$TdS = dU + PdV - \mu dN$	
Doltamon	
$\mathbf{Boltzmann}_{NL^{N_j}}$	
$\omega_i = \frac{N \cdot g_i}{N \cdot I(N - N \cdot)I}$	
$\omega_j = \frac{N! g_j^{N_j}}{N_j! (N - N_j)!}$	
$\omega_B = N! \prod rac{g_j^{N_j}}{N_j!}$	
$r = N_j$ . $N_j = N_j - \epsilon_j / kT$	
$f_j = rac{N_j}{g_j} = rac{N}{Z} e^{-\epsilon_j/kT}$	
$\mathbf{Maxwell}$ - $\mathbf{Boltzmann}$	
$egin{aligned} \mathbf{Maxwell-Boltzmann} \ \omega_j &= rac{g_j^{N_j}}{N_j!} \end{aligned}$	
$\omega_j = N_j!$	
$\omega_{MB} = \prod_{j=1}^{N_j} \frac{g_j^{N_j}}{N_j!}$	
$\omega_{MB} = \prod_{N_j!} N_j!$	
$f_j = \frac{N}{Z}e^{-\epsilon_j/kT} = (e^{(\epsilon_j - \mu)/kT})^{-1}$	
Fermi-Dirac	
$\omega_j = \frac{g_j!}{N_j!(g_j - N_j)!}$	
$N_j!(g_j-N_j)!$	
$\omega_{FD} = \prod_{i=1}^{n} \frac{g_i}{N_i!(g_i - N_i)!}$	
$\omega_{FD} = \prod_{\substack{j=1 \ N_j! (g_j - N_j)!}} \frac{g_j!}{N_j! (g_j - N_j)!}$ $f_j = (e^{(\epsilon_j - \mu)/kT} + 1)^{-1}$	
Bose-Einstein	
$(N_j + g_j - 1)!$	
$egin{aligned} & \mathbf{Bose\text{-}Einstein} \ \omega_j &= rac{(N_j + g_j - 1)!}{N_j!(g_j - 1)!} \ \omega_{BE} &= \prod rac{(N_j + g_j - 1)!}{N_j!(g_j - 1)!} \ f_j &= (e^{(\epsilon_j - \mu)/kT} - 1)^{-1} \end{aligned}$	
$\omega_{BE} = \prod \frac{(N_j + g_j - 1)!}{N_j!(g_j - 1)!}$	
$f = (c_i(\epsilon_i - \mu)/kT  1)-1$	
$f_j = (e^{-\gamma} \cdot \gamma - 1)$	
Derive Distribution	
$\ln(N!) \approx N \ln(N) - N$	
$0 = \frac{\partial}{\partial N_j} (\ln(\omega) + \alpha(\sum N_j - N) +$	
$\beta(\sum \epsilon_j N_j - U)$	
$\alpha, \beta: TdS = dU + PdV - \mu dN$	
Derive Density of States	
$n^2 = \left(\frac{8mV^{-2/3}}{h^2}\right)\epsilon \equiv R^2$	
$n^{2} = \left(\frac{1}{h^{2}}\right)\epsilon = R^{2}$	
$n = \frac{1}{8} \frac{4\pi R^3}{3}$	
$a(\epsilon)d\epsilon = \gamma \cdot \frac{dn}{d\epsilon}d\epsilon$	
$g(\epsilon)d\epsilon = \gamma_s \frac{dn}{d\epsilon} d\epsilon$	
$g(\epsilon)d\epsilon = \gamma_s \frac{4\sqrt{2}\pi V}{h^3} m^{3/2} \epsilon^{1/2} d\epsilon$	
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 $[N, T, Z, \ln(Z)]$ 

$$\begin{array}{c} MB \implies S, F, \mu \\ S = \frac{U}{T} + Nk(1 - \ln(\frac{N}{Z})) \\ F = NkT(\ln(\frac{N}{Z}) - 1) \\ \mu = kT \ln(\frac{N}{Z}) \\ Z: \gamma_s = 1 \\ Z = V(2\pi mkT/h^2)^{3/2} = Vn_Q \\ \ln(Z) = \frac{3}{2} \ln(T) + \ln(V) + \frac{3}{2} \ln\left(\frac{2\pi mk}{h^2}\right) \\ PV = NkT \\ U = \frac{3}{2}NkT \\ \lim: \lambda_{3d}^3 << \frac{V}{N}, \lambda_{dB} \equiv \left(\frac{h^2}{2\pi mkT}\right)^{1/2}, \\ (\frac{V}{N})^{1/3} = l \equiv d_{avg} \\ \text{MSD: } g(\epsilon)d\epsilon, Z, \epsilon \rightarrow \frac{1}{2}mv^2 \\ \Longrightarrow N(\epsilon)d\epsilon = 4\pi N\left(\frac{m}{2\pi kT}\right)^{3/2}v^2e^{-mv^2/2kT}dv \\ \Delta \omega_{mix} = \frac{\omega_2}{\omega_1} = \frac{N!}{N_1!N_2!}, \\ \Delta S = \dots = -Nk(x_1\ln(x_1) + x_2\ln(x_2)) \\ Z_{mono} = Z_{trans} Z_{vib}Z_{rot} \\ Z_{vib} = \frac{exp(-\theta^{vib}/2T)}{1 - exp(-\theta^{vib}/2T)} \\ Z_{rot} = \frac{T}{g^{rot}} \\ \textbf{Oscillations} \\ \epsilon_j = (j + \frac{1}{2})h\nu \\ \theta_{vib} = \frac{h\nu}{k} \\ \theta_{vib,H2} = 6000K \\ \Longrightarrow Z_{vib} \\ U_{vib} = NkT^2 \frac{\partial \ln(Z)}{\partial T} = Nk\theta(\frac{1}{2} + \frac{1}{\exp(\theta/T)-1}) \\ \approx NkT, \frac{T}{\theta} > 1 \\ T > 1 \implies C_V = Nk \\ T < 1 \implies C_V = Nk \\ T < 1 \implies C_V = Nk \\ \frac{\theta}{\theta^2} = \frac{\theta^2}{\theta^2} \\ \theta_{rot,H2} = 80K \\ g_j = 2j + 1 \\ T > > \theta_{rot} \\ \Longrightarrow U_{rot} = NkT \\ \Longrightarrow C_V, rot = Nk \\ T < \theta_{rot} = \frac{h^2}{2ik} \\ \theta_{rot,H2} = 80K \\ g_j = 2j + 1 \\ T > > \theta_{rot} \\ \Longrightarrow D(z) \approx \ln(1 + 3e^{-2\theta_{rot}/T}) \approx 3e^{-2\theta_{rot}/T} \\ \Longrightarrow C_V, rot = Nk \\ T < \theta_{rot} = \frac{h}{2} \\ \Longrightarrow C_V, rot = Nk \\ T = -NkT \ln Z \\ S = \frac{U}{T} + 3Nk \ln Z \\ \theta_E = h\nu/k, \quad \nu = \frac{1}{2\pi} \sqrt{\frac{\kappa}{\mu}} \\ U = 3Nk\theta_E(\frac{1}{2} + \frac{1}{e^{\theta_E/T}-1}) \\ T > \theta_E \implies C_V \approx 3Nk \\ T < \theta_E/T)^2 e^{-\theta_E/T} \\ \text{Debye} \\ \end{cases}$$

 $g(\nu)d\nu = 4\pi V \left| \frac{1}{c_l^3} + \frac{2}{c_t^3} \right| \nu^2 d\nu$  $N(\nu)d\nu = \left(\frac{9N}{\nu_m^3}\right) \frac{\nu^2 d\nu}{e^{h\nu/kT}-1}$  for  $\nu \le$  $\theta_D = \frac{h\nu_m}{k}$ Fermi Energy  $\mu(0) = \frac{h^2}{2m} \left(\frac{3N}{8\pi V}\right)^{2/3}$   $\epsilon_F = kT_F$  $T << T_F$  $\implies \mu \approx \mu(0) \left(1 - \frac{\pi^2}{12} \left[\frac{T}{T_F}\right]^2\right)$  $High T \implies Ideal Gas$ spatial wavefn  $N = (num) \times \frac{1}{8} \times$  $\tfrac{4}{3}\pi n_{max}^3$  $\epsilon_F = \frac{h^2}{8m} V^{-2/3} n_{max}^2$ Blackbody  $f_j = \frac{1}{e^{\epsilon_j/kT} - 1}$ Two polarization states  $g(\nu)d\nu = \frac{\$\pi V}{c^3}\nu^2 d\nu$ Planck Radiation  $u(\nu)d\nu$  $g(\nu)f(h\nu)h\nu d\nu$  $\implies u(\nu)d\nu = \frac{8\pi hV}{c^3} \left(\frac{\nu^3 d\nu}{e^{h\nu/kT}-1}\right)$  $\nu = c/\lambda \implies d\nu = (-c/\lambda^2)d\lambda$  $\lambda_{max}$  defined by  $0 = \frac{d}{d\lambda} [\lambda^5 (e^{hc/\lambda kT} -$ 1) $\implies \lambda_{max}T = 2.90 \times 10^{-3}$  $P_{rad} = (\text{Area})\sigma T^4, \sigma = 5.67 \times 10^{-8}$  $hc/\lambda kT >> 1$  $\implies u(\lambda)d\lambda = V \frac{8\pi kT}{\lambda^4} d\lambda$  $hc/\lambda kT << 1$  $\implies u(\lambda)d\lambda = V \tfrac{8\pi hc}{\lambda^5} e^{-hc/\lambda kT} d\lambda$  $Flux = (1 - A)F_S/4 = \sigma T_S^4, \sigma =$  $5.67 \times 10^{-8}$ Photon Gas  $N = 2.02 \times 10^7 T^3 V$  $\begin{array}{l} U = 7.55 \times 10^{-16} T^4 V \\ S = \frac{32 \pi^5 kV}{45} \left(\frac{kT}{hc}\right)^3 \end{array}$ Single Osc, no restriction on N  $\implies \ln(z) = -\ln(1 - e^{-h\nu/kT})$ So using  $g(\nu)d\nu = \frac{8\pi V}{c^3}\nu^2 d\nu$  get  $\ln(Z) = \frac{8\pi^5}{45} \left(\frac{kT}{hc}\right)^3 V$  $kT^2 \left(\frac{\partial \ln Z}{\partial T}\right)_{V,N}$  $\frac{8\pi^5 k}{15} \left(\frac{k}{hc}\right)^3 T^4 V$  $\mu = -kT \left( \frac{\partial \ln Z}{\partial N} \right)_{V,T} = 0$  $S = U/T + k \ln Z = \frac{32\pi^5 k}{45} \left(\frac{k}{hc}\right)^3 T^3 V$  $P = kT \left( \frac{\partial \ln Z}{\partial V} \right)_{T,N} = U/3V$ BE Condensation  $N = \int_0^\infty N(\epsilon) d\epsilon = \int_0^\infty f(\epsilon) g(\epsilon) d\epsilon$  $\implies N_{ex} = 2.612 V \left(\frac{2\pi mkT}{h^2}\right)^{3/2}$ , this also gives  $T_B$  when  $N_{ex} = N$  $U = 0.77NkT\left(\frac{T}{T_B}\right)$  $N_0 = N(1 - [T/T_C]^{3/2})$  $S = 1.28Nk[T/T_C]^{3/2}$