

Part B

An expensive piece of equipment in a laboratory is starting to show signs of age. Let X be the number of days in any week that the equipment is working and suppose that X has the following probability distribution:

x	0	1	2	3	4	5
$P(X = x)$	0.01	0.09	0.25	0.34	0.24	0.07

- a) What is the expected number of days in a week that the equipment is working?

- b) What is the standard deviation of the number of days in a week that the equipment is working?

- c) What is the expected value of the total number of days that the equipment is working over a 40-week period?

- d) Suppose the number of days in a week that the equipment is working is independent from week to week. What is the standard deviation of the total number of days that the equipment is working over a 40-week period?

Part C – Random Variables

Suppose a random person has a 1 in 3 chance of passing their driving test on each attempt. Let X be the number of attempts that a random person to pass their driving test. The probability distribution of X can be found using a simulation process. You can roll a six-sided die again and again until you see the number 5 or 6 and record the number of attempts. If you don't have a die use the `sample(1:6, 1)` in R count the number of attempts that you had to use this function to get 5 or 6. Share with your tutor the number of attempts you had to make to see 5 or 6. The tutor will share a "csv" file with you (in online tutorials) or show the values of X on white board (in face-to-face tutorials, you then need to type them and create your "csv" file).

- a) Is X a discrete or a continuous random variable?
- b) Read the "csv" file into RStudio.
- c) Use `min()` and `max()` in RStudio to find the minimum and maximum number of attempts to see number 5 or 6?
- d) What is the shape of the distribution of X . You can use `barplot()` in RStudio to create a bar chart of X to determine the shape of X .
- e) Use the `prop.table()` in RStudio to obtain the probability distribution function of X .
- f) What is probability that a randomly selected person makes at least 2 attempts to pass the driving test?
- g) Using the probability distribution of X from part c), what is the expected number of attempts taken, $E(X)$?

Part D – Binomial Distribution

- a) Suppose 56% of STAT1201 students have brown eyes and we take a random sample of 4 students. Let X be the number of students with brown eyes in the sample. What is the distribution of X ?
- b) Let X be the number of towns in which it will rain tomorrow among five neighbouring towns. Is X a Binomial random variable?

- c) Suppose 10% of people are left-handed and let X be the number of left-handed people in sample of 20 individuals. What is the probability of at least one left-handed person in the sample?
- d) Suppose a drug has a 20% chance of making a person drowsy. Out of a sample of 80 people who each take the drug, what is the probability that no more than 10 of them experience drowsiness?
- e) In Part B, we computed the expected value and standard deviation of X , the number of days in a week (Mon – Fri) that a piece of equipment is working, to be 2.92 and 1.102, respectively. The probabilities given in the table actually came from a Binomial distribution. What are the parameters of the Binomial distribution?

Part E – Normal Distribution

Based on the student survey data, suppose that pulse rates while completing the survey come from a Normal distribution with mean 71.7 bpm and standard deviation 11.7 bpm.

- a) What is the probability that a random student has a pulse rate of above 90 bpm?
- b) What is the probability that a random student has pulse rate between 60 and 80 bpm?
- c) What is the highest pulse rate of a student that would be in the bottom 10% of pulse rates?

- d) Suppose that you are taking a random sample of 4 students from this survey. What is the probability that average pulse rate of these 4 students is less than 80.5 bpm?
- e) In a random sample of 5 students, what is the probability that at least 3 of them have a pulse rate over 90 bpm while completing the survey?

You may find the following output from R useful in answering the questions on this sheet.

```
> sum(dbinom(x=3:5,size=5,prob=.0589))
[1] 0.001867087
> dbinom(x=10,size=80,prob=.2)
[1] 0.02774334
> sum(dbinom(x=1:10,size=80,prob=.2))
[1] 0.05646089
> sum(dbinom(x=0:10,size=80,prob=.2))
[1] 0.0564609
> sum(dbinom(x=11:80,size=80,prob=.2))
[1] 0.9435391
> pnorm(-2.114)
[1] 0.01725763
> pnorm(1.564)
[1] 0.9410912
> pnorm(-1.564)
[1] 0.05890878
> pnorm(0.7094017)
[1] 0.7609624
> pnorm(-1)
[1] 0.1586553
> qnorm(.1)
[1] -1.281552
> 0.05890878^3
[1] 0.0002044279
```

Reference

Several of these questions are adapted from Section 6.6 of *Mind on Statistics*, a useful source of other practice questions. See the ECP for details.