Trading the volatility surface

I collected standard options (for call and put) quotes for SPDR S&P 500 ETF Trust (Ticker: “SPY”) for all maturities. The following snapshot shows standard option quotes for SPY –

	contractSymbol	lastTradeDate	strike	lastPrice	bid	ask	change	percentChange	volume	openInterest	impliedVolatility
0	SPY230321C00332000	2023-03-20 14:22:14+00:00	332.0	59.39	65.04	65.23	0.0	0.0	8.0	4.0	2.061528
1	SPY230321C00333000	2023-03-13 16:10:40+00:00	333.0	55.88	64.05	64.23	0.0	0.0	NaN	0.0	2.035161
2	SPY230321C00340000	2023-03-20 19:46:47+00:00	340.0	53.34	57.08	57.26	0.0	0.0	4.0	4.0	1.849610
3	SPY230321C00342000	2023-03-20 13:32:52+00:00	342.0	49.19	55.01	55.19	0.0	0.0	54.0	54.0	1.760743
4	SPY230321C00345000	2023-03-20 18:40:31+00:00	345.0	47.20	52.08	52.23	0.0	0.0	23.0	13.0	1.701173
..	...	...	...	...	...	...	...	...	...	...	...
76	SPY251219P00570000	2023-02-14 14:38:15+00:00	570.0	159.75	175.00	180.00	0.0	0.0	NaN	0.0	0.171731
77	SPY251219P00600000	2023-03-06 14:30:10+00:00	600.0	195.00	200.50	205.50	0.0	0.0	2.0	2.0	0.144326
78	SPY251219P00610000	2023-02-07 17:03:46+00:00	610.0	199.44	217.00	222.00	0.0	0.0	1.0	0.0	0.207688
79	SPY251219P00630000	2023-02-02 20:15:08+00:00	630.0	215.27	223.50	228.50	0.0	0.0	NaN	0.0	0.000010
80	SPY251219P00650000	2023-02-23 20:39:26+00:00	650.0	248.00	250.50	255.50	0.0	0.0	1.0	0.0	0.165688

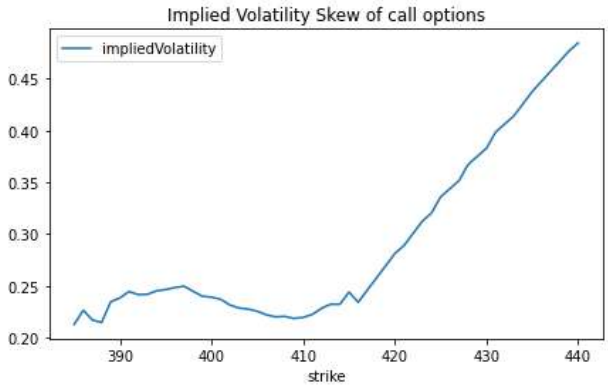
```
    inTheMoney contractSize currency optionType         expiration  \
0         True      REGULAR      USD       call 2023-03-21 23:59:59
1         True      REGULAR      USD       call 2023-03-21 23:59:59
2         True      REGULAR      USD       call 2023-03-21 23:59:59
3         True      REGULAR      USD       call 2023-03-21 23:59:59
4         True      REGULAR      USD       call 2023-03-21 23:59:59
..         ...          ...      ...        ...                 ...
76        True      REGULAR      USD        put 2025-12-19 23:59:59
77        True      REGULAR      USD        put 2025-12-19 23:59:59
78        True      REGULAR      USD        put 2025-12-19 23:59:59
79        True      REGULAR      USD        put 2025-12-19 23:59:59
80        True      REGULAR      USD        put 2025-12-19 23:59:59

    daysToExpiration
0                  1
1                  1
2                  1
3                  1
4                  1
..               ...
76              1005
77              1005
78              1005
79              1005
80              1005
```

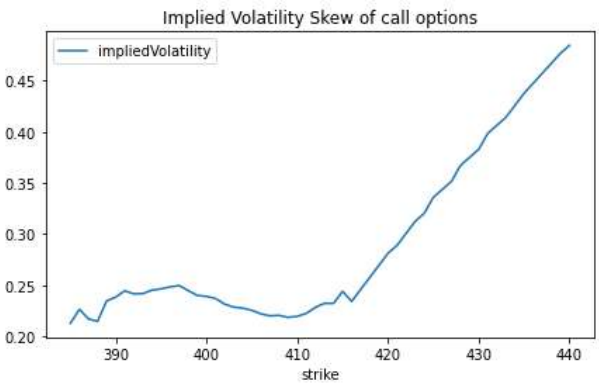
I observe that the implied volatilities of call options are significantly higher than those of put options. The last trading prices of put options are also significantly higher than those of call options. Now, let's investigate the option maturities –

```
array(['2023-03-21', '2023-03-22', '2023-03-23', '2023-03-24',
       '2023-03-27', '2023-03-28', '2023-03-29', '2023-03-30',
       '2023-03-31', '2023-04-06', '2023-04-14', '2023-04-21',
       '2023-04-28', '2023-05-19', '2023-06-16', '2023-06-30',
       '2023-07-21', '2023-08-18', '2023-09-15', '2023-09-29',
       '2023-10-20', '2023-12-15', '2023-12-29', '2024-01-19',
       '2024-03-15', '2024-06-21', '2024-12-20', '2025-01-17',
       '2025-03-21', '2025-12-19'], dtype='<U10')
```

I find that most of the call options are going to be expired much earlier than put options. Most of the option contracts are getting matured in 2023. I only find 4 maturity dates in 2024 and 3 maturity dates in 2025.

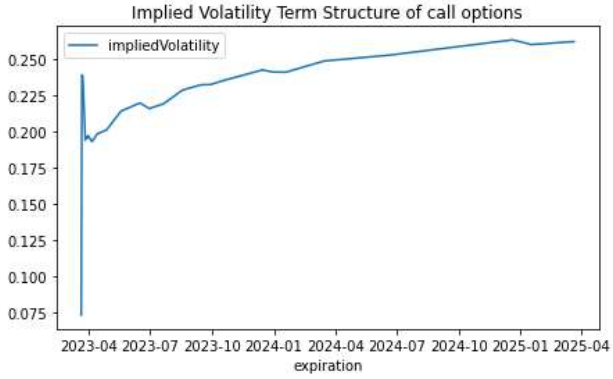
Implied Volatility Skew

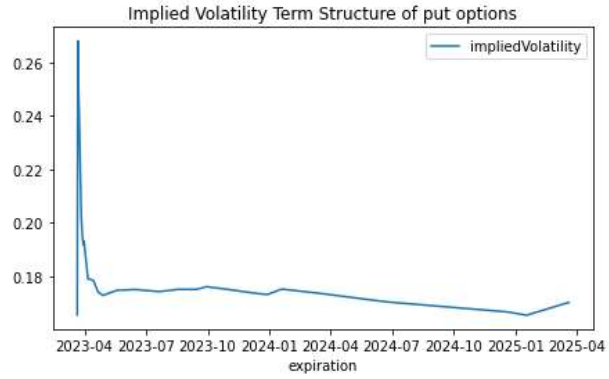




The charts show the implied volatility skew for call options and put options. Chart 1 shows that implied volatilities of call options increase till the strike prices reach 395, stay stable till 415, and then spike further. Chart 2 shows that implied volatilities of put options decline till the strike prices reaches 380, stay stable till 400 and then, spike further. So, we observed a positive relationship between implied volatility and strike price for call options and an inverse relationship between implied volatility and strike price for put options.

Implied Volatility Term Structure



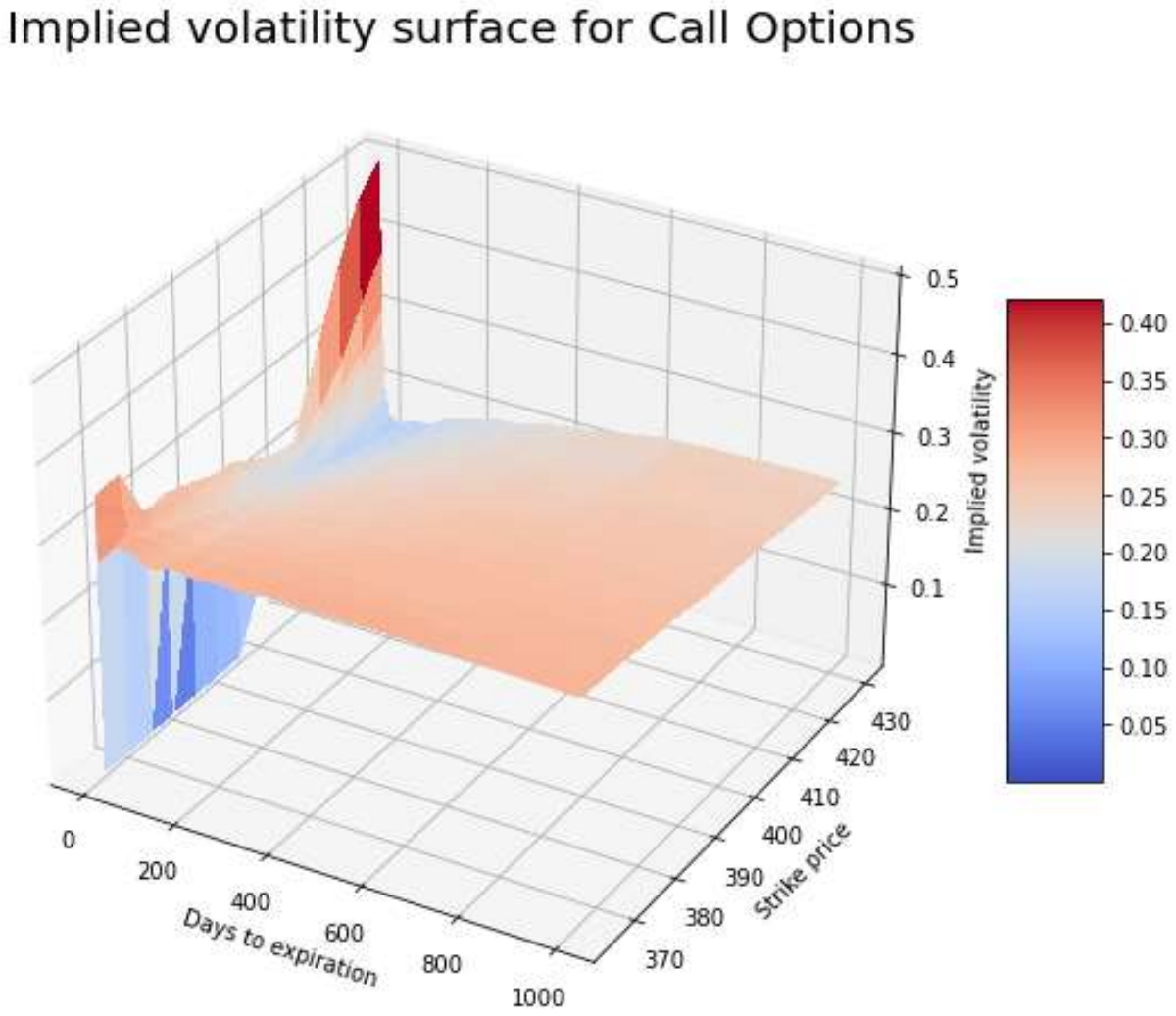


The charts show the implied volatility term structure for both call options and put options with strike price of 400. Chart 2 shows that the implied volatilities of puts at strike 400 spiked from .165 to 0.23 and then droped and continued to exhibit sluggish downward pattern with time. Chart 1 shows that the implied volatilities of calls at strike 400 spiked from 0.07 to 0.23 and then continued to exhibit gradual upward movement with time.

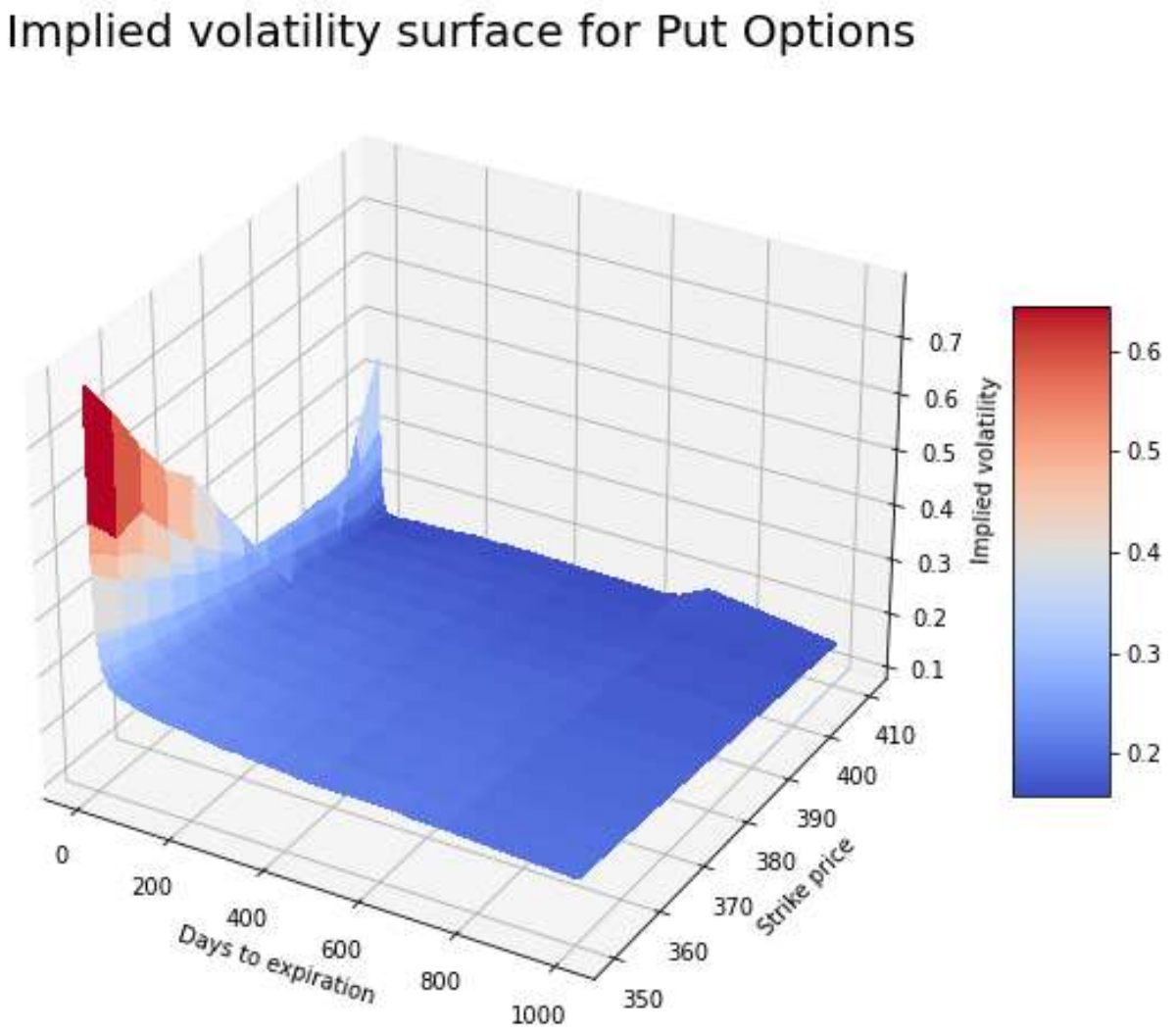
Implied Volatility Surface

The implied volatility surface refers to a three-dimensional plot that shows how the implied volatility of options varies with respect to strike price and expiration date. The surface is generated by collecting implied volatilities from various options on a particular underlying asset, such as a stock or a commodity, at different strike prices and expiration dates.

Implied volatility is the expected volatility of an underlying asset based on the prices of options that trade on that asset. The implied volatility surface is useful for traders and investors who want to analyze the market's expectations of future volatility and identify trading opportunities. The shape of the surface can reveal information about market sentiment, supply and demand dynamics, and the overall risk appetite of investors.

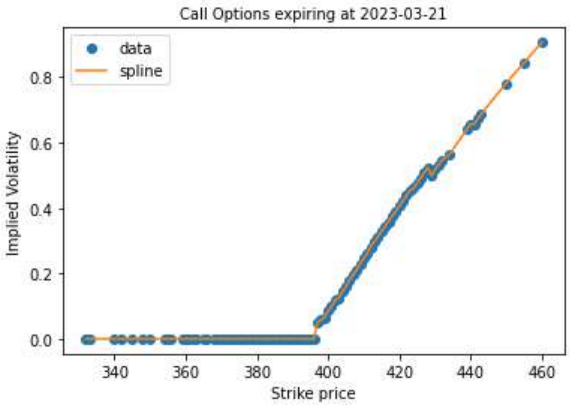


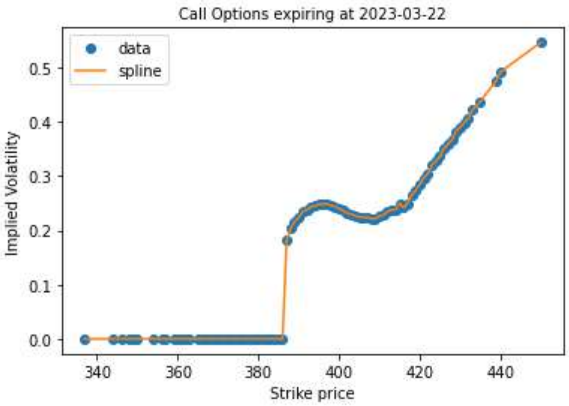
I observe that the implied volatility of call options spiked initially and continued to increase gradually with higher strike prices and longer days to expiration. From this 3-dimensional plot, I find that there is positive association of the implied volatility with both strike price and maturity for call options.



I observe that the implied volatility of put options spiked initially and continued to decline gradually with higher strike prices and longer days to expiration. From this 3-dimensional plot, I find that there is negative association of the implied volatility with both strike price and maturity for put options.

Spline model to the implied volatility curve for each of the maturity dates for call options


Call Options expiring at 2023-03-21
data
spline
Implied Volatility
Strike price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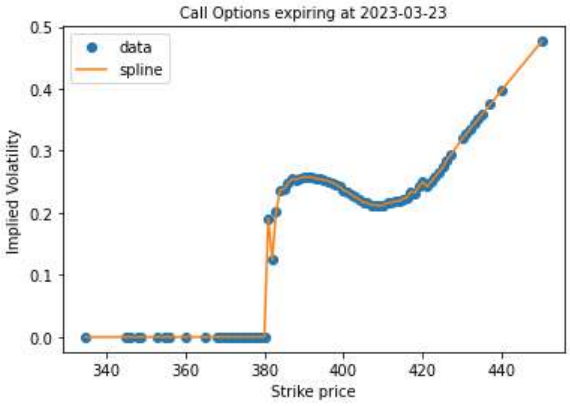
data
spline
Implied Volatility
Strike price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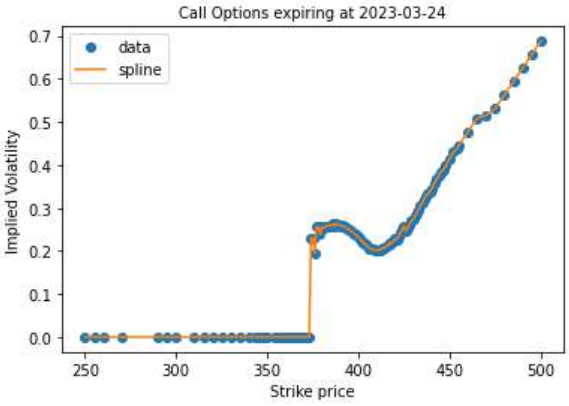
data
spline
Implied Volatility
Strike price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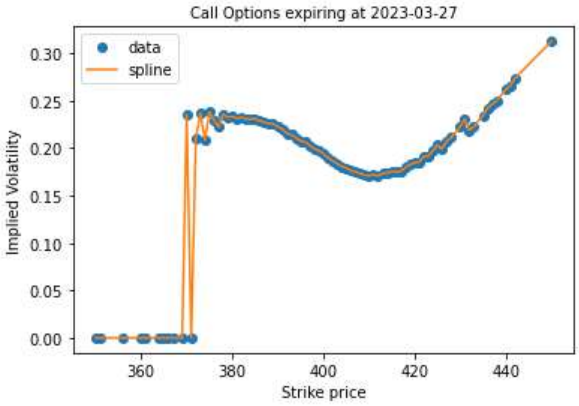
data
spline
Implied Volatility
Strike price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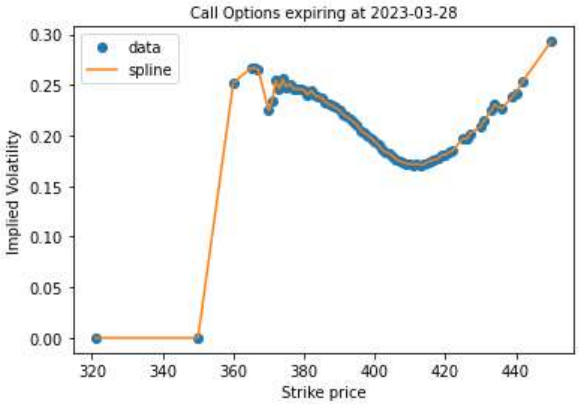
data
spline
Implied Volatility
Strike price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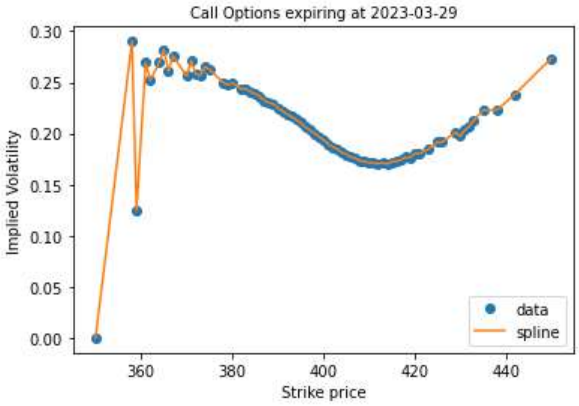
data
spline
Implied Volatility
Strike price


Call Options expiring at 2023-03-29
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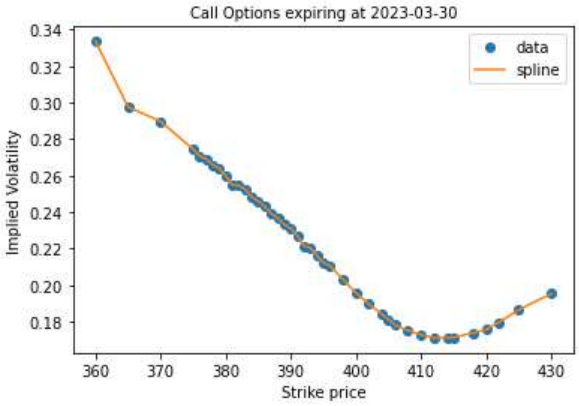
spline
Implied Volatility
Strike price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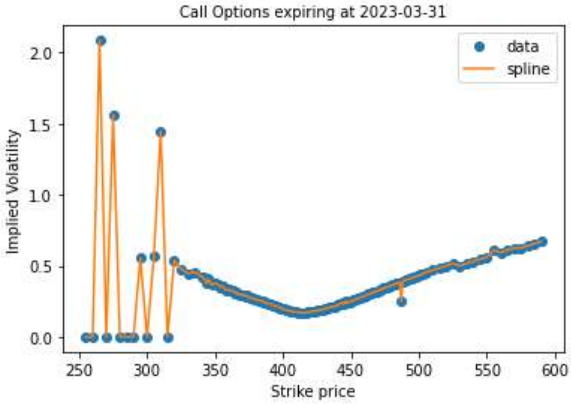
spline
Implied Volatility
Strike price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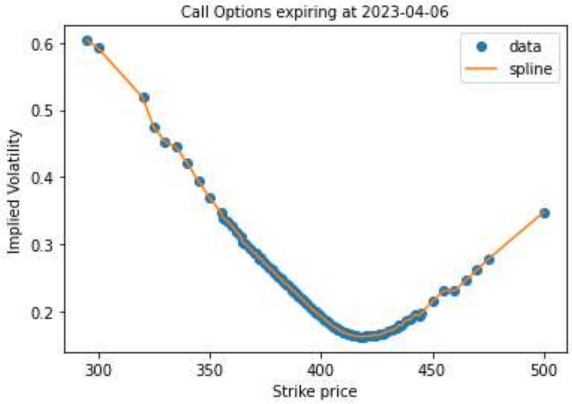
data
spline
Implied Volatility
Strike price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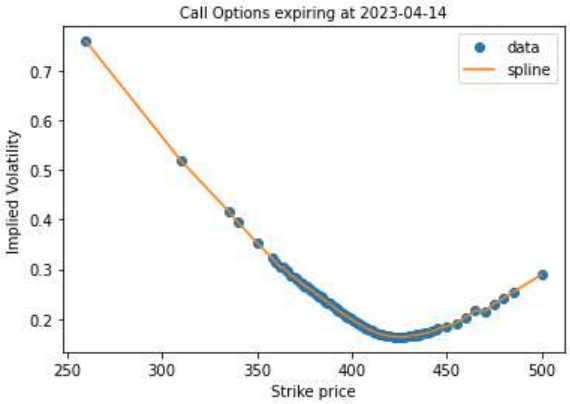
data
spline
Implied Volatility
Strike price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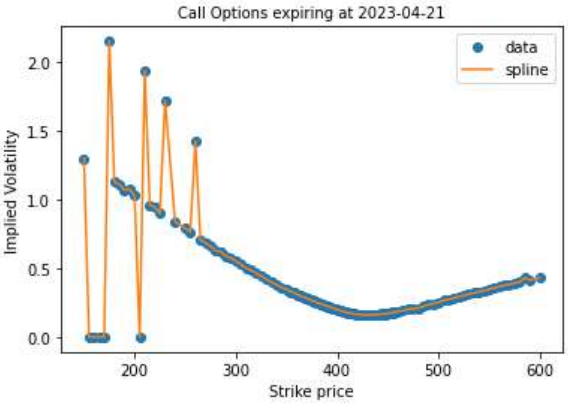
Implied Volatility
Strike price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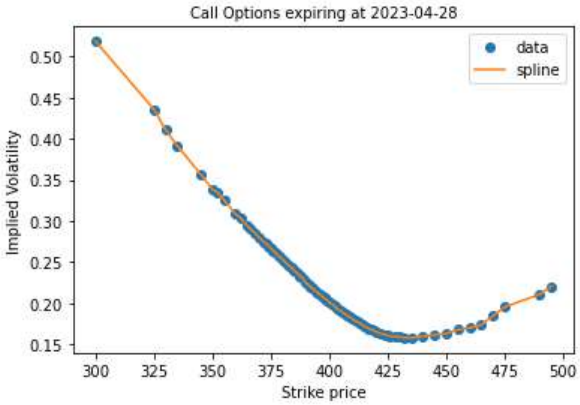
Implied Volatility
Strike price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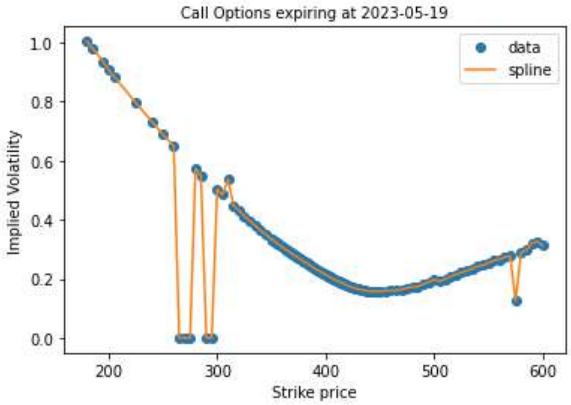
Implied Volatility
Strike price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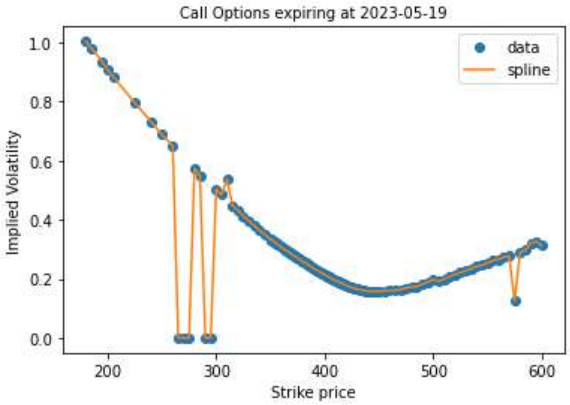
Implied Volatility
Strike price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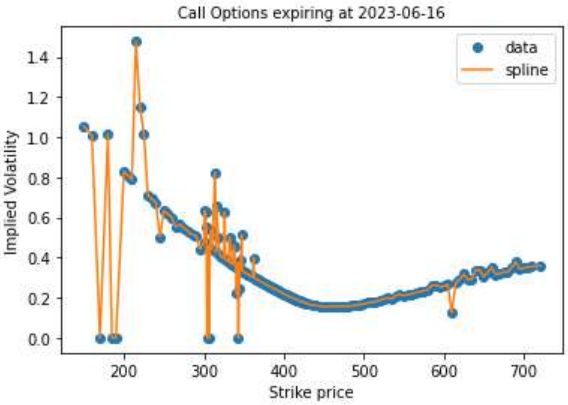
Strike price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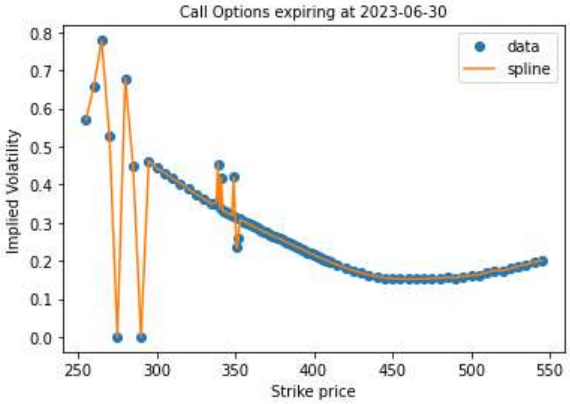
Implied Volatility
Strike price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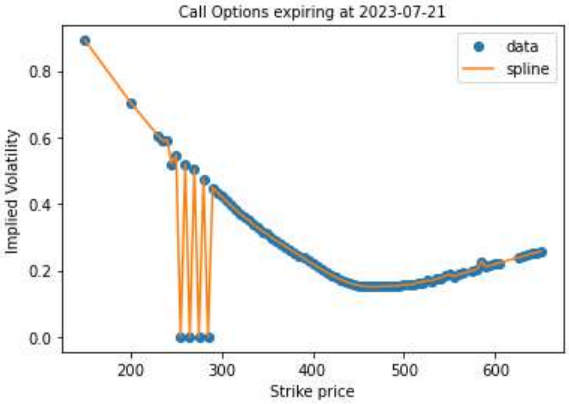
Implied Volatility
Strike price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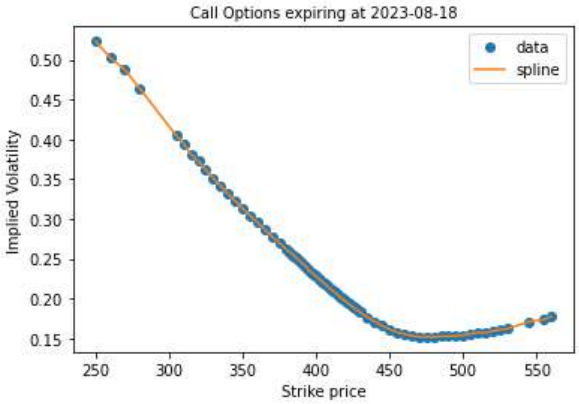
Implied Volatility
Strike price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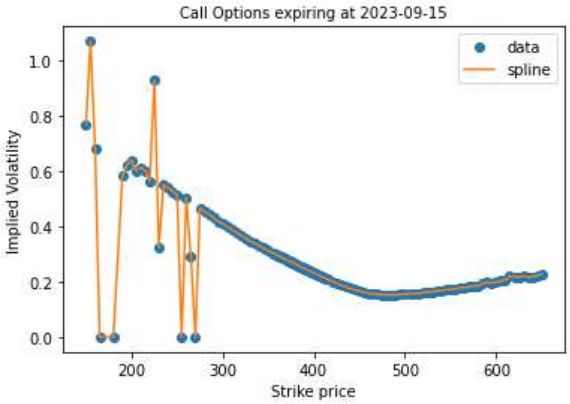
Implied Volatility
Strike price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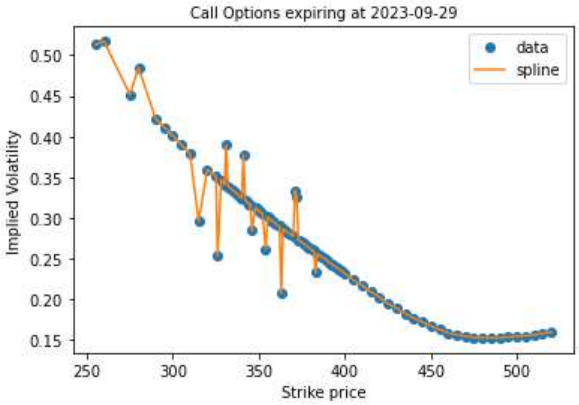
Implied Volatility
Strike price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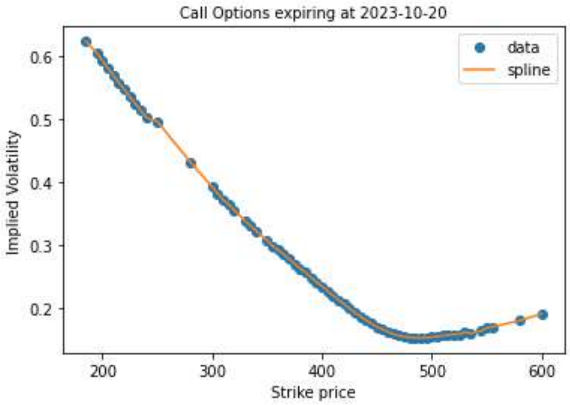
Implied Volatility
Strike price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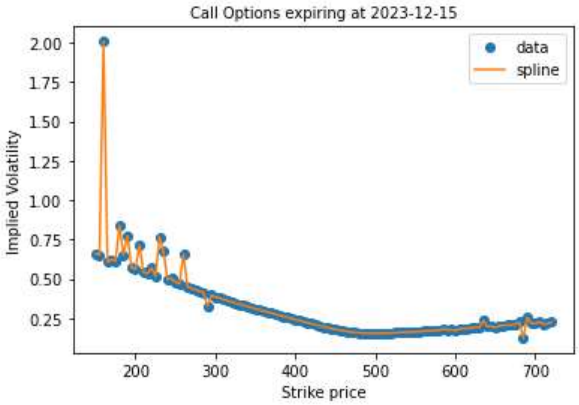
data
spline
Implied Volatility
Strike price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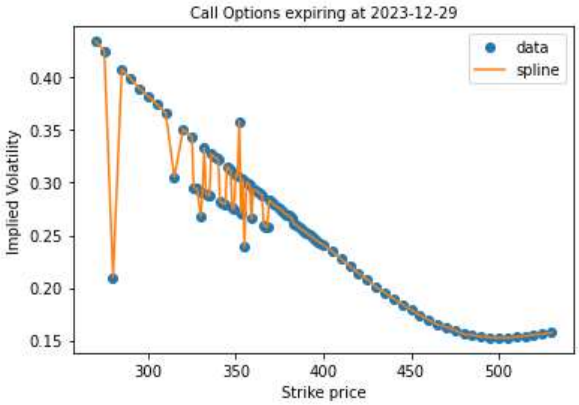
data
spline
Implied Volatility
Strike price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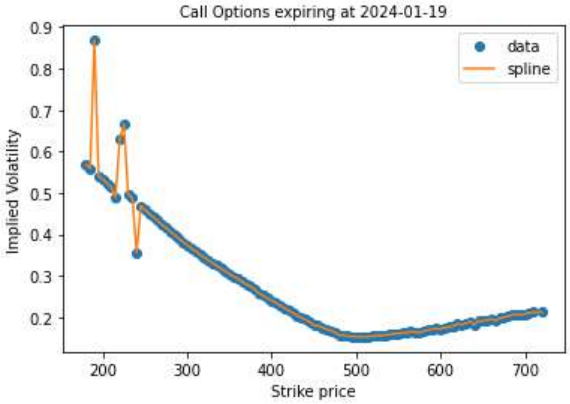
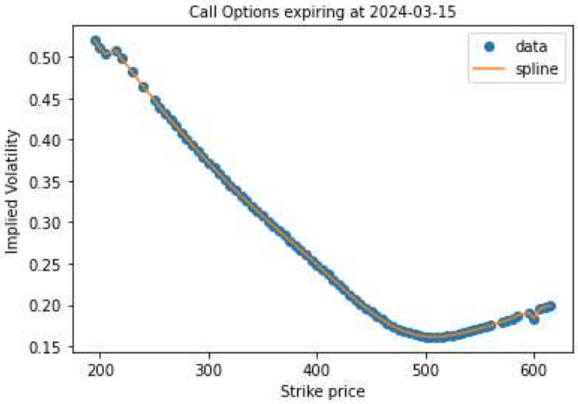
spline
Implied Volatility
Strike price


Call Options expiring at 2024-01-19
data
spline
Implied Volatility
Strike price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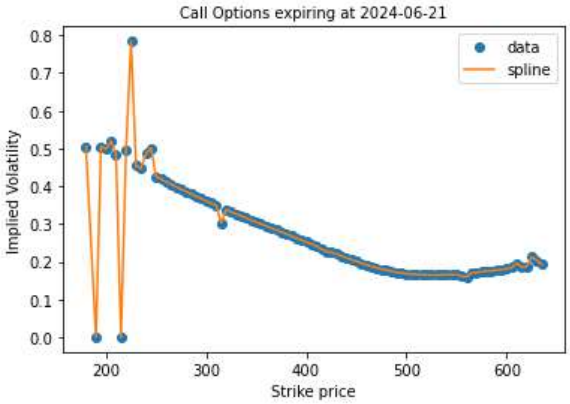
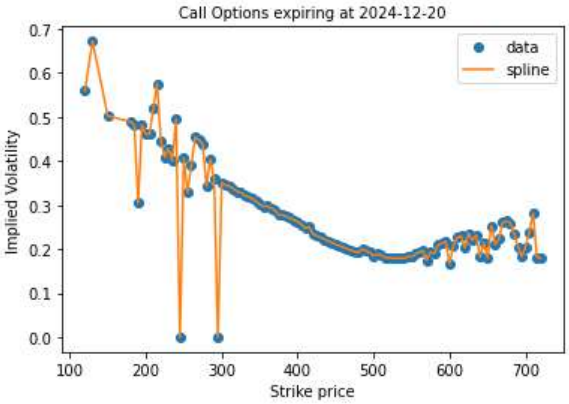
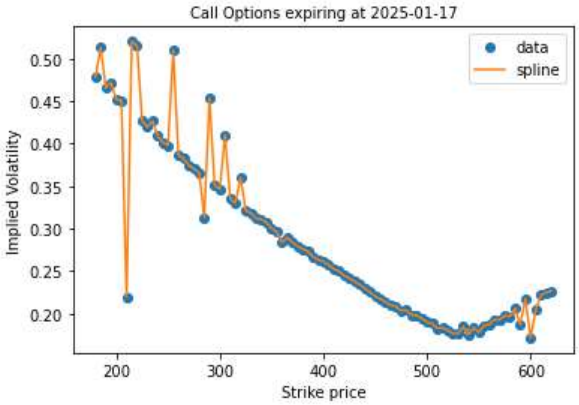
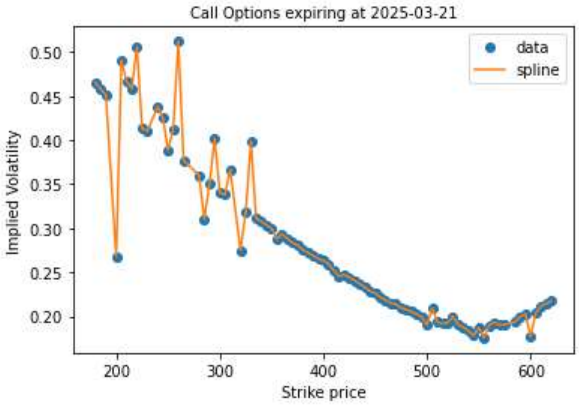
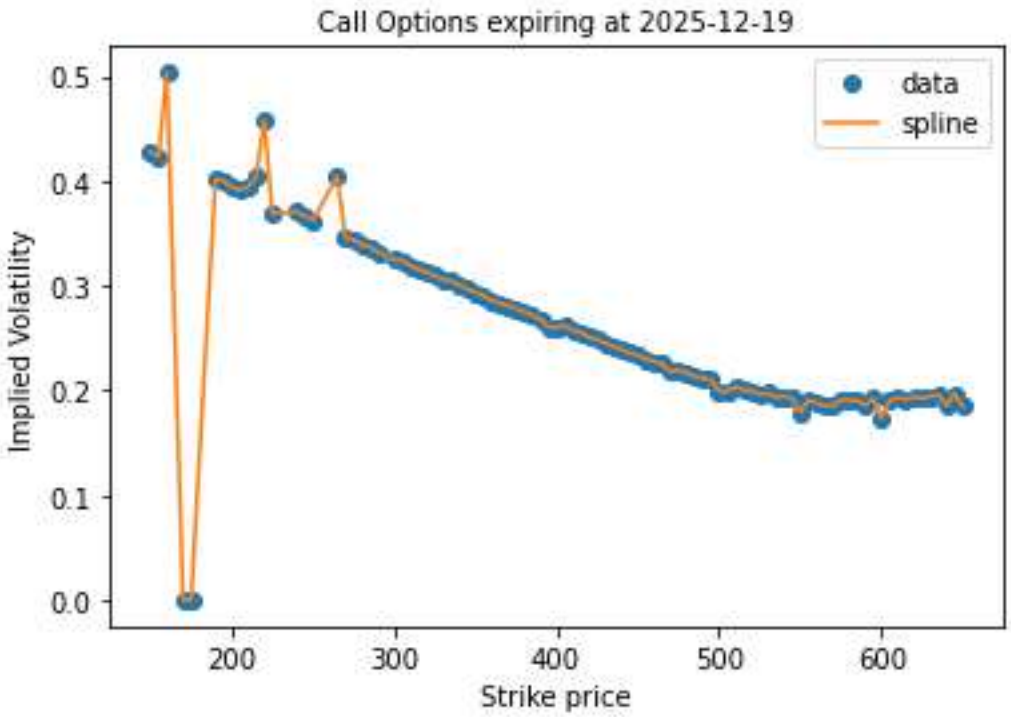
data
spline
Implied Volatility
Strike price


Call Options expiring at 2024-06-21
data
spline
Implied Volatility
Strike price

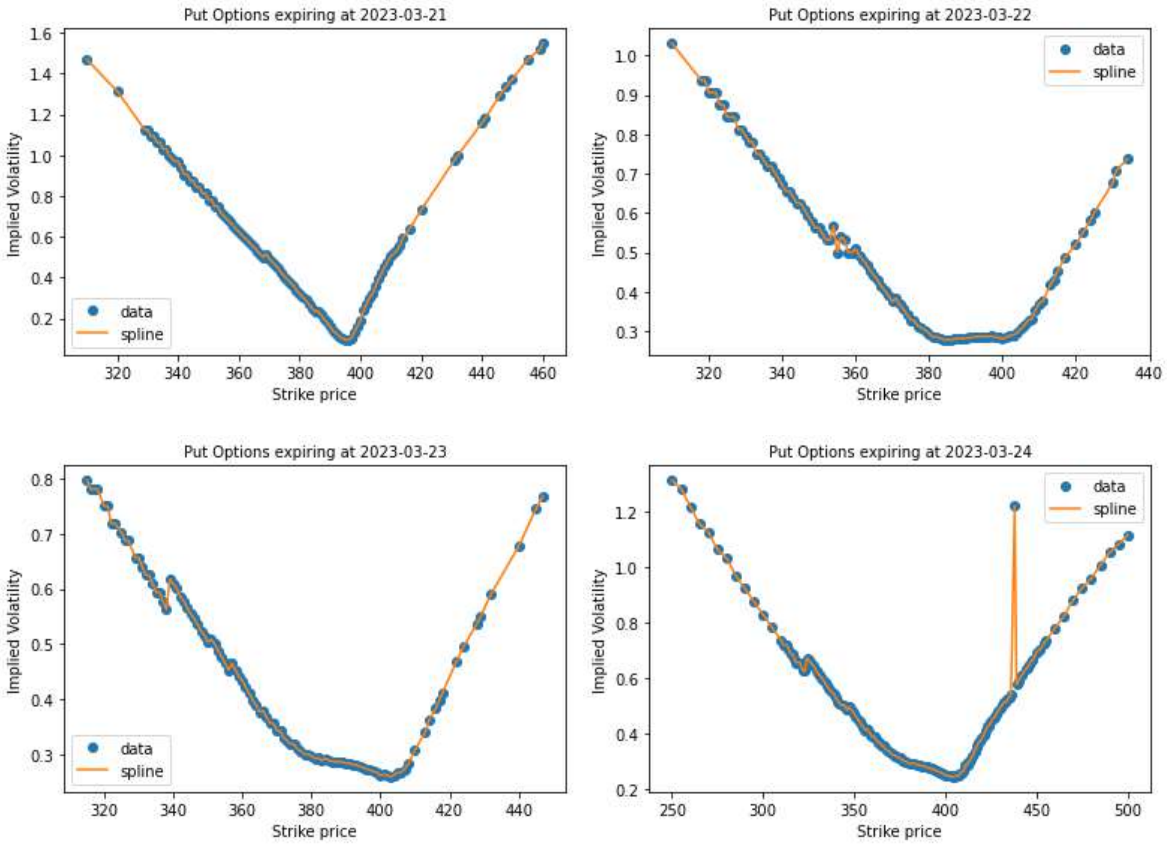

Call Options expiring at 2024-12-20
data
spline
Implied Volatility
Strike price

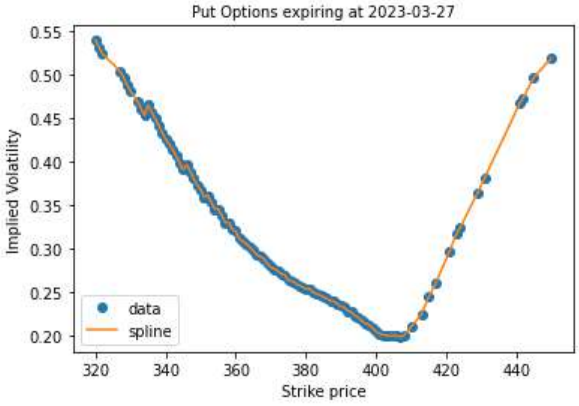

Call Options expiring at 2025-01-17
data
spline
Implied Volatility
Strike price

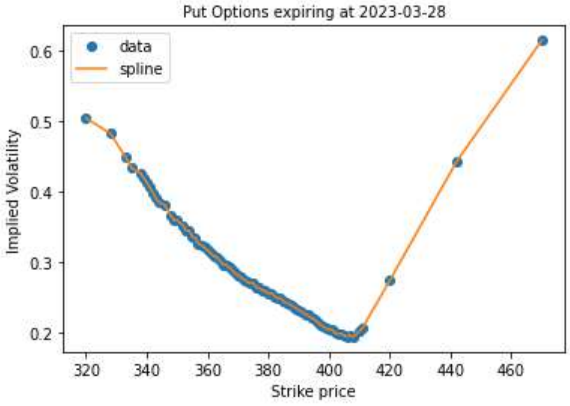

Call Options expiring at 2025-03-21
data
spline
Implied Volatility
Strike price

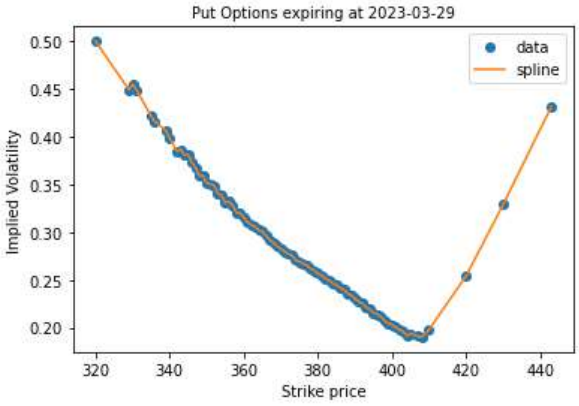


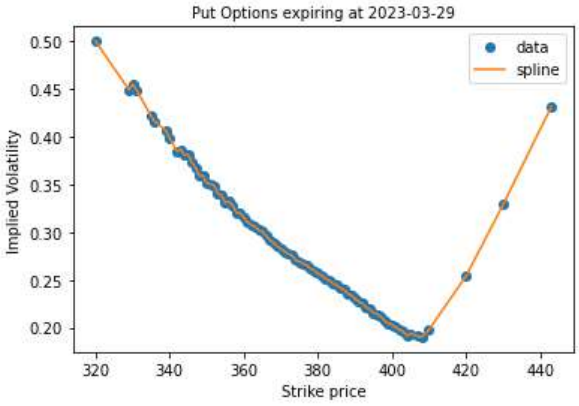
Spline model to the implied volatility curve for each of the maturity dates for put options

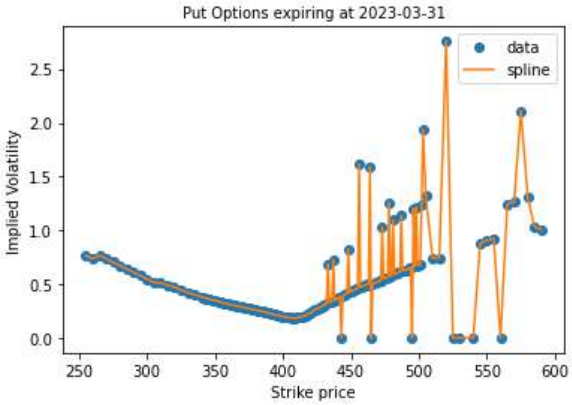



Put Options expiring at 2023-03-27
Implied Volatility
Strike price
data
spline


Put Options expiring at 2023-03-28
Implied Volatility
Strike price
data
spline


Put Options expiring at 2023-03-29
Implied Volatility
Strike price
data
spline


Put Options expiring at 2023-03-29
Implied Volatility
Strike price
data
spline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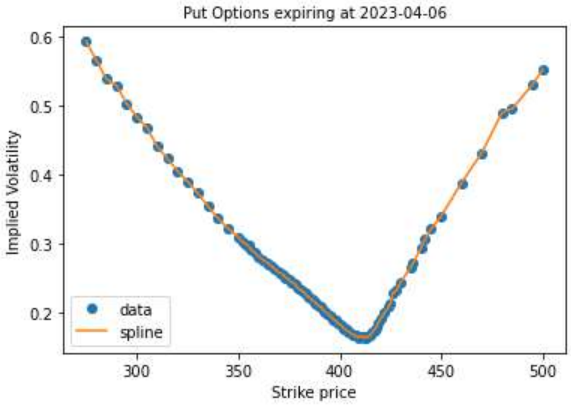
Implied Volatility
Strike price
data
spline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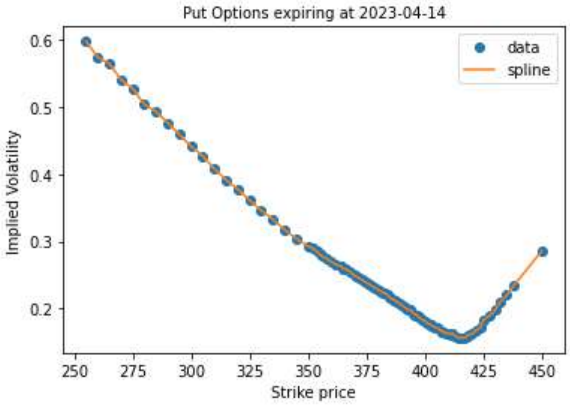
Implied Volatility
Strike price
data
spline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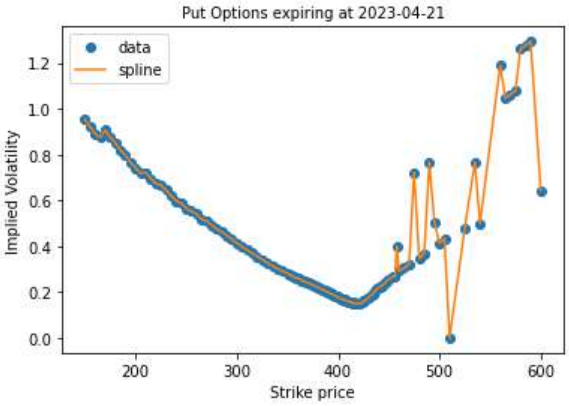
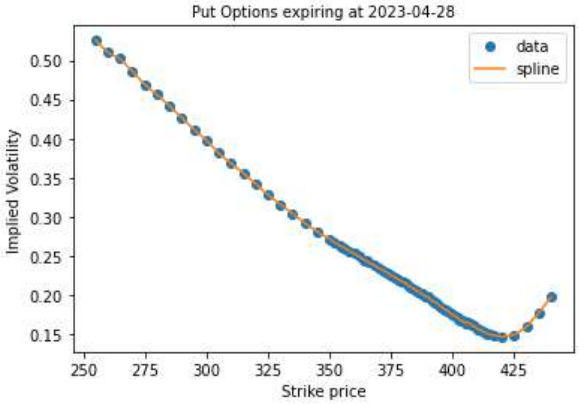
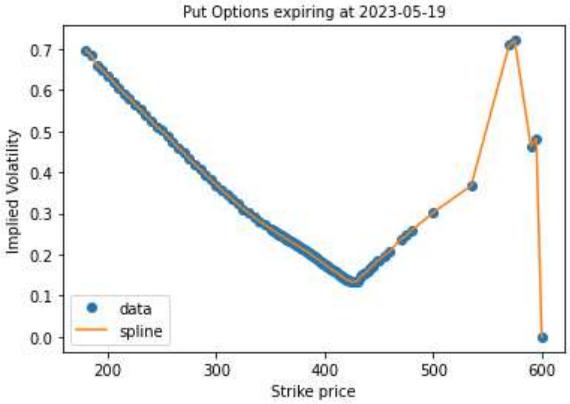
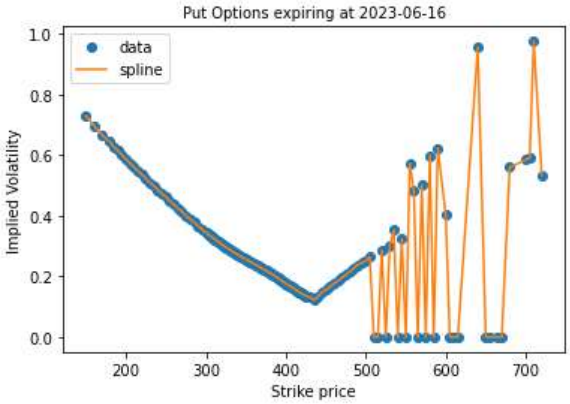
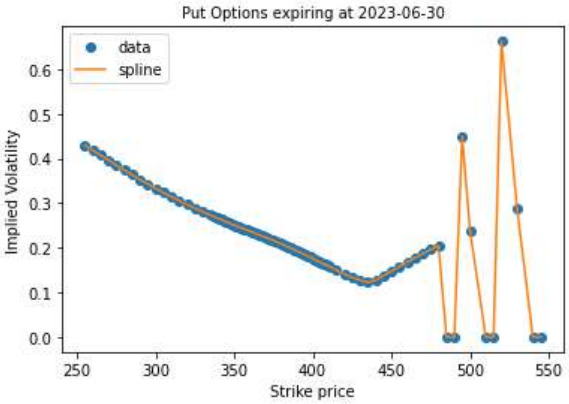
Implied Volatility
Strike price
data
spline


Put Options expiring at 2023-04-21
Implied Volatility
Strike price
data
spline


Put Options expiring at 2023-04-28
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-05-19
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-06-16
data
spline
Implied Volatility
Strike price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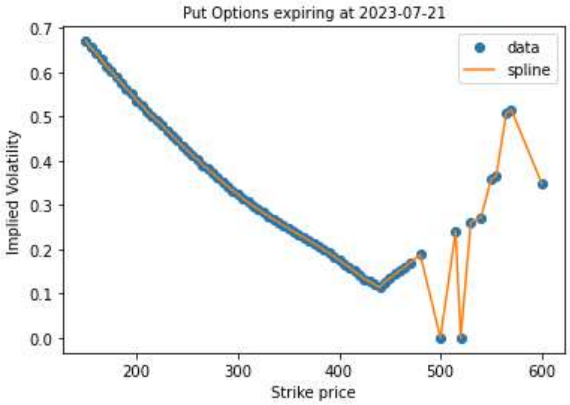
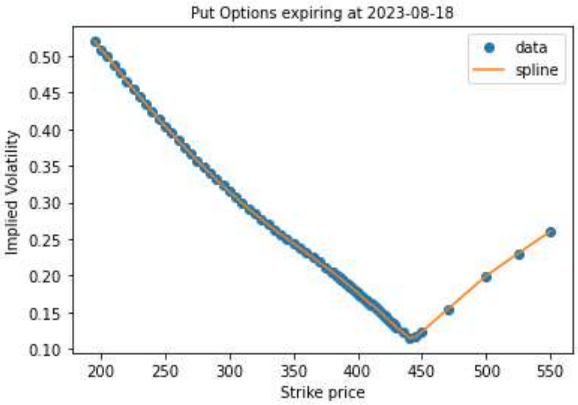
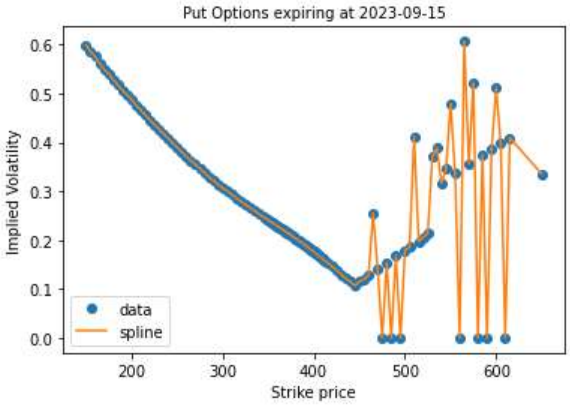
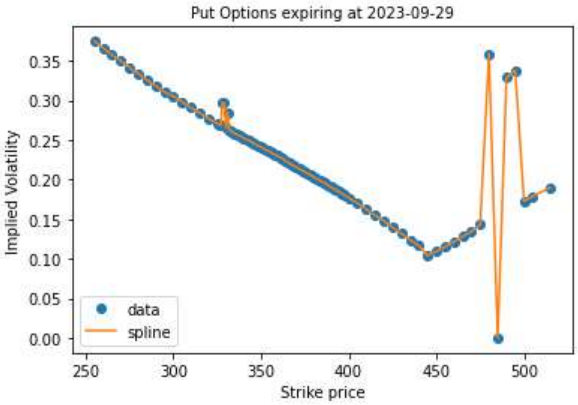
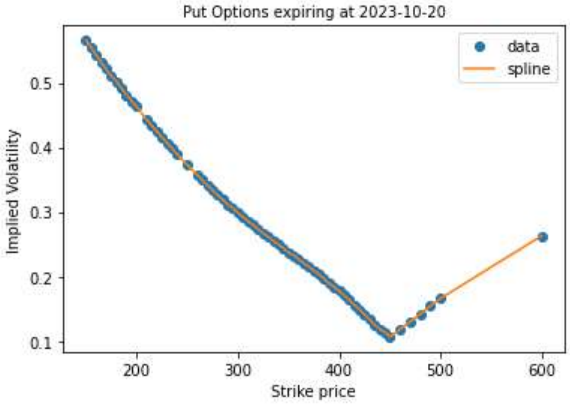
data
spline
Implied Volatility
Strike price

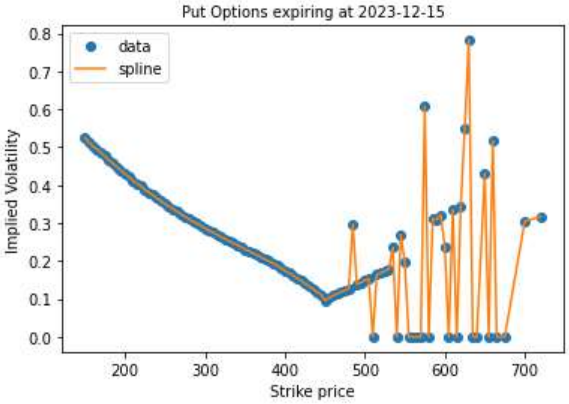

Put Options expiring at 2023-07-21
data
spline
Implied Volatility
Strike price

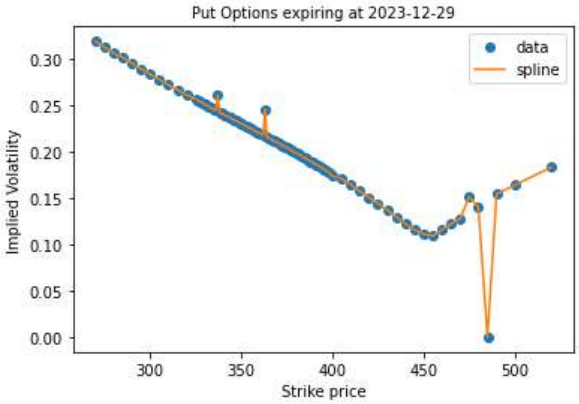

Put Options expiring at 2023-08-18
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-09-15
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-09-29
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-10-20
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-12-15
data
spline
Implied Volatility
Strike price


Put Options expiring at 2023-12-29
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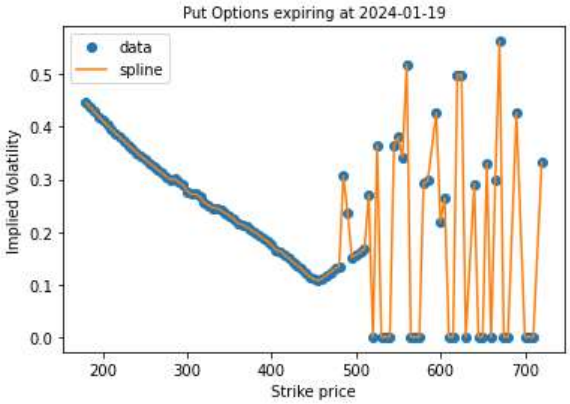
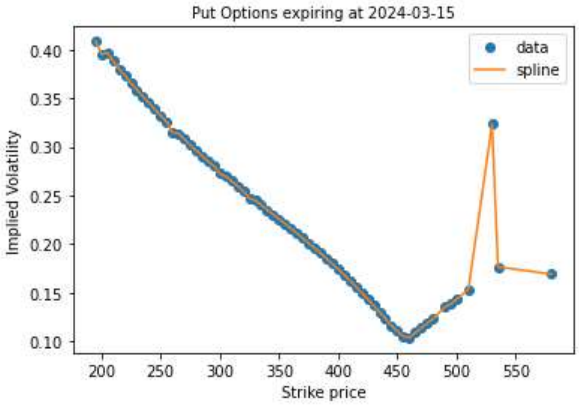
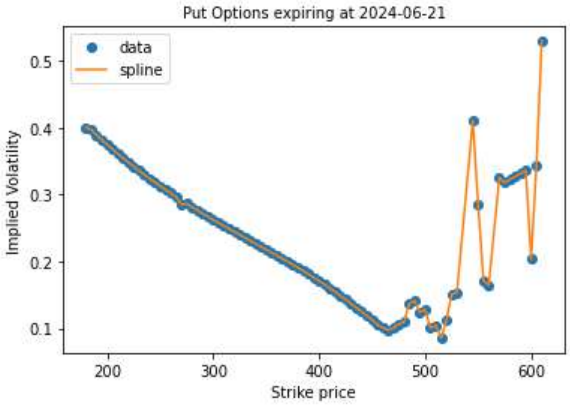
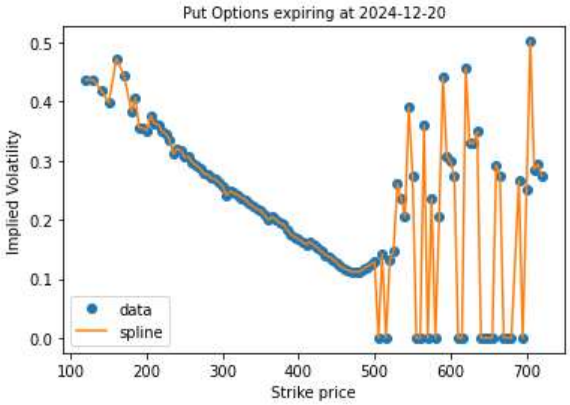
Implied Volatility
Strike price

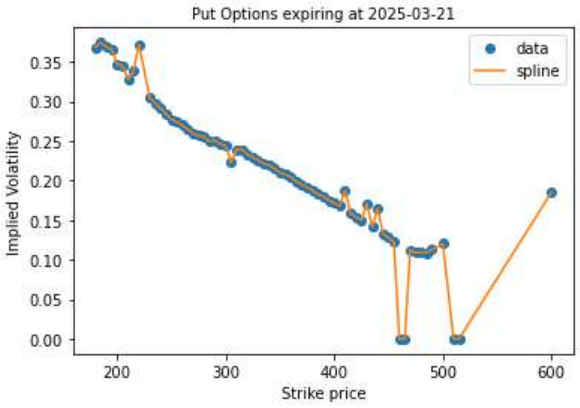

Put Options expiring at 2024-01-19
data
spline
Implied Volatility
Strike price


Put Options expiring at 2024-03-15
data
spline
Implied Volatility
Strike price


Put Options expiring at 2024-06-21
data
spline
Implied Volatility
Strike price


Put Options expiring at 2024-12-20
data
spline
Implied Volatility
Strike price

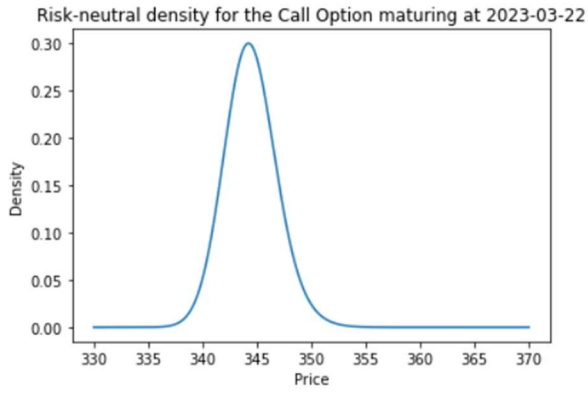

Put Options expiring at 2025-01-17
data
spline
Implied Volatility
Strike price

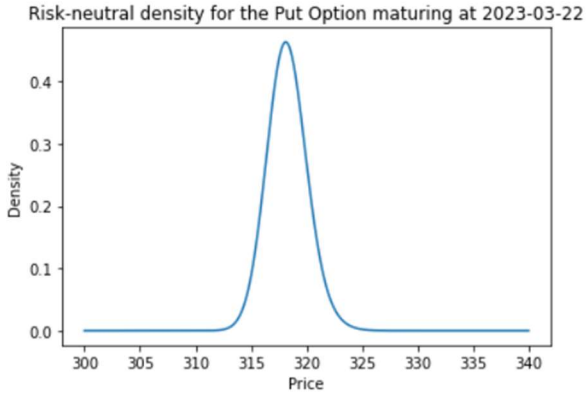


Risk-neutral density

The risk-neutral density for options is a probability distribution that reflects the market's expectations of future prices of the underlying asset, assuming that the market is risk-neutral. The risk-neutral density is a crucial concept in options pricing theory, as it allows traders and investors to calculate the fair value of an option based on the market's expectation of future prices.

The risk-neutral density is obtained by transforming the market's expectation of future prices into a probability distribution using the risk-neutral measure. The risk-neutral measure is a probability measure that is derived from the market prices of options, assuming that the market is risk-neutral. Under the risk-neutral measure, the expected return on the underlying asset is equal to the risk-free rate.





I generate the risk-neutral density distribution for both call and put options with a maturity of 2023-03-22. The findings from charts showing the risk neutral density (RND) distribution for both options are given below -

Shape of the distribution: The shape of the RND for both calls and puts reveals information about the market's expectations of future prices, such as the likelihood of a price increase or decrease and the degree of uncertainty around the expected prices.

Mean and variance: The mean and variance of the RND provide information about the expected level and volatility of future prices. A higher mean suggests a higher expected price level, while a higher variance indicates greater uncertainty or risk around the expected prices. Here, the mean strike price for call options is around 345 and for put options is between 315 and 320.

Trading program that sells over-valued options and buys undervalued options

```
Fitted volatility for call option is: 0.188

Fitted volatility for put option is: 0.304
```

```
Trade signal for call option with expiration at 2023-03-22

    expiration  daysToExpiration  Trade_signal Trade_position
0   2023-03-22                 2             1            BUY
1   2023-03-22                 2             1            BUY
2   2023-03-22                 2             1            BUY
3   2023-03-22                 2             1            BUY
4   2023-03-22                 2             1            BUY
..         ...               ...           ...            ...
82  2023-03-22                 2            -1           SELL
83  2023-03-22                 2            -1           SELL
84  2023-03-22                 2            -1           SELL
85  2023-03-22                 2            -1           SELL
86  2023-03-22                 2            -1           SELL

[87 rows x 4 columns]
```

```
Trade signal for put option with expiration at 2023-03-22

     expiration  daysToExpiration  Trade_signal Trade_position
0    2023-03-22                 2            -1           SELL
1    2023-03-22                 2            -1           SELL
2    2023-03-22                 2            -1           SELL
3    2023-03-22                 2            -1           SELL
4    2023-03-22                 2            -1           SELL
..          ...               ...           ...            ...
101  2023-03-22                 2            -1           SELL
102  2023-03-22                 2            -1           SELL
103  2023-03-22                 2            -1           SELL
104  2023-03-22                 2            -1           SELL
105  2023-03-22                 2            -1           SELL
```