Functions II Assignment

1. Let $f\colon R \to R$ be a function defined by $f(x) = \dfrac{x^2+2x+5}{x^2+x+1}$ is

   (1) one-one and into (2) one-one and onto

   (3) many-one and onto (4) many-one and into

2. $f\colon R \to R$ is a function defined by $f(x) = \dfrac{e^{|x|}-e^{-x}}{e^x-e^{-x}}$ then f is

   (1) a bijection (2) an injection only

   (3) surjection only (4) neither injection nor surjection

3. If $f\colon R \to \left[\dfrac{\pi}{6}, \dfrac{\pi}{2}\right)$ defined by $f(x) = \sin^{-1}\left(\dfrac{x^2-a}{x^2+1}\right)$ is an onto function, then the set of values of a is

   (1) $(-\infty, -1)$ (2) $(-1, \infty)$ (3) $(-\infty, 0)$ (4) $(-\infty, \infty)$

4. Let $f\colon R \to R$ be any function. Define $g\colon R \to R$ by $g(x) = |f(x)|$ for all x then g is

   (1) onto if f is onto (2) one-one if f is one-one

   (3) continuous is f is continuous (4) neither one-one nor onto

5. We have $f(x) = \begin{cases} 3, & x \le 0 \\ 3^{-x} - 3^x + 3, & x > 0 \end{cases}$ (as $\text{sgn}(e^{-x} = 1 \forall x \in R)$.

   (1) one-one and into (2) one-one and onto

   (3) many-one and onto (4) Neither one-one nor onto

6. Let $A = \{1, 2, 3, 4, 5\}$. If f is a bijective function from A to A, the the number such functions for which $f(k) \ne k,\ k = 1, 2, 3, 4, 5$ is

   (1) 5^5 (2) 120 (3) 44 (4) $5^5 - 120$

7. The set of values of a for which the function $f\colon R \to R$ given by $f(x) = x^3 + (a+2)x^2 + 3ax + 5$ is one-one is

   (1) $[-2, 4]$ (2) $(1, 3)$ (3) $(1, 4)$ (4) $(1, 5)$

8. S-I: If $f(x) = 2x^3 + 7x - 5$ then the value of $f^{-1}(4)$ is 1.
   S-II: A function $y = f(x)$ is invertible if f is one-one and onto.

   (1) Both S-I and S-II are individually true and R is the correct explanation of A

   (2) Both S-I and S-II are individually true but R is not the correct explanation of A

   (3) S-1 is true but S-II is false

   (4) S-I is false but S-II is true

9. The sum of solution(s) of the equation $2[x] + x = 6\{x\}$ where $[x]$ denotes greatest integer less than or equal to x and $\{x\}$ denotes fractional part of x is

(1) 0 (2) $\frac{5}{3}$ (3) $\frac{3}{5}$ (4) $\frac{8}{5}$

10. If $[x]$ is the greatest integer function, then $\sum_{k=1}^{4020}\left[\frac{1}{2}+\frac{k-1}{4020}\right]$ is equal to

(1) 2010 (2) 2009 (3) 2011 (4) 2005

11. $f(x) = \sin[x] + [\sin x], 0 < x < \frac{\pi}{2}$, where $[\cdot]$ represents the greatest integer function, can also be represented as

(1) $\begin{cases} 0, & 0 < x < 1 \\ 1+\sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$ (2) $\begin{cases} \frac{1}{\sqrt{2}}, & 0 < x < \frac{\pi}{4} \\ 1+\frac{1}{2}+\frac{1}{\sqrt{2}}+\frac{\sqrt{3}}{2}, & \frac{\pi}{4} \le x < \frac{\pi}{2} \end{cases}$

(3) $\begin{cases} 0, & 0 < x < 1 \\ \sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$ (4) $\begin{cases} 0, & 0 < x < \frac{\pi}{4} \\ 1, & \frac{\pi}{4} < x < 1 \\ \sin 1, & 1 \le x < \frac{\pi}{2} \end{cases}$

12. If $[x]$ and $\{x\}$ represent the integral and fractional parts of x respectively, then the value of $\sum_{r=1}^{2000}\frac{\{x+r\}}{2000}$ is

(1) x (2) $[x]$ (3) $\{x\}$ (4) $x+2001$

13. Total number of solution of $2^x + 3^x + 4^x - 5^x = 0$ is

(1) 0 (2) 1 (3) 2 (4) infinitely many

14. The of the real-valued function satisfying $f(x) + f(x+4) = f(x+2) + f(x+6)$ is

(1) 10 (2) 8 (3) 12 (4) 6

15. The period of

$$\frac{|\sin(4x)| + |\cos(4x)|}{|\sin(4x) - \cos(4x)| + |\sin(4x) + \cos(4x)|}$$

is

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{2}$ (3) $\frac{\pi}{8}$ (4) π

16. If $f(x) = \cos x + \{x\}$ where $\{\cdot\}$ is fraction part function then the period of $f(x)$ is

(1) 2π (2) 1 (3) $\frac{\pi}{2}$ (4) Does not exist

17. The period of

$$f(x) = \sin x + \tan\left(\frac{x}{2}\right) + \sin\left(\frac{x}{2}\right) + \tan\left(\frac{x}{2^2}\right) + \cdots + \sin\left(\frac{x}{2^{n-1}}\right) + \tan\left(\frac{x}{2^n}\right)$$

(1) 2π (2) $2^n\pi$ (3) $2^n\frac{\pi}{3}$ (4) $3^n\pi$

18. Period of $f(x) = \sin((\cos x) + x)$ is

(1) Does not exist (2) π (3) $\frac{\pi}{2}$ (4) 2π

19. The period of the function $\left|\sin^3\left(\frac{x}{2}\right)\right| + \left|\cos^3\left(\frac{x}{5}\right)\right|$ is

(1) 2π (2) 10π (3) 8π (4) 5π

20. Let $f\colon R \to R - \{3\}$ be a function such that for some $p > 0$,
$f(x+p) = \frac{f(x) - 5}{f(x) - 3}$ $\forall x \in R$. Then, period of f is

(1) $2p$ (2) $3p$ (3) $4p$ (4) $5p$

21. Let $f(x)$ be a real valued function with domain R such that

$$f(x+p) = 1 + \left[2 - 3f(x) + 3(f(x))^2 - (f(x))^3\right]^{\frac{1}{3}}$$

holds good $\forall x \in R$ and for some +ve constant p then the period of $f(x)$ is

(1) $\frac{p}{2}$ (2) p (3) $2p$ (4) $\frac{p}{3}$

22. If $f(a-x) = f(a+x)$ and $f(b-x) = f(b+x)$ $\forall x \in R$ where $a, b(a > b)$ are constants then the period of $f(x)$ is

(1) $2a$ (2) $2|a-b|$ (3) $3a$ (4) b

23. The period of $f(x) = [x] + [2x] + [3x] + [4x] + \cdots + [nx] - \frac{n(n+1)}{2}x$ (where $n \in N$ is

(1) n (2) 1 (3) $\frac{1}{n}$ (4) 5

24. If $f(2+x) = a + \left[1 - (f(x) - a)^4\right]^{\frac{1}{4}}$ $\forall x \in R$, then $f(x)$ is periodic with period

(1) 1 (2) 2 (3) 4 (4) 8

25. If $f\colon [-4,4] - \{-\pi, 0, \pi\} \to R$, such that $f(x) = \cot(\sin x) + \left[\frac{x^2}{|a|}\right] + \frac{\sin 2x}{x^2}$, ($[\cdot]$ denotes greatest integer function) is an odd function, then the complete set of values of a is

(1) $(-\infty, -4] \cup [4, \infty)$ (2) $(-\infty, -16) \cup (16, \infty)$

(3) $[-16, 16]$ (4) $(-\infty, -16] \cup [16, \infty)$

26. S-I: If $f(x)$ is a odd function and $g(x)$ is even function then $f(x) + g(x)$ is neither even nor odd.
S-II: Odd function is symmetrical in opposite quadrants and even function is symmetrical about the y-axis.

(1) Both S-I and S-II are individually true and R is the correct explanation of A

(2) Both S-I and S-II are individually true but R is not the correct explanation of A

(3) S-1 is true but S-II is false

(4) S-I is false but S-II is true

27. If $f(x) = \begin{cases} 1+x; & 0 \le x \le 2 \\ 3-x; & 2 < x \le 3 \end{cases}$ then $(f \circ f)(x) =$

(1) $\begin{cases} 1-x; & 0 \le x \le 2 \\ 3+x; & 2 < x \le 3 \end{cases}$

(2) $\begin{cases} 2+x; & 0 \le x \le 2 \\ 2-x; & 2 < x \le 3 \\ 4-x; & 2 < x \le 3 \end{cases}$

(3) $\begin{cases} 2+x; & 0 \le x \le 1 \\ 2-x; & 1 < x \le 2 \\ 4-x; & 2 < x \le 3 \end{cases}$

(4) does not exist

28. If for $x > 0, f(x) = (a - x^n)^{\frac{1}{n}};\ g(x) = x^2 + px + q;\ p, q \in R$ and the equation $g(x) - x = 0$ has imaginary roots then the number of real roots of the equation $g(g(x)) - f(f(x)) = 0$

(1) 0 (2) 2 (3) 4 (4) n

29. S-I: If $f(x) = \dfrac{x+1}{x-1}$ then $(f \circ f \circ f \circ f \circ f)(x) = f(x)$

S-II: $(f \circ f)(x) = x$

(1) Both S-I and S-II are individually true and R is the correct explanation of A
(2) Both S-I and S-II are individually true but R is not the correct explanation of A
(3) S-1 is true but S-II is false
(4) S-I is false but S-II is true

30. The domain of $f(x) = \dfrac{1}{x} + 2^{sin^{-1}x} + \dfrac{1}{\sqrt{x-2}}$ is

(1) $(0, \infty)$ (2) $(-\infty, 0)$ (3) $(1, 3)$ (4) ϕ

31. The domain of $f(x) = \sqrt{\ln_{(|x|-1)}(x^2 + 4x + 4)}$ is

(1) $[-3, -1] \cup [1, 2]$ (2) $(-2, -1) \cup [2, \infty]$
(3) $(-\infty, -3) \cup (-2, -1) \cup (2, \infty)$ (4) $(-\infty, \infty)$

32. The domain of $f(x) = \sqrt{\dfrac{\log_{0.3}|x-2|}{|x|}}$ is

(1) $[1, 2]$ (2) $[2, 3]$ (3) $[1, 2) \cup (2, 3]$ (4) $[0, 3]$

33. The domain of $f(x) = \log_{10}\log_{10}\log_{10}\ldots\log_{10} x$ (log n times) is

(1) $\left(10^{10^{10^{\cdot^{\cdot^{(n-2)\text{ times}}}}}}, \infty\right)$ (2) $\left(10^{n-2}, \infty\right)$

(3) $\left(10^{10^{10^{\cdot^{\cdot^{(n-1)\text{ times}}}}}}, \infty\right)$ (4) $\left(10^{10^{10^{\cdot^{\cdot^{(n-3)\text{ times}}}}}}, \infty\right)$

34. The domain of the function $f(x) = \sin^{-1}\left(\dfrac{4}{3+2\cos x}\right)$ is

(1) $\left[2n\pi - \dfrac{\pi}{3}, 2n\pi + \dfrac{\pi}{3}\right]$ (2) $\left[2n\pi - \dfrac{\pi}{6}, 2n\pi + \dfrac{\pi}{6}\right]$

(3) $\left[2n\pi - \dfrac{\pi}{2}, 2n\pi + \dfrac{\pi}{2}\right]$ (4) $\left[2n\pi - \dfrac{\pi}{3}, 2n\pi + \dfrac{\pi}{2}\right]$

35. The domain of $f(x) = \sin^{-1}\left[2 - 4x^2\right]$ (where $[\cdot]$ is G.I.F.) is

(1) $\left[-\dfrac{\sqrt{3}}{2}, \dfrac{\sqrt{3}}{2}\right]$ (2) $\left[-\dfrac{3}{2}, \dfrac{3}{2}\right] - \{0\}$

(3) $\left[-\dfrac{\sqrt{3}}{2}, 0\right) \cup \left(0, \dfrac{\sqrt{3}}{2}\right]$ (4) ϕ

36. The domain of $f(x) = \sqrt{\cos(\sin x)} + \sqrt{\log_x \{x\}}$ (where $\{\cdot\}$ is fractional part of x) is

(1) $[1, \pi)$ (2) $(0, 2\pi) - [1, \pi)$ (3) $\left(0, \dfrac{\pi}{2}\right) - \{1\}$ (4) $(0, 1)$

37. The domain of

$$\frac{[\cos^{-1}(x^4)] + |[x - 2\tan^{-1} x]| + \sqrt{\sin(\ln x)}}{\{3x^2 - 7\} + a^{\sqrt{\sin x + 3\cos x}} + \ln\cos\left(\frac{1}{\sqrt{-x^2}}\right)}$$

(where $[\cdot]$ is G.I.F. and $\{\cdot\}$ is fractional part function) is

(1) $\left(-2, \sqrt{2}\right)$ (2) $(0, 1)$ (3) $(-1, 1)$ (4) ϕ

38. The domain of $f(x) = \sin^{-1}\left(\left[2 - 3x^2\right]\right)$ (where $[\cdot]$ is G.I.F.) is

(1) $[-1, 1]$ (2) $(0, 1)$ (3) $[-1, 1] - \{0\}$ (4) ϕ

39. The domain of the function

$$f(x) = \left[9^x + 27^{\frac{2}{3}(x-2)} - 219 - 3^{2(x-1)}\right]^{\frac{1}{4}}$$

(1) $[-3, 3]$ (2) $[3, \infty)$ (3) $\left[\dfrac{5}{2}, \infty\right)$ (4) $[0, 1]$

40. If $f(x) = 2x + |x|$, $g(x) = \dfrac{1}{3}(2x - |x|)$ and $h(x) = f(g(x))$ then the domain of $\sin^{-1}(h(h(h \ldots h(x))))$ is (h is being repeated n times)

(1) $[-1, 1]$ (2) $(0, 1)$ (3) $(-1, 1)$ (4) $\left[\dfrac{1}{2}, 1\right]$

42. The range of $\tan(\log x)$

(1) $(0, \infty)$ (2) $(1, \infty)$ (3) (e, ∞) (4) $(-\infty, \infty)$

43. The range of $f(x) = \dfrac{e^{-x}}{1 + [x]}$ is (where $[\cdot]$ is G.I.F.)

(1) R (2) $R - \{0\}$ (3) $R - [-1, 0)$ (4) $[0, \infty)$

44. The range of $f(x) = \dfrac{1}{|\sin x|} + \dfrac{1}{|\cos x|}$ is

(1) $\left[2\sqrt{2}, \infty\right)$ (2) $\left(\sqrt{2}, 2\sqrt{2}\right)$ (3) $\left(0, 2\sqrt{2}\right)$ (4) $\left(2\sqrt{2}, 4\right)$

45. The range of $f(x) = \dfrac{x-[x]}{1-[x]+x}$ (where $[\cdot]$ is G.I.F.) is

(1) $\left[0, \dfrac{1}{2}\right]$ (2) $[0,1]$ (3) $\left(0, \dfrac{1}{2}\right]$ (4) $\left[0, \dfrac{1}{2}\right)$

46. Let $A = \left\{x \colon 0 \le x < \dfrac{\pi}{2}\right\}$ and $f \colon R \to A$ is an onto function given by

$f(x) = \tan^{-1}(x^2 + x + \lambda)$ where

(1) $\lambda > 0$ (2) $\lambda \le \dfrac{1}{4}$ (3) $\lambda = \dfrac{1}{4}$ (4) $\lambda \ge \dfrac{1}{8}$

47. $f(x) = |x-1|$, $f \colon R^+ \to R$ and $g(x) = e^x$, $g \colon [-1, \infty) \to R$ if the function $f \circ g(x)$ is defined, then its domain and range respectively are

(1) $(0, \infty)$ & $[0, \infty)$ (2) $[-1.\infty)$ & $[0, \infty)$

(3) $[-1, \infty)$ & $\left[1 - \dfrac{1}{e}, \infty\right)$ (4) $[-1, \infty)$ & $\left[\dfrac{1}{e} - 1, \infty\right)$

48. The range of $f(x) = \left[\dfrac{1}{\sin\{x\}}\right]$ is (where $\{\cdot\}$ is fraction part and $[\cdot]$ is G.I.F.)

(1) $\{1, -1\}$ (2) $\{0\}$ (3) N (4) Z

49. If $f(x) = \pi\left(\dfrac{\sqrt{x+7}-4}{x-9}\right)$, then range of function $y = \sin(2f(x))$ is

(1) $[0,1]$ (2) $\left(0, \dfrac{1}{\sqrt{2}}\right]$

(3) $\left(0, \dfrac{1}{\sqrt{2}}\right) \cup \left(\dfrac{1}{\sqrt{2}}, 1\right]$ (4) $(0,1]$

50. The range of $f(x) = \sqrt{a-x} + \sqrt{x-b}$ is (where $a > b > 0$)

(1) $\left[\sqrt{a-b}, \sqrt{2(a-b)}\right]$ (2) $\left[\sqrt{a-b}, \sqrt{a+b}\right]$

(3) $[a, b]$ (4) (a, b)

51. The range of $f(x) = \cot^{-1}(3x - x^2)$ is

(1) $(0, \pi)$ (2) $\left(\dfrac{\pi}{4}, \pi\right)$ (3) $\left(\dfrac{\pi}{4}, \pi\right]$ (4) $\left[\dfrac{\pi}{4}, \pi\right)$

52. The sum of the maxiumum and minimum values of

$$f(x) = \sin^{-1}(2x) + \cos^{-1}(2x) + \sec^{-1}(2x)$$

is

(1) π (2) $\dfrac{\pi}{2}$ (3) 2π (4) $\dfrac{3\pi}{2}$

53. If $f(x)=\log_{[x-1]}\left(\frac{|x|}{x}\right)$, (where $[\cdot]$ is G.I.F.) then domain and range are

(1) $(2,\infty)$, $(0,1)$ (2) $[3,\infty]$, $\{0\}$ (3) $[3,\infty]$, $\{0,1\}$ (4) $(-\infty,\infty)$, $\{0\}$

54. If $f(x)=x^3+3x^2+4x+a\sin x+b\cos x\ \forall x\in R$ is an injection then the greatest value of a^2+b^2 is

(1) 1 (2) 2 (3) $\sqrt{2}$ (4) $2\sqrt{2}$

55. The domain and range of $f(x)=\sin\left\{\log\left(\frac{\sqrt{4-x^2}}{1-x}\right)\right\}$ are

(1) $(-2,1)$ & $[-1,1]$ (2) $(1,3)$ & $[-1,1]$

(3) $[-2,1]$ & $[-1,1]$ (4) $(0,\infty)$ & $[-1,1]$

56. Consider the real valued function satisfying $2f(\sin x)+f(\cos x)=x$, then $f\left(\frac{1}{2}\right)=$

(1) 1 (2) 2 (3) 0 (4) 4

57. If $f\colon R\to R$ such that $f(x-f(y))=f(f(y))+xf(x)+f(x)-1\ \forall x,y\in R$n then $f(x)$ is

(1) $1+\frac{x^2}{4}$ (2) $1-\frac{x^2}{2}$ (3) $1+\frac{x^2}{2}$ (4) $1-\frac{x^2}{4}$

58. A function f well defined $\forall x,y\in R$ is such that $f(1)=2, f(2)=8$ and $f(x+y)-kxy=f(x)+2y^2$, where k is some constant then $f(x)$ is

(1) x^2 (2) $3x^2$ (3) $2x^2$ (4) $4x^2$

59. If f is a polynomial function satisfying $2+f(x)f(y)=f(x)+f(y)+f(xy)\forall x,y\in R$ and if $f(2)=5$ then the value of $f(f(2))=$ $(f(1)\neq 1)$

(1) 25 (2) 16 (3) 26 (4) 14

60. If f is a function such that $f(0)=2; f(1)=3$ and $f(x+2)=2f(x)-f(x+1)\ \forall x\in R$ then $f(5)=$

(1) 7 (2) 13 (3) 1 (4) 5

61. If $f(x)=(-1)^{\left[\frac{2x}{\pi}\right]}$, $g(x)=|\sin x|-|\cos x|$, and $\phi(x)=f(x)g(x)$ (where $[\cdot]$ denotes the greatest integer function), then the respective fundamental periods of $f(x)$, $g(x)$ and $\phi(x)$ are

(1) π,π,π (2) $\pi,2\pi,\pi$ (3) $\pi,\pi,\frac{\pi}{2}$ (4) $\pi,\frac{\pi}{2},\pi$

62. The domain of the function $f(x)=\frac{1}{\sqrt{{}^{10}C_{x-1}-3\times{}^{10}C_x}}$ contains the points

(1) 9, 10, 11 (2) 9, 10, 12

(3) all natural numbers (4) none of these

63. The range of $f(x)=(x+1)(x+2)(x+3)(x+4)+5$ for $x\in[-6,6]$ is

(1) $[4,5045]$ (2) $[0,5045]$ (3) $[-20,5045]$ (4) none of these