# 9 Surface Area and Volume

## Let's Study

- **Surface area of a cone**
- **Volume of a cone**
- **Surface area of a sphere**
- **Volume of a sphere**

## Let's recall.

We have learnt how to find the surface area and volume of a cuboid, a cube and a cylinder, in earlier standard.

Cuboid

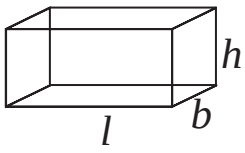



**Fig.9.1**

- Length, breadth and height of a cuboid are $l$ , $b$, $h$ respectively.

(i) Area of vertical surfaces of a cuboid = $2(l + b) \times h$

Here we have considered only 4 surfaces into consideration.

(ii) Total surface area of a cuboid = $2(lb + bh + lh)$

Here we have taken all 6 surfaces into consideration.

(iii) Volume of a cuboid $= l \times b \times h$

Cube

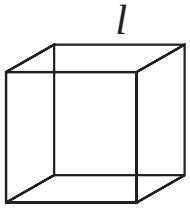



**Fig.9.2**

- If $l$ is the edge of a cube,

(i) Total surface area of a cube = $6l^2$

(ii) Area of vertical surfaces of a cube $= 4l^2$

(iii) Volume of a cube = $l^3$

Cylinder

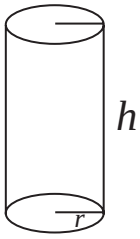



**Fig.9.3**

- Radius of cylinder is $r$ and height is $h$.

(i) Curved surface area of a cylinder = $2\pi rh$

(ii) Total surface area of a cylinder = $2\pi r(r + h)$

(iii) Volume of a cylinder = $\pi r^2 h$

## Practice Set 9.1

1. Length, breadth and height of a cuboid shape box of medicine is 20cm, 12 cm and 10 cm respectively. Find the surface area of vertical faces and total surface area of this box.
2. Total surface area of a box of cuboid shape is 500 sq. unit. Its breadth and height is 6 unit and 5 unit respectively. What is the length of that box ?
3. Side of a cube is 4.5 cm. Find the surface area of all vertical faces and total surface area of the cube.
4. Total surface area of a cube is 5400 sq. cm. Find the surface area of all vertical faces of the cube.
5. Volume of a cuboid is 34.50 cubic metre. Breadth and height of the cuboid is 1.5m and 1.15m respectively. Find its length.
6. What will be the volume of a cube having length of edge 7.5 cm ?
7. Radius of base of a cylinder is 20cm and its height is 13cm, find its curved surface area and total surface area. ($\pi = 3.14$)
8. Curved surface area of a cylinder is 1980 $cm^2$ and radius of its base is 15cm. Find the height of the cylinder. ($\pi = \frac{22}{7}$).

## Let's learn.

### Terms related to a cone and their relation

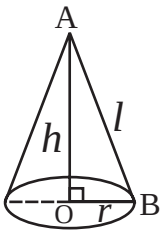



**Fig.9.4**

A cone is shown in the adjacent Fig.9.4. Centre of the circle, which is the base of the cone, is O and A is the vertex (apex) of the cone. Seg OB is a a radius and seg OA is perpendicular to the radius at O, means AO is perpendicular height of the cone. Slant height of the cone is the length of AB, which is shown by ($l$).

$\Delta$ AOB is a right angled triangle.

$\therefore$ by the Pythagoras' theorem

$$AB^2 = AO^2 + OB^2$$

$$\therefore l^2 = h^2 + r^2$$

That is, (slant height)$^2$ = (Perpendicular height)$^2$ + (Base radius)$^2$

### Surface area of a cone

A cone has two surfaces : (i) circular base and (ii) curved surface.

Out of these two we can find the area of base of a cone because we know the formula for the area of a circle.

How to find the curved surface area of a cone ? How to derive a formula for it ?

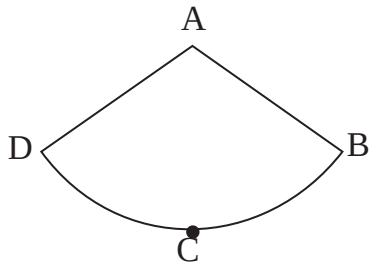



**Fig.9.5**

To find a formula for the curved surface area of a cone, let us see the net of the curved surface, which is a sector of a circle.

If a cone is cut along edge AB,we get its net as shown in fig.9.5.

Compare the figures 9.4 and 9.5

Have you noticed the following things ?

(i) Radius AB of the sector is the same as the slant height of the cones.

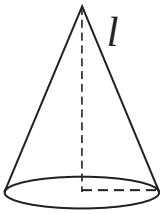
(ii) Arc BCD of the sector is the same as circumference of the base of the cone.

(iii) Curved surface area of cone = Area of sector A-BCD.

It means to find the curved surface area of a cone we have to find the area of its net that is the area of the sector.

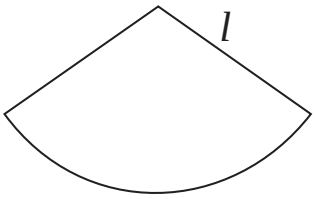
Try to understand, how it is done from the following activity.

**Activity :** Look at the following figures.



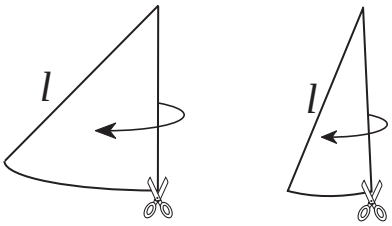

Cone

**Fig. 9.6**




Net of curved surface

**Fig. 9.7**




Pieces of the net

**Fig. 9.8**

Circumference of base of the circle $= 2\pi r$

As shown in the Fig.9.8, make pieces of the net as small as possible. Join them as shown in the Fig.9.9.

By Joining the small pieces of net of the cone, we get a rectangle ABCD approximately.

Total length of AB and CD is $2\pi r$.

$\therefore$ length of side AB of rectangle ABCD is $\pi r$ and length of side CD is also $\pi r$.

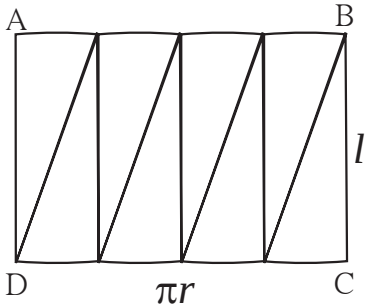



**Fig. 9.9**

Length of side BC of rectangle = slant height of cone $= l$.

Curved surface area of cone is equal to the area of the rectangle.

$\therefore$ curved surface area of cone = Area of rectangle $= AB \times BC = \pi r \times l = \pi rl$

Now, we can derive the formula for total surface area of a cone.

Total surface area of cone = Curved surface area + Area of base

$= \pi rl + \pi r^2$

$= \pi r(l + r)$

Did you notice a thing ? If a cone is not closed (Just like a cap of jocker or a cap in a birthday party) it will have only one surface, which is the curved surface. Then we get the surface area of the cone by the formula $\pi rl$.

**Activity :** Prepare a cylinder of a card sheet, keeping one of its faces open. Prepare an open cone of card sheet which will have the same base-radius and the same height as that of the cylinder.

Pour fine sand in the cone till it just fills up the cone. Empty the cone in the cylinder. Repeat the procedure till the cylinder is just filled up with sand. Note how many coneful of sand is required to fill up the cylinder.

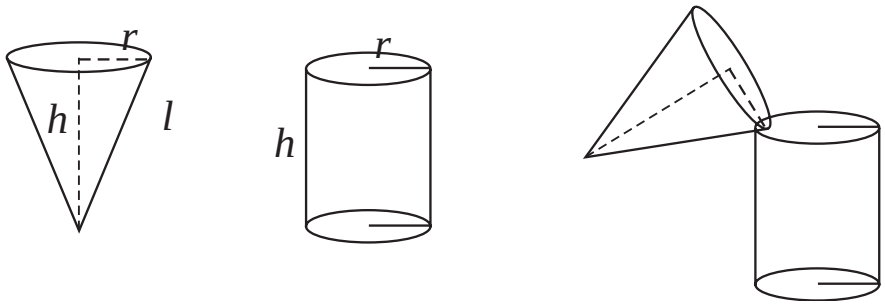



**Fig. 9.10**

To fill up the cylinder, three coneful of sand is required.

**Let's learn.**

## Volume of a cone

If the base-radii and heights of a cone and a cylinder are equal then

$3 \times$ volume of cone = volume of cylinder

$\therefore\ 3 \times$ volume of cone $= \pi r^2 h$

$\therefore$ volume of cone $= \frac{1}{3} \times \pi r^2 h$

**Remember this !**

(i) Area of base of a cone = $\pi r^2$ (ii) Curved surface area of a cone = $\pi rl$

(iii) Total surface area of a cone = $\pi r\,(l + r)$ (iv) Volume of a cone = $\frac{1}{3} \times \pi r^2 h$

**Solved Examples :**

**Ex. (1)** Radius of base ($r$) and perpendicular height ($h$) of cone is given. Find its slant height ($l$)

(i) $r = 6$ cm, $h = 8$ cm, (ii) $r = 9$ cm, $h = 12$ cm

**Solution :**

(i) $r = 6$ cm, $h = 8$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (6)^2 + (8)^2$

$\therefore l^2 = 36 + 64$

$\therefore l^2 = 100$

$\therefore l = 10$ cm

(ii) $r = 9$ cm, $h = 12$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (9)^2 + (12)^2$

$\therefore l^2 = 81 + 144$

$\therefore l^2 = 225$

$\therefore l = 15$ cm

**Ex. (2)** Find (i) the slant height, (ii) the curved surface area and (iii) total surface area of a cone, if its base radius is 12 cm and height is 16 cm. ($\pi = 3.14$)

**Solution :**

(i) $r = 12$ cm, $h = 16$ cm

$l^2 = r^2 + h^2$

$\therefore l^2 = (12)^2 + (16)^2$

$\therefore l^2 = 144 + 256$

$\therefore l^2 = 400$

$\therefore l = 20$ cm

(ii) Curved surface area $= \pi rl$

$= 3.14 \times 12 \times 20$

$= 753.6$ cm$^2$

(iii) Total surface area of cone

$= \pi r\,(l + r)$

$= 3.14 \times 12(20+12)$

$= 3.14 \times 12 \times 32$

$= 1205.76$ cm$^2$

**Ex. (3)** The total surface area of a cone is 704 sq.cm and radius of its base is 7 cm, find the slant height of the cone. ($\pi = \frac{22}{7}$)

**Solution :** Total surface area of cone $= \pi r\,(l + r)$

$\therefore \quad 704 = \frac{22}{7} \times 7\,(l + 7)$

$\therefore \quad \frac{704}{22} = l + 7$

$\therefore \quad 32 = l + 7$

$\therefore \quad 32 - 7 = l$

$\therefore \quad l = 25$ cm

**Ex. (4)** Area of the base of a cone is 1386 sq.cm and its height is 28 cm.

Find its surface area. ($\pi = \frac{22}{7}$)

**Solution :**

Area of base of cone $= \pi r^2$

$\therefore \quad 1386 = \frac{22}{7} \times r^2$

$\therefore \quad \frac{1386 \times 7}{22} = r^2$

$\therefore \quad 63 \times 7 = r^2$

$\therefore \quad 441 = r^2$

$\therefore \quad r = 21$ cm

$\therefore l^2 = (21)^2 + (28)^2$

$\therefore l^2 = 441 + 784$

$\therefore l^2 = 1225$

$\therefore l = 35$ cm

Surface area of cone $= \pi rl$

$= \frac{22}{7} \times 21 \times 35$

$= 22 \times 21 \times 5$

$= 2310$ sq. cm.

## Practice set 9.2

**1.** Perpendicular height of a cone is 12 cm and its slant height is 13 cm. Find the radius of the base of the cone.

**2.** Find the volume of a cone, if its total surface area is 7128 sq.cm and radius of base is 28 cm. ($\pi = \frac{22}{7}$)

**3.** Curved surface area of a cone is 251.2 $cm^2$ and radius of its base is 8cm. Find its slant height and perpendicular height. ($\pi = 3.14$)

**4.** What will be the cost of making a closed cone of tin sheet having radius of base 6 m and slant height 8 m if the rate of making is Rs.10 per sq.m ?

**5.** Volume of a cone is 6280 cubic cm and base radius of the cone is 30 cm. Find its perpendicular height. ($\pi = 3.14$)

**6.** Surface area of a cone is 188.4 sq.cm and its slant height is 10cm. Find its perpendicular height ($\pi = 3.14$)

**7.** Volume of a cone is 1212 $cm^3$ and its height is 24cm. Find the surface area of the cone. ($\pi = \frac{22}{7}$)

**8.** The curved surface area of a cone is 2200 sq.cm and its slant height is 50 cm. Find the total surface area of cone. ($\pi = \frac{22}{7}$)

**9.** There are 25 persons in a tent which is conical in shape. Every person needs an area of 4 sq.m. of the ground inside the tent. If height of the tent is 18m, find the volume of the tent.

**10.** In a field, dry fodder for the cattle is heaped in a conical shape. The height of the cone is 2.1m. and diameter of base is 7.2 m. Find the volume of the fodder. if it is to be covered by polythin in rainy season then how much minimum polythin sheet is needed ?

($\pi = \frac{22}{7}$ and $\sqrt{17.37} = 4.17$.)

## Let's learn.

### Surface area of a sphere

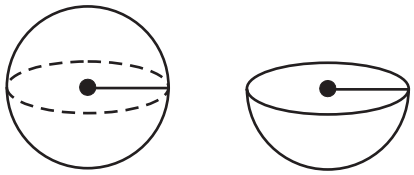


**Fig. 9.11**

Surface area of a sphere = $4\pi r^2$

$\therefore$ Surface area of a hollow hemisphere = $2\pi r^2$

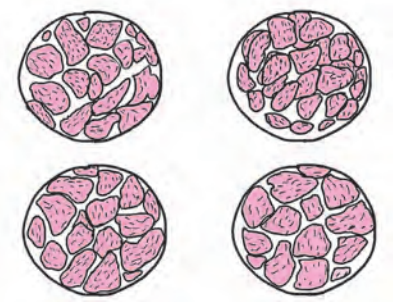
Total surface area of a solid hemisphere

= Surface area of hemisphere + Area of circle

= $2\pi r^2 + \pi r^2 = 3\pi r^2$

Take a sweet lime (Mosambe), Cut it into two equal parts.

Take one of the parts. Place its circular face on a paper. Draw its circular border. Copy three more such circles. Again, cut each half of the sweet lime into two equal parts.



Now you get 4 quarters of sweet lime. Separate the peel of a quarter part. Cut it into pieces as small as possible. Try to cover one of the circles drawn, by the small pieces.

Observe that the circle gets nearly covered.

The activity suggests that,

Curved surface area of a sphere = $4\pi r^2$

**Solved Examples :**

(1) Find the surface area of a sphere having radius 7 cm. ($\pi = \frac{22}{7}$)

**Solution :** Surface Area of sphere $= 4\pi r^2$

$= 4 \times \frac{22}{7} \times (7)^2$

$= 4 \times \frac{22}{7} \times 7 \times 7$

$= 88 \times 7$

$= 616$

Surface Area of sphere = 616 sq.cm.

(2) Find the radius of a sphere having surface area 1256sq.cm.($\pi = 3.14$)

**Solution :** Surface Area of Sphere $= 4\pi r^2$

$\therefore\ 1256 = 4 \times 3.14 \times r^2$

$\therefore\ r^2 = \frac{1256}{4 \times 3.14}$

$= \frac{31400}{314}$

$\therefore\ 100 = r^2$

$\therefore\ 10 = r$

$\therefore$ radius of the sphere is 10 cm.

**Activity :** Make a cone and a hemisphere of cardsheet such that radii of cone and hemisphere are equal and height of cone is equal to radius of the hemisphere. Fill the cone with fine sand. Pour the sand in the hemisphere. How many cones are required to fill the hemisphere completely ?

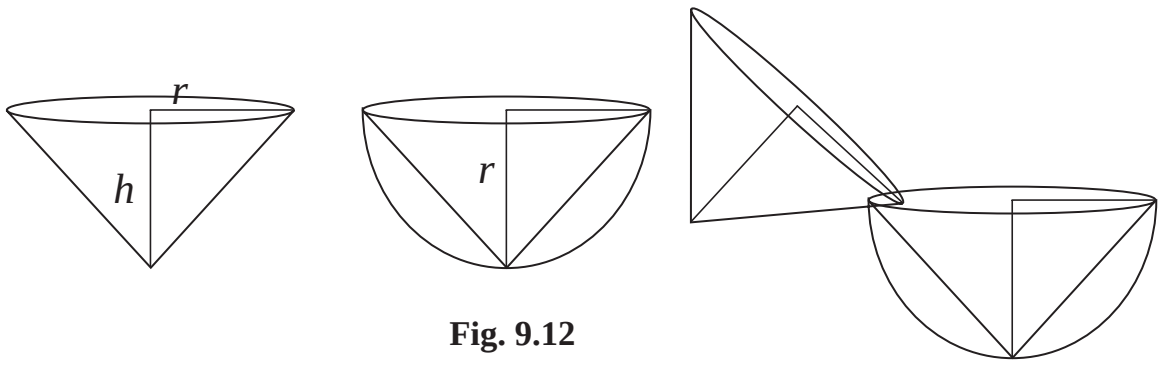



**Fig. 9.12**

Two conefull of sand is required to fill the hemisphere.

$\therefore$ 2 × volume of cone = volume of hemisphere.

$\therefore$ volume of hemisphere $= 2 \times$ volume of cone

$= 2 \times \frac{1}{3} \times \pi r^2 h$

$= 2 \times \frac{1}{3} \times \pi r^2 \times r$

$= \frac{2}{3} \pi r^3$

$\therefore$ volume of sphere

$= 2 \times$ volume of hemisphere.

$= \frac{4}{3} \pi r^3$

$\therefore$ volume of sphere $= \frac{4}{3} \pi r^3$

**Remember this !**

- Volume of hemisphere = $\frac{2}{3}\pi r^3$
- Total surface area of hemisphere = $2\pi r^2 + \pi r^2 = 3\pi r^2$

**Solved Examples :**

**Ex. (1)** Find the volume of a sphere having radius 21 cm. ($\pi = \frac{22}{7}$)

**Solution :** Volume of sphere = $\frac{4}{3}\pi r^3$

$= \frac{4}{3} \times \frac{22}{7} \times (21)^3$

$= \frac{4}{3} \times \frac{22}{7} \times 21 \times 21 \times 21$

$= 88 \times 441$

$\therefore$ volume of sphere = 38808 cubic cm.

**Ex. (2)** Find the radius of a sphere whose volume is 113040 cubic cm. ($\pi = 3.14$)

**Solution :** Volume of sphere = $\frac{4}{3}\pi r^3$

$113040 = \frac{4}{3} \times 3.14 \times r^3$

$\frac{113040 \times 3}{4 \times 3.14} = r^3$

$\frac{28260 \times 3}{3.14} = r^3$

$\therefore 9000 \times 3 = r^3$

$\therefore r^3 = 27000$

$\therefore r = 30$ cm

$\therefore$ radius of sphere is 30 cm.

**Ex. (3)** Find the volume of a sphere whose surface area is 314.sq.cm. (Take $\pi = 3.14$)

**Solution :** Surface area of sphere = $4\pi r^2$

$314 = 4 \times 3.14 \times r^2$

$\frac{314}{4 \times 3.14} = r^2$

$\frac{31400}{4 \times 314} = r^2$

$\therefore \frac{100}{4} = r^2$

$\therefore 25 = r^2$

$\therefore r = 5$ cm

Volume of sphere $= \frac{4}{3}\pi r^3$

$= \frac{4}{3} \times 3.14 \times 5^3$

$= \frac{4}{3} \times 3.14 \times 125$

$= 523.33$ cubic cm.

## Practice Set 9.3

1. Find the surface areas and volumes of spheres of the following radii.
   (i) 4 cm (ii) 9 cm (iii) 3.5 cm. ($\pi = 3.14$)
2. If the radius of a solid hemisphere is 5cm, then find its curved surface area and total surface area. ($\pi = 3.14$)
3. If the surface area of a sphere is 2826 $cm^2$ then find its volume. ($\pi = 3.14$)
4. Find the surface area of a sphere, if its volume is 38808 cubic cm. ($\pi = \frac{22}{7}$)
5. Volume of a hemisphere is 18000 $\pi$ cubic cm. Find its diameter.

## Problem set 9

1. If diameter of a road roller is 0.9 m and its length is 1.4 m, how much area of a field will be pressed in its 500 rotations ?
2. To make an open fish tank, a glass sheet of 2 mm gauge is used. The outer length, breadth and height of the tank are 60.4 cm, 40.4 cm and 40.2 cm respectively. How much maximum volume of water will be contained in it ?
3. If the ratio of radius of base and height of a cone is 5:12 and its volume is 314 cubic metre. Find its perpendicular height and slant height ($\pi = 3.14$)
4. Find the radius of a sphere if its volume is 904.32 cubic cm. ($\pi = 3.14$)
5. Total surface area of a cube is 864 sq.cm. Find its volume.
6. Find the volume of a sphere, if its surface area is 154 sq.cm.
7. Total surface area of a cone is 616 sq.cm. If the slant height of the cone is three times the radius of its base, find its slant height.
8. The inner diameter of a well is 4.20 metre and its depth is 10 metre. Find the inner surface area of the well. Find the cost of plastering it from inside at the rate Rs.52 per sq.m.
9. The length of a road roller is 2.1m and its diameter is 1.4m. For levelling a ground 500 rotations of the road roller were required. How much area of ground was levelled by the road roller? Find the cost of levelling at the rate of Rs. 7 per sq. m.

**2.** (i) $\frac{11}{2}$ (ii) $\frac{93}{20}$ (iii) 5 (iv) $\frac{2\sqrt{3}}{\sqrt{3}+1}$ (v) $\frac{3}{4}$ (vi) $\frac{\sqrt{3}}{2}$ **3.** $\frac{3}{5}$ **4.** $\frac{8}{17}$

## Problem set 8

**1.** (i) A (ii) D (iii) C (iv) D

**2.** $\sin T = \frac{12}{13}, \cos T = \frac{5}{13}, \tan T = \frac{12}{5}, \sin U = \frac{5}{13}, \cos U = \frac{12}{13}, \tan U = \frac{5}{12}$

**3.** $\sin Y = \frac{8}{17}, \cos Y = \frac{15}{17}, \tan Y = \frac{8}{15}, \sin Z = \frac{15}{17}, \cos Z = \frac{8}{17}, \tan Z = \frac{15}{8}$

**4.** $\sin\theta = \frac{7}{25}, \tan\theta = \frac{7}{24}, \sin^2\theta = \frac{49}{625}, \cos^2\theta = \frac{576}{625}$

**5.** (i) 70 (ii) 60 (iii) 50

# 9. Surface Area and Volume

## Practice set 9.1

**1.** 640 sq.cm, 1120 sq.cm. **2.** 20 Unit **3.** 81 sq.cm, 121.50 sq.cm.

**4.** 3600 sq.cm. **5.** 20 m **6.** 421.88 cubic cm

**7.** 1632.80 sq.cm, 4144.80 sq.cm. **8.** 21 cm

## Practice set 9.2

**1.** 5 cm **2.** 36960 cubic cm. **3.** 10 cm, 6 cm **4.** ₹ 2640

**5.** 15 cm **6.** 8 cm **7.** 550 sq.cm **8.** 2816 sq.cm, 9856 cubic cm

**9.** 600 cubic metre **10.** 28.51 cubic metre, 47.18 sq.m.

## Practice Set 9.3

**1.** (i) 200.96 sq.cm, 267.95 cubic cm. (ii) 1017.36 sq.cm, 3052.08 cubic cm.
(iii) 153.86 sq.m, 179.50 cubic cm.

**2.** 157 sq.cm, 235.5 sq.cm. **3.** 14130 cubic cm. **4.** 5544 sq.cm. **5.** 60 cm

## Problem set 9

**1.** 1980 sq.m. **2.** 96801.6 cubic cm. **3.** 12 m, 13 m

**4.** 6 cm **5.** 1728 cubic cm. **6.** 179.67 cubic cm.

**7.** 21 cm **8.** 132 sq.m., ₹ 6864 **9.** 4620 sq.m, ₹ 32340