Two-Qubit Dynamics with Josephson Qubits

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Background

Background

Cooper Pair Box

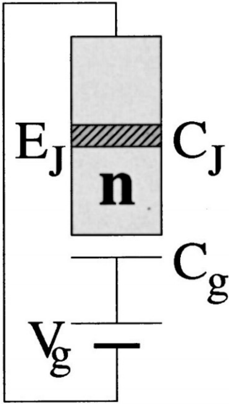


Figure : http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357

Background
SQUIDs

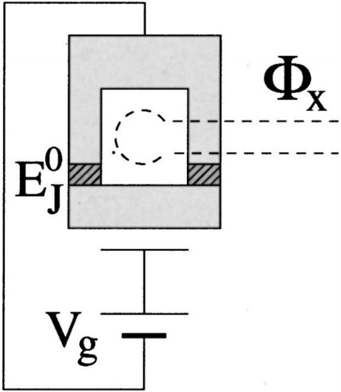


Figure : http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357

Background

Single-Qubit Charging Diagram

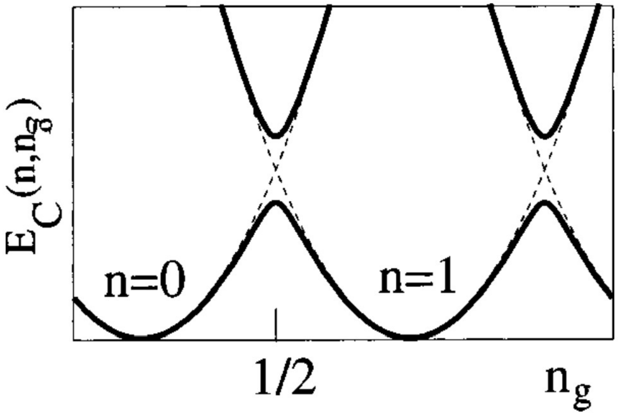
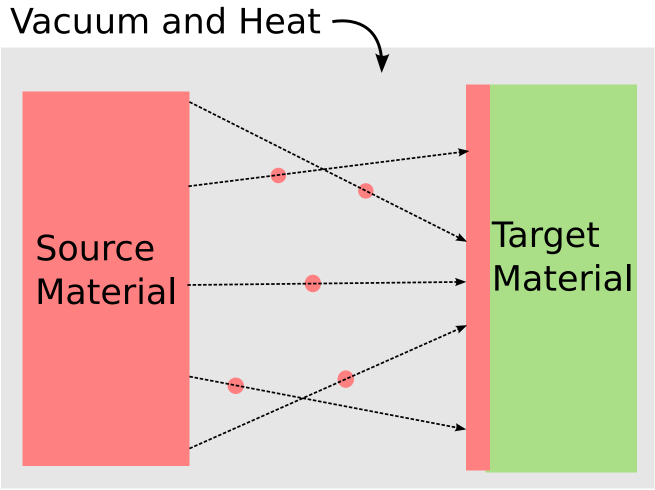


Figure : A simple caption

Fabrication Techniques

Fabrication Techniques

Evapouration (Deposition)



Fabrication Techniques

Shadow Evapouration

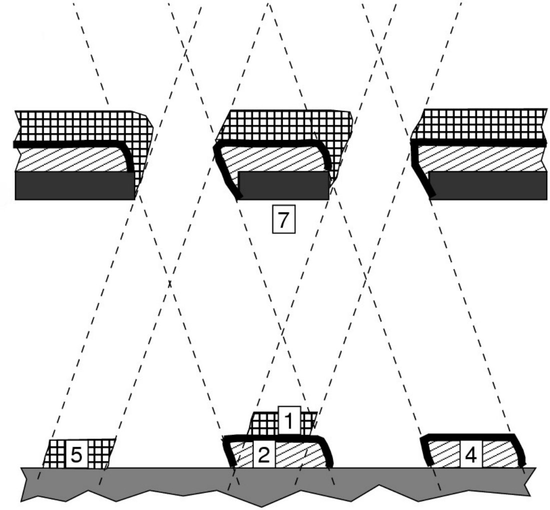
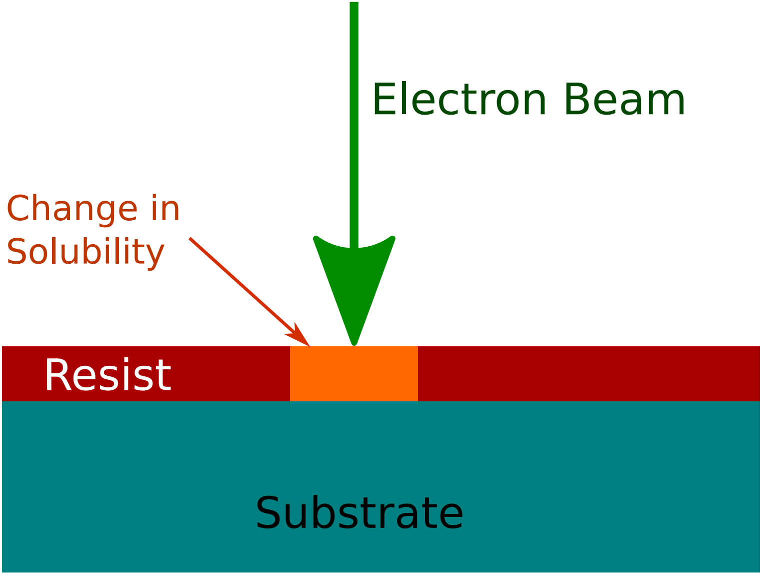


Figure : http://en.wikipedia.org/wiki/Niemeyer-Dolan_technique

Fabrication Techniques

Electron Beam Lithography (EBL)



Fabrication Techniques

Etching

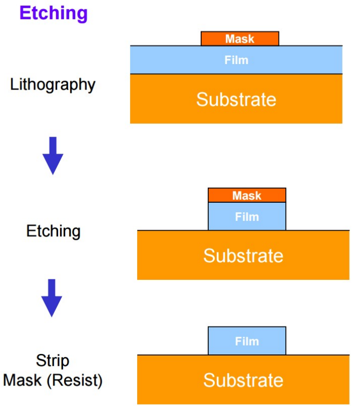


Figure : http://www.mrsec.harvard.edu/education/ap298r2004/Erli%20chen%20Fabrication%20III%20-%20Etching.pdf

Fabrication Techniques

Lift-off

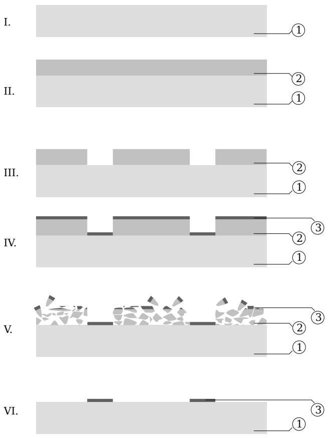


Figure : http://en.wikipedia.org/wiki/Lift-off_%28microtechnology%29

Fabrication Techniques
SEM image of a SQUID qubit

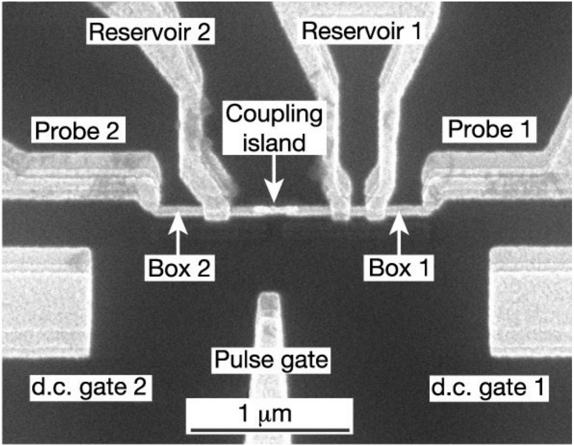
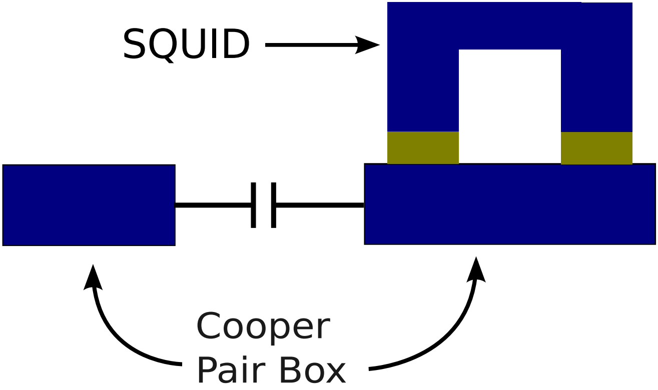


Figure : http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Coupling of Two Qubits

Coupling of Two Qubits

Basic Idea



Coupling of Two Qubits

The Circuit

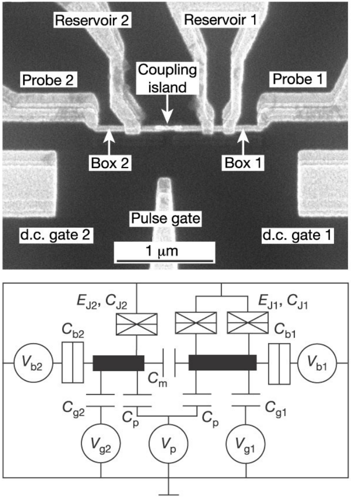


Figure : http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Coupling of Two Qubits

Charging Diagram of Two-Qubit Case

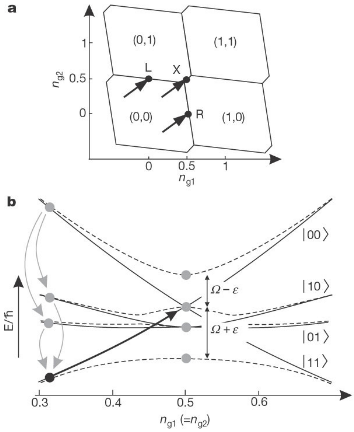


Figure : http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Coupling of Two Qubits

Points L and R

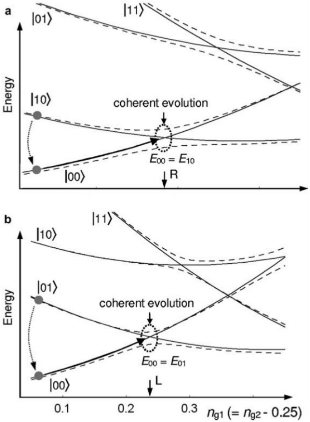


Figure : http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf

Coupling of Two Qubits

Charging Diagram Level Curves

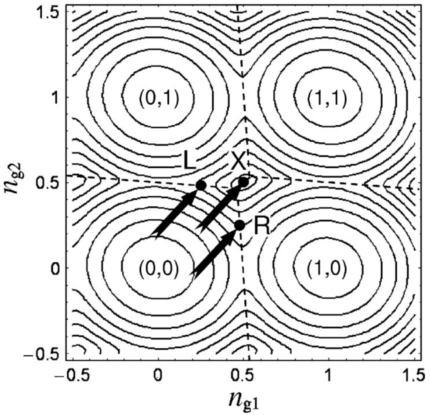


Figure : http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf

Coupling of Two Qubits

Charging-Energy Diagram

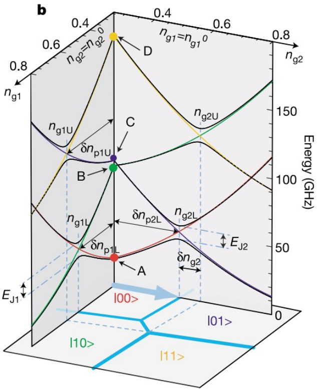


Figure : http://qudev.ethz.ch/content/courses/QSIT09/pdfs/Yamamoto2003.pdf

Coupling of Two Qubits

Theory

$$\text{Note: } E_{J1,J2} \approx E_m < E_{C1,C2}$$

Hamiltonian

$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

Where...

- $E_{n1n2} = E_{c1}(n_{g1} - n_1)^2 + E_{c2}(n_{g2} - n_2)^2 + E_m(n_{g1} - n_1)(n_{g2} - n_2)$
- E_{Ji} is the Josephson energy of the i^{th} box
- $E_{c1,c2} = 4e^2 C_{\Sigma 2,\Sigma 1}/2(C_{\Sigma 1} C_{\Sigma 2} - C_m^2)$ are the effective Cooper pair charging energies
- $C_{\Sigma i}$ is the sum of all capacitances connected to the i^{th} island
- $n_{g1,g2} = (C_{g1,g2} V_{g1,g2} + C_p V_p)/2e$ is the charge, indiced by the gate and pulse voltages, on the qubits
- $E_m = 4e^2 C_m/(C_{\Sigma 1} C_{\Sigma 2} - C_m^2)$ is the coupling energy of the qubits

Probabilities

- At the coresonance point, we have a coherent superposition state $|\psi\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle$
- ie, the qubit state probabilities are $p_1(1) = |c_2|^2 + |c_4|^2$ and $p_2(1) = |c_3|^2 + |c_4|^2$
- Now we initialize system to $|\psi\rangle = |00\rangle$
- Using this Hamiltonian and an ideal rectangular pulse of length Δt,
  $p_{1,2}(1) = \frac{1}{4}\left(2 - (1 - \chi_{1,2})cos[(\Omega + \epsilon)\Delta t] - (1 + \chi_{1,2})cos[(\Omega - \epsilon)\Delta t]\right)$
  Where...
  - $\chi_{1,2} = \frac{(E^2_{J2,J1} - E^2_{J1,J2}) + E^2_m/4}{4\hbar^2\Omega\epsilon}$
  - $\Omega = \sqrt{(E_{J1} + E_{J2})^2 + (E_m/2)^2}/2\hbar$
  - $\epsilon = \sqrt{(E_{J1} - E_{J2})^2 + (E_m/2)^2}/2\hbar$

Coupling of Two Qubits

State Readout

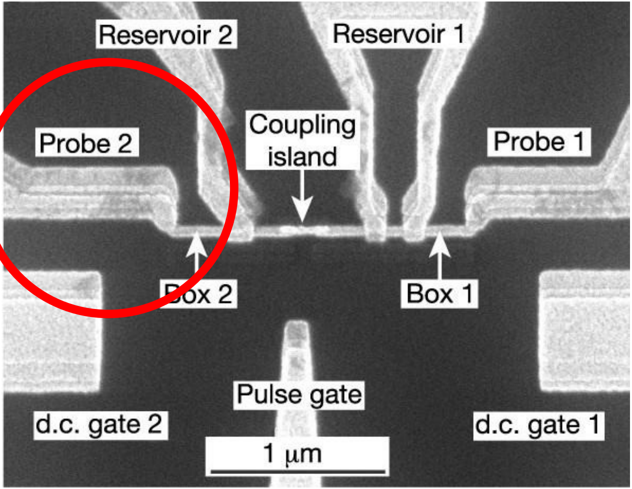


Figure : http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Coupling of Two Qubits

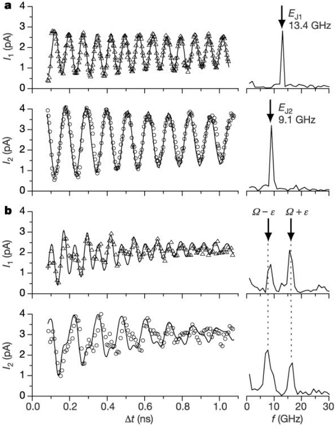
State Readout

- Probe currents are proportional to the qubit state probabilities: $I_1 \propto p_1(1) = |c_2|^2 + |c_4|^2$ and $I_2 \propto p_2(1) = |c_3|^2 + |c_4|^2$
- The current oscillation frequency is related to the Josephson energy as: $\omega_{1,2} = \frac{E_{J1,J2}}{\hbar}$

Coupling of Two Qubits

Parameter Measurements / Frequency Responses

- Tune system by bringing to point L or R, exciting oscillations in one qubit
- Get cosines with exponential decay
- Fourier transform gives frequencies, which define E_{J1} or E_{J2}
- Drive system to point X and perform same Fourier measurement
- This time, get 2 frequencies, $\Omega \pm \epsilon$



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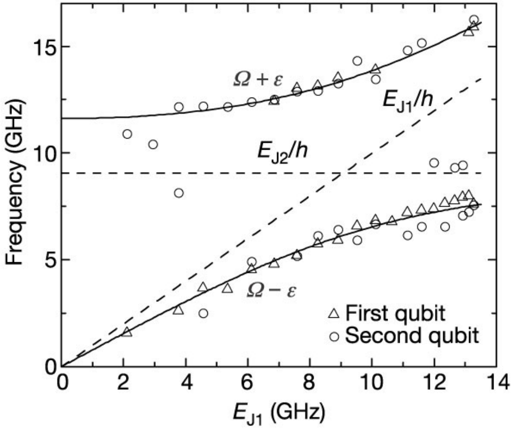
Coupling of Two Qubits

EJ1 dependence of spectrum components

Note: Frequency repulsion at $E_{J1} \approx E_{J2}$



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Coupling of Two Qubits

Decoherence

- Probe Junction
- Charge Qubit Noise, ie $n_g \to n_g + \delta n_g(t)$
- Solid State Noise
  - Thermal Noise
  - Material Imperfections
  - Charge and Flux Noise

THE END

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Reference Papers

http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html
http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf
(Figures cited individually)