Functions Assignment

1. If $f(x) = \pi\left(\frac{\sqrt{x+7}-4}{x-9}\right)$, then range of function $y = \sin(2f(x))$ is

   (1) $[0, 1]$ (2) $\left(0, \frac{1}{\sqrt{2}}\right]$

   (3) $\left(0, \frac{1}{\sqrt{2}}\right) \cup \left(\frac{1}{\sqrt{2}}, 1\right]$ (4) $(0, 1]$

2. If the range of function $f(x) = \frac{x^2+x+c}{x^2+2x+c}$, $x \in R$ is $\left[\frac{5}{6}, \frac{3}{2}\right]$, then c is equal to

   (1) -4 (2) 3 (3) 4 (4) 5

3. If a polynomial function f satisfies the relation

$$\log_2[f(x)] = \log_2\left(2 + \frac{2}{3} + \frac{2}{9} + ... + \infty\right) \cdot \log_3\left(1 + \frac{f(x)}{f\left(\frac{1}{x}\right)}\right)$$

   and $f(10) = 1001$, then value of $f(20)$ is

   (1) 2002 (2) 7999 (3) 8001 (4) 16001

4. Consider, $P = \frac{x^2-2x}{x^2+x+1}$, $Q = \frac{y-1}{y^2+y+1}$, $R = \frac{2}{z^2+z+1}$ where $x, y, z \in R$. If

   $k = [P+Q+R] - ([P]+[Q]+[R])$ then the possible value(s) of k is(are) (where $[\cdot]$ denotes greatest integer less than equal to x)

   (1) 0 (2) 1 (3) 2 (4) 3

5. Let f be a function defined in $[-2, 3]$ given as

$$f(x) = \begin{cases} 3(x+1)^{1/3}, & -2 \le x < 0 \\ -(x-1)^2, & 0 \le x < 1 \\ 2(x-1)^2 & 1 \le x < 2 \\ -x^2+4x-3, & 2 \le x \le 3 \end{cases}$$

	Column I		**Column II**
(A)	The number of integers in the range of $f(x)$ is	(p)	2
(B)	The number of integral values of x which are in the domain of $f(1-\lvert x\rvert)$, is	(q)	4
(C)	The number of integers in the range of $\lvert f(-\lvert x\rvert)\rvert$, is	(r)	6
(D)	The number of integral values of k for which the equation $f(\lvert x\rvert) = k$ has exactly four distinct solutions is	(s)	7

6. Let $f(x) = \left|x^2 - 9\right| - |x - a|$. Find the number of integers in the range of a so that $f(x) = 0$ has 4 distinct real roots.

7. The set of real values of x satisfying the equality $\left[\frac{3}{x}\right] + \left[\frac{4}{x}\right] = 5$ (where $[\cdot]$ denotes the greatest integer function) belongs to the inteval $\left(a, \frac{b}{c}\right]$ where $a, b, c \in N$ and $\frac{b}{c}$ is in its lowest form. Find the value of $a + b + c + abc$.

8. Let R be the set of real numbers and $f : R \to R$, be a differentiable function such that $|f(x) - f(y)| \le |x - y|^3 \ \forall \ x, y \in R$. If $f(10) = 100$, the value of $f(20)$ is equal to

   (1) 0 (2) 20 (3) 100 (4) 10

9. If the equation

$$\left| |x - 1| - 6 \lim_{t \to \infty} \left(\frac{\sqrt{2t^2 - t - 1} - \sqrt{t^2 - t + 1}}{t \left(\tan \frac{\pi}{8}\right)} \right) \right| = k$$

   has four distinct solutions then find the number of integral values of k.

10. Let A and B be two sets containing 2 and 3 elements respectively. Then, total number of subsets of $A \times B$ having 3 or more elements is

    (1) 42 (2) 56 (3) 54 (4) 52

11. Let the sets be $A = \{x : x \in Z^+ \text{ and } x \le 9\}, B = \{x : x \in Z \text{ and } -3 < x < 8\}$ and $C = \{x : x \text{ is a prime number}\}$, then the number of elements belonging to exactly two of the three sets A, B and C is

    (1) 3 (2) 4 (3) 6 (4) 8

12. Let f be a function from non-negative integers to non-negative integers such that $f(xy) = xf(y) + yf(x)$. It is given that $f(10) = 19$, $f(12) = 52$ and $f(15) = 26$, then $f(8)$ is equal to

    (1) 26 (2) 36 (3) 38 (4) 40

13. If a function $f(x)$ satisfies the relation $3f(x) - 5f\left(\frac{2}{x}\right) = 3 - x + x^2 \ \forall \ x \in R - \{0\}$, then the value of $f(1)$ is equal to

    (1) -2 (2) $\frac{-33}{16}$ (3) $\frac{-17}{8}$ (4) $\frac{-35}{16}$

14. The function $f(x) = \sqrt{\log_2 \log_3 \log_4 x} + \sqrt{\{x^2 + x + 1\}}$ (where $\{\}$ represents fractional function, is well defined). Then x may belong to

    (1) $[0, 32]$ (2) $[64, \infty)$ (3) $[100, \infty)$ (4) $[0, 64)$

15. Let g be a function satisfying $g(x - 2) + g(x + 2) = \sqrt{3}g(x), \ \forall \ x \in R$. Then g also satisfies the relation(s) given as

    (1) $g(x - 4) + g(x + 8) = 0$ (2) $g(x) - g(x + 12) = 0$

    (3) $g(x - 2) + g(x + 4) = 0$ (4) $g(x) - g(x + 24) = 0$

16. Domain of the function $f(x) = \log_{[x-1]}(\sin^2 \pi x)$ is $x \in [3, \infty) - \{p, p+1, p+2, \ldots, n\}$, where $[\cdot]$ denotes greatest integer function and n is a multiple of 4 and 25, then $(p) + (p+1) + (p+2) + \cdots + n$ can be

(1) 5047 (2) 20097 (3) 5050 (4) 45147

17. Let f be a real valued function which satisfies $f\left(\frac{x-3}{x+1}\right) + f\left(\frac{3+x}{1-x}\right) = x$, $|x| \neq 1$, $x \in R$. Then $f(3)$ is equal to

(1) -3 (2) 3 (3) $-\frac{11}{3}$ (4) $\frac{11}{3}$

18. Let $f(x^2 - 11x + 10) + f(x^2 - 21x + 20) = x^4 - 3x^2 + 21x + 2$, then the value of $f(0)$ is

(1) 20 (2) $\frac{21}{2}$ (3) 24 (4) 0

19. The range of the function $f(x) = \dfrac{\sin(\pi [x^2 + x + 1])}{\cos(\pi [x^2 + 3x + 2])}$ is ($[\cdot]$ represents greatest integer function)

(1) $\{0\}$ (2) $\{1, -1\}$ (3) $[-1, 1]$ (4) $(-\infty, \infty)$

20. If a function satisfies

$$(x-y)f(x+y) - (x+y)f(x-y) = 2(x^2y - y^3) \ \forall \ x, y \in R \ and \ f(1) = 5$$

then

(1) $f(x)$ is not differentiable (2) $f(x) = x^2 + 4x$

(3) $f(0) = 1$ (4) $f'(1) = 8$

21. Total number of solutions of $\sin\{x\} = \cos\{x\}$ (where $\{\cdot\}$ denotes the fractional part) in $[0, 2\pi]$ is

22. Set of exhaustive values of x satisfying

$$\left||x|^2 - x + 1\right| > \left|x^2 - 1\right|$$

(1) $(-\infty, 0) \cup \left(\frac{1}{2}, \infty\right)$ (2) $(0, 2)$

(3) $\left(\frac{1}{2}, 2\right)$ (4) $(-\infty, 0) \cup \left(\frac{1}{2}, 2\right)$

23. Consider an equation $Sgn([x]) + \lambda = x$. (where $[\cdot]$ represents greatest integer function). if n_λ denotes the number of solutions of the equation, then the value of $\frac{n_{\frac{1}{2}}}{n_1}$ is

(1) 1 (2) $\frac{3}{2}$ (3) 3 (4) $\frac{1}{2}$

24. The solution of the equation $2[x] + [3x] = 6$ (where $[\cdot]$ denotes the greatest integer function) is

(1) $\left[\frac{4}{3}, \frac{5}{3}\right)$ (2) $\left[\frac{5}{3}, 2\right)$ (3) $\left[\frac{2}{3}, \frac{5}{3}\right)$ (4) $\left[1, \frac{5}{3}\right)$

25. $A = \{x \in N : \text{HCF}(x, 12) = 1,\ x < 12\}$, $B = \{x \in N : \text{LCM}(x, 12) = 12\}$, then the number of relations from A to C, where $A \Delta C = B$, is

(1) 2^{28} (2) 2^{32} (3) 2^{24} (4) 2^{36}

26. The number of subsets of a set with 2018 elements having an odd number of elements is

(1) 2^{2015} (2) 2^{2016} (3) 2^{2017} (4) Data insufficient

27. The range of the function $f(x) = \dfrac{(x+3)^2}{x^2+1}$ is

(1) $[0, 12]$ (2) $[0, 11]$ (3) $[0, 10]$ (4) $[0, 15]$

28. The maximum value of $f(x) = |15 - 8x + |x|^2|$ in the inteval $(3.5, 4.5)$ is

(1) 0 (2) 1 (3) 2 (4) 3

29. If X and Y are two sets such that

$$n(X \cap \overline{Y}) = 12,\ \ n(\overline{X} \cap Y) = 15 \ \textit{and}\ n(X \cup Y) = 30$$

then $n(X \times Y) =$

(1) 210 (2) 270 (3) 180 (4) 300

30. Let $3^{f_1(x)} + 3^x = 9$ and $f_2(x) = \log_{\frac{1}{2}}(a + 2x - x^2)$. If maximum integral value of $f_1(x)$ is equal to the minimum value of $f_2(x)$, then a is equal to

(1) -1 (2) $-\dfrac{1}{2}$ (3) Zero (4) 1

31. Find the range of the function

$$f(x) = \log_{\frac{1}{2}}(2\sin^2 x - 2\sin x + 1)$$

(1) $\left[\log_{\frac{1}{2}} 5, 0\right]$ (2) $\left[\log_{\frac{1}{2}} 5, 1\right]$ (3) $\left(-\infty, \log_{\frac{1}{2}} 5\right]$ (4) $\left[\log_{\frac{1}{2}} 5, \infty\right)$