6. Superposition of Waves

Can you recall?

1. What is wave motion?
2. What is a wave pulse?
3. What are common properties of waves?
4. What happens when a wave propagates?
5. What are mechanical waves?
6. What are electromagnetic waves?
7. How are mechanical waves different from electromagnetic waves?
8. What are sound waves?

6.1 Introduction:

You may be familiar with different waves like water waves, sound waves, light waves, mechanical waves, electromagnetic waves etc. A mechanical wave is a disturbance produced in an elastic medium due to periodic vibrations of particles of the medium about their respective mean positions. In this process, energy and momentum are transferred from one particle to another. Thus, a wave carries or transfers energy from one point to another., but there is no transfer of matter or particles of the medium in which the wave is travelling. Another type of waves, known as electromagnetic waves, do not require material medium for their propagation; these are non-mechanical waves. We have studied sound waves (which are mechanical waves), their properties and various phenomena like echo, reverberation, Doppler effect related to these waves in earlier classes. In this Chapter, we will study mechanical waves, reflection of these waves, principle of superposition of waves, various phenomena like formation of stationary waves, beats, and their applications.

6.2 Progressive Wave:

Have you seen ripples created on the surface of water when a stone is dropped in it? The water is displaced locally where the stone actually falls in water. The disturbance slowly spreads and distant particles get disturbed from their position of rest. The wave disturbs the particles for a short duration during its path. These particles oscillate about their position of rest for a short time. They are not bodily moved from their respective positions. This disturbance caused by the stone is actually a wave pulse. It is a disturbance caused locally for a short duration.

A wave, in which the disturbance produced in the medium travels in a given direction continuously, without any damping and obstruction, from one particle to another, is a progressive wave or a travelling wave e.g., the sound wave, which is a pressure wave consisting of compressions and rarefactions travelling along the direction of propagation of the wave.

6.2.1 Properties of progressive waves:

1) Each particle in a medium executes the same type of vibration. Particles vibrate about their mean positions performing simple harmonic motion.
2) All vibrating particles of the medium have the same amplitude, period and frequency.
3) The phase, (i.e., state of vibration of a particle), changes from one particle to another.
4) No particle remains permanently at rest. Each particle comes to rest momentarily while at the extreme positions of vibration.
5) The particles attain maximum velocity when they pass through their mean positions.
6) During the propagation of wave, energy is transferred along the wave. There is no transfer of matter.
7) The wave propagates through the medium

with a certain velocity. This velocity depends upon properties of the medium.

8) Progressive waves are of two types - transverse waves and longitudinal waves.

9) In a transverse wave, vibrations of particles are perpendicular to the direction of propogation of wave and produce crests and troughs in their medium of travel. In longitudinal wave, vibrations of particles are along the direction of propagation of wave and produce compressions and rarefactions along the direction of propagation of the wave.

10) Both, the transverse as well as the longitudinal, mechanical waves can propagate through solids but only longitudinal waves can propagate through fluids.

You might recall that when a mechanical wave passes through an elastic medium, the displacement of any particle of the medium at a space point x at time t is given by the expression

$$y(x,t) = f(x - vt) \qquad \text{--- (6.1)}$$

where v is the speed at which the disturbance travels through the medium to the right (increasing x). The factor $(x - vt)$ appears because the disturbance produced at the point $x = 0$ at time t reaches the point $x = x'$ on the right at time $(t + x'/\text{v})$ or we say that the disturbance of the particle at time t at position $x = x'$ actually originated on the left side at time $(t - x'/\text{v})$. Thus, Eq. (6.1) represents a progressive wave travelling in the positive x-direction with a constant speed v. The function f depends on the motion of the source of disturbance. If the source of disturbance is performing simple harmonic motion, the wave is represented as a sine or cosine function of $(x - \text{v}t)$ multiplied by a term which will make $(x - \text{v}t)$ dimensionless. Generally we represent such a wave by the following equation

$$y(x,t) = A\sin(kx - \omega t) \qquad \text{--- (6.2)}$$

where A is the amplitude of the wave, $k = 2\pi/\lambda$ is the wave number, λ and ω are the wavelength and the angular frequency of the wave and $\text{v} = \omega/k$ is the speed. The SI units of k, λ and ω are rad m^{-1}, m and rad s^{-1} respectively. If T is the time period of oscillation, then $n = 1/T = \omega/(2\pi)$ is the frequency of oscillation measured in Hz (s^{-1}). If the wave is travelling to the left *i.e.,* along the negative x-direction, then the equation for the disturbance is

$$y(x,t) = A\sin(kx + \omega t) \qquad \text{--- (6.3)}$$

Can you tell?

What is the minimum distance between any two particles of a medium which always have the same speed if a sine wave travels through the medium?

6.3 Reflection of Waves:

When a progressive wave, travelling through a medium, reaches an interface separating two media, a certain part of the wave energy comes back in the same medium. The wave changes its direction of travel. This is called reflection of a wave from the interface.

Reflection is the phenomenon in which the sound wave traveling from one medium to another comes back in the original medium with slightly different intensity and energy. To understand the reflection of waves, we will consider three examples below.

6.3.1 Reflection of a Transverse Wave:

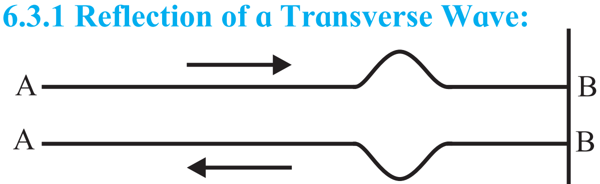


Fig. 6.1: Reflection of a wave pulse sent as a crest from a rarer medium to a denser medium.

Example 1

- Take a long light string AB. Attach one end of the string to a rigid support at B. (Here, for the wave pulse traveling on the string, the string is the rarer medium and the rigid support acts as a denser medium.)
- By giving a jerk to the free end A of the string, a crest is generated in the string.
- Observe what happens when this crest moves towards B?
- Observe what happens when the crest reaches B?

- Perform the same activity repeatedly and observe carefully. Try to find the reasons of movements in above observations.

Example 2

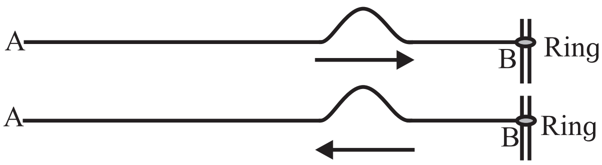


Fig.6.2: Reflection of a wave pulse sent as a crest from a denser medium to a rarer medium.

- Take a long light string AB. Attach the end B of the string to a ring which can slide easily on a vertical metal rod without friction. (Here string is the denser medium while end B attached to the sliding ring is at the interface of a rarer medium as it can move freely.)
- Give a jerk to free end A of the string.
- Observe what happens when crest reaches the point B attached to the ring.
- Try to find the reason of the observed movement.

Example 3

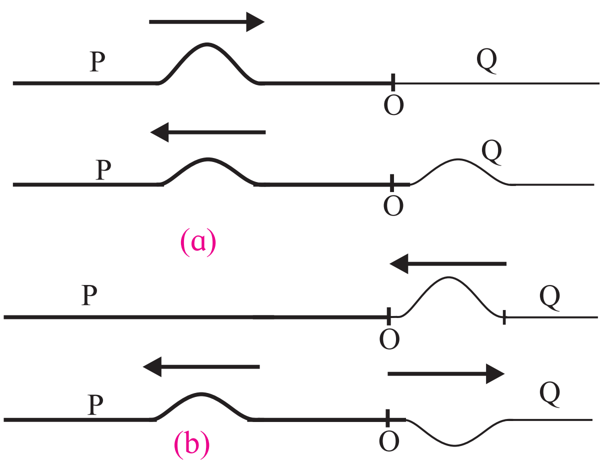


Fig. 6.3: Reflection of a crest from (a) denser medium (in this case a heavy string) and (b) rarer medium (in this case a light string).

- Take a heavy string P and a light string Q and join them. Suppose they are joined at point O. (Heavy string acts as a denser medium and light string is the rarer medium.)
- Produce a wave pulse as a crest on the heavy string P moving towards the junction O.
- Observe the part of wave pulse reflected back on the heavy string.
- Produce a wave pulse as a crest on the light string Q moving towards the junction point O.
- Observe the part of wave pulse reflected on the light string.
- What difference do you observe when the wave pulse gets reflected on the light string and when the wave pulse gets reflected on the heavy string?
- Try to find reasons behind your observations.

In example 1, when crest moves along the string towards B, it pulls the particles of string in upward direction. Similarly when the crest reaches B at rigid support, it tries to pull the point B upwards. But being a rigid support, B remains at rest and an equal and opposite reaction is produced on the string according to Newton's third law of motion. The string is pulled downwards. Thus crest gets reflected as a trough (Fig. 6.1) or a trough gets reflected as a crest. Hence from example 1, we can conclude that when transverse wave is reflected from a rigid support, i.e., from a denser medium, a crest is reflected as a trough and a trough is reflected as a crest. You have learnt in Xth and XIth Std. that there is a phase difference of π radian between the particles at a crest and at a trough. Therefore we conclude that there is a phase change of π radian on reflection from the fixed end, i.e., from a denser medium.

In example 2, we observe that when the crest reaches the point B, it pulls the ring upwards and causes the ring to move upward. The wave is seen to get reflected back as a crest and no phase change occurs on reflection from a rarer medium (Fig. 6.2).

In example 3, we find that a crest travelling from the heavy string gets reflected as a crest from the lighter string, i.e., reflection at the surface when a wave is travelling from a denser medium to a rarer medium causes a crest to be reflected as a crest (Fig. 6.3 (a)).

But in example 3 (Fig. 6.3 (b)), when a crest travels from the lighter string to the heavy string, the crest is reflected as a trough and vice versa.

6.3.2 Reflection of a Longitudinal Wave:

Consider a longitudinal wave travelling from a rarer medium to a denser medium. In a longitudinal wave compression is a high pressure region while rarefaction is a low pressure region. When compression reaches the denser medium, it tries to push the particles of that medium. But the energy of particles in the rarer medium is not sufficient to compress the particles of denser medium. According to Newton's third law of motion, an equal and opposite reaction comes into play. As a result, the particles of rarer medium get compressed. Thus, when the longitudinal wave travels from a rarer medium to a denser medium, a compression is reflected as a compression and a rarefaction is reflected as a rarefaction. There is no change of pressure phase during this reflection (Fig. 6.4).

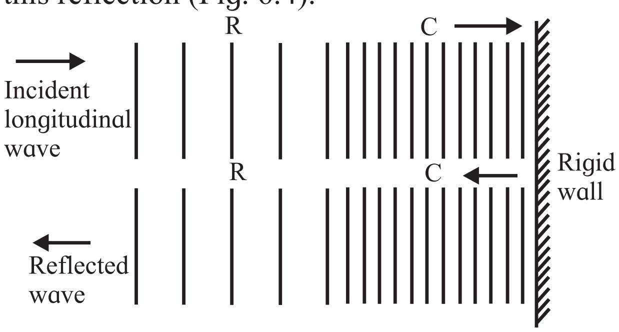


Fig. 6.4: Reflection of a longitudinal wave from a denser medium.

When longitudinal wave travels from a denser medium to a rarer medium (Fig. 6.5), a compression is reflected as a rarefaction. Here reversal of pressure phase takes place, i.e., pressure phase changes by π radians.

When compression reaches a rarer medium from denser medium, it pushes the particles of rare medium. Due to this, particles of the rarer medium get compressed and move forward and a rarefaction is left behind. Thus a compression gets reflected as a rarefaction. Similarly a rarefaction gets reflected as a compression (Fig. 6.5).

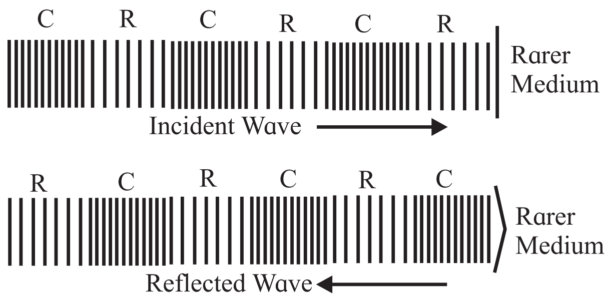


Fig. 6.5: Reflection of a longitudinal wave from a rarer medium.

6.4 Superposition of Waves:

Suppose you wish to listen to your favourite music. Is it always possible particularly when there are many other sounds from the surroundings disturbing you. How can the background sounds be blocked? Of course, the mobile lover generation uses headphones and enjoys listening to its favorite music. But you cannot avoid the background sound completely. Why?

We know that sound waves are longitudinal waves propagating through an elastic medium. When two waves travelling through a medium cross each other, each wave travels in such a way as if there is no other wave. Each wave sets the particles of the medium into simple harmonic motion. Thus each particle of the medium is set into two simple harmonic motions due to the two waves. The total displacement of the particles, at any instant of time during travelling of these waves, is the vector sum of the two displacements. This happens according to the principle of superposition of waves, which states that, **when two or more waves, travelling through a medium, pass through a common point, each wave produces its own displacement at that point, independent of the presence of the other wave. The resultant displacement at that point is equal to the vector sum of the displacements due to the individual wave at that point.** As displacement is a vector, we must add the individual displacements by considering their directions. There is no change in the shape and nature of individual waves due to superposition of waves. This principle applies to all types of waves like sound waves, light waves, waves on a string etc. and we say that interference of waves has taken place.

You might have seen singers using a special type of headphones during recording of songs. Those are active noise cancellation headphones, which is the best possible solution to avoid background sound. Active noise cancellation headphones consist of small microphones one on each earpiece. They detect the ambient noise that arrives at the ears. A special electronic circuit is built inside the earpiece to create sound waveforms exactly opposite to the arriving noise. This is called antisound. The antisound is added in the earphones so as to cancel the noise from outside. This is possible due to superposition of waves, as the displacements due to these two waves cancel each other. The phenomena of interference, beats, formation of stationary waves etc. are based on the principle of superposition of waves.

Let us consider superposition of two wave pulses in two different ways.

6.4.1 Superposition of Two Wave Pulses of Equal Amplitude and Same Phase Moving towards Each Other :

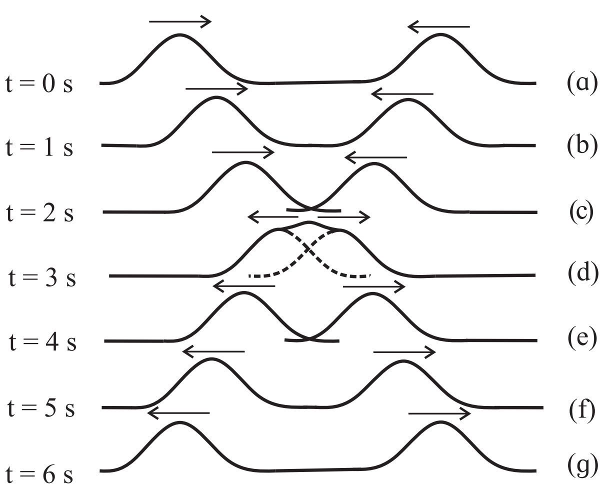


Fig. 6.6: Superposition of two wave pulses of equal amplitude and same phase moving towards each other.

The propagation of approaching wave pulses, their successive positions after every second, their superposition and their propagation after superposition are shown in Figs. 6.6 (a) to 6.6 (g). Suppose two waves cross each other between t = 2 s and t = 4 s, as shown in Figs. 6.6 (c), (d) and (e). Here the two wave pulses superpose, the resultant displacement is equal to the sum of the displacements (full line) due to individual wave pulses (dashed lines). This is constructive interference. The displacement due to wave pulses after crossing at t = 5 s and t = 6 s are shown in Figs. 6.6 (f) and (g). After crossing each other, both the wave pulses continue to maintain their individual shapes.

6.4.2 Superposition of Two Wave Pulses of Equal Amplitude and Opposite Phases Moving towards Each Other :

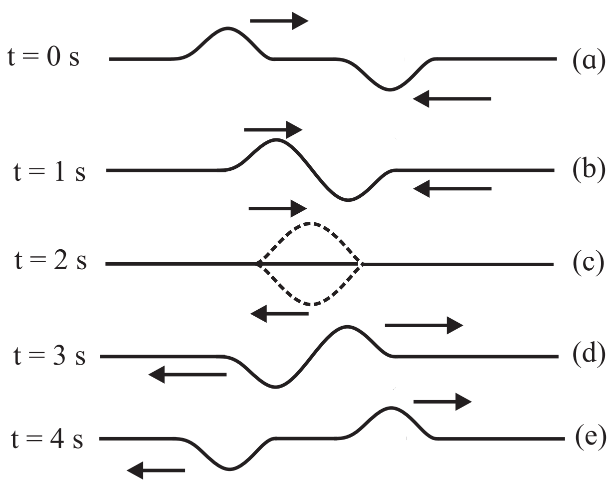


Fig. 6.7: Superposition of two wave pulses of equal amplitude and opposite phases moving towards each other.

The propagation of approaching wave pulses, their successive positions after every second, their superposition and propagation after superposition are shown in Fig. 6.7 (a) to Fig. 6.7 (e).

These wave pulses superimpose at t = 2 s and the resultant displacement (full line) is zero, due to individual displacements (dashed lines) differing in phase exactly by 180°. This is destructive interference. Displacement due to one wave pulse is cancelled by the displacement due to the other wave pulse when they cross each other (Fig. 6.7 (c)). After crossing each other, both the wave pulses continue and maintain their individual shapes.

6.4.3 Amplitude of the Resultant Wave Produced due to Superposition of Two Waves:

Consider two waves having the same frequency but different amplitudes A_1 and A_2. Let these waves differ in phase by φ. The displacement of each wave at $x = 0$ is given as

$$y_1 = A_1 \sin \omega t$$

$$y_2 = A_2 \sin(\omega t + \varphi)$$

According to the principle of superposition of waves, the resultant displacement at $x = 0$ is

$$y = y_1 + y_2$$

or, $y = A_1 \sin\omega t + A_2 \sin(\omega t + \varphi)$

$$y = A_1 \sin\omega t + A_2 \sin\omega t \cos\varphi + A_2 \cos\omega t \sin\varphi$$

$$y = (A_1 + A_2\cos\varphi)\sin \omega t + A_2\sin\varphi \cos\omega t$$

If we write

$$A_1 + A_2 \cos\varphi = A \cos\theta \qquad \text{--- (6.4)}$$

and $A_2\sin\varphi = A\sin\theta \qquad$ --- (6.5)

we get

$$y = A\cos\theta \sin\omega t + A\sin\theta \cos\omega t$$

$$\therefore \; y = A\sin(\omega t + \theta) \qquad \text{--- (6.6)}$$

This is the equation of the resultant wave. It has the same frequency as that of the interfering waves. The resultant amplitude A is given by squaring and adding Eqs. (6.4) and (6.5).

$$A^2\cos^2\theta + A^2\sin^2\theta = (A_1 + A_2\cos\varphi)^2 + A_2^{\,2}\sin^2\varphi$$

$$A^2 = A_1^{\,2} + 2A_1A_2\cos\varphi + A_2^{\,2}\cos^2\varphi + A_2^{\,2}\sin^2\varphi$$

$$\therefore \; A = \sqrt{A_1^{\,2} + 2A_1A_2\cos\varphi + A_2^{\,2}} \qquad \text{--- (6.7)}$$

Special cases:

1. When $\varphi = 0$, i.e., the waves are in phase, the resultant amplitude is

$$A = \sqrt{A_1^{\,2} + 2A_1A_2\cos 0 + A_2^{\,2}} = \sqrt{(A_1 + A_2)^2}$$
$$= A_1 + A_2$$

The resultant amplitude is maximum when $\varphi = 0$.

If the amplitudes of the waves are equal i.e., $A_1 = A_2 = A_0$ (say), then the resultant amplitude is $2A_0$.

2. When $\varphi = \pi$, i.e., the waves are out of phase, the resultant amplitude is

$$A = \sqrt{A_1^{\,2} + 2A_1A_2\cos\pi + A_2^{\,2}} = \sqrt{(A_1 - A_2)^2}$$
$$= |A_1 - A_2|$$

The resultant amplitude is minimum when $\varphi = \pi$.

If the amplitudes of the waves are equal i.e., $A_1 = A_2 = A_0$ (say), then the resultant amplitude is zero.

Thus, the maximum amplitude is the sum of the two amplitudes when the phase difference between the two waves is zero and the minimum amplitude is the difference of the two amplitudes when the phase difference between the two waves is π.

The intensities of the waves are proportional to the squares of their amplitudes. Hence, when $\varphi = 0$

$$I_{max} \propto (A_{max})^2 = (A_1 + A_2)^2 \qquad \text{--- (6.8)}$$

and when $\varphi = \pi$

$$I_{min} \propto (A_{min})^2 = (A_1 - A_2)^2 \qquad \text{--- (6.9)}$$

Therefore intensity is maximum when the two waves interfere in phase while intensity is minimum when the two waves interfere out of phase.

You will learn more about superposition of waves in Chapter 7 on Wave Optics.

Example 6.1: The displacements of two sinusoidal waves propagating through a string are given by the following equations

$$y_1 = 4\sin(20x - 30t)$$

$$y_2 = 4\sin(25x - 40t)$$

where x and y are in centimeter and t is in second.

a) Calculate the phase difference between these two waves at the points $x = 5$ cm and $t = 2$ s.

b) When these two waves interfere, what are the maximum and minimum values of the intensity?

Solution: Given

$$y_1 = 4\sin(20x - 30t)$$

and $\quad y_2 = 4\sin(25x - 40t)$

a) To find phase difference when $x = 5$ cm and $t = 2$ s:

$$y_1 = 4\sin(20\times5 - 30\times2)$$
$$= 4\sin(100 - 60) = 4\sin 40$$
$$y_2 = 4\sin(25\times5 - 40\times2)$$
$$= 4\sin(125 - 80) = 4\sin 45$$

$\therefore$ Phase difference is 5 radian because $\varphi = |45 - 40| = 5$ radian.

b) To find the maximum and minimum values of the intensity :

Amplitudes of the two waves are $A_1 = 4$ cm and $A_2 = 4$ cm,

$$\therefore I_{max} = (A_1 + A_2)^2 = (4+4)^2 = 64$$

when the phase difference is zero

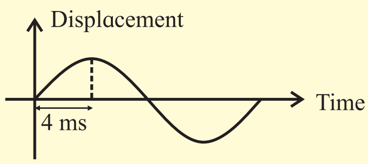
and $\quad I_{min} = (A_1 - A_2)^2 = (4-4)^2 = 0$

when the phase difference is π.

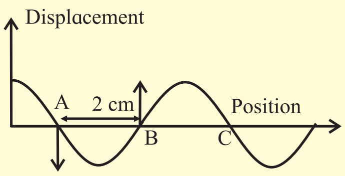
Example 6.2: A progressive wave travels on a stretched string. A particle on this string takes 4.0 ms to move from its mean position to one of its extreme positions. The distance between two consecutive points on the string which are at their mean positions (at a certain time instant) is 2.0 cm. Find the frequency, wavelength and speed of the wave.

Solution :

(a)



(b)



A particles takes 4.0×10^{-3} s to travel from its mean position to extreme position. This is a quarter of the complete oscillation as shown in Fig. (a). Hence, the particle will take $4 \times 4.0 \times 10^{-3}$ s $= 16 \times 10^{-3}$ s to complete one oscillation.

$\therefore$ frequency $n = 1/T = (1/16) \times 10^3\ s^{-1}$

$= 62.5$ Hz

As shown in Fig. (b), points A, B, and C correspond to mean positions, but the string is moving in one direction at point A and in the opposite direction at point B. Thus, out of the two consecutive particles at their mean positions, one will be moving upwards while the other will be moving downwards. The distance between them is 2.0 cm. Therefore distance between two consecutive particles moving in the same direction will be 2 × 2 cm = 4 cm. Thus the wavelength $\lambda = 4$ cm $= 0.04$ m

Speed of wave $v = n \times \lambda = 62.5 \times 0.04$

$= 2.5$ m/s.

6.5 Stationary Waves:

We have seen the superposition of two wave pulses, having same amplitudes and either same phase or opposite phases, and changes in the resultant amplitude pictorially in section 6.4. We have also derived the mathematical expression for the resultant displacement when two waves of same frequency superimpose as given by Eqs. (6.4) to (6.6). Now we are going to study an example of superposition of waves having the same amplitude and the same frequency travelling in opposite directions.

6.5.1 Formation of Stationary Waves:

Imagine a string stretched between two fixed points. If the string is pulled at the middle and released, we get what is know as a stationary wave. Releasing of string produces two progressive waves travelling in opposite directions. These waves are reflected at the fixed ends. The waves produced in the string initially and their reflected waves combine to produce stationary waves as shown in Fig. 6.8 (a).

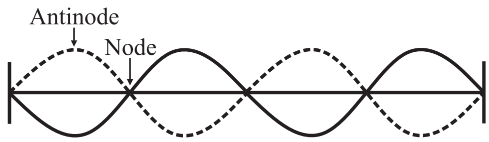


Fig. 6.8 (a): Formation of stationary waves on a string. The two sided arrows indicate the motion of the particles of the string.

6.5.2 Equation of Stationary Wave on a Stretched String:

Consider two simple harmonic progressive waves of equal amplitudes (a) and wavelength (λ) propagating on a long uniform string in opposite directions (remember $2\pi/\lambda = k$ and $2\pi n = \omega$).

The equation of wave travelling along the x-axis in the positive direction is

$$y_1 = a\sin\{2\pi\left(nt - \frac{x}{\lambda}\right)\} \quad \text{--- (6.10)}$$

The equation of wave travelling along the x-axis in the negative direction is

$$y_2 = a\sin\left\{2\pi\left(nt + \frac{x}{\lambda}\right)\right\} \quad \text{--- (6.11)}$$

When these waves interfere, the resultant displacement of particles of string is given by the principle of superposition of waves as

$$y = y_1 + y_2$$

$$y = a\sin\left\{2\pi\left(nt - \frac{x}{\lambda}\right)\right\} + a\sin\left\{2\pi\left(nt + \frac{x}{\lambda}\right)\right\}$$

By using,

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$, we get

$$y = 2a\sin(2\pi nt)\cos\frac{2\pi x}{\lambda}$$

$$y = 2a\cos\frac{2\pi x}{\lambda}\sin(2\pi nt) \quad \text{or,} \quad \text{--- (6.12)}$$

Using $2a\cos\frac{2\pi x}{\lambda} = A$ in Eq. (6.12), we get

$$y = A\sin(2\pi nt)$$

As $\omega = 2\pi n$, we get, $y = A\sin\omega t$.

This is the equation of a stationary wave which gives resultant displacement due to two simple harmonic progressive waves. It may be noted that the terms in position x and time t appear separately and not as a combination $2\pi(nt \pm x/\lambda)$.

Hence, the wave is not a progressive wave. x is present only in the expression for the amplitude. The amplitude of the resultant wave is given as $A = 2a\cos\frac{2\pi x}{\lambda}$. It is a periodic function of x i.e., the amplitude is varying periodically in space. The amplitudes are different for different particles but each point on the string oscillates with the same frequency ω (same as that of the individual progressive wave). All the particles of the string pass through their mean positions simultaneously twice during each vibration. The string as a whole is vibrating with frequency ω with different amplitudes at different points. The wave is not moving either to the left or to the right. We therefore call such a wave a **stationary wave or a standing wave**. Particles move so fast that the visual effect is formation of loops. It is therefore customary to represent stationary waves as loops. In case of a string tied at both the ends, loops are seen when a stationary wave is formed because each progressive wave on a string is a traverse wave. *When two identical waves travelling along the same path in opposite directions interfere with each other, resultant wave is called stationary wave.*

Condition for node:

Nodes are the points of minimum displacement. This is possible if the amplitude is minimum (zero), i.e.,

$$2a\cos\frac{2\pi x}{\lambda} = 0,$$

$$\text{or,}\ \cos\frac{2\pi x}{\lambda} = 0,$$

$$\text{or,}\ \frac{2\pi x}{\lambda} = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \ldots\ldots\ldots$$

$$\therefore x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, \ldots\ldots\ldots$$

i.e., $x = (2p-1)\frac{\lambda}{4}$ where $p = 1, 2, 3, \ldots\ldots\ldots$

The distance between two successive nodes is $\frac{\lambda}{2}$.

Condition for antinode:

Antinodes are the points of maximum displacement,

$$\text{i.e.,}\ A = \pm 2a$$

$$\therefore 2a\cos\frac{2\pi x}{\lambda} = \pm 2a$$

$$\text{or,}\ \cos\frac{2\pi x}{\lambda} = \pm 1$$

$$\therefore \frac{2\pi x}{\lambda} = 0, \pi, 2\pi, 3\pi \ldots$$

$$\text{or, } x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, \ldots..$$

i.e., $x = \frac{\lambda p}{2}$ where $p =$ 0, 1, 2,3….

The distance between two successive antinodes is $\frac{\lambda}{2}$. Nodes and antinodes are formed alternately. Therefore, the distance between a node and an adjacent antinode is $\frac{\lambda}{4}$.

When $\sin \omega t = 1$, at that instant of time, all the particles for which $\cos kx$ is positive have their maximum displacement in positive direction. At the same instant, all the particles for which $\cos kx$ is negative have their maximum displacement in negative direction. When $\sin \omega t = 0$, all the particles cross their mean positions, some of them moving in the positive direction and some in the negative direction.

Longitudinal waves e.g. sound waves travelling in a tube /pipe of finite length are relected at the ends in the same way as transverse waves along a string are reflected at the ends. Interference between these waves travelling in opposite directions gives rise to standing waves as shown in Fig. 6.8 (b). We represent longitudinal stationary wave by a loop but the actual motion of the particles is along the length of the loop and not perpendiculat to it.

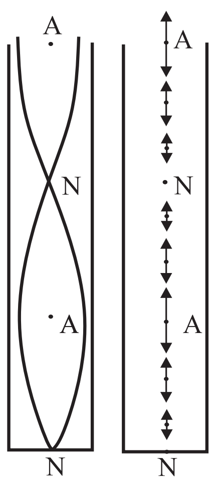


Fig. 6.8 (b): Figure on the left shows standing waves in a conventional way while figure on the right shows the actual oscillations of material particles for a longitudinal stationary wave. Points A and N denote antinodes and nodes respectively.

6.5.3 Properties of Stationary Waves:

1. Stationary waves are produced due to superposition of two identical waves (either transverse or longitudinal waves) traveling through a medium along the same path in opposite directions.
2. If two identical transverse progressive waves superimpose or interfere, the resultant wave is a transverse stationary wave as shown in Fig. 6.8 (a).
   - When a transverse stationary wave is produced on a string, some points on the string are motionless. The points which do not move are called **nodes**.
   - There are some points on the string which oscillate with greatest amplitude (say A). They are called **antinodes**.
   - Points between the nodes and antinodes vibrate with values of amplitudes between 0 and A.
3. If two identical longitudinal progressive waves superimpose or interfere, the resultant wave is a longitudinal stationary wave. Figure 6.8 (b) shows a stationary sound wave produced in a pipe closed at one end.
   - The points, at which the amplitude of the particles of the medium is minimum (zero), are called nodes.
   - The points, at which the amplitude of the particles of the medium is maximum (say A), are called antinodes.
   - Points between the nodes and antinodes vibrate with values of amplitudes between 0 and A
4. The distance between two consecutive nodes is $\frac{\lambda}{2}$ and the distance between two consecutive antinodes is $\frac{\lambda}{2}$.
5. Nodes and antinodes are produced alternately. The distance between a node and an adjacent antinode is $\frac{\lambda}{4}$.
6. The amplitude of vibration varies periodically in space. All points vibrate with the same frequency.
7. Though all the particles (except those at the nodes) possess energy, there is no propagation of energy. The wave is localized and its velocity is zero. Therefore, we call it a stationary wave.

8. All the particles between adjacent nodes (i.e., in one loop) vibrate in phase. There is no progressive change of phase from one particle to another particle. All the particles in the same loop are in the same phase of oscillation, which reverses for the adjacent loop.

Musical instruments such as violin, *tanpura*, are based on the principle of formation of stationary waves or standing waves.

Example 6.3: Find the distance between two successive nodes in a stationary wave on a string vibrating with frequency 64 Hz. The velocity of progressive wave that resulted in the stationary wave is 48 m s^{-1} .

Solution: Given:

Speed of wave = v = 48 m s^{-1}

Frequency n = 64 Hz

We have $v = n\lambda$

$$\therefore \lambda = \frac{v}{n} = \frac{48}{64} = 0.75\text{m}$$

We know that distance between successive nodes

$$= \frac{\lambda}{2} = \frac{0.75}{2} = 0.375 \text{ m}$$

6.5.4 Comparison of Progressive Waves and Stationary Waves:

1. In a progressive wave, the disturbance travels form one region to the other with definite velocity. In stationary waves, disturbance remains in the region where it is produced, velocity of the wave is zero.
2. In progressive waves, amplitudes of all particles are same but in stationary waves, amplitudes of particles are different.
3. In a stationary wave, all the particles cross their mean positions simultaneously but in a progressive wave, this does not happen.
4. In progressive waves, all the particles are moving while in stationary waves particles at the position of nodes are always at rest.
5. Energy is transmitted from one region to another in progressive waves but in stationary waves there is no transfer of energy.
6. All particles between two consecutive nodes are moving in the same direction and are in phase while those in adjacent loops are moving in opposite directions and differ in phase by 180° in stationary waves but in a progressive wave, phases of adjacent particles are different.

Do you know?

- What happens if a simple pendulum is pulled aside and released?
- What happens when a guitar string is plucked?
- Have you noticed vibrations in a drill machine or in a washing machine? How do they differ from vibrations in the above two cases?
- A vibrating tuning fork of certain frequency is held in contact with table top and vibrations are noticed and then another vibrating tuning fork of different frequency is held on table top. Are the vibrations produced in the table top the same for both the tuning forks? Why?

6.6 Free and Forced Vibrations:

The frequency at which an object tends to vibrate when hit, plucked or somehow disturbed is known as its natural frequency. In these vibrations, object is not under the influence of any outside force.

When a simple pendulum is pulled aside and released, it performs free vibrations with its natural frequency. Similarly when a string of guitar is plucked at some point it performs free vibrations with its natural frequency.

In free vibration, the body at first is given an initial displacement and the force is then withdrawn. The body starts vibrating and continues the motion on its own. No external force acts on the body further to keep it in motion.

Free vibration of a system means that the system vibrates at its natural frequency. In case of free vibrations, a body continuously loses energy due to frictional resistance of surrounding medium. Therefore, the amplitude of vibrations goes on decreasing, the vibrations of the body eventually stop and the body comes to rest.

The vibrations in a drill machine and in a washing machine are forced vibrations. Also the vibrations produced in the table top due to tuning forks of two different frequencies are different as they are forced vibrations due to two tuning forks of different frequencies.

In forced vibrations, an external periodic force is applied on a body whose natural period is different from the period of the force. The body is made to vibrate with a frequency equal to that of the externally impressed force. The amplitude of forced vibrations depends upon the difference between the frequency of external periodic force and the natural frequency of the body. If this difference is small, the amplitude of forced vibrations is large and vice versa. If the frequencies exactly match, it is termed as resonance and the amplitude of vibration is maximum.

An object vibrating with its natural frequency can cause another nearby object to vibrate. The second object absorbs the energy transmitted by the first object and starts vibrating if the natural frequencies of the two objects match. You have seen the example of two simple pendula supported from a string in the previous chapter. The second object is said to undergo forced vibrations. Strings or air columns can also undergo forced oscillations if the frequency of the external source of sound is close to the natural frequency of the system. Resonance is said to occur and we hear a louder sound.

6.7 Harmonics and Overtones:

When a string or an air column is set into vibrations by some means, the waves are reflected from the ends and stationary waves can be formed. An important condition to form stationary waves depends on the boundary conditions that constrain the possible wavelengths or frequencies of vibration of the system. These are called the natural frequencies of normal modes of oscillations. The minimum of these frequencies is termed the fundamental frequency or the first harmonic. The corresponding mode of oscillations is called the fundamental mode or fundamental tone. The term overtone is used to represent higher frequencies. The first frequency higher than the fundamental frequency is called the first overtone, the next higher frequency is the second overtone and so on. The term 'harmonic' is used when the frequency of a particular overtone is an integral multiple of the fundamental frequency. In strings and air columns, the frequencies of overtones are integral multiples of the fundamental frequencies, hence they are termed as harmonics. But all harmonics may not be present in a given sound. The overtones are only those multiples of fundamental frequency which are actually present in a given sound. The harmonics may or may not be present in the sound so produced.

To understand the concept of harmonics and overtones, let us study vibrations of air column.

6.7.1 End Correction:

When an air column vibrates either in a pipe closed at one end or open at both ends, boundary conditions demand that there is always an antinode at the open end(s) (since the particles of the medium are comparatively free) and a node at the closed end (since there is hardly any freedom for the particles to move). The antinode is not formed exactly at the open end but it is slightly beyond the open end as air is more free to vibrate there in comparison to the air inside the pipe. Also as air particles in the plane of open end of the pipe are not free to move in all directions, reflection takes place at the plane at small distance outside the pipe.

The distance between the open end of the pipe and the position of antinode is called the end correction. According to Reynold, to the first approximation, the end correction *at an end* is given by $e = 0.3d$, where d is the inner diameter of the pipe. Thus the length L of air column is different from the length l of the pipe.

For a pipe closed at one end

The corrected length of air column L = length of air column in pipe l + end correction at the open end.

$$\therefore L = l + e \qquad \text{--- (6.13)}$$

For a pipe open at both ends

The corrected length of air column L = length of air column in pipe l + end corrections at both the ends.

$$\therefore L = l + 2e \qquad \text{--- (6.14)}$$

6.7.2 Vibrations of air column in a pipe closed at one end:

Consider a long cylindrical tube closed at one end. It consists of an air column with rigid boundary at one end. When a vibrating tuning fork is held near the open end of the closed pipe, sound waves are sent by the fork inside the tube. Longitudinal waves traveling along a pipe of finite length are reflected at the ends as transverse waves are refelcted at the fixed ends of a string. The phase of the reflected wave depends on whether the end of the pipe is open or closed and how wide or narrow the pipe is in compansion to the wavelength of longitudinal wave like a sound wave.

At the closed end there is least freedom for motion of air particles. Thus, there must be a node at the closed end. The particles little beyond the open end are most free to vibrate. As a result, an antinode must be formed little beyond the open end. The length l of pipe and length L of air column are shown separately in all the figures (refer Figs. 6.9 and 6.10).

The first mode of vibrations of air column closed at one end is as shown in Fig. 6.9 (a).

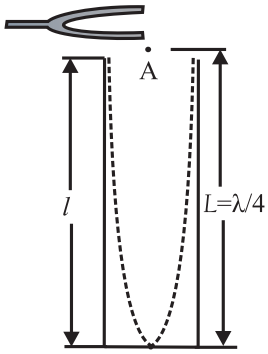


Fig. 6.9 (a): Set-up for generating vibrations of air column in a pipe closed at one end. The distance of the antinode from the open end of the pipe has been exaggerated.

This is the simplest mode of vibration of air column closed at one end, known as the fundamental mode.

$\therefore$ Length of air column

$$L = \frac{\lambda}{4} \quad \text{and} \quad \lambda = 4L$$

where λ is the wavelength of fundamental mode of vibrations in air column. If n is the fundamental frequency, we have

$$v = n\lambda \qquad \text{--- (6.15)}$$

$$\therefore n = \frac{v}{\lambda}$$

$$\therefore n = \frac{v}{4L} = \frac{v}{4(l+e)} \qquad \text{--- (6.16)}$$

The fundamental frequency is also known as the first harmonic. It is the lowest frequency of vibration in air column in a pipe closed at one end.

The next mode of vibrations of air column closed at one end is as shown in Fig. 6.9 (b). Here the air column is made to vibrate in such a way (as shown in Fig. 6.9 (b)) that it contains a node at the closed end, an antinode at the open end with one more node and antinode in between. If n_1 is the frequency and λ_1 is the wavelength of wave in this mode of vibrations in air column, we have, the length of the air column $L = \frac{3\lambda_1}{4}$

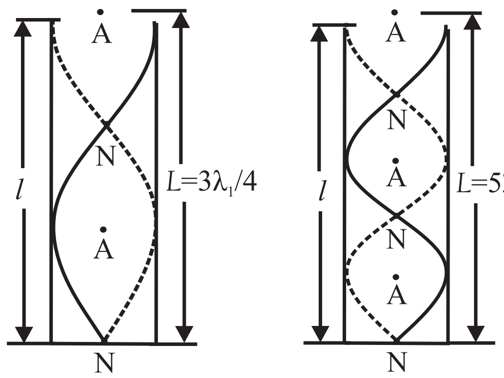
$$\therefore \lambda_1 = \frac{4L}{3} = \frac{4(l+e)}{3} \qquad \text{--- (6.17)}$$

The velocity in the second mode is given as $v = n_1\lambda_1$

$$\therefore n_1 = \frac{v}{\lambda_1} = \frac{3v}{4L} = \frac{3v}{4(l+e)}$$

$$\therefore \ n_1 = 3n \qquad \text{--- (6.18)}$$

This frequency is the third harmonic. It is the first overtone. Remember that the overtones are always numbered sequentially.



(b) (c)

Fig. 6.9 (b) and (c): First and second overtones for vibrations of air column in a pipe closed at one end. The distance of the antinode from the open end of the pipe has been exaggerated.

The next higher mode of vibrations of air column closed at one end is as shown in Fig. 6.9 (c). Here the same air column is made to vibrate in such a way that it contains a node at the closed end, an antinode at the open end with two more nodes and antinodes in between. If n_2 is the frequency and λ_2 is the wavelength of the wave in this mode of vibrations in air column, we have

Length of air column $L=\dfrac{5\lambda_2}{4}$

$$\therefore \lambda_2 = \frac{4L}{5} = \frac{4(l+e)}{5} \qquad \text{--- (6.19)}$$

The velocity this mode is given as

$$\mathrm{v} = n_2\lambda_2$$

$$\therefore n_2 = \frac{\mathrm{v}}{\lambda_2} = \frac{5V}{4L} = \frac{5V}{4(l+e)} \qquad \therefore n_2 = 5n \qquad \text{-- (6.20)}$$

This frequency is the fifth harmonic. It is the second overtone.

Continuing in a similar way, for the p^{th} overtone we get the frequency n_p as

$$n_p = (2p+1)n. \qquad \text{-- (6.21)}$$

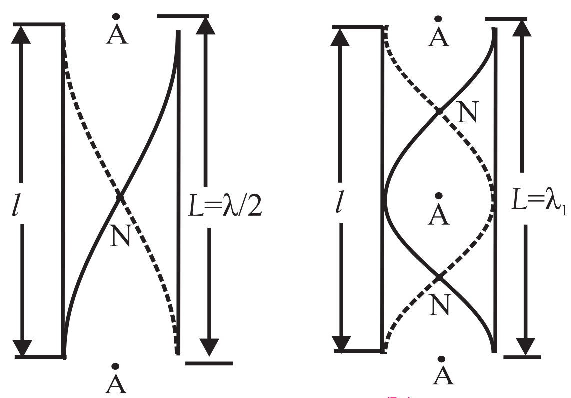
Thus for a pipe closed at one end only odd harmonics are present and even harmonics are absent.

6.7.3 Vibrations of air column in a pipe open at both ends:

In this case boundary conditions are such that an antinode is present at each open end. When a source of sound like a tuning fork is held near one end of the pipe, it sends the waves inside the pipe.

Even though both the ends of the pipe are open, the air inside the pipe is still bound by the wall of the tube. As a result, the air inside the pipe is little denser than the air outside. When the waves travel to the other open end, there is partial reflection at the open end. The partially reflected waves superimpose with the incident waves. Under suitable conditions, stationary waves will be formed. There is maximum freedom for motion of air column at both the ends as pipe is open at both ends.

Suppose a compression produced by a tuning fork travels through the air column. It



(a) (b)

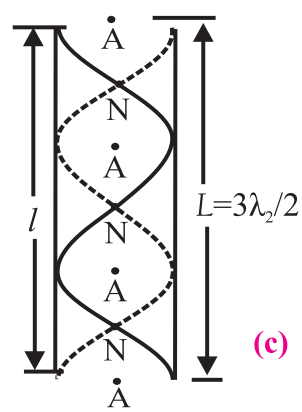


Fig. 6.10: First three modes of vibrations of air column in a pipe open at both ends. The distance of the antinodes from the open ends of the pipe has been exaggerated.

gets reflected as a rarefaction at open end. The rarefaction moves back and gets reflected as compression at the other end. It suffers second reflection at open end near the source and then interferes with the wave coming in by a path difference of $2L$.

The different modes of vibrations of air column in pipe open at both ends are shown in Fig. 6.10 (a), (b) and (c). The fundamental tone or mode of vibrations of air column open at both ends is as shown in Fig. 6.10 (a). There are two antinodes at two open ends and one node between them.

$\therefore$ Length of air column $= L = \frac{\lambda}{2}$ or, $\lambda = 2L$

and $v = n\lambda = 2nL$ ----(6.22)

$$\therefore n = \frac{v}{\lambda} = \frac{v}{2L} = \frac{v}{2(l+2e)} \quad \text{----(6.23)}$$

This is the fundamental frequency or the first harmonic. It is the lowest frequency of vibration.

The next possible mode of vibrations of air column open at both ends is as shown in Fig. 6.10 (b). Three antinodes and two nodes are formed.

$\therefore$ Length of air column $= L = \lambda_1$

i.e., $\lambda_1 = L = (l+2e)$ ----(6.24)

If n_1 and λ_1 are frequency and wavelength of this mode of vibration of air column respectively, then

$$v = n_1\lambda_1$$

$$n_1 = \frac{v}{\lambda_1} = \frac{v}{L} = \frac{v}{(l+2e)}$$

$$\therefore n_1 = 2n \quad \text{--- (6.25)}$$

This is the frequency of second harmonic or first overtone.

In the next possible mode of vibrations of air column open at both ends (as shown in Fig. 6.10 (c)), four antinodes and three nodes are formed.

$$\therefore \text{Length of air column} = L = \frac{3\lambda_2}{2}$$

$$\therefore \lambda_2 = \frac{2L}{3} = \frac{2(l+2e)}{3} \quad \text{--- (6.26)}$$

If n_2 and λ_2 are the frequency and wavelength of this mode of vibration of air column respectively, then $v = n_2\lambda_2$

$$\therefore n_2 = \frac{v}{\lambda_2} = \frac{3v}{2L} = \frac{3v}{2(l+2e)}$$

$$\therefore n_2 = 3n \quad \text{--- (6.27)}$$

This is the frequency of third harmonic or second overtone.

Thus all harmonics are present as overtones in the modes of vibration of air column open at both ends.

Continuing in this manner, the frequency n_p for p^{th} overtone is,

$$n_p = (p+1)n \quad \text{--- (6.28)}$$

where n is the fundamental frequency and p = 0,1,2,3…

It may be noted that

1. Sound produced by an open pipe contains all harmonics. Its quality is richer than that produced by a closed pipe.
2. Fundamental frequency of vibration of air column in an open pipe is double that of the fundamental frequency of vibration in a closed pipe of the same length.

Using the formula and knowing values of n, l and end correction velocity of sound in air at room temperature can be calculated. As discussed earlier, the antinodes are formed little beyond the open ends of the pipe. It is however not possible to locate the positions of the antinodes precisely. Therefore, in experiments, the length of the pipe is measured and end corrections are incorporated.

6.7.4 Practical Determination of End Connection:

An exact method to determine the end correction, using two pipes of same diameter but different lengths l_1 and l_2, with fundamental frequencies n_1 and n_2 respectively, is as follows.

For a pipe open at both ends:

$$v = 2n_1L_1 = 2n_2L_2 \quad \text{using Eq. (6.22)}$$

$$\therefore n_1L_1 = n_2L_2$$

$$\therefore n_1(l_1+2e) = n_2(l_2+2e)$$

$$\therefore \quad e = \frac{n_1l_1 - n_2l_2}{2(n_2-n_1)} \text{ or } \frac{n_2l_2 - n_1l_1}{2(n_1-n_2)} \quad \text{--- (6.29)}$$

For a pipe closed at one end:

$$v = 4n_1L_1 = 4n_2L_2$$

$$\therefore n_1L_1 = n_2L_2$$

$$\therefore n_1(l_1+e) = n_2(l_2+e)$$

$$\therefore e = \frac{n_1l_1 - n_2l_2}{(n_2-n_1)} \text{ or } \frac{n_2l_2 - n_1l_1}{(n_1-n_2)} \quad \text{--- (6.30)}$$

Remember this

For correct value of end correction, the inner diameter of pipe must be uniform throughout its length. It may be noted that effect of flow of air and effect of temperature of air outside the tube has been neglected.

Example 6.4: An air column is of length 17 cm long. Calculate the frequency of 5th overtone if the air column is (a) closed at one end and (b) open at both ends. (Velocity of sound in air = 340 ms^{-1}).

Solution: Given

Length of air column = 17cm = 0.17m

Overtone number $p = 5$ and velocity of sound in air = 340 ms^{-1}.

For an air column closed at one end,

Fundamental frequency $n^c = \frac{v}{4L} = \frac{340}{4 \times 0.17}$

$= 500$ Hz

and frequency of p^{th} overtone $n_p^c = (2p+1)n^c$

$\therefore$ for fifth overtone $n_5^c = (2 \times 5 + 1) \times 500$

$= 5500$ Hz

For an air column open at both ends,

Fundamental frequency $n^o = \frac{v}{2L} = \frac{340}{2 \times 0.17}$

$n = 1000$ Hz

and frequency of p^{th} overtone $n_p^o = (p+1)n^o$

$\therefore$ for fifth overtone $n_5^o = (5+1) \times 1000$

$n_5^o = 6000$ Hz

Example 6.5 : A closed pipe and an open pipe have the same length. Show that no mode of the closed pipe has the same wavelength as any mode of the open pipe.

Solution: For a closed pipe, the frequency of allowed modes is given by $n_p^c = (2p+1)n^c$ where $n^c = \frac{v}{4L}$ (superscript c and o refer to closed and open pipes respectively) using Eqs. (6.21) and (6.16), where p is any integer.

$\therefore \lambda_p^c = \frac{4L}{2p+1}$, where p is any integer.

On the other hand, for an open pipe, the frequency of allowed modes is given by,

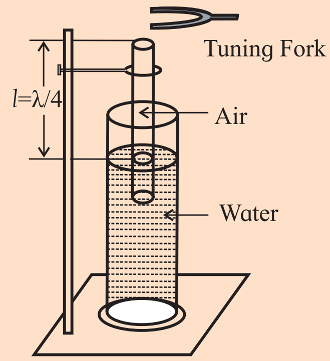
$n^o_m = (m + 1)n^o$ and $n^o = v/2L$, using Eqs. (6.28) and (6.23), where m is any integer.

$$\therefore \lambda_m^o = \frac{2L}{m+1}$$

If $\lambda_p^c = \lambda_m^o$, it would mean $\frac{4L}{2p+1} = \frac{2L}{m+1}$.

Or, 2 (m +1) = 2p + 1 which is not possible. Hence the two pipes cannot have modes with the same frequency or wavelength.

Activity

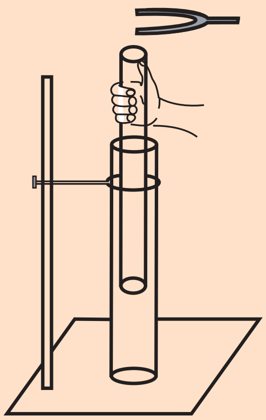


Take a glass tube open at both ends and clamp it so that its one end dips into a glass cylinder containing water as shown in the accompanying figure. By changing the position of the tube at the clamp, you can adjust the length of the air column in the tube. Hold a vibrating tuning fork of frequency 488 Hz or 512 Hz just above the open end of the tube and make the air column vibrate. What is the difference between the sounds that you hear? The sound will be louder. This is an example of resonance. This set-up is a resonance tube. Note the heights of the air column when you hear louder sound. Interpret your observations.

Take another tuning fork of the same frequency as the first one. Vibrate them together above the open end of the tube. Do you hear beats? If the two tuning forks are of same frequency, you should not hear beats. In practice, due to usage, frequencies change and in most of the cases, you will hear beats. If you do not hear beats, there can be two reasons : (i) frequencies of the two forks are exactly same or (ii) the frequencies are very much different (difference greater than 6-7 Hz) and we cannot recognize the beats. Then wind a piece of thread around the tong of one of the tuning fork so that its frequency changes slightly. Try to hear the beats. By changing the position of the thread, vary the frequency and note down your observations systematically. What information you get from this activity?

Activity

Take two pipes of slightly different diameters, open at both the ends, so that one pipe can be moved freely inside the other. Keep the wider pipe fixed by clamping on a stand and move the other pipe up and down by hand as shown in the accompanying figure. Use a tuning fork of frequency 320 Hz or 288 Hz and keep it above the open end of the fixed pipe. Move the inner tube and try to hear the various sound patterns and write down your observations. Try to analyze the results based on the knowledge you have from the sound pattern formed with a pipe open at both ends.



6.7.5 Vibrations Produced in a String:

Consider a string of length l stretched between two rigid supports. The linear density (mass per unit length of string) is m and the tension T acts on the string due to stretching. If it is made to vibrate by plucking or by using a vibrator like a tuning fork, a transverse wave can be produced along the string.

When the wave reaches to the fixed ends of the string, it gets reflected with change of phase by π radians. The reflected waves interfere with the incident wave and stationary waves are formed along the string. The string vibrates with different modes of vibrations.

If a string is stretched between two rigid supports and is plucked at its centre, the string vibrates as shown in Fig 6.11 (a). It consists of an antinode formed at the centre and nodes at the two ends with one loop formed along its length. If λ is the wavelength and l is the length of the string, we get

Length of loop $= \dfrac{\lambda}{2} = l$

$$\therefore \lambda = 2l$$

The frequency of vibrations of the string,

$$n = \frac{v}{\lambda} = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \left(\because v = \sqrt{\frac{T}{m}}\right)$$

This is the lowest frequency with which the string can vibrate. It is the fundamental frequency of vibrations or the first harmonic.

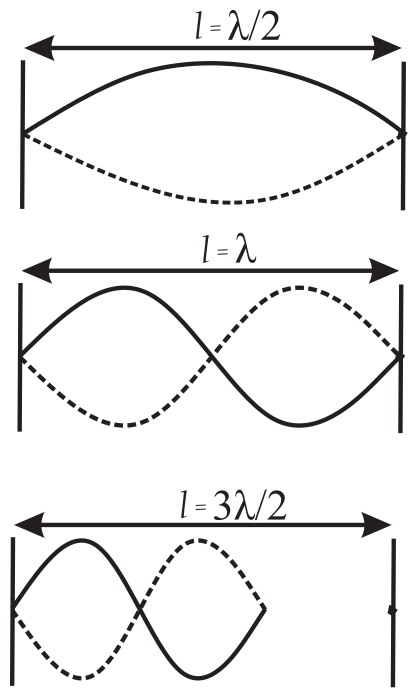


Fig. 6.11: Different modes of vibrations of a stretched string.

If the centre of the string is prevented from vibrating by touching it with a light object and string is plucked at a point midway between one of the segments, the string vibrates as shown in Fig. 6.11 (b).

Two loops are formed in this mode of vibrations. There is a node at the centre of the string and at its both ends. If λ_1 is wavelength of vibrations, the length of one loop $= \dfrac{\lambda_1}{2} = \dfrac{l}{2}$

$$\therefore \lambda_1 = l$$

Thus, the frequency of vibrations is given as

$$n_1 = \frac{1}{\lambda_1}\sqrt{\frac{T}{m}}$$

$$n_1 = \frac{1}{l}\sqrt{\frac{T}{m}}$$

Comparing with fundamental frequency we get that $n_1 = 2n$.

Thus the frequency of the first overtone or second harmonic is equal to twice the fundamental frequency.

The string is made to vibrate in such a way that three loops are formed along the string as shown in Fig. 6.11 (c). If λ_2 is the wavelength here, the length of one loop is $\frac{\lambda_2}{2} = \frac{l}{3}$

$$\therefore \lambda_2 = \frac{2l}{3}$$

Therefore the frequency of vibrations is

$$n_2 = \frac{1}{\lambda_2}\sqrt{\frac{T}{m}}$$

$$n_2 = \frac{3}{2l}\sqrt{\frac{T}{m}}$$

Comparing with fundamental frequency, we get that $n_2 = 3n$.

Thus frequency of second overtone or third harmonic is equal to thrice the fundamental frequency. Similarly for higher modes of vibrations of the string, the frequencies of vibrations are as $4n$, $5n$, $6n$…etc. Thus all harmonics are present in case of a stretched string and the frequencies are given by

$$n_p = pn \qquad \text{--- (6.31)}$$

Example 6.6: A string is fixed at both ends. What is the ratio of the frequency of the first harmonic to that of the second harmonic?

Solution: For a string of length l fixed at both ends, the wavelengths of the first and second harmonics are given as $l = \lambda/2$ and $l = \lambda_1$ respectively. Hence the ratio of their frequencies is

$$\frac{n}{n_1} = \frac{v/\lambda}{v/\lambda_1} = \frac{\lambda_1}{\lambda} = \frac{l}{2l} = \frac{1}{2}$$

Example 6.7: The velocity of a transverse wave on a string of length 0.5 m is 225 m/s. (a) What is the fundamental frequency of a standing wave on this string if both ends are kept fixed? (b) While this string is vibrating in the fundamental harmonic, what is the wavelength of sound produced in air if the velocity of sound in air is 330 m/s?

Solution: The wavelength of the fundamental mode is $\lambda = 2l$, hence the fundamental frequency is

$$n = \frac{v}{\lambda} = \frac{v}{2l} = \frac{225\,\text{m/s}}{2\times 0.5\,\text{m}} = 225\,\text{s}^{-1} = 225\ \text{Hz}$$

While the string is vibrating in the fundamental harmonic, the frequency of the sound produced by the string will be same as the fundamental frequency of the string. The wavelength of sound produced is $= \frac{v_s}{n} = \frac{330\ \text{m/s}}{225\,\text{s}^{-1}} = 1.467$ m.

6.7.6 Laws of a Vibrating String :

The fundamental frequency of a vibrating string under tension is given as

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}} \qquad \text{--- (6.32)}$$

From this formula, three laws of vibrating string can be given as follows:

1) Law of length: The fundamental frequency of vibrations of a string is inversely proportional to the length of the vibrating string, if tension and mass per unit length are constant.

$$n \propto \frac{1}{l}, \text{ if } T \text{ and } m \text{ are constant.} \quad \text{--- (6.33)}$$

2) Law of tension: The fundamental frequency of vibrations of a string is directly proportional to the square root of tension, if vibrating length and mass per unit length are constant.

$$n \propto \sqrt{T}, \text{ if } l \text{ and } m \text{ are constant.} \quad \text{--- (6.34)}$$

3) Law of linear density: The fundamental frequency of vibrations of a string is inversely proportional to the square root of mass per unit length (linear density), if the tension and vibrating length of the string are constant.

$$n \propto \frac{1}{\sqrt{m}}, \text{ if } T \text{ and } l \text{ are constant.} \quad \text{--- (6.35)}$$

If r is the radius and ρ is the density of material of string, linear density is given as

Linear density = mass per unit length

$$= \text{volume per unit length} \times \text{density}$$

$$= (\pi r^2 l/l)\rho$$

As $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant, we get

$$n \propto \frac{1}{\sqrt{\pi r^2 \rho}}$$

$$\therefore \ n \propto \frac{1}{\sqrt{\rho}} \text{ and } n \propto \frac{1}{r} \quad \text{--- (6.36)}$$

Thus the fundamental frequency of vibrations of a stretched string is inversely proportional to (i) the radius of string and (ii) the square root of the density of the material of vibrating string.

Example 6.8: A string 105 cm long is fixed at one end. Transverse vibrations of frequency 15 Hz are imposed at the free end. A stationary wave, produced in the string, consists of 3 loops. Calculate the speed of progressive waves which have produced the stationary wave in the string.

Solution: Given

Length of string = l = 105 cm = 3 loops

$$\therefore \ l = 3\frac{\lambda}{2}$$

$$\therefore \lambda = \frac{2}{3}l = \frac{2}{3} \times 105 = 70\,\text{cm} = 0.70 \text{ m}$$

Speed of wave = $\text{v} = n\lambda$

$$\text{v} = 15 \times 0.70 = 10.50 \text{ m s}^{-1}$$

6.8 Sonometer:

A sonometer consists of a hollow rectangular wooden box called the sound box. The sound box is used to make a larger mass of air vibrate so that the sound produced by the vibrating string (metal wire in this case) gets amplified. The same principle is applied in stringed instruments such as the violin, guitar, *tanpura* etc. There are two bridges P and Q along the width of the box which can be moved parallel to the length of box. A metal wire of uniform cross-section runs along the length of the box over the bridges. It is fixed at one end and its other end passes over a pulley. A hanger with suitable slotted weights can be attached to the free end of wire. By changing the weights, the tension in the wire can be varied. The movable bridges allow us to change the vibrating length AB of the wire.

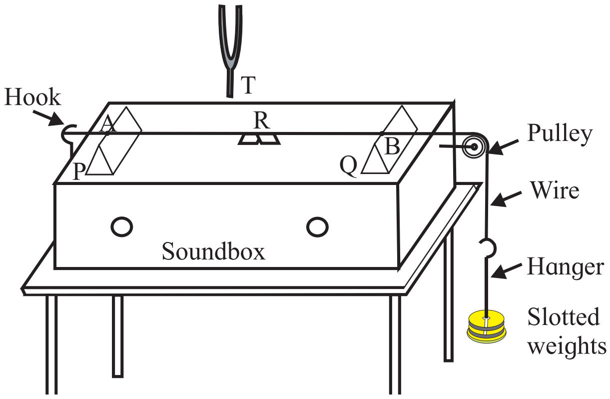


Fig. 6.12: Experimental set-up of a sonometer.

If the wire is plucked at a point midway between the bridges, transverse waves are produced in the wire. Stationary waves are produced between the two bridges due to reflection of transverse wave at the bridges and their superposition. Thus portion AB of the wire between the two bridges P and Q is the vibrating length. Wire can also be made to vibrate by holding a vibrating tuning fork near it. The frequency of vibration is then same as that of the tuning fork. If this frequency happens to be one of the natural frequencies of the wire, standing waves with large amplitude are set up in the wire since the two vibrate in resonance.

To identify the resonance, a small piece of paper, known as the rider R, is placed over the wire at a point in the middle of the length AB as determined by the position of the bridges P and Q. If the frequency of the tuning fork and of the fundamental mode of vibration of the wire match (this is achieved by adjusting the length AB of wire using the bridges P and Q), the paper rider happens to be at the antinode and flies off the wire.

Sonometer can be used to verify the laws of a vibrating string.

1) Verification of first law of a vibrating string:

By measuring length of wire and its mass, the mass per unit length (m) of wire is determined. Then the wire is stretched on the sonometer and the hanger is suspended from its free end. A suitable tension (T) is applied to the wire by placing slotted weights on the hanger. The length of wire (l_1) vibrating with the same frequency (n_1) as that of the tuning fork is determined as follows.

A light paper rider is placed on the wire midway between the bridges. The tuning fork is set into vibrations by striking on a rubber pad. The stem of tuning fork is held in contact with the sonometer box. By changing distance between the bridges without disturbing paper rider, frequency of vibrations of wire is changed. When the frequency of vibrations of wire becomes exactly equal to the frequency of tuning fork, the wire vibrates with maximum amplitude and the paper rider is thrown off.

In this way a set of tuning forks having different frequencies n_1, n_2, n_3, ...are used and corresponding vibrating lengths of wire are noted as l_1, l_2, l_3..........by keeping the tension T constant . We will observe that

$n_1 l_1 = n_2 l_2 = n_3 l_3 = \ldots\ldots$= constant, for constant value of tension (T) and mass per unit length (m).

$\therefore nl =$ constant

i.e., $n \propto \frac{1}{l}$, if T and m are constant.

Thus, the first law of a vibrating string is verified by using a sonometer.

2) Verification of second law of a vibrating string:

The vibrating length (l) of the given wire of mass per unit length (m) is kept constant for verification of second law. By changing the tension the same length is made to vibrate in unison with different tuning forks of various frequencies. If tensions T_1, T_2, T_3....... correspond to frequencies n_1, n_2, n_3,.........etc. we will observe that.

$$\frac{n_1}{\sqrt{T_1}} = \frac{n_2}{\sqrt{T_2}} = \frac{n_3}{\sqrt{T_3}} = \ldots\ldots\ldots = \text{constant}$$

$$\text{or } \frac{n}{\sqrt{T}} = \text{constant}$$

$\therefore n \propto \sqrt{T}$ if l and m are constant. This is the second law of a vibrating string.

3) Verification of third law of a vibrating string:

For verification of third law of a vibrating string, two wires having different masses per unit lengths m_1 and m_2 (linear densities) are used. The first wire is subjected to suitable tension and made to vibrate in unison with given tuning fork. The vibrating length is noted as (l_1). Using the same fork, the second wire is made to vibrate under the same tension and the vibrating length (l_2) is determined. Thus the frequency of vibration of the two wires is kept same under same applied tension T. It is found that,

$$l_1\sqrt{m_1} = l_2\sqrt{m_2}$$

$$l\sqrt{m} = \text{constant}$$

But by first law of a vibrating string, $n \propto \frac{1}{l}$

Therefore we get that, $n \propto \frac{1}{\sqrt{m}}$, if T and l are constant. This is the third law of vibrating string.

In this way, laws of a vibrating string are verified by using a sonometer.

Example 6.9: A sonometer wire of length 50 cm is stretched by keeping weights equivalent of 3.5 kg. The fundamental frequency of vibration is 125 Hz. Determine the linear density of the wire.

Solution: Given, $l = 50$ cm $= 0.5$ m , $T = 3.5$ kg × 9.8 m/s^2 = 34.3 N, n = 125 Hz

$$n = \frac{1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore n^2 = \frac{1}{4l^2}\frac{T}{m}$$

$$\therefore m = \frac{T}{4n^2l^2}$$

$$\therefore m = \frac{34.3}{4\times(125)^2\times(0.5)^2}$$

$$\therefore m = 2.195\times10^{-3}\ \text{kg m}^{-1}$$

Example 6.10: Two wires of the same material and the same cross section are stretched on a sonometer in succession. Length of one wire is 60 cm and that of the other is 30 cm. An unknown load is applied to the first wire and second wire is loaded with 1.5 kg. If both the wires vibrate with the same fundamental frequencies, calculate the unknown load.

Solution: Two wires are given to be of the same material and having the same cross section,

$$\therefore m_1 = m_2 = m$$

Same fundamental frequency, $n_1 = n_2 = n$

$l_1 = 60$ cm $= 0.6$ m, $l_2 = 30$ cm $= 0.3$ m, $T_2 = 1.5 \times 9.8$ N

For the first wire, $n_1 = \frac{1}{2l_1}\sqrt{\frac{T_1}{m_1}}$

For the second wire, $n_2 = \frac{1}{2l_2}\sqrt{\frac{T_2}{m_2}}$

$$\therefore \frac{n_1}{n_2} = \frac{l_2}{l_1}\sqrt{\frac{T_1 \times m_2}{T_2 \times m_1}}$$

$$\therefore \frac{n}{n} = \frac{0.3}{0.6}\sqrt{\frac{T_1 \times m}{1.5 \times 9.8 \times m}}$$

$$\therefore 1 = \frac{1}{2}\sqrt{\frac{T_1}{1.5 \times 9.8}}$$

$$\therefore 2 = \sqrt{\frac{T_1}{1.5 \times 9.8}}$$

or, $4 = \frac{T_1}{1.5 \times 9.8}$ $\quad\therefore T_1 = 6 \times 9.8$ N

$\therefore$ Applied load = 6 kg.

Example 6.11: A wire has linear density 4.0×10^{-3} kg/m. It is stretched between two rigid supports with a tension of 360 N. The wire resonates at a frequency of 420 Hz and 490 Hz in two successive modes. Find the length of the wire.

Solution: Given $m = 4.0 \times 10^{-3}$ kg/m, $T =$ 360 N. Let the wire vibrate at 420 Hz and 490 Hz in its p^{th} and $(p+1)^{th}$ harmonics.

Then $n_p = p.n$ where $n = \frac{1}{2l}\sqrt{\frac{T}{m}}$ is the fundamental frequency

$$420\text{ Hz} = \frac{p}{2l}\sqrt{\frac{T}{m}} \quad \text{and} \quad 490\text{ Hz} = \frac{p+1}{2l}\sqrt{\frac{T}{m}}$$

$$\therefore \frac{490}{420} = \frac{p+1}{p}$$

or, $p = 6$

Using this value of p, for the frequency of p^{th} harmonic, we get

$$420\text{ Hz} = \frac{6}{2l}\sqrt{\frac{360\text{ N}}{4.0 \times 10^{-3}\text{ kg/m}}} = \frac{900}{l}\text{ m/s}$$

$\therefore l = 900/420$ m $= 2.143$ m

6.9 Beats:

This is an interesting phenomenon based on the principle of superposition of waves. When there is superposition of two sound waves, having same amplitude but slightly different frequencies, travelling in the same direction, the intensity of sound varies periodically with time. This phenomenon is known as production of beats.

The occurrences of maximum intensity are called waxing and those of minimum intensity are called waning. One waxing and successive waning together constitute one beat. The number of beats heard per second is called beat frequency.

6.9.1 Analytical method to determine beat frequency:

Consider two sound waves, having same amplitude and slightly different frequencies n_1 and n_2. Let us assume that they arrive in phase at some point x of the medium. The displacement due to each wave at any instant of time at that point is given as

$$y_1 = a\sin\left\{2\pi\left(n_1 t - \frac{x}{\lambda_1}\right)\right\}$$

$$y_2 = a\sin\left\{2\pi\left(n_2 t - \frac{x}{\lambda_2}\right)\right\}$$

Let us assume for simplicity that the listener is at $x = 0$.

$$\therefore \quad y_1 = a\sin(2\pi n_1 t)$$

and $\quad y_2 = a\sin(2\pi n_2 t)$

According to the principle of superposition of waves,

$$y = y_1 + y_2$$

$$\therefore \; y = a\sin(2\pi n_1 t) + a\sin(2\pi n_2 t)$$

or,

$$y = 2a\sin\left[2\pi\left(\frac{n_1+n_2}{2}\right)t\right]\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] \quad \text{--- (6.37)}$$

[By using formula,

$\sin C + \sin D = 2\sin\left(\frac{C+D}{2}\right)\cos\left(\frac{C-D}{2}\right)$]

Rearranging the above equation, we get

$$y = 2a\cos\left[\frac{2\pi(n_1-n_2)}{2}t\right]\sin\left[\frac{2\pi(n_1+n_2)}{2}t\right]$$

Substituting $2a\cos\left[\frac{2\pi(n_1-n_2)}{2}t\right] = A$

and $\frac{n_1+n_2}{2} = n$, we get

$$y = A\sin(2\pi n t) \quad \text{--- (6.38)}$$

This is the equation of a progressive wave having frequency n and amplitude A. The frequency n is the mean of the frequencies n_1 and n_2 of arriving waves while the amplitude A varies periodically with time.

The intensity of sound is proportional to the square of the amplitude. Hence the resultant intensity will be maximum when the amplitude is maximum.

For maximum amplitude (waxing),

$$A = \pm 2a$$

$$\therefore \quad 2a\cos\left[\frac{2\pi(n_1-n_2)}{2}\right]t = \pm 2a$$

$$\text{or,} \; \cos\left[\frac{2\pi(n_1-n_2)}{2}\right]t = \pm 1$$

$$\text{i.e.,} \left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0, \pi, 2\pi, 3\pi, \ldots\ldots$$

$$\therefore t = 0, \frac{1}{n_1-n_2}, \frac{2}{n_1-n_2}, \frac{3}{n_1-n_2}, \ldots.$$

Thus, the time interval between two successive maxima of sound is always $\frac{1}{n_1-n_2}$.

Hence the period of beats is $T = \frac{1}{n_1-n_2}$.

The number of waxing heard per second is the reciprocal of period of waxing.

$\therefore$ frequency of beats, $N = n_1 - n_2$ --- (6.39)

The intensity of sound will be minimum when amplitude is zero (waning):

For minimum amplitude, $A = 0$,

$$\therefore \; 2a\cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0$$

$$\text{or,} \quad \cos\left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = 0$$

$$\therefore \left[2\pi\left(\frac{n_1-n_2}{2}\right)t\right] = \frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2} \ldots\ldots$$

$$\therefore t = \frac{1}{2(n_1-n_2)}, \frac{3}{2(n_1-n_2)}, \frac{5}{2(n_1-n_2)},$$

Therefore time interval between two successive minima is also $\frac{1}{(n_1-n_2)}$, which is expected.

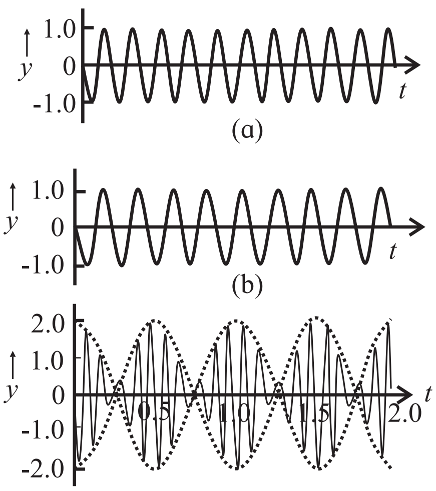


Fig. 6.13: Superposition of two harmonic waves of nearly equal frequencies resulting in the formation of beats.

By comparing the instances of successive waxing and waning, we come to know that waxing and waning occur alternately with equal frequency.

The variation in the loudness of sound that goes up and down is the phenomenon of formation of beats. It can be considered as superposition of waves and formation of standing waves in time at one point in space where waves of slightly different frequencies are passing. The two waves are in and out of phase giving constructive and destructive interference. The interval between two maximum sound intensities is the time period of beats.

Remember this

We can hear beats if the frequency difference between the two superimposed waves is very small (practically less than 6-7 Hz, for normal human ear). At frequencies higher than these, individual beats cannot be distinguished from the sound that is produced.

Activity

- Take two tuning forks of the same frequency.
- Put some wax on the prongs of one of the forks.
- Vibrate both the tuning forks and keep them side by side.
- Listen to the periodic vibrations of loudness of resulting sound.
- How many beats have you heard in one minute?
- Can you guess whether frequency of tuning fork is increased or decreased by applying wax on the prong?
- How you can find the new frequency of the fork after applying wax on it.

6.9.2 Applications of beats :

1] The phenomenon of beats is used for matching the frequencies of different musical instruments by artists. They go on tuning until no beats are heard by their sensitive ears. When beat frequency becomes equal to zero, the musical instruments are in unison with each other i.e., their frequencies are identical and the effect of playing such instruments together gives a pleasant music.

2] The speed of an airplane can be determined by using Doppler RADAR.

When a source of sound and the listener are in relative motion, the listener detects a sound whose frequency is different from the actual or original frequency of the sound source. This is Doppler effect.

A microwave signal (pulse) of known frequency is sent towards the moving airplane. Principle of Doppler effect giving the apparent frequency when the source and observer are in relative motion applies twice, once for the signal sent by the microwave source and received by the airplane and second time when the signal is reflected by the airplane and is received back at the microwave source. Phenomenon of beats, arising due to the difference in frequencies produced by the source and received at the source after reflection from the air plane, allows us to calculate the velocity of the air plane.

The same principle is used by traffic police to determine the speed of a vehicle to check whether speed limit is exceeded. Sonar (**So**und **n**avigation **a**nd **r**anging) works on similar principle for determining speed of submarines using a sound source and sensitive microphones.

Doppler ultrasonography and echo cardiogram work on similar principle. Doctors use an analogous set up to assess the direction and speed of blood flow in a human body and identify circulation problems. Measurement of the dimension of the blood vessels can be used to estimate the volume flow rate. Ultrasound beams also determine phase shifts to diagnose vascular problems in arteries and veins.

3] Unknown frequency of a sound note can be determined by using the phenomenon of beats. Initially the sound notes of known and unknown frequency are heard simultaneously. The known frequency from a source of adjustable frequency is adjusted in such a way that the beat frequency reduces to zero. At this stage frequencies of both the sound notes become equal. Hence unknown frequency can be determined.

Example 6.12: Two sound waves having wavelengths 81cm and 82.5 cm produce 8 beats per second. Calculate the speed of sound in air.

Solution: Given

$$\lambda_1 = 81 \text{ cm} = 0.81 \text{ m}$$
$$\lambda_2 = 82.5 \text{ cm} = 0.825 \text{ m}$$
$$n_1 = \frac{v}{\lambda_1} = \frac{v}{0.81}$$
$$n_2 = \frac{v}{\lambda_2} = \frac{v}{0.825}$$

Here $\lambda_1 < \lambda_2, \therefore n_1 > n_2$.

As 8 beats are produced per second,

$$n_1 - n_2 = 8$$
$$\therefore \frac{v}{\lambda_1} - \frac{v}{\lambda_2} = 8$$
$$\therefore v\left[\frac{1}{\lambda_1} - \frac{1}{\lambda_2}\right] = 8$$
$$\therefore v\left[\frac{1}{0.81} - \frac{1}{0.825}\right] = 8$$
$$\therefore v = 356.4$$

Example 6.13: Two tuning forks having frequencies 320 Hz and 340 Hz are sounded together to produce sound waves. The velocity of sound in air is 326.4 m s^{-1}. Find the difference in wavelength of these waves.

Solution: Given

$n_1 = 320$ Hz, $n_2 = 340$ Hz, v = 326.4 m s^{-1}.

$$v = n_1\lambda_1 = n_2\lambda_2$$

Here, $n_1 < n_2$, $\therefore \lambda_1 > \lambda_2$

$$\therefore \lambda_1 - \lambda_2 = \frac{v}{n_1} - \frac{v}{n_2}$$
$$\therefore \lambda_1 - \lambda_2 = v\left[\frac{1}{n_1} - \frac{1}{n_2}\right]$$
$$\therefore \lambda_1 - \lambda_2 = 326.4\left[\frac{1}{320} - \frac{1}{340}\right]$$
$$\therefore \lambda_1 - \lambda_2 = 0.06 \text{ m}$$

6.10 Characteristics of Sound:

Sound has three characteristics: loudness, pitch and quality.

1. Loudness: Loudness is the human perception to intensity of sound. We know that when a sound wave travels through a medium, there are regions of compressions and rarefactions. Thus there are changes in pressure. When a sound is heard, say by a human, the wave exerts pressure on the human ear. The pressure variation is related to the amplitude and hence to the intensity. Depending on the sound produced, the variation in this pressure is from 28 Pa for the loudest tolerable sound to 2.0×10^{-5} Pa for the feeblest sound like a whisper that can be heard by a human. Intensity is a measurable quantity while the sensation of hearing or loudness is very subjective. It is therefore important to find out how does a sound of intensity I affect a detectable change ΔI in the intensity for the human ear to note. It is known that the value of such ΔI depends linearly on intensity I and this fact allows humans to deal with a large variation in intensity.

The response of human ear to sound is exponential and not linear. It depends upon the amount of energy crossing unit area around a point per unit time. Intensity is proportional to the square of amplitude. It also depends upon various other factors like distance of source from the listener, the motion of air, density of medium, the surface area of sounding body etc. The presence of other resonant objects around the sounding body also affects loudness of sound.

Scientifically, sound is specified not by its intensity but by the sound level β (expressed in *decibles* (dB)), defined as

$$\beta = 10 \log_{10}\left(\frac{I}{I_0}\right), \qquad \text{--- (6.40)}$$

where I_0 is a minimum reference intensity (10^{-12} W/m^2) that a normal human ear can hear. Sound levels are then expressed in decibel (dB). When $I = I_0$, $\beta = 0$, thus the standard reference intensity has measure of sound level 0 dB. The unit of difference in loudness is ***bel***. You have studied about this unit in XIth Std.

$$1\,decibel = \frac{1}{10}\,bel$$

As mentioned above, minimum audible sound is denoted by 0 dB while whispering and normal speech have levels 10 dB and 60 dB respectively at a distance of approximately 1 m from the source. The intensity level of maximum tolerable sound for a human ear is around 120 dB.

Loudness is different at different frequencies, even for the same intensity. For measuring loudness the unit ***phon*** is used. Phon is a measure of loudness. It is equal to the loudness in decibel of any equally loud pure tone of frequency 1000Hz.

2. Pitch: It is a sensation of sound which helps the listener to distinguish between a high frequency and a low frequency note. Pitch is the human perception to frequency- higher frequency denotes higher pitch. The pitch of a female voice is higher than that of a male voice.

3. Quality or timbre: Normally sound generated by a source has a number of frequency components with different amplitudes. Quality of sound is that characteristic which enables us to distinguish between two sounds of same pitch and loudness. We can recognize the voice of a person or an instrument due to its quality of sound. Quality depends on number of overtones present in the sound along with a given frequency.

A sound which produces a pleasing sensation to the ear is a musical sound. It is produced by regular and periodic vibrations without any sudden change in loudness. Musical sound has certain well-defined frequencies with sizable amplitude; these are normally harmonics of a fundamental frequency. A mixture of sounds of different frequencies which do not have any relation with each other produces what we call a noise. Noise therefore is not pleasant to hear. If in addition, it is loud, it may cause headaches.

A sequence of frequencies which have a specific relationship with each other is called a musical scale. Normally both in Indian classical music and western classical music, eight frequencies, in specific ratio, form an octave, each frequency denoting a specific note. In a given octave frequency increases along sa re ga ma pa dha ni så (as well as along Do Re Mi Fa So La Ti Dò). An example of values of frequencies is 240, 270, 300, 320, 360, 400, 450, 480 Hz respectively.

6.11 Musical instruments:

Audible waves originate in vibrating strings, vibrating air columns and vibrating plates and membranes. Accordingly, musical instruments are classified into three main types. (a) Stringed instruments (b) wind instruments (c) percussion instruments.

a) Stringed instruments: consist of stretched strings. Sound is produced by plucking of strings. The strings are tuned to certain frequencies by adjusting tension in them. They are further of three different types.

1) Plucked string type: In these instruments string is plucked by fingers, e.g., *tanpura*, *sitar*, guitar, *veena*, etc.

2) Bowed string type: In these instruments, a string is played by bowing, e.g., violin, *sarangi*.

3) Struck string type: the string is struck by a stick , e.g. *santoor*, piano.

b) Wind instruments: These instruments consist of air column. Sound is produced by setting vibrations of air column. They are further of three different types

1) Freewind type: In these instruments free brass reeds are vibrated by air. The air is either blown or compressed. e.g., mouth organ, harmonium etc.

2) Edge type: In these instruments air is blown against an edge. e.g., Flute.

3) Reedpipes: They may consists of single or double reeds and also instruments without reeds .e.g., saxophone, clarinet (single reed), bassoon (double reed), bugle (without reed).

c) Percussion instruments: In these instruments sound is produced by setting vibrations in a stretched membrane. e.g., tabla, drum, dhol, mridangam, sambal, daphali,etc. These instruments sometimes also consist of metal plates which produce sound when they are struck against each other or with a beater. e.g., cymbals (i.e., jhanja), xylophone, etc.

A blow on the membrane or plate or plucking of string produces vibrations with one fundamental and many overtones. Superposition of several natural modes of oscillations with different amplitudes and hence intensities characterize different musical instruments. We can thus distinguish the instruments by their sounds.

Production of different notes by musical instrument depends on the creation of stationary waves. For a stringed instrument such as guitar or sitar, the two ends of the string are fixed. Depending on where the string is plucked, stationary waves of various modes can be produced, plucking at the midpoint produces the minimum frequency or the fundamental mode of vibration. In wind instruments, air column is made to vibrate by blowing. By changing the length of air column, note can be changed. In wind instrument like flute, holes can be uncovered to change the vibrations of air column this changes the pattern of nodes and antinodes.

In practice, sound produced is made up of several stationary waves having different patterns of nodes and antinodes. Musicians skill lies in stimulating the string or air column to produce the desired mixture of frequencies.

Do you know?

Sir C.V. Raman, the great physicist and the first Noble Laureate of India, had done research on the Indian classical musical instruments such as *mridangam* and *tabla*. Read more about his research work in this field from website: https://www.livehistoryindia.com c.v.ramans work on Indian music.

Internet my friend

- https://www.acs.psu.edu/drussell/Demos/superposition/superposition.html
- https://www.acs.psu.edu/drussell/demos.html
- https://www.google.com/search?client=firefox-b-d&q=superposition+of+waves
- https://www.youtube.com/watch?v=J_Oto3mUIuk
- https://www.youtube.com/watch?v=GsP5LqGtkwE
- https://www.acs.psu.edu/drussell/Demos/StandingWaves/StandingWaves.html
- https://www.physicsclassroom.com/class/waves/Lesson-4/Formation-of-Standing-Waves
- https://www.physicsclassroom.com/class/waves/Lesson-4/Formation-of-Standing-Waves
- https://www.youtube.com/watch?v=-D9UlPcJSRM
- https://www.youtube.com/watch?v=jHjXNFmm8y4
- https://www.youtube.com/watch?v=BWqyXHKhaZ8
- https://physics.info/waves-standing/
- https://www.youtube.com/watch?v=nrJrV_Gn_Cw&t=661s

Exercises

1. Choose the correct option.

i) When an air column in a pipe closed at one end vibrates such that three nodes are formed in it, the frequency of its vibrations is …….times the fundamental frequency.

(A) 2 (B) 3 (C) 4 (D) 5

ii) If two open organ pipes of length 50 cm and 51 cm sounded together produce 7 beats per second, the speed of sound is.

(A) 307 m/s (B) 327m/s
(C) 350m/s (D) 357m/s

iii) The tension in a piano wire is increased by 25%. Its frequency becomes ….. times the original frequency.

(A) 0.8 (B) 1.12 (C) 1.25 (D) 1.56

iv) Which of the following equations represents a wave travelling along the y-axis?

(A) $x = A\sin(ky - \omega t)$
(B) $y = A\sin(kx - \omega t)$
(C) $y = A\sin(ky)\cos(\omega t)$
(D) $y = A\cos(ky)\sin(\omega t)$

v) A standing wave is produced on a string fixed at one end with the other end free. The length of the string

(A) must be an odd integral multiple of λ/4.
(B) must be an odd integral multiple of λ/2.
(C) must be an odd integral multiple of λ.
(D) must be an even integral multiple of λ.

2. Answer in brief.

i) A wave is represented by an equation y = A sin (Bx + Ct). Given that the constants A, B and C are positive, can you tell in which direction the wave is moving?

ii) A string is fixed at the two ends and is vibrating in its fundamental mode. It is known that the two ends will be at rest. Apart from these, is there any position on the string which can be touched so as not to disturb the motion of the string? What will be the answer to this question if the string is vibrating in its first and second overtones?

iii) What are harmonics and overtones?

iv) For a stationary wave set up in a string having both ends fixed, what is the ratio of the fundamental frequency to the second harmonic?

v) The amplitude of a wave is represented by

$$y = 0.2\sin 4\pi\left[\frac{t}{0.08} - \frac{x}{0.8}\right] \text{ in SI units.}$$

Find (a) wavelength, (b) frequency and (c) amplitude of the wave.

[(a) 0.4 m (b) 25 Hz (c) 0.2 m]

3. State the characteristics of progressive waves.

4. State the characteristics of stationary waves.

5. Derive an expression for equation of stationary wave on a stretched string.

6. Find the amplitude of the resultant wave produced due to interference of two waves given as $y_1 = A_1\sin\omega t \quad y_2 = A_2\sin(\omega t + \varphi)$

7. State the laws of vibrating strings and explain how they can be verified using a sonometer.

8. Show that only odd harmonics are present in the vibrations of air column in a pipe closed at one end.

9. Prove that all harmonics are present in the vibrations of the air column in a pipe open at both ends.

10. A wave of frequency 500 Hz is travelling with a speed of 350 m/s.
(a) What is the phase difference between two displacements at a certain point at times 1.0 ms apart? (b) what will be the smallest distance between two points which are 45° out of phase at an instant of time?

[Ans : π, 8.75 cm]

11. A sound wave in a certain fluid medium is reflected at an obstacle to form a standing wave. The distance between two successive nodes is 3.75 cm. If the velocity of sound is 1500 m/s, find the frequency.

[Ans : 20 kHz]

12. Two sources of sound are separated by a distance 4 m. They both emit sound with the same amplitude and frequency (330 Hz), but they are 180º out of phase. At what points between the two sources, will the sound intensity be maximum? (Take velocity of sound to be 330 m/s)

[Ans: ± 0.25, ± 0.75, ± 1.25 and ± 1.75 m from the point at the center]

13. Two sound waves travel at a speed of 330 m/s. If their frequencies are also identical and are equal to 540 Hz, what will be the phase difference between the waves at points 3.5 m from one source and 3 m from the other if the sources are in phase?

[Ans : 1.636 π]

14. Two wires of the same material and same cross section are stretched on a sonometer. One wire is loaded with 1.5 kg and another is loaded with 6 kg. The vibrating length of first wire is 60 cm and its fundamental frequency of vibration is the same as that of the second wire. Calculate vibrating length of the other wire.

[Ans: 1.2 m]

15. A pipe closed at one end can produce overtones at frequencies 640 Hz, 896 Hz and 1152 Hz. Calculate the fundamental frequency.

[Ans: 128 Hz]

16. A standing wave is produced in a tube open at both ends. The fundamental frequency is 300 Hz. What is the length of tube in the fundamental mode? (speed of the sound= 340 m s^{-1}). [Ans: 0.5666 m]

17. Find the fundamental, first overtone and second overtone frequencies of a pipe, open at both the ends, of length 25 cm if the speed of sound in air is 330 m/s.

[Ans: 660 Hz, 1320 Hz, 1980 Hz]

18. A pipe open at both the ends has a fundamental frequency of 600 Hz. The first overtone of a pipe closed at one end has the same frequency as the first overtone of the open pipe. How long are the two pipes? (Take velocity of sound to be 330 m/s)

[Ans : 27.5 cm, 20.625 cm]

19. A string 1m long is fixed at one end. Transverse vibrations of frequency 15 Hz are imposed at the free end. Due to this, a stationary wave with four complete loops, is produced on the string. Find the speed of the progressive wave which produces the stationary wave.[Hint: Remember that the free end is an antinode.]

[Ans: 6.67 m s^{-1}]

20. A violin string vibrates with fundamental frequency of 440Hz. What are the frequencies of first and second overtones?

[Ans: 880 Hz, 1320 Hz]

21. A set of 8 tuning forks is arranged in a series of increasing order of frequencies. Each fork gives 4 beats per second with the next one and the frequency of last fork is twice that of the first. Calculate the frequencies of the first and the last fork.

[Ans: 28 Hz, 56 Hz]

22. A sonometer wire is stretched by tension of 40 N. It vibrates in unison with a tuning fork of frequency 384 Hz. How many numbers of beats get produced in two seconds if the tension in the wire is decreased by 1.24 N?

[Ans: 12 beats]

23. A sonometer wire of length 0.5 m is stretched by a weight of 5 kg. The fundamental frequency of vibration is 100 Hz. Calculate linear density of wire.

[Ans: 4.9×10^{-3} kg/m]

24. The string of a guitar is 80 cm long and has a fundamental frequency of 112 Hz. If a guitarist wishes to produce a frequency of 160 Hz, where should the person press the string? [Ans : 56 cm from one end]

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