OLYMPIAD PROBLEMS 2006-2007

DIVISION E

WITH ANSWERS AND SOLUTIONS

MATH OLYMPIADS

Since 1979

Mathematical Olympiads for Elementary and Middle Schools

A Nonprofit Public Foundation

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Mathematical Olympiads

NOVEMBER 14, 2006

for Elementary and Middle Schools

1A *Time: 3 minutes*
Choose any number between 32 and 56.
Add 20. Subtract 17. Add 13. Subtract your original number.
What is the resulting number?

1B *Time: 5 minutes*
The sum of three consecutive natural numbers is 15 more than the greatest of them. What is the greatest of the three numbers?

1C *Time: 6 minutes*
Each of three colored cups covers one of three objects.

1) **The red cup is somewhere to the left of the white cup.**
2) **The coin is somewhere to the left of the bean.**
3) **The gray cup is somewhere to the right of the shell.**
4) **The bean is somewhere to the right of the gray cup.**

Under which color cup is the shell?

1D *Time: 5 minutes*
A rectangle has a perimeter of 2 meters and a length of 70 centimeters. Find the area of the rectangle in square centimeters.

1E *Time: 6 minutes*
Asha has 5 more 40¢ stamps than 30¢ stamps. The total value of her 40¢ stamps is \$5.20 more than that of her 30¢ stamps. How many 40¢ stamps does Asha have?

Division **E**

Mathematical Olympiads

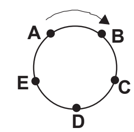
DECEMBER 12, 2006

for Elementary and Middle Schools

Contest **2**

2A ***Time: 3 minutes***

An ant travels around the circle in the direction shown. It touches each of the labeled points in order. The first three points that the ant touches are A, B, and C, in that order. What is the 28th point that the ant touches?



2B ***Time: 4 minutes***

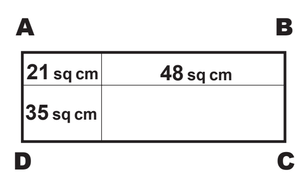
Gina is taller than Henry but shorter than Jennie. Ivan is taller than Katie but shorter than Gina. Who is the tallest of these five people?

2C ***Time: 5 minutes***

Mr. Jackson was born on January 1, 1970. His daughter Lea was born on January 1, 1992. In what year was Mr. Jackson exactly three times as old as Lea?

2D ***Time: 6 minutes***

Rectangle ABCD is split into four smaller rectangles as shown. Each side of each rectangle is a whole number of centimeters. The areas of three of the small rectangles are shown. What is the area of rectangle ABCD, in sq cm?



2E ***Time: 7 minutes***

Amy began with 5 marbles for every 3 marbles that Tara had. After Amy gave 1 marble to Tara, Amy ended with 3 marbles for every 2 that Tara had then. How many marbles did Amy begin with?

3A *Time: 4 minutes*
One natural number is 4 times as great as a second natural number. The product of the two numbers is 36. What is the sum of the two numbers?

3B *Time: 5 minutes*
Mrs. Saada is between 50 and 80 years old. If you divide her age by 9, the remainder is 1. If you divide her age by 4, the remainder is 1. How old is Mrs. Saada?

3C *Time: 5 minutes*
The floor of a rectangular room is completely covered with square tiles. The room is 9 tiles long and 5 tiles wide. Find the number of tiles that touch the walls of the room.

3D *Time: 6 minutes*
One day Kevin counts 50 cars that pass his house. One-fifth of the cars contain more than one person. Of the cars containing only one person, three-fifths are driven by women. Of the cars containing just one person, how many cars are driven by men?

3E *Time: 7 minutes*
What is the smallest prime number that is a divisor of the following after all the arithmetic is completed?

$$(13 \times 17) + (19 \times 23)$$

4A *Time: 4 minutes*
Suppose today is Tuesday. In all, how many Fridays are there in the next 53 days?

4B *Time: 5 minutes*
In the addition at the right, different letters represent different digits. What is the two-digit number **HA**?

$$\begin{array}{r} A \\ A \\ +\ \ H \\ \hline H\,A \end{array}$$

4C *Time: 6 minutes*
The room numbers on one side of a hotel hall are odd. They are numbered from 11 through 59 inclusive. Kristen is in one of these rooms. Express as a fraction the probability that Kristen's room number is divisible by 5.

4D *Time: 6 minutes*
A circle, a rectangle, and a triangle are drawn on the same sheet of paper. No side of the rectangle is also all or part of a side of the triangle. What is the greatest possible number of points of intersection?

4E *Time: 6 minutes*
The pages of a book are numbered consecutively, starting with page 1. It takes 258 digits to number all the pages. What is the last page number?

5A *Time: 4 minutes*
My book is open. I see two pages. The sum of the page numbers is 245. What is the next page number?

5B *Time: 5 minutes*
The average height of four adults is 180 cm. Two of the adults are each 170 cm tall, and the third is 185 cm tall. How tall, in cm, is the fourth adult?

5C *Time: 7 minutes*
The number on Mr. Kay's license plate has three digits. The product of the digits is 216. Their sum is 19. What is the greatest three-digit number that could be on the license plate?

5D *Time: 7 minutes*
Ana starts with 8 and counts by 5s. Her first three numbers are 8, 13, and 18. Josh starts with a whole number other than 8 and counts by a whole number other than 5. Some of his numbers are 11, 32, and 46, but he does not start with 11. 24 is not one of his numbers. What number does Josh start with?

5E *Time: 5 minutes*
The table shown is filled so that each row and each column contains each of the numbers 1, 2, 3, and 4 exactly once. Find the number in the square marked "**X**".

			1
	2		
		X	
1			4

ANSWERS AND SOLUTIONS

Note: Number in parentheses indicates percent of all competitors with a correct answer.

OLYMPIAD 1 — NOVEMBER 14, 2006

Answers: [1A] **16** [1B] **9** [1C] **Red** [1D] **2100** [1E] **37**

71% correct

1A METHOD 1: *Strategy: Rearrange the numbers to simplify the problem.*
Since the only operations are addition and subtraction, the number chosen is subtracted out. It is only necessary to start with 20, subtract 17 and add 13.
$\square + 20 - 17 + 13 - \square = (\square - \square) + (20 - 17 + 13) = 16$. **The result is 16.**

METHOD 2: *Strategy: Choose a number between 32 and 56.*
Choose any number (such as 40) between 32 and 56 and perform the indicated operations. $40 + 20 - 17 + 13 - 40 = 16$. It is wise to repeat, choosing at least two other numbers, in order to make sure 16 is the only result.

40%

1B METHOD 1: *Strategy: Find the sum of the two smaller numbers.*
Since the sum of the three numbers is 15 more than the greatest of them, then the sum of the two smaller numbers is 15. The numbers are consecutive, so the two smaller numbers are 7 and 8. **The greatest** (third) **number is 9.**

METHOD 2: *Strategy: Use algebra.*
Let M denote the middle number. The 3 consecutive numbers are $M - 1$, M, and $M + 1$.
Then $(M - 1) + M + (M + 1) = 15 + (M + 1)$.
Solving, $M = 8$. The greatest of 7, 8, and 9 is 9.

METHOD 3: *Strategy: Guess, check and revise in an organized way.*
Test 1, 2, 3: The difference between $1 + 2 + 3$ and $15 + 3$ is 12, so add 6 to each number. Then $7 + 8 + 9 = 15 + 9$ and the greatest number is 9.

> ***Follow–Ups:*** *(1) If the sum of three consecutive even numbers is 18 more than the smallest number, what are the three numbers?* [6,8,10] *(2) The sum of seven consecutive natural numbers is 56. Find the fourth (middle) number.* [8]

56%

1C *Strategy: First determine the order of the cups.*
From statement (1) the order of the cups, from left to right is RWG, RGW, or GRW. From statement (3) the gray cup is not leftmost and from statement (4) it is not rightmost. These eliminate GRW and RWG, respectively. Then the cup order is RGW. Statement (3) says that the shell is to the left of the gray cup, so **the shell is under the red cup.**
(*A similar method would start by first determining the order of the objects.*)

20%

1D *Strategy: Express all distances in the same unit of measure.*
The perimeter of the rectangle is 2 meters = 200 cm. Thus the sum of the length and width (called the semiperimeter) is 100 cm. Since the length is 70 cm, the width is 30 cm. **The area of the rectangle is** $70 \times 30 =$ **2100 sq cm.**

12%

1E **METHOD 1:** *<u>Strategy</u>: Pair each 30¢-stamp with a 40¢-stamp.*
All but five of the 40¢-stamps can be paired with 30¢-stamps. The value of these "extra" five 40¢-stamps is \$2.00. Then the value of the pairs is the remaining \$3.20 of the given difference. Since, in each pair, the difference in value is 10¢, there are 32 pairs. Then **Asha has** thirty-two 30¢-stamps and **thirty-seven 40¢-stamps.**

Checking, (37 × 40¢) – (32 × 30¢) = \$14.80 – \$9.60 = \$5.20.

METHOD 2: *<u>Strategy</u>: Make an organized list of simpler cases and find a pattern.*

1	**Number of 30¢ stamps**	1	2	3	…	
2	**Number of 40¢ stamps**	6	7	8	…	?
3	**Value of 30¢ stamps**	\$0.30	\$0.60	\$0.90	…	
4	**Value of 40¢ stamps**	\$2.40	\$2.80	\$3.20	…	
5	**Difference in value**	\$2.10	\$2.20	\$2.30	…	\$5.20

For each additional 30¢-stamp, the difference in values increases by 10¢, as indicated by line 5. To change the difference in value from \$2.30 to \$5.20, 290¢ ÷ 10¢ = 29 additional stamps of each type are needed. Asha has (3 + 29) = thirty-two 30¢-stamps and (8 + 29) = thirty-seven 40¢-stamps.

METHOD 3: *<u>Strategy</u>: Use algebra.*
Let x = the number of 30¢-stamps that Asha has.
Then $x + 5$ = the number of 40¢-stamps.
$30x$ = the value (in cents) of the 30¢-stamps and
$40(x + 5)$ = the value (in cents) of the 40¢-stamps.

$$\mathbf{40(x + 5) = 30x + 520.}$$

Solving, $x = 32$, so $x + 5 = 37$. Asha has thirty-seven 40¢-stamps.

Follow-Up: Monica has \$10.00 in quarters, nickels, and dimes. She has 6 more quarters than dimes and 10 more nickels than dimes. How many of each coin does she have? [30 nickels, 20 dimes, 26 quarters]

OLYMPIAD 2 — DECEMBER 12, 2006

Answers: [2A] **C** [2B] **Jennie** [2C] **2003** [2D] **184** [2E] **25**

83% correct

2A **METHOD 1:** *<u>Strategy</u>: Count by complete circuits of the circle.*
The letter E is touched every 5 points beginning with the fifth point. Thus E is the 25th touch and **C is the 28th point the ant touches.**

METHOD 2: *<u>Strategy</u>: Count by individual points.*
The points in order are **ABCDE ABCDE …** . The 28th point that the ant touches is C.

Follow-Ups: *(1) What is the 528th point the ant touches?* [C] *(2) Suppose the ant touches every second point, beginning with A. The first four points are A, C, E, B. What is the 28th point the ant touches? The 2006th?* [E, A]

92%

2B **METHOD 1:** *Strategy: Draw a diagram.*

Letters farther to the right represent taller people.

- Gina is taller than Henry but shorter than Jennie: __ H __ G __ J __
- Ivan is taller than Katie but shorter than Gina: __ K __ I __ G __

Gina appears on both lines. Only Jennie is to the right of Gina, so **Jennie is the tallest.**

METHOD 2: *Strategy: Use reasoning.*

From the first sentence, Jennie is the tallest of the three people named. From the second sentence, Gina is the tallest of the three people named. Since Jennie is taller than Gina, Jennie is the tallest.

42%

2C *Strategy: Start with the difference in their ages.*

1992 –1970 = 22. Mr. Jackson is 22 years older than Lea.

METHOD 1: *Strategy: Express this difference in terms of Lea's age.*

In the year in question, Mr. Jackson's age can be expressed as (Lea's age) + (Lea's age) + (Lea's age). The difference in their ages, 22 years, is then twice Lea's age, so Lea is 11 years old. Mr. Jackson is 11 + 22 = 33 years old. Eleven years after 1992 is 2003, as is 33 years after 1970. **The year was 2003.**

METHOD 2: *Strategy: Make a chart listing their ages each year.*

Mr. Jackson's age in the required year is 3 times Lea's age. The chart lists multiples of 3 for his ages and then subtracts 22 years to get her corresponding ages.

Mr. J's age	24	27	30	**33**
Lea's age	2	5	8	**11**
Is Mr. J. 3 times as old?	No	No	No	**Yes**
Year	**1994**	**1997**	**2000**	**2003**

The only time that his age was three times hers was when he was 33 years and she was 11 years. The year was 2003.

METHOD 3: *Strategy: Use algebra.*

Let L be Lea's age when her father's age is 3 times as great.

Her father's age is both L + 22 and 3L.

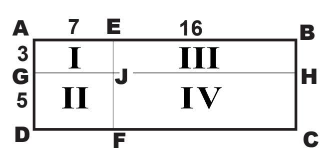
$3L = L + 22$

Solving, $L = 11$. Lea is 11 years old in the year 1992 + 11 or 2003.

FOLLOW-UP: Dave is four times as old as Jeff. In 10 years, he will be twice as old. How old are they now? [Dave is 20; Jeff is 5.]

16%

2D *Strategy: Consider the possible dimensions of the small rectangles.*



Rectangle **I**, with area 21 sq cm, is either 1 cm by 21 cm, or 3 cm by 7 cm. Rectangle **II**, with area 35 sq cm, is either 1 cm by 35 cm, or 5 cm by 7 cm. The common side, $\overline{GJ}$, of both rectangles is then either 1 cm or 7 cm in length. If GJ = 1 cm, then EJ = 21 cm. But 21 is not a factor of 48 and cannot be the length of a side of rectangle **III**. Thus GJ = AE = DF = 7 cm and AG = EJ = BH = 3 cm. Then EB = JH = FC = 48 ÷ 3 = 16 cm, GD = JF = HC = 5 cm. Finally AB = 23 and AD =8.

With a length of 23 and width of 8, **the area rectangle ABCD is 184 sq cm.**

19%

2E **METHOD 1:** <u>Strategy</u>: *Group the marbles two different ways.*
Before the exchange: group all the marbles by 8s, of which 5 are Amy's and 3 are Tara's. After the exchange: group the marbles by 5s, of which 3 are Amy's and 2 are Tara's. Thus, the total number of marbles remains constant and is a multiple of both 8 and 5, that is, of 40: 40, 80, 120, and so on.

Assume there is the smallest total of 40 marbles, Amy having 25 and Tara 15. After Amy gives 1 marble to Tara, Amy has 24 and Tara has 16 marbles. Amy's marbles can be arranged in 8 groups of 3 and Tara's in 8 groups of 2. This satisfies all conditions of the problem. **Amy started with 25 marbles.** There is no need to check 80, 120, and so on.

METHOD 2: <u>Strategy</u>: *Make a table of possible numbers of marbles for each girl.*
Consider all number pairs in a ratio of 5 to 3 ("before"). In each case, look for a ratio of 3 to 2 after trading 1 marble ("after").

Number before exchange			Number after exchange	
Amy	Tara		Amy	Tara
5	3		4	4
10	6		9	7
15	9		14	10
20	12		19	13
25	**15**		**24**	**16**

This occurs when Amy ends with 24 and Tara with 16 marbles. **Amy started with 25 marbles.**

Follow-Up: *Louise has $\frac{7}{8}$ as many grapes as Cindy. After Louise eats 10 grapes, she has $\frac{2}{3}$ as many grapes as Cindy. How many grapes does Cindy have?* [48; Hint: what number does not change?]

OLYMPIAD 3 — JANUARY 9, 2007

Answers: [3A] **15** [3B] **73** [3C] **24** [3D] **16** [3E] **2**

55% correct

3A **METHOD 1:** <u>Strategy</u>: *Divide the larger factor by 4.*
As a result, the two numbers are equal with a product of 9. Then the smaller number is 3, the larger number is 12, and **the sum of the two numbers is 15.**

METHOD 2: <u>Strategy</u>: *List the factor pairs of 36.*
The factor pairs of 36 are: 1 & 36, 2 & 18, 3 & 12, 4 & 9, 6 & 6. The only pair in which one factor is four times the other is 3 and 12. Their sum is 15.

METHOD 3: <u>Strategy</u>: *Apply the Associative Property of Multiplication.*
Use 4 as one of the factors of 36.

$$\begin{aligned} 36 &= 4 \times 9 \\ &= 4 \times (3 \times 3) \\ &= (4 \times 3) \times 3 \\ &= 12 \times 3. \end{aligned}$$

The sum of the numbers is 15.

63%

3B **METHOD 1:** *<u>Strategy</u>: Consider Mrs. Saada's age last year.*
<u>Last</u> year her age was a multiple of both 9 and 4 and thus a multiple of 36. The only multiple of 36 between 50 and 80 is 72. Then *<u>this</u>* year **Mrs. Saada is 73 years old.**

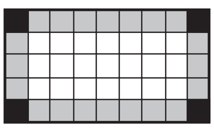
METHOD 2: *<u>Strategy</u>: List the numbers that satisfy one of the conditions.*
It is faster to divide by 9 than 4 because 9 generates fewer results. The only numbers between 50 and 80 that are 1 more than a multiple of 9 are 55, 64, and 73. Of these, only 73 is one more than a multiple of 1. Mrs. Saada is 73 years old.

FOLLOW-UP: *Kai and his mother are younger than 10 and 50 years old, respectively. When the sum of their ages is divided by 5, the remainder is 2. When each of their ages is divided by 7, there is no remainder. How old is Kai's mother?* [35]

46%

3C **METHOD 1:** *<u>Strategy</u>: Measure the perimeter of the room in tiles.*
The perimeter of the room is $2 \times 9 + 2 \times 5 = 28$ tiles. This counts each of the four corner tiles twice. To compensate, subtract 4. **24 tiles touch the walls.**

METHOD 2: *<u>Strategy</u>: From a diagram, count the tiles touching the edges.*



24 tiles touch the walls.

24%

3D *<u>Strategy</u>: Determine how many people are in each group.*
$\frac{5}{5}$ represents a whole amount. Of the 50 cars, the remaining $\frac{4}{5}$ contain just one person. Of these 40 cars, the remaining $\frac{2}{5}$ are driven by men. Thus, **16 cars are driven by men.**

FOLLOW-UP: *One day, Sharon goes to the mall and spends $\frac{1}{2}$ her money on a pair of jeans. She spends $\frac{1}{3}$ of what she has left on a CD and then $\frac{1}{4}$ of what's left on lunch. If she returns home with \$15.00, how much did she start with?* [\$60]

39%

3E **METHOD 1:** *<u>Strategy</u>: Determine whether the result is even or odd.*
Find a way to avoid all that computation. The product of two odd numbers is odd, and the sum of two odd numbers is even. Therefore, $(13 \times 17) + (19 \times 23)$ is even. The least prime number, 2, is a factor of every even number. **The smallest prime number that divides the given expression is 2.**

METHOD 2: *<u>Strategy</u>: Do the arithmetic and then factor.*
$(13 \times 17) + (19 \times 23) = 221 + 437 = 658$. $658 = 2 \times 7 \times 47$. Observe that once the factor of 2 is found, it is not necessary to factor further to know that the least prime number that divides the sum is 2.

FOLLOW–UPS: *(1) What is the greatest prime number that divides $(17 \times 13) + (17 \times 23)$?* [17] *(2) What is the greatest prime number that divides $(17 \times 25) + (17 \times 20) + (17 \times 12)$?* [19; the expression = 17×57] *(3) What prime numbers divide $(14 \times 31) - (7 \times 17)$?* [7, 5, 3; because $14 \times 31 = 7 \times 62$]

OLYMPIAD 4 FEBRUARY 6, 2007

Answers: [4A] **8** [4B] **19** [4C] $\frac{5}{25}$ or $\frac{1}{5}$ [4D] **20** [4E] **122**

67% correct

4A METHOD 1: *<u>Strategy</u>: Determine the number of whole weeks.*
Every 7 days from "today" will be a Tuesday. In 49 days from today, 7 weeks will have passed, which includes 7 Fridays. Since the 49th day from today is a Tuesday, the 52nd day from today is another Friday. **There are 8 Fridays in the next 53 days.**

METHOD 2: *<u>Strategy</u>: Count Fridays.*
Since "today" is Tuesday, the 3rd day will be a Friday, as will the 10th, 17th, 24th, … , 52nd days from today. There are 8 Fridays in the next 53 days.

51%

4B *<u>Strategy</u>: Use reasoning.*
The largest possible sum of three single-digit numbers is 27, so H is either 1 or 2. Suppose H is 2. Then A + A + 2 is 20 + A. This is impossible for any digit A. So H is 1. Since A + A + 1 = 10 + A, A is 9. **The two-digit number HA is 19.**

Follow-Ups: *(1) EM and ME are two-digit numbers. Their product is 252 EM is 9 more than ME. Find the values of E and M.* [E= 2, M = 1] *(2) See pages 115-118 of Creative Problem Solving in School Mathematics, 2nd Edition, for more cryptarithms.*

26%

4C METHOD 1: *<u>Strategy</u>: Count the odd numbers from 11 to 59 inclusive.*
There are 49 whole numbers from 11 through 59. Since the list starts and ends with an odd number, it contains one more odd number than even number. Thus there are 25 odd and 24 even numbers. There are five odd multiples of 5 between 11 and 59: 15, 25, 35, 45, and 55. **The probability of Kristen being in an odd-numbered room is $\frac{5}{25}$ or $\frac{1}{5}$.**

METHOD 2: *<u>Strategy</u>: Split the room numbers up into decades.*
Consider the five decades 11-19, 21-29, 31-39, 41-49, and 51-59. Each decade has 5 odd room numbers, for a total of 25 odd-numbered rooms. In each decade only the room number ending in 5 is divisible by 5. The probability of Kristen being in an odd-numbered room is $\frac{5}{25}$ or $\frac{1}{5}$.

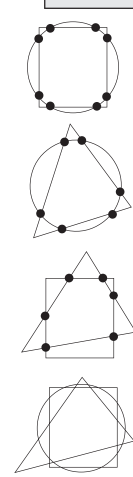
5%

4D *<u>Strategy</u>: Consider each possible pair of figures separately.*
A circle may cross each side of a square in at most 2 points, for a total of 8 points.

A circle may cross each side of a triangle in at most 2 points, for a total of 6 points.

Each side of a triangle may cross in at most 2 sides of a square, for a total of 6 points.

It is possible to draw the figures so that all points are different, so **the greatest number of points where two or more of them may cross is** 8 + 6 + 6 = **20.**



11%

4E *Strategy: Count the digits used for one-digit numbers first, then two-digit numbers, etc.*

Pages	# of digits used	# of digits remaining
1-9	9×1 = 9	258 – 9 = 249
10-99	90×2 = 180	249 – 180 = 69

All the rest are three-digit numbers. 69 ÷ 3 = 23. So the 23rd number following 99 is the last page of the book. **The last page number is 122.**

FOLLOW-UP: (1) How many digits are used to number 1200 pages? [3693] *(2) How many 2's are used to number the pages of a book that starts on page 1 and ends on page 227?* [79]

OLYMPIAD 5 — MARCH 6, 2007

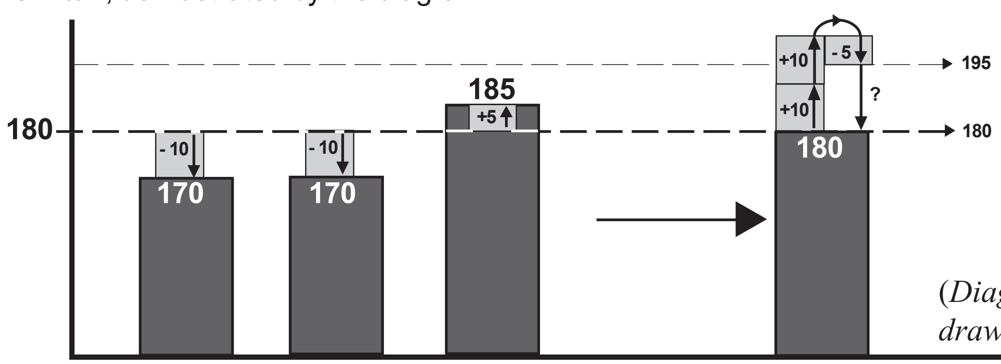
Answers: [5A] **124** [5B] **195** [5C] **964** [5D] **4** [5E] **4**

77% correct

5A METHOD 1: *Strategy: Find the average.*
The page numbers in a book are consecutive. Half of 245 is 122.5, so the page numbers that you see are 122 and 123. **The next page number is 124.**

44%

5B METHOD 1: *Strategy: Compare each height with the average.*
The heights of the first two adults are each 10 cm less than the average. The height of the third adult is 5 cm *more* than the average. The sum of these three heights is 15 cm less than the average. T**he fourth adult is** then 15 cm more than the average, or **195 cm tall**, as illustrated by the diagram.



(*Diagram is not drawn to scale.*)

METHOD 2: *Strategy: Find the total of the four heights.*
The four heights have an average of 180, so their total is 180 × 4 = 720 cm. The sum of the three known heights is 525 cm, so the fourth adult's height is 720 – 525 = 195 cm.

FOLLOW-UPS: (1) Four of Samantha's jars contain an average of 35 pennies and the other three contain an average of 42 pennies. What is the average number of pennies in all seven jars? [38] *(2) In a recipe for a certain three-bean salad, you mix 4 ounces of a 20¢ bean, 5 ounces of a 38¢ bean, and 3 ounces of a 50¢ bean. What is the cost per ounce of the salad?* [35¢]

37%

5C **METHOD 1:** *Strategy: Examine the factors of 216.*
216 = (2 × 2 × 2) × (3 × 3 × 3). The only single-digit products are 1; 2, 2 × 2, and 2 × 2 × 2; 3 and 3 × 3; and 2 × 3. First check if 9 can be the hundreds digit. For the largest plate number, write the digits in descending order.

216 = 9 × 24 = 9 × 8 × 3 or 9 × 6 × 4. The digit-sum of 983 is 20 and the digit-sum of 964 is 19. Thus **the greatest license plate number is 964.**

METHOD 2: *Strategy: List all numbers whose digit-sum is 19.*
List the digits in each potential plate number in descending order to make the resulting number as great as possible. Then multiply the digits for each.

Plate number	991	982	973	964	955	883	874	865	775	766
Digit-product	81	144	189	216	225	192	224	240	245	252

The greatest license plate number is 964.

Follow-Up: (1) How many three-digit plate numbers have a digit-product of 216 and a digit-sum of 19? [6] *(2) How many three-digit numbers have a digit-product of 336? 96?* [6; 18] *(3) How many numbers have a digit-product of 12 if 1 is not a digit and it may have any number of digits?* [6]

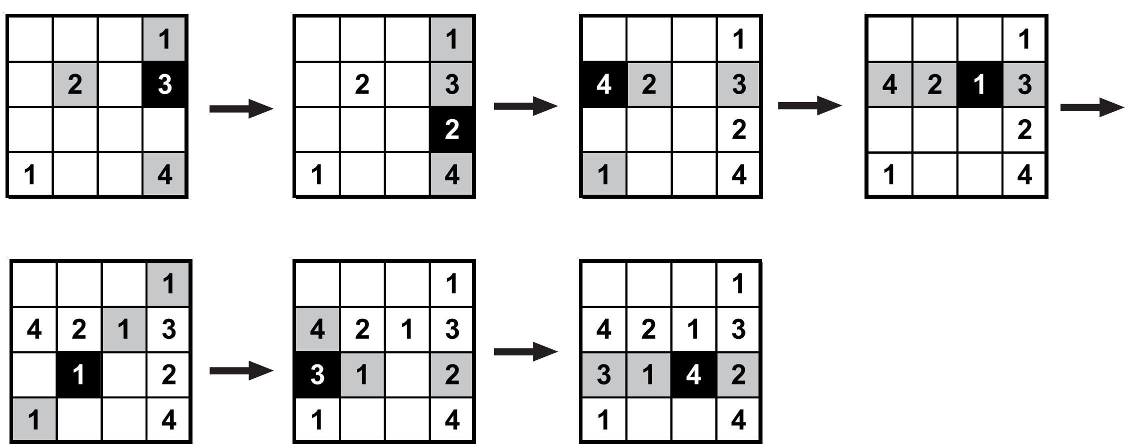
38%

5D *Strategy: Use the differences between the terms.*
32 – 11 = 21 and 46 – 32 = 14. Thus, the number by which Josh is counting — call that his "jump" — is a factor of both 14 and 21. The only two common factors are 1 and 7. The jump cannot be 1, or else 24 would be one of his numbers. So Josh's jump is 7. Since his starting number is a whole number other than 11, the second number is 11, and therefore **the whole number Josh starts with is 4.**

Follow-Up: The first number that is in both Ana's and Josh's lists is 18. What is the second number that is in both lists? [53] *The fifth number?* [158]

69%

5E *Strategy: Find a box for which only one value is possible.*
There are many ways to do this problem. Here is one. Each entry in a black box is derived from those in the shaded boxes.



The number in the square originally marked "X" is 4.

