6 Trigonometry

Let's study.

- Trigonometric ratios
- Trigonometric identities
- Angle of elevation and angle of depression
- Problems based on heights and distances

Let's recall.

1. Fill in the blanks with reference to figure 6.1 .

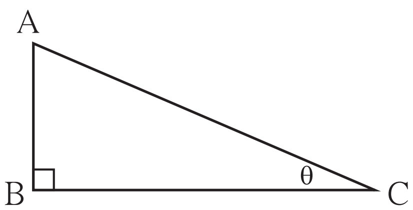


Fig. 6.1

$\sin\theta = \dfrac{\boxed{\quad}}{\boxed{\quad}}$, $\cos\theta = \dfrac{\boxed{\quad}}{\boxed{\quad}}$,

$\tan\theta = \dfrac{\boxed{\quad}}{\boxed{\quad}}$

2. Complete the relations in ratios given below .

(i) $\dfrac{\sin\theta}{\cos\theta} = \boxed{\quad}$ (ii) $\sin\theta = \cos(90 - \boxed{\quad})$

(iii) $\cos\theta = \sin(90 - \boxed{\quad})$ (iv) $\tan\theta \times \tan(90 - \theta) = \boxed{\quad}$

3. Complete the equation.

$\sin^2\theta + \cos^2\theta = \boxed{\quad}$

4. Write the values of the following trigonometric ratios.

(i) $\sin 30^\circ = \dfrac{1}{\boxed{\quad}}$ (ii) $\cos 30^\circ = \dfrac{\boxed{\quad}}{\boxed{\quad}}$ (iii) $\tan 30^\circ = \dfrac{\boxed{\quad}}{\boxed{\quad}}$

(iv) $\sin 60^\circ = \dfrac{\boxed{\quad}}{\boxed{\quad}}$ (v) $\cos 45^\circ = \dfrac{\boxed{\quad}}{\boxed{\quad}}$ (vi) $\tan 45^\circ = \boxed{\quad}$

In std IX, we have studied some trigonometric ratios of some acute angles. Now we are going to study some more trigonometric ratios of acute angles.

Let's learn.

cosec, sec and cot ratios

Multiplicative inverse or the reciprocal of sine ratio is called cosecant ratio. It is written in brief as cosec. $\therefore$ $\operatorname{cosec}\theta = \dfrac{1}{\sin\theta}$

Similarly, multiplicative inverses or reciprocals of cosine and tangent ratios are called "secant" and "cotangent" ratios respectively. They are written in brief as sec and cot.

$\therefore$ $\sec\theta = \dfrac{1}{\cos\theta}$ and $\cot\theta = \dfrac{1}{\tan\theta}$

In figure 6.2,

$$\sin\theta = \frac{AB}{AC}$$

$$\therefore \operatorname{cosec}\theta = \frac{1}{\sin\theta} = \frac{1}{\frac{AB}{AC}} = \frac{AC}{AB}$$

It means,

$$\operatorname{cosec}\theta = \frac{\text{hypotenuse}}{\text{opposite side}}$$

$$\tan\theta = \frac{AB}{BC}$$

$$\therefore \quad \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{AB}{BC}}$$

$$\cot\theta = \frac{BC}{AB} = \frac{\text{adjacent side}}{\text{opposite side}}$$

$$\cos\theta = \frac{BC}{AC}$$

$$\sec\theta = \frac{1}{\cos\theta} = \frac{1}{\frac{BC}{AC}} = \frac{AC}{BC}$$

It means,

$$\sec\theta = \frac{\text{hypotenuse}}{\text{adjacent side}}$$

You know that,

$$\tan\theta = \frac{\sin\theta}{\cos\theta}$$

$$\therefore \cot\theta = \frac{1}{\tan\theta} = \frac{1}{\frac{\sin\theta}{\cos\theta}} = \frac{\cos\theta}{\sin\theta}$$

$$\therefore \cot\theta = \frac{\cos\theta}{\sin\theta}$$

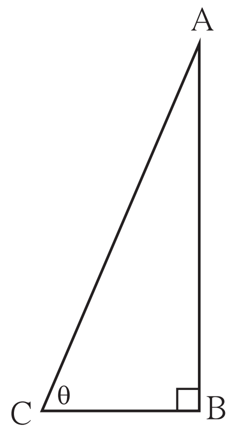


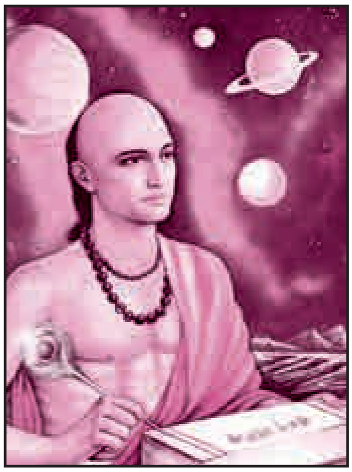
Fig. 6.2

Remember this !

The relation between the trigonometric ratios, according to the definitions of cosec, sec and cot ratios

- $\frac{1}{\sin\theta} = \text{cosec } \theta \qquad \therefore \sin\theta \times \text{cosec } \theta = 1$
- $\frac{1}{\cos\theta} = \sec\theta \qquad \therefore \cos\theta \times \sec\theta = 1$
- $\frac{1}{\tan\theta} = \cot\theta \qquad \therefore \tan\theta \times \cot\theta = 1$

For more information :



The great Indian mathematician Aryabhata was born in 476 A.D. in Kusumpur which was near Patna in Bihar. He has done important work in Arithmetic, Algebra and Geometry. In the book 'Aryabhatiya' he has written many mathematical formulae. For example,

(1) In an Arithmetic Progression, formulae for n^{th} term and the sum of first n terms.

(2) The formula to approximate $\sqrt{2}$

(3) The correct value of π upto four decimals, $\pi = 3.1416$.

In the study of Astronomy he used trigonometry and the sine ratio of an angle for the first time.

Comparing with the mathematics in the rest of the world at that time, his work was great and was studied all over India and was carried to Europe through Middle East.

Most observers at that time believed that the earth is immovable and the Sun, the Moon and stars move arround the earth. But Aryabhata noted that when we travel in a boat on the river, objects like trees, houses on the bank appear to move in the opposite direction. 'Similarly', he said 'the Sun, Moon and the stars are observed by people on the earth to be moving in the opposite direction while in reality the Earth moves !'

On 19 April 1975, India sent the first satellite in the space and it was named 'Aryabhata' to commemorate the great Mathematician of India.

✲ The table of the values of trigonometric ratios of angles 0°, 30°, 45°, 60° and 90°.

Trigonometric ratio	Angle (θ)				
	0°	30°	45°	60°	90°
$\sin\theta$	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0
$\tan\theta$	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not defined
$\operatorname{cosec}\theta = \frac{1}{\sin\theta}$	Not defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1
$\sec\theta = \frac{1}{\cos\theta}$	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not defined
$\cot\theta = \frac{1}{\tan\theta}$	Not defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0

Let's learn.

Trigonometric identities

In the figure 6.3, Δ ABC is a right angled triangle, $\angle B = 90°$

(i) $\sin\theta = \frac{BC}{AC}$ (ii) $\cos\theta = \frac{AB}{AC}$

(iii) $\tan\theta = \frac{BC}{AB}$ (iv) $\operatorname{cosec}\theta = \frac{AC}{BC}$

(v) $\sec\theta = \frac{AC}{AB}$ (vi) $\cot\theta = \frac{AB}{BC}$

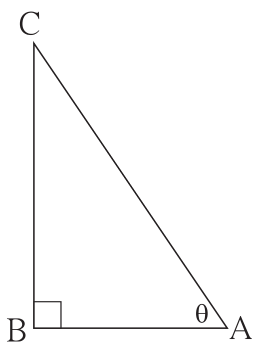


Fig. 6.3

By Pythagoras therom,

$BC^2 + AB^2 = AC^2$ . . . . .(I)

Dividing both the sides of (1) by AC^2

$$\frac{BC^2 + AB^2}{AC^2} = \frac{AC^2}{AC^2}$$

$\therefore \ \dfrac{BC^2}{AC^2} + \dfrac{AB^2}{AC^2} = 1$

$\therefore \ \left(\dfrac{BC}{AC}\right)^2 + \left(\dfrac{AB}{AC}\right)^2 = 1$

$\therefore (\sin\theta)^2 + (\cos\theta)^2 = 1$ .... [$(\sin\theta)^2$ is written as $\sin^2\theta$ and $(\cos\theta)^2$ is written as $\cos^2\theta$.]

$\mathbf{\sin^2\theta + \cos^2\theta = 1}$ .... (II)

Now dividing both the sides of equation (II) by $\sin^2\theta$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$\mathbf{1 + \cot^2\theta = \operatorname{cosec}^2\theta}$ .... (III)

Dividing both the sides of equation (II) by $\cos^2\theta$

$$\frac{\sin^2\theta}{\cos^2\theta} + \frac{\cos^2\theta}{\cos^2\theta} = \frac{1}{\cos^2\theta}$$

$\tan^2\theta + 1 = \sec^2\theta$

$\mathbf{1 + \tan^2\theta = \sec^2\theta}$ .... (IV)

Relations (II),(III), and (IV) are fundamental trigonometric identities.

Solved Examples

Ex. (1) If $\sin\theta = \dfrac{20}{29}$ then find $\cos\theta$

Solution : **Method I**

We have

$\sin^2\theta + \cos^2\theta = 1$

$\left(\dfrac{20}{29}\right)^2 + \cos^2\theta = 1$

$\dfrac{400}{841} + \cos^2\theta = 1$

$\cos^2\theta = 1 - \dfrac{400}{841}$

$= \dfrac{441}{841}$

Taking square root of both sides.

$\cos\theta = \dfrac{21}{29}$

Method II

$\sin\theta = \dfrac{20}{29}$

from figure, $\sin\theta = \dfrac{AB}{AC}$

$\therefore$ AB = $20k$ and AC = $29k$

Let BC = x.

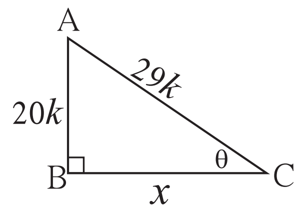


Fig. 6.4

According to Pythagoras therom,

$AB^2 + BC^2 = AC^2$

$(20k)^2 + x^2 = (29k)^2$

$400k^2 + x^2 = 841k^2$

$x^2 = 841k^2 - 400k^2$

$= 441k^2$

$\therefore \quad x = 21k$

$\therefore \cos\theta = \dfrac{BC}{AC} = \dfrac{21k}{29k} = \dfrac{21}{29}$

Ex. (2) If $\sec\theta = \frac{25}{7}$, find the value of $\tan\theta$.

Solution : Method I

we have ,

$$1+\tan^2\theta = \sec^2\theta$$

$$\therefore \quad 1+\tan^2\theta = \left(\frac{25}{7}\right)^2$$

$$\therefore \quad \tan^2\theta = \frac{625}{49} - 1$$

$$= \frac{625-49}{49}$$

$$= \frac{576}{49}$$

$$\therefore \quad \tan\theta = \frac{24}{7}$$

Method II

from the figure,

$$\sec\theta = \frac{PR}{PQ}$$

$$\therefore \quad PQ = 7k,\ PR = 25k$$

According to Pythagoras therom,

$$PQ^2 + QR^2 = PR^2$$

$$\therefore \quad (7k)^2 + QR^2 = (25k)^2$$

$$\therefore \quad QR^2 = 625k^2 - 49k^2 = 576k^2$$

$$\therefore \quad QR = 24k$$

Now, $\tan\theta = \frac{QR}{PQ} = \frac{24k}{7k} = \frac{24}{7}$

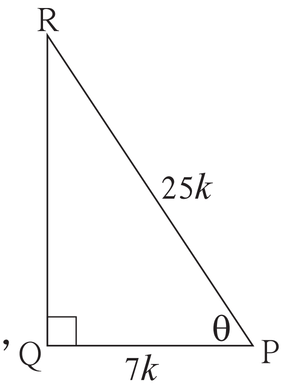


Fig. 6.5

Ex. (3) If $5\sin\theta - 12\cos\theta = 0$, find the values of $\sec\theta$ and $\mathrm{cosec}\theta$.

Solution : $5\sin\theta - 12\cos\theta = 0$

$$\therefore \ 5\sin\theta = 12\cos\theta$$

$$\therefore \ \frac{\sin\theta}{\cos\theta} = \frac{12}{5}$$

$$\therefore \ \tan\theta = \frac{12}{5}$$

we have,

$$1+\tan^2\theta = \sec^2\theta$$

$$\therefore 1+\left(\frac{12}{5}\right)^2 = \sec^2\theta$$

$$\therefore \ 1 + \frac{144}{25} = \sec^2\theta$$

$$\therefore \ \frac{25+144}{25} = \sec^2\theta$$

$$\therefore \quad \sec^2\theta = \frac{169}{25}$$

$$\therefore \quad \sec\theta = \frac{13}{5}$$

$$\therefore \cos\theta = \frac{5}{13}$$

Now, $\sin^2\theta + \cos^2\theta = 1$

$$\therefore \ \sin^2\theta = 1 - \cos^2\theta$$

$$\therefore \sin^2\theta = 1 - \left(\frac{5}{13}\right)^2$$

$$= 1 - \frac{25}{169}$$

$$= \frac{144}{169}$$

$$\therefore \ \sin\theta = \frac{12}{13}$$

$$\therefore \ \mathrm{cosec}\theta = \frac{13}{12}$$

Ex. (4) $\cos\theta = \frac{\sqrt{3}}{2}$ then find the value of $\frac{1-\sec\theta}{1+\operatorname{cosec}\theta}$.

Solution : Method I

$\cos\theta = \frac{\sqrt{3}}{2}$ $\therefore \sec\theta = \frac{2}{\sqrt{3}}$

$\sin^2\theta + \cos^2\theta = 1$

$\therefore \sin^2\theta + \left(\frac{\sqrt{3}}{2}\right)^2 = 1$

$\therefore \sin^2\theta = 1 - \frac{3}{4} = \frac{1}{4}$

$\therefore \sin\theta = \frac{1}{2}$ $\therefore \operatorname{cosec}\theta = 2$

$$\therefore \frac{1-\sec\theta}{1+\operatorname{cosec}\theta} = \frac{1-\frac{2}{\sqrt{3}}}{1+2}$$

$$= \frac{\frac{\sqrt{3}-2}{\sqrt{3}}}{3}$$

$$= \frac{\sqrt{3}-2}{3\sqrt{3}}$$

Method II

$\cos\theta = \frac{\sqrt{3}}{2}$

we know that, $\cos 30^\circ = \frac{\sqrt{3}}{2}$.

$\therefore \theta = 30^\circ$

$\therefore \sec\theta = \sec 30^\circ = \frac{2}{\sqrt{3}}$

$\operatorname{cosec}\theta = \operatorname{cosec} 30^\circ = 2$

$$\therefore \frac{1-\sec\theta}{1+\operatorname{cosec}\theta} = \frac{1-\frac{2}{\sqrt{3}}}{1+2}$$

$$= \frac{\frac{\sqrt{3}-2}{\sqrt{3}}}{3}$$

$$= \frac{\sqrt{3}-2}{3\sqrt{3}}$$

Ex. (5) Show that $\sec x + \tan x = \sqrt{\frac{1+\sin x}{1-\sin x}}$

Solution : $\sec x + \tan x = \frac{1}{\cos x} + \frac{\sin x}{\cos x}$

$$= \frac{1+\sin x}{\cos x}$$

$$= \sqrt{\frac{(1+\sin x)^2}{\cos^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)(1+\sin x)}{1-\sin^2 x}}$$

$$= \sqrt{\frac{(1+\sin x)(1+\sin x)}{(1-\sin x)(1+\sin x)}}$$

$$= \sqrt{\frac{1+\sin x}{1-\sin x}}$$

Ex. (6) Eliminate θ from given equations.

$x = a \cot\theta - b \operatorname{cosec}\theta$

$y = a \cot\theta + b \operatorname{cosec}\theta$

Solution : $x = a \cot\theta - b \operatorname{cosec}\theta$ .... (I)

$y = a \cot\theta + b \operatorname{cosec}\theta$ .... (II)

Adding equations (I) and (II).

$x + y = 2a \cot\theta$

$\therefore \cot\theta = \dfrac{x+y}{2a}$ .... (III)

Subtracting equation (II) from (I) ,

$y - x = 2b \operatorname{cosec}\theta$

$\therefore \operatorname{cosec}\theta = \dfrac{y-x}{2b}$ .... (IV)

Now, $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

$$\therefore \left(\frac{y-x}{2b}\right)^2 - \left(\frac{y+x}{2a}\right)^2 = 1$$

$$\therefore \frac{(y-x)^2}{4b^2} - \frac{(y+x)^2}{4a^2} = 1$$

$$\text{or } \left(\frac{y-x}{b}\right)^2 - \left(\frac{y+x}{a}\right)^2 = 4$$

Practice set 6.1

1. If $\sin\theta = \dfrac{7}{25}$, find the values of $\cos\theta$ and $\tan\theta$.

2. If $\tan\theta = \dfrac{3}{4}$, find the values of $\sec\theta$ and $\cos\theta$.

3. If $\cot\theta = \dfrac{40}{9}$, find the values of $\operatorname{cosec}\theta$ and $\sin\theta$.

4. If $5\sec\theta - 12\operatorname{cosec}\theta = 0$, find the values of $\sec\theta$, $\cos\theta$ and $\sin\theta$.

5. If $\tan\theta = 1$ then, find the values of $\dfrac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta}$.

6. Prove that:

   (1) $\dfrac{\sin^2\theta}{\cos\theta} + \cos\theta = \sec\theta$

   (2) $\cos^2\theta(1 + \tan^2\theta) = 1$

(3) $\sqrt{\dfrac{1-\sin\theta}{1+\sin\theta}} = \sec\theta - \tan\theta$

(4) $(\sec\theta - \cos\theta)(\cot\theta + \tan\theta) = \tan\theta\sec\theta$

(5) $\cot\theta + \tan\theta = \operatorname{cosec}\theta\sec\theta$

(6) $\dfrac{1}{\sec\theta-\tan\theta} = \sec\theta + \tan\theta$

(7) $\sec^4\theta - \cos^4\theta = 1 - 2\cos^2\theta$

(8) $\sec\theta + \tan\theta = \dfrac{\cos\theta}{1-\sin\theta}$

(9) If $\tan\theta + \dfrac{1}{\tan\theta} = 2$, then show that $\tan^2\theta + \dfrac{1}{\tan^2\theta} = 2$

(10) $\dfrac{\tan A}{(1+\tan^2 A)^2} + \dfrac{\cot A}{(1+\cot^2 A)^2} = \sin A\cos A$

(11) $\sec^4 A(1-\sin^4 A) - 2\tan^2 A = 1$

(12) $\dfrac{\tan\theta}{\sec\theta-1} = \dfrac{\tan\theta+\sec\theta+1}{\tan\theta+\sec\theta-1}$

Let's learn.

Application of trigonometry

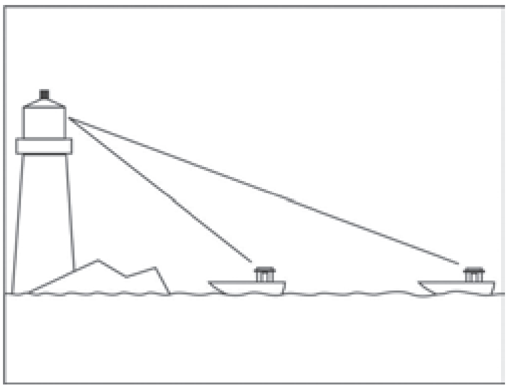


Fig. 6.6

Many times we need to know the height of a tower, building, tree or distance of a ship from the lighthouse or width of a river etc.

We cannot measure them actually but we can find them with the help of trigonometric ratios.

For the purpose of computation, we draw a rough sketch to show the given information. 'Trees', 'hills' or 'towers' are vertical objects, so we shall represent them in the figure by segments which are perpendicular to the ground. We will not consider height of the observer and we shall normally regard observer's line of vision to be parallel to the horizontal level.

Let us study a few related terms.

(i) Line of vision : If the observer is standing at the location 'A', looking at an object 'B' then the line AB is called line of the vision.

(ii) Angle of elevation :

If an observer at A, observes the point B which is at a level higher than A and AM is the horizontal line, then $\angle$ BAM is called the angle of elevation.

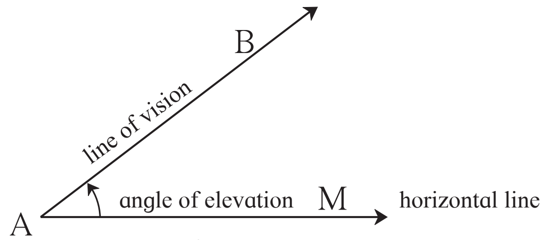


Fig. 6.7

(iii) Angle of depression :

If an observer at A, observes the point C which is at a level lower than A and AM is the horizontal line, the $\angle$ MAC is called the angle of depression.

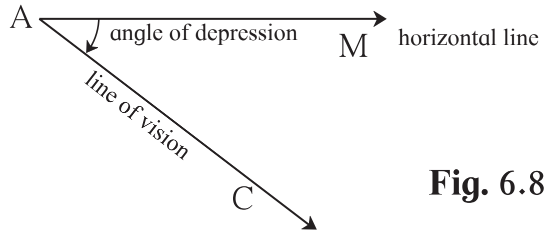


Fig. 6.8

When we see above the horizontal line, the angle formed is the angle of elevation. When we see below the horizontal line, the angle formed is the angle of depression.

Solved Examples

Ex. (1) An observer at a distance of 10 m from a tree looks at the top of the tree, the angle of elevation is 60°. What is the height of the tree ? ($\sqrt{3}$ = 1.73)

Solution : In figure 6.9, AB = h = height of the tree.

BC = 10 m, distance of the observer from the tree .

Angle of elevation (θ) = $\angle$ BCA = 60°

from figure, $\tan\theta = \dfrac{AB}{BC}$ .... (I)

$\tan 60° = \sqrt{3}$ .... (II)

$\therefore \dfrac{AB}{BC} = \sqrt{3}$ .... from equation (I)and (II)

$\therefore$ AB = BC$\sqrt{3}$ = 10$\sqrt{3}$

$\therefore$ AB = 10 × 1.73 = 17.3 m

$\therefore$ height of the tree is 17.3m.

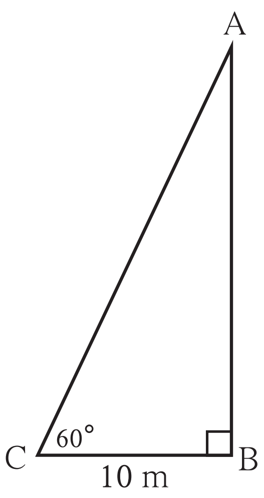


Fig. 6.9

Ex. (2) From the top of a building, an observer is looking at a scooter parked at some distance away, makes an angle of depression of $30°$. If the height of the building is 40m, find how far the scooter is from the building. ($\sqrt{3}$ = 1.73)

Solution: In the figure 6.10, AB is the building. A scooter is at C which is 'x' m away from the building.

In figure, 'A' is the position of the observer.

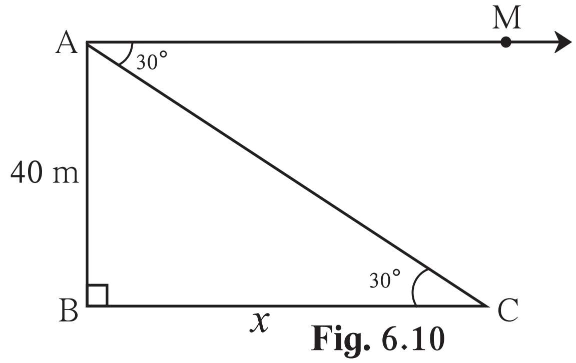


Fig. 6.10

AM is the horizontal line and $\angle$ MAC is the angle of depression.

$\angle$ MAC and $\angle$ ACB are alternate angles.

from fig, $\tan 30° = \frac{AB}{BC}$

$$\therefore \quad \frac{1}{\sqrt{3}} = \frac{40}{x}$$

$$\therefore \quad x = 40\sqrt{3}$$

$$= 40 \times 1.73$$

$$= 69.20 \text{ m.}$$

$\therefore$ the scooter is 69.20 m. away from the building.

Ex. (3) To find the width of the river, a man observes the top of a tower on the opposite bank making an angle of elevation of $61°$. When he moves 50m backword from bank and observes the same top of the tower, his line of vision makes an angle of elevation of $35°$. Find the height of the tower and width of the river. ($\tan 61° = 1.8$, $\tan 35° = 0.7$)

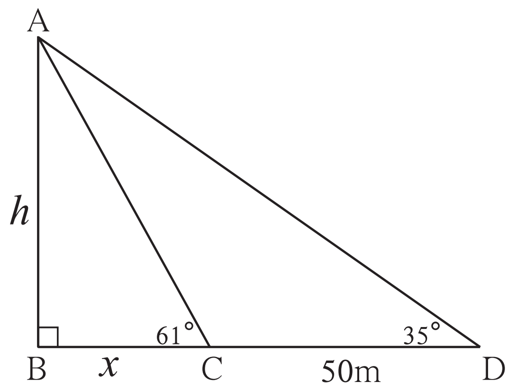


Fig. 6.11

Solution : seg AB shows the tower on the opposite bank. 'A' is the top of the tower and seg BC shows the width of the river. Let 'h' be the height of the tower and 'x' be the width of the river.

from figure, $\tan 61° = \frac{h}{x}$

$$\therefore\ 1.8 = \frac{h}{x}$$

$$h = 1.8 \times x$$

$10h = 18x$ .... (I)..... multipling by 10

In right angled Δ ABD,

$$\tan 35 = \frac{h}{x + 50}$$

$$0.7 = \frac{h}{x + 50}$$

$$\therefore\ h = 0.7\,(x + 50)$$

$\therefore\ 10h = 7\,(x + 50)$ .... (II)

$\therefore$ from equations (I) and (II) ,

$$18x = 7(x + 50)$$

$$\therefore\ 18x = 7x + 350$$

$$\therefore\ 11x = 350$$

$$\therefore\ x = \frac{350}{11} = 31.82$$

Now, $h = 1.8x = 1.8 \times 31.82$

$= 57.28$ m.

$\therefore$ width of the river = 31.82 m and height of tower = 57.28 m

Ex. (4) Roshani saw an eagle on the top of a tree at an angle of elevation of 61°, while she was standing at the door of her house. She went on the terrace of the house so that she could see it clearly. The terrace was at a height of 4m. While observing the eagle from there the angle of elevation was 52°. At what height from the ground was the eagle ? (Find the answer correct upto nearest integer)

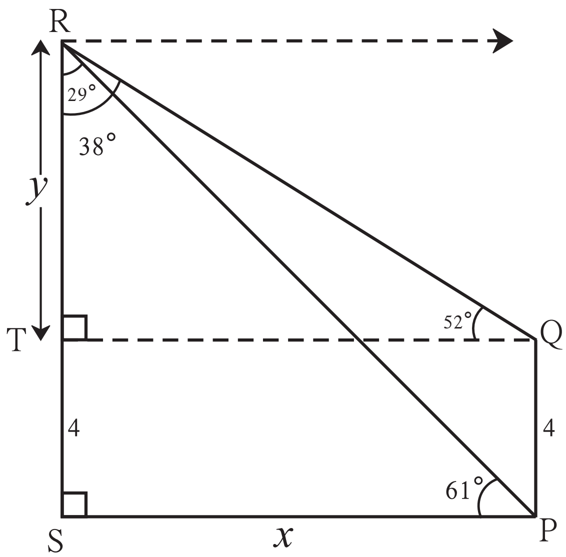


Fig. 6.12

($\tan 61^\circ = 1.80$, $\tan 52^\circ = 1.28$, $\tan 29^\circ = 0.55$, $\tan 38^\circ = 0.78$)

Solution : In figure 6.12, PQ is the house and SR is the tree. The eagle is at R.

Draw seg $QT \perp$ seg RS.

$\therefore \square$ TSPQ is a rectangle.

Let SP = x and TR = y

Now in Δ RSP, $\angle$ PRS = $90^\circ - 61^\circ = 29^\circ$

and in Δ RTQ, $\angle$ QRT = $90^\circ - 52^\circ = 38^\circ$

$\therefore \tan \angle PRS = \tan 29^\circ = \frac{SP}{RS}$

$\therefore 0.55 = \frac{x}{y+4}$

$\therefore x = 0.55(y + 4)$ .... (I)

Similarly, $\tan \angle QRT = \frac{TQ}{RT}$

$\therefore \tan 38^\circ = \frac{x}{y}$ .... [$\because$ SP = TQ = x]

$\therefore 0.78 = \frac{x}{y}$

$\therefore x = 0.78y$ .... (II)

$\therefore 0.78y = 0.55(y + 4)$ .... from (I) and (II)

$\therefore 78y = 55(y + 4)$

$\therefore 78y = 55y + 220$

$\therefore 23y = 220$

$\therefore y = 9.565 = 10$ (upto nearest integer)

$\therefore$ RS = $y + 4 = 10 + 4 = 14$

$\therefore$ the eagle was at a height of 14 metre from the ground.

Ex. (5) A tree was broken due to storm. Its broken upper part was so inclined that its top touched the ground making an angle of $30^\circ$with the ground.The distance from the foot of the tree and the point where the top touched the ground was 10 metre. What was the height of the tree.

Solution: As shown in figure 6.13, suppose AB is the tree. It was broken at 'C' and its top touched at 'D'.

$\angle$ CDB = 30°, BD = 10 m, BC = x m

CA= CD = y m

In right angled Δ CDB,

$$\tan 30° = \frac{BC}{BD}$$

$$\frac{1}{\sqrt{3}} = \frac{x}{10}$$

$$x = \frac{10}{\sqrt{3}}$$

$$y = \frac{20}{\sqrt{3}}$$

$$x + y = \frac{10}{\sqrt{3}} + \frac{20}{\sqrt{3}}$$

$$= \frac{30}{\sqrt{3}}$$

$$x + y = 10\sqrt{3}$$

$\therefore$ height of the tree was $10\sqrt{3}$ m.

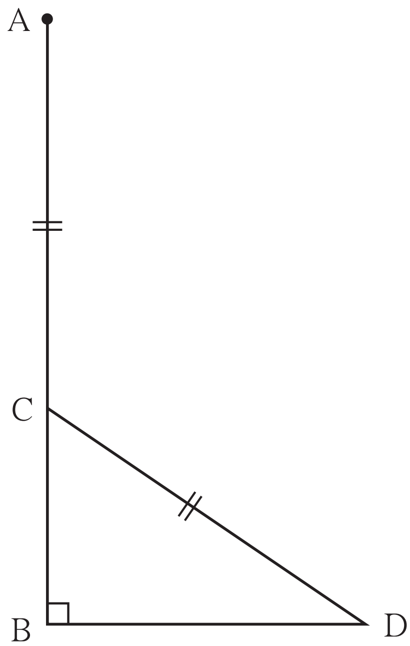


Fig. 6.13

Practice set 6.2

1. A person is standing at a distance of 80m from a church looking at its top. The angle of elevation is of 45°. Find the height of the church.
2. From the top of a lighthouse, an observer looking at a ship makes angle of depression of 60°. If the height of the lighthouse is 90 metre, then find how far the ship is from the lighthouse. ($\sqrt{3}$ =1.73)
3. Two buildings are facing each other on a road of width 12 metre. From the top of the first building, which is 10 metre high, the angle of elevation of the top of the second is found to be 60°. What is the height of the second building?
4. Two poles of heights 18 metre and 7 metre are erected on a ground. The length of the wire fastened at their tops in 22 metre. Find the angle made by the wire with the horizontal.
5. A storm broke a tree and the treetop rested 20 m from the base of the tree, making an angle of 60° with the horizontal. Find the height of the tree.
6. A kite is flying at a height of 60 m above the ground. The string attached to the kite is tied at the ground. It makes an angle of 60° with the ground. Assuming that the string is straight, find the length of the string. ($\sqrt{3}$ =1.73)

Problem set 6

1. Choose the correct alternative answer for the following questions.

(1) $\sin\theta\ \mathrm{cosec}\theta$ = ?

(A) 1 (B) 0 (C) $\frac{1}{2}$ (D) $\sqrt{2}$

(2) $\mathrm{cosec}45°$ = ?

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{2}{\sqrt{3}}$

(3) $1 + \tan^2\theta$ = ?

(A) $\cot^2\theta$ (B) $\mathrm{cosec}^2\theta$ (C) $\sec^2\theta$ (D) $\tan^2\theta$

(4) When we see at a higher level , from the horizontal line,angle formed is....... .

(A) angle of elevation. (B) angle of depression.

(C) 0 (D) straight angle.

2. If $\sin\theta = \frac{11}{61}$, find the values of $\cos\theta$ using trigonometric identity.

3. If $\tan\theta = 2$, find the values of other trigonometric ratios.

4. If $\sec\theta = \frac{13}{12}$, find the values of other trigonometric ratios.

5. Prove the following.

(1) $\sec\theta\,(1 - \sin\theta)\,(\sec\theta + \tan\theta) = 1$

(2) $(\sec\theta + \tan\theta)\,(1 - \sin\theta) = \cos\theta$

(3) $\sec^2\theta + \mathrm{cosec}^2\theta = \sec^2\theta \times \mathrm{cosec}^2\theta$

(4) $\cot^2\theta - \tan^2\theta = \mathrm{cosec}^2\theta - \sec^2\theta$

(5) $\tan^4\theta + \tan^2\theta = \sec^4\theta - \sec^2\theta$

(6) $\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta} = 2\sec^2\theta$

(7) $\sec^6 x - \tan^6 x = 1 + 3\sec^2 x \times \tan^2 x$

(8) $\frac{\tan\theta}{\sec\theta + 1} = \frac{\sec\theta - 1}{\tan\theta}$

(9) $\frac{\tan^3\theta - 1}{\tan\theta - 1} = \sec^2\theta + \tan\theta$

(10) $\frac{\sin\theta - \cos\theta + 1}{\sin\theta + \cos\theta - 1} = \frac{1}{\sec\theta - \tan\theta}$

6. A boy standing at a distance of 48 meters from a building observes the top of the building and makes an angle of elevation of 30°.Find the height of the building.

7. From the top of the light house, an observer looks at a ship and finds the angle of depression to be 30°.If the height of the light-house is 100 meters, then find how far the ship is from the light-house.

8. Two buildings are in front of each other on a road of width 15 meters. From the top of the first building, having a height of 12 meter, the angle of elevation of the top of the second building is 30°.What is the height of the second building?

9. A ladder on the platform of a fire brigade van can be elevated at an angle of 70° to the maximum. The length of the ladder can be extended upto 20m. If the platform is 2m above the ground, find the maximum height from the ground upto which the ladder can reach. (sin 70° = 0.94)

10 *. While landing at an airport, a pilot made an angle of depression of 20°. Average speed of the plane was 200 km/hr. The plane reached the ground after 54 seconds. Find the height at which the plane was when it started landing. (sin 20° = 0.342)

PV6XPH

5. $(-7, 0)$ 6. (1) $a\sqrt{2}$ (2) 13 (3) $5a$ **7.** $\left(-\frac{1}{3}, \frac{2}{3}\right)$

8. (1) Yes, scalene triangle (2) No. (3) Yes, equilateral triangle 9. $k = 5$

13. 5, $2\sqrt{13}$, $\sqrt{37}$ **14.** (1, 3) **16.** $\left(\frac{25}{6}, \frac{13}{6}\right)$, radius = $\frac{13\sqrt{2}}{6}$ **17.** (7, 3)

18. Parallelogram **19.** A(20, 10), P(16, 12), R(8, 16), B(0, 20). **20.** (3, −2)

21. (7, 6) and (3, 6) **22.** 10 and 0

Chapter 6 Trigonometry

Practice set 6.1

1. $\cos\theta = \frac{24}{25}$; $\tan\theta = \frac{7}{24}$ **2.** $\sec\theta = \frac{5}{4}$; $\cos\theta = \frac{4}{5}$

3. $\operatorname{cosec}\theta = \frac{41}{9}$; $\sin\theta = \frac{9}{41}$ **4.** $\sec\theta = \frac{13}{5}$; $\cos\theta = \frac{5}{13}$; $\sin\theta = \frac{12}{13}$

5. $\frac{\sin\theta + \cos\theta}{\sec\theta + \operatorname{cosec}\theta} = \frac{1}{2}$

Practice set 6.2

1. Height of the church is 80 metre.
2. The ship is 51.90 metre away from the ligthhouse.
3. Height of the second building is $(10 + 12\sqrt{3})$ metre.
4. Angle made by the wire with the horizontal line is 30°.
5. Height of the tree is $(40 + 20\sqrt{3})$ metre.
6. The length of the string is 69.20 metre.

Problem set 6

1. (1) A (2) B (3) C (4) A

2. $\cos 60 = \frac{60}{61}$ **3.** $\sin\theta = \frac{2}{\sqrt{5}}$; $\cos\theta = \frac{1}{\sqrt{5}}$; $\operatorname{cosec}\theta = \frac{\sqrt{5}}{2}$; $\sec\theta = \sqrt{5}$; $\cot\theta = \frac{1}{2}$

4. $\sin\theta = \frac{5}{13}$; $\cos\theta = \frac{12}{13}$; $\operatorname{cosec}\theta = \frac{13}{5}$; $\tan\theta = \frac{5}{12}$; $\cot\theta = \frac{12}{5}$

6. Height of the building is $16\sqrt{3}$ metre.
7. The ship is $100\sqrt{3}$ metre away from the ligthhouse.
8. Height of the second buliding is $(12 + 15\sqrt{3})$ metre.
9. The maximum height that ladder can reach is 20.80 metre.

10. the plane was 1026 metre high at the time of landing.

Chapter 7 Mensuration

Practice set 7.1

1. 11.79 cm^3 **2.** 113.04 cm^3 **3.** 1413 sq.cm (by taking $\pi = 3.14$) **4.** 616 sq.cm
5. 21 cm **6.** 12 jugs **7.** 5 cm **8.** 273π sq.cm **9.** 20 tablets
10. 94.20 cm^3, 103.62 sq.cm **11.** 5538.96 sq.cm, 38772.72 cm^3
12. 1468.67π cm^3

Practice set 7.2

1. 10.780 litre **2.** (1) 628 sq.cm (2) 1356.48 sq.cm (3) 1984.48 cm^3

Practice set 7.3

1. 47.1 sq.cm **2.** 25.12 cm **3.** 3.85 sq.cm **4.** 214 sq.cm **5.** 4 cm
6. (1) 154 sq.cm (2) 25.7 sq.cm (3) 128.3 sq.cm **7.** 10.2 sq.cm
8. 7.3 cm ; 22 cm **9.** (1) $90°$ (2) 22 cm
10.(1) 12.83 sq.cm (2) 89.83 sq.cm (3) 115.5 sq.cm **11.** 3.5 cm
12. x = 154 sq.cm ; y = 38.5 sq.cm ; z = 101.5 sq.cm
13. (1) 84.87 sq.cm (2) 25.67 sq.cm (3) 77.01 sq.cm (4) 7.86 sq.cm

Practice set 7.4

1. 3.72 sq.cm **2.** 9.08 sq.cm **3.** 0.65625 sq.unit **4.** 20 cm
5. 20.43 sq.cm ; 686.07 sq.cm

Problem set 7

1. (1) A, (2) D, (3) B, (4) B, (5) A, (6) A, (7) D, (8) C.
2. 20.35 litre **3.** 7830 balls **4.** 2800 coins (by taking $\pi = \frac{22}{7}$) **5.** Rs. 6336
6. 452.16 sq.cm ; 3385.94 gm **7.** 2640 sq.cm **8.** 108 metre
9. $150°$; 5π cm **10.** 39.28 sq.cm