Two-Qubit Dynamics with Josephson Qubits

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Background

Background

Cooper Pair Box

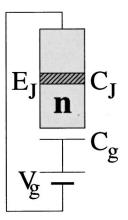


Figure: http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357

Background SQUIDs

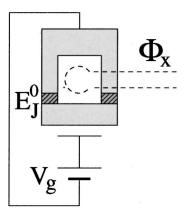


Figure: http://journals.aps.org/rmp/pdf/10.1103/RevModPhys.73.357

Background

Single-Qubit Charging Diagram

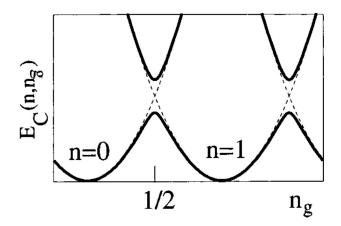
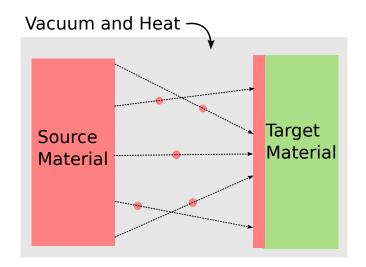


Figure: A simple caption

Evapouration (Deposition)



Shadow Evapouration

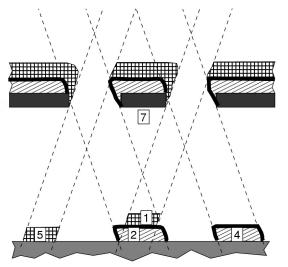
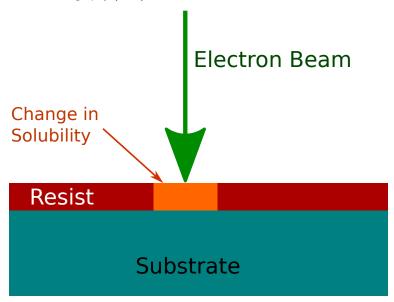


Figure: http://en.wikipedia.org/wiki/Niemeyer-Dolan_technique

Fabrication Techniques Electron Beam Lithography (EBL)



Fabrication Techniques Etching

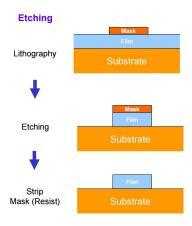


Figure: http://www.mrsec.harvard.edu/education/ap298r2004/Erli%20chen%20Fabrication%20III%20-%20Etching.pdf

Lift-off

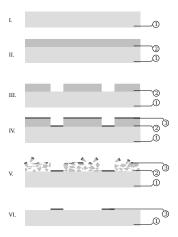


Figure: http://en.wikipedia.org/wiki/Lift-off_%28microtechnology%29

SEM image of a SQUID qubit

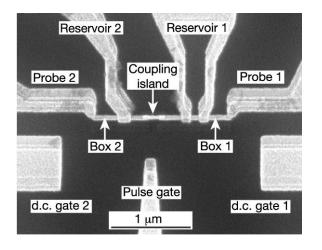
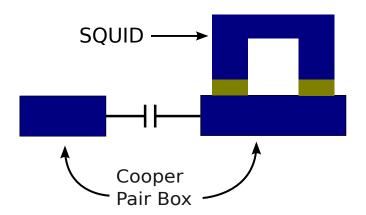


Figure: http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Basic Idea



The Circuit

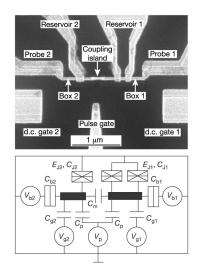


Figure: http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Charging Diagram of Two-Qubit Case

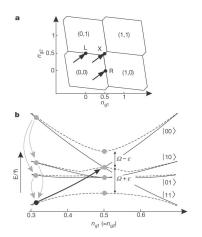


Figure: http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Points L and R

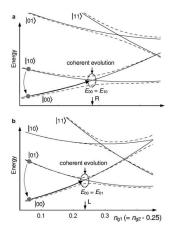


Figure: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf

Charging Diagram Level Curves

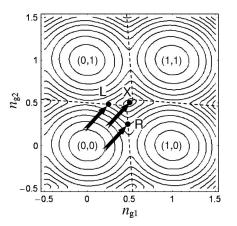


Figure: http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098&rep=rep1&type=pdf

Charging-Energy Diagram

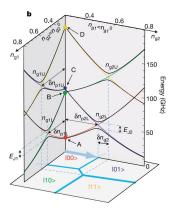


Figure: http://qudev.ethz.ch/content/courses/QSIT09/pdfs/Yamamoto2003.pdf

Theory

Note:
$$E_{J1,J2} \approx E_m < E_{C1,C2}$$

Hamiltonian

$$H = \begin{bmatrix} E_{00} & -\frac{1}{2}E_{J1} & -\frac{1}{2}E_{J2} & 0\\ -\frac{1}{2}E_{J1} & E_{10} & 0 & -\frac{1}{2}E_{J2}\\ -\frac{1}{2}E_{J2} & 0 & E_{01} & -\frac{1}{2}E_{J1}\\ 0 & -\frac{1}{2}E_{J2} & -\frac{1}{2}E_{J1} & E_{11} \end{bmatrix}$$

Where...

- $E_{n1n2} = E_{c1}(n_{g1} n_1)^2 + E_{c2}(n_{g2} n_2)^2 + E_m(n_{g1} n_1)(n_{g2} n_2)$
- $ightharpoonup E_{Ji}$ is the Josephson energy of the i^{th} box
- ▶ $E_{c1,c2} = 4e^2 C_{\Sigma 2,\Sigma 1}/2(C_{\Sigma 1}C_{\Sigma 2} C_m^2)$ are the effective Cooper pair charging energies
- $ightharpoonup C_{\Sigma i}$ is the sum of all capacitances connected to the i^{th} island
- ▶ $n_{g1,g2} = (C_{g1,g2}V_{g1,g2} + C_{\rho}V_{\rho})/2e$ is the charge, indiced by the gate and pulse voltages, on the qubits
- $E_m = 4e^2 C_m/(C_{\Sigma 1}C_{\Sigma 2}-C_m^2)$ is the coupling energy of the qubits

Probabilities

- At the coresonance point, we have a coherent superposition state
- ie, the qubit state probabilities are $p_1(1) = |c_2|^2 + |c_4|^2$ and $p_2(1) = |c_3|^2 + |c_4|^2$
- Now we initialize system to $|\psi\rangle=|00\rangle$

 $|\psi\rangle = c_1 |00\rangle + c_2 |10\rangle + c_3 |01\rangle + c_4 |11\rangle$

lacktriangle Using this Hamiltonian and an ideal rectangular pulse of length Δt ,

$$p_{1,2}(1) = \frac{1}{4} \left(2 - (1 - \chi_{1,2}) cos[(\Omega + \epsilon) \Delta t] - (1 + \chi_{1,2}) cos[(\Omega - \epsilon) \Delta t] \right)$$

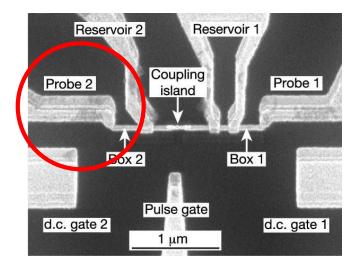
Where...

$$\chi_{1,2} = \frac{(E_{J_2,J_1}^2 - E_{J_1,J_2}^2) + E_m^2/4}{4\hbar^2 \Omega \epsilon}$$

•
$$\Omega = \sqrt{(E_{J1} + E_{J2})^2 + (E_m/2)^2}/2\hbar$$

•
$$\epsilon = \sqrt{(E_{J1} - E_{J2})^2 + (E_m/2)^2}/2\hbar$$

State Readout



 $Figure: \ http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html$

State Readout

$$|\psi\rangle=c_1|00\rangle+c_2|10\rangle+c_3|01\rangle+c_4|11\rangle$$

- Probe currents are proportional to the qubit state probabilities: $I_1 \propto p_1(1) = |c_2|^2 + |c_4|^2$ and $I_2 \propto p_2(1) = |c_3|^2 + |c_4|^2$
- ▶ The current oscillation frequency is related to the Josephson energy as: $\omega_{1,2} = \frac{E_{J1,J2}}{\hbar}$

Parameter Measurements / Frequency Responses

- Tune system by bringing to point L or R, exciting oscillations in one qubit
- Get cosines with exponential decay
- Fourier transform gives frequencies, which define E_{J1} or E_{J2}
- Drive system to point X and perform same Fourier measurement
- This time, get 2 frequencies, $\Omega \pm \epsilon$

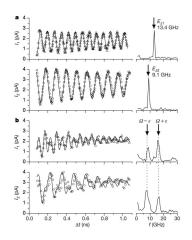


Figure: http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

EJ1 dependence of spectrum components

Note: Frequency repulsion at $E_{J1} \approx E_{J2}$

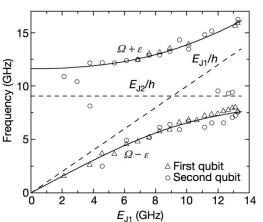


Figure: http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html

Decoherence

- Probe Junction
- ▶ Charge Qubit Noise, ie $n_g \rightarrow n_g + \delta n_g(t)$
- Solid State Noise
 - Thermal Noise
 - Material Imperfections
 - Charge and Flux Noise

THE END

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Reference Papers

 $\label{lem:http://www.nature.com/nature/journal/v421/n6925/full/nature01365.html $$ $$ http://citeseerx.ist.psu.edu/viewdoc/download?doi=10.1.1.193.5098\&rep=rep1&type=pdf (Figures cited individually) $$$