2 Pythagoras Theorem

Let's study.

- Pythagorean triplet
- Theorem of geometric mean
- Application of Pythagoras theorem
- Similarity and right angled triangles
- Pythagoras theorem
- Apollonius theorem

Let's recall.

Pythagoras theorem :

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaning two sides.

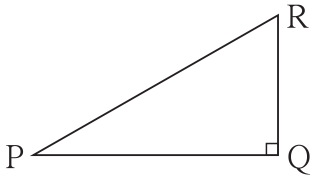

Fig. 2.1

In Δ PQR $\angle$ PQR = 90°

$l(PR)^2 = l(PQ)^2 + l(QR)^2$

We will write this as,

$PR^2 = PQ^2 + QR^2$

The lengths PQ, QR and PR of Δ PQR can also be shown by letters r, p and q. With this convention, refering to figure 2.1, Pythagoras theorem can also be stated as $q^2 = p^2 + r^2$.

Pythagorean Triplet :

In a triplet of natural numbers, if the square of the largest number is equal to the sum of the squares of the remaining two numbers then the triplet is called Pythagorean triplet.

For Example: In the triplet (11, 60, 61) ,

$11^2 = 121$, $\quad 60^2 = 3600$, $\quad 61^2 = 3721$ $\quad$ and $\quad 121 + 3600 = 3721$

The square of the largest number is equal to the sum of the squares of the other two numbers.

$\therefore$ 11, 60, 61 is a Pythagorean triplet.

Verify that (3, 4, 5), (5, 12, 13), (8, 15, 17), (24, 25, 7) are Pythagorean triplets.

Numbers in Pythagorean triplet can be written in any order.

For more information

Formula for Pythagorean triplet:

If a, b, c are natural numbers and $a > b$, then $[(a^2 + b^2),(a^2 - b^2),(2ab)]$ is Pythagorean triplet.

$\because\ (a^2 + b^2)^2 = a^4 + 2a^2b^2 + b^4$ .... (I)

$(a^2 - b^2) = a^4 - 2a^2b^2 + b^4$ .... (II)

$(2ab)^2 = 4a^2b^2$ .... (III)

$\therefore$ by (I), (II) and (III) , $(a^2 + b^2)^2 = (a^2 - b^2)^2 + (2ab)^2$

$\therefore [(a^2 + b^2), (a^2 - b^2), (2ab)]$ is Pythagorean Triplet.

This formula can be used to get various Pythagorean triplets.

For example, if we take $a = 5$ and $b = 3$,

$a^2 + b^2 = 34$, $a^2 - b^2 = 16$, $2ab = 30$.

Check that (34, 16, 30) is a Pythagorean triplet.

Assign different values to a and b and obtain 5 Pythagorean triplet.

Last year we have studied the properties of right angled triangle with the angles $30° - 60° - 90°$ and $45° - 45° - 90°$.

(I) Property of $30°-60°-90°$ triangle.

If acute angles of a right angled triangle are $30°$ and $60°$, then the side opposite $30°$angle is half of the hypotenuse and the side opposite to $60°$ angle is $\frac{\sqrt{3}}{2}$ times the hypotenuse.

See figure 2.2. In Δ LMN, $\angle L = 30°$, $\angle N = 60°$, $\angle M = 90°$

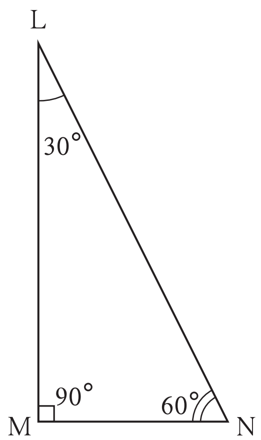

Fig. 2.2

$\therefore$ side opposite $30°$angle = MN = $\frac{1}{2} \times$ LN

side opposite $60°$angle = LM = $\frac{\sqrt{3}}{2} \times$ LN

If LN = 6 cm, we will find MN and LM.

$MN = \frac{1}{2} \times LN$

$= \frac{1}{2} \times 6$

$= 3$ cm

$LM = \frac{\sqrt{3}}{2} \times LN$

$= \frac{\sqrt{3}}{2} \times 6$

$= 3\sqrt{3}$ cm

(II) Property of $45^\circ - 45^\circ - 90^\circ$

If the acute angles of a right angled triangle are 45° and 45°, then each of the perpendicular sides is $\frac{1}{\sqrt{2}}$ times the hypotenuse.

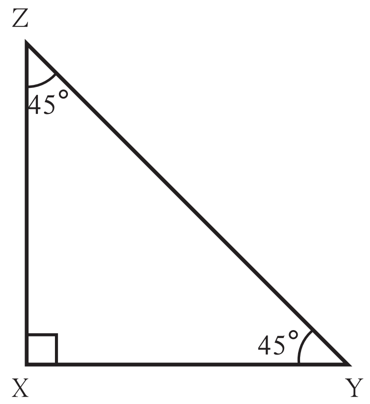

Fig. 2.3

See Figure 2.3. In Δ XYZ,

$$XY = \frac{1}{\sqrt{2}} \times ZY$$

$$XZ = \frac{1}{\sqrt{2}} \times ZY$$

$$\therefore \quad XY = XZ = \frac{1}{\sqrt{2}} \times ZY$$

If $ZY = 3\sqrt{2}$ cm then we will find XY and ZX

$$XY = XZ = \frac{1}{\sqrt{2}} \times 3\sqrt{2}$$

$$XY = XZ = 3\text{cm}$$

In 7th standard we have studied theorem of Pythagoras using areas of four right angled triangles and a square. We can prove the theorem by an alternative method.

Activity:

Take two congruent right angled triangles. Take another isosceles right angled triangle whose congruent sides are equal to the hypotenuse of the two congruent right angled triangles. Join these triangles to form a trapezium

Area of the trapezium = $\frac{1}{2}$ × (sum of the lengths of parallel sides) × height

Using this formula, equating the area of trapezium with the sum of areas of the three right angled triangles we can prove the theorem of Pythagoras.

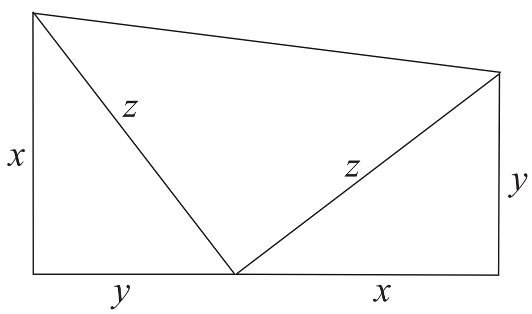

Fig. 2.4

Let's learn.

Now we will give the proof of Pythagoras theorem based on properties of similar triangles. For this, we will study right angled similar triangles.

Similarity and right angled triangle

Theorem : In a right angled triangle, if the altitude is drawn to the hypotenuse, then the two triangles formed are similar to the original triangle and to each other.

Given : In Δ ABC, $\angle$ ABC = 90°,
seg BD $\perp$ seg AC, A-D-C

To prove: Δ ADB ~ Δ ABC
Δ BDC ~ Δ ABC
Δ ADB ~ Δ BDC

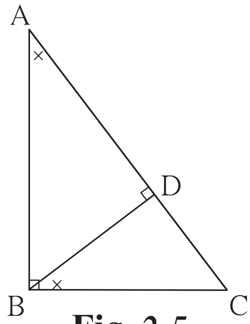

Fig. 2.5

Proof : In Δ ADB and Δ ABC
$\angle$ DAB $\cong$ $\angle$ BAC ...(common angle)
$\angle$ ADB $\cong$ $\angle$ ABC ... (each 90°)
Δ ADB ~ Δ ABC ... (AA test)... (I)

In Δ BDC and Δ ABC
$\angle$ BCD $\cong$ $\angle$ ACB .....(common angle)
$\angle$ BDC $\cong$ $\angle$ ABC ..... (each 90°)
Δ BDC ~ Δ ABC ..... (AA test) ... (II)

$\therefore$ Δ ADB ~ Δ BDC from (I) and (II) ....(III)

$\therefore$ from (I), (II) and (III), Δ ADB ~ Δ BDC ~ Δ ABC ....(transitivity)

Theorem of geometric mean

In a right angled triangle, the perpendicular segment to the hypotenuse from the opposite vertex, is the geometric mean of the segments into which the hypotenuse is divided.

Proof : In right angled triangle PQR, seg QS $\perp$ hypotenuse PR

Δ QSR ~ Δ PSQ .... (similarity of right triangles)

$$\frac{QS}{PS} = \frac{SR}{SQ}$$

$$\frac{QS}{PS} = \frac{SR}{QS}$$

QS^2 = PS × SR

$\therefore$ seg QS is the 'geometric mean' of seg PS and SR.

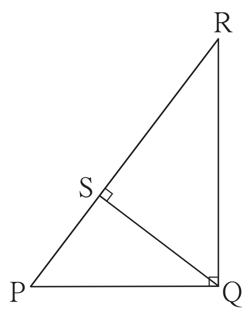

Fig. 2.6

Pythagoras Theorem

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

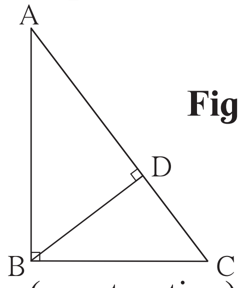

Fig. 2.7

Given : In Δ ABC, $\angle$ABC = 90°

To prove : $AC^2 = AB^2 + BC^2$

Construction : Draw perpendicular seg BD on side AC.

A-D-C.

Proof : In right angled Δ ABC, seg BD $\perp$ hypotenuse AC ..... (construction)

$\therefore \Delta ABC \sim \Delta ADB \sim \Delta BDC$ ..... (similarity of right angled triangles)

$\Delta ABC \sim \Delta ADB$

$$\frac{AB}{AD} = \frac{BC}{DB} = \frac{AC}{AB}$$ - corresponding sides

$$\frac{AB}{AD} = \frac{AC}{AB}$$

$AB^2 = AD \times AC$ .... (I)

Similarly, $\Delta ABC \sim \Delta BDC$

$$\frac{AB}{BD} = \frac{BC}{DC} = \frac{AC}{BC}$$ -corresponding sides

$$\frac{BC}{DC} = \frac{AC}{BC}$$

$BC^2 = DC \times AC$ .... (II)

Adding (I) and (II)

$$AB^2 + BC^2 = AD \times AC + DC \times AC$$

$$= AC\,(AD + DC)$$

$$= AC \times AC \text{ .... (A-D-C)}$$

$\therefore AB^2 + BC^2 = AC^2$

$\therefore AC^2 = AB^2 + BC^2$

Converse of Pythagoras theorem

In a triangle if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle.

Given : In Δ ABC, $AC^2 = AB^2 + BC^2$

To prove : $\angle$ ABC = 90°

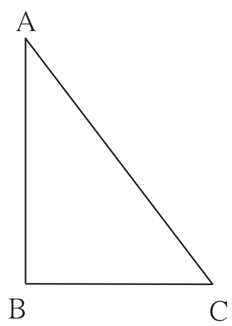

Fig. 2.8

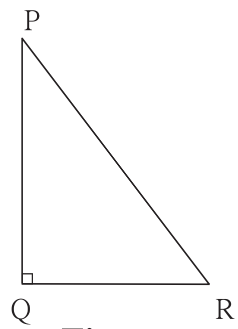

Fig. 2.9

Construction : Draw Δ PQR such that, AB = PQ, BC = QR , $\angle$ PQR = 90°.

Proof : In Δ PQR, $\angle$ Q = 90°

$PR^2 = PQ^2 + QR^2$ .... (Pythagoras theorem)

$= AB^2 + BC^2$ .... (construction) ......(I)

$= AC^2$ .... (given) ......(II)

$\therefore PR^2 = AC^2$

$\therefore$ PR = AC .... (III)

$\therefore \Delta ABC \cong \Delta PQR$ .... (SSS test)

$\therefore \angle ABC = \angle PQR = 90°$

Remember this!

(1) (a) Similarity and right angled triangle

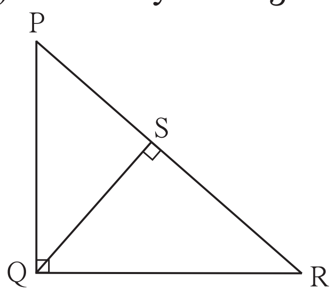

Fig. 2.10

In ΔPQR $\angle$ Q = 90° , seg QS $\perp$ seg PR, $\Delta PQR \sim \Delta PSQ \sim \Delta QSR$. Thus all the right angled triangles in the figure are similar to one another.

(b) Theorem of geometric mean

In the above figure, $\Delta PSQ \sim \Delta QSR$

$\therefore QS^2 = PS \times SR$

$\therefore$ seg QS is the geometric mean of seg PS and seg SR

(2) Pythagoras Theorem:

In a right angled triangle, the square of the hypotenuse is equal to the sum of the squares of remaining two sides.

(3) Converse of Pythagoras Theorem:

In a triangle, if the square of one side is equal to the sum of the squares of the remaining two sides, then the triangle is a right angled triangle

(4) Let us remember one more very useful property.

In a right angled triangle, if one side is half of the hypotenuse then the angle opposite to that side is 30°.

This property is the converse of 30°-60°-90° theorem.

Ex. (1) See fig 2.11. In Δ ABC, $\angle$ B= 90°, $\angle$ A= 30°, AC=14, then find AB and BC

Solution :

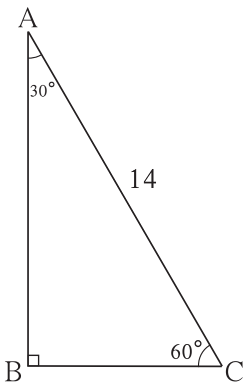

Fig. 2.11

In Δ ABC,

$\angle B = 90°$, $\angle A = 30°$, $\therefore$ $\angle C = 60°$

By 30°– 60°– 90° theorem,

$BC = \frac{1}{2} \times AC$

$BC = \frac{1}{2} \times 14$

$BC = 7$

$AB = \frac{\sqrt{3}}{2} \times AC$

$AB = \frac{\sqrt{3}}{2} \times 14$

$AB = 7\sqrt{3}$

Ex. (2) See fig 2.12, In Δ ABC, seg AD $\perp$ seg BC, $\angle C = 45°$, BD = 5 and AC = $8\sqrt{2}$ then find AD and BC.

Solution : In Δ ADC

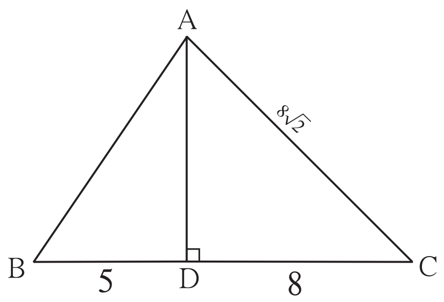

Fig. 2.12

$\angle ADC = 90°$, $\angle C = 45°$, $\therefore$ $\angle DAC = 45°$

$AD = DC = \frac{1}{\sqrt{2}} \times 8\sqrt{2}$ ... by 45°–45°–90° theorem

$DC = 8$ $\therefore$ $AD = 8$

$BC = BD + DC$

$= 5 + 8$

$BC = 13$

Ex. (3) In fig 2.13, $\angle$ PQR = 90°,seg QN $\perp$ seg PR, PN = 9, NR = 16. Find QN.

Solution : In Δ PQR, seg QN $\perp$ seg PR

$NQ^2 = PN \times NR$ ... theorem of geometric mean

$\therefore NQ = \sqrt{PN \times NR}$

$= \sqrt{9 \times 16}$

$= 3 \times 4$

$= 12$

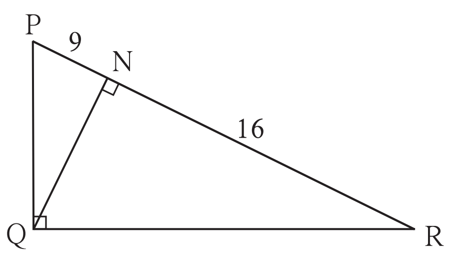

Fig. 2.13

Ex. (4) See figure 2.14.In Δ PQR, $\angle$ PQR = 90°, seg QS $\perp$ seg PR then find x, y, z.

Solution : In Δ PQR, $\angle$ PQR = 90°, seg QS $\perp$ seg PR

$$QS = \sqrt{PS \times SR} \quad \text{.......... (theorem of geometric mean)}$$

$$= \sqrt{10 \times 8}$$

$$= \sqrt{5 \times 2 \times 8}$$

$$= \sqrt{5 \times 16}$$

$$= 4\sqrt{5}$$

$$\therefore \ x = 4\sqrt{5}$$

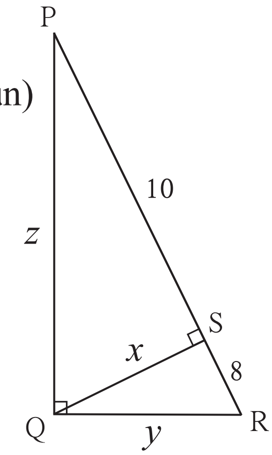

Fig. 2.14

In Δ QSR, by Pythagoras theorem

$$QR^2 = QS^2 + SR^2$$

$$= \left(4\sqrt{5}\right)^2 + 8^2$$

$$= 16 \times 5 + 64$$

$$= 80 + 64$$

$$= 144$$

$$\therefore QR = 12$$

In Δ PSQ, by Pythagoras theorem

$$PQ^2 = QS^2 + PS^2$$

$$= \left(4\sqrt{5}\right)^2 + 10^2$$

$$= 16 \times 5 + 100$$

$$= 80 + 100$$

$$= 180$$

$$= 36 \times 5$$

$$\therefore \ PQ = 6\sqrt{5}$$

Hence $x = 4\sqrt{5}$, $y = 12$, $z = 6\sqrt{5}$

Ex. (5) In the right angled triangle, sides making right angle are 9 cm and 12 cm. Find the length of the hypotenuse

Solution: In Δ PQR ,$\angle$ Q = 90°

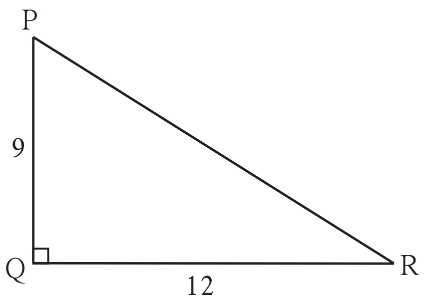

Fig. 2.15

$$PR^2 = PQ^2 + QR^2 \text{ (Pythagoras theorem)}$$

$$= 9^2 + 12^2$$

$$= 81 + 144$$

$$PR^2 = 225$$

$$PR = 15$$

Hypotenuse= 15 cm

Ex. (6) In Δ LMN , l = 5, m = 13, n = 12. State whether ΔLMN is a right angled triangle or not.

Solution : $l = 5, \; m = 13, \quad n = 12$

$l^2 = 25, \quad m^2 = 169, \quad n^2 = 144$

$\therefore \; m^2 = l^2 + n^2$

$\therefore$ by converse of Pythagoras theorem Δ LMN is a right angled triangle.

Ex. (7) See fig 2.16. In Δ ABC, seg AD $\perp$ seg BC. Prove that:

$AB^2 + CD^2 = BD^2 + AC^2$

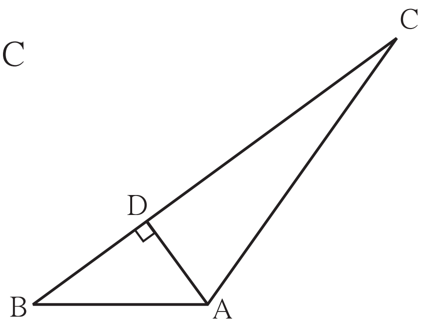

Fig. 2.16

Solution : According to Pythagoras theorem, in Δ ADC

$AC^2 = AD^2 + CD^2$

$\therefore \; AD^2 = AC^2 - CD^2 \; \ldots \text{(I)}$

In Δ ADB

$AB^2 = AD^2 + BD^2$

$\therefore \; AD^2 = AB^2 - BD^2 \; \ldots \text{(II)}$

$\therefore \; AB^2 - BD^2 = AC^2 - CD^2$ ....from I and II

$\therefore \; AB^2 + CD^2 = AC^2 + BD^2$

Practice set 2.1

1. Identify, with reason, which of the following are Pythagorean triplets.

(i)(3, 5, 4) (ii)(4, 9, 12) (iii)(5, 12, 13)

(iv) (24, 70, 74) (v)(10, 24, 27) (vi)(11, 60, 61)

2. In figure 2.17, $\angle$ MNP = 90°, seg NQ $\perp$seg MP, MQ = 9, QP = 4, find NQ.

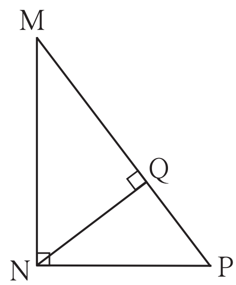

Fig. 2.17

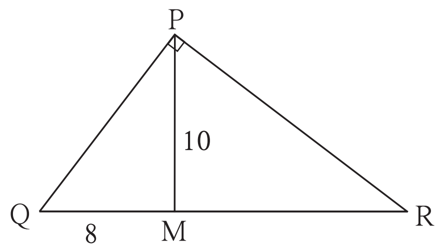

Fig. 2.18

3. In figure 2.18, $\angle$ QPR = 90°, seg PM $\perp$ seg QR and Q-M-R, PM = 10, QM = 8, find QR.

4. See figure 2.19. Find RP and PS using the information given in Δ PSR.

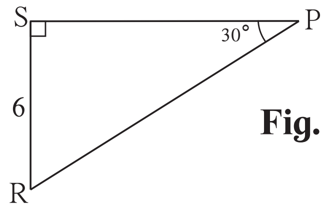

Fig. 2.19

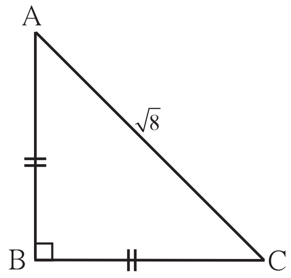

Fig. 2.20

5. For finding AB and BC with the help of information given in figure 2.20, complete following activity.

AB = BC .... $\boxed{}$

$\therefore \quad \angle$ BAC = $\boxed{}$

$\therefore$ AB = BC = $\boxed{} \times$ AC

= $\boxed{} \times \sqrt{8}$

= $\boxed{} \times 2\sqrt{2}$

= $\boxed{}$

6. Find the side and perimeter of a square whose diagonal is 10 cm.

7. In figure 2.21, $\angle$ DFE = 90°, FG $\perp$ ED, If GD = 8, FG = 12, find (1) EG (2) FD and (3) EF

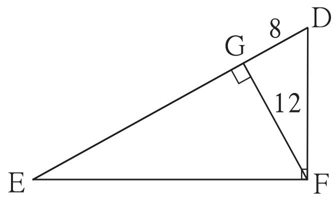

Fig. 2.21

8. Find the diagonal of a rectangle whose length is 35 cm and breadth is 12 cm.

9★. In the figure 2.22, M is the midpoint of QR. $\angle$ PRQ = 90°. Prove that, $PQ^2 = 4PM^2 - 3PR^2$

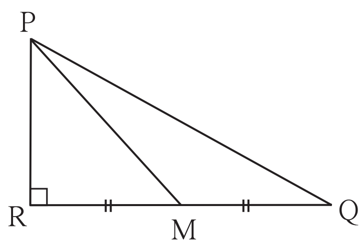

Fig. 2.22

10★. Walls of two buildings on either side of a street are parellel to each other. A ladder 5.8 m long is placed on the street such that its top just reaches the window of a building at the height of 4 m. On turning the ladder over to the other side of the street , its top touches the window of the other building at a height 4.2 m. Find the width of the street.

Let's learn.

Application of Pythagoras theorem

In Pythagoras theorem, the relation between hypotenuse and sides making right angle i.e. the relation between side opposite to right angle and the remaining two sides is given.

In a triangle, relation between the side opposite to acute angle and remaining two sides and relation of the side opposite to obtuse angle with remaining two sides can be determined with the help of Pythagoras theorem. Study these relations from the following examples.

Ex. (1) In Δ ABC, $\angle$ C is an acute angle, seg AD $\perp$ seg BC. Prove that:

$AB^2 = BC^2 + AC^2 - 2BC \times DC$

In the given figure let AB = c, AC = b, AD = p, BC = a, DC = x,

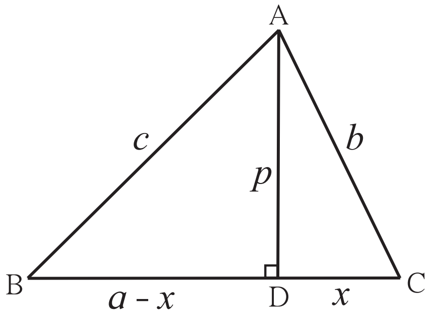

Fig. 2.23

$\therefore$ BD = $a - x$

In Δ ADB , by Pythagoras theorem

$c^2 = (a-x)^2 + \boxed{\quad}$

$c^2 = a^2 - 2ax + x^2 + \boxed{\quad}$ .... (I)

In Δ ADC, by Pythagoras theorem

$b^2 = p^2 + \boxed{\quad}$

$p^2 = b^2 - \boxed{\quad}$ .... (II)

Substituting value of p^2 from (II) in (I),

$c^2 = a^2 - 2ax + x^2 + b^2 - x^2$

$\therefore c^2 = a^2 + b^2 - 2ax$

$\therefore AB^2 = BC^2 + AC^2 - 2BC \times DC$

Ex. (2) In Δ ABC, $\angle$ ACB is obtuse angle, seg AD $\perp$ seg BC. Prove that:

$AB^2 = BC^2 + AC^2 + 2BC \times CD$

In the figure seg AD $\perp$ seg BC

Let AD = p, AC = b, AB = c,

BC = a and DC = x.

DB = $a + x$

In Δ ADB, by Pythagoras theorem,

$c^2 = (a + x)^2 + p^2$

$c^2 = a^2 + 2ax + x^2 + p^2$ .... (I)

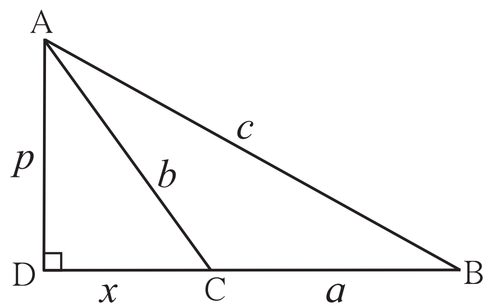

Fig. 2.24

Similarly, in Δ ADC

$$b^2 = x^2 + p^2$$

$\therefore \quad p^2 = b^2 - x^2$ .... (II)

$\therefore$ substituting the value of p^2 from (II) in (I)

$\therefore \quad c^2 = a^2 + 2ax + b^2$

$\therefore$ $AB^2 = BC^2 + AC^2 + 2BC \times CD$

Apollonius theorem

In Δ ABC, if M is the midpoint of side BC, then $AB^2 + AC^2 = 2AM^2 + 2BM^2$

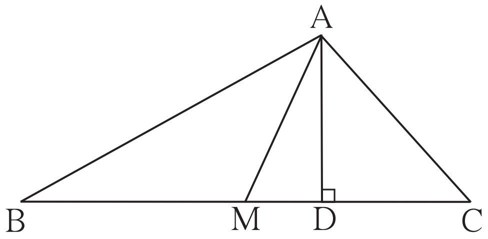

Fig. 2.25

Given : In Δ ABC, M is the midpoint of side BC.

To prove : $AB^2 + AC^2 = 2AM^2 + 2BM^2$

Construction: Draw seg AD $\perp$ seg BC

Proof : If seg AM is not perpendicular to seg BC then out of $\angle$AMB and $\angle$AMC one is obtuse angle and the other is acute angle

In the figure, $\angle$AMB is obtuse angle and $\angle$AMC is acute angle.

From examples (1) and (2) above,

$AB^2 = AM^2 + MB^2 + 2BM \times MD$ ..... (I)

and $AC^2 = AM^2 + MC^2 - 2MC \times MD$

$\therefore$ $AC^2 = AM^2 + MB^2 - 2BM \times MD$ ($\because$ BM = MC) ....(II)

$\therefore$ adding (I) and (II)

$AB^2 + AC^2 = 2AM^2 + 2BM^2$

Write the proof yourself if seg AM $\perp$ seg BC.

From this example we can see the relation among the sides and medians of a triangle.

This is known as Apollonius theorem.

Solved Examples

Ex. (1) In the figure 2.26, seg PM is a median of ΔPQR. PM = 9 and $PQ^2 + PR^2 = 290$, then find QR.

Solution : In Δ PQR , seg PM is a median.

M is the midpoint of seg QR .

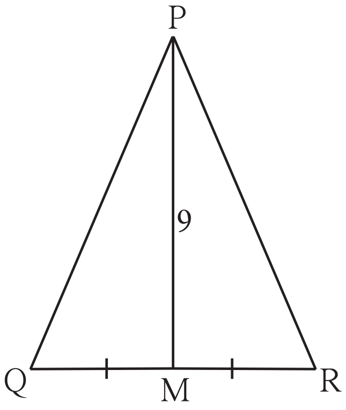

Fig. 2.26

$$QM = MR = \frac{1}{2}QR$$

$PQ^2 + PR^2 = 2PM^2 + 2QM^2$ (by Apollonius theorem)

$$290 = 2 \times 9^2 + 2QM^2$$
$$290 = 2 \times 81 + 2QM^2$$
$$290 = 162 + 2QM^2$$
$$2QM^2 = 290 - 162$$
$$2QM^2 = 128$$
$$QM^2 = 64$$
$$QM = 8$$
$$\therefore \quad QR = 2 \times QM$$
$$= 2 \times 8$$
$$= 16$$

Ex. (2) Prove that, the sum of the squares of the diagonals of a rhombus is equal to the sum of the squares of the sides.

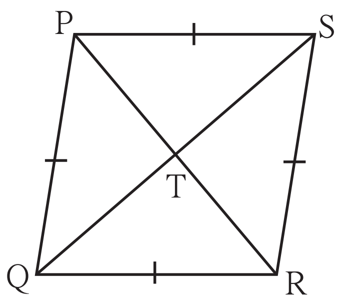

Fig. 2.27

Given : $\square$PQRS is a rhombus. Diagonals PR and SQ intersect each other at point T

To prove : $PS^2 + SR^2 + QR^2 + PQ^2 = PR^2 + QS^2$

Proof : Diagonals of a rhombus bisect each other .

$\therefore$ by Apollonius' theorem,

$PQ^2 + PS^2 = 2PT^2 + 2QT^2$ .... (I)

$QR^2 + SR^2 = 2RT^2 + 2QT^2$ .... (II)

$\therefore$ adding (I) and (II) ,

$PQ^2 + PS^2 + QR^2 + SR^2 = 2(PT^2 + RT^2) + 4QT^2$

$= 2(PT^2 + PT^2) + 4QT^2$ .... (RT = PT)

$= 4PT^2 + 4QT^2$

$= (2PT)^2 + (2QT)^2$

$= PR^2 + QS^2$

(The above proof can be written using Pythagoras theorem also.)

Practice set 2.2

1. In Δ PQR, point S is the midpoint of side QR.If PQ = 11,PR = 17, PS =13, find QR.

2. In Δ ABC, AB = 10, AC = 7, BC = 9 then find the length of the median drawn from point C to side AB

3. In the figure 2.28 seg PS is the median of Δ PQR and PT $\perp$ QR. Prove that,

(1) $PR^2 = PS^2 + QR \times ST + \left(\frac{QR}{2}\right)^2$

ii) $PQ^2 = PS^2 - QR \times ST + \left(\frac{QR}{2}\right)^2$

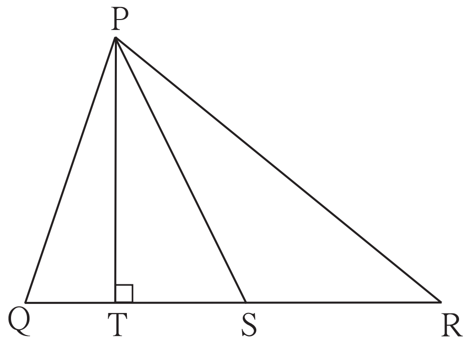

Fig. 2.28

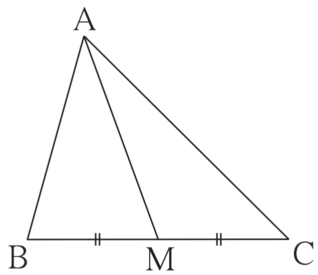

Fig. 2.29

4. In Δ ABC, point M is the midpoint of side BC.

If, $AB^2 + AC^2 = 290\text{ cm}^2$,

AM = 8 cm, find BC.

5★. In figure 2.30, point T is in the interior of rectangle PQRS,

Prove that, $TS^2 + TQ^2 = TP^2 + TR^2$

(As shown in the figure, draw seg AB || side SR and A-T-B)

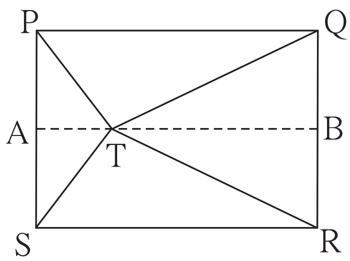

Fig. 2.30

Problem set 2

1. Some questions and their alternative answers are given. Select the correct alternative.

(1) Out of the following which is the Pythagorean triplet?

(A) (1, 5, 10) (B) (3, 4, 5) (C) (2, 2, 2) (D) (5, 5, 2)

(2) In a right angled triangle, if sum of the squares of the sides making right angle is 169 then what is the length of the hypotenuse?

(A) 15 (B) 13 (C) 5 (D) 12

(3) Out of the dates given below which date constitutes a Pythagorean triplet ?

(A) 15/08/17 (B) 16/08/16 (C) 3/5/17 (D) 4/9/15

(4) If a, b, c are sides of a triangle and $a^2 + b^2 = c^2$, name the type of triangle.

(A) Obtuse angled triangle (B) Acute angled triangle

(C) Right angled triangle (D) Equilateral triangle

(5) Find perimeter of a square if its diagonal is $10\sqrt{2}$ cm.

(A)10 cm (B) $40\sqrt{2}$ cm (C) 20 cm (D) 40 cm

(6) Altitude on the hypotenuse of a right angled triangle divides it in two parts of lengths 4 cm and 9 cm. Find the length of the altitude.

(A) 9 cm (B) 4 cm (C) 6 cm (D) $2\sqrt{6}$ cm

(7) Height and base of a right angled triangle are 24 cm and 18 cm find the length of its hypotenus

(A) 24 cm (B) 30 cm (C) 15 cm (D) 18 cm

(8) In Δ ABC, AB = $6\sqrt{3}$ cm, AC = 12 cm, BC = 6 cm. Find measure of $\angle$A.

(A) 30° (B) 60° (C) 90° (D) 45°

2. Solve the following examples.

(1) Find the height of an equilateral triangle having side $2a$.

(2) Do sides 7 cm , 24 cm, 25 cm form a right angled triangle ? Give reason.

(3) Find the length a diagonal of a rectangle having sides 11 cm and 60cm.

(4) Find the length of the hypotenuse of a right angled triangle if remaining sides are 9 cm and 12 cm.

(5) A side of an isosceles right angled triangle is x. Find its hypotenuse.

(6) In Δ PQR; PQ = $\sqrt{8}$, QR = $\sqrt{5}$, PR = $\sqrt{3}$. Is Δ PQR a right angled triangle ? If yes, which angle is of 90° ?

3. In Δ RST, $\angle$ S = 90°, $\angle$ T = 30°, RT = 12 cm then find RS and ST.

4. Find the diagonal of a rectangle whose length is 16 cm and area is 192 sq.cm.

5★. Find the length of the side and perimeter of an equilateral triangle whose height is $\sqrt{3}$ cm.

6. In Δ ABC seg AP is a median. If BC = 18, $AB^2 + AC^2 = 260$ Find AP.

7★. Δ ABC is an equilateral triangle. Point P is on base BC such that PC = $\frac{1}{3}$ BC, if AB = 6 cm find AP.

8. From the information given in the figure 2.31, prove that PM = PN = $\sqrt{3} \times a$

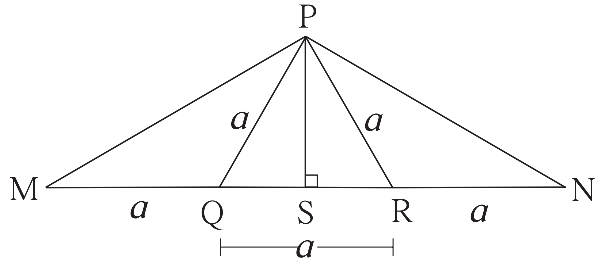

Fig. 2.31

9. Prove that the sum of the squares of the diagonals of a parallelogram is equal to the sum of the squares of its sides.

10. Pranali and Prasad started walking to the East and to the North respectively, from the same point and at the same speed. After 2 hours distance between them was $15\sqrt{2}$ km. Find their speed per hour.

11★. In Δ ABC, $\angle$ BAC = 90°, seg BL and seg CM are medians of Δ ABC. Then prove that: $4(BL^2 + CM^2) = 5\, BC^2$

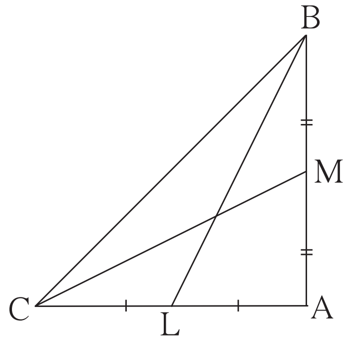

Fig. 2.32

12. Sum of the squares of adjacent sides of a parallelogram is 130 sq.cm and length of one of its diagonals is14 cm. Find the length of the other diagonal.

13. In Δ ABC, seg AD $\perp$ seg BC DB = 3CD. Prove that : $2AB^2 = 2AC^2 + BC^2$

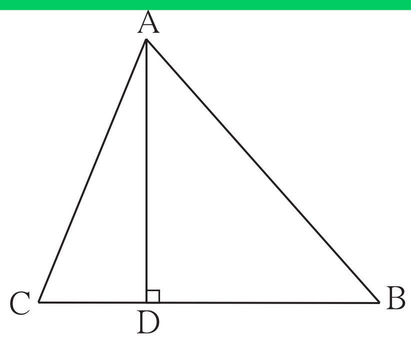

Fig. 2.33

14★. In an isosceles triangle, length of the congruent sides is 13 cm and its base is 10 cm. Find the distance between the vertex opposite the base and the centroid.

15. In a trapezium ABCD,

seg AB || seg DC

seg BD $\perp$ seg AD,

seg AC $\perp$ seg BC,

If AD = 15, BC = 15

and AB = 25. Find A($\square$ ABCD)

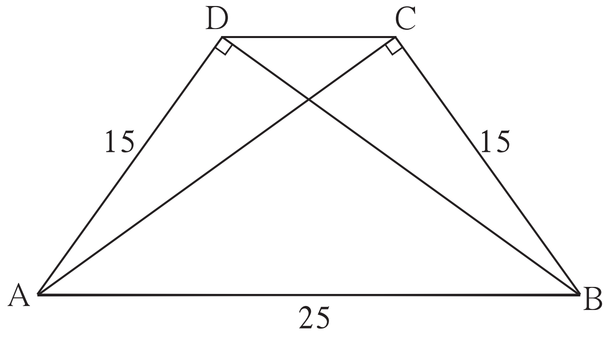

Fig. 2.34

16★. In the figure 2.35, Δ PQR is an equilatral triangle. Point S is on seg QR such that

QS = $\frac{1}{3}$ QR.

Prove that : 9 PS^2 = 7 PQ^2

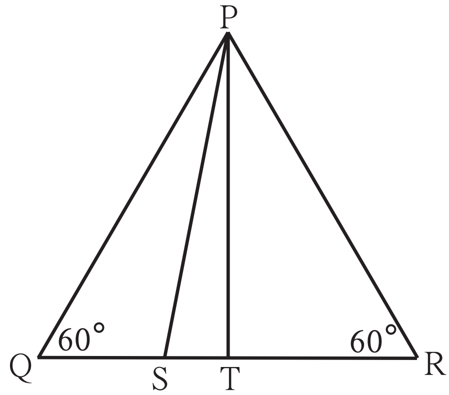

Fig. 2.35

17★. Seg PM is a median of Δ PQR. If PQ = 40, PR = 42 and PM = 29, find QR.

18. Seg AM is a median of Δ ABC. If AB = 22, AC = 34, BC = 24, find AM

ICT Tools or Links

Obtain information on 'the life of Pythagoras' from the internet. Prepare a slide show.

ANSWERS

Chapter 1 Similarity

Practice set 1.1

1. $\frac{3}{4}$ **2.** $\frac{1}{2}$ **3.** 3 **4.** 1:1 **5.** (1) $\frac{BQ}{BC}$, (2) $\frac{PQ}{AD}$, (3) $\frac{BC}{DC}$, (4) $\frac{DC \times AD}{QC \times PQ}$

Practice set 1.2

1. (1) is a bisector. (2) is not a bisector. (3) is a bisector.

2. $\frac{PN}{NR} = \frac{PM}{MQ} = \frac{3}{2}$, therefore line NM || side RQ **3.** QP = 3.5 **5.** BQ = 17.5

6. QP = 22.4 **7.** $x = 6$; AE = 18 **8.** LT = 4.8 **9.** $x = 10$

10. Given, XQ, PD, Given, $\frac{\boxed{XR}}{\boxed{RF}} = \frac{\boxed{XQ}}{\boxed{QE}}$, Basic propotionality theorem, $\frac{\boxed{XP}}{\boxed{PD}} = \frac{\boxed{XR}}{\boxed{RF}}$

Practice set 1.3

1. Δ ABC ~ Δ EDC, AA test **2.** Δ PQR ~ Δ LMN ; SSS test of similarity

3. 12 metre **4.** AC = 10.5 **6.** OD = 4.5

Practice set 1.4

1. Ratio of areas = 9 : 25 **2.** $\boxed{PQ^2}$, $\frac{\boxed{4}}{\boxed{9}}$ **3.** $\boxed{A(\Delta PQR)}$, $\frac{4}{5}$

4. MN = 15 **5.** 20 cm **6.** $4\sqrt{2}$

7. $\boxed{PF}$; $\boxed{x} + \boxed{2x}$; $\boxed{\angle FPQ}$; $\boxed{\angle FQP}$; $\frac{DF^2}{PF^2}$; $\boxed{20}$; $\boxed{45}$; $\boxed{45} - \boxed{20}$; $\boxed{\text{25 sq. unit}}$

Problem set 1

1. (1) (B), (2) (B), (3) (B), (4) (D), (5) (A)

2. $\frac{7}{13}$, $\frac{7}{20}$, $\frac{13}{20}$ **3.** 9 cm **4.** $\frac{3}{4}$ **5.** 11 cm **6.** $\frac{25}{81}$ **7.** 4

8. PQ = 80, QR = $\frac{280}{3}$, RS = $\frac{320}{3}$ **9.** $\frac{\boxed{PM}}{\boxed{MQ}} = \frac{\boxed{PX}}{\boxed{XQ}}$, $\frac{\boxed{PM}}{\boxed{MR}} = \frac{\boxed{PY}}{\boxed{YR}}$,

10. $\frac{AX}{XY} = \frac{3}{2}$ **12.** $\frac{\boxed{3}}{\boxed{2}}$, $\frac{\boxed{3}+\boxed{2}}{\boxed{2}}$, $\frac{\boxed{5}}{\boxed{3}}$, $\boxed{AA}$, $\frac{\boxed{5}}{\boxed{3}}$, $\boxed{15}$

Chapter 2 Pythagoras Theorem

Practice set 2.1

1. Pythagorean triplets ; (1), (3), (4), (6) **2.** NQ = 6 **3.** QR = 20.5

4. RP = 12, PS = $6\sqrt{3}$

5. $\boxed{\text{side opoosite to congruent angles}}$, $\boxed{45°}$, $\boxed{\frac{1}{\sqrt{2}}}$, $\boxed{\frac{1}{\sqrt{2}}}$, $\boxed{\frac{1}{\sqrt{2}}}$, $\boxed{2}$

6. side = $5\sqrt{2}$ cm, perimeter = $20\sqrt{2}$ cm **7.** (1) 18 (2) $4\sqrt{13}$ (3) $6\sqrt{13}$ **8.** 37 cm
10. 8.2 metre.

Practice set 2.2

1. 12 **2.** $2\sqrt{10}$ **4.** 18 cm

Problem set 2

1. (1) (B), (2) (B), (3) (A), (4) (C), (5) (D), (6) (C), (7) (B), (8) (A).
2. (1) $a\sqrt{3}$, (2) form a right angled triangle. (3) 61 cm, (4) 15 cm, (5) $x\sqrt{2}$, (6) $\angle$ PRQ.
3. RS = 6 cm, ST = $6\sqrt{3}$ cm **4.** 20 cm **5.** side = 2 cm, perimeter = 6 cm
6. 7 **7.** AP = $2\sqrt{7}$ cm **10.** 7.5 km / hr **12.** 8 cm **14.** 8 cm
15. 192 sq.unit **17.** 58 **18.** 26

Chapter 3 Circle

Practice set 3.1

1. (1) 90°, tangent-radius theorem (2) 6 cm ; perpendicular distance
(3) $6\sqrt{2}$ cm (4) 45°
2. (1) $5\sqrt{3}$ cm (2) 30° (3) 60° **4.** 9 cm

Practice set 3.2

1. 1.3 cm 2. 9.7 cm **4.** (3) 110° **5.** $4\sqrt{6}$ cm

Practice set 3.3

1. m(arc DE) = 90°, m(arc DEF) = 160°

Practice set 3.4

1. (1) 60° (2) 30° (3) 60° (4) 300° **2.** (1) 70° (2) 220° (3) 110° (4) 55°
3. $\angle$R = 92°; $\angle$N = 88° **7.** 44° **8.** 121°

Practice set 3.5

1. PS = 18; RS = 10, **2.** (1) 7.5 (2) 12 or 6
3. (1) 18 (2) 10 (3) 5 **4.** 4

Problem set 3

1. (1) D (2) B (3) B (4) C (5) B (6) D (7) A (8) B (9) A (10) C.
2. (1) 9 cm (2) in the interior of the circle (3) 2 locations, 12 cm
3. (1) 6 (2) $\angle$K = 30°; $\angle$M = 60° **5.** 10 **6.** (1) 9 cm (2) 6.5 cm