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MATH 4441

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Homework 8

1.
See code

The error norm for Gauss-Seidel after a single iteration was 31.82, and the error norm for Jacobi (after 2 iterations) was 6.624, so Jacobi seems to be converging more slowly after two iterations than Gauss-Seidel after a single iteration.

2.

For the matrix to be positive, $a > 0$. Then, because the convergence of the Jacobi iteration sequence requires that $\rho(A) < 1$. The eigenvalues of A are : $1 - a$ and $2a + 1$. For the absolute value of the maximum eigenvalue to be $< 1$, a cannot be any value, as the maximum eigenvalue is $> 1$ for every value of a. Therefore, a can be any value greater than zero.

3.a.
$M = \alpha^{-1}$
B is the iteration matrix, and $B = I - \alpha A$

3.b.i.
For the iteration matrix to converge, $\rho(B) < 1$. B = I - $M^{-1}A$, and by part A, we find that B = I - $\alpha$A. For $\rho(B)$ to be less than 1, $max|\lambda| = |I - \lambda(I - \alpha A)| < 1$, which can be guaranteed if $\alpha = \lambda$.

3.b.ii.
By theorem 3, as A is symmetric positive definite, ( I use f because I cannot find the greek letter used in the notes) $g(\alpha) = f(x_k + \alpha p_k) = \frac{1}{2}(x_k + \alpha p_k)^T A(x_k + \alpha p_k) - (x_k + \alpha p_k)^T b$, which is what we want to minimize for $\alpha$.

By substituting alpha into the equations, we get:
$g(\frac{2}{\lambda_1+\lambda_n}) = f(x_k + \alpha p_k) = \frac{1}{2}(x_k + \frac{2}{\lambda_1+\lambda_n}p_k)^T A(x_k + \frac{2}{\lambda_1+\lambda_n}p_k) - (x_k + \frac{2}{\lambda_1+\lambda_n}p_k)^T b$
...