Random Questions

1. Let $f(x)$ be defined as:

$$f(x) = \begin{cases} x^2 \sin\left(\frac{1}{x}\right) & x \neq 0 \\ 0 & x = 0 \end{cases}$$

If the function $f(x)$ is continuos then $f'(0)$ is:

(1) 0 (2) 1 (3) -1 (4) Not defined

2. The number of integral roots of the equation

$$x^8 - 24x^7 - 18x^5 + 39x^2 + 1155 = 0$$

(1) 0 (2) 2 (3) 4 (4) 6

3. Let $x_1, x_2, x_3, \ldots, x_k$ be the divisors of positive number n (including 1 and n). If $x_1 + x_2 + x_3 + \cdots + x_k = 2022$, then $\sum_{i=1}^{k} \left(\frac{1}{x_i}\right)$ is equal to:

(1) $\frac{2022}{k}$ (2) $\frac{2022}{n}$ (3) $\frac{1}{n}$ (4) $\frac{1}{2022}$

4. Suppose that f satisfies the equation $f(x+y) = f(x) + f(y) + x^2y + xy^2$ $\forall x, y \in R$. Suppose further that

$$\lim_{x \to 0} \frac{f(x)}{x} = 1$$

Find $f'(x)$

5. Solve for x,

$$x^{\ln x} = x \ln x$$

6. Consider the function $f(x) = a^{a^x} - x$. If the equation $f(x) = 0$ has exactly 2 roots. Then range of a is

(1) $a \in (0, 1)$ (2) $a \in \left(1, e^{\frac{1}{e}}\right)$

(3) $a \in (0, 1) \cup \left(1, e^{\frac{1}{e}}\right)$ (4) No such a exists

7. Solve for x,

$$2^{x^6} + 2^{x^2} = 2^{x^4+1}$$