

# Two-Qubit Dynamics with Josephson Qubits

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April 2, 2015

# Background

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## Cooper Pair Box

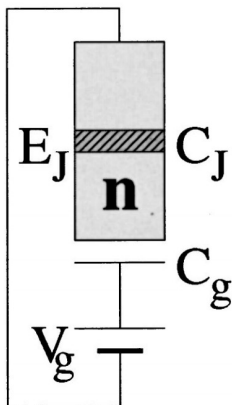


Figure : <http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357>

# Background

## SQUIDs

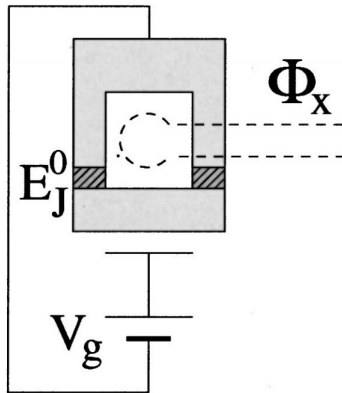


Figure : <http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357>

# Background

## Single-Qubit Charging Diagram

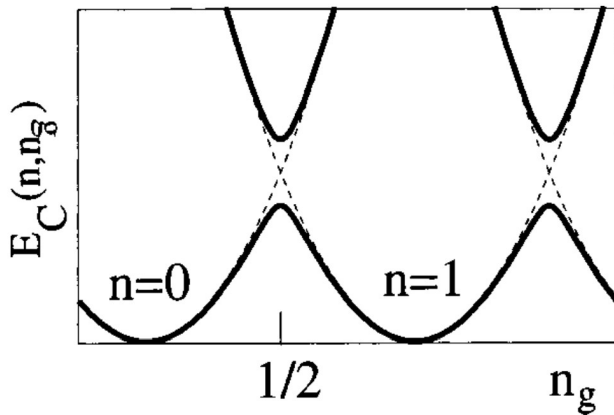


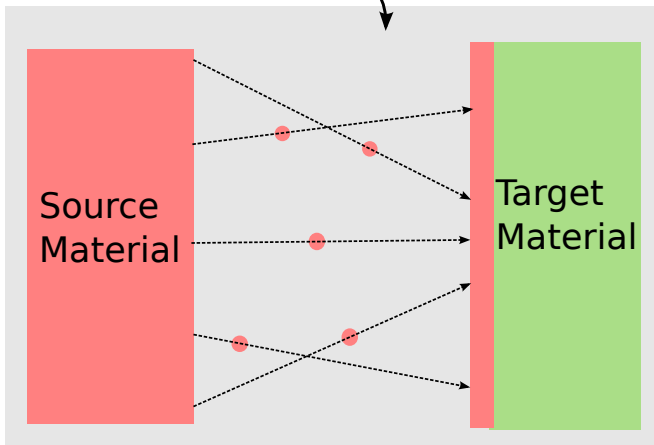
Figure : A simple caption

# Fabrication Techniques

# Fabrication Techniques

## Evaporation (Deposition)

Vacuum and Heat



# Fabrication Techniques

## Shadow Evaporation

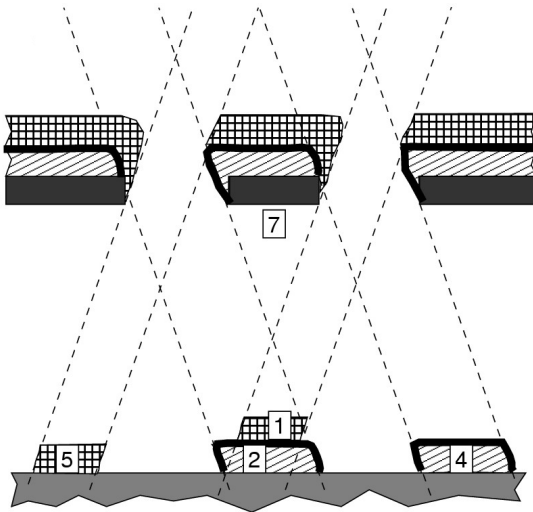
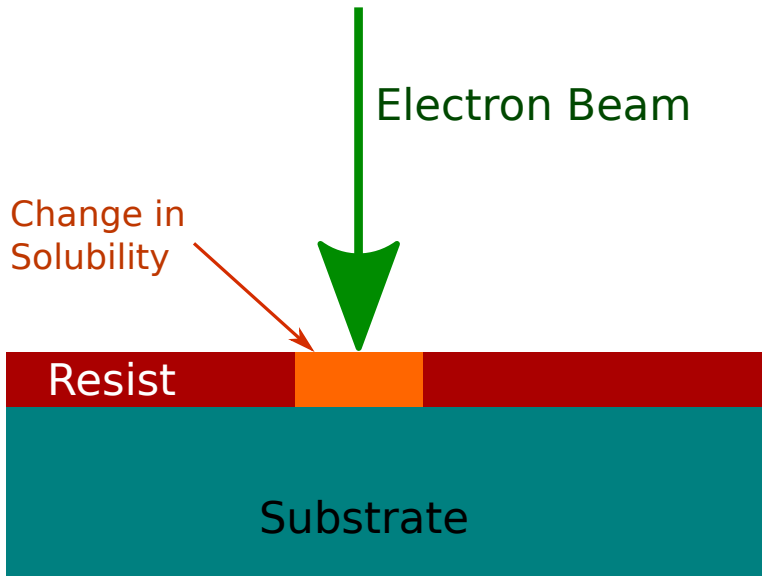


Figure : [http://en.wikipedia.org/wiki/Niemeyer-Dolan\\_technique](http://en.wikipedia.org/wiki/Niemeyer-Dolan_technique)



# Fabrication Techniques

## Electron Beam Lithography (EBL)



# Fabrication Techniques

## Etching

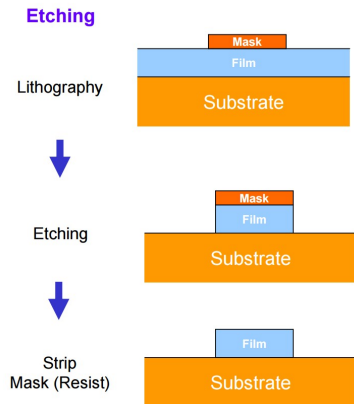


Figure : <http://www.mrsec.harvard.edu/education/ap298r2004/Erli%20chen%20Fabrication%20III%20-%20Etching.pdf>

# Fabrication Techniques

## Lift-off

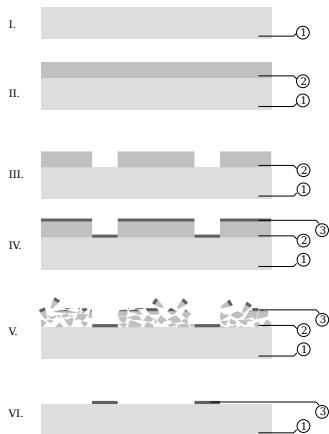


Figure : [http://en.wikipedia.org/wiki/Lift-off\\_%28microtechnology%29](http://en.wikipedia.org/wiki/Lift-off_%28microtechnology%29)

# Fabrication Techniques

SEM image of a SQUID qubit

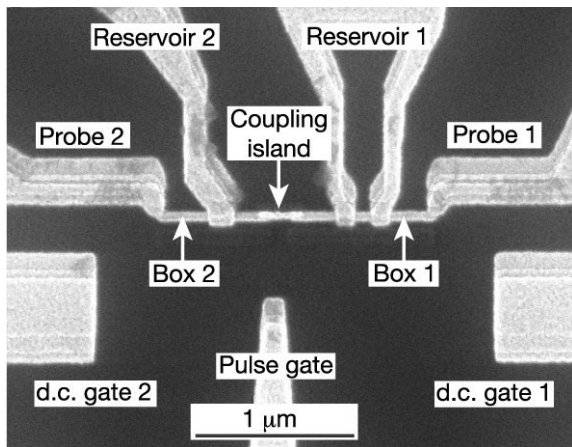
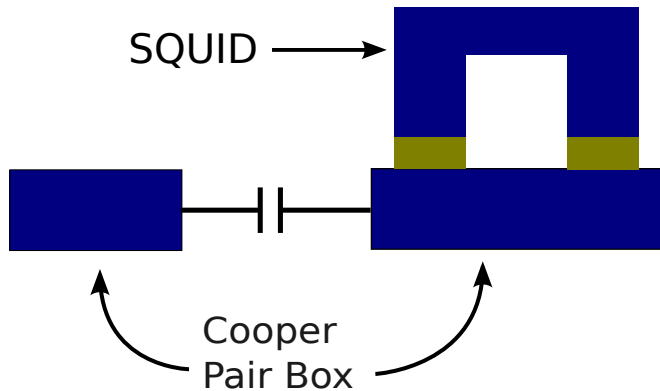


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>

# Coupling of Two Qubits

# Coupling of Two Qubits

## Basic Idea



# Coupling of Two Qubits

## The Circuit

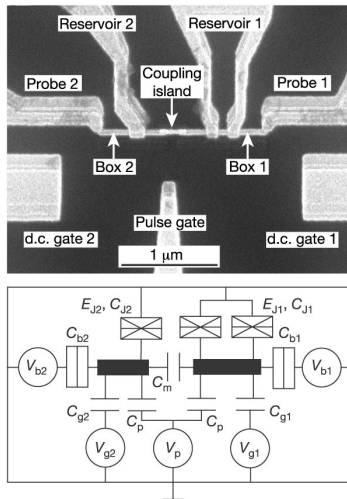


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>

# Coupling of Two Qubits

## Charging Diagram of Two-Qubit Case

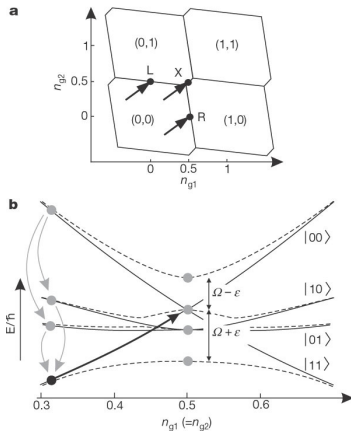


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>



# Coupling of Two Qubits

Points L and R

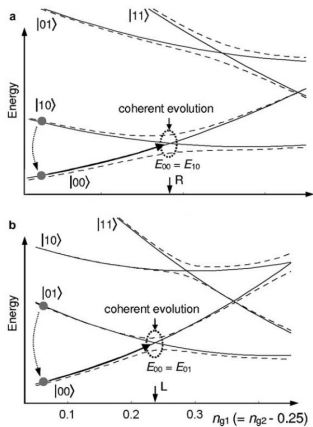


Figure : <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf>

# Coupling of Two Qubits

## Charging Diagram Level Curves

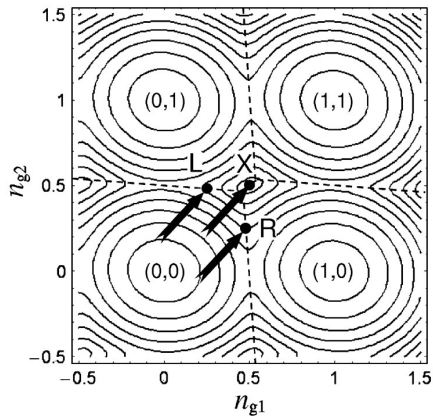


Figure : <http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf>

# Coupling of Two Qubits

## Charging-Energy Diagram

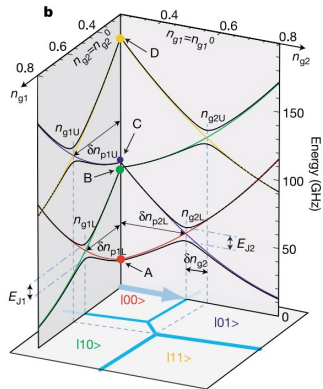


Figure : <http://qudev.ethz.ch/content/courses/QSIT09/pdfs/Yamamoto2003.pdf>

# Coupling of Two Qubits

## Theory

### Hamiltonian

$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0 \\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2} \\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1} \\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

Where...

- ▶  $E_{n1n2} = E_{c1}(n_{g1} - n_1)^2 + E_{c2}(n_{g2} - n_2)^2 + E_m(n_{g1} - n_1)(n_{g2} - n_2)$
- ▶  $E_{Ji}$  is the Josephson energy of the  $i^{th}$  box
- ▶  $E_{c1,c2} = 4e^2 C_{\Sigma 2, \Sigma 1} / 2(C_{\Sigma 1} C_{\Sigma 2} - C_m^2)$  are the effective Cooper pair charging energies
- ▶  $C_{\Sigma i}$  is the sum of all capacitances connected to the  $i^{th}$  island
- ▶  $n_{g1,g2} = (C_{g1,g2} V_{g1,g2} + C_p V_p) / 2e$  is the charge, indexed by the gate and pulse voltages, on the qubits
- ▶  $E_m = 4e^2 C_m / (C_{\Sigma 1} C_{\Sigma 2} - C_m^2)$  is the coupling energy of the qubits

# Coupling of Two Qubits

## Theory

### Probabilities

- ▶ At the coresonance point, we have a coherent superposition state

$$|\psi\rangle = c_1|00\rangle + c_2|10\rangle + c_3|01\rangle + c_4|11\rangle$$

- ▶ ie, the qubit state probabilities are  $p_1(1) = |c_2|^2 + |c_4|^2$  and  $p_2(1) = |c_3|^2 + |c_4|^2$

- ▶ Now we initialize system to  $|\psi\rangle = |00\rangle$

- ▶ Using this Hamiltonian and an ideal rectangular pulse of length  $\Delta t$ ,

$$p_{1,2}(1) = \frac{1}{4} \left( 2 - (1 - \chi_{1,2})\cos[(\Omega + \epsilon)\Delta t] - (1 + \chi_{1,2})\cos[(\Omega - \epsilon)\Delta t] \right)$$

Where...

- ▶  $\chi_{1,2} = \frac{(E_{J2,J1}^2 - E_{J1,J2}^2) + E_m^2/4}{4\hbar^2\Omega\epsilon}$
- ▶  $\Omega = \sqrt{(E_{J1} + E_{J2})^2 + (E_m/2)^2}/2\hbar$
- ▶  $\epsilon = \sqrt{(E_{J1} - E_{J2})^2 + (E_m/2)^2}/2\hbar$

# Coupling of Two Qubits

## State Readout

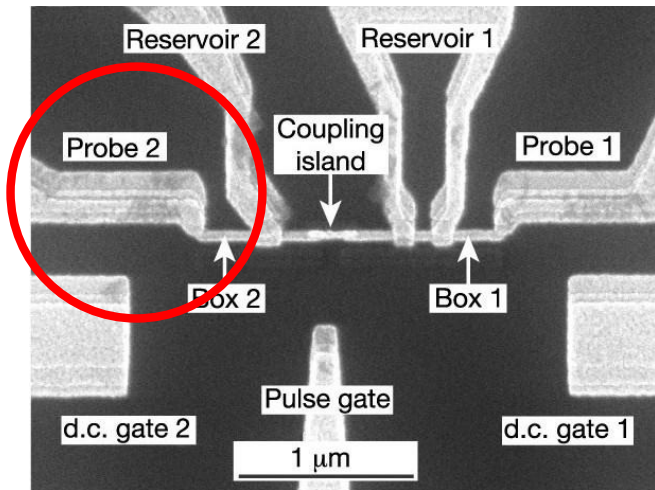


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>

# Coupling of Two Qubits

## State Readout

- ▶ Probe currents are proportional to the qubit state probabilities:  $I_1 \propto p_1(1) = |c_2|^2 + |c_4|^2$  and  $I_2 \propto p_2(1) = |c_3|^2 + |c_4|^2$
- ▶ The current oscillation frequency is related to the Josephson energy as:  $\omega_{1,2} = \frac{E_{J1,J2}}{\hbar}$

# Coupling of Two Qubits

## Parameter Measurements / Frequency Responses

- ▶ Tune system by bringing to point L or R, exciting oscillations in one qubit
- ▶ Get cosines with exponential decay
- ▶ Fourier transform gives frequencies, which define  $E_{J1}$  or  $E_{J2}$
- ▶ Drive system to point X and perform same Fourier measurement
- ▶ This time, get 2 frequencies,  $\Omega \pm \epsilon$

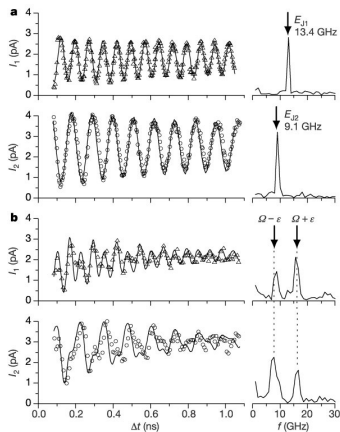


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>



# Coupling of Two Qubits

## $E_{J1}$ dependence of spectrum components

Note: Frequency repulsion at  $E_{J1} \approx E_{J2}$

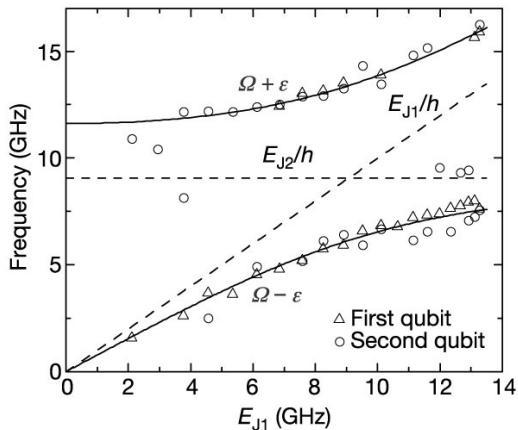


Figure : <http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>

# Coupling of Two Qubits

## Decoherence

- ▶ Probe Junction
- ▶ Charge Qubit Noise, ie  $n_g \rightarrow n_g + \delta n_g(t)$
- ▶ Solid State Noise
  - ▶ Thermal Noise
  - ▶ Material Imperfections
  - ▶ Charge and Flux Noise

# THE END

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## Reference Papers

<http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html>

<http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf>

(Figures cited individually)