Complex & Quadratic Assignment

1. If the quadratic expression $px^2 + |2p-3|x - 6$ is positive for exactly two integral values of x then (where $[\cdot]$ is greatest integer function)

   (1) $[p] = -1$ (2) $[p] = -2$ (3) $p = \left(-\frac{3}{4}, -\frac{3}{5}\right]$ (4) $p \in (-2, -1]$

2. If $lx^{17} + mx^{16} + 1$ is divisible by $x^2 - x - 1$ then

   (1) l is divisible by 3 (2) l is divisible by 7

   (3) l is divisible by 47 (4) l is divisible by 21

   Let α, β, γ, δ be the roots of $x^4 + ax^3 + bx^2 + cx + d = 0$ if $(\alpha + \beta) = (\gamma + \delta)$ and a, b, c, $d \in R$, then answer Q.3 & 4

3. The correct options is/are

   (1) If $a = 2$, then $b - c \neq 2$ (2) If $a = 2$, then $b - c = 2$

   (3) If $a = 1$, then $b - 2c \neq 1$ (4) If $a = 1$, then $b - 2c = \frac{1}{4}$

4. If $b + c = 1$ and $a \neq -2$ then

   (1) $b \leq \frac{3}{4}$ (2) $b \geq \frac{3}{4}$ (3) $c \leq \frac{1}{4}$ (4) $c \geq \frac{1}{4}$

Question: For any complex number z, $z = |z|\left[\cos(\arg(z)) + i\sin(\arg(z))\right]$. Choose the correct answer(s)

5. If z is any non-zero complex number, then

   (1) $\left|\frac{z}{|z|} - 1\right| \leq |\arg(z)|$ (2) $\left|\frac{z}{|z|} - 1\right| > |\arg(z)|$

   (3) $|z-1| > ||z|-1| + |z||\arg(z)|$ (4) $|z-1| \leq ||z|-1| + |z||\arg(z)|$

6. If z is any non-zero complex number such that $\arg\left(z^{\frac{3}{8}}\right) = \frac{1}{2}\arg\left(z^2 + \overline{z}z^{\frac{1}{2}}\right)$, then z

   (1) Must be purely imaginary and non-unimodular complex number
   (2) Could be unimodular complex number
   (3) Could be purely read complex number
   (4) Must be purely imaginary complex number

Question: Let $e^{\ln[1+\{xyz\}]}$, $\log_y x$, $\log_z y$ and $\log_x z^{-15}$ be the first four terms of an A.P. with common difference d, where all terms of the A.P. are read and defined (where $[\cdot]$ and $\{\cdot\}$ represents greatest integer function and fractional part function) then answer the following questions

7. Which of the following interval(s) contain d?

(1) $[0, \infty)$ (2) $(-\infty, 0]$ (3) $[-10, 20]$ (4) $[10, 20]$

8. The value of $\sum_{k=1}^{\infty} \frac{k}{\left(\frac{x}{z^3} + xy + yz^3\right)^k}$ is less than or equal to

(1) 1 (2) 2 (3) $\frac{1}{2}$ (4) $\frac{2}{3}$

Question: α, β, γ are the roots of a cubic equation $ax^3 + bx^2 + cx + d = 0$ then $\alpha + \beta + \gamma = -\frac{b}{a}$, $\alpha\beta + \beta\gamma + \gamma\alpha = \frac{c}{a}$, $\alpha\beta\gamma = -\frac{d}{a}$.
Let the number p and q are positive and the roots of the equation $x^3 - 4px + 3q = 0$ are real. Let α is a root of this given cubic equation of minimum absolute value then

9. The roots of the given cubic equation must be

(1) Two positive and one negative (2) All positive

(3) All negative (4) Two negative and one positive

10. The range of α is

(1) $-\frac{3q}{4p} < \alpha < \frac{q}{2p}$ (2) $-\frac{9q}{8p} < \alpha < \frac{3q}{4p}$

(3) $\frac{3q}{4p} < \alpha < \frac{9q}{8p}$ (4) $\frac{q}{p} < \alpha < \frac{9q}{8p}$

Question: If x_1, x_2, $x_3 \ldots x_n$ are all positive and $m \in R$, then

$$\frac{x_1^m + x_2^m + x_3^m + \cdots + x_n^m}{n} \geq \left(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}\right)^m.$$

If $m \in R - (0, 1)$ and

$$\frac{x_1^m + x_2^m + x_3^m + \cdots + x_n^m}{n} \leq \left(\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n}\right)^m \text{ if } 0 < m < 1$$

also, $\frac{x_1 + x_2 + x_3 + \cdots + x_n}{n} \geq (x_1 \cdot x_2 \cdot x_3 \cdots x_n)^{\frac{1}{n}} \geq \frac{n}{\frac{1}{x_1} + \frac{1}{x_2} + \frac{1}{x_3} + \cdots + \frac{1}{x_n}}$. Then answer the following questions.

11. If $x_1 > 0$, $x_2 > 0$, $x_3 > 0$ and $x_1 + x_2 + x_3 = 1$. then the minimum value of

$$\frac{x_1}{3 - x_1} + \frac{x_2}{3 - x_2} + \frac{x_3}{3 - x_3} \text{ is}$$

(1) $\frac{3}{8}$ (2) $\frac{5}{8}$ (3) $\frac{7}{8}$ (4) $\frac{3}{4}$