AIATS Functions Assignment

1. Range of the function $f(x) = \log_2(2 - \log_{\sqrt{2}}(16\sin^2 x + 1))$ is:

   (1) $[0, 1]$ (2) $(-\infty, 1]$ (3) $[-1, 1]$ (4) $(-\infty, \infty)$

2. Let A be the greatest value of the function $f(x) = \log_x[x]$, (where $[\cdot]$ denotes greatest integer function) and B be the least value of the function $g(x) = |\sin x| + |\cos x|$, then:

   (1) $A > B$ (2) $A < B$ (3) $A = B$ (4) $2A + B = 4$

3. Solution of the inequation $\{x\}(\{x\} - 1)(\{x\} + 2) \geq 0$
   (where $\{\cdot\}$ denotes fractional part function) is:

   (1) $x \in (-2, 1)$ (2) $x \in I$ (3) $x \in [0, 1)$ (4) $x \in [-2, 0)$

4. The range of function $f(x) = [1+\sin x] + \left[2 + \sin\frac{x}{2}\right] + \left[3 + \sin\frac{x}{3}\right] + \cdots + \left[n + \sin\frac{x}{n}\right]$ $\forall\, x \in [0, \pi], n \in N$ ($[\cdot]$ denotes greatest integer function) is:

   (1) $\left\{\frac{n^2+n-2}{2}, \frac{n(n+1)}{2}\right\}$ (2) $\left\{\frac{n(n+1)}{2}\right\}$

   (3) $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}, \frac{n^2+n+4}{2}\right\}$ (4) $\left\{\frac{n(n+1)}{2}, \frac{n^2+n+2}{2}\right\}$

5. If $f(x)$ and $g(x)$ are two functions such that $f(x) = [x] + [-x]$ and $g(x) = \{x\} \forall\, x \in R$ and $h(x) = f(g(x))$; then which of the following is incorrect?
   ($[\cdot]$ denotes greatest integer function and $\{\cdot\}$ denotes fractional part function)

   (1) $f(x)$ and $h(x)$ are identical functions
   (2) $f(x) = g(x)$ has no solution
   (3) $f(x) + h(x) > 0$ has no solution
   (4) $f(x) - h(x)$ is a periodic function

6. Number of elements in the range set of $f(x) = \left[\frac{x}{15}\right]\left[-\frac{15}{x}\right] \forall\, x \in (0, 90)$; (where $[\cdot]$ denotes greatest integer function):

   (1) 5 (2) 6 (3) 7 (4) Infinite

7. If $|f(x) + 6 - x^2| = |f(x)| + |4 - x^2| + 2$, then $f(x)$ is necessarily non-negative for:

   (1) $x \in [-2, 2]$ (2) $x \in (-\infty, -2) \cup (2, \infty)$

   (3) $x \in \left[-\sqrt{6}, \sqrt{6}\right]$ (4) $x \in [-5, -2] \cup [2, 5]$

8. The number of solutions of the equation $[y + [y]] = 2\cos x$ is:
   (where $y = \frac{1}{3}[\sin x + [\sin x + [\sin x]]]$ and $[\cdot]$ = greatest integer function)

   (1) 0 (2) 1 (3) 2 (4) Infinite

9. $f(x) = \{x\} + \{x+1\} + \{x+2\} + \cdots + \{x+99\}$, then $\left[f(\sqrt{2})\right]$, (where $\{\cdot\}$ denotes fractional part function and $[\cdot]$ denotes the greatest integer function) is equal to:

(1) 5050 (2) 4950 (3) 41 (4) 14

10. Let $f(x)$ be a polynomial of degree 5 with leading coefficient unity such that $f(1) = 5$, $f(2) = 4$, $f(3) = 3$, $f(4) = 2$, $f(5) = 1$. Then $f(6)$ is equal to:

(1) 0 (2) 24 (3) 120 (4) 720

11. Let $f : A \to B$ be a function such that $f(x) = \sqrt{x-2} + \sqrt{4-x}$ is invertible, then which of the following is not possible?

(1) $A = [3, 4]$ (2) $A = [2, 3]$ (3) $A = [2, 2\sqrt{3}]$ (4) $A = [2, 2\sqrt{2}]$

12. The number of positive integral values of x satisfying $\left[\frac{x}{9}\right] = \left[\frac{x}{11}\right]$ is:

(1) 21 (2) 22 (3) 23 (4) 24

13. The solution set of the equation $[x]^2 + [x+1] - 3 = 0$, where $[\cdot]$ represents greatest integer function is:

(1) $[-1, 0) \cup [1, 2)$ (2) $[-2, -1) \cup [1, 2)$ (3) $[1, 2)$ (4) $[-3, -2) \cup [2, 3)$

14. If complete solution set of $e^{-x} \le 4 - x$ is $[\alpha, \beta]$, then $[\alpha] + [\beta]$ is equal to: (where $[\cdot]$ denotes greatest integer function)

(1) 0 (2) 2 (3) 1 (4) 4

15. Range of $f(x) = \sqrt{\sin(\log_7(\cos(\sin x)))}$ is:

(1) $[0, 1)$ (2) $\{0, 1\}$ (3) $\{0\}$ (4) $[1, 7]$

If domain of $y = f(x)$ is $x \in [-3, 2]$, then domain of $y = f(|[x]|)$: (where $[\cdot]$ denotes greatest integer function)

(1) $[-3, 2]$ (2) $[-2, 3)$ (3) $[-3, 3]$ (4) $[-2, 3]$

16. Let $f : R - \left\{\frac{3}{2}\right\} \to R$, $f(x) = \frac{3x+5}{2x-3}$. Let $f_1(x) = f(x), f_n(x) = f(f_{n-1}(x))$ for $n \ge 2$, $n \in N$, then $f_{2008}(x) + f_{2009}(x) =$

(1) $\frac{2x^2+5}{2x-3}$ (2) $\frac{x^2+5}{2x-3}$ (3) $\frac{2x^2-5}{2x-3}$ (4) $\frac{x^2-5}{2x-3}$

17. Range of the function, $f(x) = \frac{(1+x+x^2)(1+x^4)}{x^3}$, for $x > 0$ is:

(1) $[0, \infty)$ (2) $[2, \infty)$ (3) $[4, \infty)$ (4) $[6, \infty)$

18. If $f(x) = \sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$; $x \in R$, then range of $f(x)$ is given by:

(1) $[-1, 1]$ (2) $[0, 1]$ (3) $(-1, 1)$ (4) None of these

19. Consider all functions $f : \{1, 2, 3, 4\} \to \{1, 2, 3, 4\}$ which are one-one, onto and satisfy the following property:
If $f(k)$ is odd then $f(k+1)$ is even, $k = 1, 2, 3$.
The number of such functions is:

(1) 4 (2) 8 (3) 12 (4) 16

20. Let $f : R \to R$ and $f(x) = \dfrac{x(x^4+1)(x+1)+x^4+2}{x^2+x+1}$, then $f(x)$ is:

(1) One-one, into (2) Many-one, onto (3) One-one, onto (4) Many-one, into

21. Let $f(x)$ be defined as:

$$f(x) = \begin{cases} |x| & 0 \leq x < 1 \\ |x-1| + |x-2| & 1 \leq x < 2 \\ |x-3| & 2 \leq x < 3 \end{cases}$$

The range of function $g(x) = \sin(7(f(x)))$ is:

(1) $[0, 1]$ (2) $[-1, 0]$ (3) $\left[-\frac{1}{2}, \frac{1}{2}\right]$ (4) $[-1, 1]$

22. If $[x]^2 - 7[x] + 10 < 0$ and $4[y]^2 - 16[y] + 7 < 0$, then $[x+y]$ cannot be ($[\cdot]$ denotes greatest integer function):

(1) 7 (2) 8 (3) 9 (4) both (b) and (c)

23. The function $f(x)$ satisfy the equation $f(1-x) + 2f(x) = 3x \; \forall \, x \in R$, then $f(0) =$

(1) -2 (2) -1 (3) 0 (4) 1

24. The number of integral values of x in the domain of function f defined as $f(x) = \sqrt{\ln|\ln|x||} + \sqrt{7|x| - |x|^2 - 10}$ is:

(1) 5 (2) 6 (3) 7 (4) 8