ASSIGNMENT 3 - PART 1

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Parameters

Roll number used to calculate x and y: 2019113013

x = $1 - ((3013\%30) + 1)/100 = 0.86$
y = $(13\%4) + 1 = 2$

Observation Probabilities:

Observation	Probability
P (Observation = RED | State = RED)	0.90
P (Observation = GREEN | State = RED)	0.10
P (Observation = GREEN | State = GREEN)	0.85
P (Observation = RED | State = GREEN)	0.15

Initial States: S_1, S_3, S_6 (red) with equal probability

Calculation

We define our belief state array as [$b(S_1)$, $b(S_2)$, $b(S_3)$, $b(S_4)$, $b(S_5)$, $b(S_6)$]

Initial Belief State:
[$\frac{1}{3}$, 0, $\frac{1}{3}$, 0, 0, $\frac{1}{3}$]

Step 1

Agent took the action Right and observed Green

$P(Green|Green) = 0.85, P(Green|Red) = 0.10$

$P(Green|Right) = [(0.86 \times 0.85 + 0.14 \times 0.10) \times \frac{1}{3}] + [(0.86 \times 0.10 + 0.14 \times 0.85) \times \frac{1}{3}] + [(0.86 \times 0.10 + 0.14 \times 0.85) \times \frac{1}{3}] = 0.60$

From S_1, going `right` can lead to S_1 (red) or S_2 (green).

$P(S_1) = 0.14, P(S_2) = 0.86$

$b'(S_1) = \frac{0.14 \times 0.10}{0.60} \times \frac{1}{3} \approx 0.00777778$

$b'(S_2) = \frac{0.86 \times 0.85}{0.60} \times \frac{1}{3} \approx 0.40611111$

From S_3, going `right` can lead to S_4 (green) or S_2 (green).

$P(S_2) = 0.14, P(S_4) = 0.86$

$b'(S_2) = \frac{0.14 \times 0.85}{0.60} \times \frac{1}{3} \approx 0.06611111$

$b'(S_4) = \frac{0.86 \times 0.85}{0.60} \times \frac{1}{3} \approx 0.40611111$

From S_6 going `right` can lead to S_6 (red) or S_5 (green).

$P(S_5) = 0.14, P(S_6) = 0.86$

$b'(S_5) = \frac{0.14 \times 0.85}{0.60} \times \frac{1}{3} \approx 0.06611111$

$b'(S_6) = \frac{0.86 \times 0.10}{0.60} \times \frac{1}{3} \approx 0.04777778$

Belief State after Step 1:
[0.00777778, 0.47222222, 0.0, 0.40611111, 0.06611111, 0.04777778]

Step 2

Agent took the action Left and observed Red.

$P(Red|Red) = 0.90, P(Red|Green) = 0.15$

$P(Red|Left) = (0.86 \times 0.90 + 0.14 \times 0.15) \times b(S_1) + 0.90 \times b(S_2) + 0 + (0.86 \times 0.90 + 0.14 \times 0.15) \times b(S_4) + (0.86 \times 0.15 + 0.14 \times 0.90) \times b(S_5) + (0.86 \times 0.15 + 0.14 \times 0.90) \times b(S_6)$

$= 0.795 \times 0.0777778 + 0.90 \times 0.47222222 + 0.795 \times 0.40611111 + 0.255 \times 0.06611111 + 0.255 \times 0.04777778$

$= 0.7831$

From S_1 going `left` can lead to S_1 (red) or S_2 (green).

$P(S_1) = 0.86, P(S_2) = 0.14$

$b'(S_1) = \frac{0.86 \times 0.90}{0.7831} \times 0.00777778 \approx 0.00768740$

$b'(S_2) = \frac{0.14 \times 0.15}{0.7831} \times 0.00777778 \approx 0.00020858$

From S_2 going `left` can lead to S_1 (red) or S_3 (red).

$P(S_1) = 0.86, P(S_3) = 0.14$

$b'(S_1) = \frac{0.86 \times 0.90}{0.7831} \times 0.47222222 \approx 0.46673477$

$b'(S_3) = \frac{0.14 \times 0.90}{0.7831} \times 0.47222222 \approx 0.07598008$

From S_4 going `left` can lead to S_3 (red) or S_5 (green).

$P(S_3) = 0.86, P(S_5) = 0.14$

$b'(S_3) = \frac{0.86 \times 0.90}{0.7831} \times 0.40611111 \approx 0.40139191$

$b'(S_5) = \frac{0.14 \times 0.15}{0.7831} \times 0.40611111 \approx 0.01089048$

From S_5, going `left` can lead to S_4 (red) and S_6 (green).

$P(S_4) = 0.86, P(S_6) = 0.14$

$b'(S_4) = \frac{0.86 \times 0.15}{0.7831} \times 0.06611111 \approx 0.01089071$

$b'(S_6) = \frac{0.14 \times 0.90}{0.7831} \times 0.06611111 \approx 0.01063744$

From S_6, going `left` can lead to S_5 (green) or S_6 (red).

$P(S_5) = 0.86, P(S_6) = 0.14$

$b'(S_5) = \frac{0.86 \times 0.15}{0.7831} \times 0.04777778 \approx 0.00787043$

$b'(S_6) = \frac{0.14 \times 0.90}{0.7831} \times 0.04777778 \approx 0.00768756$

Belief State after Step 2:
$[0.47443227, 0.00020858, 0.47738214, 0.01089071, 0.01876131, 0.01832500]$

Step 3

Agent took the action Left and observed Green.

$P(Green|Green) = 0.85, P(Green|Red) = 0.10$

$P(Green|Left) = 0.205 \times b(S_1) + 0.10 \times b(S_2) + 0.85 \times b(S_3) + 0.205 \times b(S_4) + 0.745 \times b(S_5) + 0.745 \times b(S_6)$

$= 0.532916$

From S_1 going `left` can lead to S_1 (red) or S_2 (green).

$P(S_1) = 0.86, P(S_2) = 0.14$

$b'(S_1) = \frac{0.86 \times 0.10}{0.532916} \times 0.47443227 \approx 0.07656211$

$b'(S_2) = \frac{0.14 \times 0.10}{0.532916} \times 0.47443227 \approx 0.01246360$

From S_2 going `left` can lead to S_1 (red) or S_3 (red).

$P(S_1) = 0.86, P(S_3) = 0.14$

$b'(S_1) = \frac{0.86 \times 0.10}{0.532916} \times 0.00020858 \approx 0.00003366$

$b'(S_3) = \frac{0.14 \times 0.10}{0.532916} \times 0.00020858 \approx 0.00000548$

From S_3 going `left` can lead to S_2 (green) or S_4 (green).

$P(S_2) = 0.86, P(S_4) = 0.14$

$b'(S_2) = \frac{0.86 \times 0.85}{0.532916} \times 0.47738214 \approx 0.65482430$

$b'(S_4) = \frac{0.14 \times 0.85}{0.532916} \times 0.47738214 \approx 0.10659930$

From S_4 going `left` can lead to S_3 (red) or S_5 (green).

$P(S_3) = 0.86, P(S_5) = 0.14$

$b'(S_3) = \frac{0.86 \times 0.10}{0.532916} \times 0.01089071 \approx 0.00175750$

$b'(S_5) = \frac{0.14 \times 0.85}{0.532916} \times 0.01089071 \approx 0.00243189$

From S_5, going `left` can lead to S_4 (red) and S_6 (green).

$P(S_4) = 0.86, P(S_6) = 0.14$

$b'(S_4) = \frac{0.86 \times 0.10}{0.532916} \times 0.01876131 \approx 0.00302763$

$b'(S_6) = \frac{0.14 \times 0.85}{0.532916} \times 0.01876131 \approx 0.00418940$

From S_6, going `left` can lead to S_5 (green) or S_6 (red).

$P(S_5) = 0.86, P(S_6) = 0.14$

$b'(S_5) = \frac{0.86 \times 0.85}{0.532916} \times 0.01832500 \approx 0.02513637$

$b'(S_6) = \frac{0.14 \times 0.10}{0.532916} \times 0.01832500 \approx 0.00048141$

Belief State after Step 3:
[0.07659575, 0.76076463, 0.00176298, 0.13233411, 0.02756825, 0.00097428]