

# Introduction to Problem Solving

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Notes



# Introduction

- Jitender Punia (Jectu)
- full time Instructor + Program Management work
- 4.5 years of teaching experience.

# Few terms that you shall see/hear throughout the course!

## 1. PSP (Problem Solving Percentage) -

### Solved Assignment Problems / Total Open Assignment Problems

- There are two types of section - Assignment and Additional. Assignment section consists of implementation of the problems done in class. PSP is calculated based on only Assignment Problems
- Additional Problems are slight modifications of assignment problems, they are not part of PSP but once you're done with assignment, we highly recommend to complete additional problems as well.
- Try to keep PSP least 85% no matter what. It shall really help you stay focused and we have seen in the past that people with  $\geq 85\%$ , do well in interviews.

## 2. Attendance

- Try to maintain at least **75% attendance** either through classes or by watching recordings.
- Though I will recommend you to come to classes regularly because otherwise it may create backlogs.
- So, I expect all of you to attend live classes and if for any reasons you are unable to, then please send me a message stating the reason.



## Next Month

- Introduction to Problem Solving
- Time Complexity
- Introduction to Arrays
- Prefix Sum
- Carry Forward
- Subarrays
- 2D Matrices
- Sorting Basics
- Strings Basics
- Bit Manipulation Basics
- Interview Problems
- Contest [ covers full Intermediate DSA ]



## Contest will be organised after Intermediate Module

- It will be for 1.5 hours and will be conducted within class duration followed by Contest Discussion (Instructor shall be discussing contest problems).
- It will consist of 3 questions and we expect you to solve  $\geq 2$  problems. If for any reason you are unable to solve, then we shall also be having re-attempts as well.  
( We will provide more info on re-attempts moving forward)
- Contests are critical to retaining what you have learnt and measuring where you need improvement. Please take contests seriously.

## FAQs

- Notes will be uploaded after the class.
- Assignment will be unlocked after the class ends.
- There is no deadline for assignments.
- If asking a questions, ask in public chat.
- If answering a questions, answer in private chat.

Question

To : Everyone

Answering

To : Jitender.  
(private)



**Factor**  $\rightarrow$   $i$  is a factor of  $N$  if  $i$  divides  $N$  completely.  
i.e. the remainder is 0.

**< Question > :** Given  $N$ . Find the count of factors of  $N$ . ( $N > 0$ )

1.  $N = 24$

1, 2, 3, 4, 6, 8, 12, 24

count of factors = 8.

2.  $N = 10$

[1, 10]

1, 2, 5, 10

count of factors = 4.

%  $\rightarrow$   $10 \% 2 \rightarrow 0$   
 $10 \% 3 \rightarrow 1$   
 $10 \% 4 \rightarrow 2$   
 $10 \% 5 \rightarrow 0$



idea → iterate on all the numbers from 1 to  $N$  and check if that no. divides  $N$  completely.

$N \geq 1$ .

```

int countFactors (int n) {
    count = 0;
    for ( i = 1; i ≤ N; i++) { // i → 1 to N
        if ( N % i == 0 ) {
            count++;
        }
    }
    return count;
}

```

→ (Next section)

Assumption:  $10^8$  iterations are executed in 1 sec.

$N$	no. of iterations	execution time
$10^8$	$10^8$	1 sec.
$10^9$	$10^9$	10 sec.
$10^{18}$	$10^{18}$	<u><math>10^{10}</math> sec.</u> $\approx$ <u>317 years.</u>

$10^8$  itr → 1 sec

h/e<sup>x</sup> → 1<sup>st</sup> Gen → 2<sup>nd</sup> Gen → 3<sup>rd</sup> Gen → 4<sup>th</sup> / 5<sup>th</sup>.



# Optimisation

$i * j = N$  , Both  $i$  and  $j$  are factors of  $N$ .

$j = \frac{N}{i}$  , Both  $i$  and  $\frac{N}{i}$  are factors of  $N$ .

↓

If  $i$  is a factor, then  $\frac{N}{i}$  is also a factor of  $N$ .

$N = 24$

$i$	$\frac{N}{i}$
1	24
2	12
3	8
4	6
6	4
8	3
12	2
24	1

$N = 100$

$i$	$\frac{N}{i}$
1	100
2	50
4	25
5	20
10	10
20	5
25	4
50	2
100	1

- After a certain value, factors are repeating.

$$i \leq \frac{N}{i} \Rightarrow i * i \leq N$$

$$\Rightarrow \underline{i \leq \sqrt{N}}$$





&lt;/&gt; Code

```
int countFactors2 (int N) {
```

```
    count = 0;
```

```
    for (i = 1; i * i ≤ N; i++) {
```

```
        if (N % i == 0) { // i, N/i are factors of N
```

```
            if (i == N/i) { count++; }
```

```
            else { count += 2; }
```

```
    }
    return count;
```

}

N → 100

<u>i</u>	<u>N/i</u>
1	100
2	50
4	25
5	20
<u>10</u>	<u>10</u>

count++

count → ~~8~~ <sup>8</sup> ~~24~~

Assumption: →  $10^8$  iterations are executed in 1 sec.

<u>N</u>	no. of iterations	execution time
$10^{18}$	$10^9$	<u>10 sec.</u>

∴ Observation is the key ingredient to improve Problem Solving.



**Prime Numbers**  $\Rightarrow$  no's having only 2 factors.  
 $\downarrow$   
 [1 and no. itself]

no  $\rightarrow$  10, 11, 23, 2, 25, 27, 31  $\rightarrow$  ans = 4

**< Question > :** Given a number N. Check if it is prime or not.

**< / > Pseudo Code**

```

boolean checkPrime (int n) {
    int count = countFactors2(n);
    if (count == 2) {
        return true;
    }
    else {
        return false;
    }
}
  
```

1 is neither prime nor composite no.  
 $\downarrow$   $\downarrow$   
 exactly 2 factors  $> 2$  factors.



// Gauss. (4<sup>th</sup> class)

$$S = 1 + 2 + 3 + 4 + \dots + 98 + 99 + 100$$

$$S = 100 + 99 + 98 + 97 + \dots + 3 + 2 + 1$$

+

$$2.S = \underbrace{101 + 101 + 101 + 101 + \dots + 101 + 101 + 101}_{100 \text{ times}}$$

$$2.S = 101 \times 100$$

$$S = \frac{101 \times \cancel{100}^{50}}{2} = \underline{\underline{5050}}$$

Sum of first N natural no's →

$$S = 1 + 2 + 3 + \dots + (N-2) + (N-1) + N$$

$$S = N + (N-1) + (N-2) + \dots + 3 + 2 + 1$$

$$2.S = \underbrace{(N+1) + (N+1) + \dots + (N+1)}_{N \text{ times}}$$

$$\boxed{S = \frac{N(N+1)}{2}}$$



# Range

1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11

$$[1, 11] \rightarrow 11 - 1 + 1 \Rightarrow \underline{\underline{11}}$$

$$[3, 10] \rightarrow 10 - 3 + 1 = \underline{8}$$

$$[a, b] \rightarrow \underline{\underline{b - a + 1}}$$

$$[a, b) \rightarrow b - a$$

$$(a, b] \rightarrow b - a$$

$$(a, b) \rightarrow b - a - 1$$



# What is an iteration?

⇒ No. of times a loop runs.

## Quiz- 1

```
for(i=1; i≤N; i++){  
    if(i==N) {break}  
}
```

$i \rightarrow [1, N]$

⇒ N iterations

## Quiz- 2

```
s=0;  
for(int i=0; i≤100; i++){  
    s=s+i+i2  
}
```

$i : [0, 100]$

iterations →  $100 - 0 + 1 = \underline{101}$



### Quiz- 3

```
for(i=1; i≤N; i++){  
    if(i%2==0){  
        print(i);  
    }  
}  
  
for(j=1; j≤m; j++){  
    if(j%2==0){  
        print(j);  
    }  
}
```

$i : [1, N] \rightarrow N \text{ iterations}$

+

$j : [1, m] \rightarrow m \text{ iterations}$

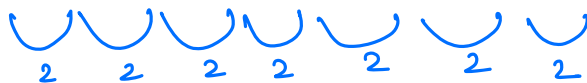
$\Rightarrow \underline{N+m \text{ iterations.}}$



# Geometric Progression

- Series where the ratio of two consecutive terms remains same.

5, 10, 20, 40, 80, 160, 320, 640



first term  $\rightarrow a$  , common ratio  $\rightarrow r$

a,  $a \cdot r$ ,  $a \cdot r^2$ ,  $ar^3$ ,  $ar^4$ ,  $ar^5$ ,  $ar^6$ , ... —  $\frac{ar^{n-1}}{n^{\text{th}} \text{ term}}$

$$\left\{ \begin{array}{l} \text{Sum of first } n \text{ terms of G.P.} \\ \frac{a \cdot (r^n - 1)}{(r - 1)} \end{array} \right\}$$

$$rS = ar + ar^2 + ar^3 + \dots + ar^{N-1} + ar^N \quad \text{--- (1)}$$

$$S = a + ar + ar^2 + \dots + ar^{N-2} + ar^{N-1} \quad \text{--- (2)}$$

---


$$rS - S = ar^N - a$$


---

$$S(r-1) = a(r^N - 1)$$

$$\left\{ S = \frac{a(r^N - 1)}{(r - 1)} \right\}$$

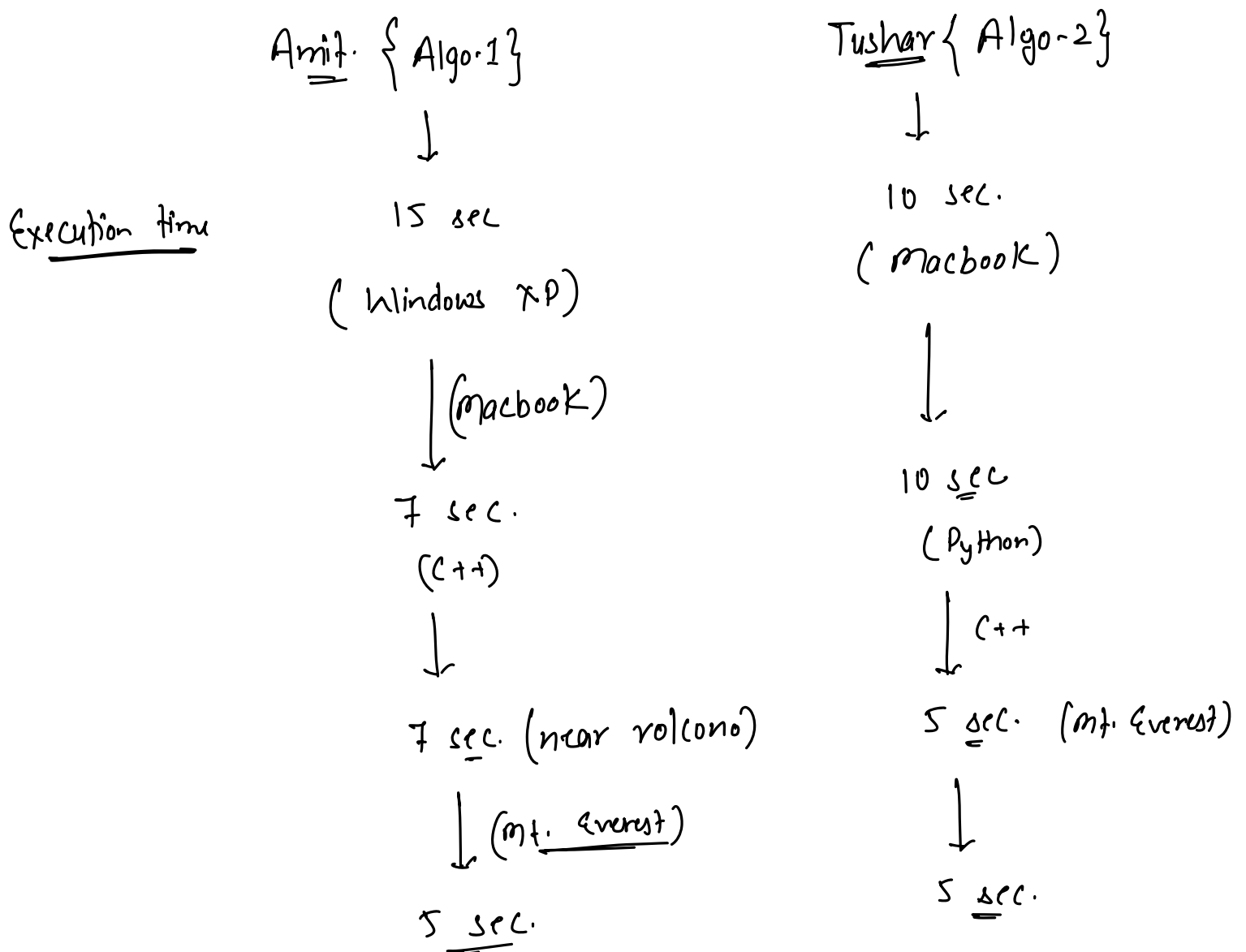
$$\left\{ \begin{array}{l} a \rightarrow 1^{\text{st}} \text{ term} \\ r \rightarrow \text{common ratio} \\ N \rightarrow \text{no. of terms} \end{array} \right\}$$





# How to compare two algorithms?

$N = 10^8$ . Given  $N$  elements, sort the elements in increasing order.

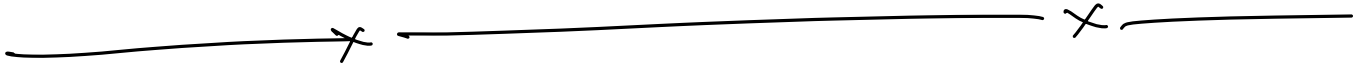


{ Conclusion → We can't use execution time to compare 2 algorithms. }

How should we compare then?

↓

No. of iterations.



3 sessions → Advanced DSA-1

→ Permutation & Combination

→ Prime

→ GCD.