

Functions II Assignment

1. Let $f: R \rightarrow R$ be a function defined by $f(x) = \frac{x^2 + 2x + 5}{x^2 + x + 1}$ is
 - (1) one-one and into
 - (2) one-one and onto
 - (3) many-one and onto
 - (4) many-one and into
2. $f: R \rightarrow R$ is a function defined by $f(x) = \frac{e^{|x|} - e^{-x}}{e^x - e^{-x}}$ then f is
 - (1) a bijection
 - (2) an injection only
 - (3) surjection only
 - (4) neither injection nor surjection
3. If $f: R \rightarrow \left[\frac{\pi}{6}, \frac{\pi}{2}\right)$ defined by $f(x) = \sin^{-1}\left(\frac{x^2 - a}{x^2 + 1}\right)$ is an onto function, then the set of values of a is
 - (1) $(-\infty, -1)$
 - (2) $(-1, \infty)$
 - (3) $(-\infty, 0)$
 - (4) $(-\infty, \infty)$
4. Let $f: R \rightarrow R$ be any function. Define $g: R \rightarrow R$ by $g(x) = |f(x)|$ for all x then g is
 - (1) onto if f is onto
 - (2) one-one if f is one-one
 - (3) continuous if f is continuous
 - (4) neither one-one nor onto
5. We have $f(x) = \begin{cases} 3, & x \leq 0 \\ 3^{-x} - 3^x + 3, & x > 0 \end{cases}$ (as $\text{sgn}(e^{-x}) = 1 \forall x \in R$).
 - (1) one-one and into
 - (2) one-one and onto
 - (3) many-one and onto
 - (4) Neither one-one nor onto
6. Let $A = \{1, 2, 3, 4, 5\}$. If f is a bijective function from A to A , then the number of such functions for which $f(k) \neq k$, $k = 1, 2, 3, 4, 5$ is
 - (1) 5^5
 - (2) 120
 - (3) 44
 - (4) $5^5 - 120$
7. The set of values of a for which the function $f: R \rightarrow R$ given by $f(x) = x^3 + (a + 2)x^2 + 3ax + 5$ is one-one is
 - (1) $[-2, 4]$
 - (2) $(1, 3)$
 - (3) $(1, 4)$
 - (4) $(1, 5)$
8. S-I: If $f(x) = 2x^3 + 7x - 5$ then the value of $f^{-1}(4)$ is 1.
S-II: A function $y = f(x)$ is invertible if f is one-one and onto.
 - (1) Both S-I and S-II are individually true and R is the correct explanation of A
 - (2) Both S-I and S-II are individually true but R is not the correct explanation of A
 - (3) S-I is true but S-II is false
 - (4) S-I is false but S-II is true

9. The sum of solution(s) of the equation $2[x] + x = 6\{x\}$ where $[x]$ denotes greatest integer less than or equal to x and $\{x\}$ denotes fractional part of x is

(1) 0 (2) $\frac{5}{3}$ (3) $\frac{3}{5}$ (4) $\frac{8}{5}$

10. If $[x]$ is the greatest integer function, then $\sum_{k=1}^{4020} \left[\frac{1}{2} + \frac{k-1}{4020} \right]$ is equal to

(1) 2010 (2) 2009 (3) 2011 (4) 2005

11. $f(x) = \sin[x] + [\sin x]$, $0 < x < \frac{\pi}{2}$, where $[\cdot]$ represents the greatest integer function, can also be represented as

(1) $\begin{cases} 0, & 0 < x < 1 \\ 1 + \sin 1, & 1 \leq x < \frac{\pi}{2} \end{cases}$ (2) $\begin{cases} \frac{1}{\sqrt{2}}, & 0 < x < \frac{\pi}{4} \\ 1 + \frac{1}{2} + \frac{1}{\sqrt{2}} + \frac{\sqrt{3}}{2}, & \frac{\pi}{4} \leq x < \frac{\pi}{2} \end{cases}$

(3) $\begin{cases} 0, & 0 < x < 1 \\ \sin 1, & 1 \leq x < \frac{\pi}{2} \end{cases}$ (4) $\begin{cases} 0, & 0 < x < \frac{\pi}{4} \\ 1, & \frac{\pi}{4} < x < 1 \\ \sin 1, & 1 \leq x < \frac{\pi}{2} \end{cases}$

12. If $[x]$ and $\{x\}$ represent the integral and fractional parts of x respectively, then the value of $\sum_{r=1}^{2000} \frac{\{x+r\}}{2000}$ is

(1) x (2) $[x]$ (3) $\{x\}$ (4) $x + 2001$

13. Total number of solution of $2^x + 3^x + 4^x - 5^x = 0$ is

(1) 0 (2) 1 (3) 2 (4) infinitely many

14. The of the real-valued function satisfying $f(x) + f(x+4) = f(x+2) + f(x+6)$ is

(1) 10 (2) 8 (3) 12 (4) 6

15. The period of

$$\frac{|\sin(4x)| + |\cos(4x)|}{|\sin(4x) - \cos(4x)| + |\sin(4x) + \cos(4x)|}$$

is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) π

16. If $f(x) = \cos x + \{x\}$ where $\{\cdot\}$ is fractional part function then the period of $f(x)$ is

(1) 2π (2) 1 (3) $\frac{\pi}{2}$ (4) Does not exist

17. The period of

$$f(x) = \sin x + \tan\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2^2}\right) + \cdots + \sin\left(\frac{x}{2^{n-1}}\right) + \tan\left(\frac{x}{2^n}\right)$$

(1) 2π (2) $2^n\pi$ (3) $2^n\frac{\pi}{3}$ (4) $3^n\pi$

18. Period of $f(x) = \sin((\cos x) + x)$ is
 (1) Does not exist (2) π (3) $\frac{\pi}{2}$ (4) 2π
19. The period of the function $\left| \sin^3\left(\frac{x}{2}\right) \right| + \left| \cos^5\left(\frac{x}{5}\right) \right|$ is
 (1) 2π (2) 10π (3) 8π (4) 5π
20. Let $f: R \rightarrow R - \{3\}$ be a function such that for some $p > 0$,
 $f(x+p) = \frac{f(x)-5}{f(x)-3} \quad \forall x \in R$. Then, period of f is
 (1) $2p$ (2) $3p$ (3) $4p$ (4) $5p$
21. Let $f(x)$ be a real valued function with domain R such that

$$f(x+p) = 1 + [2 - 3f(x) + 3(f(x))^2 - (f(x))^3]^{\frac{1}{3}}$$
 holds good $\forall x \in R$ and for some +ve constant p then the period of $f(x)$ is
 (1) $\frac{p}{2}$ (2) p (3) $2p$ (4) $\frac{p}{3}$
22. If $f(a-x) = f(a+x)$ and $f(b-x) = f(b+x) \quad \forall x \in R$ where $a, b (a > b)$ are constants then the period of $f(x)$ is
 (1) $2a$ (2) $2|a-b|$ (3) $3a$ (4) b
23. The period of $f(x) = [x] + [2x] + [3x] + [4x] + \dots + [nx] - \frac{n(n+1)}{2}x$ (where $n \in N$ is
 (1) n (2) 1 (3) $\frac{1}{n}$ (4) 5
24. If $f(2+x) = a + [1 - (f(x) - a)^4]^{\frac{1}{4}} \quad \forall x \in R$, then $f(x)$ is periodic with period
 (1) 1 (2) 2 (3) 4 (4) 8
25. If $f: [-4, 4] - \{-\pi, 0, \pi\} \rightarrow R$, such that $f(x) = \cot(\sin x) + \left[\frac{x^2}{|a|} \right] + \frac{\sin 2x}{x^2}$, ($[\cdot]$ denotes greatest integer function) is an odd function, then the complete set of values of a is
 (1) $(-\infty, -4] \cup [4, \infty)$ (2) $(-\infty, -16) \cup (16, \infty)$
 (3) $[-16, 16]$ (4) $(-\infty, -16] \cup [16, \infty)$
26. S-I: If $f(x)$ is a odd function and $g(x)$ is even function then $f(x) + g(x)$ is neither even nor odd.
 S-II: Odd function is symmetrical in opposite quadrants and even function is symmetrical about the y-axis.
 (1) Both S-I and S-II are individually true and R is the correct explanation of A
 (2) Both S-I and S-II are individually true but R is not the correct explanation of A
 (3) S-I is true but S-II is false
 (4) S-I is false but S-II is true

27. If $f(x) = \begin{cases} 1+x; & 0 \leq x \leq 2 \\ 3-x; & 2 < x \leq 3 \end{cases}$ then $(f \circ f)(x) =$

(1) $\begin{cases} 1-x; & 0 \leq x \leq 2 \\ 3+x; & 2 < x \leq 3 \end{cases}$ (2) $\begin{cases} 2+x; & 0 \leq x \leq 2 \\ 2-x; & 2 < x \leq 3 \\ 4-x; & 2 < x \leq 3 \end{cases}$

(3) $\begin{cases} 2+x; & 0 \leq x \leq 1 \\ 2-x; & 1 < x \leq 2 \\ 4-x; & 2 < x \leq 3 \end{cases}$ (4) does not exist

28. If for $x > 0$, $f(x) = (a - x^n)^{\frac{1}{n}}$; $g(x) = x^2 + px + q$; $p, q \in R$ and the equation $g(x) - x = 0$ has imaginary roots then the number of real roots of the equation $g(g(x)) - f(f(x)) = 0$

(1) 0 (2) 2 (3) 4 (4) n

29. S-I: If $f(x) = \frac{x+1}{x-1}$ then $(f \circ f \circ f \circ f \circ f)(x) = f(x)$

S-II: $(f \circ f)(x) = x$

- (1) Both S-I and S-II are individually true and R is the correct explanation of A
 (2) Both S-I and S-II are individually true but R is not the correct explanation of A
 (3) S-I is true but S-II is false
 (4) S-I is false but S-II is true

30. The domain of $f(x) = \frac{1}{x} + 2^{\sin^{-1} x} + \frac{1}{\sqrt{x-2}}$ is

(1) $(0, \infty)$ (2) $(-\infty, 0)$ (3) $(1, 3)$ (4) ϕ

31. The domain of $f(x) = \sqrt{\ln_{(|x|-1)}(x^2 + 4x + 4)}$ is

(1) $[-3, -1] \cup [1, 2]$ (2) $(-2, -1) \cup [2, \infty)$
 (3) $(-\infty, -3] \cup (-2, -1) \cup (2, \infty)$ (4) $(-\infty, \infty)$

32. The domain of $f(x) = \sqrt{\frac{\log_{0.3} |x-2|}{|x|}}$ is

(1) $[1, 2]$ (2) $[2, 3]$ (3) $[1, 2) \cup (2, 3]$ (4) $[0, 3]$

33. The domain of $f(x) = \log_{10} \log_{10} \log_{10} \dots \log_{10} x$ (log n times) is

(1) $\left(10^{10^{\dots(n-2) \text{ times}}}, \infty\right)$ (2) $(10^{n-2}, \infty)$
 (3) $\left(10^{10^{\dots(n-1) \text{ times}}}, \infty\right)$ (4) $\left(10^{10^{\dots(n-3) \text{ times}}}, \infty\right)$

34. The domain of the function $f(x) = \sin^{-1} \left(\frac{4}{3 + 2 \cos x} \right)$ is
- (1) $\left[2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{3} \right]$ (2) $\left[2n\pi - \frac{\pi}{6}, 2n\pi + \frac{\pi}{6} \right]$
 (3) $\left[2n\pi - \frac{\pi}{2}, 2n\pi + \frac{\pi}{2} \right]$ (4) $\left[2n\pi - \frac{\pi}{3}, 2n\pi + \frac{\pi}{2} \right]$
35. The domain of $f(x) = \sin^{-1} [2 - 4x^2]$ (where $[\cdot]$ is G.I.F.) is
- (1) $\left[-\frac{\sqrt{3}}{2}, \frac{\sqrt{3}}{2} \right]$ (2) $\left[-\frac{3}{2}, \frac{3}{2} \right] - \{0\}$
 (3) $\left[-\frac{\sqrt{3}}{2}, 0 \right) \cup \left(0, \frac{\sqrt{3}}{2} \right]$ (4) ϕ
36. The domain of $f(x) = \sqrt{\cos(\sin x)} + \sqrt{\log_x \{x\}}$ (where $\{ \cdot \}$ is fractional part of x) is
- (1) $[1, \pi)$ (2) $(0, 2\pi) - [1, \pi)$ (3) $\left(0, \frac{\pi}{2} \right) - \{1\}$ (4) $(0, 1)$
37. The domain of
- $$\frac{[\cos^{-1}(x^4)] + |[x - 2 \tan^{-1} x]| + \sqrt{\sin(\ln x)}}{\{3x^2 - 7\} + a^{\sqrt{\sin x + 3 \cos x}} + \ln \cos \left(\frac{1}{\sqrt{-x^2}} \right)}$$
- (where $[\cdot]$ is G.I.F. and $\{ \cdot \}$ is fractional part function) is
- (1) $(-2, \sqrt{2})$ (2) $(0, 1)$ (3) $(-1, 1)$ (4) ϕ
38. The domain of $f(x) = \sin^{-1} ([2 - 3x^2])$ (where $[\cdot]$ is G.I.F.) is
- (1) $[-1, 1]$ (2) $(0, 1)$ (3) $[-1, 1] - \{0\}$ (4) ϕ
39. The domain of the function
- $$f(x) = \left[9^x + 27^{\frac{2}{3}(x-2)} - 219 - 3^{2(x-1)} \right]^{\frac{1}{4}}$$
- (1) $[-3, 3]$ (2) $[3, \infty)$ (3) $\left[\frac{5}{2}, \infty \right)$ (4) $[0, 1]$
40. If $f(x) = 2x + |x|$, $g(x) = \frac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$ then the domain of $\sin^{-1}(h(h(h \dots h(x))))$ is (h is being repeated n times)
- (1) $[-1, 1]$ (2) $(0, 1)$ (3) $(-1, 1)$ (4) $\left[\frac{1}{2}, 1 \right]$
42. The range of $\tan(\log x)$
- (1) $(0, \infty)$ (2) $(1, \infty)$ (3) (e, ∞) (4) $(-\infty, \infty)$
43. The range of $f(x) = \frac{e^{-x}}{1 + [x]}$ is (where $[\cdot]$ is G.I.F.)
- (1) R (2) $R - \{0\}$ (3) $R - [-1, 0)$ (4) $[0, \infty)$

44. The range of $f(x) = \frac{1}{|\sin x|} + \frac{1}{|\cos x|}$ is
 (1) $[2\sqrt{2}, \infty)$ (2) $(\sqrt{2}, 2\sqrt{2})$ (3) $(0, 2\sqrt{2})$ (4) $(2\sqrt{2}, 4)$
45. The range of $f(x) = \frac{x - [x]}{1 - [x] + x}$ (where $[\cdot]$ is G.I.F.) is
 (1) $\left[0, \frac{1}{2}\right]$ (2) $[0, 1]$ (3) $\left(0, \frac{1}{2}\right]$ (4) $\left[0, \frac{1}{2}\right)$
46. Let $A = \left\{x: 0 \leq x < \frac{\pi}{2}\right\}$ and $f: R \rightarrow A$ is an onto function given by
 $f(x) = \tan^{-1}(x^2 + x + \lambda)$ where
 (1) $\lambda > 0$ (2) $\lambda \leq \frac{1}{4}$ (3) $\lambda = \frac{1}{4}$ (4) $\lambda \geq \frac{1}{8}$
47. $f(x) = |x - 1|$, $f: R^+ \rightarrow R$ and $g(x) = e^x$, $g: [-1, \infty) \rightarrow R$ if the function $(f \circ g)(x)$ is defined, then its domain and range respectively are
 (1) $(0, \infty) \& [0, \infty)$ (2) $[-1, \infty) \& [0, \infty)$
 (3) $[-1, \infty) \& \left[1 - \frac{1}{e}, \infty\right)$ (4) $[-1, \infty) \& \left[\frac{1}{e} - 1, \infty\right)$
48. The range of $f(x) = \left[\frac{1}{\sin \{x\}}\right]$ is (where $\{\cdot\}$ is fractional part and $[\cdot]$ is G.I.F.)
 (1) $\{1, -1\}$ (2) $\{0\}$ (3) N (4) Z
49. If $f(x) = \pi \left(\frac{\sqrt{x+7}-4}{x-9}\right)$, then range of function $y = \sin(2f(x))$ is
 (1) $[0, 1]$ (2) $\left(0, \frac{1}{\sqrt{2}}\right]$
 (3) $\left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$ (4) $(0, 1]$
50. The range of $f(x) = \sqrt{a-x} + \sqrt{x-b}$ is (where $a > b > 0$)
 (1) $[\sqrt{a-b}, \sqrt{2(a-b)}]$ (2) $[\sqrt{a-b}, \sqrt{a+b}]$
 (3) $[a, b]$ (4) (a, b)
51. The range of $f(x) = \cot^{-1}(2x - x^2)$ is
 (1) $(0, \pi)$ (2) $\left(\frac{\pi}{4}, \pi\right)$ (3) $\left(\frac{\pi}{4}, \pi\right]$ (4) $\left[\frac{\pi}{4}, \pi\right)$
52. The sum of the maximum and minimum values of
 $f(x) = \sin^{-1}(2x) + \cos^{-1}(2x) + \sec^{-1}(2x)$
 is
 (1) π (2) $\frac{\pi}{2}$ (3) 2π (4) $\frac{3\pi}{2}$

53. If $f(x) = \log_{[x-1]} \left(\frac{|x|}{x} \right)$, (where $[\cdot]$ is G.I.F.) then domain and range are
 (1) $(2, \infty), (0, 1)$ (2) $[3, \infty), \{0\}$ (3) $[3, \infty), \{0, 1\}$ (4) $(-\infty, \infty), \{0\}$
54. If $f(x) = x^3 + 3x^2 + 4x + a \sin x + b \cos x \quad \forall x \in R$ is an injection then the greatest value of $a^2 + b^2$ is
 (1) 1 (2) 2 (3) $\sqrt{2}$ (4) $2\sqrt{2}$
55. The domain and range of $f(x) = \sin \left\{ \log \left(\frac{\sqrt{4-x^2}}{1-x} \right) \right\}$ are
 (1) $(-2, 1) \& [-1, 1]$ (2) $(1, 3) \& [-1, 1]$
 (3) $[-2, 1] \& [-1, 1]$ (4) $(0, \infty) \& [-1, 1]$
56. Consider the real valued function satisfying $2f(\sin x) + f(\cos x) = x$, then $f\left(\frac{1}{2}\right) =$
 (1) 1 (2) 2 (3) 0 (4) 4
57. If $f: R \rightarrow R$ such that $f(x - f(y)) = f(f(y)) + xf(x) + f(x) - 1 \quad \forall x, y \in R$ then $f(x)$ is
 (1) $1 + \frac{x^2}{4}$ (2) $1 - \frac{x^2}{2}$ (3) $1 + \frac{x^2}{2}$ (4) $1 - \frac{x^2}{4}$
58. A function f well defined $\forall x, y \in R$ is such that $f(1) = 2, f(2) = 8$ and $f(x+y) - kxy = f(x) + 2y^2$, where k is some constant then $f(x)$ is
 (1) x^2 (2) $3x^2$ (3) $2x^2$ (4) $4x^2$
59. If f is a polynomial function satisfying $2 + f(x)f(y) = f(x) + f(y) + f(xy) \quad \forall x, y \in R$ and if $f(2) = 5$ then the value of $f(f(2)) =$ $(f(1) \neq 1)$
 (1) 25 (2) 16 (3) 26 (4) 14
60. If f is a function such that $f(0) = 2; f(1) = 3$ and $f(x+2) = 2f(x) - f(x+1) \quad \forall x \in R$ then $f(5) =$
 (1) 7 (2) 13 (3) 1 (4) 5
61. If $f(x) = (-1)^{\left[\frac{2x}{\pi}\right]}$, $g(x) = |\sin x| - |\cos x|$, and $\phi(x) = f(x)g(x)$ (where $[\cdot]$ denotes the greatest integer function), then the respective fundamental periods of $f(x)$, $g(x)$ and $\phi(x)$ are
 (1) π, π, π (2) $\pi, 2\pi, \pi$ (3) $\pi, \pi, \frac{\pi}{2}$ (4) $\pi, \frac{\pi}{2}, \pi$
62. The domain of the function $f(x) = \frac{1}{\sqrt{{}^{10}C_{x-1} - 3 \times {}^{10}C_x}}$ contains the points
 (1) 9, 10, 11 (2) 9, 10, 12
 (3) all natural numbers (4) none of these
63. The range of $f(x) = (x+1)(x+2)(x+3)(x+4) + 5$ for $x \in [-6, 6]$ is
 (1) $[4, 5045]$ (2) $[0, 5045]$ (3) $[-20, 5045]$ (4) none of these