STAT1201 – Summer Semester 2022

Lecture 5 - Statistical Inference

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Lecture 5: Statistical Inference

In this lecture, you will practice

- Confidence Interval estimation
- One sample t-test
- Type I and Type II errors in hypothesis testing
- Power of an experiment

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What is Statistical Inference?

Process of drawing conclusions about the population parameters and the reliability of statistical relationships based on sample information.

There are two main parts: Confidence Intervals and Hypothesis testing.

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What is Statistical Inference? cont…

Suppose that you take a random sample of 20 STAT1201 Summer 2022 students and measure their heights and calculate the sample average height ($\overline{X}$). Say $\overline{X}$ = 156cm. Based on the sample information you can make inferences about height of all STAT1201 Summer 2022 students. For example, you can say,

- The average height of STAT1201 Summer 2022 students is 156cm. That is μ = 156cm. This is a point estimate for the population mean.
- The average height of STAT1201 Summer 2022 students lies between 152cm and 160cm. That is $152 \leq \mu \leq 160$. This is an interval estimate for the population mean.
- The average height of STAT1201 Summer 2022 students is greater than 158cm. That us $\mu > 158$. This is a hypothesis test for the population mean.

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What is Statistical Inference? cont…

Poll Question 1

An interval estimate provides more information about a population parameter than does a point estimate.

a)True

b)Flase

Confidence Interval(CI) estimates

- CI is a range of values that is likely to include the population parameter with a certain level of confidence.
- Has a lower limit (LL) and an upper limit (UL).
- Takes into consideration the variation in sample statistics from sample to sample.

Level of Confidence (LOC) - How confident you will be the interval contain the unknown population parameter. LOC is always less than 100%.

Examples:

95% CI for μ implies that 95% of all confidence intervals, constructed using all possible samples will contain the population mean.

99% CI for μ implies that 99% of all confidence intervals, constructed using all possible samples will contain the population mean.

Confidence Interval(CI) estimates

CI estimates for a population parameter takes the following form

point estimate $\pm$ Margin of Error

MOE = Margin of Error

Example: 95% CI for the population mean (μ) is written as

$$\overline{X} \pm MOE$$

That is, LL = $\overline{X} - MOE$ and UL = $\overline{X} + MOE$

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Confidence Interval(CI) estimates

Poll Question 2

You are taking a random sample of 10 lizards living in one Island in Australia and estimate the 95% CI for the mean length of all lizards living in that Island. The lower and upper limits of the CI are 42.3cm and 53.7cm respectively. What is the MOE?

1. 4.23
2. 5.37
3. 48
4. 5.7

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Confidence Interval(CI) estimates cont…

In case of 95% CI for the population mean;

$$\overline{X} \pm Z^* \frac{\sigma}{\sqrt{n}}$$

MOE = $Z^* \frac{\sigma}{\sqrt{n}}$ where Z^* is called the critical value (Z scores that have a certain probability to their right).

The value Z^* depends on the level of confidence (LOC).

When we find Z^* we need to deduct half of (1-LOC) since the CI is spread around the mean with equal lengths to the left and right.
For 90% CI, Z^* = qnorm(0.95) = 1.644854
For 95% CI, Z^* = qnorm(0.975) = 1.959964
For 99% CI, Z^* = qnorm(0.995) = 2.575829
Note —> (1-LOC) is called the level of significance (α). We will use this in hypothesis testing.

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Confidence Interval(CI) estimates cont…

Poll Question 3

Which of the following is true?

1. As sample size increases MOE increases
2. As standard deviation increases MOE decreases
3. As LOC increases MOE increases
4. As LOC increases MOE decreases

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Confidence Interval(CI) estimates cont…

Example: You are taking a random sample of 16 lizards living in one Island in Australia. The sample mean body length of those is 45cm. From a previous research the population standard deviation of the body length is known and it is 8cm. What is the 95% CI for the mean body length of lizards living in this island?

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Confidence Interval(CI) estimates cont…

However, in most of the cases, we don't know σ, the population standard deviation. Using sample information we can find sample standard deviation (s).

The estimated standard deviation for the sample mean = $\frac{s}{\sqrt{n}}$

This $\frac{s}{\sqrt{n}}$ is called the standard error of the sample mean.

$$\text{se}(\overline{x}) = \frac{s}{\sqrt{n}}$$

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Confidence Interval(CI) estimates cont…

The $\frac{s}{\sqrt{n}}$ will be different for each sample since ***s*** will be different for each sample.
We follow similar process like we did in Lecture 4 to find the distribution of the sample mean. However, now standardized values are calculated using sample standard deviations instead of constant population standard deviation. The distribution of this standardized values are no longer normally distributed. They have fatter tails than a standard normal (Z) distribution. We call this new distribution "Student's *t* Distribution". This distribution was developed by William Gosset.
That is;

T = $\frac{\overline{X}-\mu}{\frac{s}{\sqrt{n}}}$ and T ~ $t_{(n-1)}$

In general, *t* values are calculated from sample size of *n* will have the $t_{(n-1)}$ distribution. *(n-1)* is called degrees of freedom of the *t*-distribution.

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Confidence Interval(CI) estimates cont…

Thus CI for the population mean when σ is unknown is:

$$\overline{x} \pm t^*_{(n-1)}\frac{s}{\sqrt{n}}$$

$t^*_{(n-1)}$ is the number of standard errors required for the desired level of confidence in the $t_{(n-1)}$ distribution.

Example:

You are taking a random sample of 16 lizards living in one Island in Australia. The sample mean and standard deviation are 45cm and 8.4cm. Estimate the 95% CI for the mean length of all lizards living in that Island.

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Confidence Interval(CI) estimates cont…

Poll Question 4

You are taking a random sample of 25 lizards living in one Island in Australia. The sample mean and standard deviation are 45cm and 8.4cm. What is the degrees of freedom of *t* distribution to calculate 99% CI for the population mean?

1. 25
2. 24
3. 12
4. 13

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Choosing Sample Size

Consider the example we discussed to calculate the 95% CI for μ when σ is known (given below again).

You are taking a random sample of 16 lizards living in one Island in Australia. The sample mean body length of those is 45cm. From a previous research the population standard deviation of the body length is known and it is 8cm. What is the 95% CI for the mean body length of lizards living in this island?

The calculated MOE was 3.919928

That is, $Z^* \frac{\sigma}{\sqrt{n}}$ = 3.919928

Now suppose that you need to reduce the MOE to 2cm. What is the sample size need to construct 95% CI for μ?

MOE = $Z^* \frac{\sigma}{\sqrt{n}}$

Rearranging; n = $(\frac{Z^* \sigma}{MOE})^2$

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Choosing Sample Size cont…

$n = (\frac{Z^{*}\sigma}{MOE})^2$

Thus, $n = (\frac{1.959964*8}{2})^2$

n = 61.46334.

Add one to the integer part irrespective of the value decimal point to obtain the required sample size. This is because MOE must be less than or equal to a specified value.

The required sample size = 62.

Check by yourself the value of MOE if n = 61 and if n = 62

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Hypothesis Testing

A hypothesis test is a claim (assumption, belief, assertion) about an unknown population parameter. We discussed the language of hypothesis testing in Lecture 1.

Test for the population mean

Example: One of your friends claim that the average height of STAT1201 Summer 2022 students is 165cm (i.e $H_0: \mu = 165$). We will use one sample t test to test this claim.

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One Sample *t* test

To test the population mean, the null and alternative hypotheses is written as follows according to the nature of the experiment or study.

Case 1: to test whether population mean (μ) not equals to a specific value say (μ_0)

$H_0: \mu = \mu_0$ Vs $H_1: \mu \neq \mu_0$

Case 2: to test whether population mean (μ) is greater than to a specific value say (μ_0)

$H_0: \mu = \mu_0$ Vs $H_1: \mu > \mu_0$

Case 3: to test whether population mean (μ) is less than to a specific value say (μ_0)

$H_0: \mu = \mu_0$ Vs $H_1: \mu < \mu_0$

Case 1 is an example of a tow tail test. Cases 2 and 3 are examples of one tail test. Case 2, upper tail test and Case 3, lower tail test.

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One Sample *t* test cont…

The general method for a test of $H_0: \mu = \mu_0$ uses the *t* statistic as follows.

$$t_{stat} = \frac{\overline{x} - \mu_0}{se(\overline{x})}$$

$$t_{stat} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

This t_{stat} has a *t* distribution with *(n-1)* degrees of freedom (df).

Example:

A researcher claims that the average breath holding time for young males is 55 seconds. To test this claim the breath holding times of randomly selected 20 males aged between 18 and 25 years was measured.

The sample mean breath holding time was 60.8 seconds with a sample standard deviation was 10.91 seconds. Use one sample *t* test to test the researcher's claim.

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One Sample *t* test cont…Example

$H_0: \mu = 55$ Vs $H_0: \mu \neq 55$

$$t_{stat} = \frac{\overline{x} - \mu_0}{s/\sqrt{n}}$$

$$t_{stat} = \frac{60.8 - 55}{10.91/\sqrt{20}}$$

$t_{stat} = \frac{5.8}{2.43955}$ = 2.377488

We need to find the *p*-value to make a conclusion. *p*-value is the Probability of obtaining a result equal to or more extreme than was observed in the data.

We consider the *t* distribution with 19 df.

Considering the t_{19} distribution, $\frac{p-value}{2}$ can be found considering the area beyond the t_{stat}.

Using R; $\frac{p-value}{2}$ = 1 - pt(2.377488, df=19).

$\frac{p-value}{2}$ = 1 - 0.9859561 = 0.01404394

p-value = 2*0.01404394 = 0.02808788

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One Sample *t* test cont…Example

Conclusion: We have moderate evidence to conclude that the average breath holding time for young male adults is different from 55 seconds.

Instead suppose that you need to test whether the mean breath holding time of young adults is greater than 55 seconds. How do you write the null and alternative hypothesis.

$H_0: \mu = 55$ Vs $H_1: \mu > 55$

What is the t_{stat}?

What is the *p* - value?

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One Sample *t* test

Poll Question 5

Consider the breath holding time example we just discussed. Using a different sample of size 20 you found that the t_{stat} is -2.377 to test whether mean breath holding time of young adults is less than 55 seconds. You can conclude that there is

1. strong evidence to conclude that mean breath holding time is less than 55 seconds (p-value = 0.0140)
2. no evidence to conclude that mean breath holding time is less than 55 seconds (p-value = 0.9860)
3. moderate to conclude that mean breath holding time is less than 55 seconds (p-value = 0.0280)
4. moderate to conclude that mean breath holding time is less than 55 seconds (p-value = 0.0140)

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One Sample *t* test Assumptions

1. The data (variable) are continuous
2. The population distribution of data is normally distributed
3. The observations are independent

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Hypothesis testing Decisions - Types of Errors

Possible Outcomes from Decisions

	Actual (reality) Situation	
Statistical Decision	H_0 True	H_0 False
Do Not Reject H_0	✓	**Type II Error**
Reject H_0	**Type I Error**	✓

Type I Error is caused due to rejecting a true null hypothesis.

The probability of making a Type I error is denoted by α. Also called the level of significance.

Type II Error is caused due to not rejecting a false null hypothesis. The probability of making type II Error is denoted by β.

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Hypothesis testing Decisions - Types of Errors

Type I and Type II Errors Probabilities (in brackets)

Possible Outcomes from Decisions

	Actual (reality) Situation	
Statistical Decision	H_0 True	H_0 False
Do Not Reject H_0	✓ $(1-\alpha)$	II (β)
Reject H_0	I (α)	✓ $(1-\beta)$

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Hypothesis testing Decisions - Types of Errors

Probability of making Type I and II errors are both conditional probabilities.

Not able to make both errors at the same time.

Increasing α will decrease β.

Power of the Hypothesis Test (1-β)

Statistical power, or the power of a hypothesis test is the probability that the test correctly rejects the false null hypothesis.

Power (1-β) = P(Reject H_0 | H_0 is false).

Increasing sample size makes the hypothesis test more sensitive. That is more likely to reject the null hypothesis when it is, in fact, false. Thus, it increases the power of the test.

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Hypothesis testing Decisions - Types of Errors

Poll Question 6

In hypothesis testing, increasing sample size will lead to

1. increase the probability of type I Error
2. decrease the probability of type I Error
3. increase the probability of type II Error
4. decrease the probability of type II Error

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Statistical Power Analysis

- Low statistical power: Large risk of committing type II errors
- High statistical power: Small risk of committing type II errors
- Experimental results with too low statistical power will lead to invalid conclusion.
- Therefore a minimum level of statistical power must be sought.
- It is common to design experiments with a statistical power of 80% (i.e. 0.8). That is, 20% probability of encountering Type II Error.

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Next …

Reminders

Quizzes 4 and 5 are now open.

Lecture 6 – Ethical case Studies by Dr Julian Lamont

Thursday, 15 December 2022

at 12:00 via Zoom (818 1453 7986)

Lecture 7 - Comparing two populations

Tuesday, 20 December 2022

at 12:00 via Zoom (818 1453 7986)

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