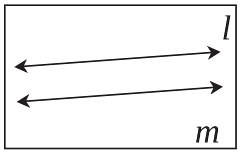
2 Parallel Lines

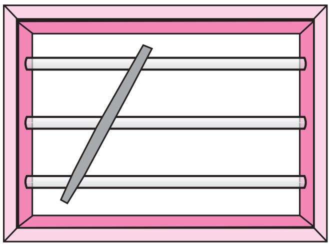
Let's study.

- Properties of angles formed by parallel lines and its transversal
- Tests of parallelness of two lines
- Use of properties of parallel lines

Let's recall.

Parallel lines : The lines which are coplanar and do not intersect each other are called parallel lines.





Hold a stick across the horizontal parallel bars of a window as shown in the figure. How many angles are formed ?

- Do you recall the pairs of angles formed by two lines and their transversal ?

  In figure 2.1, line *n* is a transversal of line *l* and line *m*.

  Here, in all 8 angles are formed. Pairs of angles formed out of these angles are as follows :

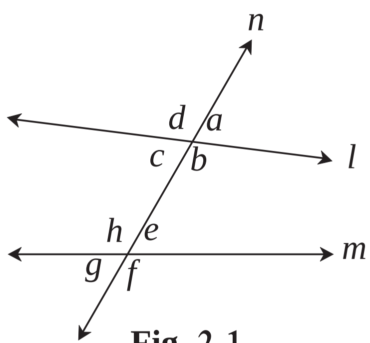


Fig. 2.1

Pairs of corresponding angles

(i) $\angle d$, $\angle h$

(ii) $\angle a$, $\square$

(iii) $\angle c$, $\square$

(iv) $\angle b$, $\square$

Pairs of alternate interior angles

(i) $\angle c$, $\angle e$ (ii) $\angle b$, $\angle h$

Pairs of alternate exerior angles

(i) $\angle d$, $\angle f$

(ii) $\angle a$, $\angle g$

Pairs of interior angles on the same side of the transversal

(i) $\angle c$, $\angle h$

(ii) $\angle b$, $\angle e$

Some important properties :

(1) When two lines intersect, the pairs of opposite angles formed are congruent.

(2) The angles in a linear pair are supplementary.

(3) When one pair of corresponding angles is congruent, then all the remaining pairs of corresponding angles are congruent.

(4) When one pair of alternate angles is congruent, then all the remaining pairs of alternate angles are congruent.

(5) When one pair of interior angles on one side of the transversal is supplementary, then the other pair of interior angles is also supplementary.

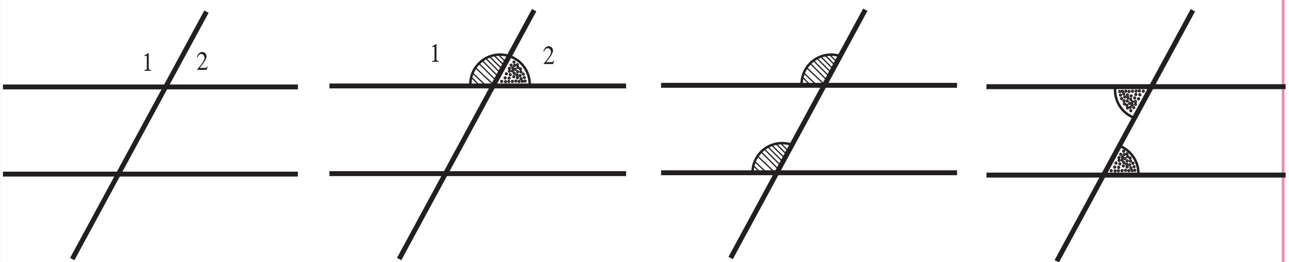
Let's learn.

Properties of parallel lines

Activity

To verify the properties of angles formed by a transversal of two parallel lines.

Take a piece of thick coloured paper. Draw a pair of parallel lines and a transversal on it. Paste straight sticks on the lines. Eight angles will be formed. Cut pieces of coloured paper, as shown in the figure, which will just fit at the corners of $\angle 1$ and $\angle 2$. Place the pieces near different pairs of corresponding angles, alternate angles and interior angles and verify their properties.



Let's learn.

We have verified the properties of angles formed by a transversal of two parallel lines. Let us now prove the properties using Euclid's famous fifth postulate given below.

If sum of two interior angles formed on one side of a transversal of two lines is less than two right angles then the lines produced in that direction intersect each other.

Interior angle theorem

Theorem : If two parallel lines are intersected by a transversal, the interior angles on either side of the transversal are supplementary.

Given : line $l \parallel$ line m and line n is their transversal. Hence as shown in the figure $\angle a$, $\angle b$ are interior angles formed on one side and $\angle c$, $\angle d$ are interior angles formed on other side of the transversal.

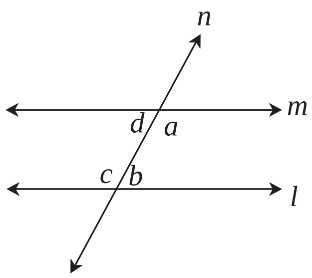


Fig. 2.2

To prove : $\angle a + \angle b = 180^\circ$

$\angle d + \angle c = 180^\circ$

Proof : Three possibilities arise regarding the sum of measures of $\angle a$ and $\angle b$.

(i) $\angle a + \angle b < 180^\circ$ (ii) $\angle a + \angle b > 180^\circ$ (iii) $\angle a + \angle b = 180^\circ$

Let us assume that the possibility (i) $\angle a + \angle b < 180^\circ$ is true.

Then according to Euclid's postulate, if the line l and line m are produced will intersect each other on the side of the transversal where $\angle a$ and $\angle b$ are formed.

But line l and line m are parallel lines ....given

$\therefore\ \angle a + \angle b < 180^\circ$ impossible ....(I)

Now let us suppose that $\angle a + \angle b > 180^\circ$ is true.

$\therefore\ \angle a + \angle b > 180^\circ$

But $\angle a + \angle d = 180^\circ$

and $\angle c + \angle b = 180^\circ$ . . . . . angles in linear pairs

$\therefore\ \angle a + \angle d + \angle b + \angle c = 180^\circ + 180^\circ = 360^\circ$

$\therefore \angle c + \angle d = 360^\circ - (\angle a + \angle b)$

If $\angle a + \angle b > 180^\circ$ then $[360^\circ - (\angle a + \angle b)] < 180^\circ$

$\therefore \angle c + \angle d < 180^\circ$

$\therefore$ In that case line l and line m produced will intersect each other on the same side of the transversal where $\angle c$ and $\angle d$ are formed.

$\therefore \angle c + \angle d < 180$ is impossible.

That is $\angle a + \angle b > 180^\circ$ is impossible...... (II)

$\therefore$ the remaining possibility,

$\angle a + \angle b = 180^\circ$ is true......from (I) and (II)

$\therefore$ $\angle a + \angle b = 180^\circ$ Similarly, $\angle c + \angle d = 180^\circ$

Note that, in this proof, because of the contradictions we have denied the possibilities $\angle a + \angle b > 180^\circ$ and $\angle a + \angle b < 180^\circ$.

Therefore, this proof is an example of indirect proof.

Corresponding angles and alternate angles theorems

Theorem : The corresponding angles formed by a transversal of two parallel lines are of equal measure.

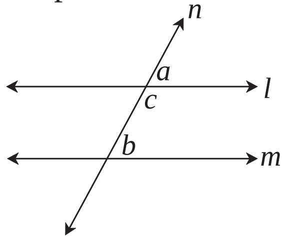


Fig. 2.3

Given : line l || line m
line n is a transversal.

To prove : $\angle a = \angle b$

Proof : $\angle a + \angle c = 180^\circ$ ....(I) angles in linear pair

$\angle b + \angle c = 180^\circ$ ....(II) property of interior angles of parallel lines

$\angle a + \angle c = \angle b + \angle c$ ....from (I) and (II)

$\therefore$ $\angle a = \angle b$

Theorem : The alternate angles formed by a transversal of two parallel lines are of equal measures.

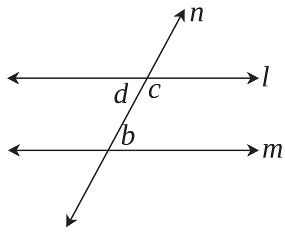


Fig. 2.4

Given : line l || line m
line n is a transversal.

To prove : $\angle d = \angle b$

Proof : $\angle d + \angle c = 180^\circ$................(I) angles in linear pair

$\angle c + \angle b = 180^\circ$ ....(II) property of interior angles of parallel line

$\angle d + \angle c = \angle c + \angle b$................from (I) and (II)

$\therefore$ $\angle d = \angle b$

Practice set 2.1

1. In figure 2.5, line RP || line MS and line DK is their transversal. $\angle DHP = 85^\circ$
Find the measures of following angles.
(i) $\angle RHD$ (ii) $\angle PHG$
(iii) $\angle HGS$ (iv) $\angle MGK$

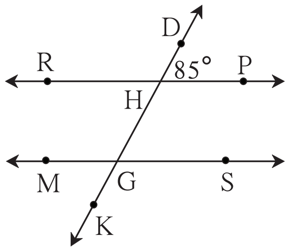


Fig. 2.5

2. In figure 2.6, line p || line q and line l and line m are transversals. Measures of some angles are shown. Hence find the measures of $\angle a$, $\angle b$, $\angle c$, $\angle d$.

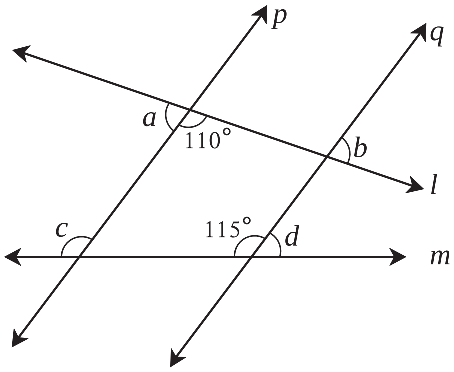


Fig. 2.6

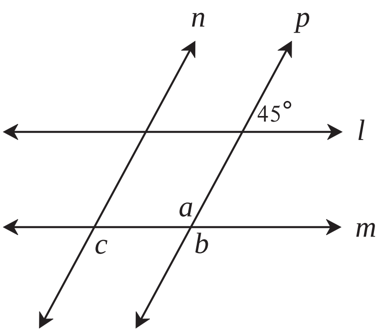


Fig. 2.7

3. In figure 2.7, line l || line m and line n || line p. Find $\angle a$, $\angle b$, $\angle c$ from the given measure of an angle.

4*. In figure 2.8, sides of $\angle PQR$ and $\angle XYZ$ are parallel to each other. Prove that, $\angle PQR \cong \angle XYZ$

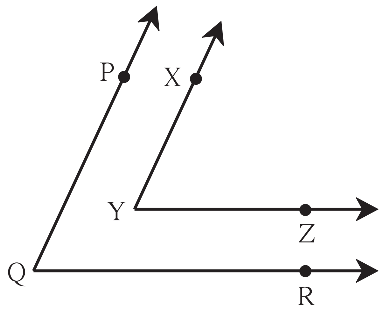


Fig. 2.8

5. In figure 2.9, line AB || line CD and line PQ is transversal. Measure of one of the angles is given.
Hence find the measures of the following angles.

(i) $\angle$ART (ii) $\angle$CTQ

(iii) $\angle$DTQ (iv) $\angle$PRB

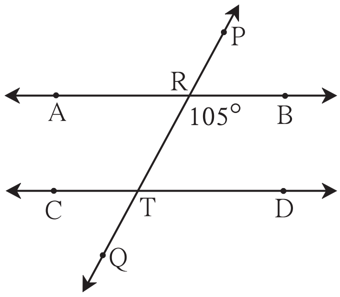


Fig. 2.9

Let's learn.

Use of properties of parallel lines

Let us prove a property of a triangle using the properties of angles made by a transversal of parallel lines.

Theorem : The sum of measures of all angles of a triangle is 180°.

Given : Δ ABC is any triangle.

To prove : $\angle$ABC + $\angle$ACB + $\angle$BAC = 180°.

Construction : Draw a line parallel to seg BC and passing through A. On the line take points P and Q such that, P - A - Q.

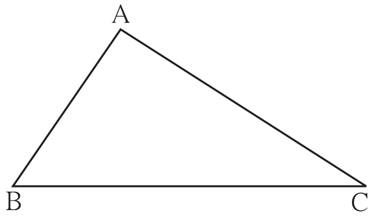


Fig. 2.10

Proof : Line PQ || line BC and seg AB is a transversal.

$\therefore$ $\angle$ABC = $\angle$PAB.......alternate angles.....(I)

line PQ || line BC and seg AC is a transversal.

$\therefore$ $\angle$ACB = $\angle$QAC.......alternate angles.....(II)

$\therefore$ From I and II ,

$\angle$ABC + $\angle$ACB = $\angle$PAB + $\angle$QAC . . . (III)

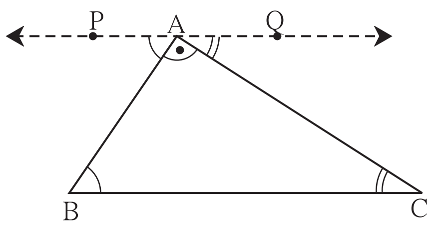


Fig. 2.11

Adding $\angle$BAC to both sides of (III).

$\angle$ABC + $\angle$ACB + $\angle$BAC = $\angle$PAB + $\angle$QAC + $\angle$BAC

= $\angle$PAB + $\angle$BAC + $\angle$QAC

= $\angle$PAC + $\angle$QAC ...($\because$ $\angle$PAB + $\angle$BAC = $\angle$PAC)

= 180° ...Angles in linear pair

That is, sum of measures of all three angles of a triangle is 180°.

Let's discuss.

In fig. 2.12, How will you decide whether line l and line m are parallel or not ?

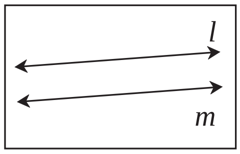


Fig. 2.12

Let's learn.

Tests for parallel lines

Whether given two lines are parallel or not can be decided by examining the angles formed by a transversal of the lines.

(1) If the interior angles on the same side of a transversal are supplementary then the lines are parallel.

(2) If one of the pairs of alternate angles is congruent then the lines are parallel.

(3) If one of the pairs of corresponding angles is congruent then the lines are parallel.

Interior angles test

Theorem : If the interior angles formed by a transversal of two distinct lines are supplementary, then the two lines are parallel.

Given : Line XY is a transversal of line AB and line CD.

$\angle BPQ + \angle PQD = 180°$

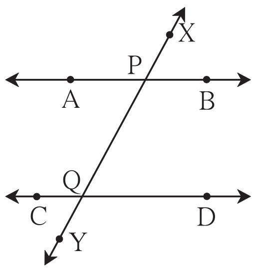


Fig. 2.13

To prove : line AB $\|$ line CD

Proof : We are going to give an indirect proof.

Let us suppose that the statement to be proved is wrong. That is, we assume,

line AB and line CD are not parallel,

means line AB and CD intersect at point T.

So Δ PQT is formed.

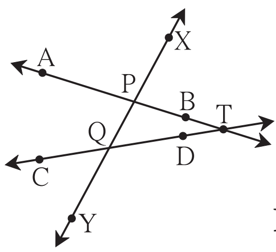


Fig. 2.14

$\therefore$ $\angle TPQ + \angle PQT + \angle PTQ = 180°$ ....sum of angles of a triangle

but $\angle TPQ + \angle PQT = 180°$ ....given

That is the sum of two angles of the triangle is $180°$.

But sum of three angles of a triangle is $180°$.

$\therefore$ $\angle PTQ = 0°$.

$\therefore$ line PT and line QT means line AB and line CD are not distinct lines.

But, we are given that line AB and line CD are distinct lines.

$\therefore$ we arrive at a contradiction.

$\therefore$ our assumption is wrong. Hence line AB and line CD are parallel.

Thus it is proved that if the interior angles formed by a transversal are supplementary, then the lines are parallel.

This property is called **interior angles test** of parallel lines.

Alternate angles test

Theorem : If a pair of alternate angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of alternate angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180°$ . ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore \angle b + \angle c = 180°$

But $\angle b$ and $\angle c$ are interior angles on the same side of the transversal.

$\therefore$ line $l \parallel$ line m .... interior angles test

This property is called the **alternate angles test** of parallel lines.

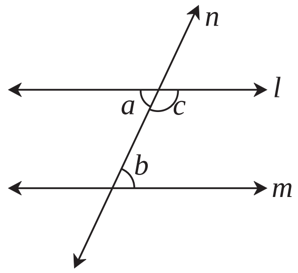


Fig. 2.15

Corresponding angles Test

Theorem : If a pair of corresponding angles formed by a transversal of two lines is congruent then the two lines are parallel.

Given : Line n is a transversal of line l and line m.

$\angle a$ and $\angle b$ is a congruent pair of corresponding angles.

That is, $\angle a = \angle b$

To prove : line $l \parallel$ line m

Proof : $\angle a + \angle c = 180°$ ....angles in linear pair

$\angle a = \angle b$ .... given

$\therefore \angle b + \angle c = 180°$

That is a pair of interior angles on the same side of the transversal is congruent.

$\therefore$ line $l \parallel$ line m ....interior angles test

This property is called the **corresponding angles test** of parallel lines.

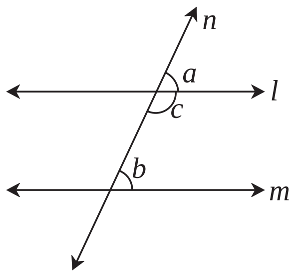


Fig. 2.16

Corollary I : If a line is perpendicular to two lines in a plane, then the two lines are parallel to each other.

Given : Line $n \perp$ line l and line $n \perp$ line m

To prove : line $l \parallel$ line m

Proof : line $n \perp$ line l and line $n \perp$ line m ...given

$\therefore \angle a = \angle c = 90°$

$\angle a$ and $\angle c$ are corresponding angles formed by transversal n of line l and line m.

$\therefore$ line $l \parallel$ line m ....corresponding angles test

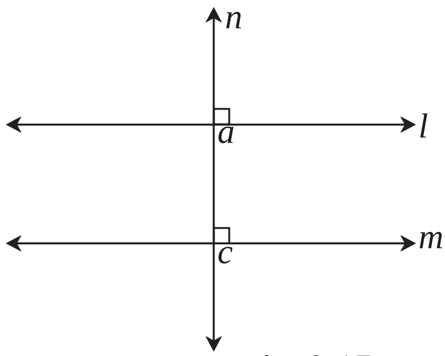


Fig. 2.17

Corollary II : If two lines in a plane are parallel to a third line in the plane then those two lines are parallel to each other. Write the proof of the corollary.

Practice set 2.2

1. In figure 2.18, $y = 108°$ and $x = 71°$
Are the lines m and n parallel ? Justify ?

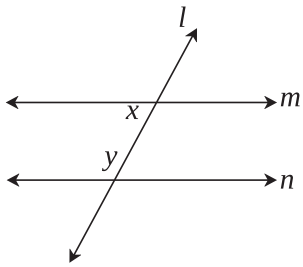


Fig. 2.18

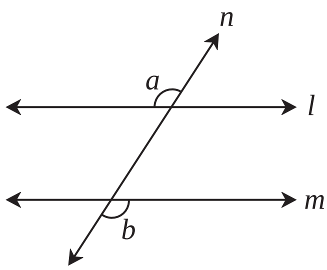


Fig. 2.19

2. In figure 2.19, if $\angle a \cong \angle b$ then prove that line $l \parallel$ line m.

3. In figure 2.20, if $\angle a \cong \angle b$ and $\angle x \cong \angle y$ then prove that line $l \parallel$ line n.

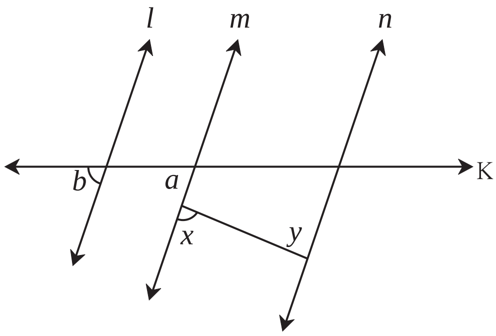


Fig. 2.20

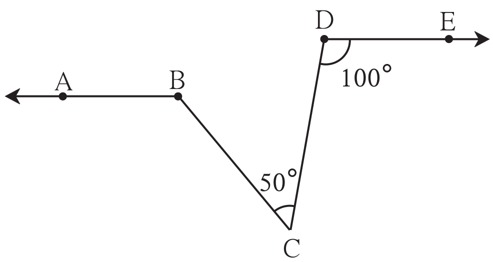


Fig. 2.21

4. In figure 2.21, if ray BA $\parallel$ ray DE, $\angle C = 50°$ and $\angle D = 100°$. Find the measure of $\angle ABC$.
(Hint **:** Draw a line passing through point C and parallel to line AB.)

5.

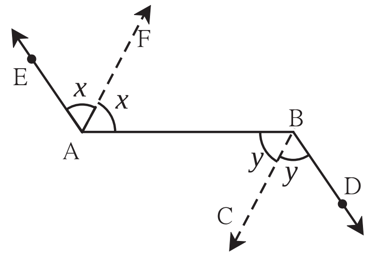


Fig. 2.22

In figure 2.22, ray AE || ray BD, ray AF is the bisector of $\angle$EAB and ray BC is the bisector of $\angle$ABD. Prove that line AF || line BC.

6. A transversal EF of line AB and line CD intersects the lines at point P and Q respectively. Ray PR and ray QS are parallel and bisectors of $\angle$BPQ and $\angle$PQC respectively.
Prove that line AB || line CD.

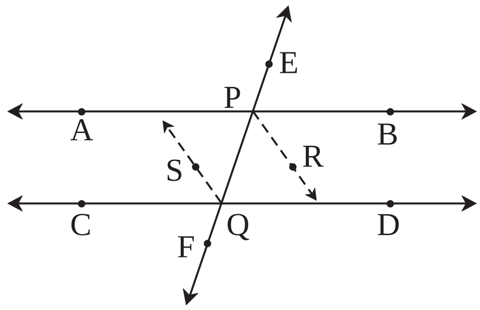


Fig. 2.23

Problem set 2

1. Select the correct alternative and fill in the blanks in the following statements.

(i) If a transversal intersects two parallel lines then the sum of interior angles on the same side of the transversal is ....

(A) 0° (B) 90° (C) 180° (D) 360°

(ii) The number of angles formed by a transversal of two lines is ....

(A) 2 (B) 4 (C) 8 (D) 16

(iii) A transversal intersects two parallel lines. If the measure of one of the angles is 40° then the measure of its corresponding angle is ....

(A) 40° (B) 140° (C) 50° (D) 180°

(iv) In Δ ABC, $\angle A = 76^\circ$, $\angle B = 48^\circ$, $\therefore$ $\angle C =$ ....

(A) 66° (B) 56° (C) 124° (D) 28°

(v) Two parallel lines are intersected by a transversal. If measure of one of the alternate interior angles is 75° then the measure of the other angle is ....

(A) 105° (B) 15° (C) 75° (D) 45°

2*. Ray PQ and ray PR are perpendicular to each other. Points B and A are in the interior and exterior of $\angle$QPR respectively. Ray PB and ray PA are perpendicular to each other. Draw a figure showing all these rays and write -

(i) A pair of complementary angles (ii) A pair of supplementary angles.

(iii) A pair of congruent angles.

3. Prove that, if a line is perpendicular to one of the two parallel lines, then it is perpendicular to the other line also.

4. In figure 2.24, measures of some angles are shown. Using the measures find the measures of $\angle x$ and $\angle y$ and hence show that line $l \parallel$ line m.

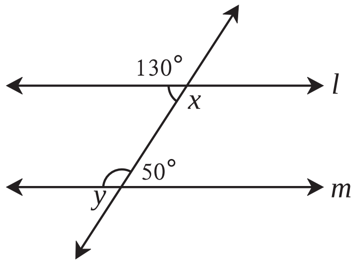


Fig. 2.24

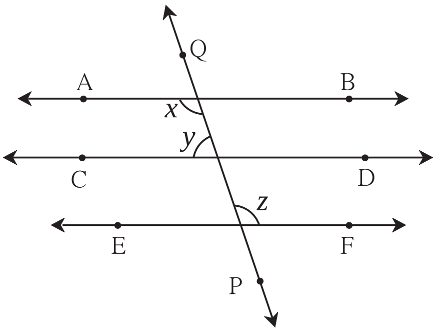


Fig. 2.25

5. Line AB $\parallel$ line CD $\parallel$ line EF and line QP is their transversal. If $y : z = 3 : 7$ then find the measure of $\angle x$. (See figure 2.25.)

6. In figure 2.26, if line $q \parallel$ line r, line p is their transversal and if $a = 80°$ find the values of f and g.

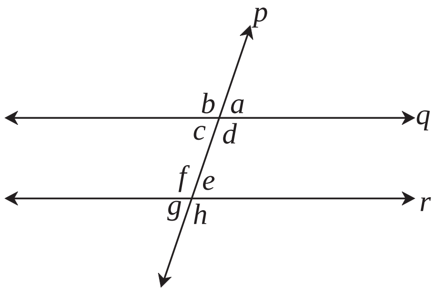


Fig. 2.26

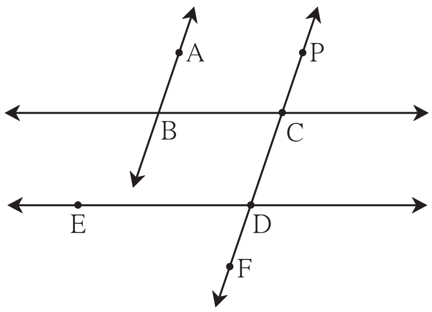


Fig. 2.27

7. In figure 2.27, if line AB $\parallel$ line CF and line BC $\parallel$ line ED then prove that $\angle$ABC = $\angle$FDE.

8. In figure 2.28, line PS is a transversal of parallel line AB and line CD. If Ray QX, ray QY, ray RX, ray RY are angle bisectors, then prove that $\square$ QXRY is a rectangle.

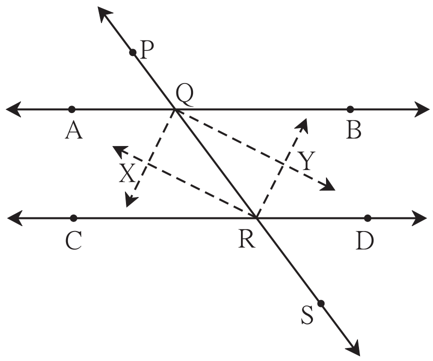


Fig. 2.28

2. Parallel Lines

Practice set 2.1

1. (i) 95° (ii) 95° (iii) 85° (iv) 85°

2. $\angle a = 70^\circ, \angle b = 70^\circ, \angle c = 115^\circ, \angle d = 65^\circ$

3. $\angle a = 135^\circ, \angle b = 135^\circ, \angle c = 135^\circ$

5. (i) 75° (ii) 75° (iii) 105° (iv) 75°

Practice set 2.2

1. No. **4.** $\angle ABC = 130^\circ$

Problem set 2

1. (i) C (ii) C (iii) A (iv) B (v) C **4.** $x = 130^\circ$ $y = 50^\circ$

5. $x = 126^{0\circ}$ **6.** $f = 100^\circ$ $g = 80^\circ$

3. Triangles

Practice set 3.1

1. 110° **2.** 45° **3.** $80^\circ, 60^\circ, 40^\circ$ **4.** $30^\circ, 60^\circ, 90^\circ$

5. $60^\circ, 80^\circ, 40^\circ$ **6.** $\angle DRE = 70^\circ, \angle ARE = 110^\circ$

7. $\angle AOB = 125^\circ$ **9.** $30^\circ, 70^\circ, 80^\circ$

Practice set 3.2

1. (i) SSC Test (ii) SAS Test (iii) ASA Test (iv) Hypotenuse Side Test.

2. (i) ASA Test, $\angle BAC \cong \angle QPR$,side AB $\cong$ side PQ, side AC $\cong$ side PR
(ii) SAS Test, $\angle TPQ \cong \angle TSR$, $\angle TQP \cong \angle TRS$, side PQ $\cong$ side SR

3. Hypotenuse Side Test, $\angle ACB \cong \angle QRP$, $\angle ABC \cong \angle QPR$, side AC $\cong$ side QR

4. SSS Test, $\angle MLN \cong \angle MPN$, $\angle LMN \cong \angle MNP$, $\angle LNM \cong \angle PMN$

Practice set 3.3

1. $x = 50^\circ$, $y = 60^\circ$, $m\angle ABD = 110^\circ$, $m\angle ACD = 110^\circ$.

2. 7.5 Units **3.** 6.5 Units **4.** l(PG) = 5 cm , l(PT) = 7.5 cm

Practice set 3.4

1. 2 cm **2.** 28° **3.** $\angle QPR, \angle PQR$ **4.** greatest side NA, smallest side FN

Practice set 3.5

1. $\dfrac{XY}{LM} = \dfrac{YZ}{MN} = \dfrac{XZ}{LN}$, $\angle X \cong \angle L$, $\angle Y \cong \angle M$, $\angle Z \cong \angle N$

2. l(QR) = 12 cm, l(PR) = 10 cm