5. APPLICATION OF DEFINITE INTEGRATION

Let us Study

- Area under the curve
    - Area bounded by the curve, axis and given lines
    - Area between two curves.

Let us Recall

- In previous chapter, we have studied definition of definite integral as limit of a sum. Geometrically $\int_a^b f(x)\,dx$ gives the area A under the curve $y = f(x)$ with $f(x) > 0$ and bounded by the X−axis and the lines $x = a$, $x = b$; and is given by

$$\int_a^b f(x)\,dx = \phi(b) - \phi(a)$$

where $\int f(x)\,dx = \phi(x)$

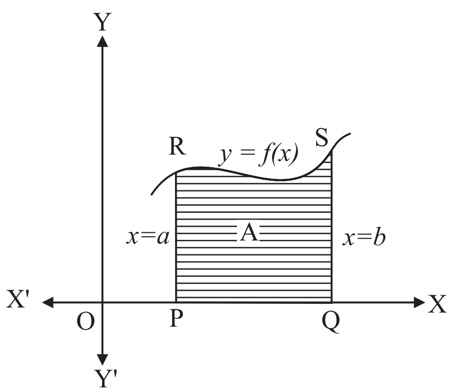

Fig. 5.1

This is also known as fundamental theorem of integral calculus.

We shall find the area under the curve by using definite integral.

5.1 Area under the curve :

For evaluation of area bounded by certain curves, we need to know the nature of the curves and their graphs. We should also be able to draw sketch of the curves.

5.1.1 Area under a curve :

The curve $y = f(x)$ is continuous in $[a, b]$ and $f(x) \geq 0$ in $[a, b]$.

1. The area shaded in figure 5.2 is bounded by the curve $y = f(x)$, X−axis and the lines $x = a$, $x = b$ and is given by the definite integral $\int_{x=a}^{x=b} (y)\,dx$

   A = area of the shaded region.

   $$A = \int_a^b f(x)\,dx$$

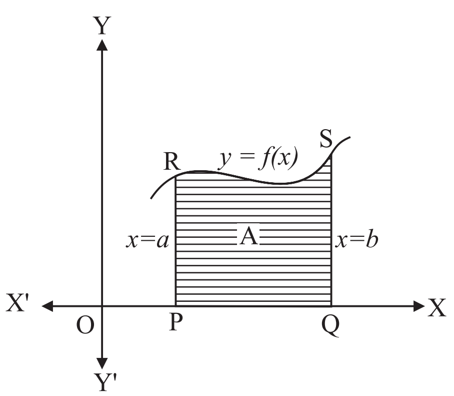

Fig. 5.2

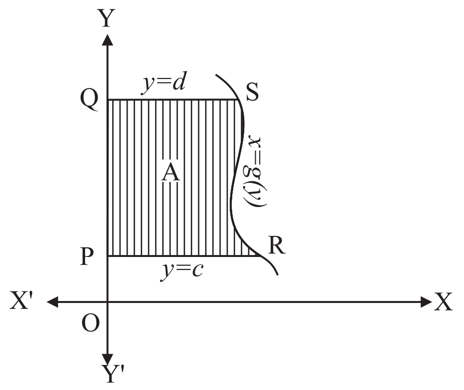

Fig. 5.3

2. The area A, bounded by the curve $x = g(y)$, Y axis and the lines $y = c$ and $y = d$ is given by

$$A = \int_{y=c}^{d} x\, dy$$

$$= \int_{y=c}^{y=d} g(y)\, dx$$

SOLVED EXAMPLE

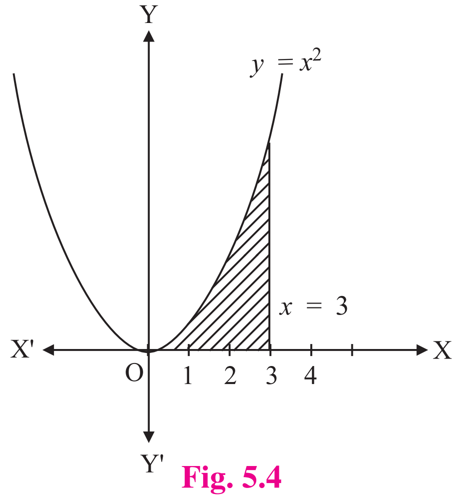

Fig. 5.4

Ex. 1 : Find the area bounded by the curve $y = x^2$, the Y axis the X axis and $x = 3$.

Solution : The required area $A = \int_{x=0}^{3} y\, dx$

$$A = \int_{0}^{3} x^2\, dx$$

$$= \left[\frac{x^3}{3}\right]_0^3$$

$$A = 9 - 0$$

$$= 9 \text{ sq.units}$$

5.1.2 Area between two curves :

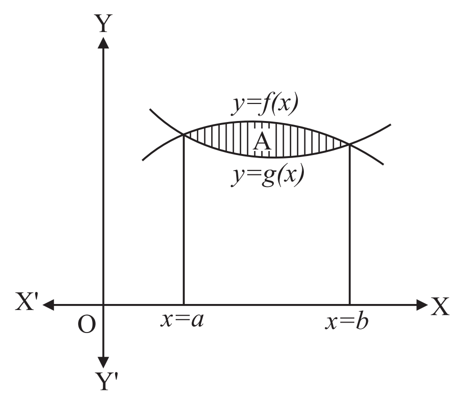

Fig. 5.5

Let $y = f(x)$ and $y = g(x)$ be the equations of the two curves as shown in fig 5.5.

Let A be the area bounded by the curves $y = f(x)$ and $y = g(x)$

$$A = |A_1 - A_2| \quad \text{where}$$

A_1 = Area bounded by the curve $y = f(x)$, X-axis and $x = a$, $x = b$.

A_2 = Area bounded by the curve $y = g(x)$, X-axis and $x = a$, $x = b$.

The point of intersection of the curves $y = f(x)$ and $y = g(x)$ can be obtained by solving their equations simultaneously.

$\therefore$ The required area $\qquad A = \left| \int_a^b f(x)\, dx - \int_a^b g(x)\, dx \right|$

SOLVED EXAMPLES

Ex. 1 : Find the area of the region bounded by the curves $y^2 = 9x$ and $x^2 = 9y$.

Solution : The equations of the curves are

$y^2 = 9x \, \ldots\ldots \text{(I)}$

and $\quad x^2 = 9y \, \ldots\ldots \text{(II)}$

Squaring equation (II)

$x^4 = 81y^2$

$x^4 = 81\,(9x) \quad \ldots \text{by (1)}$

$x^4 = 729\, x$

$\therefore \quad x\,(x^3 - 9^3) = 0$

i.e. $\quad x\,(x^3 - 9^3) = 0$

$\Rightarrow \quad x = 0 \quad$ or $\quad x = 9$

From equation (II), $\quad y = 0 \quad$ or $\quad y = 9$

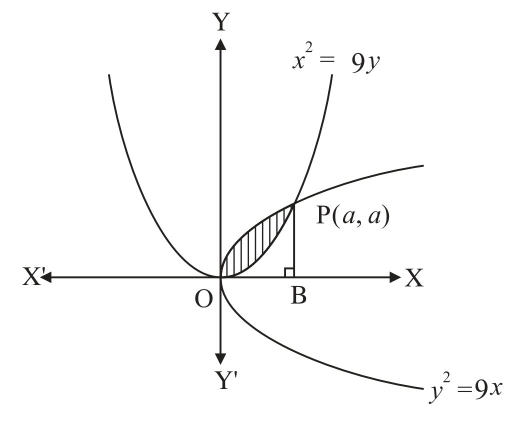

Fig. 5.6

$\therefore$ The points of intersection of the curves are (0, 0), (9, 9).

$$\therefore \text{ Required area } \quad A = \int_0^9 \sqrt{9x}\, dx - \int_0^9 \frac{x^2}{9}\, dx$$

$$= \left[3 \cdot \frac{2}{3} \cdot x^{\frac{3}{2}}\right]_0^9 - \left[\frac{1}{9} \cdot \frac{x^3}{3}\right]_0^9$$

$$= 2 \cdot 9^{\frac{3}{2}} - 27$$

$$A = 54 - 27$$

$$= 27 \text{ sq.units}$$

Now, we will see how to find the area bounded by the curve $y = f(x)$, X-axis and lines $x = a$, $x = b$ if $f(x)$ is negative i.e. $f(x) \leq 0$ in $[\, a, b\,]$.

Ex. 2 : Find the area bounded by the curve $y = -x^2$, X−axis and lines $x = 1$ and $x = 4$.

Solution : Let A be the area bounded by the curve $y = -x^2$, X−axis and $1 \leq x \leq 4$.

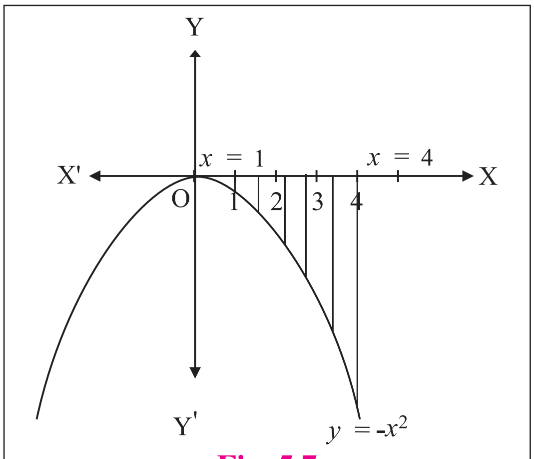

Fig. 5.7

$$\text{The required area} \quad A = \int_1^4 y\, dx$$

$$= \int_1^4 -x^2 \; dx$$

$$= \left[-\frac{x^3}{3} \right]_1^4$$

$$= -\frac{64}{3} + \frac{1}{3}$$

$$A = -21,$$

But we consider the area to be positive.

$\therefore$ A $= \left| -21 \right|$ sq.units $= 21$ square units.

Thus, if $f(x) \leq 0$ or $f(x) \geq 0$ in $[\, a, b\,]$ then the area enclosed between $y = f(x)$, X−axis and $x = a, x = b$ is $\left| \int_a^b f(x) \cdot dx \right|$.

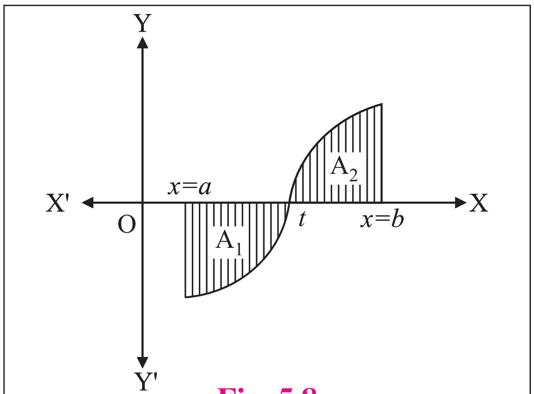

Fig. 5.8

If the area A is divided into two parts A_1 and A_2 such that

A_1 is the part of $a \leq x \leq t$ where $f(x) \leq 0$ and

A_2 is the part of $a \leq x \leq t$ where $f(x) \geq 0$

then in A_1, the required area is below the X−axis

and in A_2, the required area is above the X-axis.

$$\text{Now the total area} \quad A = A_1 + A_2$$

$$= \left| \int_a^t f(x)\, dx \right| + \left| \int_t^b f(x)\, dx \right|$$

Ex. 3 : Find the area bounded by the line $y = x$, X axis and the lines $x = -1$ and $x = 4$.

Solution : Consider the area A, bounded by straight line $y = x$, X axis and $x = -1$, $x = 4$.

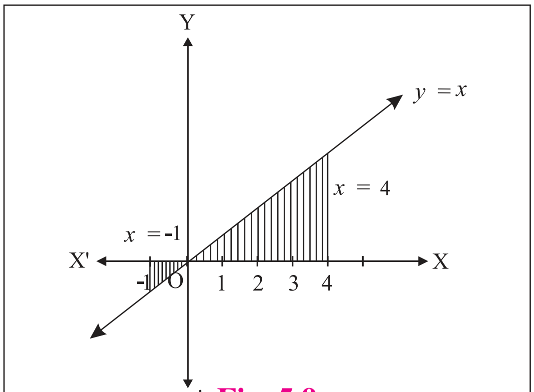

Fig. 5.9

From figure 5.9, A is divided into A_1 and A_2

$$\text{The required area} \quad A_1 = \int_{-1}^0 y\, dx = \int_{-1}^0 x \; dx$$

$$= \left[\frac{x^2}{2} \right]_{-1}^0$$

$$= 0 - \frac{1}{2}$$

$$A_1 = -\frac{1}{2} \text{ square units.}$$

But area is always positive.

$\therefore \quad A_1 = \left| -\frac{1}{2} \right| \text{sq.units} = \frac{1}{2}$ square units.

$$A_2 = \int_0^4 y\,dx = \int_0^4 x\,dx = \left[\frac{x^2}{2}\right]_0^4 = \frac{4^2}{2} = 8 \text{ square units.}$$

$\therefore \quad$ Required area $\quad A = A_1 + A_2 = \frac{1}{2} + 8 = \frac{17}{2}$ sq.units

Ex. 4 : Find the area enclosed between the X-axis and the curve $y = \sin x$ for values of x between 0 to 2π.

Solution : The area enclosed between the curve and the X-axis consists of equal area lying alternatively above and below X-axis which are respectively positive and negative.

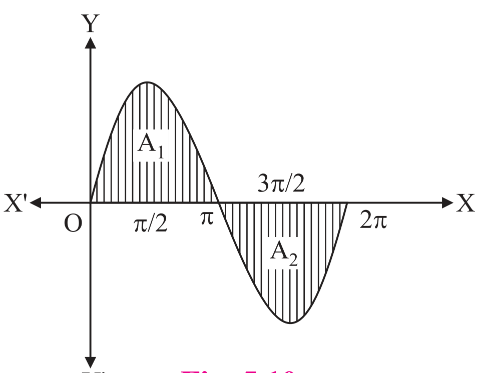

Fig. 5.10

1) Area $\quad A_1 \quad =$ area lying above the X-axis

$$= \int_0^{\pi} \sin x \; dx = \left[-\cos x\right]_0^{\pi}$$

$$= -[\cos\pi - \cos 0] = -(-1-1)$$

$$A_1 = 2$$

2) Area $\quad A_2 \quad =$ area lying below the X-axis $= \int_{\pi}^{2\pi} \sin x\,dx = \left[-\cos x\right]_{\pi}^{2\pi} = [-\cos 2\pi - \cos\pi]$

$$= -[-1-(-1)]$$

$$A_2 = -2$$

$\therefore \quad$ Total area $= A_1 + |A_2| = 2 + |(-2)| = 4$ sq.units.

Activity :

Ex. 5 : Find the area enclosed between $y = \sin x$ and X-axis between 0 and 4π.

Ex. 6 : Find the area enclosed between $y = \cos x$ and X-axis between the limits :

(i) $\quad 0 \le x \le \frac{\pi}{2}$

(ii) $\quad \frac{\pi}{2} \le x \le \pi$

(iii) $\quad 0 \le x \le \pi$

SOLVED EXAMPLES

Ex. 1 : Using integration, find the area of the region bounded by the line $2y + x = 8$, X–axis and the lines $x = 2$ and $x = 4$.

Solution : The required region is bounded by the lines $2y + x = 8$, and $x = 2$, $x = 4$ and X–axis.

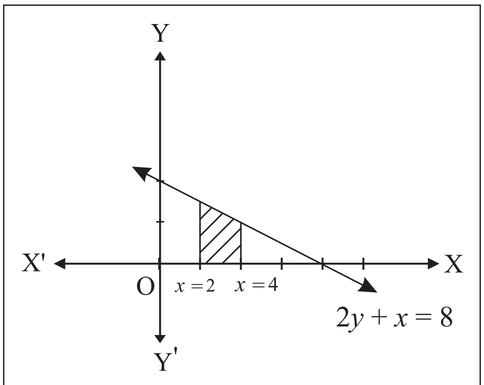

Fig. 5.11

$\therefore \quad y = \frac{1}{2}(8 - x)$ and the limits are $x = 2$, $x = 4$.

$$\begin{aligned}
\text{Required area} &= \text{Area of the shaded region} \\
&= \int_{x=2}^{4} y\, dx \\
&= \int_{2}^{4} \frac{1}{2}(8 - x)\, dx \\
&= \frac{1}{2}\left[8x - \frac{x^2}{2}\right]_2^4 \\
&= \frac{1}{2}\left[\left(8\cdot(4) - \frac{4^2}{2}\right) - \left(8\cdot(2) - \frac{2^2}{2}\right)\right] \\
&= 5 \text{ sq. units.}
\end{aligned}$$

Ex. 2 : Find the area of the regions bounded by the following curve, the X–axis and the given lines :

(i) $y = x^2$, $x = 1$, $x = 2$

(ii) $y^2 = 4x$, $x = 1$, $x = 4$, $y \geq 0$

(iii) $y = \sin x$, $x = -\frac{\pi}{2}$, $x = \frac{\pi}{2}$

Solution : Let A be the required area

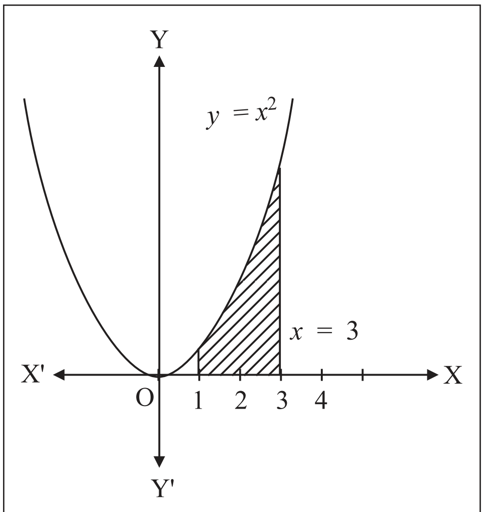

Fig. 5.12

$$\begin{aligned}
\text{(i)} \quad A &= \int_1^3 y\, dx \\
&= \int_1^3 x^2\, dx \\
&= \frac{1}{3}\left[x^3\right]_1^3 \\
&= \frac{1}{3}[27 - 1] \\
A &= \frac{26}{3} \text{ sq. units.}
\end{aligned}$$

(ii) $A = \int_1^4 y\, dx$

$= \int_1^4 2\sqrt{x}\ dx$

$= 2\int_1^4 x^{\frac{1}{2}}\, dx$

$= 2 \cdot \frac{2}{3}\left[x^{\frac{3}{2}}\right]_1^4 \qquad = \frac{4}{3}\left[4^{\frac{3}{2}} - 1\right]$

$A = \frac{28}{3}$ sq. units.

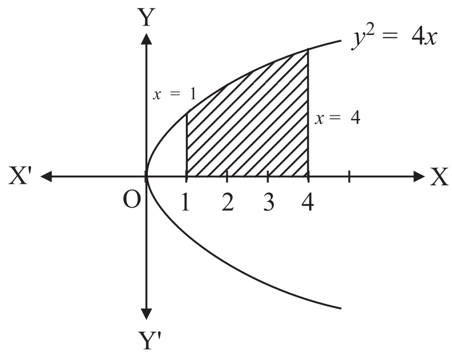

Fig. 5.13

(iii) $A = \int_{-\pi/2}^{\pi/2} y\ dx$

$= \int_{-\pi/2}^{\pi/2} \sin x\, dx$

$= \left|\int_{-\pi/2}^{0} \sin x\ \ dx\right| + \left|\int_{0}^{\pi/2} \sin x\ \ dx\right|$

$= \left|\left[-\cos x\right]_{-\frac{\pi}{2}}^{0}\right| + \left|\left[-\cos x\right]_0^{\frac{\pi}{2}}\right|$

$= \left|-\left[\cos 0 - \cos\left(\frac{\pi}{2}\right)\right]\right| + \left|\left[-\cos\left(\frac{\pi}{2}\right) + \cos 0\right]\right|$

$= |\,[-1-0] + [\,0+1\,] \;=\; 1+1$

$A = 2$ sq. units.

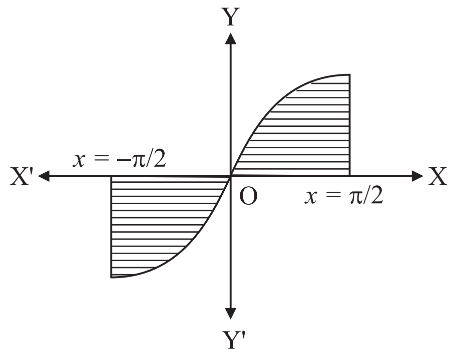

Fig. 5.14

Ex. 3 : Find the area of the region bounded by the parabola $y^2 = 16x$ and the line $x = 4$.

Solution : $y^2 = 16x \qquad \Rightarrow \qquad y = \pm 4\sqrt{x}$

A = Area POCP + Area QOCQ

= 2 (Area POCP)

$= 2\int_0^4 y\ \ dx$

$= 2\int_0^4 4\sqrt{x}\ dx$

$A = 8 \cdot \frac{2}{3}\left[x^{\frac{3}{2}}\right]_0^4 \qquad = \frac{16}{3} \times 8$

$A = \frac{128}{3}$ sq. units.

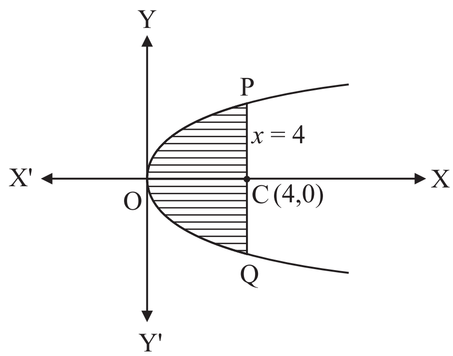

Fig. 5.15

Ex. 4 : Find the area of the region bounded by the curves $x^2 = 16y$, $y = 1$, $y = 4$ and the Y-axis, lying in the first quadrant.

Solution :

Required area $= \int_1^4 x \; dy$

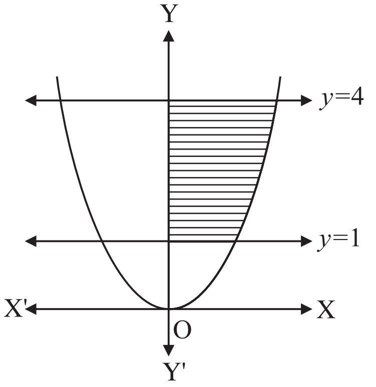

Fig. 5.16

$$A = \int_1^4 \sqrt{16\,y} \; dy$$

$$= 4\int_1^4 \sqrt{y} \;\; dy$$

$$= 4 \cdot \left[\frac{2}{3} \cdot y^{\frac{3}{2}}\right]_1^4$$

$$= \frac{8}{3} \times [\, 8 - 1 \,]$$

$$A = \frac{56}{3} \text{ sq. units.}$$

Ex. 5 : Find the area of the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$.

Solution : By the symmetry of the ellipse, required area of the ellipse is 4 times the area of the region OPQO. For this region the limit of integration are $x = 0$ and $x = a$.

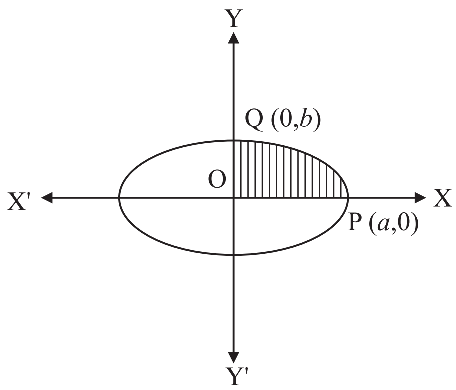

Fig. 5.17

From the equation of ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1$

$$\frac{y^2}{b^2} = 1 - \frac{x^2}{a^2}$$

$$y^2 = b^2 \cdot \left(\frac{a^2 - x^2}{a^2}\right)$$

$$y = \frac{b}{a} \cdot \sqrt{a^2 - x^2} \qquad \text{, In first quadrant, } y > 0$$

$$A = 4 \int_{x=0}^{a} y \, dx$$

$$= \int_0^a \frac{b}{a} \cdot \sqrt{a^2 - x^2} \;\; dx$$

$$= \frac{4b}{a} \cdot \left[\frac{x}{a} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1}\frac{x}{a}\right]_0^a$$

$$= \frac{4b}{a} \cdot \left[\frac{a^2}{2} \cdot \frac{\pi}{2} - 0\right]_0^a$$

$$A = \pi ab \text{ sq. units}$$

Ex. 6 : Find the area of the region lying between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$ where $a > 0$.

Solution : The equations of the parabolas are

$$y^2 = 4ax \qquad \ldots \text{(I)}$$

and $$x^2 = 4ay \qquad \ldots \text{(II)}$$

From (ii) $y = \dfrac{x^2}{4a}$ substitute in (I)

$$\left(\frac{x^2}{4a}\right)^2 = 4ax$$

$$\Rightarrow x^4 = 64a^3x$$

$$\therefore \quad x\,(x^3 - 64a^3) = 0$$

$$\therefore \quad x\,[x^3 - (4a^3)] = 0$$

$$\therefore \quad x = 0 \text{ and } x = 4a \qquad \therefore \quad y = 0 \text{ and } y = 4a$$

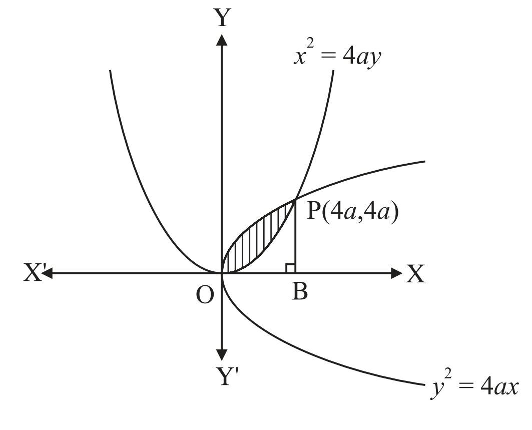

Fig. 5.18

The point of intersection of curves are O (0, 0), P ($4a$, $4a$)

$\therefore$ The required area is in the first quadrant and it is

A = area under the parabola ($y^2 = 4ax$) $-$ area under the parabola ($x^2 = 4ay$)

$$A = \int_0^{4a} \sqrt{4ax}\, dx - \int_0^{4a} \frac{x^2}{4a}\, dx \qquad = \sqrt{4a}\int_0^{4a} x^{\frac{1}{2}}\, dx - \int_0^{4a} \frac{x^2}{4a}\, dx$$

$$= 2\sqrt{a}\cdot\left[\frac{2}{3}\cdot x^{\frac{3}{2}}\right]_0^{4a} - \frac{1}{4a}\cdot\left[\frac{x^3}{3}\right]_0^{4a}$$

$$= \frac{4}{3}\sqrt{a}\cdot\left[4a\,\sqrt{4a} - \frac{1}{4a}\cdot 64a^3\right] = \frac{32}{3}\,a^2 - \frac{16}{3}\,a^2 \qquad \therefore\ A = \frac{16}{3}\,a^2 \text{ sq. units.}$$

Ex. 7 : Find the area of the region bounded by the curve $y = x^2$ and the line $y = 4$.

Solution : Required area A = 2 × area of OPQO

$$\therefore \quad A = \int_0^4 x\cdot dy$$

$$A = 2\cdot\int_0^4 \sqrt{y}\cdot dy$$

$$= 2\cdot\left[\frac{2}{3}\cdot y^{\frac{3}{2}}\right]_0^4 \qquad = \left(\frac{4}{3}\times 4^{\frac{3}{2}}\right)$$

$$= \frac{4}{3}\times 8$$

$$A = \frac{32}{3} \text{ sq. units.}$$

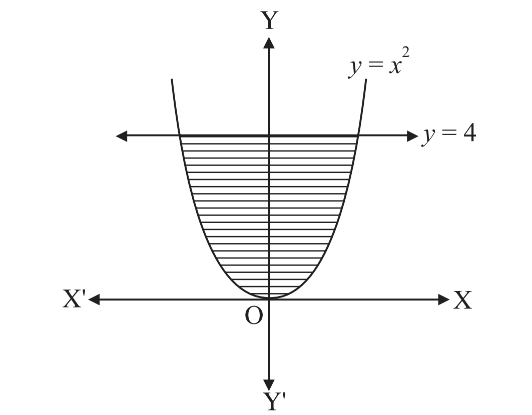

Fig. 5.19

Ex. 8 : Find the area of sector bounded by the circle $x^2 + y^2 = 16$ and the line $y = x$ in the first quadrant.

Solution : Required area A = A (ΔOCB) + A (region ABC)

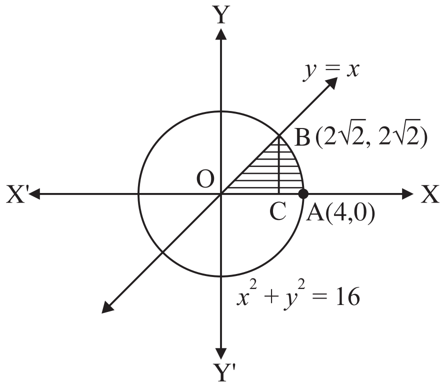

Fig. 5.20

To find,

The point of intersection of $\quad x^2 + y^2 = 16 \quad \ldots$ (I)

and line $\quad y = x \quad \ldots$ (II)

Substitute (II) in (I)

$$x^2 + x^2 = 16$$
$$2x^2 = 16$$
$$x^2 = 8$$
$$x = \pm 2\sqrt{2}, \qquad y = \pm 2\sqrt{2}$$

The point of intersection is B $(2\sqrt{2}, 2\sqrt{2})$

$$A = \int_0^{2\sqrt{2}} x \ dx + \int_{2\sqrt{2}}^{0} \sqrt{16 - x^2} \ dx = \frac{1}{2}\left[x^2\right]_0^{2\sqrt{2}} + \left[\frac{x}{2}\sqrt{16 - x^2} + \frac{16}{2}\sin^{-1}\frac{x}{4}\right]_{2\sqrt{2}}^{4}$$

$$= \frac{1}{2} \cdot (2\sqrt{2})^2 + \left[8 \sin^{-1} 1 - \left(\frac{2\sqrt{2}}{2}\sqrt{8} + 8 \sin^{-1}\frac{1}{2}\right)\right]$$

$$= 4 + 8 \cdot \frac{\pi}{2} - 4 - 8 \cdot \frac{\pi}{4} \qquad \therefore \quad A = 2\pi \text{ sq. units.}$$

Note that, the required area is $\frac{1}{8}$ times the area of the circle given.

EXERCISE 5.1

(1) Find the area of the region bounded by the following curves, X- axis and the given lines:

(i) $y = 2x, x = 0, x = 5$

(ii) $x = 2y, y = 0, y = 4$

(iii) $x = 0, \ x = 5, y = 0, y = 4$

(iv) $y = \sin x, \ x = 0, x = \frac{\pi}{2}$

(v) $xy = 2, \ x = 1, x = 4$

(vi) $y^2 = x, x = 0, x = 4$

(vii) $y^2 = 16x$ and $x = 0, x = 4$

(2) Find the area of the region bounded by the parabola :

(i) $y^2 = 16x$ and its latus rectum.

(ii) $y = 4 - x^2$ and the X-axis

(3) Find the area of the region included between:

(i) $y^2 = 2x$, line $y = 2x$

(ii) $y^2 = 4x$, line $y = x$

(iii) $y = x^2$ and the line $y = 4x$

(iv) $y^2 = 4ax$ and the line $y = x$

(v) $y = x^2 + 3$ and the line $y = x + 3$

Let us Remember

- The area A, bounded by the curve $y = f(x)$, X-axis and the lines $x = a$ and $x = b$ is given by

  $A = \int_a^b f(x)\, dx = \int_{x=a}^{x=b} f(x)\, dx$

  If the area A lies below the X-axis, then A is negative and in this case we take $|A|$.

- The area A of the region bounded by the curve $x = g(y)$, the Y axis, and the lines $y = c$ and $y = d$ is given by

  $A = \int_{y=c}^{d} x\, dy = \int_{y=c}^{d} g(y)\, dy$

- **Tracing of curve :**

  (i) X-axis is an axis of symmetry for a curve C, if $(x, y) \in C \Leftrightarrow (x, -y) \in C$.

  (ii) Y-axis is an axis of symmetry for a curve C, if $(x, y) \in C \Leftrightarrow (-x, y) \in C$.

  (iii) If replacing x and y by $-x$ and $-y$ respectively, the equation of the curve is unchanged then the curve is symmetric about X-axis and Y-axis.

MISCELLANEOUS EXERCISE 5

(I) Choose the correct option from the given alternatives :

(1) The area bounded by the region $1 \le x \le 5$ and $2 \le y \le 5$ is given by

(A) 12 sq. units (B) 8 sq. units (C) 25 sq. units (D) 32 sq. units

(2) The area of the region enclosed by the curve $y = \frac{1}{x}$, and the lines $x = e$, $x = e^2$ is given by

(A) 1 sq. unit (B) $\frac{1}{2}$ sq. unit (C) $\frac{3}{2}$ sq. units (D) $\frac{5}{2}$ sq. units

(3) The area bounded by the curve $y = x^3$, the X-axis and the lines $x = -2$ and $x = 1$ is

(A) -9 sq. units (B) $-\frac{15}{4}$ sq. units (C) $\frac{15}{4}$ sq. units (D) $\frac{17}{4}$ sq. units

(4) The area enclosed between the parabola $y^2 = 4x$ and line $y = 2x$ is

(A) $\frac{2}{3}$ sq. units (B) $\frac{1}{3}$ sq. units (C) $\frac{1}{4}$ sq. units (D) $\frac{3}{4}$ sq. units

(5) The area of the region bounded between the line $x = 4$ and the parabola $y^2 = 16x$ is

(A) $\frac{128}{3}$ sq. units (B) $\frac{108}{3}$ sq. units (C) $\frac{118}{3}$ sq. units (D) $\frac{218}{3}$ sq. units

(6) The area of the region bounded by $y = \cos x$, Y-axis and the lines $x = 0$, $x = 2\pi$ is

(A) 1 sq. unit (B) 2 sq. units (C) 3 sq. units (D) 4 sq. units

(7) The area bounded by the parabola $y^2 = 8x$ the X-axis and the latus rectum is

(A) $\frac{31}{3}$ sq. units (B) $\frac{32}{3}$ sq. units (C) $\frac{32\sqrt{2}}{3}$ sq. units (D) $\frac{16}{3}$ sq. units

(8) The area under the curve $y = 2\sqrt{x}$, enclosed between the lines $x = 0$ and $x = 1$ is

(A) 4 sq. units (B) $\frac{3}{4}$ sq. units (C) $\frac{2}{3}$ sq. units (D) $\frac{4}{3}$ sq. units

(9) The area of the circle $x^2 + y^2 = 25$ in first quadrant is

(A) $\frac{25\pi}{3}$ sq. units (B) 5π sq. units (C) 5 sq. units (D) 3 sq. units

(10) The area of the region bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ is

(A) ab sq. units (B) πab sq. units (C) $\frac{\pi}{ab}$ sq. units (D) πa^2 sq. units

(11) The area bounded by the parabola $y^2 = x$ and the line $2y = x$ is

(A) $\frac{4}{3}$ sq. units (B) 1 sq. units (C) $\frac{2}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

(12) The area enclosed between the curve $y = \cos 3x$, $0 \le x \le \frac{\pi}{6}$ and the X-axis is

(A) $\frac{1}{2}$ sq. units (B) 1 sq. units (C) $\frac{2}{3}$ sq. units (D) $\frac{1}{3}$ sq. units

(13) The area bounded by $y = \sqrt{x}$ and line $x = 2y + 3$, X-axis in first quadrant is

(A) $2\sqrt{3}$ sq. units (B) 9 sq. units (C) $\frac{34}{3}$ sq. units (D) 18 sq. units

(14) The area bounded by the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ and the line $\frac{x}{a} + \frac{y}{b} = 1$ is

(A) $\pi ab - 2ab$ (B) $\frac{\pi ab}{4} - \frac{ab}{2}$ (C) $\pi ab - ab$ (D) πab

(15) The area bounded by the parabola $y = x^2$ and the line $y = x$ is

(A) $\frac{1}{2}$ (B) $\frac{1}{3}$ (C) $\frac{1}{6}$ (D) $\frac{1}{12}$

(16) The area enclosed between the two parabolas $y^2 = 4x$ and $y = x$ is

(A) $\frac{8}{3}$ (B) $\frac{32}{3}$ (C) $\frac{16}{3}$ (D) $\frac{4}{3}$

(17) The area bounded by the curve $y = \tan x$, X-axis and the line $x = \frac{\pi}{4}$ is

(A) $\frac{1}{3}\log 2$ (B) $\log 2$ (C) $2\log 2$ (D) $3 \cdot \log 2$

(18) The area of the region bounded by $x^2 = 16y$, $y = 1$, $y = 4$ and $x = 0$ in the first quadrant, is

(A) $\frac{7}{3}$ (B) $\frac{8}{3}$ (C) $\frac{64}{3}$ (D) $\frac{56}{3}$

(19) The area of the region included between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$, $(a > 0)$ is given by

(A) $\frac{16\,a^2}{3}$ (B) $\frac{8\,a^2}{3}$ (C) $\frac{4\,a^2}{3}$ (D) $\frac{32\,a^2}{3}$

(20) The area of the region included between the line $x + y = 1$ and the circle $x^2 + y^2 = 1$ is

(A) $\frac{\pi}{2} - 1$ (B) $\pi - 2$ (C) $\frac{\pi}{4} - \frac{1}{2}$ (D) $\pi - \frac{1}{2}$

(II) Solve the following :

(1) Find the area of the region bounded by the following curve, the X-axis and the given lines

(i) $0 \le x \le 5$, $0 \le y \le 2$ (ii) $y = \sin x$, $x = 0$, $x = \pi$ (iii) $y = \sin x$, $x = 0$, $x = \frac{\pi}{3}$

(2) Find the area of the circle $x^2 + y^2 = 9$, using integration.

(3) Find the area of the ellipse $\frac{x^2}{25} + \frac{y^2}{16} = 1$ using integration.

(4) Find the area of the region lying between the parabolas.

(i) $y^2 = 4x$ and $x^2 = 4y$ (ii) $4y^2 = 9x$ and $3x^2 = 16y$ (iii) $y^2 = x$ and $x^2 = y$

(5) Find the area of the region in first quadrant bounded by the circle $x^2 + y^2 = 4$ and the x axis and the line $x = y\sqrt{3}$.

(6) Find the area of the region bounded by the parabola $y^2 = x$ and the line $y = x$ in the first quadrant.

(7) Find the area enclosed between the circle $x^2 + y^2 = 1$ and the line $x + y = 1$, lying in the first quadrant.

(8) Find the area of the region bounded by the curve $(y - 1)^2 = 4(x + 1)$ and the line $y = (x - 1)$.

(9) Find the area of the region bounded by the straight line $2y = 5x + 7$, X–axis and $x = 2$, $x = 5$.

(10) Find the area of the region bounded by the curve $y = 4x^2$, Y-axis and the lines $y = 1$, $y = 4$.

5. APPLICATION OF DEFINITE INTEGRAL

EXERCISE 5.1

(1) (i) 25 (ii) 16 (iii) 20

(iv) 1 (v) 2 log 4 (vi) $\frac{32}{3}$

(vii) $\frac{128}{3}$ sq. units

(2) (i) $\frac{128}{3}$ (ii) $\frac{16}{3}$

(3) (i) $\frac{1}{12}$ (ii) $\frac{8}{3}$ (iii) $\frac{32}{3}$

(iv) $8\frac{a^2}{3}$ (v) $\frac{1}{6}$

MISCELLANEOUS EXERCISE 5

(I)

1	2	3	4	5	6	7	8	9	10
A	A	C	B	A	D	B	D	A	B

11	12	13	14	15	16	17	18	19	20
A	D	B	B	C	C	A	D	A	C

(II)

1. (i) 10 (ii) 2 (iii) $\frac{1}{2}$

2. 9π

3. 20π

4. (i) $\frac{16}{3}$ (ii) $\frac{8}{3}$ (iii) $\frac{1}{3}$

5. $\frac{\pi}{3}$

6. $\frac{1}{6}$

7. $\frac{\pi}{4}-\frac{1}{2}$

8. $\frac{56}{3}$

9. $36\frac{3}{4}$

10. $\frac{7}{3}$

6. DIFFERENTIAL EQUATIONS

EXERCISE 6.1

(1) (i) 2, 1 (ii) 2, 3 (iii) 1, 2

(iv) 3, 1 (v) 2, 1 (vi) 3, 2

(vii) 2, not definded (viii) 2, 2

(ix) 3, 3 (x) 2, 1

EXERCISE 6.2

(1) (i) $2x^3 + 3xy^2\frac{dy}{dx} - y^3 = 0$

(ii) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - y\frac{dy}{dx} = 0$

(iii) $x^2\frac{d^2y}{dx^2} + x\frac{dy}{dx} + y = 0$ (iv) $8\left(\frac{dy}{dx}\right)^3 - 27y = 0$

(v) $\frac{d^2y}{dx^2} - 25y = 0$ (vi) $2\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

(vii) $(x^2+xy)\frac{dy}{dx} + y = 0$ (viii) $\frac{d^2y}{dx^2} - 7\frac{dy}{dx} + 10y = 0$

(ix) $xy\frac{d^2y}{dx^2} + x\left(\frac{dy}{dx}\right)^2 - 2y\frac{dy}{dx} = 0$

(x) $\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 5y = 0$

(2) $y = \frac{dy}{dx}(x-a)$ (3) $2a\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^3 = 0$

(4) $x + 4y\frac{dy}{dx} = 0$ (5) $3\frac{dy}{dx} + 2 = 0$

(6) $81\left(\frac{d^2y}{dx^2}\right)^2 = \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^3$

(7) $y\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

EXERCISE 6.3

(2) (i) $\tan^{-1} y = \tan^{-1} x + c$

(ii) $2e^{-3y} + 3e^{2x} = c$ (iii) $x = cy$

(iv) $\tan x \cdot \tan y = c$ (v) $\sin y \cdot \cos x = c$

(vi) $y = -kx + c$

(vii) $2(x^2 + y^2) + 2(x\sin 2x + y\sin 2y) + \cos 2y + \cos 2x + c = 0$

(viii) $2y^2 \tan^{-1} x + 1 = cy^2$

(ix) $4e^x + 3e^{-2y} = c$

(x) $3e^x + 3e^{-y} + x^3 = c$

(3) (i) $(1 + e^x)^3 \tan y = 0$

(ii) $(1 + x^2)(1 - y^2) = 5$

(iii) $y = ex\log x$ (iv) $(\sin x)(e^y + 1) = \sqrt{2}$

(v) $2(2 + e^y) = 3(x + 1)$

(vi) $\cos\left(\frac{y-2}{x}\right) = a$

(4) (i) $\tan\left(\frac{x+y}{2}\right) = x + c$

(ii) $c + 2y = a\log\left(\frac{x-y-a}{x-y+a}\right)$

(iii) $\sin(x^2 + y^2) + 2x = c$

(iv) $x = \tan(x - 2y) + c$

(v) $(2x - y) - \log(x - y + 2) + 1 = 0$

EXERCISE 6.4

(1) $\cos\left(\frac{y}{x}\right) dy = \log(x) + c$

(2) $x^2 - y^2 = cx$ (3) $x + 2ye^{\frac{x}{y}} = c$

(4) $xy^2 = c^2(x + 2y)$ (5) $x^2 + y^2 = cx$

(6) $y = c(x + y)^3 + x$

(7) $x\left[1 - \cos\left(\frac{y}{x}\right)\right] = \sin\left(\frac{y}{x}\right)$

(8) $x + ye^{\frac{x}{y}} = c$ (9) $\log(y) + \frac{y}{x} = c$

(10) $x^2y = 4$ (11) $x^2 + y^2 = x^4$

(12) $\tan^{-1}\left(\frac{y}{x}\right) = \log(x) + c$

(13) $(3x + y)^3(x + y)^2 = c$

(14) $c = \log(x) + \frac{x}{x+y}$ (15) $x^2 - y^2 = cx$

EXERCISE 6.5

1. (i) $\frac{x^5}{5} - \frac{3x^2}{2} - xy = c$

(ii) $ye^{\tan x} = e^{\tan x}(\tan x - 1) + c$

(iii) $x = y(c + y^2)$

(iv) $y(\sec x + \tan x) = \sec x + \tan x - x + c$

(v) $x^2 y = \frac{x^4 \log x}{4} - \frac{x^4}{16} + c$

(vi) $x + y + 1 = ce^y$

(vii) $2y = (x + a)^5 + 2c(x + a)^3$

(viii) $r\sin^2\theta + \frac{\sin^4\theta}{2} = c$

(ix) $\frac{y^3}{3} = xy + c$

(x) $y = \sqrt{1 - x^2} + c(1 - x^2)$

(xi) $y = \frac{1}{2}e^{\tan^{-1}x} + c\,e^{-\tan^{-1}x}$

2. $3(x+3y) = 2(1-e^{3x})$

3. $4x^2 + 9y^2 = 36$

4. $y = 4 - x - 2e^x$

5. $1 + y = 2e^{\frac{x^2}{2}}$

EXERCISE 6.6

1. 8 times of original.

2. 95·4 years

3. 36·36°c

4. 5656

5. $\dfrac{\log 3}{k}$

6. $\dfrac{27}{5}$ gms

7. $(3000)\left(\dfrac{4}{9}\right)^{\frac{t}{40}}$

8. 1 hour

10. $r = 3 - t$

11. 27,182

12. $\left(10 - \dfrac{p}{10}\right)^2 \%$

MISCELLANEOUS EXERCISE 6

(I)

1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
D	A	C	B	A	D	C	B	C	D	B	A	B	B	B

(II) (1) (i) 2, 1 (ii) 3, 10 (iii) 2 , 3 (iv) 1.4 (v) 4, not defined

(3) (i) $xy\dfrac{d^2y}{dx^2} + x\left(\dfrac{dy}{dx}\right)^2 - 2y\dfrac{dy}{dx} = 0$ (ii) $\dfrac{d^2y}{dx^2} + y = 0$ (iii) $(y-a)\dfrac{d^2y}{dx^2} + \left(\dfrac{dy}{dx}\right)^2 = 0$

(iv) $2x^2y\dfrac{d^2y}{dx^2} + 2x^2\left(\dfrac{dy}{dx}\right)^2 + y = 0$ (v) $\dfrac{d^2y}{dx^2} - 9y = 0$

(4) (i) $2xy\dfrac{dy}{dx} + x^2 - y^2 = 0$ (ii) $2b\dfrac{d^2y}{dx^2} - 1 = 0$ (iii) $x + 4y\dfrac{dy}{dx} = 0$ (iv) $2\dfrac{dy}{dx} - 3 = 0$

(v) $4y\dfrac{dy}{dx} - 9x = 0$

(5) (i) $2e^{-3y} + 3e^{2x} + 6c = 0$ (ii) $\log(y) = \dfrac{x^3}{3} + x + c$ (iii) $y = \dfrac{x}{2}\log(x^2) + 2 + cx$

(iv) $y = 1 + x\log x + cx$ (v) $y = x^2 + c\cdot\operatorname{cosec} x$ (vi) $x\log y = (\log y)^2 + c$

(vii) $4xe^{2y} + 5e^{-y} = c$

(6) (i) $ex\log x - y = 0$ (ii) $x = 2y^2$ (iii) $y\operatorname{cosec}^2 x + 2 = 4\sin 2x$

(iv) $\log\sqrt{x^2+y^2} + \tan^{-1}\left(\dfrac{y}{x}\right) = \dfrac{\pi}{4}$ (v) $x + 2ye^{\frac{x}{y}} = 2$

(8) $x^2 + y^2 = 4x + 5$ (9) $r = (63t + 27)^{\frac{1}{3}}$ (10) $\dfrac{20}{9}$ years