Nicholas Garrett

Professor Zhu

MATH 4441

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Homework 8

1.
See code

The error norm for Gauss-Seidel was 0.9720, and the error norm for Jacobi (after 2 iterations) was 0.0288, so Jacobi seems to be converging faster between the two.

2.

The variable a has to be between 0 (to be positive) and $\frac{1}{2}$.
In this range, the Jacobi iteration does not converge and the error accelerates into larger and larger numbers.

3.a.
$M = \alpha^{-1}$
B is the iteration matrix, and $B = I - \alpha A$
...
3.b.i.
By definition of eigenvalues, $Ax = \lambda x$

And as $A = (I - \alpha A)$

$\Rightarrow (I - \alpha A)x = \lambda x$
$\Rightarrow (Ix - \alpha Ax = (Ix - \alpha\lambda x) = (I - \alpha\lambda)x$

$\rho(B) = \rho(I - \alpha A) = (1 - \alpha\lambda)$, which must be ¡ 1 for this to converge.
$(1 - \alpha\lambda) < 1$
$\Rightarrow -\alpha\lambda < 1$
For this to occur, $\alpha > 0$

3.b.ii.
By theorem 3, as A is symmetric positive definite, (I use f because I cannot find the greek letter used in the notes) $g(\alpha) = f(x_k + \alpha p_k) = \frac{1}{2}(x_k + \alpha p_k)^T A(x_k + \alpha p_k) - (x_k + \alpha p_k)^T b$, which is what we want

to minimize for α.

By substituting alpha into the equations, we get:
$g(\frac{2}{\lambda_1+\lambda_n}) = f(x_k + \alpha p_k) = \frac{1}{2}(x_k + \frac{2}{\lambda_1+\lambda_n}p_k)^T A(x_k + \frac{2}{\lambda_1+\lambda_n}p_k) - (x_k + \frac{2}{\lambda_1+\lambda_n}p_k)^T b$

...