STAT1201 - Summer Semester, 2022

Lecture 13 (Based on Module 12) - Nonparametric Methods

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Introduction

- In parametric statistics, we make assumptions about the distribution of population from which the sample was taken. For example, in two sample t-test, we assume that population data are normally distributed.
- In contrast, nonparametric statistics are not based on the assumptions. That is data can be collected from a population that is not followed a specific distribution.
- Nonparametric tests are typically more robust in presence of outliers or strong skewness.
- Parametric tests are generally more powerful. However, there are situations that parametric tests cannot be used due to various factors. For example, outcomes are ordinal, ranked or contnuous variables that are not normally distributed or presence of outliers.

Sign test

- Sign test is an alternative to a one sample t-test or paired t-test.
- The test does not assume that the data comes from a particular distribution, like normal distribution.
- Can also be used for categorical data.

Sign test - Example

The following table shows the data comparing the growth of cross fertilised and self fertilised plants from an experiment (Darwin, 1992).

Table 1: Differences in heights (cm) between 15 pairs of cross and self fertilised plants.

15.6 | -21.3 | 2.5 | 5.1 | 1.9 | 7.3 | 8.9 | 13.0 | 4.4 | 9.2 | 17.8 | 7.6 | 23.8 | 19.1 | -15.2

Note: In this example two sets of plants are considered as matched pairs.

The researcher calims that the cross-fertilised plants would grow more vigorously than self fertilised plants.

Sign test - Example cont... (using t test for the differences)

** Poll Question 1**

If we define $\mu =$ population mean growth difference of cross and self fertilised plants, we can use one sample t-test to test the researcher's claim. The appropriate null and alternative hypotheses are:

a) $H_0 : \mu = 0$ Vs $H_1 : \mu > 0$

b) $H_0 : \mu = 0$ Vs $H_1 : \mu < 0$

c) $H_0 : \mu > 0$ Vs $H_1 : \mu < 0$

d) $H_0 : \mu = 0$ Vs $H_1 : \mu \neq 0$

Sign test - Example cont ... (using t test for the differences)

Using R:

```r
HD = c(15.6 , -21.3 , 2.5 , 5.1  , 1.9 , 7.3 , 8.9 , 13.0
       , 4.4 , 9.2 , 17.8 , 7.6 , 23.8 , 19.1 , -15.2)
t.test(HD, alternative = "greater")
```

```
    One Sample t-test

data:  HD
t = 2.147, df = 14, p-value = 0.0249
alternative hypothesis: true mean is greater than 0
95 percent confidence interval:
 1.194   Inf
sample estimates:
mean of x
 6.646667
```

Sign test - Example cont...

How can we use the sign test for this data to answer the same question?

Table 1: Differences in heights (cm)

15.6 | -21.3 | 2.5 | 5.1 | 1.9 | 7.3 | 8.9 | 13.0 | 4.4 | 9.2 | 17.8 | 7.6 | 23.8 | 19.1 | -15.2

Define p = P(cross fertilised plants taller)

H_0 : p $=$ 0.5 vs H_1 : p $>$ 0.5

If H_0 is true, each pair would have a 0.5 chance of having cross fertilised plants taller.

Define X = Number of pairs that the cross fertilised plant is taller

$X \sim \text{Binom}(15, 0.5)$

Sign test - Example cont…

From the data table 1, we observe that X = 13 (because 13 pairs show positive differences)

$P(X \geq 13) = P(X = 13) + P(X = 14) + P(X = 15)$

H_0 : p = 0.5 vs H_1 : p > 0.5

```
sum(dbinom(13:15, prob = 0.5, 15))
```

```
[1] 0.003692627
```

$P(X \geq 13) = 0.0037$

Thus p-value = 0.0037

This p-value is the probability of getting a result as extreme or more extreme than 13.

p-value < 0.01. There is strong evidence to conclude that cross fertilised plants are growing better than self fertilised plants.

Sign test - Example cont...

Note

Sign test can also be viewed as a test for median.

If zero (0) is the median difference in plant growth, then 50% of values above 0 and 50% below 0.

H_0 : median difference in plant growth is 0

H_1 : median difference in plant growth is greater than zero (or positive)

Signed - Rank test (also called Wilcoxon Signed- Rank test)

Nonparametric test equivalent to the paired t test. That is a test for paired or matched data.

More powerful than the sign test. Uses more information from the sets of scores than the sign test.

Signed - Rank test looks not only the sign of each difference, but also the magnitude.

Signed - Rank test Example

Consider the example used for the sign test.

Table 1: Differences in heights (cm) between 15 pairs of cross and self fertilised plants.

15.6	-21.3	2.5	5.1	1.9	7.3	8.9	13.0	4.4	9.2	17.8	7.6	23.8	19.1	-15.2

Take the absolute differences and then rank them

Table 2: Absolute differences in heights

15.6	21.3	2.5	5.1	1.9	7.3	8.9	13.0	4.4	9.2	17.8	7.6	23.8	19.1	15.2

Table 3: Ranked absolute differences in heights

11	14	2	4	1	5	7	9	3	8	12	6	15	13	10

Signed - Rank test Example

The Signed - Rank statistic, S

S = sum of the ranks corresponding to positive differences

$S = 11 + 2 + 4 + 1 + 5 + 7 + 9 + 3 + 8 + 12 + 6 + 15 + 13$

$S = 96$

H_0 : median difference in plant growth is 0

H_1 : median difference in plant growth is positive

Thus, p-value = $P(S \geq 96)$

To find the p-value we use normal approximation.

Signed - Rank test Example

Normal approximation

If the number of pairs is such that $\frac{n(n+1)}{2}$ is large enough (> 20), a normal appximation can be used with

$E(S) = \frac{n(n+1)}{4}$ and $sd(S) = \sqrt{\frac{n(n+1)(2n+1)}{24}}$

In this example, $E(S) = \frac{15(15+1)}{4} = 60$

$sd(S) = \sqrt{\frac{15(16)(31)}{24}} = 17.61$

Using $Z = \frac{S-E(S)}{sd(S)}$, we find $z = \frac{96-60}{17.61} = 2.04$

$P(S \geq 96) = P(Z \geq 2.04) = 0.021$

p-value $= 0.0206752$

There is moderate evidence to conclude that median difference in plant growth is positive. That is, cross fertilised plants are growing better than self fertilised plants.

Note for sign test and signed-Rank test

Both of these tests can be either one tail or two tail. In our example, we considered one tail test only.

If we want to test that cross fertilised plants and self fertilised plants growing differently (or at different rates), we can write the null and alternative hypotheses as below.

H_0 : median difference in plant growth is 0

H_1 : median difference in plant growth is not zero

Note for sign test and signed-Rank test

Poll Question 2

If the researcher wants to test cross fertilised and self fertilised plants grow at different rates, the p-value would be (Hint: the p-value to test the cross fertilised plants are growing better than self fertilised plants is 0.0207):

a) 0.0207

b) 0.0414

c) 0.0103

d) 0.9586

Rank-Sum test (also called Wilcoxon Rank-Sum test)

Wilcoxon (1945) describes a nonparametric test for comparing two independent samples.

This test can be described as the noparametric version of the two sample t-test.

Rank-Sum test also referred to as the Mann-Whitney test.

In general, we compare whether the population distributions of the two samples are similar in shape.

H_0 : There is no difference between the population distributions

H_1 : The population distributions are different (i.e. one distribution is shifted to the left or right of the other)

Rank-Sum test (Wilcoxon Rank-Sum test) - Example

A study was conductd to see the effect of caffeinated drinks on pulse rate. 20 subjects were randomly selected and 10 subjects (sample 1) were given caffeinated cola while other 10 subjects (sample 2) were given decaffeinated cola. The two samples are independent of one another. The objective of the study was to test whether caffeine will have an impact on increase in pulse rates.

Note: You used two sample t-test in Lecture 7 to answer this question and compared the population means. However, the validity of inferences were subject to the assumption of the two populations that the samples were taken are normally distributed.

Now we are going to use Wilcoxon Rank-Sum test and test whether caffeine increase pulse rates.

Rank-Sum test (Wilcoxon Rank-Sum test) - Example

Table 1: Change in pulse rates (bpm)

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
Caff	17	22	21	16	6	-2	27	15	16	20
Decaff	4	10	7	-9	5	4	5	7	6	12

.

The null and alternative hypotheses are:

H_0 : There is no difference between the population distributions of the caffeinated and decaffeinated groups

H_1 : The population distribution of the caffeinated sample shifted to the right of the decaffeinated group

Rank-Sum test (Wilcoxon Rank-Sum test) - Example

First rank data from the smallest to the largest.

Table 2: Ranked pulse rates

	[,1]	[,2]	[,3]	[,4]	[,5]	[,6]	[,7]	[,8]	[,9]	[,10]
Caff	16.0	19	18.0	14.5	7.5	2.0	20.0	13.0	14.5	17
Decaff	3.5	11	9.5	1.0	5.5	3.5	5.5	9.5	7.5	12

You can see that test is based on the combined sample. Each observation has a rank.

Define W = sum of the ranks for observations from caffeinated sample.

$W = 16 + 19 + 18 + 14.5 + 7.5 + 2 + 20 + 13 + 14.5 + 17$

$W = 141.5$

Rank-Sum test (Wilcoxon Rank-Sum test) - Example

If the caffeinated subjects tended to have higher increase in pulse rate, the distribution of caffeinated sample would shift to the right. That is, W would tend to be bigger.

Thus, p-value = P(W$\geq$ 141.5)

This p-value can be found using Wilcoxon Rank-Sum critical values table. However, we are not going to use those tables and we will use normal approximation like we did for Signed-Rank test.

We define n_1 as the sample size whose ranks are summing. Then n_2 is the sample size of the other group.

Thus, $n_1 = 10$ and $n_2 = 10$ in this example.

We assume that W has a normal distribution with

E(W) $= \frac{n_1(n_1+n_2+1)}{2}$ and sd(W) $=\sqrt{\frac{n_1 n_2(n_1+n_2+1)}{12}}$

Rank-Sum test (Wilcoxon Rank-Sum test) - Example

E(W) $= \frac{10(10+10+1)}{2} = 105$

sd(W) $=\sqrt{\frac{10*10(10+10+1)}{12}} = 13.23$

Using Z $= \frac{W-E(W)}{sd(W)}$, we find z $= \frac{141.5-105}{13.23} = 2.76$

P(W$\geq$ 141.5) = P(Z$\geq$ 2.76) = 0.003 (Note: using R function 1-pnorm(2.76))

Thus, p-value = 0.003

There is strong evidence to conclude that caffeine does tend to give higher increase in pulse rate.

Next

Reminders

Quizzes 9 and 10 are now open.

Research project due date is 25 Jan 2023 at 3:00 pm.Only one student in each group needs to submit the video.

Lecture 14 - Revision - Thursday, 19 Jan 2023

Teaching evaluations are now open and will close at 11:59pm on Friday, 03 Feb 2023.