Complex & Quadratic Assignment

1. If the quadratic expression $px^2 + |2p-3|x - 6$ is positive for exactly two integral values of x then (where $[\cdot]$ is greatest integer function)

   (1) $[p] = -1$ (2) $[p] = -2$ (3) $p = \left(-\frac{3}{4}, -\frac{3}{5}\right]$ (4) $p \in (-2, -1]$

2. If $lx^{17} + mx^{16} + 1$ is divisible by $x^2 - x - 1$ then

   (1) l is divisible by 3 (2) l is divisible by 7

   (3) l is divisible by 47 (4) l is divisible by 21

Question: Let α, β, γ, δ be the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ if $(\alpha + \beta) = (\gamma + \delta)$ and a, b, c, $d \in R$, then

3. The correct options is/are

   (1) If $a = 2$, then $b - c \neq 2$ (2) If $a = 2$, then $b - c = 1$

   (3) If $a = 1$, then $b - 2c \neq 1$ (4) If $a = 1$, then $b - 2c = \frac{1}{4}$

4. If $b + c = 1$ and $a \neq -2$ then

   (1) $b \leq \frac{3}{4}$ (2) $b \geq \frac{3}{4}$ (3) $c \leq \frac{1}{4}$ (4) $c \geq \frac{1}{4}$

Question: For any complex number z, $z = |z|\left[\cos(\arg(z)) + i\sin(\arg(z))\right]$. Choose the correct answer(s)

5. If z is any non-zero complex number, then

   (1) $\left|\frac{z}{|z|} - 1\right| \leq |\arg(z)|$ (2) $\left|\frac{z}{|z|} - 1\right| > |\arg(z)|$

   (3) $|z - 1| > ||z| - 1| + |z||\arg(z)|$ (4) $|z - 1| \leq ||z| - 1| + |z||\arg(z)|$

6. If z is any non-zero complex number such that $\arg\left(z^{\frac{3}{8}}\right) = \frac{1}{2}\arg\left(z^2 + \overline{z}z^{\frac{1}{2}}\right)$, then z

   (1) Must be purely imaginary and non-unimodular complex number
   (2) Could be unimodular complex number
   (3) Could be purely real complex number
   (4) Must be purely imaginary complex number

Question: Let $e^{\ln[1+\{xyz\}]}$, $\log_y x$, $\log_z y$ and $\log_x z^{-15}$ be the first four terms of an A.P. with common difference d, where all terms of the A.P. are real and defined (where $[\cdot]$ and $\{\cdot\}$ represents greatest integer function and fractional part function) then answer the following questions

7. Which of the following interval(s) contain d?

(1) $[0, \infty)$ (2) $(-\infty, 0]$ (3) $[-10, 20]$ (4) $[10, 20]$

8. The value of $\displaystyle\sum_{k=1}^{\infty} \frac{k}{\left(\frac{x}{z^3} + xy + yz^3\right)^k}$ is less than or equal to

(1) 1 (2) 2 (3) $\dfrac{1}{2}$ (4) $\dfrac{2}{3}$

Question: α, β, γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ then $\alpha + \beta + \gamma = -\dfrac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \dfrac{c}{a}$, $\alpha\beta\gamma = -\dfrac{d}{a}$.
Let the number p and q are positive and the roots of the equation $x^3 - 4px + 3q = 0$ are real. Let α is a root of this given cubic equation of minimum absolute value then

9. The roots of the given cubic equation must be

(1) Two positive and one negative (2) All positive

(3) All negative (4) Two negative and one positive

10. The range of α is

(1) $-\dfrac{3q}{4p} < \alpha < \dfrac{q}{2p}$ (2) $-\dfrac{9q}{8p} < \alpha < \dfrac{3q}{4p}$

(3) $\dfrac{3q}{4p} < \alpha < \dfrac{9q}{8p}$ (4) $\dfrac{q}{p} < \alpha < \dfrac{9q}{8p}$

Question: If x_1, x_2, $x_3 \ldots x_n$ are all positive and $m \in R$, then

$$\frac{x_1^m + x_2^m + x_3^m + \cdots + x_n^m}{n} \geq \left(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}\right)^m \text{ if } m \in R - (0,1)$$

and

$$\frac{x_1^m + x_2^m + x_3^m + \cdots + x_n^m}{n} \leq \left(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}\right)^m \text{ if } 0 < m < 1$$

also, $\dfrac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \geq (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{\frac{1}{n}} \geq \dfrac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}$. Then answer the following questions.

11. If $x_1 > 0$, $x_2 > 0$, $x_3 > 0$ and $x_1 + x_2 + x_3 = 1$, then the minimum value of

$$\frac{x_1}{3 - x_1} + \frac{x_2}{3 - x_2} + \frac{x_3}{3 - x_3} \text{ is}$$

(1) $\dfrac{3}{8}$ (2) $\dfrac{5}{8}$ (3) $\dfrac{7}{8}$ (4) $\dfrac{3}{4}$

12. If $x_1 + x_2 + x_3 = 3$ and $x_1 > 0$, $x_2 > 0$, $x_3 > 0$ then the minimum value of

$$\left(\frac{3}{x_1} - 1\right)\left(\frac{3}{x_2} - 1\right)\left(\frac{3}{x_3} - 1\right) \text{ is}$$

(1) 3 (2) 4 (3) 7 (4) 8

Question: Let the cubic equation $ax^3+bx^2+cx+d=0$ is shown by $P(a, b, c, d, x) = 0$. Now answer the following questions.

13. If the equation $P(1, 0, 1, 7, x) = 0$ has roots x_1, x_2, x_3, then the equation whose roots are $(x_1 - x_2)^2$, $(x_2 - x_3)^2$, $(x_3 - x_1)^2$ is

(1) $P(1, 6, 9, 1327, x) = 0$ (2) $P(1, 1, 9, 127, x) = 0$

(3) $P(1, 6, 9, 127, x) = 0$ (4) $P(6, 9, 1, 127, x) = 0$

14. If the roots of the equation $P(6, -11, 6, -1, x) = 0$ are in HP and equation $P(1, -6, 11, -6, x) = 0$ have roots α, β, γ, then $\alpha + \beta^2 + \gamma^3$ can be

(1) 27 (2) 9 (3) 32 (4) 12

Question: If Locus of point $P(z)$ in complex plane is $|z + z_1| + |z + z_2| = 4$ where A represents z_1, as $(1, 0)$ and B represents z_2 as $(-1, 0)$ and $Q(w)$ is variable point inside the locus of P such that all internal angle bisectors of triange PAB concurrent at Q and if $|w - w_1| + |w - w_2| = 2$, then

15. The value of $|w_1| + |w_2|$ is equal to

(1) $\frac{2}{\sqrt{3}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $2\sqrt{\frac{2}{3}}$ (4) $\frac{2\sqrt{2}}{3}$

16. If minimum value of $|w - z_1| + |w - z_2|$ is equal to m, then $[m]$ (where $[x]$ denotes greatest integer function) is

(1) 1 (2) 2 (3) 3 (4) 4

Question: Let $f(x) = x^3 + x + 1$ suppose g is a cubic polynomial such that $g(0) = -1$ and the roots of $g(x) = 0$ are square of the roots of $f(x) = 0$. Then

17. The equation $f(x) = 0$ has

(1) At least one positive root (2) At least two negative roots

(3) Exactly one negative root (4) Exactly two positive roots

18. The polynomial $g(x^2)$ is identical with

(1) $f(x^2)$ (2) $(f(x))^2$ (3) $2f(x)f(-x)$ (4) $-f(x)f(-x)$

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19. If the equation $|z|(z+1)^8 = z^8|z+1|$ where $z \in C$ and $z(z+1) \neq 0$ has distinct roots $z_1, z_2, \ldots z_n$ (where $n \in N$) then which of the following is/are true?

(1) $z_1, z_2, \ldots z_n$ are concyclic point

(2) $z_1, z_2, \ldots z_n$ are collinear point

(3) $\sum_{r=1}^{n} \text{Re}(z_r) = -\frac{7}{2}$

(4) $\sum_{r=1}^{n} \text{Im}(z_r) = 0$

20. Let z be a complex number satisfying equation $z^n = (\overline{z})^m$ where $n, m \in N$ then

(1) If $n = m$ then number of solutions will be finite

(2) If $n = m$ then number of solutions will be infinite

(3) If $n \neq m$ then number of solutions will be $n + m$

(4) If $n \neq m$ then number of solutions will be $n + m + 1$

Question: Consider the quadratic equation $ax^2 - (2a-1)x + a - 3 = 0$

21. If both roots are real and are of opposite sign, then complete set of values of a is

(1) (0,3)

(2) $(-\infty, 0) \cup (3, \infty)$

(3) $\left\{\frac{1}{2}\right\}$

(4) $\{1, 3\}$

22. If a is given as $\frac{n^2+n}{2}$, where n is natural number then both the roots are necessarily

(1) Integers (2) Rational number (3) Even integers (4) Odd integers

As per the data of previous question both the root will lie in interval

(1) $[-1, 2]$ (2) $[1, \infty]$ (3) $[1, 2]$ (4) $\left[\frac{1}{2}, 2\right]$

Question: $f(x) = x^3 + ax^2 + bx + c$ has 3 distinct positive integral roots α, β, γ such that $\alpha < \beta < \gamma$. Also $g(x) = x^2 - 2x + 79$ such that $f(g(x)) = 0$ has no real roots. Also $f(78) = 77$.

23. $\alpha + \beta + \gamma$ is

(1) A prime number

(2) A multiple of 5

(3) A perfect square

(4) Divisible by exactly 4 natural numbers

24. c is equal to

(1) -366289 (2) -364089 (3) Odd number (4) Multiple of 3

25. Choose the incorrect statement

(1) Out of α, β, γ exactly 2 are prime

(2) $(\alpha - 3)$ is a perfect square

(3) Number of divisors of γ is 2

(4) $\beta\gamma$ is a perfect square

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26. Let p, q, r are the roots of the equation $x^3 + ax^2 + bx + c = 0$ and α, β, γ are the roots of the equation $x^3 + 2x^2(3p + a) + 4x(3p^2 + 2ap + b) + 8(p^3 + ap^2 + bp + c) = 0$

(1) One out of α, β, γ is $2q - 2p$

(2) Sum of the roots α, β and γ is $2(q + r) - p$

(3) One of the root out of α, β, γ is $2p - q$

(4) Product of roots α, β, γ is 0

27. Let z be a complex number on the locus $\dfrac{z - i}{z + i} = e^{i\theta}(\theta \in R)$, such that $|z - 3 - 2i| + |z + 1 - 3i|$ is minimum, then which of the following statement(s) is(are) correct?

(1) $\arg(z) = 0$ (2) $\arg(z) = \pi$ (3) $|z| = \dfrac{7}{5}$ (4) $z = 7$