Rendering Lightning Effects

CS775-project

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[Overview]

We aim to simulate physically-based lightning models in this project. We will be using the Dielectric Breakdown Model(DBM) since it is very close to what we want to achieve. We provide a solution to the DBM, as presented in the paper, i.e. one that can be implemented. But this model has the complexity of the order of #grid_cells in the scene. To tackle this we reproduce another algorithm presented in another similar paper by the same author. Finally, to solve the problem of animating and rendering, we use bloom filters to create a glow like effect.

[Reference Paper]

- http://gamma.cs.unc.edu/LIGHTNING/lightning.pdf

[Problem Statement]

Theoretical challenges:

- Simulating electrical discharge phenomenon like lightning
- Choosing a physical model for the simulation
- Tuning parameters of the physical model to get lightning effects from the general solution to such model
- Setting charge configurations (i.e. specify boundary conditions to the model)
- Explore solving (exact and approximate) techniques to the boundary condition specified model
- Research and develop faster (in computation time) algorithms for the same.
- Provide possible extensions to the model and implementation described above

Practical challenges:

- Choose appropriate modeling techniques, platforms, etc.

- Generating arcs geometry and graph structure efficiently in terms of space and time. Here we present two methods to achieve this.
    - Physics-Based Model(Dielectric Breakdown Model)
    - Mathematical Model(Monte-carlo methods)
- Render the lightning arcs in the scene
- Research on producing glowing effects via bloom filters or other convolutional filters etc.
- Efficient computation of convolution with kernel over space
- Create different charge maps(configurations) for testing algorithm
- Compare visual effect with an alternative rendering technique developed independently

What we propose to do differently from reference paper?

- Using an alternate method of solving Laplace equation
- Mathematical Model to generate lightning arcs
- Using Post Processing techniques to render glow effects.

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[Solution Strategy]

1.Physics-Based Model

The shape that the lightning arc resembles is closely modeled by the Laplacian growth phenomena.
In the beginning, we break our 3D-scene into grid cells with each grid cell assigned some potential. We compute the growing arc in a series of iteration steps from the starting cell where initially $Ø = 0$.

In each iteration we do the following :

To get the potential distribution in a charged configuration we need to solve the Laplace Equation

$$\nabla^2 Ø = 0$$

And we need to specify boundary configurations according to our charge configuration.

Once the potentials at the grid cells have been obtained from the solution to the Laplace Equation we will use the Dielectric Breakdown Model to predict the site of the next breakdown.

We choose the site of the next breakdown using the probability formula

$$p_i = (\text{Ø}_i)^{\eta} \;\; / \, \Sigma^{n}_{j=0} \, (\text{Ø}_j)^{\eta}$$

Where i is a cell in the list of adjacent cells, and n is the total number of cells in the list. The η term is a useful parameter to control the growth pattern of the arc.

The chosen cell is then set as Ø = 0 for the grid cell. This is then incorporated into the boundary condition used for solving the potential configuration in the next step. **Also, an edge is created between the previous and current growth site.**

This series of iterations will give us the first arc i.e. the stepped leader(as referred to in the paper). We also need to generate subsequent arcs in the same direction, which is what happens naturally; these are called dart leaders. This is due to the conducting passageway provided in the medium to the generation of the first arc.

This can be hypothesized due to the existence of residual positive charge along the stepped leader path.
Thus we make changes to the Laplace equation to be solved for the subsequent arc generated

$$\nabla^2 \text{Ø} = -4\pi\rho \qquad \text{… } \textit{Poisson Equation}$$

Where ρ is the charge configuration obtained from the previous iteration of stepped leader i.e. an intuitive explanation would be creating a second grid of values storing the charge in each cell computed using the potential configuration of the grid at the end of the first iteration.

The paper does not present a solution or away in terms of computer graphics to do this.

The algorithm explained above has O(G * n) order of complexity where G is the number of grid cells and n is the number of iterations. This could prove very costly if we are rendering the lightning in a 3D scene, i.e. solving the Laplace in 3D for each iteration could prove costly.

Approximations to solving the Laplacian growth equation can be made to boost the time complexity[2]. We look at the Laplacian in spherical polar coordinates

$$\nabla^2 \equiv \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial}{\partial r}\right) + \frac{1}{r^2\sin^2\phi}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r^2\sin\phi}\frac{\partial}{\partial\phi}\left(\sin\phi\frac{\partial}{\partial\phi}\right).$$

And boundary conditions as

$$\varphi(R1,\theta,\Phi) = 0$$
$$\varphi(R2,\theta,\Phi) = 1$$

Here

R1 is the inner sphere radius

R2 is outer sphere radius

We can drop angular terms due to symmetry.
the solution to this becomes

$$\varphi = 1 - R1/r$$

Now, this is a fairly straightforward formula that can be used easily in the implementation of laplacian growth.

2.Mathematical Model

This model considers lightning as a stochastic natural phenomenon, hence Monte-Carlo methods are employed[1]. This, however, limits the model to only specific cases like a lightning strike from the cloud on earth because of its non-determinism.

The model is as follows:-

$$P(I) = 0.5 * erfc(u)$$
$$erfc(u) = 1 - 2/\sqrt{\pi} * \Sigma^{\infty}_{n=0}((-1)^n.u^{2n+1})/n!(2n+1)$$
$$u = (log(I) - log(I_u))/(\sqrt{2}.\sigma_{log})$$

I – peak value of the lightning current

P(I) – the probability that a lightning current peak value I will be exceeded

Using these we get a distribution of current. We sample a current value from this distribution. This value will be used to calculate strike distance using the empirical formula

$$R = A.I^{\alpha}$$

Here A = 10 and α=0.65.

We create arcs by creating strikes one after another starting from a given point towards a given plane.
Given a point (x,y,z) we generate (x',y',z') as

$$x' = x + R.\sin\Theta.\cos\Phi$$
$$y' = y + R.\sin\Theta.\sin\Phi$$
$$z' = z + R.\cos\Theta$$

Where Θ and Φ are randomly drawn from $[0, \pi/2]$ and $[0, 2\pi]$.

[Rendering]

- Having constructed the strike as a tree-like structure. We first assign luminance value to these edges.
- For glowing effects, we do post-processing on the scene rendered and apply a bloom filter.
- Steps are as follows
  - Get the final rendered scene
  - Extract bright light areas from the image
  - Apply blurring filters on it
  - Merge the blurred and original scene

Depending on the ease of computation and result we will choose between whether to do it in OpenGL or via Ray-Tracing.

References

1. https://www.researchgate.net/publication/224347603_Mathematical_Model_of_Lightning_Stroke_Development
2. http://gamma.cs.unc.edu/FRAC/laplacian_large.pdf
3. http://gamma.cs.unc.edu/LIGHTNING/lightning.pdf