Simulation project

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1 Develop a model of the system

Factors:
Number of Canadian agents. 17 levels, $\{0, 1, \ldots, 16\}$.
Strategy used. 2 levels, $\{Strategy1, Strategy2\}$

My design is a full factorial design. I reduce the level of Number of Canadian agents to 10 levels — from $\{0, 1, .., 16\}$ to $\{4, 5, ..., 13\}$, cause when number of Canadian agents is less than 4 or more than 13, the time passengers spend is unbearable to test.

Assumptions I made:
1. There are exactly 700 passengers and the 455 of them are Canadians and 245 of them are visitors.
2. Passengers arrival every 1200/700 seconds 3. There is no scenario that when the agent have processed one passenger in the queue, the next passenger has not arrived.

Detailed model is showed below.
Nc - number of Canadian passengers.
Nv - number of visitor passengers.
λc - arrival rate of Canadians.
λv - arrival rate of visitors.
γ - passenger arrival interval.
Mc - number of Canadian agents.
Mv - number of visitor agents.
sc - service time of one Canadian agent.
sv - service time of one visitor agent.
μc - service rate of one Canadian agent.
μv - service rate of one visitor agent.
Tw - waiting time of a passenger.
T - the total time a passenger spend in the Customs and Immigration area.

As we know,
$Mc + Mv = 16$
$Nc = 700 * 65\% = 455$; $Nv = 700 * 35\% = 245$
$\gamma = \frac{20*60sec}{Nc+Nv-1} = 1.717sec$
$\frac{1}{\mu c} = sc = 40sec$; $\frac{1}{\mu v} sv = 75sec$
The goal is to minimize $\overline{T}$.

1.1 Strategy1

As showed in Fig. 1. The average arrival rate of Canadian queue is $\overline{\lambda c} = Nc/20 * 60sec$; The average arrival rate of visitor queue is $\overline{\lambda v} = Nv/20 * 60sec$.

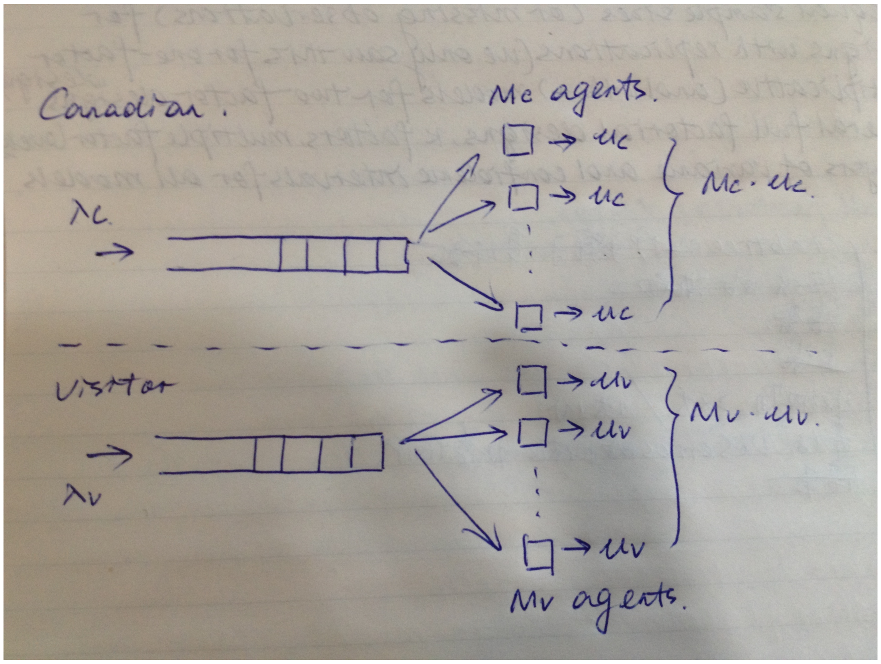

Figure 1: Strategy1

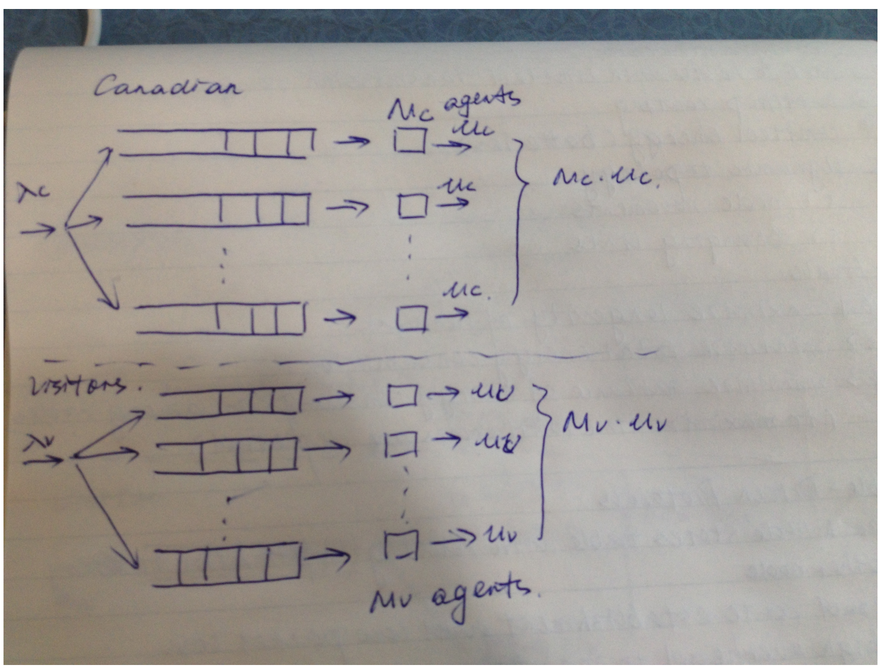

Figure 2: Strategy2

For Canadian passenger i, arriving time is t. The processing time of this first i passengers is $\frac{i}{\mu c * Mc}$. So the time Canadian i spend in the Customs and Immigration area is $\frac{i}{\mu c * Mc} - t$

For visitor passenger i, arriving time is t. The processing time of this first i passengers is $\frac{i}{\mu v * Mv}$. So the time visitor i spend in the Customs and Immigration area is $\frac{i}{\mu v * Mv} - t$

$$\overline{T} = (\sum_{i=1}^{Nc} Ti + \sum_{i=1}^{Nv} Ti)/700$$

$$= [\sum_{i=1}^{455}(\frac{i}{\mu c * Nc} - ti) + \sum_{i=1}^{245}(\frac{i}{\mu v * Nv} - ti)]/700$$

$$= \{\sum_{i=1}^{455}[\frac{i}{\frac{1}{40sec} * Nc} - \frac{i-1}{455-1} * (20 * 60)sec] + \sum_{i=1}^{245}[\frac{i}{\frac{1}{75sec} * (16 - Nc)} - \frac{i-1}{245-1} * (20 * 60)sec]\}/700$$

$$= [\frac{40sec}{Nc}\sum_{i=1}^{455} i - \frac{1200sec}{454}\sum_{i=1}^{455}(i-1) + \frac{75sec}{16 - Nc}\sum_{i=1}^{245} i - \frac{1200sec}{244}\sum_{i=1}^{245}(i-1)]/700$$

$$= [\frac{40sec}{Nc} * \frac{456 * 455}{2} - \frac{1200sec}{454} * \frac{454 * 455}{2} + \frac{75sec}{16 - Nc} * \frac{246 * 245}{2} + \frac{1200sec}{244} * \frac{244 * 245}{2}]/700$$

$$= \frac{40 * 456 * 455sec}{2 * 700} * Nc^{-1} + \frac{75 * 246 * 245sec}{2 * 700} * (16 - Nc)^{-1} - \frac{1200 * 455 + 1200 * 245}{2 * 700} sec$$

$$= 5928 * Nc^{-1} + 3228.75 * (16 - Nc)^{-1} - 600$$

So, the theoretically average time is:

Table 1: Strategy one simulation results

Canadian agent number	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
ave time(sec)	1151.06	879.12	710.88	605.61	544.59	519.92	530.93	584.66	701.19	932.25

1.2 Strategy2

As showed in Fig. 2. The average arrival rate of each Canadian queue is $\overline{\lambda c} = Nc/20 * 60sec/Mc$; The average arrival rate of visitor queue is $\overline{\lambda v} = Nv/20 * 60sec/Mv$.

The only difference between Strategy2 and Strategy1 is when passengers arrive, they join the shortest queue. The result of Strategy2 should the same as Strategy1 in math.

However in fact, there must be a situation that some agents(and queues) are idle while other agents(queues) are busy. So the minimum mean time in the Customs and Immigration area should be no less(usually longer) than Strategy 1. This difference can only be found by simulation.

2 Simulation program

2.1 Class

I use four classes.

Passenger: this class presents a single passenger. If this passenger is a Canadian, the time him arrives, the time him departures, and the time him needs to be serviced are recorded there.

WaitingQueue: If this is strategy one, then it will generate 2 waiting queues, otherwise, it will generate 16 waiting queues.

Agent: this class presents a single agent. It will get passenger from the waiting queue and serve them.

Agents: schedule all the agents.

2.2 Thread

There are 18 threads. One main thread, one thread for adding the passengers to the waiting queue, and another 16 threads for each agent to process passengers.

2.3 Exponential distribution

As we know the cumulative distribution function of exponential distribution is:

$$F(x; \lambda) = 1 - e^{-\lambda x}, x \geq 0$$

$$F^{-1}(x) = -\frac{ln(1 - P(X \leq x))}{\lambda}$$

So suppose we need to generate 245 numbers, which is an exponential distribution of $\lambda = 75000$, then we can divide the probability to intervals of $1/245 = 0.0041$, corresponding, $F^{-1}(x)$ is also divided into intervals. According to the definition of CDF, there must be a number between each interval. for example, there must be a number between $F^{-1}(1 * 0.0041)$ and $F^{-1}(2 * 0.0041)$

Now we need to get a random number between each interval. How I did this is to genterate a random number *random* between 0 and 10000. Let $leftBound$ and $rightBound$ denote the boundary of a interval (how to get the boundary of interval has been introduced above). Then $leftBound + random/10000 * (rightBound - leftBound)$ is what we want.

In this way we can generate a set of numbers that are exponentially distributed.

2.4 Generate passengers

Accordong to the requirement, each passenger has 65% possibility to be a Canadian and 35% to be a visitor, and their service time should be unordered.

However, the service time we get above is exponentially distributed and is in ascending order. So when I generate 700 passengers and give each a mark—*mark* 1,say, 1,2...700. At the same time I read the file sim-project.14-1.data.txt and assign each number to a passenger—*mark* 2. Then I sort the passengers by *mark* 2 and set the first 455 passengers as Canadians and the other 245 visitors, at the same time I assigned the ordered service time to the passengers. Then I sorted the passengers by *mark* 1. Now we have unordered passengers.

3 Run simulation

3.1 Strategy 1

Table 2: Strategy one simulation results

Canadian agent number	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
Replication 1(sec)	1148.54	876.55	706.8	604.15	541.59	514.73	523.08	574.22	685.55	904.56
Replication 2(sec)	1033.42	889.01	711.32	597.2	549.83	513.23	520.96	576.52	683.97	904.31
Replication 3(sec)	1021.68	890.34	699.55	606.12	545.6	515.64	523.47	574.77	686.21	906.75
ave time(sec)	1067.88	885.3	705.89	606.12	545.67	514.53	522.5	575.17	685.24	905.21

Table 3: Strategy one simulation results

Canadian agent number	**4**	**5**	**6**	**7**	**8**	**9**	**10**	**11**	**12**	**13**
average time(sec)			1088.34	825.8	636.57	540.02	546.01	591.91	710.72	937.05
average time(sec)			1120.56	1125.8	655.59	538.79	549.23	600.33	708.17	951.71
average time(sec)			1079.1	865.72	630.28	541.05	551.29	594.86	713.76	941.19
average time(sec)			1096	939.11	640.81	539.95	548.84	595.7	710.88	943.32

3.2 Strategy 2

From Table 2, the result we get from strategy 1 is almost the same as the expected average time we just calculated.

(I did not have time to run the program when Canadian agent number = 4,5, but the results are definite longer than 9 Canadian agents.)

From Table 3, we can see that the result we get from strategy 2 is bigger than we calculated.

4 Queueing model

As we have calculated in Section 1.1,

$$\overline{T} = 5928 * Nc^{-1} + 3228.75 * (16 - Nc)^{-1} - 600$$

Then,

$$\overline{T}(Nc)' = 5928 * (-1) * Nc^{-2} + 3228.75 * (-1) * (16 - Nc)^{-2} * (-1)$$

when

$$\overline{T}(Nc)' = 0$$

$$2699.25 * Nc^2 - 5928 * 32 * Nc + 5928 * 256 = 0$$

$$Nc = 9.21$$

$$\overline{T}(9) = \frac{5928}{9} + \frac{3228.75}{7} - 600 = 519.92sec$$

So 9 Canadian agents and 7 visitor agents can minimize the mean time in the Customs and Immigration area for passengers. And the minimum time is 519.92 seconds. This result is the same as the simulation result.

5 Recommendation

9 Canadian agents and 7 visitor agents can minimize the mean time in the Customs and Immigration area for passengers.

Besides, I discussed the simulation program with my classmates. My program use multi-thread, so the running time is almost the same as real situation while their program is event-driven. They think if I have time to run a program, I can use the time to observe the real situation. In my point of view, my program is a complete simulation, but it is more flexible than the real situation. For example, I can make a little change to my program to simulate other arrival pattern / service pattern, but in real situation, we can observe all the arrival pattern/ service pattern.