ON DISCRETE SYMMETRIES OF ROBOTIC SYSTESMS:

A DATA-DRIVEN AND GROUP-THEORETIC ANALYSIS

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Symmetries in Dynamical Systems

A symmetry is defined as an energy-preserving transformation of the system state configuration

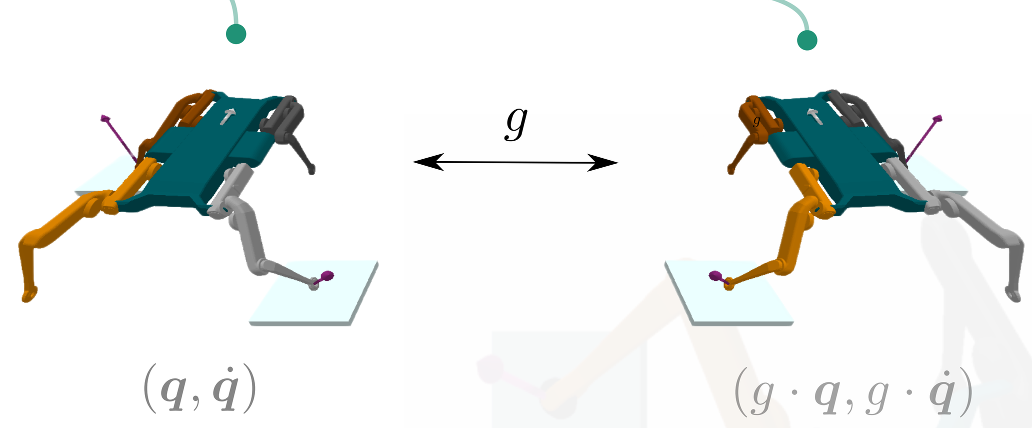
Symmetry transformation/action

Lagrangian function

Symmetry Group

$$\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \mathcal{L}(g\cdot\boldsymbol{q}, g\cdot\dot{\boldsymbol{q}}) \;|\forall g \in \mathcal{G}, \boldsymbol{q}\in \mathrm{Q}, \dot{\boldsymbol{q}} \in \mathrm{T}_{\boldsymbol{q}}\mathrm{Q}.$$

System state · Generalized coordinates · Symmetric state · Generalized velocities



The symmetries of a system extend to symmetries in the system's dynamics or equations of motion. Formally we say that a symmetric dynamical system has equivariant equations of motion:

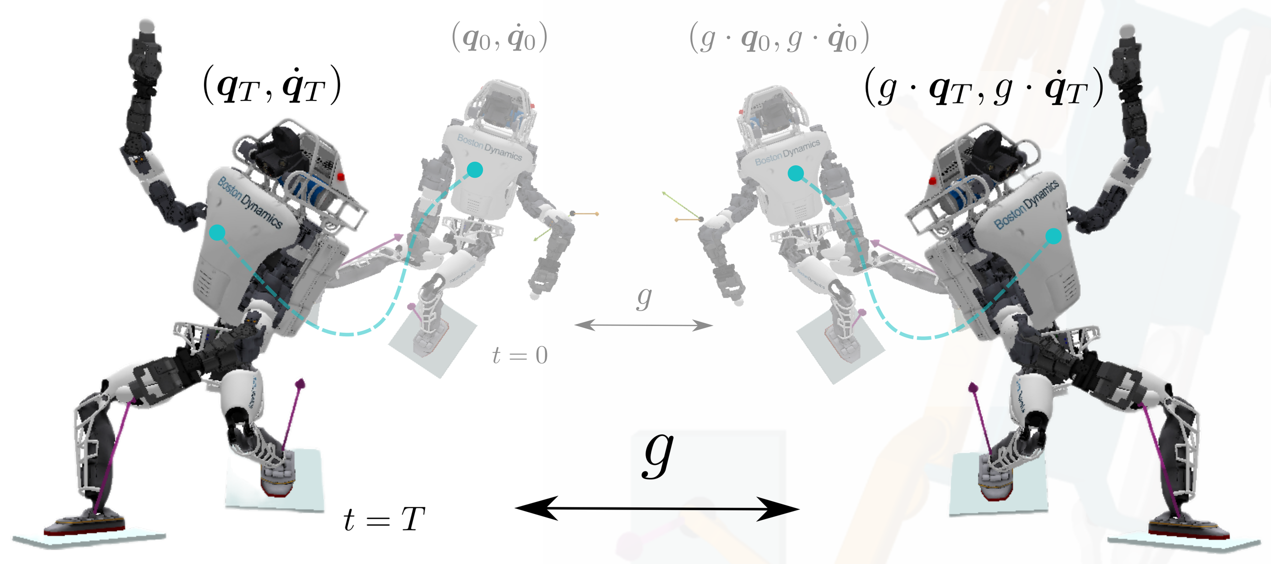
$$g\cdot[\underbrace{\mathbf{M}(\boldsymbol{q})\ddot{\boldsymbol{q}}}_{Inertial} - \underbrace{\boldsymbol{\tau}(\boldsymbol{q},\dot{\boldsymbol{q}})}_{Moving}] = \underbrace{\mathbf{M}(g\cdot\boldsymbol{q})g\cdot\ddot{\boldsymbol{q}}}_{Inertial} - \underbrace{\boldsymbol{\tau}(g\cdot\boldsymbol{q}, g\cdot\dot{\boldsymbol{q}})}_{Moving} = \mathbf{0}$$
$$|\;\forall g\in\mathcal{G},\; \boldsymbol{q}\in\mathrm{Q},\; \dot{\boldsymbol{q}}\in\mathrm{T}_{\boldsymbol{q}}\mathrm{Q}.$$

$\mathbf{M}(\boldsymbol{q}) : \mathrm{Q} \to \mathbb{R}^{n\times n}$ — Generalized Mass Matrix function

$\boldsymbol{\tau}(\boldsymbol{q},\dot{\boldsymbol{q}}) : \mathrm{Q}\times\mathrm{T}_{\boldsymbol{q}}\mathrm{Q} \to \mathbb{R}^n$ — Generalized moving forces

- Friction and damping
- Contact forces
- Joint-torques
- Control torques

This implies that for every trajectory of evolution of the system dynamics, there exists a symmetric equivalent trajectory for each symmetry of the system



The trajectories above remain related by the symmetry transformation if both intertial and moving forces remain equivariant functions at all points in time

$$|\;\forall g\in\mathcal{G},\; \boldsymbol{q}\in\mathrm{Q},\; \dot{\boldsymbol{q}}\in\mathrm{T}_{\boldsymbol{q}}\mathrm{Q}$$
$$\mathbf{M}(g\cdot\boldsymbol{q}) = g\mathbf{M}(\boldsymbol{q})g^{-1} \;\wedge\; g\cdot\boldsymbol{\tau}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \boldsymbol{\tau}(g\cdot\boldsymbol{q}, g\cdot\dot{\boldsymbol{q}})$$

Equivariant generalized mass matrix

- **Implies constraints in kinematic parameters** (e.g., links lengths)
- **Implies constraints in dynamic parameters** (mass and inertia)
- **Identifying** property of symmetric dynamical systems
- In practice inertial forces are only **approximatedly equivariant.**

Equivariant generalized moving forces

- **Equivariant control policy**
- Symmetric joint position and velocity constraints
- Symmetric actuator dynamics
- Equivariance is violated with the introduction of a **symmetry-breaking force** (e.g. contact)

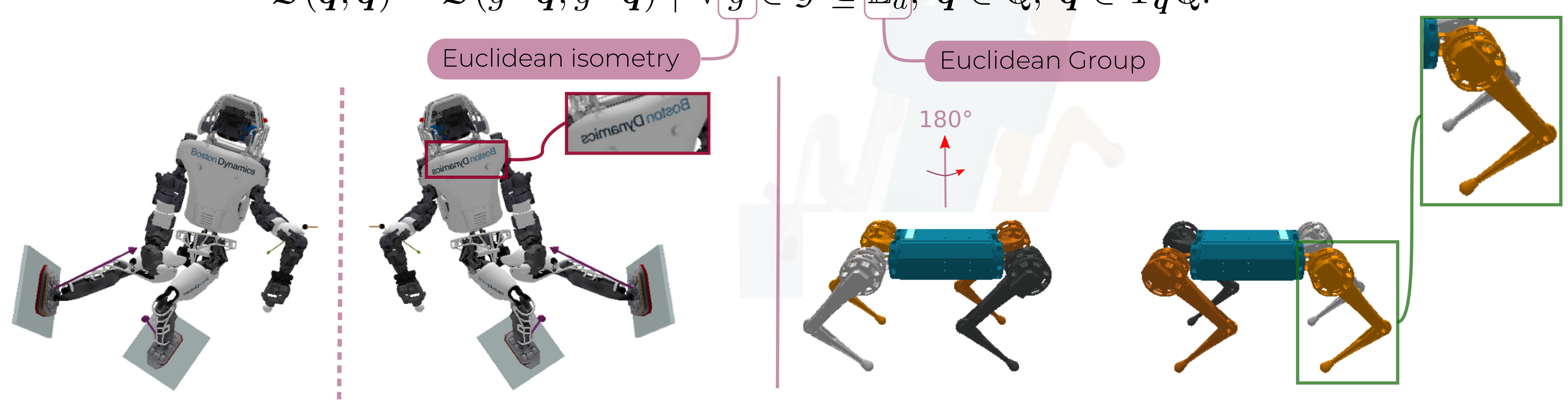
MorphoSymm · arXiv · YouTube

Symmetries due to Euclidean isometries

Most dynamical systems are symmetric to some **set** of rotations, reflections, and translations, i.e., to some set of Euclidean isometries

$$\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \mathcal{L}(\bar{g}\cdot\boldsymbol{q}, \bar{g}\cdot\dot{\boldsymbol{q}}) \;|\; \forall \bar{g} \in \mathcal{G} \subseteq \mathbb{E}_d,\; \boldsymbol{q}\in\mathrm{Q},\; \dot{\boldsymbol{q}}\in\mathrm{T}_{\boldsymbol{q}}\mathrm{Q}.$$

Euclidean isometry · Euclidean Group



True reflections of space · Continuous rotations in space

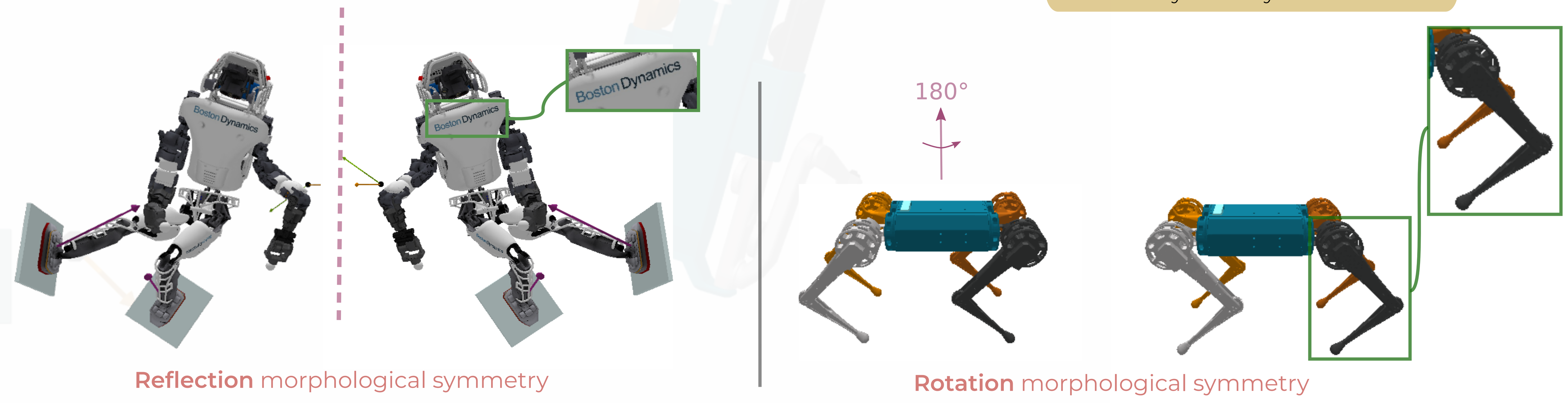
Discrete Morphological Symmetries (DMSs)

A DMS imitates some Euclidean isometries with feasible discrete transformations in the system internal configuration:

DMS · Euclidean Isometry

$$\mathcal{L}(\boldsymbol{q},\dot{\boldsymbol{q}}) = \mathcal{L}(\bar{g}\cdot\boldsymbol{q}, \bar{g}\cdot\dot{\boldsymbol{q}}) = \mathcal{L}(g\cdot\boldsymbol{q}, g\cdot\dot{\boldsymbol{q}}) \;|\, g\in\mathcal{G},\; \bar{g}\in\mathbb{E}_d,\; \forall\; g\cdot\boldsymbol{q}, \boldsymbol{q}\in\mathrm{Q},\; g\cdot\dot{\boldsymbol{q}}, \dot{\boldsymbol{q}}\in\mathrm{T}_{\boldsymbol{q}}\mathrm{Q}.$$

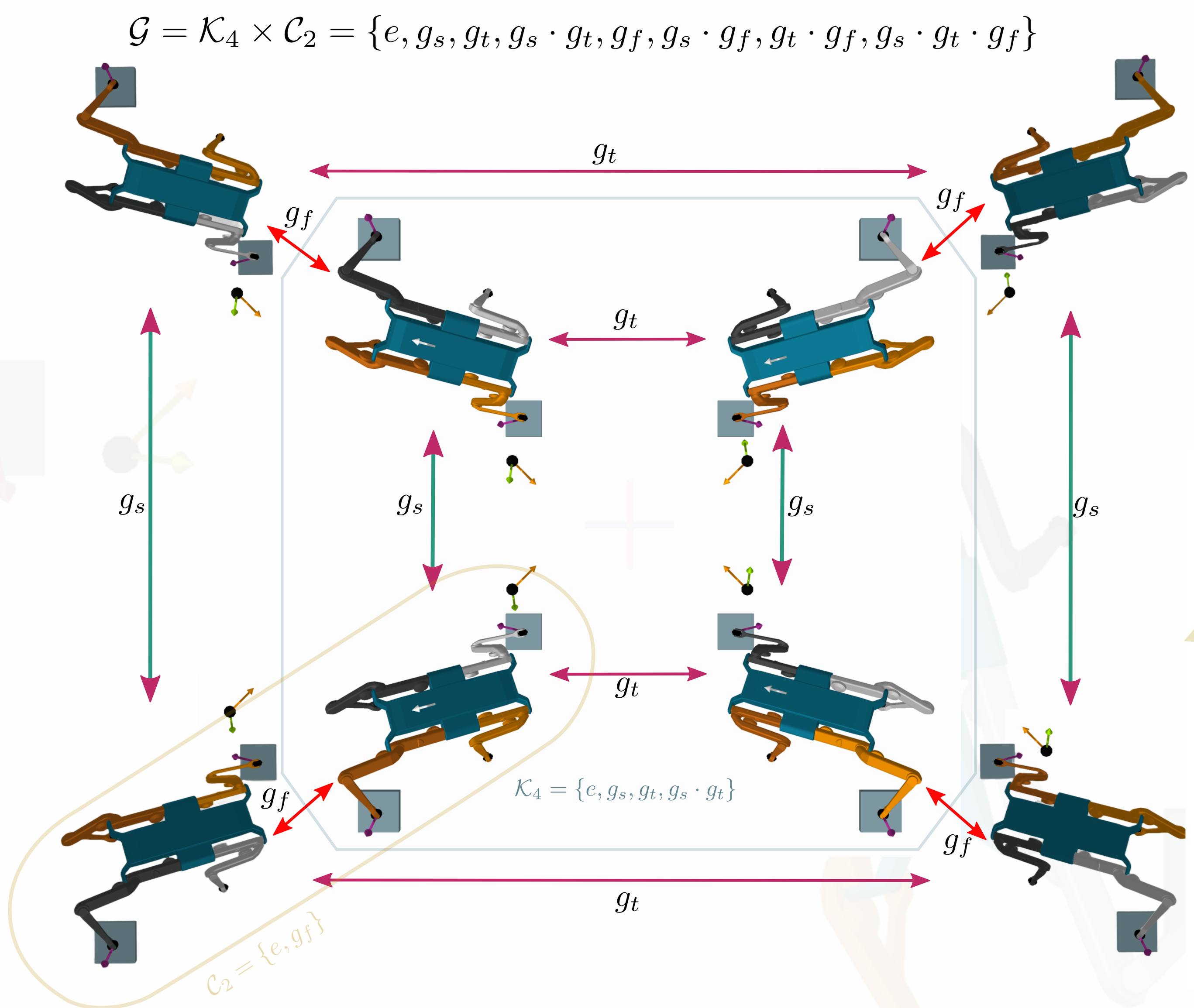
Feasible symmetry transformation



Reflection morphological symmetry · **Rotation** morphological symmetry

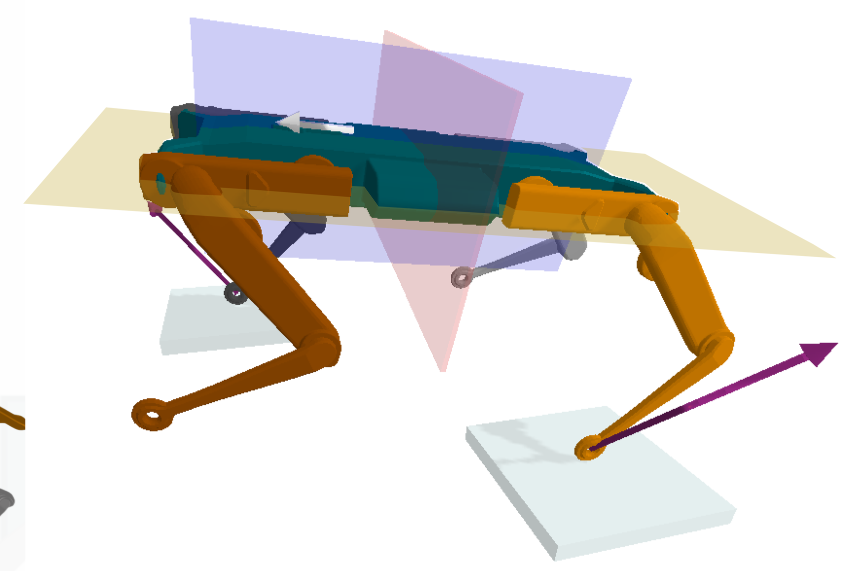
Morphological Symmetry Group of the quadruped Solo

$$\mathcal{G} = \mathcal{K}_4 \times \mathcal{C}_2 = \{e, g_s, g_t, g_s\cdot g_t, g_f, g_s\cdot g_f, g_t\cdot g_f, g_s\cdot g_t\cdot g_f\}$$



The quadruped Solo has a symmetry group of order four, due to:

1. The three symmetry planes of its trunk.
2. The modulatiry of the Kinematic tree (all legs are identical or relfections of one another)
3. Symmetric position and velocity constraints of leg's joints

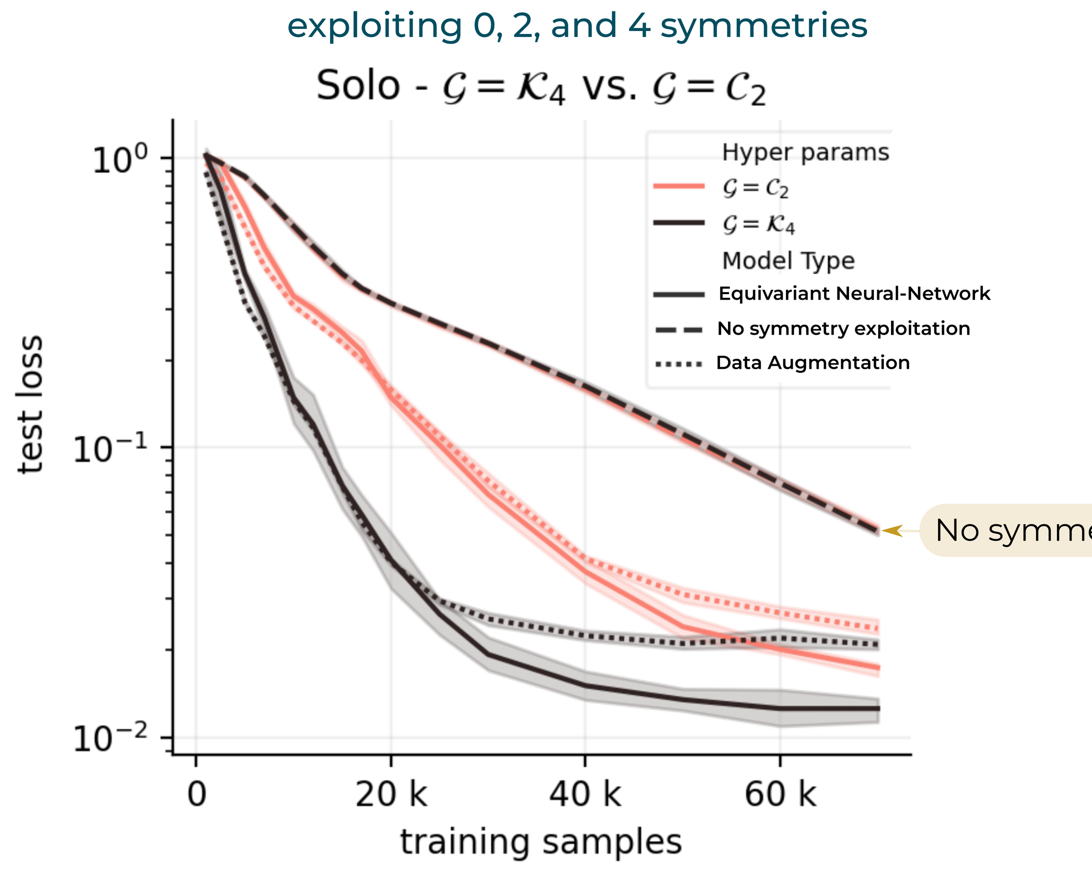


Exploiting Morphological Symmetries in Learning applications

We can exploit morphological symmetries in data-driven applications in two manners:

1. Data augmentation.
2. Equivariant Function Approximators.

Robot solo Center of Mass momentum regression exploiting 0, 2, and 4 symmetries



Static-friction contact classification for the Mini-Cheetah robot using real-world data.