**Table 7.3: Specific heat capacity of some substances at room temperature and atmospheric pressure.**

Substance	Specific heat capacity ( $\text{J kg}^{-1} \text{K}^{-1}$ )
Steel	120
Lead	128
Gold	129
Tungsten	134.4
Silver	234
Copper	387
Iron	448
Carbon	506.5
Glass	837
Aluminium	903.0
Kerosene	2118
Paraffin oil	2130
Alcohol (ethyl)	2400
Ethanol	2500
Water	4186.0

The SI unit of specific heat capacity is  $\text{J/kg } ^{\circ}\text{C}$  or  $\text{J/kg K}$  and C.G.S. unit is  $\text{erg/g } ^{\circ}\text{C}$  or  $\text{erg/g K}$ . The specific heat capacity is a property of the substance and weakly depends on its temperature. Except for very low temperatures, the specific heat capacity is almost constant for all practical purposes.

If the amount of substance is specified in terms of moles  $\mu$  instead of mass  $m$  in kg, then the specific heat is called molar specific heat ( $C$ ) and is given by

$$C = \frac{1}{\mu} \frac{\Delta Q}{\Delta T} \quad \text{--- (7.27)}$$

The SI unit of molar specific heat capacity is  $\text{J/mol } ^{\circ}\text{C}$  or  $\text{J/mol K}$ . Like specific heat, molar specific heat also depends on the nature of the substance and its temperature. Table 7.3 lists the values of specific heat capacity for some common materials.

From Table 7.3, it can be seen that water has the highest specific heat capacity compared to other substances. For this reason, water is used as a coolant in automobile radiators as well as in a heater in hot water bags.

### 7.6.2 Specific Heat Capacity of Gas:

In case of a gas, slight change in temperature is accompanied with considerable changes in both, the volume and the pressure. If gas is heated at constant pressure, volume changes and therefore some work is done on the surroundings during expansion requiring additional heat. As a result, specific heat at constant pressure ( $S_p$ ) is greater than specific heat at constant volume ( $S_v$ ). It is thus necessary to define two principal specific heat capacities for a gas.

#### Principal specific heat capacities of gases:

- The principal specific heat capacity of a gas at constant volume ( $S_v$ ) is defined as the quantity of heat absorbed or released for the rise or fall of temperature of unit mass of a gas through  $1 \text{ K}$  (or  $1^{\circ}\text{C}$ ) when its volume is kept constant.
- The principal specific heat capacity of a gas at constant pressure ( $S_p$ ) is defined as the quantity of heat absorbed or released for the rise or fall of temperature of unit mass of a gas through  $1 \text{ K}$  ( $1^{\circ}\text{C}$ ) when its pressure is kept constant.

#### Molar specific heat capacities of gases:

- Molar specific heat capacity of a gas at constant volume ( $C_v$ ) is defined as the quantity of heat absorbed or released for the rise or fall of temperature of one mole of the gas through  $1 \text{ K}$  (or  $1^{\circ}\text{C}$ ), when its volume is kept constant.

- b) Molar specific heat capacity of a gas at constant pressure ( $C_p$ ) is defined as the quantity of heat absorbed or released for the rise or fall of temperature of one mole of the gas through 1K (or 1°C), when its pressure is kept constant.

### Relation between Principal and Molar Specific Heat Capacities:

A relation between principal specific heat capacity and molar specific heat capacity is given by the following expression

Molar specific heat capacity = Molecular weight  $\times$  principal specific heat capacity.

$$\text{i.e. } C_p = \mu \times S_p \text{ and } C_v = \mu \times S_v$$

where  $\mu$  is the molecular weight of the gas.

Table 7.4 lists values of molar specific heat capacity for some commonly known gases.

**Table 7.4: Molar specific heat capacity of some gases.**

Gas	$C_p$ (J mol <sup>-1</sup> K <sup>-1</sup> )	$C_v$ (J mol <sup>-1</sup> K <sup>-1</sup> )
He	20.8	12.5
H <sub>2</sub>	28.8	20.4
N <sub>2</sub>	29.1	20.8
O <sub>2</sub>	29.4	21.1
CO <sub>2</sub>	37.0	28.5

### 7.6.3 Heat Equation:

If a substance has a specific heat capacity of 1000 J/kg °C, it means that heat energy of 1000 J raises the temperature of 1 kg of that substance by 1°C or 6000 J will raise the temperature of 2 kg of the substance by 3 °C. If the temperature of 2 kg mass of the substance falls by 3 °C, the heat given out would also be 6000 J. In general we can write the heat equation as

Heat received or given out ( $Q$ ) = mass ( $m$ )  $\times$  temperature change ( $\Delta t$ )  $\times$  specific heat capacity ( $s$ ).

$$\text{or } Q = m \times \Delta T \times s \quad \text{--- (7.28)}$$

**Example 7.13:** If the temperature of 4 kg mass of a material of specific heat capacity 300 J/kg °C rises from 20 °C to 30 °C. Find the heat received.

**Solution:**

$$Q = 4 \text{ kg} \times (30-20) ^\circ\text{C} \times 300 \text{ J/kg } ^\circ\text{C}$$

$$= 4 \times 10 \times 300 \text{ J}$$

$$\therefore Q = 12000 \text{ J}$$

### 7.6.4 Heat Capacity (Thermal Capacity):

Heat capacity or thermal capacity of a body is the quantity of heat needed to raise or lower the temperature of the whole body by 1°C (or 1K).

$\therefore$  Thermal heat capacity can be written as Heat received or given out

$$= \text{mass} \times 1 \times \text{specific heat capacity}$$

$$\text{Heat capacity} = Q = m \times s \quad \text{--- (7.29)}$$

Heat capacity (thermal capacity) is measured in J/°C.

**Example 7.14:** Find thermal capacity for a copper block of mass 0.2 kg, if specific heat capacity of copper is 290 J/kg °C.

**Solution:** Given

$$m = 0.2 \text{ kg}$$

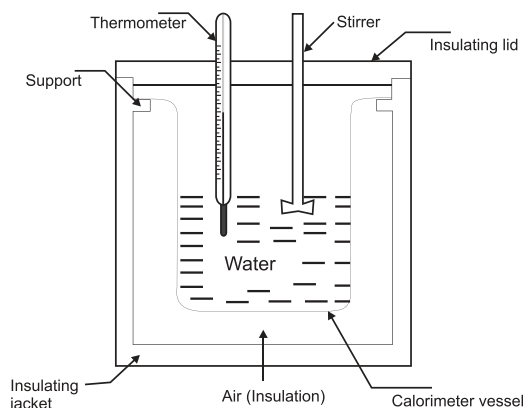
$$s = 290 \text{ J/kg } ^\circ\text{C}$$

$$\text{Thermal capacity} = m \times s = 0.2 \text{ kg} \times 290 \text{ J/kg } ^\circ\text{C} = 58 \text{ J/ } ^\circ\text{C}$$

### 7.7 Calorimetry:

Calorimetry is an experimental technique for the quantitative measurement of heat exchange. To make such measurement a calorimeter is used. Figure 7.8 shows a simple water calorimeter.

It consists of cylindrical vessel made of copper or aluminium and provided with a stirrer and a lid. The calorimeter is well-insulated to prohibit any transfer of heat into or out of the calorimeter.



**Fig. 7.8: Calorimeter.**

One important use of calorimeter is to determine the specific heat of a substance using

the principle of conservation of energy. Here we are dealing with heat energy and the system is isolated from surroundings. Therefore, heat gained is equal to the heat lost.

In the technique known as the “method of mixtures”, a sample 'A' of the substance is heated to a high temperature which is accurately measured. The sample 'A' is then placed quickly in the calorimeter containing water. The contents are stirred constantly until the mixture attains a final common temperature. The heat lost by the sample 'A' will be gained by the water and the calorimeter. The specific heat of the sample 'A' of the substance can be calculated as under:

Let

$m_1$  = mass of the sample 'A'

$m_2$  = mass of the calorimeter and the stirrer

$m_3$  = mass of the water in calorimeter

$s_1$  = specific heat capacity of the substance of sample 'A'

$s_2$  = specific heat capacity of the material of calorimeter (and stirrer)

$s_3$  = specific heat capacity of water

$T_1$  = initial temperature of the sample 'A'

$T_2$  = initial temperature of the calorimeter stirrer and water

$T$  = final temperature of the combined system

We have the data as following:

Heat lost by the sample 'A' =  $m_1 s_1 (T_1 - T)$

Heat gained by the calorimeter and the stirrer  
=  $m_2 s_2 (T - T_2)$

Heat gained by the water =  $m_3 s_3 (T - T_2)$

Assuming no loss of heat to the surroundings, the heat lost by the sample goes into the calorimeter, stirrer and water. Thus writing heat equation as,

$$m_1 s_1 (T_1 - T) = m_2 s_2 (T - T_2) + m_3 s_3 (T - T_2) \quad \text{---(7.30)}$$

Knowing the specific heat capacity of water ( $s_3 = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$ ) and copper ( $s_2 = 387 \text{ J kg}^{-1} \text{ K}^{-1}$ ) being the material of the calorimeter and the stirrer, one can calculate specific heat capacity ( $s_1$ ) of material of sample 'A', from Eq. (7.30) as

$$s_1 = \frac{(m_2 s_2 + m_3 s_3)(T - T_2)}{m_1 (T_1 - T)} \quad \text{--- (7.31)}$$

Also, one can find specific heat capacity of water or any liquid using the following expression, if the specific heat capacity of the material of calorimeter and sample is known

$$s_3 = \frac{m_1 s_1 (T_1 - T)}{m_3 (T - T_2)} - \frac{m_2 s_2}{m_3} \quad \text{--- (7.32)}$$

**Note** - In the experiment, the heat from the solid sample 'A' is given to the liquid and therefore the sample should be denser than the liquid, so that sample does not float on the liquid.

**Example 7.15:** A sphere of aluminium of 0.06 kg is placed for sufficient time in a vessel containing boiling water so that the sphere is at 100 °C. It is then immediately transferred to 0.12 kg copper calorimeter containing 0.30 kg of water at 25 °C. The temperature of water rises and attains a steady state at 28 °C. Calculate the specific heat capacity of aluminium. (Specific heat capacity of water,  $s_w = 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ , specific heat capacity of copper  $s_{Cu} = 0.387 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$ )

**Solution :** Given

Mass of aluminium sphere =  $m_1 = 0.06 \text{ kg}$

Mass of copper calorimeter =  $m_2 = 0.12 \text{ kg}$

Mass of water in calorimeter

=  $m_3 = 0.30 \text{ kg}$

Specific heat capacity of copper

=  $s_{Cu} = s_2 = 0.387 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

Specific heat capacity of water

=  $s_w = s_3 = 4.18 \times 10^3 \text{ J kg}^{-1} \text{ K}^{-1}$

Initial temperature of aluminium sphere

=  $T_1 = 100^\circ\text{C}$

Initial temperature of calorimeter and water =  $T_2 = 25^\circ\text{C}$

Final temperature of the mixture

=  $T = 28^\circ\text{C}$

We have

$$\begin{aligned}
 s_1 &= \frac{(m_2 s_2 + m_3 s_3)(T - T_2)}{m_1 (T_1 - T)} \\
 &= \frac{[(0.12 \times 387) + (0.30 \times 4180)](28 - 25)}{(0.06)(100 - 28)} \\
 &= \frac{(46.44 + 1254) \times 3}{(0.06) \times 72} = \frac{3901.32}{4.32} \\
 &= 903.08 \text{ J kg}^{-1} \text{ K}^{-1}
 \end{aligned}$$

$\therefore$  Specific heat capacity of aluminium is  $903.08 \text{ J kg}^{-1} \text{ K}^{-1}$ .

### 7.8 Change of State:

Matter normally exists in three states: solid, liquid and gas. A transition from one of these states to another is called a change of state. Two common changes of states are solid to liquid and liquid to gas (and vice versa). These changes can occur when exchange of heat takes place between the substance and its surroundings.



#### Activity

##### To understand the process of change of state

Take some cubes of ice in a beaker. Note the temperature of ice ( $0^\circ\text{C}$ ). Start heating it slowly on a constant heat source. Note the temperature after every minute. Continuously stir the mixture of water and ice. Observe the change in temperature. Continue heating even after the whole of ice gets converted into water. Observe the change in temperature as before till vapours start coming out. Plot the graph of temperature (along y-axis) versus time (along x-axis). Obtain a graph of temperature versus time as shown in Fig. 7.9.

#### Analysis of observations :

##### 1) From point A to B:

There is no change in temperature from point A to point B, this means the temperature of the ice bath does not change even though heat is being continuously supplied. That is the temperature remains constant until the entire amount of the ice melts. The heat supplied is being utilised in changing the state from solid

(ice) to liquid (water).

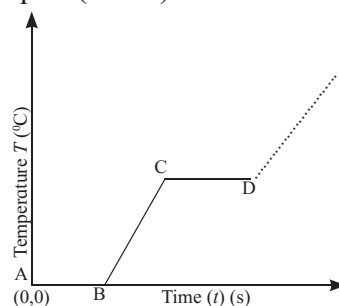


Fig. 7.9 : Variation of temperature with time.

- The change of state from solid to liquid is called melting and from liquid to solid is called solidification.
- Both the solid and liquid states of the substance co-exist in thermal equilibrium during the change of states from solid to liquid or vice versa.
- The temperature at which the solid and the liquid states of the substance are in thermal equilibrium with each other is called the melting point of solid (here ice) or freezing point of liquid (here water). It is the characteristic of the substance and also depends on pressure.
- The melting point of a substance at one standard atmospheric pressure is called its normal melting point.
- At one standard atmospheric pressure, the freezing point of water and melting point of ice is  $0^\circ\text{C}$  or  $32^\circ\text{F}$ . The freezing point describes the liquid to solid transition while melting point describes solid-to-liquid transition.

##### 2) From point B to D:

The temperature begins to rise from point B to point C, i.e., after the whole of ice gets converted into water and we continue further heating. We see that temperature begins to rise. The temperature keeps on rising till it reaches point C i.e., nearly  $100^\circ\text{C}$ . Then it again becomes steady. It is observed that the temperature remains constant until the entire amount of the liquid is converted into vapour. The heat supplied is now being utilized to change water from liquid state to vapour or gaseous state.

- The change of state from liquid to vapour



is called vapourisation while that from vapour to liquid is called condensation.

- b) Both the liquid and vapour states of the substance coexist in thermal equilibrium during the change of state from liquid to vapour.
- c) The temperature at which the liquid and the vapour states of the substance coexist is called the boiling point of liquid, here water or steam point. This is also the temperature at which water vapour condenses to form water.
- d) The boiling point of a substance at one standard atmospheric pressure is called its normal boiling point.



### Can you tell?

1. What after point D in graph ? Can steam be hotter than  $100^{\circ}\text{C}$  ?
2. Why steam at  $100^{\circ}\text{C}$  causes more harm to our skin than water at  $100^{\circ}\text{C}$ ?



### Do you know ?

You must have seen that water spilled on floor dries up after some time. Where does the water disappear? It is converted into water vapour and mixes with air. We say that water has evaporated. You also know that water can be converted into water vapour if you heat the water till its boiling point. **What is then the difference between boiling and evaporation?**

Both evaporation and boiling involve change of state, evaporation can occur at any temperature but boiling takes place at a fixed temperature for a given pressure, unique for each liquid. Evaporation takes place from the surface of liquid while boiling occurs in the whole liquid.

As you know, molecules in a liquid are moving about randomly. The average kinetic energy of the molecules decides the temperature of the liquid. However, all molecules do not move with the same speed. One with higher kinetic energy may escape from the surface region by overcoming the interatomic forces. This process can take place at any temperature. This is evaporation. If the temperature of the liquid is higher, more is the average kinetic energy. Since the number of molecules is fixed, it implies that the number of fast moving molecules

is more. Hence the rate of losing such molecules to atmosphere will be larger. Thus, higher is the temperature of the liquid, greater is the rate of evaporation. Since faster molecules are lost, the average kinetic energy of the liquid is reduced and hence the temperature of the liquid is lowered. Hence the phenomenon of evaporation gives a cooling effect to the remaining liquid. Since evaporation takes place from the surface of a liquid, the rate of evaporation is more if the area exposed is more and if the temperature of the liquid is higher.

You might have seen that if your mother wants her sari/clothes to dry faster, she does not fold them. More is the area exposed, faster is the drying because the water gets evaporated faster. The presence of wind or strong breeze and content of water vapour in the atmosphere are two other important factors determining the drying of clothes but we do not refer to them here.

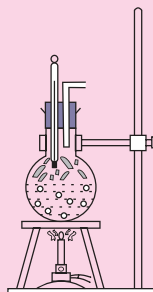
Before giving an injection to a patient, normally a spirit swab is used to disinfect the region. We feel a cooling effect on our skin due to evaporation of the spirit as explained before.



### Activity

#### Activity to understand the dependence of boiling point on pressure

Take a round bottom flask, more than half filled with water. Keep it over a burner and fix a thermometer and steam outlet through the cork of the flask as shown in figure. As water in the flask gets heated, note that first the air, which was dissolved in the water comes out as small bubbles. Later bubbles of steam form at the bottom but as they rise to the cooler water near the top, they condense and disappear. Finally, as the temperature of the entire mass of the water reaches  $100^{\circ}\text{C}$ , bubbles of steam reach the surface and boiling is said to occur. The steam in the flask may not be visible but as it comes out of the flask, it condenses as tiny droplets of water giving a foggy appearance.



If now the steam outlet is closed for a few seconds to increase the pressure in the

flask, you will notice that boiling stops. More heat would be required to raise the temperature (depending on the increase in pressure) before boiling starts again. Thus boiling point increases with increase in pressure.

Let us now remove the burner. Allow water to cool to about  $80^{\circ}\text{C}$ . Remove the thermometers and steam outlet. Close the flask with a air tight cork. Keep the flask turned upside down on a stand. Pour ice-cold water on the flask. Water vapours in the flask condense reducing the pressure on the water surface inside the flask. Water begins to boil again, now at a lower temperature. Thus boiling point decreases with decrease in pressure and increases with increase in pressure.



**Can you tell?**

1. Why cooking is difficult at high altitude?
2. Why cooking is faster in pressure cooker?

### 7.8.1 Sublimation:

Have you seen what happens when camphor is burnt? All substances do not pass through the three states: solid-liquid-gas. There are certain substances which normally pass from the solid to the vapour state directly and vice versa. The change from solid state to vapour state without passing through the liquid state is called sublimation and the substance is said to sublime. Dry ice (solid  $\text{CO}_2$ ) and iodine sublime. During the sublimation process, both the solid and vapour states of a substance coexist in thermal equilibrium. Most substances sublime at very low pressures.

### 7.8.2 Phase Diagram:

A pressure - temperature (PT) diagram often called a phase diagram, is particularly convenient for comparing different phases of a substance.

A phase is a homogeneous composition of a material. A substance can exist in different phases in solid state, e.g., you are familiar with two phases of carbon- graphite and diamond. Both are solids but the regular geometric

arrangement of carbon atoms is different in the two cases. Figure 7.10 shows the phase diagram of water and  $\text{CO}_2$ . Let us try to understand the diagram.

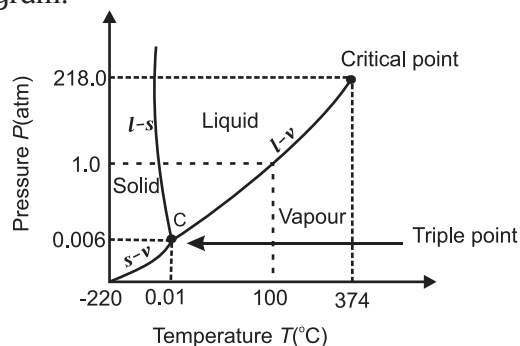


Fig. 7.10 (a): Phase diagram of water (not to scale).

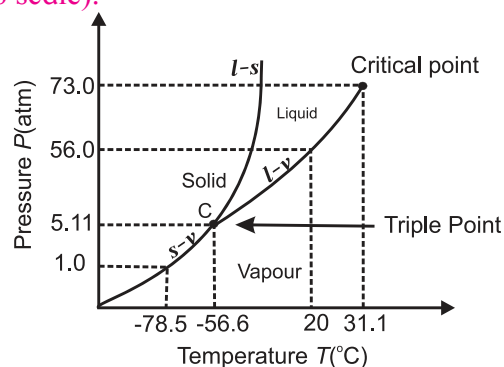


Fig. 7.10 (b): Phase diagram of  $\text{CO}_2$  (not to scale).

**i) Vapourisation curve  $I-v$ :** The curve labelled  $I-v$  represents those points where the liquid and vapour phases are in equilibrium. Thus it is a graph of boiling point versus pressure. Note that the curves correctly show that at a pressure of 1 atmosphere, the boiling points of water is  $100^{\circ}\text{C}$  and that the boiling point is lowered for a decreased pressure.

**ii) Fusion curve  $I-s$ :** The curve  $I-s$  represents the points where the solid and liquid phases coexist in equilibrium. Thus it is a graph of the freezing point versus pressure. At one standard atmosphere pressure, the freezing point of water is  $0^{\circ}\text{C}$  as shown in Fig. 7.10 (a). Also notice that at a pressure of one standard atmosphere water is in the liquid phase if the temperature is between  $0^{\circ}\text{C}$  and  $100^{\circ}\text{C}$  but is in the solid or vapour phase if the temperature is below  $0^{\circ}\text{C}$  or above  $100^{\circ}\text{C}$ . Note that  $I-s$  curve for water slopes upward to the left i.e., fusion curve of water has a slightly negative slope. This is true

only of substances that expand upon freezing. However, for most materials like  $\text{CO}_2$ , the  $l$ - $s$  curve slopes upwards to the right i.e., fusion curve has a positive slope. The melting point of  $\text{CO}_2$  is  $-56^\circ\text{C}$  at higher pressure of 5.11 atm.

**iii) Sublimation curve  $s$  -  $v$ :** The curve labelled  $s$  -  $v$  is the sublimation point versus pressure curve. Water sublimates at pressure less than 0.0060 atmosphere, while carbon dioxide, which in the solid state is called dry ice, sublimates even at atmospheric pressure at temperature as low as  $-78^\circ\text{C}$ .

**iv) Triple point:** The temperature and pressure at which the fusion curve, the vapourisation curve and the sublimation curve meet and all the three phases of a substance coexist is called the triple point of the substance. That is, the triple point of water is that point where water in solid, liquid and gaseous states coexist in equilibrium and this occurs only at a unique temperature and pressure. The triple point of water is  $273.16\text{ K}$  and  $6.11 \times 10^{-3}\text{ Pa}$  and that of  $\text{CO}_2$  is  $-56.6^\circ\text{C}$  and  $5.1 \times 10^{-5}\text{ Pa}$ .

### 7.8.3 Gas and Vapour:

The terms gas and vapour are sometimes used quite randomly. Therefore, it is important to understand the difference between the two. A gas cannot be liquefied by pressure alone, no matter how high the pressure is. In order to liquefy a gas, it must be cooled to a certain temperature. This temperature is called critical temperature.

Critical temperatures for some common gases and water vapour are given in Table 7.5. Thus, nitrogen must be cooled below  $-147^\circ\text{C}$  to liquefy it by pressure.

**Table 7.5: Critical Temperatures of some common gases and water vapour.**

Gas	Critical Temperature	
	( $^\circ\text{C}$ )	( $\text{K}$ )
Air	-190	83
$\text{N}_2$	-147	126
$\text{O}_2$	-118	155
$\text{CO}_2$	31.1	241.9
Water vapour	374	647

Gas and vapour can thus be defined as-

- 1) A substance which is in the gaseous phase and is above its critical temperature is called a gas.
- 2) A substance which is in the gaseous phase and is below its critical temperature is called a vapour.

Vapour can be liquefied simply by increasing the pressure, while gas cannot. Vapour also exerts pressure like a gas.

### 7.8.4 Latent Heat:

Whenever there is a change in the state of a substance, heat is either absorbed or given out but there is no change in the temperature of the substance.

**Latent heat of a substance is the quantity of heat required to change the state of unit mass of the substance without changing its temperature.**

Thus if mass  $m$  of a substance undergoes a change from one state to the other then the quantity of heat absorbed or released is given by  $Q = mL$  --- (7.33)

where  $L$  is known as latent heat and is characteristic of the substance. Its SI unit is  $\text{J kg}^{-1}$ . The value of  $L$  depends on the pressure and is usually quoted at one standard atmospheric pressure.

**The quantity of heat required to convert unit mass of a substance from its solid state to the liquid state, at its melting point, without any change in its temperature is called its latent heat of fusion ( $L_f$ ).**

**The quantity of heat required to convert unit mass of a substance from its liquid state to vapour state, at its boiling point without any change in its temperature is called its latent heat of vapourization ( $L_v$ ).**

A plot of temperature versus heat energy for a given quantity of water is shown in Fig. 7.11.

From Fig. 7.11, we see that when heat is added (or removed) during a change of state, the temperature remains constant. Also the slopes of the phase lines are not all the same, which indicates that specific heats of the various states are not equal. For water the latent heat of fusion

and vaporisation are  $L_f = 3.33 \times 10^5 \text{ J kg}^{-1}$  and  $L_v = 22.6 \times 10^5 \text{ J kg}^{-1}$  respectively. That is  $3.33 \times 10^5 \text{ J}$  of heat is needed to melt 1 kg of ice at  $0^\circ \text{C}$  and  $22.6 \times 10^5 \text{ J}$  of heat is needed to convert 1 kg of water to steam at  $100^\circ \text{C}$ . Hence, steam at  $100^\circ \text{C}$  carries  $22.6 \times 10^5 \text{ J kg}^{-1}$  more heat than water at  $100^\circ \text{C}$ . This is why burns from steam are usually more serious than those from boiling water. Melting points, boiling point and latent heats for various substances are given Table 7.6.

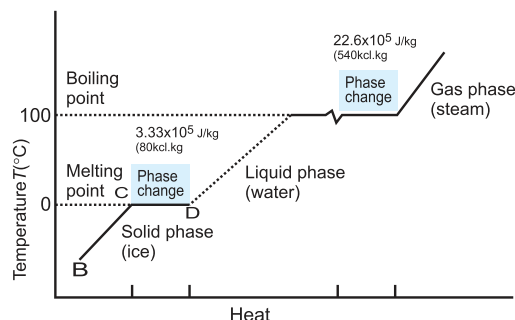


Fig. 7.11: Temperature versus heat for water at one standard atmospheric pressure (not to scale).

**Table 7.6 : Temperature of change of state and latent heats for various substances at one standard atmosphere pressure.**

Substance	Melting point ( $^\circ \text{C}$ )	$L_f$ ( $\times 10^5 \text{ J kg}^{-1}$ )	Boiling point ( $^\circ \text{C}$ )	$L_v$ ( $\times 10^5 \text{ J kg}^{-1}$ )
Gold	1063	0.645	2660	15.8
Lead	328	0.25	1744	8.67
Water	0	3.33	100	22.6
Ethyl alcohol	-114	1.0	78	8.5
Mercury	-39	0.12	357	2.7
Nitrogen	-210	0.26	-196	2.0
Oxygen	-219	0.14	-183	2.1

**Example 7.16:** When 0.1 kg of ice at  $0^\circ \text{C}$  is mixed with 0.32 kg of water at  $35^\circ \text{C}$  in a container. The resulting temperature of the mixture is  $7.8^\circ \text{C}$ . Calculate the heat of fusion of ice ( $s_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$ ).

**Solution:** Given

$$m_{\text{ice}} = 0.1 \text{ kg}$$

$$m_{\text{water}} = 0.32 \text{ kg}$$

$$T_{\text{ice}} = 0^\circ \text{C}$$

$$T_{\text{water}} = 35^\circ \text{C}$$

$$T_F = 7.8^\circ \text{C}$$

$$s_{\text{water}} = 4186 \text{ J kg}^{-1} \text{ K}^{-1}$$

Heat lost by water

$$= m_{\text{water}} s_{\text{water}} (T_F - T_{\text{water}})$$

$$= 0.32 \text{ kg} \times 4186 \text{ J} \times (7.8 - 35)^\circ \text{C}$$

= - 36434.944 J (here negative sign indicates loss of heat energy)

$$\text{Heat required to melt ice} = m_{\text{ice}} L_f = 0.1 \times L_f$$

Heat required to raise temperature of water (from ice) to final temperature

$$= m_{\text{ice}} s (T - T_{\text{ice}})$$

$$= 0.1 \text{ kg} \times 4186 \text{ J} \times (7.8 - 0)^\circ \text{C}$$

$$= 3265.08 \text{ J}$$

Head lost = Heat gained

$$36434.944 = 0.1 L_f + 3265.08$$

$$L_f = \frac{36434.944 - 3265.08}{0.1} = 3316.9864$$

$$= 3.31698 \times 10^5 \text{ J kg}^{-1}$$



**Do you know ?**

The latent heat of vapourization is much larger than the latent heat of fusion. The energy required to completely separate the molecules or atoms is greater than the energy needed to break the rigidity (rigid bonds between the molecules or atoms) in solids. Also when the liquid is converted into vapour, it expands. Work has to be done against the surrounding atmosphere to allow this expansion.



## 7.9 Heat Transfer:

Heat may be transferred from one point of body to another in three different ways- by conduction, convection and radiation. Heat transfers through solids by conduction. In this process, heat is passed on from one molecule to other molecule but the molecules do not leave their mean positions. Liquids and gases are heated by convection. In this process, there is a bodily movement of the heated molecules. In order to transfer heat by conduction and convection a material medium is required. However transfer of heat by radiation does not need any medium. Radiation of heat energy takes place by EM waves that travel with a speed of  $3 \times 10^8 \text{ ms}^{-1}$  in the space/vacuum. The energy from the Sun comes to us by radiation. It may be noted that conduction is a slow process of heat transfer while convection is a rapid process. However radiation is the fastest process because the transfer of heat takes place at the speed of light.

### 7.9.1 Conduction:

Conduction is the process by which heat flows from the hot end to the cold end of a solid body without any net bodily movement of the particles of the body.

Heat passes through solids by conduction only. When one end of a metal rod is placed in a flame while the other end is held in hand, the end held in hand slowly gets hotter, although it itself is not in direct contact with flame. We say that heat has been conducted from the hot end to the cold end. When one end of the rod is heated, the molecules there vibrate faster. As they collide with their slow moving neighbours, they transfer some of their energy by collision to these molecules which in turn transfer energy to their neighbouring molecules still farther down the length of the rod. Thus the energy of thermal motion is transferred by molecular collisions down the rod. The transfer of heat continues till the two ends of the rod are at the same temperature in principle but this will take infinite time. Normally various sections of the rod will attain a temperature which remains constant but not same through out the length of

the rod. This method of heat transfer is called conduction.

Those solid substances which conduct heat easily are called good conductors of heat e.g. silver, copper, aluminium, brass etc. All metals are good conductors of heat. Those substances which do not conduct heat easily are called bad conductors of heat e.g. wood, cloth, air, paper, etc. In general, good conductors of heat are also good conductors of electricity. Similarly bad conductors of heat are bad conductors of electricity also.

#### 7.9.1.1 Thermal Conductivity:

Thermal conductivity of a solid is a measure of the ability of the solid to conduct heat through it. Thus good conductors of heat have higher thermal conductivity than bad conductors.

Suppose that one end of a metal rod is heated (see Fig 7.12 (a)). The heat flows by conduction from hot end to the cold end. As a result the temperature of every section of the rod starts increasing. Under this condition, the rod is said to be in a variable temperature state. After some time the temperature at each section of the rod becomes steady i.e. does not change. Note that temperature of each cross-section of the rod is constant but not the same. This is called steady state condition. Under steady state condition, the temperature at points within the rod decreases uniformly with distance from the hot end to the cold end. The fall of temperature with distance between the ends of the rod in the direction of flow of heat, is called temperature gradient.

$$\therefore \text{Temperature gradient} = \frac{T_1 - T_2}{x}$$

where  $T_1$  = temperature of hot end

$T_2$  = temperature of cold end

$x$  = length of the rod

#### 7.9.1.2 Coefficient of Thermal Conductivity:

Consider a cube of each side  $x$  and each face of cross-sectional area  $A$ . Suppose its opposite faces are maintained at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ) as shown in Fig. 7.12 (b). Experiments show that under steady state condition, the

quantity of heat ' $Q$ ' that flows from the hot face to the cold face is

- directly proportional to the cross-sectional area  $A$  of the face. i.e.,  $Q \propto A$
- directly proportional to the temperature difference between the two faces i.e.,  $Q \propto (T_1 - T_2)$
- directly proportional to time  $t$  (in seconds) for which heat flows i.e.  $Q \propto t$
- inversely proportional to the perpendicular distance  $x$  between hot and cold faces i.e.,  $Q \propto 1/x$

Combining the above four factors, we have the quantity of heat

$$Q \propto \frac{A(T_1 - T_2)t}{x}$$

$$\therefore Q = \frac{kA(T_1 - T_2)t}{x} \quad \text{--- (7.34)}$$

where  $k$  is a constant of proportionality and is called coefficient of thermal conductivity. Its value depends upon the nature of the material.

If  $A = 1 \text{ m}^2$ ,  $T_1 - T_2 = 1^\circ\text{C}$  (or  $1 \text{ K}$ ),  $t = 1 \text{ s}$  and  $x = 1 \text{ m}$ , then from Eq. (7.34),  $Q = k$ .

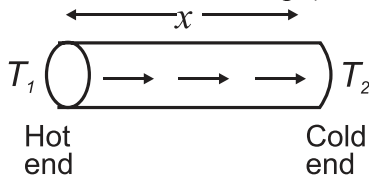


Fig 7.12 (a): Section of a metal bar in the steady state.

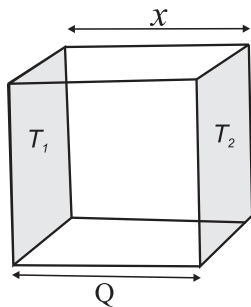


Fig 7.12 (b): Section of a cube in the steady state.

Thus the **coefficient of thermal conductivity of a material is defined as the quantity of heat that flows in one second between the opposite faces of a cube of side 1 m, the faces being kept at a temperature difference of  $1^\circ\text{C}$  (or  $1 \text{ K}$ ).**

From Eq. (7.34), we have

$$k = \frac{Qx}{A(T_1 - T_2)t} \quad \text{--- (7.35)}$$

SI unit of coefficient of thermal conductivity  $k$  is  $\text{J s}^{-1} \text{ m}^{-1} ^\circ\text{C}^{-1}$  or  $\text{J s}^{-1} \text{ m}^{-1} \text{ K}^{-1}$  and its dimensions are  $[\text{L}^1 \text{ M}^1 \text{ T}^3 \text{ K}^{-1}]$ .

From Eq. (7.34), we also have

$$\frac{Q}{t} = \frac{kA(T_1 - T_2)}{x} \quad \text{--- (7.36)}$$

The quantity  $Q/t$ , denoted by  $P_{\text{cond}}$ , is the time rate of heat flow (i.e. heat flow per second) from the hotter face to the colder face, at right angles to the faces. Its SI unit is watt (W). SI unit of  $k$  can therefore be written as  $\text{W m}^{-1} ^\circ\text{C}^{-1}$  or  $\text{W m}^{-1} \text{ K}^{-1}$ .

Using calculus, Eq. (7.36) may be written as

$$\frac{dQ}{dt} = -kA \frac{dT}{dx}$$

where  $\frac{dT}{dx}$  is the temperature gradient.

The negative sign indicates that heat flow is in the direction of decreasing temperature.

If  $A = 1 \text{ m}^2$  and  $\frac{dT}{dx} = 1$ , then  $\frac{dQ}{dt} = k$  (numerically).

Hence the **coefficient of thermal conductivity of a material may also be defined as the rate of flow of heat per unit area per unit temperature gradient when the heat flow is at right angles to the faces of a thin parallel-sided slab of material.**

The coefficients of thermal conductivity of some materials are given Table 7.7.

### 7.9.1.3 Thermal Resistance ( $R_T$ ):

Conduction rate  $P_{\text{cond}}$  is the amount of energy transferred per unit time through a slab of area  $A$  and thickness  $x$ , the two sides of the slab being at temperatures  $T_1$  and  $T_2$  ( $T_1 > T_2$ ), and is given by Eq. (7.36)

$$P_{\text{cond}} = \frac{Q}{t} = kA \frac{T_1 - T_2}{x} \quad \text{--- (7.37)}$$

As discussed earlier,  $k$  depends on the material of the slab. A material that readily transfers heat energy by conduction is a good thermal conductor and has high value of  $k$ .

**Table 7.7: Coefficient of thermal conductivity ( $k$ ).**

Substance	Coefficient of thermal conductivity ( $\text{J s}^{-1} \text{m}^{-1} \text{K}^{-1}$ )
Silver	406
Copper	385
Aluminium	205
Steel	50.2
Insulating brick	0.15
Glass	0.8
Brick and concrete	0.8
Water	0.8
Wood	0.04-0.12
Air at 0 °C	0.024

In western countries, where the temperature falls below 0 °C in winter season, insulating the house from the surroundings is very important. In our country, if we wish to carry cold drinks with us for picnic or wish to bring ice-cream from the shop to our house, we need to keep them in containers (made up of say thermocol) that are poor thermal conductors. Hence the concept of thermal resistance  $R_T$ , similar to electrical resistance, is introduced. The opposition of a body, to the flow of heat through it, is called thermal resistance. The greater the thermal conductivity of a material, the smaller is its thermal resistance and vice versa. Thus bad thermal conductors are those which have high thermal resistance.

From Eq. (7.37)

$$\frac{(T_1 - T_2)}{P_{\text{cond}}} = \frac{x}{kA}$$

We know that when a current flows through a conductor, the ratio  $V/I$  is called the electrical resistance of the conductor where  $V$  is the electrical potential difference between the ends of the conductor and  $I$  is the current or rate of flow of charge. In Eq. (7.38),  $(T_1 - T_2)$  is the temperature difference between the ends of the conductor and  $P_{\text{cond}}$  is the rate of flow of heat. Therefore in analogy with electrical resistance,  $(T_1 - T_2)/P_{\text{cond}}$  is called thermal resistance  $R_T$  of the material i.e.,

$$\text{Thermal resistance } R_T = \frac{x}{kA}$$

The SI unit of thermal resistance is °C s/

kcal or °C s/J and its dimensional formula is  $[\text{M}^{-1} \text{L}^{-2} \text{T}^3 \text{K}^1]$ .

The lower the thermal conductivity  $k$ , the higher is the thermal resistance.  $R_T$  A material with high  $R_T$  value is a poor thermal conductor and is a good thermal insulator. Thermal resistivity  $\rho_T$  is the reciprocal of thermal conductivity  $k$  and is characteristic of a material while thermal resistance is that of slab (or of rod) and depends on the material and on the thickness of slab (or length of rod).

**Example 7.17:** What is the rate of energy loss in watt per square metre through a glass window 5 mm thick if outside temperature is -20 °C and inside temperature is 25 °C? ( $k_{\text{glass}} = 1 \text{ W/m K}$ )

**Solution :** Given

$$k_{\text{glass}} = 1 \text{ W/m K}$$

$$T_1 = 25 \text{ °C}$$

$$T_2 = -20 \text{ °C}$$

$$x = 5 \text{ mm} = 5 \times 10^{-3} \text{ m}$$

$$\therefore T_1 - T_2 = 25 - (-20) \text{ °C} = 45 \text{ K}$$

$$\text{We have } P_{\text{cond}} = \frac{Q}{t} = kA \frac{T_1 - T_2}{x}.$$

$\therefore$  The energy loss per square metre,

$$\frac{P_{\text{cond}}}{A} = k \frac{T_1 - T_2}{x}$$

$$= 1 \text{ W m}^{-1} \text{ K}^{-1} \times 45 \text{ K} / (5 \times 10^{-3} \text{ m})$$

$$= 9 \times 10^3 \text{ W/m}^2$$

#### 7.9.1.4 Applications of Thermal Conductivity:

- Cooking utensils are made of metals but are provided with handles of bad conductors.

Since metals are good conductors of heat, heat can be easily conducted through the base of the utensils. The handles of utensils are made of bad conductors of heat (e.g., wood, ebonite etc.) so that they can not conduct heat from the utensils to our hands.

- Thick walls are used in the construction of cold storage rooms. Brick is a bad conductor of heat so that it reduces the flow of heat from the surroundings to the rooms. Still better heat insulation is obtained by using hollow bricks. Air

being a poorer conductor than a brick, it further avoids the conduction of heat from outside.

- iii) To prevent ice from melting it is wrapped in a gunny bag. A gunny bag is a poor conductor of heat and reduces the heat flow from outside to ice. Moreover, the air filled in the interspaces of a gunny bag, being very bad conductor of heat, further avoids the conduction of heat from outside.

Low thermal conductivity can also be a disadvantage. When hot water is poured in a glass beaker the inner surface of the glass expands on heating. Since glass is a bad conductor of heat, the heat from inside does not reach the outside surface so quickly. Hence the outer surface does not expand thereby causing a crack in the glass.

**Example 7.18:** The temperature difference between two sides of an iron plate, 2 cm thick, is 10 °C. Heat is transmitted through the plate at the rate of 600 kcal per minute per square metre at steady state. Find the thermal conductivity of iron.

**Solution:** Given

$$\frac{Q}{At} = 600 \text{ kcal / min m}^2 = \frac{600}{60} = 10 \text{ kcal / s m}^2$$

$$x = 2 \text{ cm} = 2 \times 10^{-2} \text{ m}$$

$$T_1 - T_2 = 10^\circ\text{C}$$

From Eq.(7.34), we have

$$Q = \frac{kA(T_1 - T_2)t}{x}$$

$$\therefore k = \frac{Q}{At} \frac{x}{T_1 - T_2}$$

$$= \frac{10 \text{ kcal / s m}^2 \times 2 \times 10^{-2} \text{ m}}{10^\circ\text{C}}$$

$$= 0.02 \text{ kcal / m s }^\circ\text{C}$$

**Example 7.19:** Calculate the rate of loss of heat through a glass window of area 1000 cm<sup>2</sup> and thickness of 4 mm, when temperature inside is 27 °C and outside is -5 °C. Coefficient of thermal conductivity of glass is 0.022 cal/ s cm °C.

**Solution :** Given

$$A = 1000 \text{ cm}^2 = 1000 \times 10^{-4} \text{ m}^2$$

$$k = 0.022 \text{ cal/ s cm }^\circ\text{C} = 0.022 \times 10^2 \text{ cal/m }^\circ\text{C}$$

$$x = 4 \text{ mm} = 0.4 \times 10^{-2} \text{ m}$$

$$T_1 = 27^\circ\text{C}, T_2 = -5^\circ\text{C}$$

From Eq. (7.34), we have

$$Q = \frac{kA(T_1 - T_2)t}{x}$$

$$\therefore \frac{Q}{t} = \frac{kA(T_1 - T_2)}{x}$$

$$= \frac{0.022 \times 10^2 \times 1000 \times 10^{-4} \times (27 - (-5))}{0.4 \times 10^{-2}}$$

$$= 1.76 \times 10^3 \text{ cal / s} = 1.76 \text{ kcal / s}$$

### 7.9.2 Convection:

We have seen that heat is transmitted through solids by conduction wherein energy is transferred from one molecule to another but the molecules themselves vibrating with larger amplitude do not leave their mean positions. But in convection, heat is transmitted from one point to another by the actual bodily movement of the heated (energised) molecules within the fluid.

In liquids and gases heat is transmitted by convection because their molecules are quite free to move about. The mechanism of heat transfer by convection in liquids and gases is described below.

Consider water being heated in a vessel from below. The water at the bottom of the vessel is heated first and consequently its density decreases i.e., water molecules at the bottom are separated farther apart. These hot molecules have high kinetic energy and rise upward to cold region while the molecules from cold region come down to take their place. Thus each molecule at the bottom gets heated and rises then cools and descends. This action sets up the flow of water molecules called convection currents. The convection currents transfer heat to the entire mass of water. Note that transfer of heat is by the bodily/ physical movement of the water molecules.



**Always remember:**

The process by which heat is transmitted through a substance from one point to another due to the actual bodily movement of the heated particles of the substance is called convection.

**7.9.2.1 Applications of Convection:****i) Heating and cooling of rooms**

The mechanism of heating a room by a heater is entirely based on convection. The air molecules in immediate contact with the heater are heated up. These air molecules acquire sufficient energy and rise upward. The cool air at the top being denser moves down to take their place. This cool air in turn gets heated and moves upward. In this way, convection currents are set up in the room which transfer heat to different parts of the room. The same principle but in opposite direction is used to cool a room by an air-conditioner.

**ii) Cooling of transformers**

Due to current flowing in the windings of the transformer, enormous heat is produced. Therefore, transformer is always kept in a tank containing oil. The oil in contact with transformer body heats up, creating convection currents. The warm oil comes in contact with the cooler tank, gives heat to it and descends to the bottom. It again warms up to rise upward. This process is repeated again and again. The heat of the transformer body is thus carried away by convection to the cooler tank. The cooler tank, in turn loses its heat by convection to the surrounding air.

**7.9.2.2 Free and Forced Convection:**

- i) When a hot body is in contact with air under ordinary conditions, like air around a firewood, the air removes heat from the body by a process called free or natural convection. Land and sea breezes are also formed as a result of free convection currents in air.
- ii) The convection process can be accelerated by employing a fan to create a rapid

circulation of fresh air. This is called forced convection. Example in section 7.9.2.1 are of forced convection, namely, heat convector, air conditioner, heat radiators in IC engine etc.

**7.9.3 Radiation:**

**The transfer of heat energy from one place to another via emission of EM energy (in a straight line with the speed of light) without heating the intervening medium is called radiation.**

For transfer of heat by radiation, molecules are not needed i.e. medium is not required. The fact that Earth receives large quantities of heat from the Sun shows that heat can pass through empty space (i.e., vacuum) between the Sun and the atmosphere that surrounds the Earth. In fact, transfer of heat by radiation has the same properties as light (or EM wave).

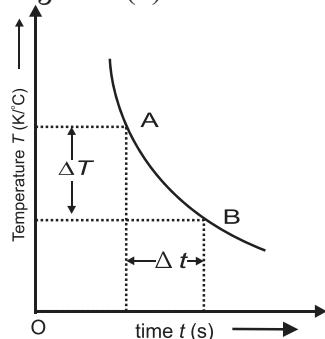
A natural question arises as to how heat transfer occurs in the absence of a medium (i.e., molecules). All objects possess thermal energy due to their temperature  $T$  ( $T > 0 \text{ K}$ ). The rapidly moving molecules of a hot body emit EM waves travelling with the velocity of light. These are called **thermal radiations**. These carry energy with them and transfer it to the low-speed molecules of a cold body on which they fall. This results in an increase in the molecular motion of the cold body and its temperature rises. Thus transfer of heat by radiation is a two-fold process- the conversion of thermal energy into waves and reconversion of waves into thermal energy by the body on which they fall. We will learn about EM waves in Chapter 13.

**7.10 Newton's Laws of Cooling:**

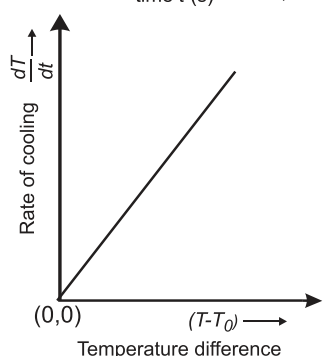
If hot water in a vessel is kept on table, it begins to cool gradually. To study how a given body can cool on exchanging heat with its surroundings, following experiment is performed.

A calorimeter is filled up to two third of its capacity with boiling water and is covered. A thermometer is fixed through a hole in the lid and its position is adjusted so that the bulb of the thermometer is fully immersed in water.

The calorimeter vessel is kept in a constant temperature enclosure or just in open air since room temperature will not change much during experiment. The temperature on the thermometer is noted at one minute interval until the temperature of water decreases by about 25 °C. A graph of temperature  $T$  (along  $y$ -axis) is plotted against time  $t$  (along  $x$ -axis). This graph is called cooling curve (Fig 7.13 (a)). From this graph you can infer how the cooling of hot water depends on the difference of its temperature from that of its surroundings. You will also notice that initially the rate of cooling is higher and it decreases as the temperature of the water falls. A tangent is drawn to the curve at suitable points on the curve. The slope of each tangent ( $dT/dt$ ) gives the rate of fall of temperature at that temperature. Taking (0,0) as the origin, if a graph of  $dT/dt$  is plotted against corresponding temperature difference ( $T-T_0$ ), the curve is a straight line as shown in Fig 7.13 (b).



**Fig 7.13 (a):**  
Temperature versus time graph.  $\lim_{\Delta t \rightarrow 0} \frac{\Delta T}{\Delta t}$  gives the slope of the tangent drawn to the curve at point A and indicates the rate of fall of temperature.



**Fig 7.13 (b):**  
Rate of change of temperature versus time graph.

The above activity shows that a hot body loses heat to its surroundings in the form of heat radiation. The rate of loss of heat depends on the difference in the temperature of the body and its surroundings. Newton was the first to study the relation between the heat lost by a body in a given enclosure and its temperature in a systematic manner.

According to Newton's law of cooling the rate of loss of heat  $dT/dt$  of the body is directly proportional to the difference of temperature ( $T - T_0$ ) of the body and the surroundings provided the difference in temperatures is small. Mathematically this may be expressed as

$$\frac{dT}{dt} \propto (T - T_0)$$

$$\therefore \frac{dT}{dt} = C (T - T_0) \text{ --- (7.39)}$$

where  $C$  is constant of proportionality.

**Example 7.22:** A metal sphere cools at the rate of 1.6 °C/min when its temperature is 70°C. At what rate will it cool when its temperature is 40°C. The temperature of surroundings is 30°C.

**Solution:** Given  $T_1 = 70^\circ \text{C}$   
 $T_2 = 40^\circ \text{C}$   
 $T_0 = 30^\circ \text{C}$

$$\left( \frac{dT}{dt} \right)_1 = 1.6^\circ \text{C/min}$$

According to Newton's law of cooling, if  $C$  is the constant of proportionality

$$\left( \frac{dT}{dt} \right)_1 = C (T_1 - T_0)$$

$$\text{or, } 1.6 = C (70 - 30)$$

$$\therefore C = \frac{1.6}{40} = 0.04 / \text{min}$$

$$\text{Also } \left( \frac{dT}{dt} \right)_2 = C (T_2 - T_0)$$

$$= 0.04(40 - 30) = 0.4^\circ \text{C/min}$$

Thus the rate of cooling drops by a factor of four when the difference in temperature of the metal sphere and its surroundings drops by a factor of four.



#### Internet my friend

1. <https://hyperphysics.phy-astr.gsu.edu/hbase/hframe.html>
2. <https://youtu.be/7ZKHc5J6R5Q>
3. <https://physics.info/expansion>



## Exercises

### 1. Choose the correct option.

- i) Range of temperature in a clinical thermometer, which measures the temperature of human body, is  
(A) 70 °C to 100 °C  
(B) 34 °C to 42 °C  
(C) 0 °F to 100 °F  
(D) 34 °F to 80 °F
- ii) A glass bottle completely filled with water is kept in the freezer. Why does it crack?  
(A) Bottle gets contracted  
(B) Bottle is expanded  
(C) Water expands on freezing  
(D) Water contracts on freezing
- iii) If two temperatures differ by 25 °C on Celsius scale, the difference in temperature on Fahrenheit scale is  
(A) 65°                      (B) 45°  
(C) 38°                      (D) 25°
- iv) If  $\alpha$ ,  $\beta$  and  $\gamma$  are coefficients of linear, area  $l$  and volume expansion of a solid then  
(A)  $\alpha:\beta:\gamma$  1:3:2                      (B)  $\alpha:\beta:\gamma$  3:1:2  
(C)  $\alpha:\beta:\gamma$  2:3:1                      (D)  $\alpha:\beta:\gamma$  3:1:2
- v) Consider the following statements-  
(I) The coefficient of linear expansion has dimension  $K^{-1}$   
(II) The coefficient of volume expansion has dimension  $K^{-1}$   
(A) I and II are both correct  
(B) I is correct but II is wrong  
(C) II is correct but I is wrong  
(D) I and II are both wrong
- vi) Water falls from a height of 200 m. What is the difference in temperature between the water at the top and bottom of a water fall given that specific heat of water is  $4200 \text{ J kg}^{-1} \text{ }^\circ\text{C}^{-1}$ ?  
(A) 0.96°C                      (B) 1.02°C  
(C) 0.46°C                      (D) 1.16°C

### 2. Answer the following questions.

- i) Clearly state the difference between heat and temperature?
- ii) How a thermometer is calibrated?
- iii) What are different scales of temperature? What is the relation between them?

- iv) What is absolute zero?
- v) Derive the relation between three coefficients of thermal expansion.
- vi) State applications of thermal expansion.
- vii) Why do we generally consider two specific heats for a gas?
- viii) Are freezing point and melting point same with respect to change of state? Comment.
- ix) Define (i) Sublimation (ii) Triple point.
- x) Explain the term 'steady state'.
- xi) Define coefficient of thermal conductivity. Derive its expression.
- xii) Give any four applications of thermal conductivity in every day life.
- xiii) Explain the term thermal resistance. State its SI unit and dimensions.
- xiv) How heat transfer occurs through radiation in absence of a medium?
- xv) State Newton's law of cooling and explain how it can be experimentally verified.
- xvi) What is thermal stress? Give an example of disadvantages of thermal stress in practical use?
- xvii) Which materials can be used as thermal insulators and why?

### 3. Solve the following problems.

- i) A glass flask has volume  $1 \times 10^{-4} \text{ m}^3$ . It is filled with a liquid at 30 °C. If the temperature of the system is raised to 100 °C, how much of the liquid will overflow. (Coefficient of volume expansion of glass is  $1.2 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1}$  while that of the liquid is  $75 \times 10^{-5} \text{ (}^\circ\text{C)}^{-1}$ ).

[Ans :  $516.6 \times 10^{-8} \text{ m}^3$ ]

- ii) Which will require more energy, heating a 2.0 kg block of lead by 30 K or heating a 4.0 kg block of copper by 5 K? ( $s_{\text{lead}} = 128 \text{ J kg}^{-1} \text{ K}^{-1}$ ,  $s_{\text{copper}} = 387 \text{ J kg}^{-1} \text{ K}^{-1}$ )  
[Ans : copper]

- iii) Specific latent heat of vaporization of water is  $2.26 \times 10^6 \text{ J/kg}$ . Calculate the energy needed to change 5.0 g of water

- into steam at 100 °C.  
[Ans :  $11.3 \times 10^3$  J]
- iv) A metal sphere cools at the rate of 0.05 °C/s when its temperature is 70 °C and at the rate of 0.025 °C/s when its temperature is 50 °C. Determine the temperature of the surroundings and find the rate of cooling when the temperature of the metal sphere is 40 °C.  
[Ans : 30 °C, 0.0125 °C/s]
- v) The volume of a gas varied linearly with absolute temperature if its pressure is held constant. Suppose the gas does not liquefy even at very low temperatures, at what temperature the volume of the gas will be ideally zero?  
[Ans : -273.15 °C]
- vi) In olden days, while laying the rails for trains, small gaps used to be left between the rail sections to allow for thermal expansion. Suppose the rails are laid at room temperature 27 °C. If maximum temperature in the region is 45 °C and the length of each rail section is 10 m, what should be the gap left given that  $\alpha = 1.2 \times 10^{-5} \text{ K}^{-1}$  for the material of the rail section?  
[Ans : 2.16 mm]
- vii) A blacksmith fixes iron ring on the rim of the wooden wheel of a bullock cart. The diameter of the wooden rim and the iron ring are 1.5 m and 1.47 m respectively at room temperature of 27 °C. To what temperature the iron ring should be heated so that it can fit the rim of the wheel ( $\alpha_{\text{iron}} = 1.2 \times 10^{-5} \text{ K}^{-1}$ ).  
[Ans: 1727.7 °C]
- viii) In a random temperature scale X, water boils at 200 °X and freezes at 20 °X. Find the boiling point of a liquid in this scale if it boils at 62 °C.  
[Ans: 131.6°X]
- ix) A gas at 900°C is cooled until both its pressure and volume are halved. Calculate its final temperature.  
[Ans: 293.29K]
- x) An aluminium rod and iron rod show 1.5 m difference in their lengths when heated at all temperature. What are their lengths at 0 °C if coefficient of linear expansion for aluminium is  $24.5 \times 10^{-6} / ^\circ\text{C}$  and for iron is  $11.9 \times 10^{-6} / ^\circ\text{C}$   
[Ans: 1.417m, 2.917m]
- xi) What is the specific heat of a metal if 50 cal of heat is needed to raise 6 kg of the metal from 20°C to 62 °C ?  
[Ans:  $s = 0.198 \text{ cal/kg } ^\circ\text{C}$ ]
- xii) The rate of flow of heat through a copper rod with temperature difference 30 °C is 1500 cal/s. Find the thermal resistance of copper rod.  
[Ans 0.02 °C/scal]
- xiii) In electric kettle takes 20 minutes to heat a certain quantity of water from 0°C to boiling point. It requires 90 minutes to turn all the water at 100°C into steam. Find the latent heat of vaporisation. (Specific heat of water = 1cal/g°C)  
[Ans: 450 cal/g]
- xiv) Find the temperature difference between two sides of a steel plate 4 cm thick, when heat is transmitted through the plate at the rate of 400 k cal per minute per square metre at steady state. Thermal conductivity of steel is 0.026 kcal/m s K.  
[Ans: 10.26°C]
- xv) A metal sphere cools from 80 °C to 60 °C in 6 min. How much time will it take to cool from 60 °C to 40 °C if the room temperature is 30°C?  
[Ans: 10 min]
- \*\*\*