

Complex & Quadratic Assignment

1. If the quadratic expression $px^2 + |2p - 3|x - 6$ is positive for exactly two integral values of x then (where $[\cdot]$ is greatest integer function)

(1) $[p] = -1$ (2) $[p] = -2$ (3) $p = \left(-\frac{3}{4}, -\frac{3}{5}\right]$ (4) $p \in (-2, -1]$

2. If $lx^{17} + mx^{16} + 1$ is divisible by $x^2 - x - 1$ then

(1) l is divisible by 3 (2) l is divisible by 7
(3) l is divisible by 47 (4) l is divisible by 21

Question: Let $\alpha, \beta, \gamma, \delta$ be the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ if $(\alpha + \beta) = (\gamma + \delta)$ and $a, b, c, d \in R$, then

3. The correct options is/are

(1) If $a = 2$, then $b - c \neq 2$ (2) If $a = 2$, then $b - c = 1$
(3) If $a = 1$, then $b - 2c \neq 1$ (4) If $a = 1$, then $b - 2c = \frac{1}{4}$

4. If $b + c = 1$ and $a \neq -2$ then

(1) $b \leq \frac{3}{4}$ (2) $b \geq \frac{3}{4}$ (3) $c \leq \frac{1}{4}$ (4) $c \geq \frac{1}{4}$

Question: For any complex number z , $z = |z| [\cos(\arg(z)) + i \sin(\arg(z))]$. Choose the correct answer(s)

5. If z is any non-zero complex number, then

(1) $\left| \frac{z}{|z|} - 1 \right| \leq |\arg(z)|$ (2) $\left| \frac{z}{|z|} - 1 \right| > |\arg(z)|$
(3) $|z - 1| > ||z| - 1| + |z| |\arg(z)|$ (4) $|z - 1| \leq ||z| - 1| + |z| |\arg(z)|$

6. If z is any non-zero complex number such that $\arg\left(z^{\frac{3}{8}}\right) = \frac{1}{2} \arg\left(z^2 + \bar{z}z^{\frac{1}{2}}\right)$, then z

(1) Must be purely imaginary and non-unimodular complex number
(2) Could be unimodular complex number
(3) Could be purely real complex number
(4) Must be purely imaginary complex number

Question: Let $e^{\ln[1+\{xyz\}]}$, $\log_y x$, $\log_z y$ and $\log_x z^{-15}$ be the first four terms of an A.P. with common difference d , where all terms of the A.P. are real and defined (where $[\cdot]$ and $\{\cdot\}$ represents greatest integer function and fractional part function) then answer the following questions

7. Which of the following interval(s) contain d ?
- (1) $[0, \infty)$ (2) $(-\infty, 0]$ (3) $[-10, 20]$ (4) $[10, 20]$
8. The value of $\sum_{k=1}^{\infty} \frac{k}{\left(\frac{x}{z^3} + xy + yz^3\right)^k}$ is less than or equal to
- (1) 1 (2) 2 (3) $\frac{1}{2}$ (4) $\frac{2}{3}$
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Question: α, β, γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ then $\alpha + \beta + \gamma = -\frac{b}{a}, \alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}, \alpha\beta\gamma = -\frac{d}{a}$.

Let the number p and q are positive and the roots of the equation $x^3 - 4px + 3q = 0$ are real. Let α is a root of this given cubic equation of minimum absolute value then

9. The roots of the given cubic equation must be
- (1) Two positive and one negative (2) All positive
(3) All negative (4) Two negative and one positive
10. The range of α is
- (1) $-\frac{3q}{4p} < \alpha < \frac{q}{2p}$ (2) $-\frac{9q}{8p} < \alpha < \frac{3q}{4p}$
(3) $\frac{3q}{4p} < \alpha < \frac{9q}{8p}$ (4) $\frac{q}{p} < \alpha < \frac{9q}{8p}$
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Question: If $x_1, x_2, x_3 \dots x_n$ are all positive and $m \in R$, then

$$\frac{x_1^m + x_2^m + x_3^m + \dots + x_n^m}{n} \geq \left(\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \right)^m \text{ if } m \in R - (0, 1)$$

and

$$\frac{x_1^m + x_2^m + x_3^m + \dots + x_n^m}{n} \leq \left(\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \right)^m \text{ if } 0 < m < 1$$

also, $\frac{x_1 + x_2 + x_3 + \dots + x_n}{n} \geq (x_1 \cdot x_2 \cdot x_3 \dots x_n)^{\frac{1}{n}} \geq \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \dots + \frac{1}{x_n}}$. Then answer the following questions.

11. If $x_1 > 0, x_2 > 0, x_3 > 0$ and $x_1 + x_2 + x_3 = 1$, then the minimum value of

$$\frac{x_1}{3 - x_1} + \frac{x_2}{3 - x_2} + \frac{x_3}{3 - x_3} \text{ is}$$

- (1) $\frac{3}{8}$ (2) $\frac{5}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{4}$

12. If $x_1 + x_2 + x_3 = 3$ and $x_1 > 0, x_2 > 0, x_3 > 0$ then the minimum value of

$$\left(\frac{3}{x_1} - 1\right) \left(\frac{3}{x_2} - 1\right) \left(\frac{3}{x_3} - 1\right) \text{ is}$$

- (1) 3 (2) 4 (3) 7 (4) 8
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Question: Let the cubic equation $ax^3 + bx^2 + cx + d = 0$ is shown by $P(a, b, c, d, x) = 0$. Now answer the following questions.

13. If the equation $P(1, 0, 1, 7, x) = 0$ has roots x_1, x_2, x_3 , then the equation whose roots are $(x_1 - x_2)^2, (x_2 - x_3)^2, (x_3 - x_1)^2$ is

- (1) $P(1, 6, 9, 1327, x) = 0$ (2) $P(1, 1, 9, 127, x) = 0$
 (3) $P(1, 6, 9, 127, x) = 0$ (4) $P(6, 9, 1, 127, x) = 0$

14. If the roots of the equation $P(6, -11, 6, -1, x) = 0$ are in HP and equation $P(1, -6, 11, -6, x) = 0$ have roots α, β, γ , then $\alpha + \beta^2 + \gamma^3$ can be

- (1) 27 (2) 9 (3) 32 (4) 12
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Question: If Locus of point $P(z)$ in complex plane is $|z + z_1| + |z + z_2| = 4$ where A represents z_1 , as $(1, 0)$ and B represents z_2 as $(-1, 0)$ and $Q(w)$ is variable point inside the locus of P such that all internal angle bisectors of triangle PAB concurrent at Q and if $|w - w_1| + |w - w_2| = 2$, then

15. The value of $|w_1| + |w_2|$ is equal to

- (1) $\frac{2}{\sqrt{3}}$ (2) $\sqrt{\frac{2}{3}}$ (3) $2\sqrt{\frac{2}{3}}$ (4) $\frac{2\sqrt{2}}{3}$

16. If minimum value of $|w - z_1| + |w - z_2|$ is equal to m , then $[m]$ (where $[x]$ denotes greatest integer function) is

- (1) 1 (2) 2 (3) 3 (4) 4
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Question: Let $f(x) = x^3 + x + 1$ suppose g is a cubic polynomial such that $g(0) = -1$ and the roots of $g(x) = 0$ are square of the roots of $f(x) = 0$. Then

17. The equation $f(x) = 0$ has

- (1) At least one positive root (2) At least two negative roots
 (3) Exactly one negative root (4) Exactly two positive roots

18. The polynomial $g(x^2)$ is identical with

- (1) $f(x^2)$ (2) $(f(x))^2$ (3) $2f(x)f(-x)$ (4) $-f(x)f(-x)$
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19. If the equation $|z|(z+1)^8 = z^8|z+1|$ where $z \in \mathbb{C}$ and $z(z+1) \neq 0$ has distinct roots z_1, z_2, \dots, z_n (where $n \in \mathbb{N}$) then which of the following is/are true?
- (1) z_1, z_2, \dots, z_n are concyclic point (2) z_1, z_2, \dots, z_n are collinear point
- (3) $\sum_{r=1}^n \operatorname{Re}(z_r) = -\frac{7}{2}$ (4) $\sum_{r=1}^n \operatorname{Im}(z_r) = 0$
20. Let z be a complex number satisfying equation $z^n = (\bar{z})^m$ where $n, m \in \mathbb{N}$ then
- (1) If $n = m$ then number of solutions will be finite
 (2) If $n = m$ then number of solutions will be infinite
 (3) If $n \neq m$ then number of solutions will be $n + m$
 (4) If $n \neq m$ then number of solutions will be $n + m + 1$
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Question: Consider the quadratic equation $ax^2 - (2a - 1)x + a - 3 = 0$

21. If both roots are real and are of opposite sign, then complete set of values of a is
- (1) $(0, 3)$ (2) $(-\infty, 0) \cup (3, \infty)$
 (3) $\left\{\frac{1}{2}\right\}$ (4) $\{1, 3\}$
22. If a is given as $\frac{n^2 + n}{2}$, where n is natural number then both the roots are necessarily
- (1) Integers (2) Rational number (3) Even integers (4) Odd integers

As per the data of previous question both the root will lie in interval

- (1) $[-1, 2]$ (2) $[1, \infty]$ (3) $[1, 2]$ (4) $\left[\frac{1}{2}, 2\right]$
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Question: $f(x) = x^3 + ax^2 + bx + c$ has 3 distinct positive integral roots α, β, γ such that $\alpha < \beta < \gamma$. Also $g(x) = x^2 - 2x + 79$ such that $f(g(x)) = 0$ has no real roots. Also $f(78) = 77$.

23. $\alpha + \beta + \gamma$ is
- (1) A prime number (2) A multiple of 5
 (3) A perfect square (4) Divisible by exactly 4 natural numbers
24. c is equal to
- (1) -366289 (2) -364089 (3) Odd number (4) Multiple of 3
25. Choose the incorrect statement
- (1) Out of α, β, γ exactly 2 are prime (2) $(\alpha - 3)$ is a perfect square
 (3) Number of divisors of γ is 2 (4) $\beta\gamma$ is a perfect square
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26. Let p, q, r are the roots of the equation $x^3 + ax^2 + bx + c = 0$ and α, β, γ are the roots of the equation $x^3 + 2x^2(3p + a) + 4x(3p^2 + 2ap + b) + 8(p^3 + ap^2 + bp + c) = 0$
- (1) One out of α, β, γ is $2q - 2p$
 - (2) Sum of the roots α, β and γ is $2(q + r) - p$
 - (3) One of the root out of α, β, γ is $2p - q$
 - (4) Product of roots α, β, γ is 0
27. Let z be a complex number on the locus $\frac{z-i}{z+i} = e^{i\theta} (\theta \in R)$, such that $|z - 3 - 2i| + |z + 1 - 3i|$ is minimum, then which of the following statement(s) is(are) correct?
- (1) $\arg(z) = 0$
 - (2) $\arg(z) = \pi$
 - (3) $|z| = \frac{7}{5}$
 - (4) $z = 7$