

# Functions Assignment

1. If  $f(x) = \pi \left( \frac{\sqrt{x+7} - 4}{x-9} \right)$ , then range of function  $y = \sin(2f(x))$  is

- (1)  $[0, 1]$  (2)  $\left(0, \frac{1}{\sqrt{2}}\right]$   
 (3)  $\left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$  (4)  $(0, 1]$

2. If the range of function  $f(x) = \frac{x^2 + x + c}{x^2 + 2x + c}$ ,  $x \in R$  is  $\left[\frac{5}{6}, \frac{3}{2}\right]$ , then  $c$  is equal to

- (1)  $-4$  (2)  $3$  (3)  $4$  (4)  $5$

3. If a polynomial function  $f$  satisfies the relation

$$\log_2 [f(x)] = \log_2 \left( 2 + \frac{2}{3} + \frac{2}{9} + \dots + \infty \right) \cdot \log_3 \left( 1 + \frac{f(x)}{f\left(\frac{1}{x}\right)} \right)$$

and  $f(10) = 1001$ , then value of  $f(20)$  is

- (1) 2002 (2) 7999 (3) 8001 (4) 16001

4. Consider,  $P = \frac{x^2 - 2x}{x^2 + x + 1}$ ,  $Q = \frac{y - 1}{y^2 + y + 1}$ ,  $R = \frac{2}{z^2 + z + 1}$  where  $x, y, z \in R$ . If

$k = [P + Q + R] - ([P] + [Q] + [R])$  then the possible value(s) of  $k$  is(are) (where  $[\cdot]$  denotes greatest integer less than equal to  $x$ )

- (1) 0 (2) 1 (3) 2 (4) 3

5. Let  $f$  be a function defined in  $[-2, 3]$  given as

$$f(x) = \begin{cases} 3(x+1)^{1/3}, & -2 \leq x < 0 \\ -(x-1)^2, & 0 \leq x < 1 \\ 2(x-1)^2, & 1 \leq x < 2 \\ -x^2 + 4x - 3, & 2 \leq x \leq 3 \end{cases}$$

## Column I

## Column II

- (A) The number of integers in the range of  $f(x)$  is (p) 2  
 (B) The number of integral values of  $x$  which are in the domain of  $f(1 - |x|)$ , is (q) 4  
 (C) The number of integers in the range of  $|f(-|x|)|$ , is (r) 6  
 (D) The number of integral values of  $k$  for which the equation  $f(|x|) = k$  has exactly four distinct solutions is (s) 7

6. Let  $f(x) = |x^2 - 9| - |x - a|$ . Find the number of integers in the range of  $a$  so that  $f(x) = 0$  has 4 distinct real roots.
7. The set of real values of  $x$  satisfying the equality  $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$  (where  $[\cdot]$  denotes the greatest integer function) belongs to the interval  $\left(a, \frac{b}{c}\right]$  where  $a, b, c \in N$  and  $\frac{b}{c}$  is in its lowest form. Find the value of  $a + b + c + abc$ .
8. Let  $R$  be the set of real numbers and  $f : R \rightarrow R$ , be a differentiable function such that  $|f(x) - f(y)| \leq |x - y|^3 \forall x, y \in R$ . If  $f(10) = 100$ , the value of  $f(20)$  is equal to  
 (1) 0 (2) 20 (3) 100 (4) 10
9. If the equation  

$$\left| |x - 1| - 6 \lim_{t \rightarrow \infty} \left( \frac{\sqrt{2t^2 - t - 1} - \sqrt{t^2 - t + 1}}{t \left( \tan \frac{\pi}{8} \right)} \right) \right| = k$$
 has four distinct solutions then find the number of integral values of  $k$ .
10. Let  $A$  and  $B$  be two sets containing 2 and 3 elements respectively. Then, total number of subsets of  $A \times B$  having 3 or more elements is  
 (1) 42 (2) 56 (3) 54 (4) 52
11. Let the sets be  $A = \{x : x \in Z^+ \text{ and } x \leq 9\}$ ,  $B = \{x : x \in Z \text{ and } -3 < x < 8\}$  and  $C = \{x : x \text{ is a prime number}\}$ , then the number of elements belonging to exactly two of the three sets  $A, B$  and  $C$  is  
 (1) 3 (2) 4 (3) 6 (4) 8
12. Let  $f$  be a function from non-negative integers to non-negative integers such that  $f(xy) = xf(y) + yf(x)$ . It is given that  $f(10) = 19$ ,  $f(12) = 52$  and  $f(15) = 26$ , then  $f(8)$  is equal to  
 (1) 26 (2) 36 (3) 38 (4) 40
13. If a function  $f(x)$  satisfies the relation  $3f(x) - 5f\left(\frac{2}{x}\right) = 3 - x + x^2 \forall x \in R - \{0\}$ , then the value of  $f(1)$  is equal to  
 (1)  $-2$  (2)  $\frac{-33}{16}$  (3)  $\frac{-17}{8}$  (4)  $\frac{-35}{16}$
14. The function  $f(x) = \sqrt{\log_2 \log_3 \log_4 x} + \sqrt{\{x^2 + x + 1\}}$  (where  $\{\}$  represents fractional function, is well defined). Then  $x$  may belong to  
 (1)  $[0, 32]$  (2)  $[64, \infty)$  (3)  $[100, \infty)$  (4)  $[0, 64]$
15. Let  $g$  be a function satisfying  $g(x - 2) + g(x + 2) = \sqrt{3}g(x)$ ,  $\forall x \in R$ . Then  $g$  also satisfies the relation(s) given as  
 (1)  $g(x - 4) + g(x + 8) = 0$  (2)  $g(x) - g(x + 12) = 0$   
 (3)  $g(x - 2) + g(x + 4) = 0$  (4)  $g(x) - g(x + 24) = 0$

16. Domain of the function  $f(x) = \log_{[x-1]}(\sin^2 \pi x)$  is  $x \in [3, \infty) - \{p, p+1, p+2, \dots, n\}$ , where  $[\cdot]$  denotes greatest integer function and  $n$  is a multiple of 4 and 25, then  $(p) + (p+1) + (p+2) + \dots + n$  can be

(1) 5047                      (2) 20097                      (3) 5050                      (4) 45147

17. Let  $f$  be a real valued function which satisfies  $f\left(\frac{x-3}{x+1}\right) + f\left(\frac{3+x}{1-x}\right) = x$ ,  $|x| \neq 1$ ,  $x \in R$ . Then  $f(3)$  is equal to

(1)  $-3$                       (2)  $3$                       (3)  $-\frac{11}{3}$                       (4)  $\frac{11}{3}$

18. Let  $f(x^2 - 11x + 10) + f(x^2 - 21x + 20) = x^4 - 3x^2 + 21x + 2$ , then the value of  $f(0)$  is

(1) 20                      (2)  $\frac{21}{2}$                       (3) 24                      (4) 0

19. The range of the function  $f(x) = \frac{\sin(\pi [x^2 + x + 1])}{\cos(\pi [x^2 + 3x + 2])}$  is ( $[\cdot]$  represents greatest integer function)

(1)  $\{0\}$                       (2)  $\{1, -1\}$                       (3)  $[-1, 1]$                       (4)  $(-\infty, \infty)$

20. If a function satisfies

$$(x-y)f(x+y) - (x+y)f(x-y) = 2(x^2y - y^3) \quad \forall x, y \in R \text{ and } f(1) = 5$$

then

(1)  $f(x)$  is not differentiable                      (2)  $f(x) = x^2 + 4x$   
 (3)  $f(0) = 1$                       (4)  $f'(1) = 8$

21. Total number of solutions of  $\sin\{x\} = \cos\{x\}$  (where  $\{\cdot\}$  denotes the fractional part) in  $[0, 2\pi]$  is

22. Set of exhaustive values of  $x$  satisfying

$$||x|^2 - x + 1| > |x^2 - 1|$$

(1)  $(-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$                       (2)  $(0, 2)$   
 (3)  $\left(\frac{1}{2}, 2\right)$                       (4)  $(-\infty, 0) \cup \left(\frac{1}{2}, 2\right)$

23. Consider an equation  $Sgn([x]) + \lambda = x$ . (where  $[\cdot]$  represents greatest integer function). if  $n_\lambda$  denotes the number of solutions of the equation, then the value of  $\frac{n_{\frac{1}{2}}}{n_1}$  is

(1) 1                      (2)  $\frac{3}{2}$                       (3) 3                      (4)  $\frac{1}{2}$

24. The solution of the equation  $2[x] + [3x] = 6$  (where  $[\cdot]$  denotes the greatest integer function) is

(1)  $\left[\frac{4}{3}, \frac{5}{3}\right)$       (2)  $\left[\frac{5}{3}, 2\right)$       (3)  $\left[\frac{2}{3}, \frac{5}{3}\right)$       (4)  $\left[1, \frac{5}{3}\right)$

25.  $A = \{x \in N : \text{HCF}(x, 12) = 1, x < 12\}$ ,  $B = \{x \in N : \text{LCM}(x, 12) = 12\}$ , then the number of relations from  $A$  to  $C$ , where  $A \Delta C = B$ , is

(1)  $2^{28}$       (2)  $2^{32}$       (3)  $2^{24}$       (4)  $2^{36}$

26. The number of subsets of a set with 2018 elements having an odd number of elements is

(1)  $2^{2015}$       (2)  $2^{2016}$       (3)  $2^{2017}$       (4) Data insufficient

27. The range of the function  $f(x) = \frac{(x+3)^2}{x^2+1}$  is

(1)  $[0, 12]$       (2)  $[0, 11]$       (3)  $[0, 10]$       (4)  $[0, 15]$

28. The maximum value of  $f(x) = |15 - 8x + |x|^2|$  in the interval  $(3.5, 4.5)$  is

(1) 0      (2) 1      (3) 2      (4) 3

29. If  $X$  and  $Y$  are two sets such that

$$n(X \cap \bar{Y}) = 12, \quad n(\bar{X} \cap Y) = 15 \text{ and } n(X \cup Y) = 30$$

then  $n(X \times Y) =$

(1) 210      (2) 270      (3) 180      (4) 300

30. Let  $3^{f_1(x)} + 3^x = 9$  and  $f_2(x) = \log_{\frac{1}{2}}(a + 2x - x^2)$ . If maximum integral value of  $f_1(x)$  is equal to the minimum value of  $f_2(x)$ , then  $a$  is equal to

(1)  $-1$       (2)  $-\frac{1}{2}$       (3) Zero      (4) 1

31. Find the range of the function

$$f(x) = \log_{\frac{1}{2}}(2 \sin^2 x - 2 \sin x + 1)$$

(1)  $\left[\log_{\frac{1}{2}} 5, 0\right]$       (2)  $\left[\log_{\frac{1}{2}} 5, 1\right]$       (3)  $\left(-\infty, \log_{\frac{1}{2}} 5\right]$       (4)  $\left[\log_{\frac{1}{2}} 5, \infty\right)$